$p_\perp$ fluctuations, correlations, factorization breaking

Piotr Bożek

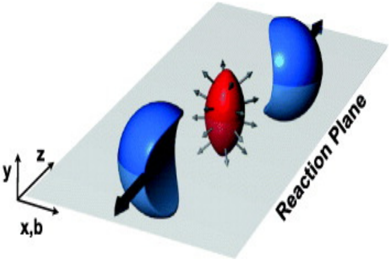
AGH University of Science and Technology, Kraków

AGH

NATIONAL SCIENCE CENTRE POLAND

asymmetry in the transverse plane at finite impact parameter

eccentricity - $\epsilon_2 = -\frac{\int dxdy(x^2-y^2)\rho(x,y)}{\int dxdy(x^2+y^2)\rho(x,y)}$



Snellings 2011

larger gradient and stronger flow in-plane - $v_2 > 0$ - **elliptic flow**

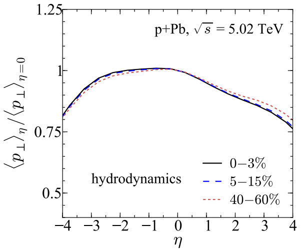
$$\frac{dN}{d\phi} \propto 1 + 2v_2 cos(2\phi)$$

$$\epsilon_2 + \mathrm{HYDRO\ RESPONSE} \longrightarrow \mathrm{v}_2$$

Event Plane (Reaction plane) must be reconstructed in each event

Mean $p_\perp$ reflects the transverse collective flow

- stronger expansion $\longrightarrow$ larger flow $\longrightarrow$ larger $[p_\perp]$
- overall flow constrains evolution time, EOS, bulk viscosity



PB, Bzdak, Skokov, 1309.7358

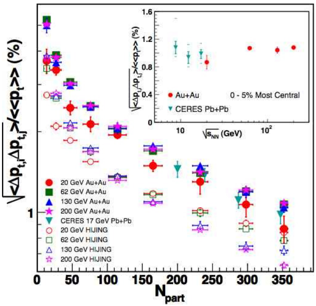
- flow fluctuations $\leftrightarrow$ $[p_\perp]$ fluctuations
- **transverse flow fluctuations $\longrightarrow$ additional information**

Mean transverse momentum in an event

$[p_\perp] = \frac{1}{N}\sum_{i=1}^{N} p_\perp^i$

Fluctuates from event to event

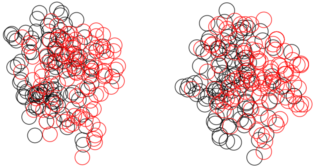
How to measure



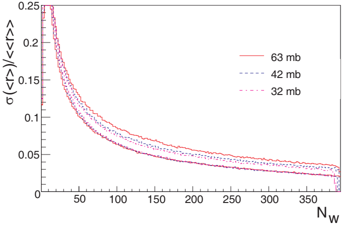
- statistical (thermal) fluctuations of p_T must be subtracted

$$\frac{\langle \Delta p_i \Delta p_j \rangle}{\langle [p_\perp] \rangle^2} = \frac{C_{p_\perp}}{\langle [p_\perp] \rangle^2} = \frac{\frac{1}{N(N-1)} \sum_{i \neq j} \langle (p_i - \langle [p] \rangle)(p_j - \langle [p] \rangle) \rangle}{\langle [p_\perp] \rangle^2}$$

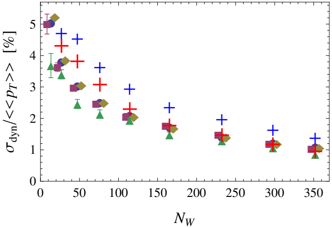
Size fluctuations $\leftrightarrow$ $p_\perp$ fluctuations



size fluctuations



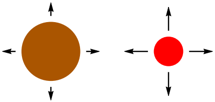
transverse flow fluctuations

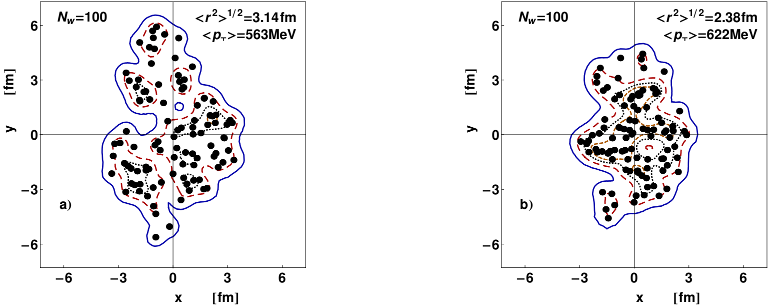


proposed by Broniowski et al. Phys.Rev. C80 (2009) 051902 :

two-shots calculation

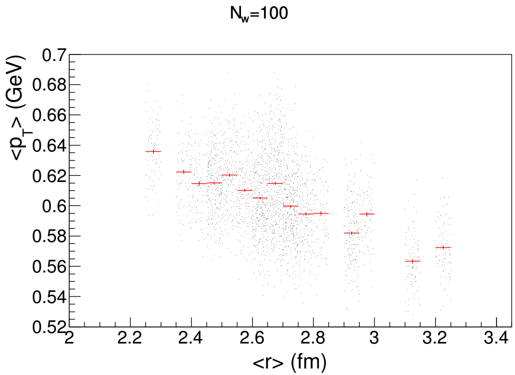
Size fluctuations $\leftrightarrow$ $p_\perp$ fluctuations





PB, Broniowski 1203.1810

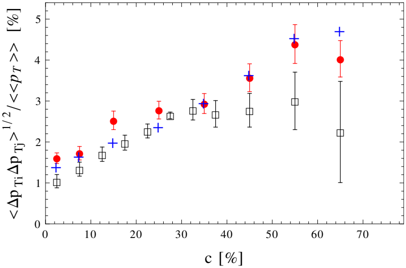
Physical and statistical fluctuations

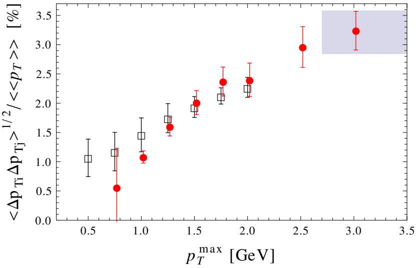


$$Var(p_\perp)_{dyn} = C_{p_\perp} = \frac{1}{N(N-1)} \sum_{i \neq j} \langle (p_i - \langle [p] \rangle)(p_j - \langle [p] \rangle) \rangle$$

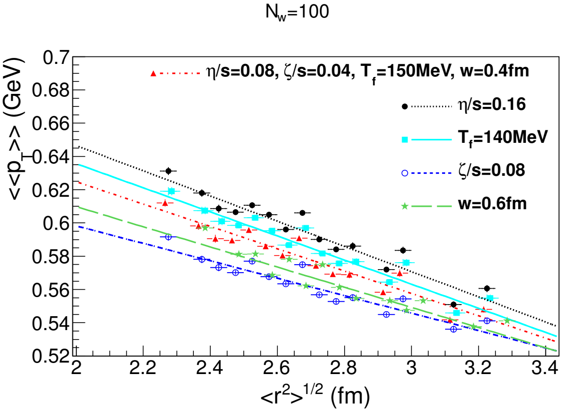
measures the variance of the “collective” $p_\perp$ (red points)

PHENIX data vs. hydro.



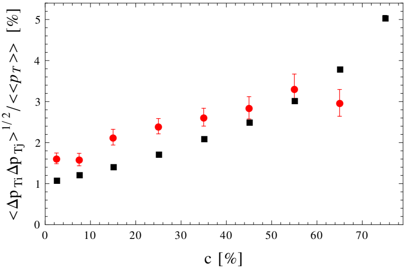


Viscosity effects on hydro response



$$\frac{\Delta p}{p} \simeq 0.4 \frac{\Delta r}{r}$$

- size fl. $\leftrightarrow$ $p_\perp$ fluctuations
- hydro. response not modified by
  - viscosity
  - T_F
  - smearing
  - core-corona
  - P_{tot} conservation
  - centrality def.



- too much fluctuations?

nucleon Glauber model $\longrightarrow$ quark Glauber model

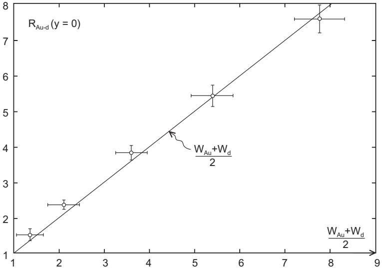
Wounded nucleon model

$$\frac{dN^{AB}_{ch}}{d\eta} = N^{AB}_{part} \frac{dN^{pp}_{ch}}{d\eta}$$

- full scaling (Bialas, Bleszynski, Czyz, 1976)

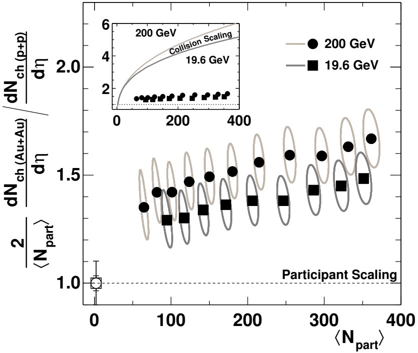
$$\frac{dN^{AB}_{ch}}{d\eta} \propto N^{AB}_{part}$$

- partial scaling (with centrality)



very good (full) scaling for d-Au at RHIC (Bialas, Czyz, 2004)

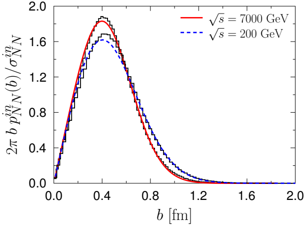
Broken scaling for A-A



- dependence on centrality
- p-p point too low

Wounded quark model - pp scattering

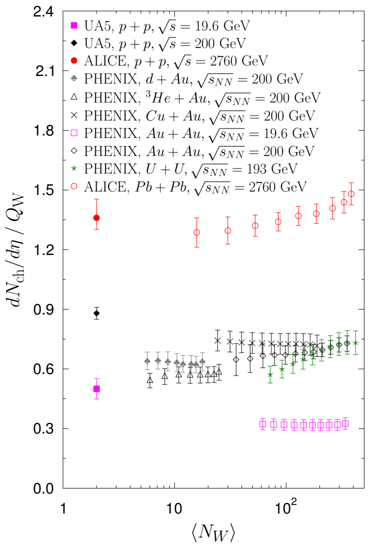
- three quarks distributed in each nucleon $\rho(r) \simeq e^{-r/b}$
- recentering
- Gaussian Q-Q wounding profile
- parameters fitted to reproduce N-N scattering



(200GeV, $\sigma_{QQ} = 7$mb, $r_{QQ} = 0.29$fm) (7000GeV, $\sigma_{QQ} = 14.3$mb, $r_{QQ} = 0.30$fm)

- small change of nucleon size with $\sqrt{s}$
- increase of σ_{QQ} with $\sqrt{s}$

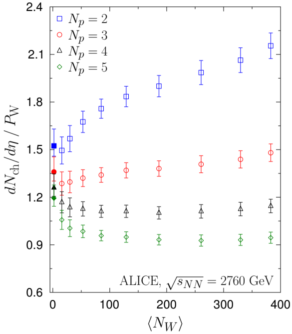
Wounded quark scaling in AA



- very good (full) scaling at LHC
- approximate scaling at RHIC

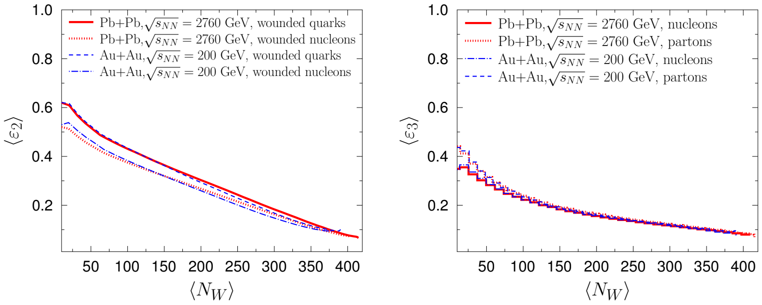
LHC - 3 partons , RHIC - 2 partons ?

How many partons?



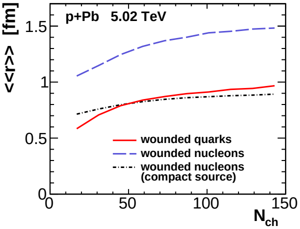
- wounded quark scaling changes with effective number of partons
- for each N_p N-N scattering profile reproduced
- number of partons increases with energy ?

Eccentricities in AA

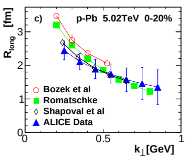


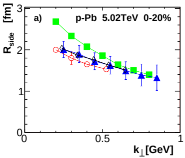
- very small effect of subnucleonic structure on eccentricities !
- similar as in wounded nucleon model with binary contribution

Fireball size in p-Pb

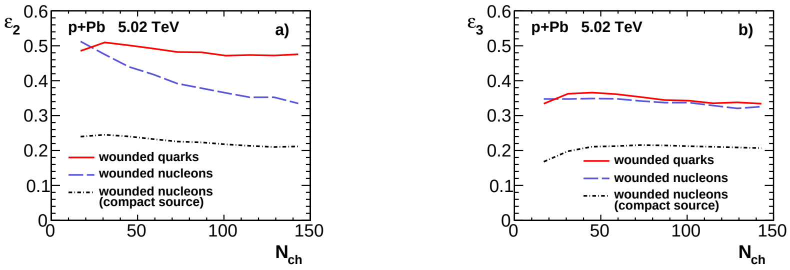


- wounded quark model gives small fireball size
- *compact source* consistent with p-Pb data (HBT, $< p_\perp >$)



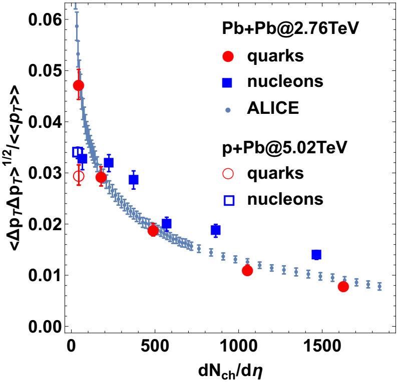


Fireball eccentricities in p-Pb



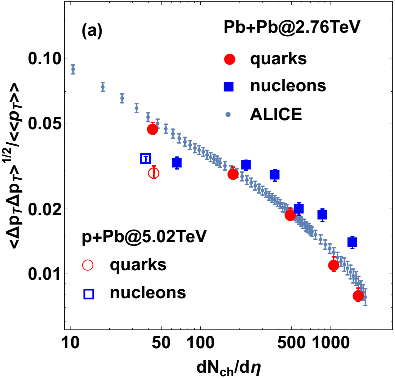
- significant eccentricities in p-Pb
- consistent with experimental observation of v_2 and v_3 in p-Pb

$p_\perp$ fluctuation quark Glauber model initial conditions



Quark Glauber model gives better description of initial volume fluctuations

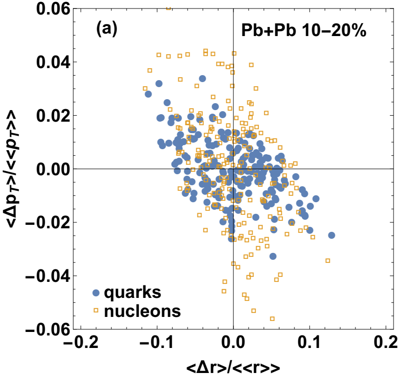
Same in log scale

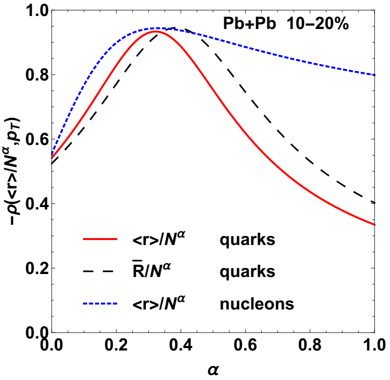


more than simple $N^{-1/2}$ scaling

both experiment and theory $\longrightarrow$ not minijets

Size - $p_\perp$ correlation

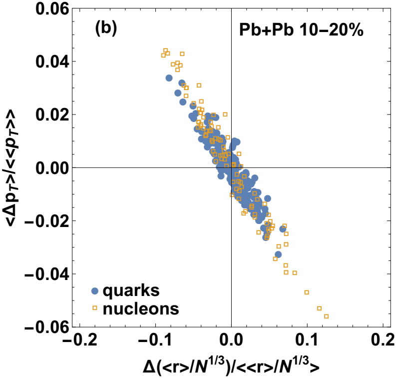




$\frac{N_q^\alpha}{<r>}$ - predictor of the final $p_\perp$

consistent with predictor of Mazellauskas-Teaney, PRC 2016

(Size+Multiplicity) - $p_\perp$ correlation

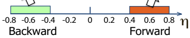


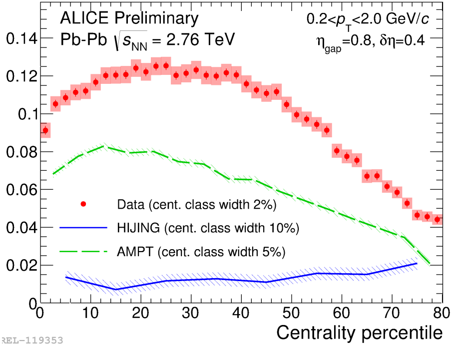
$\frac{N_q^{1/3}}{<r>}$ - predictor of the final $p_\perp$

$p_\perp - p_\perp$ correlation in rapidity - ALICE preliminary

$$b_{\rm corr} = \frac{\langle FB\rangle - \langle F\rangle\langle B\rangle}{\langle F^2\rangle - \langle F\rangle^2}$$

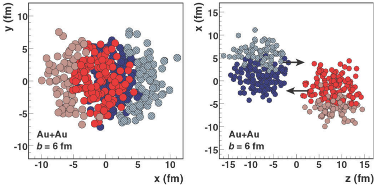
$$B \equiv \overline{p_{\rm T}}_B = \frac{\sum_{i=1}^{n_B} p_{\rm T}^{(i)}}{n_B} \qquad F \equiv \overline{p_{\rm T}}_F = \frac{\sum_{j=1}^{n_F} p_{\rm T}^{(j)}}{n_F}$$





event generators have problems to reproduce data

Forward and backward asymmetry

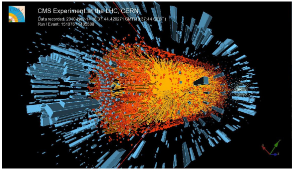


Ann.Rev.Nucl.Part.Sci. 57 (2007) 205

- Glauber Monte Carlo model $\longrightarrow$ different forward and backward distributions

- different fireball shape at forward and backward rapidities

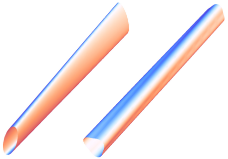
multiplicity-multiplicity correlations



dozens of years, hundreds of papers

many effects sum up ...

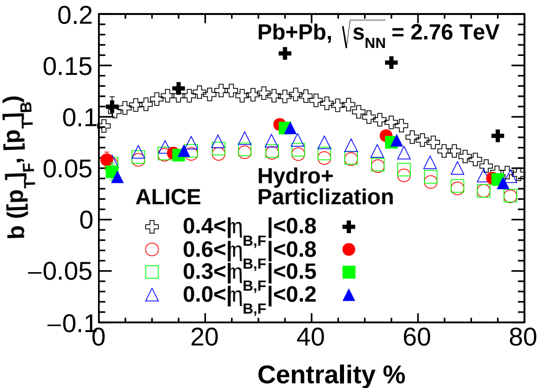
flow angle-flow angle correlations



PB, W. Broniowski, J.Moreira : 1011.3354

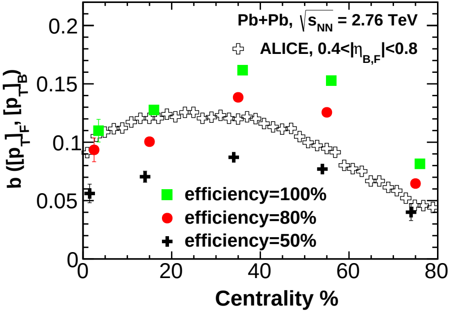
experiment and theory picks up momentum

$p_\perp - p_\perp$ correlation in rapidity - hydro



reasonable description of the data

$p_\perp - p_\perp$ correlation coefficient - ill defined



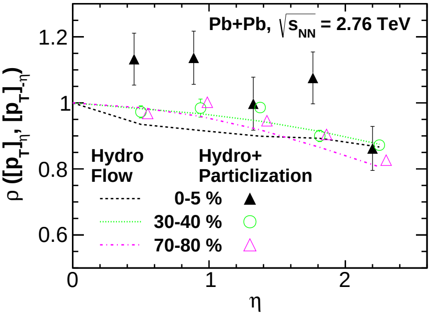
$$b = \frac{< [p_\perp]_A [p_\perp]_B > - < [p_\perp]_A >< [p_\perp]_B >}{\sqrt{(< p_A^2 > - < p_A >^2)(< p_B^2 > - < p_B >^2)}} = \frac{\dots}{\sqrt{\frac{1}{n_A^2} \sum_{ij} p_i^A p_j^A \dots}}$$

sensitive to acceptance, particle multiplicity

dominated by statistical fluctuations!

$[p_\perp]-[p_\perp]$ correlation coefficient

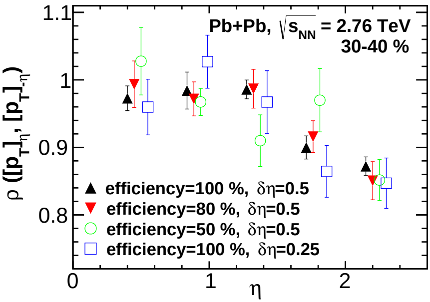
$$\frac{<[p_\perp]_A[p_\perp]_B> - <[p_\perp]_A><[p_\perp]_B>}{\sqrt{C^A_{p_\perp} C^B_{p_\perp}}} = \frac{\dots}{\sqrt{\frac{1}{n_A(n_A-1)}\sum_{i\neq j} p_i^A p_j^A \dots}}$$



$$\rho([p_T],[p_T]) \simeq 1$$

in the current model - strong correlations

$[p_\perp] - [p_\perp]$ correlation coefficient

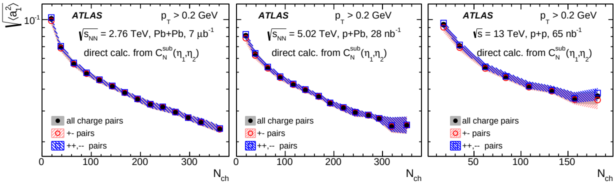


insensitive to acceptance, efficiency, multiplicity

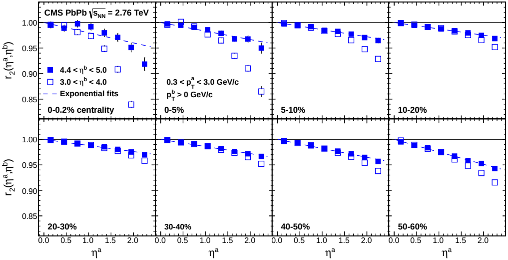
true measure of flow-flow correlations

Small decorrelation expected!

FB multiplicity fluctuations $\rho_2(\eta_1, \eta_2) \simeq < \rho(\eta_1) >< \rho(\eta_2) > (1 + a_1 a_1 \frac{\eta_1}{Y} \frac{\eta_2}{Y})$



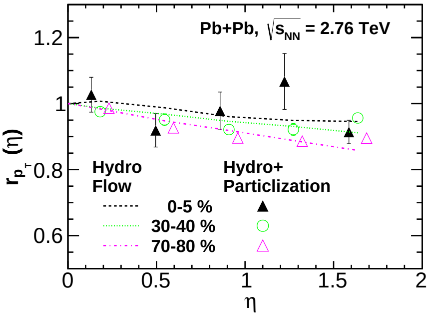
Azimuthal flow decorrelations (3-bin measure)



small decorrelation of flow and multiplicity in pseudorapidity

3-bin measure of $[p_\perp]$ decorrelation

$r_{p_T}(\Delta\eta) = \frac{Cov([p_T],[p_T])(\eta+\Delta\eta)}{Cov([p_T],[p_T])(\eta-\Delta\eta)}$ **Measure of $[p_T]$ decorrelation in pseudorapidity**

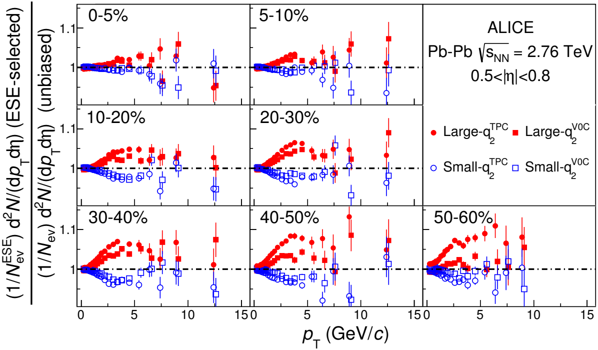


Strong $[p_\perp]-[p_\perp]$ correlations? - should be measured

Effect of elliptic flow on other observables

event shape engineering

select events with large q_2 $\longrightarrow$ harder spectra



difficult to interpret

Correlation of elliptic flow with other observables

covariance

$$cov(v_n\{2\}^2, \mathcal{O}) = \langle \frac{1}{N_{pairs}} \sum_{i \neq k} e^{in\phi_i} e^{-in\phi_k} (\mathcal{O} - \langle \mathcal{O} \rangle) \rangle$$

correlation coefficient

$$\rho(v_n\{2\}^2, \mathcal{O}) = \frac{cov(v_n\{2\}^2, [p_\perp])}{\sqrt{Var(v_n^2)_{dyn} Var(\mathcal{O})_{dyn}}}$$

for $[p_\perp]$

$$\rho(v_n\{2\}^2, [p_\perp]) = \frac{cov(v_n\{2\}^2, [p_\perp])}{\sqrt{Var(v_n^2)_{dyn} Var(p_\perp)_{dyn}}} \ .$$

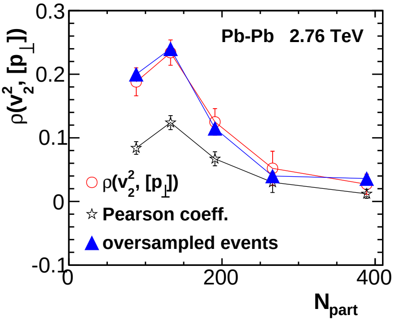
Note: variances exclude selfcorrelations

$$Var(v_n^2)_{dyn} = v_n\{2\}^4 - v_n\{4\}^4 \ .$$

$$Var(p_\perp)_{dyn} = C_{p_\perp} = \frac{1}{N(N-1)} \sum_{i \neq j} \langle (p_i - \langle [p] \rangle)(p_j - \langle [p] \rangle) \rangle$$

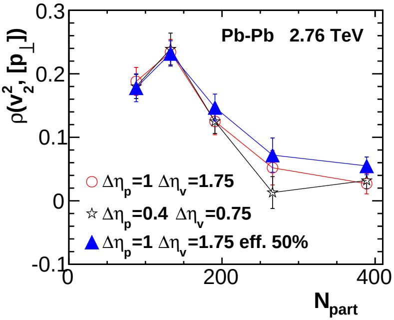
Elliptic flow - $p_\perp$

$$\rho(v_3\{2\}^2, [p_\perp])$$



positive correlation between v_2 and $[p_\perp]$

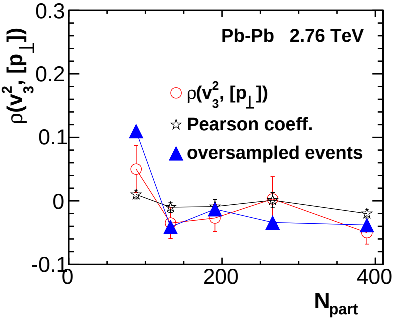
Acceptance - efficiency effects



correct definition of the correlation coefficient

Triangular flow - $p_\perp$

$$\rho(v_3\{2\}^2, [p_\perp])$$



no correlation between v_3 and $[p_\perp]$

Comparing to experiment
Correcting for correlations with multiplicity !

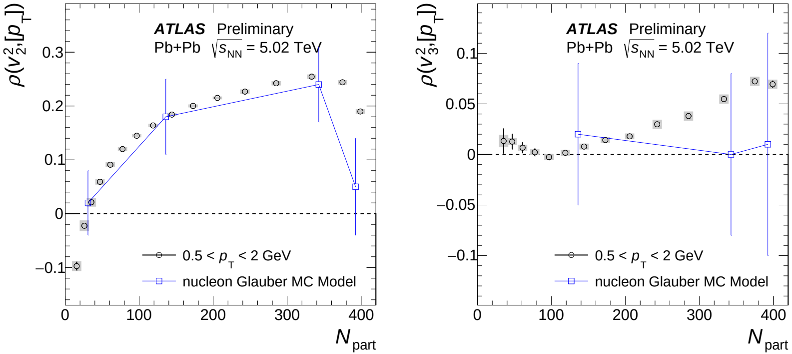
$\rho(v_n\{2\}^2,[p_\perp]) \neq 0$
but also
$\rho(v_n\{2\}^2,N) \neq 0 \quad , \quad \rho([p_\perp],N) \neq 0$

How to calculate the correlation at fixed multiplicity

Partial correlation coefficient (Olszewski, Broniowski 1706.01532)

$$\rho(v_n\{2\}^2,[p_\perp]) \simeq \frac{\rho(v_n\{2\}^2,[p_\perp]) - \rho(v_n\{2\}^2,N)\rho([p_\perp],N)}{\sqrt{1-\rho(v_n\{2\}^2,N)^2}\sqrt{1-\rho([p_\perp],N)^2}}$$

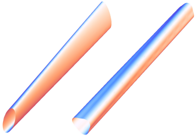
ATLAS results



Factorization breaking

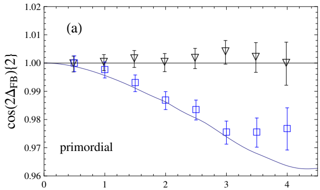
Flow at a and b does not factorize

$$r_n(a,b) = \frac{\langle q_n(a) q_n^\star(b)\rangle}{\sqrt{\langle q_n(a) q_n^\star(a)\rangle \langle q_n(b) q_n^\star(b)\rangle}} \neq 1$$



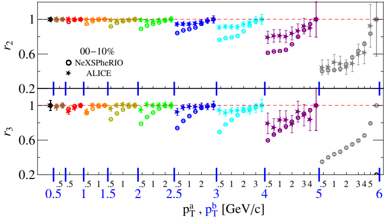
$$q_n(a) = \frac{1}{N}\sum_{j\in a} exp^{in(\phi_j)} = v_n(a) exp^{in\Psi_n(a)}$$

factorization breaking in η



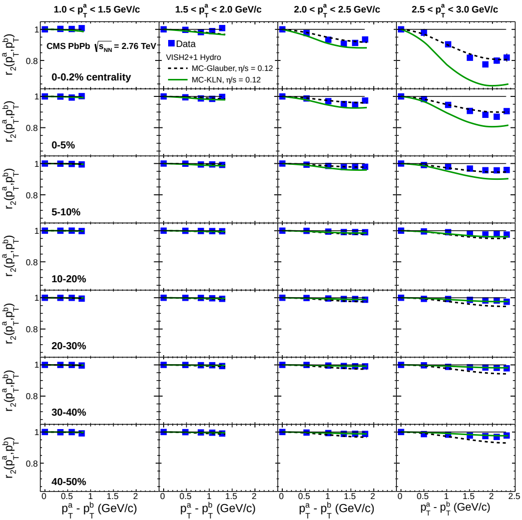
PB, Broniowski, Moreira 1011.3354

factorization breaking in $p_\perp$

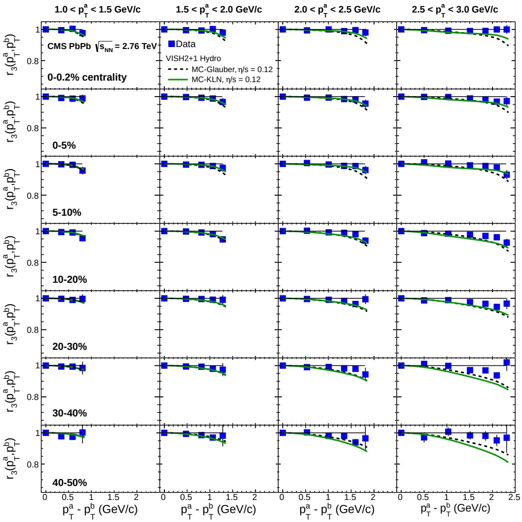


Gardim, Grassi, Luzum, Ollitrault 1211.0989

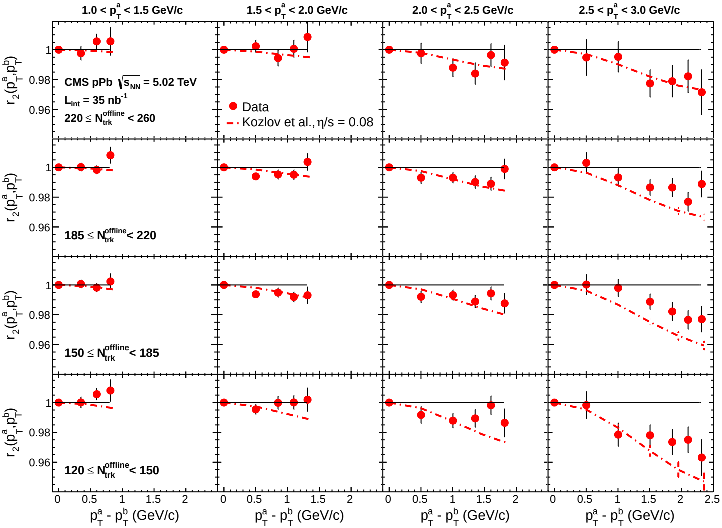
CMS results - PbPb



CMS results - PbPb

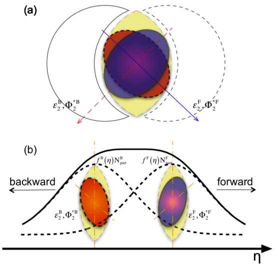


CMS results - pPb



Flow asymmetry + Twist angle

$$r_2(\eta) = 1 - 2F_n\eta = 1 - 2F_n^{asy}\eta - 2F_n^{twi}\eta$$



the two can be separated using
3-bin and 4-bin correlators **ATLAS** 1709.02301

$$R_n(\eta) = \frac{< q_n(-\eta_{ref})q_n^\star(\eta)q_n(-\eta)q_n^\star(\eta_{ref}) >}{< q_n(-\eta_{ref})q_n^\star(-\eta)q_n(\eta)q_n^\star(\eta_{ref}) >} \simeq 1 - 2F_{n,2}^{twi}\eta$$

flow angle decorrelation

$$r_{n,2}(\eta) = \frac{< q(-\eta)^2 q^\star(\eta_{ref})^2 >}{< q(\eta)^2 q^\star(\eta_{ref})^2 >} \simeq 1 - 2F_{n,2}^{asy}\eta - 2F_{n,2}^{twi}\eta$$

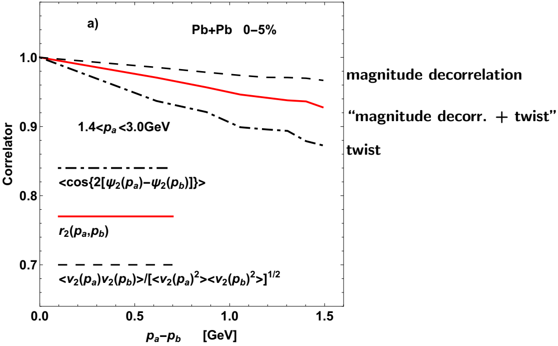
flow angle+flow magnitude decorrelation

Jia, Huo 1403.6077

talk by W. Broniowski

$r_{n,2}(\eta)$ first measured by CMS 1503.01692

twist angle and flow magnitude decorrelation
3+1D hydro model

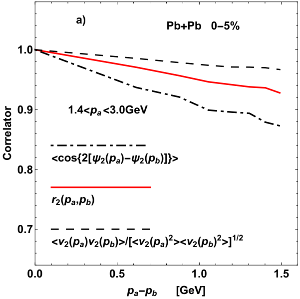


surprising result: "inverted hierarchy"

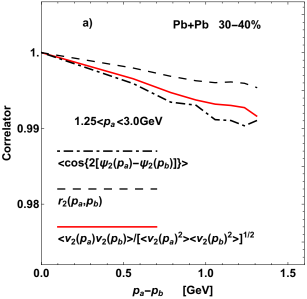
magnitude decorr. $<$ "magnitude decorrelation + twist" $<$ twist

central versus peripheral

$0-5\%$



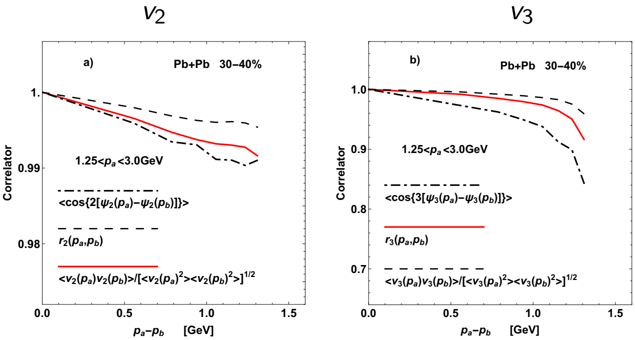
$30-40\%$



the "inverted hierarchy" effect is stronger in central collisions

large elliptic flow in semi-central collisions $\rightarrow$ less fluctuations

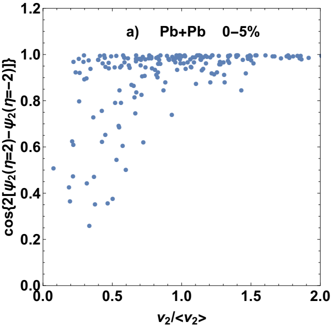
elliptic versus triangular



the "inverted hierarchy" effect is stronger for v_3

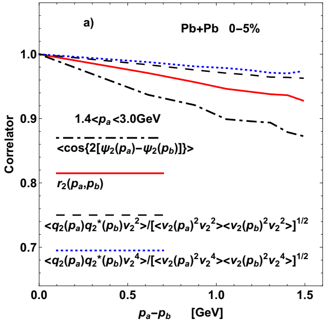
triangular flow - fluctuation dominated

Correlation between flow magnitude and twist angle



- **strong** correlation between flow magnitude and twist angle
- events with large flow have smaller twist angle
- twist angle measure $< cos(\Delta\Psi_2) > \propto (v_2)^0$
  "magnitude decorr.+twist" $< q_2(\eta) q_2^\star(\eta_{ref}) > \propto (v_2)^2$
- different weighting by (v_2) powers explains "inverted hierarchy"

Correlators weighted by powers of v_n

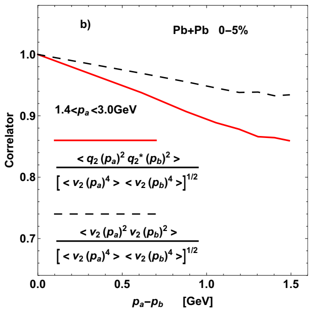


- hierarchy of correlators consistent with expectations

$$\frac{\langle q_n(p_a)q_n^\star(p_b)\rangle}{\sqrt{\langle q_n(p_a)q_n^\star(p_b)\rangle\langle q_n(p_b)q_n^\star(p_b)\rangle}} < \frac{\langle q_n(p_a)q_n^\star(p_b)v_n^2\rangle}{\sqrt{\langle q_n(p_a)q_n^\star(p_a)v_n^2\rangle\langle q_n(p_b)q_n^\star(p_b)v_n^2\rangle}} < \frac{\langle q_n(p_a)q_n^\star(p_b)v_n^4\rangle}{\sqrt{\langle q_n(p_a)q_n^\star(p_a)v_n^4\rangle\langle q_n(p_b)q_n^\star(p_b)v_n^4\rangle}}$$

- the correlation between flow magnitude and twist can be measured experimentally

Measuring separately magnitude and angle decorrelation



- flow magnitude factorization breaking (square)

$$r_n^{v_n^2}(p_a, p_b)) = \frac{\langle v_n^2(p_a) v_n^2(p_b) \rangle}{\sqrt{\langle v_n^4(p_a) \rangle \langle v_n^4(p_b) \rangle}}$$

- angle+magnitude factorization breaking

$$\frac{\langle q_n(p_a)^2 q_n^\star(p_b)^2 \rangle}{\sqrt{\langle q_n(p_a)^2 q_n^\star(p_a)^2 \rangle \langle q_n(p_b)^2 q_n^\star(p_b)^2 \rangle}}$$

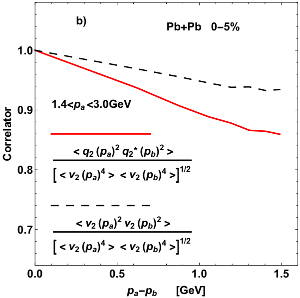
(angle+magnitude f. b.) $\simeq$ (twist angle f. b.)(flow magnitude f. b.)

Note: same effective power of q_n

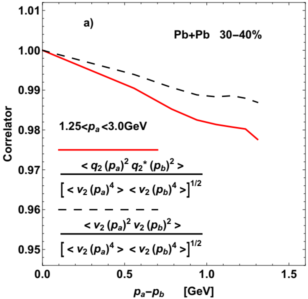
(similar factorization decomposition measured by ATLAS for η)

central versus peripheral

$0 - 5\%$

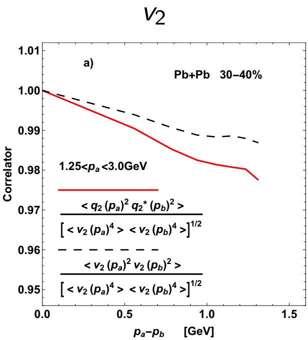


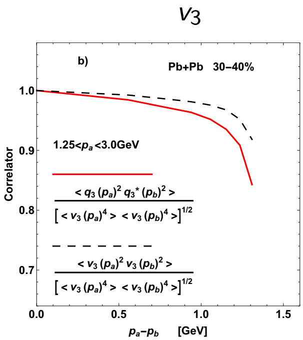
$30 - 40\%$



angle decorrelation $\simeq$ magnitude decorrelation $\simeq \frac{1}{2}$ flow decorrelation

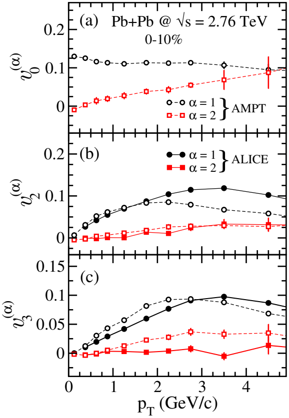
elliptic versus triangular





angle decorrelation $\simeq$ magnitude decorrelation $\simeq \frac{1}{2}$ flow decorrelation

Principal component analysis



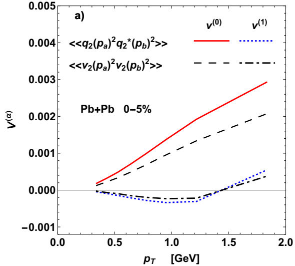
$$\langle q_n(p_a) q_n^\star(p_b)\rangle = \lambda^{(0)}\Psi^{(0)(p_a)}\Psi^{(0)}(p_b) + \lambda^{(1)}\Psi^{(1)}(p_a)\Psi^{(1)}(p_b) + \ldots$$

subleading modes break factorization

$$r_n(p_a,p_b) = 1 - \frac{1}{2}\left|\frac{\sqrt{\lambda^{(1)}}\Psi^{(1)}(p_a)}{\sqrt{\lambda^{(0)}}\Psi^{(0)}(p_a)} - \frac{\sqrt{\lambda^{(1)}}\Psi^{(1)}(p_b)}{\sqrt{\lambda^{(0)}}\Psi^{(0)}(p_b)}\right|^2 < 1$$

Bhalerao, Ollitrault, Pal, Teaney 1410.7739

Principal component analysis for higher order correlators



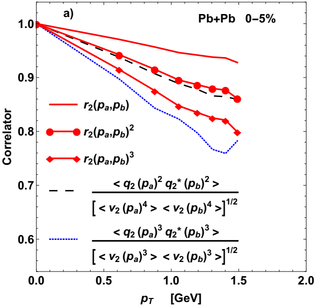
$$<< q_n(p_a)^2 q_n^\star(p_b)^2 >>=< q_n(p_a)^2 q_n^\star(p_b)^2 >$$

$$<< v_n(p_a)^2 v_n p_b)^2 >>=< v_n(p_a)^2 v_n(p_b)^2 > - < v_n(p_a)^2 >< v_n(p_b)^2 >$$

$$= v^{(0)}(p_a)v^{(0)}(p_b) + v^{(1)}(p_a)v^{(1)}(p_b) + \ldots$$

similar shape of eigenvectors $\longrightarrow$ similar shape of factorization breaking

factorization breaking for higher powers of flow



- flow factorization breaking

$$r_n(p_a, p_b) = \frac{\langle q_n(p_a) q_n^\star(p_b)\rangle}{\sqrt{\langle q_n(p_a) q_n^\star(p_a)\rangle \langle q_n(p_b) q_n^\star(p_b)\rangle}}$$

- flow^2 factorization breaking

$$\frac{\langle q_n(p_a)^2 q_n^\star(p_b)^2\rangle}{\sqrt{\langle q_n(p_a)^2 q_n^\star(p_a)^2\rangle \langle q_n(p_b)^2 q_n^\star(p_b)^2\rangle}}$$

- flow^3 factorization breaking

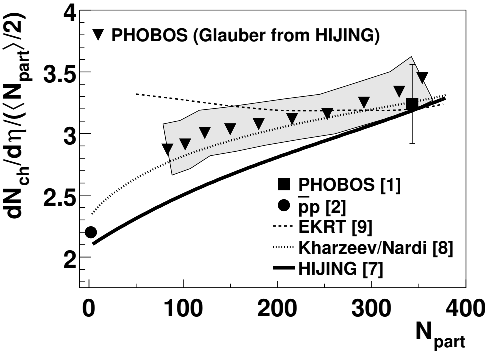
$$\frac{\langle q_n(p_a)^3 q_n^\star(p_b)^3\rangle}{\sqrt{\langle q_n(p_a)^3 q_n^\star(p_a)^3\rangle \langle q_n(p_b)^3 q_n^\star(p_b)^3\rangle}}$$

a way to measure higher moments of the decorrelation

- p_T fluctuations
- p_T correlations
- p_T-flow correlations
- factorization breaking
  flow decorrelation, angle decorrelation, magnitude decorrelation
- higher order factorization breaking

- **Measure**
- **Calculate**
  hydro $\leftrightarrow$ cascade $\leftrightarrow$ CGC

Two component model



$$\frac{dN_{ch}}{d\eta} \propto \frac{1-\alpha}{2} N_{part} + \alpha N_{coll}$$

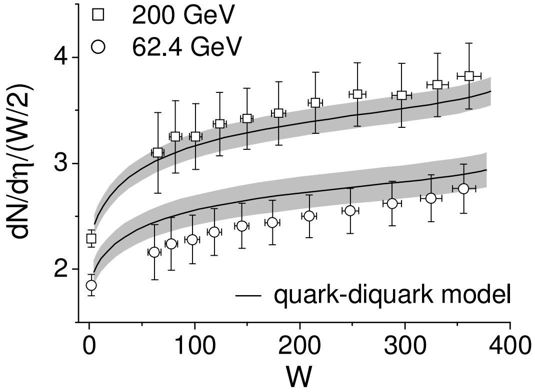
- binary (N_{coll}) contribution $\alpha = 0.1 - 0.2$

Kharzeev, Nardi, 2000

- maybe due to hard processes

-hard to incorporate in models of initial fireball

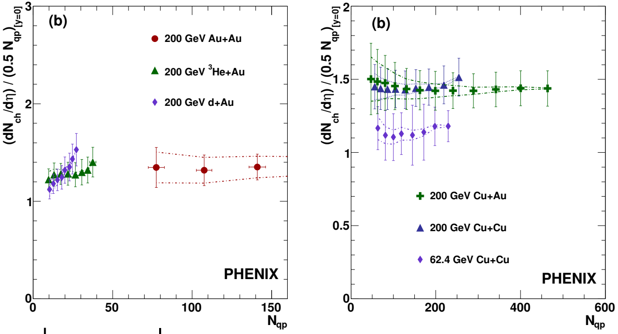
quark-diquark model



- subnucleonic structure !
- wounded quark model (Bialas, Czyz, Furmanski 1977, + ...many others)
- quark-diquark model fitted to p-p scattering
- helps in describing RHIC A-A data (Bialas, Bzdak, 2006)

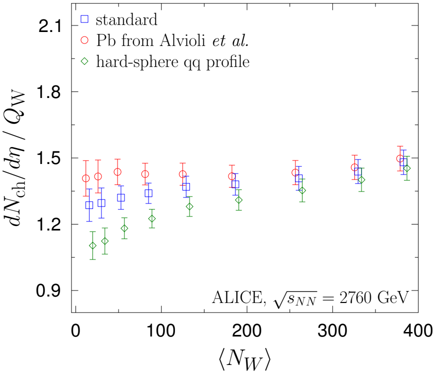
Constituent quark model - PHENIX

PHENIX 2015



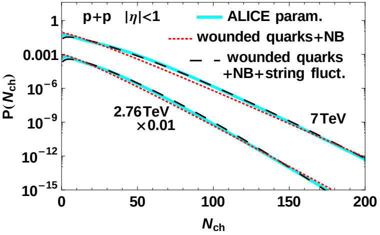
- three quarks per nucleon
- Q distribution in N from electron-proton
- hard-sphere Q-Q scattering (8.17mb at 200GeV)
- fairly good scaling with N_Q, problem with p-p point
- recent (2016) calculations : Lacey et al., Zheng et al. , Loizides, Mitchell et al.

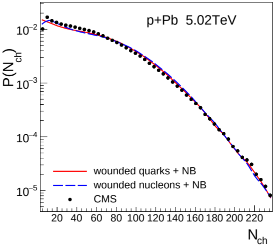
N-N profile matters



- bulk properties sensitive to modeling of N-N scattering

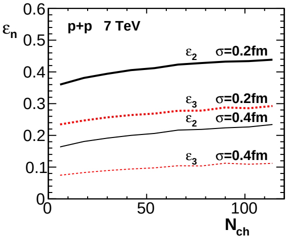
Multiplicity distribution p-A, p-p

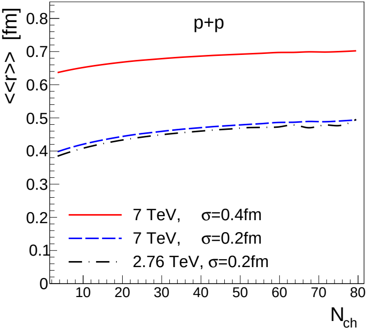




- overlaid negative binomial distribution for each wounded quark

p-p scattering





- significant eccentricities in p-p
- small size of the interaction region 0.4fm

- Wounded quark Glauber model for pp, pA AA
- Quark distribution in nuclei and Q-Q scattering adjusted to reproduce N-N scattering
- Particle production scales with number of wounded quarks at LHC
- Semi-microscopic description of subnucleonic structure in p-Pb, consistent with experimental data
- Small deformed interaction region in p-p
- Indication of an increase of the effective number of partons with $\sqrt{s}$

Caution - additional fluctuation may change the results