Odd Harmonics from the Classical Gluon Fields

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based on work with V. Skokov, arXiv:1802.08166 [hep-ph];
with D. Wertepny arXiv:1212.1195 [hep-ph];
with G. Chirilli and D. Wertepny, arXiv:1501.03106 [hep-ph]

Outline

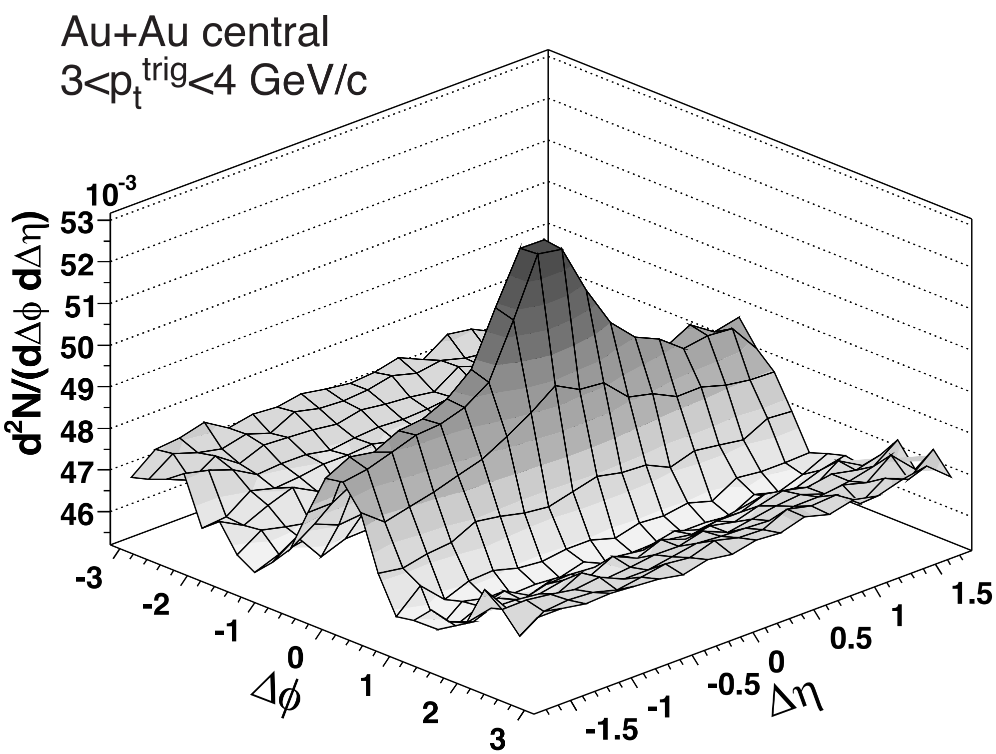
- Introduction: long-range rapidity correlations.
- The problem. Where are the odd harmonics?
- An aside: Single inclusive gluon production in saturation physics: first saturation correction in the projectile
- Solution: odd harmonics are generated by the higher-order saturation corrections in the interactions with the projectile <u>and</u> the target. At the very least, the odd harmonics require three scatterings in the projectile and three scatterings in the target.

Introduction: Long-range rapidity correlations

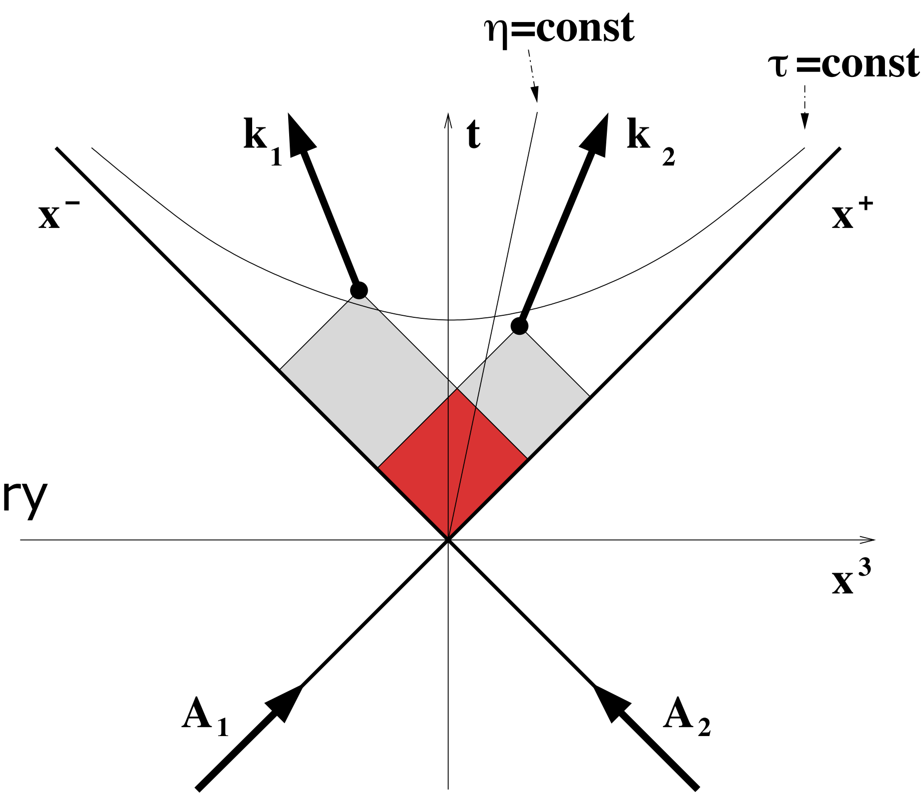
YK, D. Wertepny,
arXiv:1212.1195 [hep-ph], arXiv:1310.6701 [hep-ph]

Ridge in heavy ion collisions

- Heavy ion collisions, along with high-multiplicity p+p and p+A collisions, are known to have long-range rapidity correlations, known as ‘the ridge’:



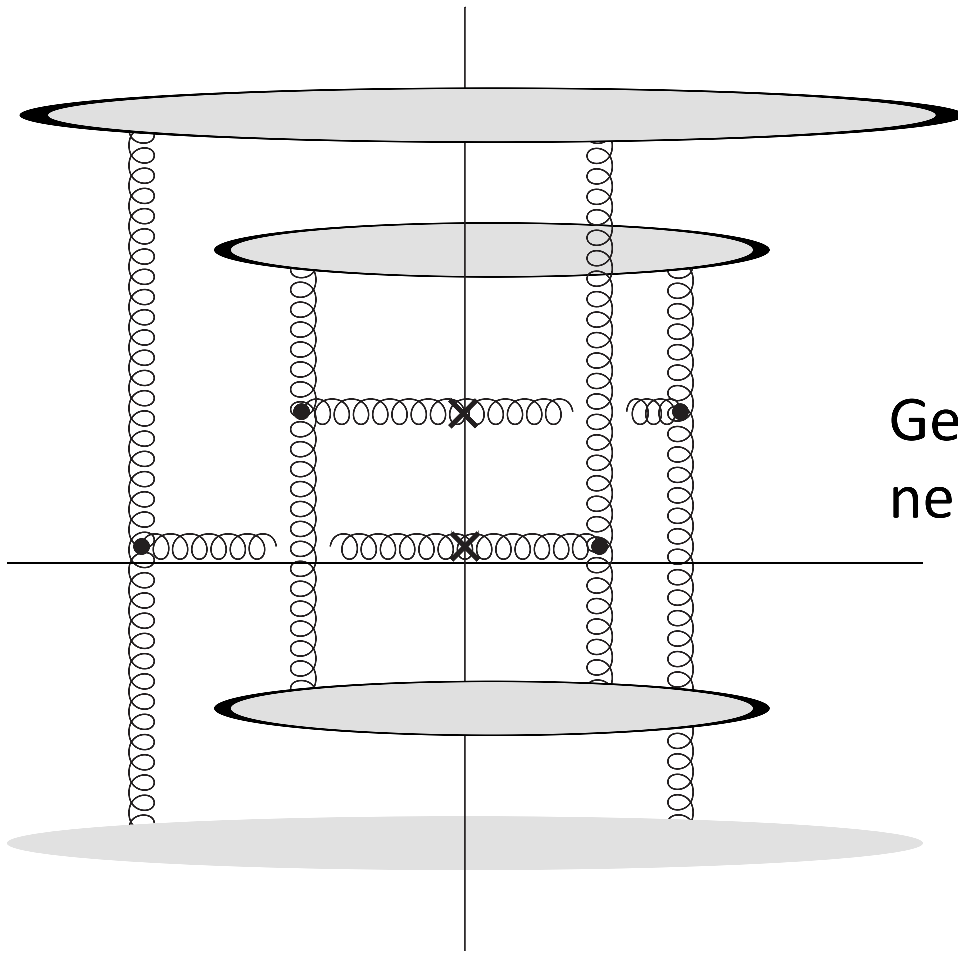
Origin of rapidity correlations



Causality demands that long-range rapidity correlations originate at very early times (cf. explanation of the CMB homogeneity in the Universe)

Gavin, McLerran, Moschelli '08;
Dumitru, Gelis, McLerran, Venugopalan '08.

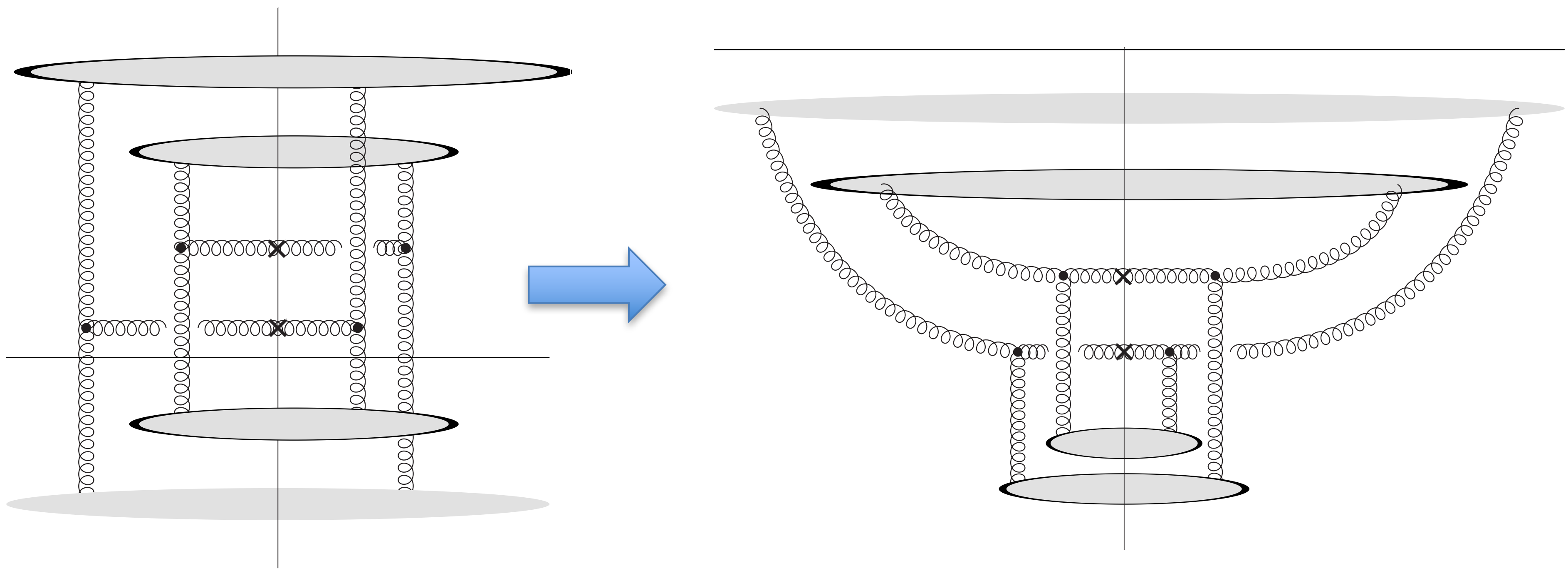
Glasma graphs



Generate back-to-back and near-side azimuthal correlations.

Dumitru, Gelis, McLerran, Venugopalan ‘08.

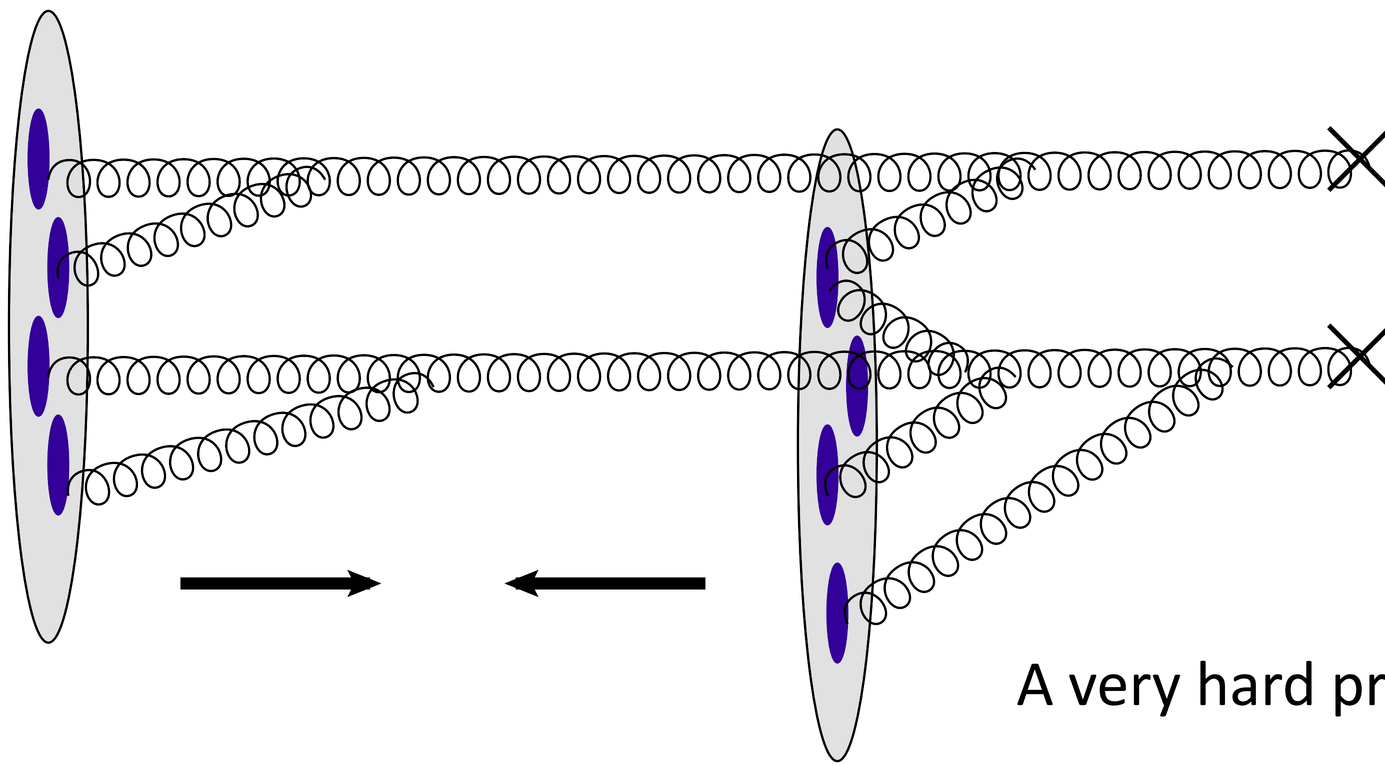
Glasma graphs in LC gauge



Glasma graphs are one of the many rescattering diagrams when two nucleons with a gluon each scatter on a nuclear target.

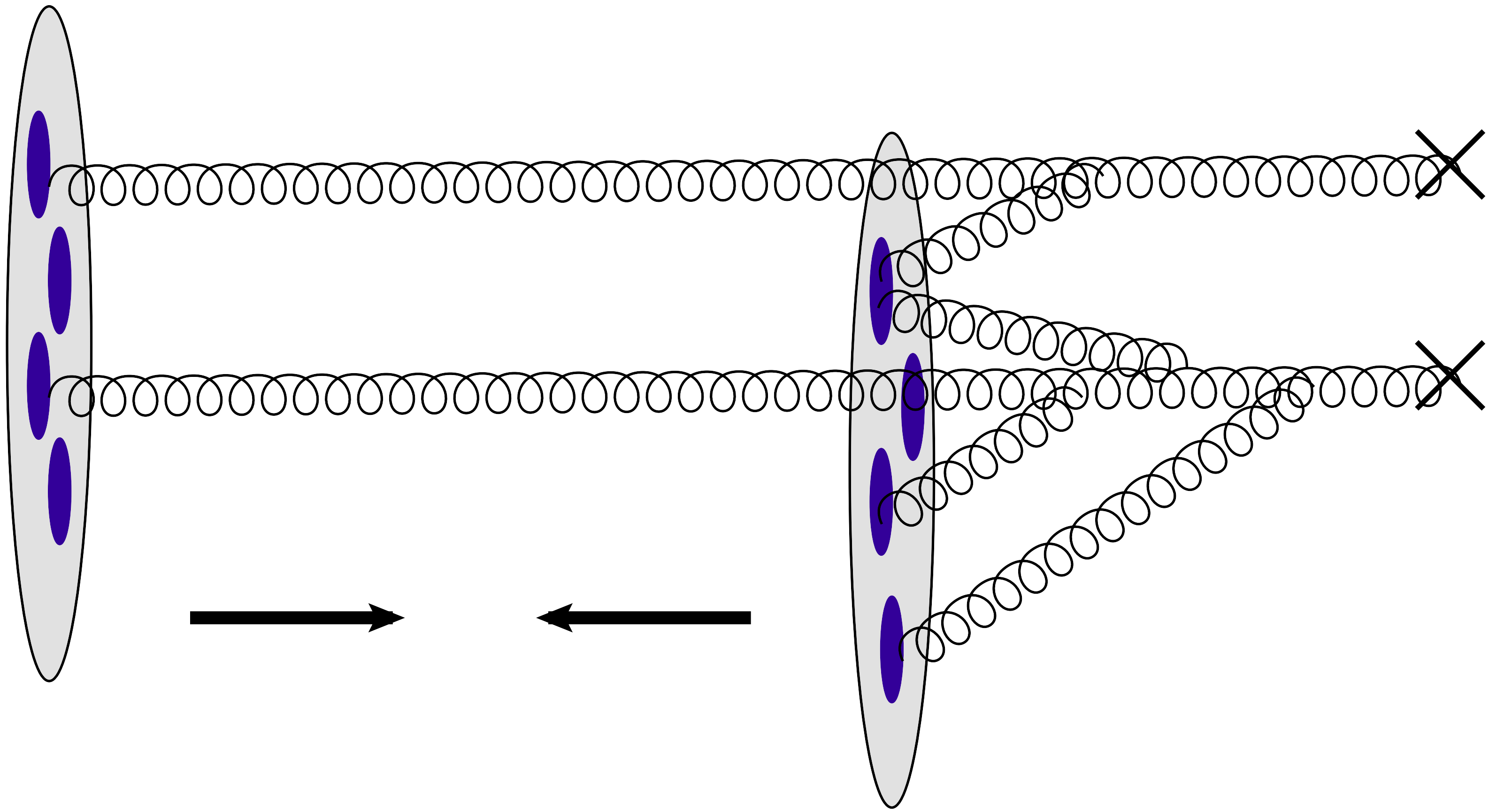
What to calculate?

- To systematically include Glasma graphs in the CGC formalism it would be great to solve the two-gluon inclusive production problem in the MV model, that is, including multiple rescatterings in both nuclei to all orders (the two produced gluons only talk to each other through sources):



A very hard problem!

Heavy-Light Ion Collisions



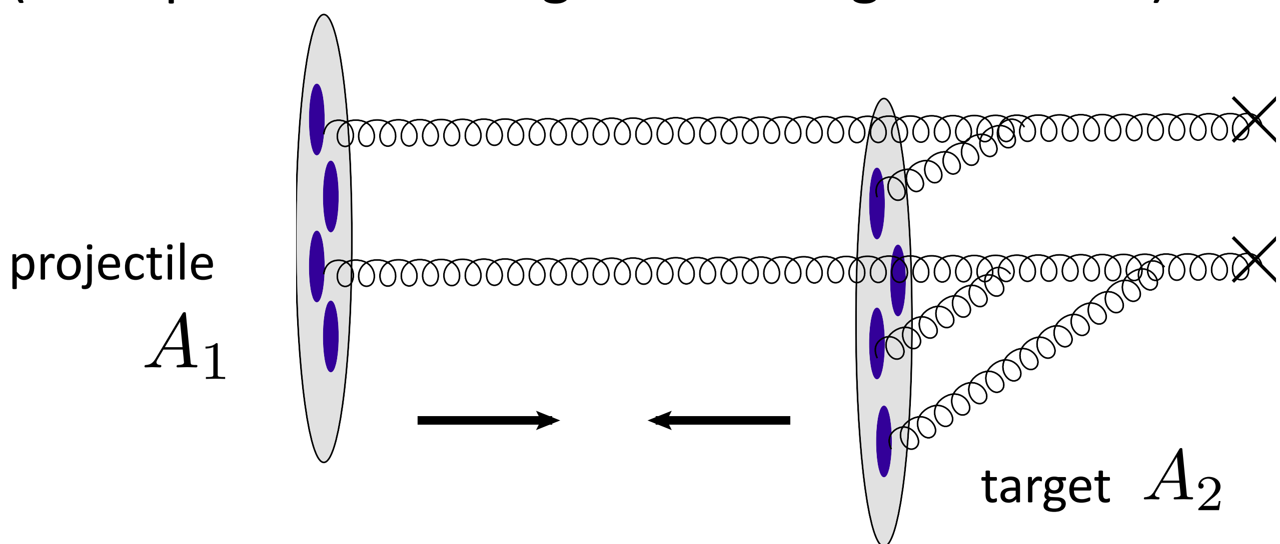
Little steps for the little feet: consider multiple rescatterings only in one of the two nuclei.

Two-gluon production

- We want to calculate two gluon production in A_1+A_2 collisions with 1 << A_1 << A_2 resumming all powers of

$$\alpha_s^2 \, A_2^{1/3} \sim 1 \qquad \text{while} \qquad \alpha_s^2 \, A_1^{1/3} \ll 1$$

(multiple rescatterings in the target nucleus)



- The gluons come from different nucleons in the projectile nucleus as A_1>>1 and this is enhanced compared to emission from the same nucleon.

Applicability region

- The saturation scales of the two nuclei are very different:

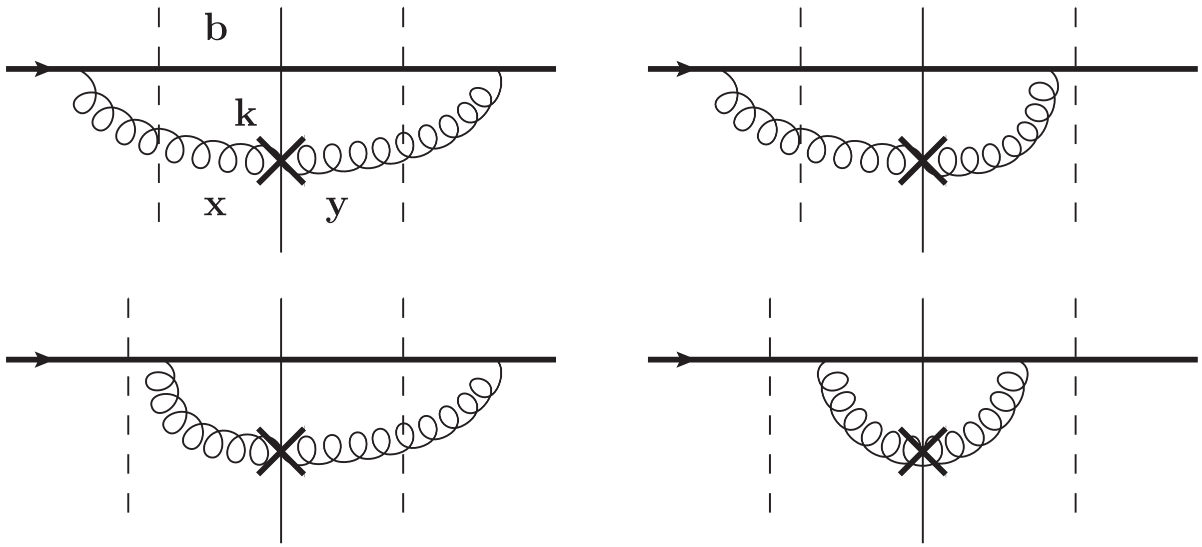
$$\Lambda_{QCD} \ll Q_{s1} \ll Q_{s2}$$

- We are working above the saturation scale of the smaller nucleus: $k_T \gg Q_{s1}$
- We thus sum all multiple rescatterings in the larger nucleus, $Q_{s2}/k_T \sim 1$, staying at the lowest non-trivial order in $Q_{s1}/k_T << 1$.
- Multiple interactions with the same nucleon in either nucleus are suppressed by $\Lambda_{QCD}/k_T <<< 1$.

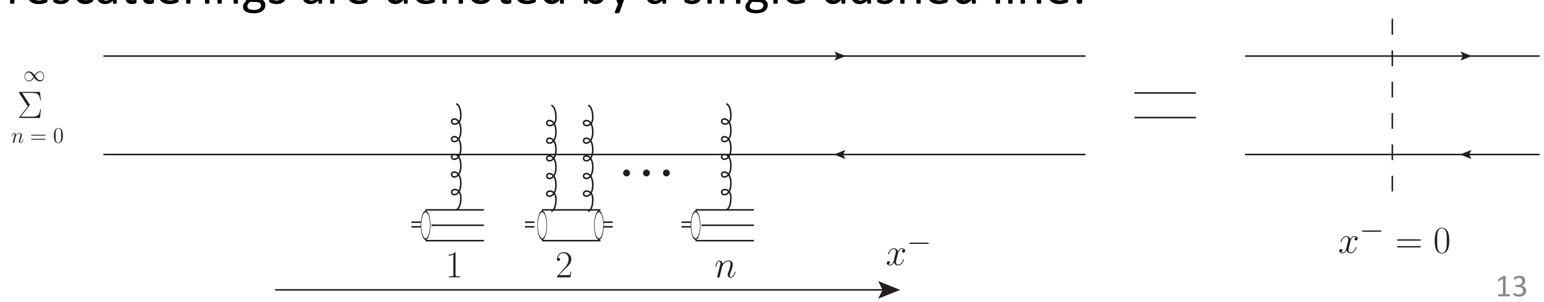
(i) Single gluon production in pA

Single gluon production in pA

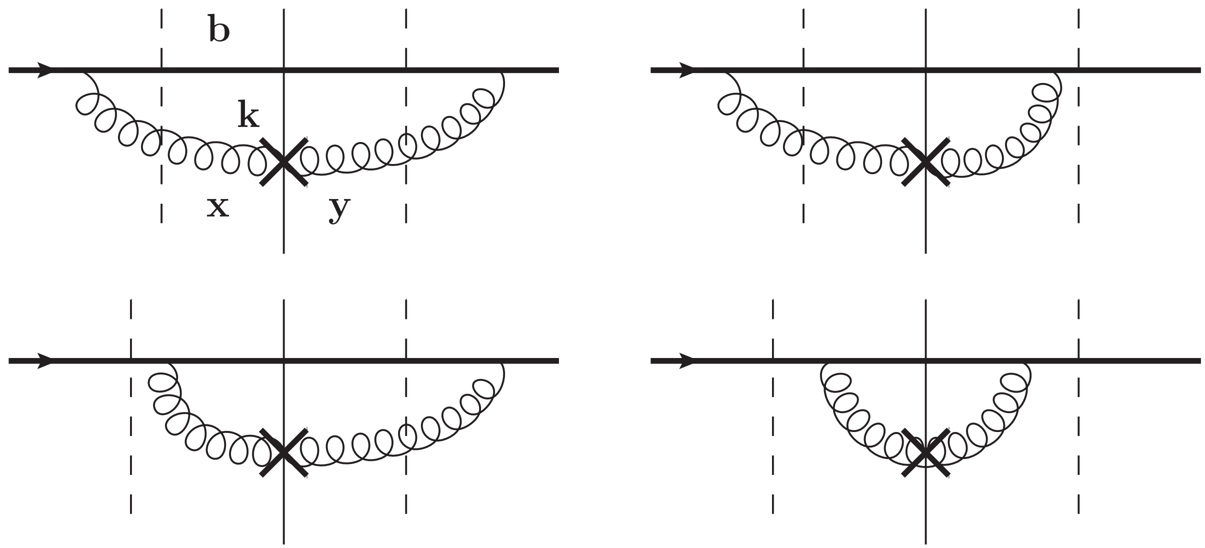
Model the proton by a single quark (can be easily improved upon).
The diagrams are shown below (Yu.K., A. Mueller ’97):



Multiple rescatterings are denoted by a single dashed line:



Single gluon production in pA



The gluon production cross section can be readily written as (U = Wilson line in **adjoint** representation, represents gluon interactions with the target)

$$\left\langle \frac{d\sigma^{pA_2}}{d^2k\,dy\,d^2b} \right\rangle = \frac{\alpha_s\,C_F}{4\,\pi^4} \int d^2x\,d^2y\;e^{-i\,\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\;\frac{\mathbf{x}-\mathbf{b}}{|\mathbf{x}-\mathbf{b}|^2}\cdot\frac{\mathbf{y}-\mathbf{b}}{|\mathbf{y}-\mathbf{b}|^2}$$

$$\times\;\left\langle \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}}U_{\mathbf{y}}^\dagger]\;-\;\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}}U_{\mathbf{b}}^\dagger]\;-\;\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}}U_{\mathbf{y}}^\dagger]\;+\;1 \right\rangle$$

Forward dipole amplitude

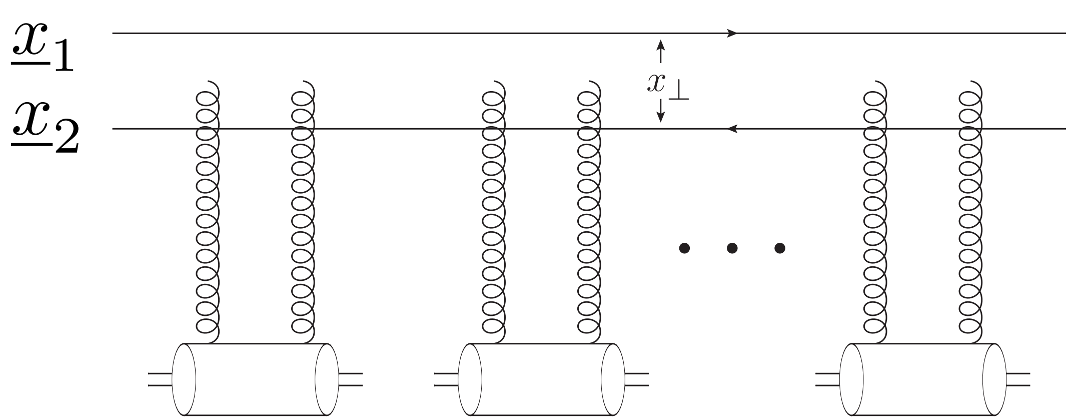
- The eikonal quark propagator is given by the Wilson line

$$V(\underline{x}) = \mathrm{P}\exp\left[i\,g\int\limits_{-\infty}^{\infty} dx^{+}\,A^{-}(x^{+}, x^{-}=0, \underline{x})\right]$$

with the light cone coordinates $$x^{\pm} = \frac{t \pm z}{\sqrt{2}}$$

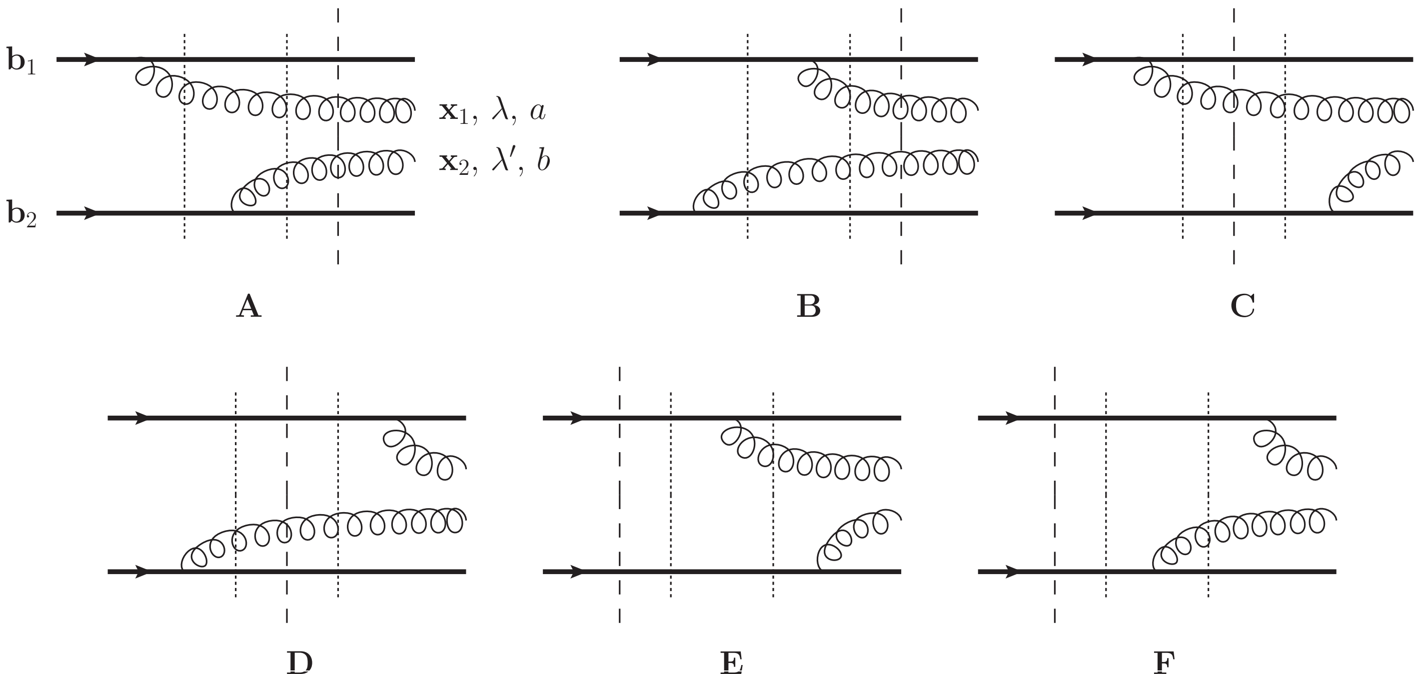
- The quark dipole scattering amplitude is

$$N(\underline{x}_1, \underline{x}_2) = 1 - \frac{1}{N_c}\,\langle \mathrm{tr}\left[V(\underline{x}_1)\,V^{\dagger}(\underline{x}_2)\right]\rangle$$



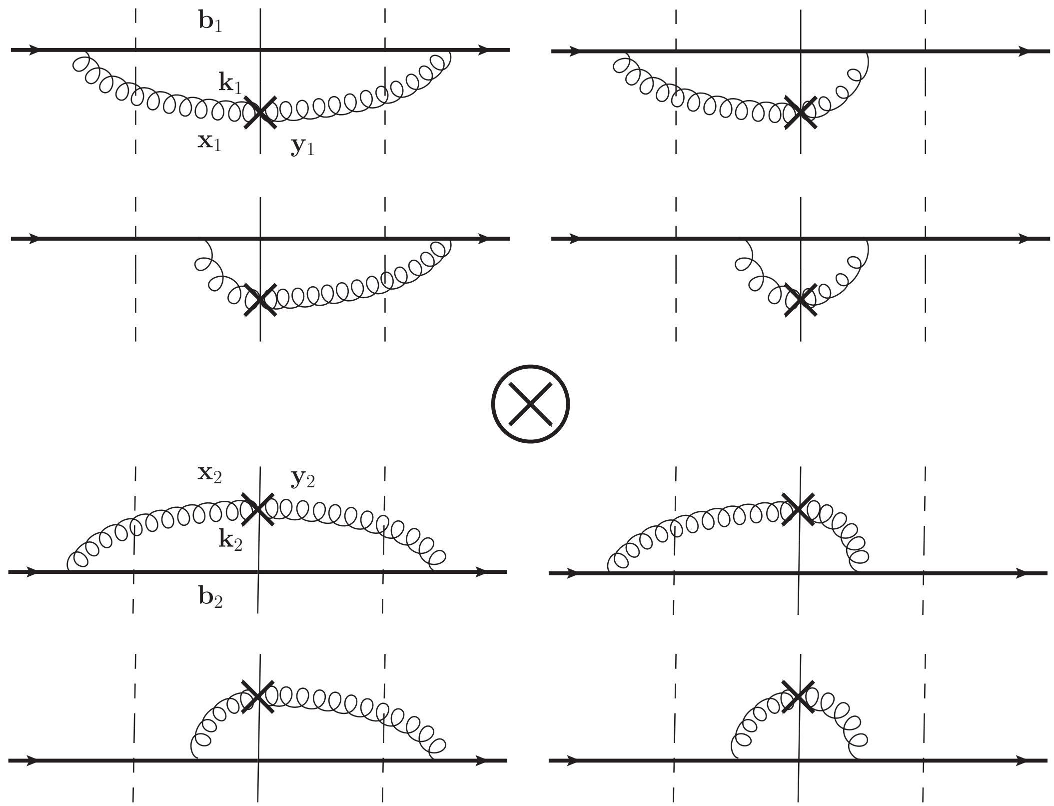
(ii) Two-gluon production in heavy-light ion collisions

The process



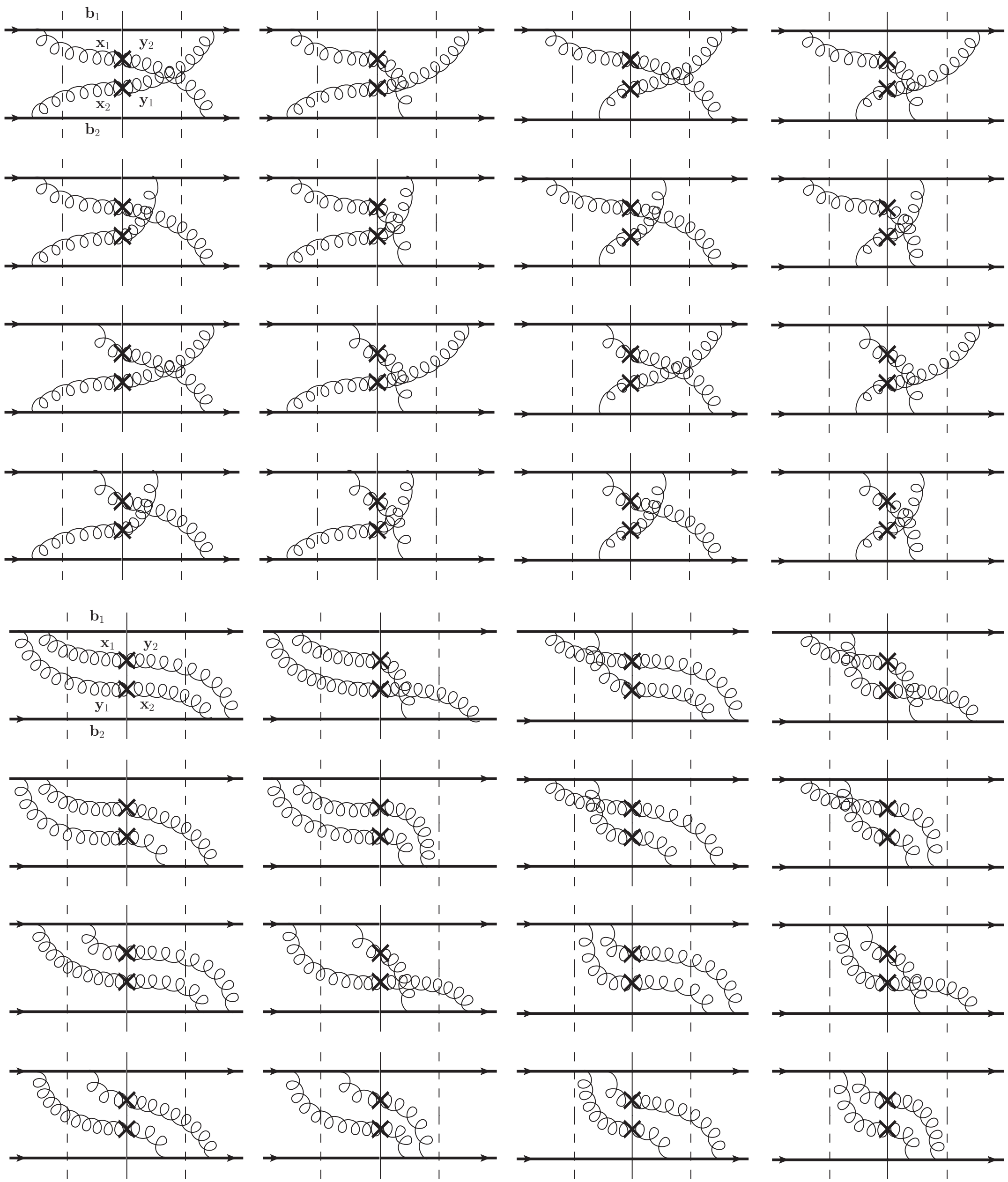
Solid horizontal lines = quarks in the incoming nucleons.
Dashed vertical line = interaction with the target.
Dotted vertical lines = energy denominators (ignore).

Amplitude squared



This contribution to two-gluon production looks like one-gluon production squared, with the target averaging applied to both.

Amplitude squared

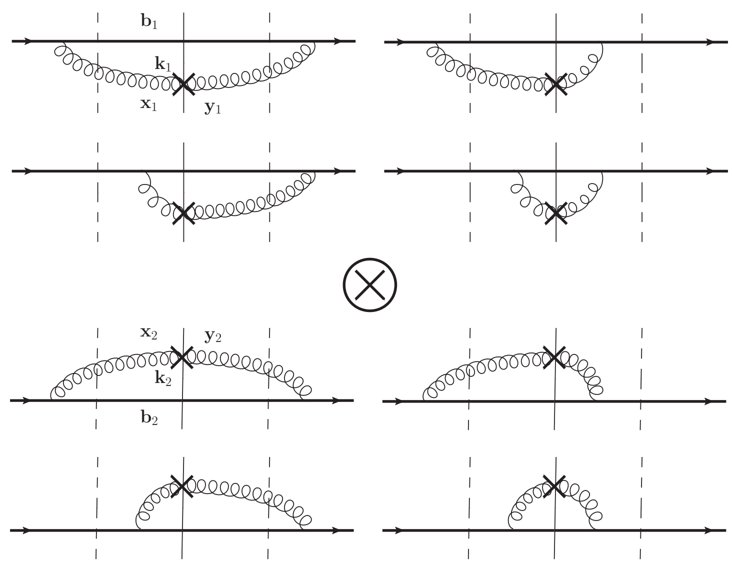


These contributions to two-gluon production contain cross-talk between the emissions from different nucleons.

Two-gluon production cross section

- "Squaring" the single gluon production cross section yields

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2\, C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$
$$\times\ \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\ \frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$
$$\times\ \Bigg\langle\left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{y}_1}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{b}_1}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_1}U^\dagger_{\mathbf{y}_1}]\ +\ 1\right)$$
$$\times\ \left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{y}_2}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{b}_2}]\ -\ \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_2}U^\dagger_{\mathbf{y}_2}]\ +\ 1\right)\Bigg\rangle$$



(cf. Kovner & Lublinsky, '12)

Two-gluon production cross section

- The "crossed" diagrams give

$$\frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2$$

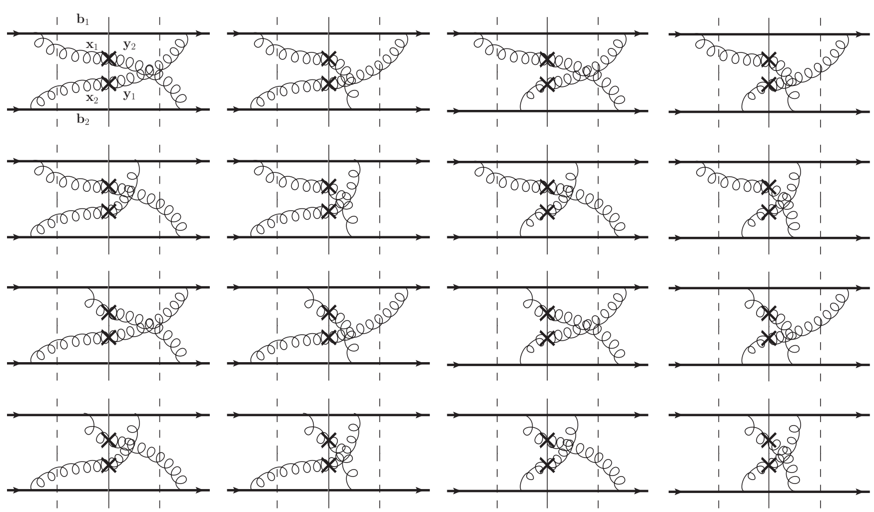
$$\times \left[e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} + e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)+i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} \right]$$

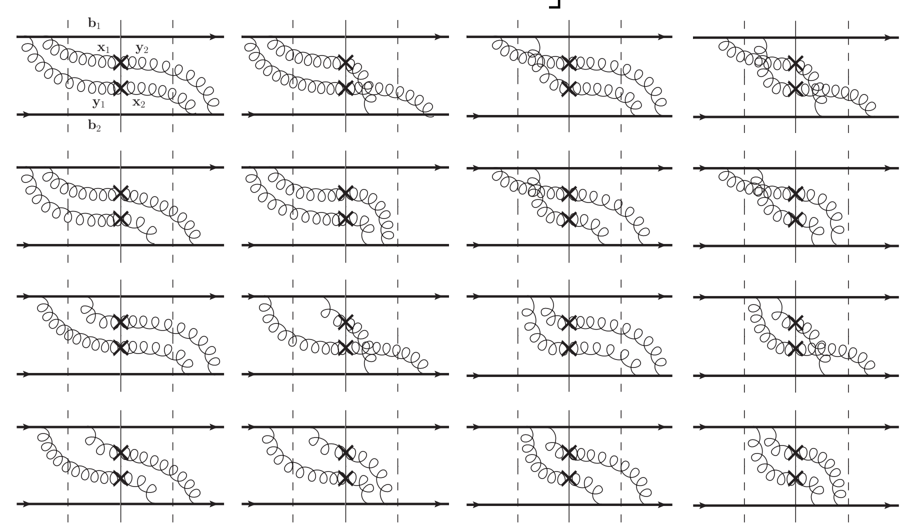
$$\times \frac{16\,\alpha_s^2}{\pi^2}\,\frac{C_F}{2N_c}\,\frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}\,\frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}$$

$$\times \Big[Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) + S_G(\mathbf{x}_1,\mathbf{y}_1) - Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{y}_2)$$

$$+ Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{b}_2) + Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{x}_1,\mathbf{b}_1) - Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) + Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2)$$

$$+ Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{b}_1,\mathbf{y}_1) + S_G(\mathbf{x}_2,\mathbf{y}_2) - S_G(\mathbf{x}_2,\mathbf{b}_2) - S_G(\mathbf{b}_2,\mathbf{y}_2) + 1 \Big]$$





Two-gluon production cross section

- The "crossed" diagrams give

$$
\frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2
$$

$$
\times \left[e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} + e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)+i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} \right]
$$

$$
\times \frac{16\,\alpha_s^2}{\pi^2}\,\frac{C_F}{2N_c}\,\frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}\,\frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}
$$

$$
\times \Big[Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) + S_G(\mathbf{x}_1,\mathbf{y}_1) - Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{y}_2)
$$

$$
+ Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{b}_2) + Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{x}_1,\mathbf{b}_1) - Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) + Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2)
$$

$$
+ Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{b}_1,\mathbf{y}_1) + S_G(\mathbf{x}_2,\mathbf{y}_2) - S_G(\mathbf{x}_2,\mathbf{b}_2) - S_G(\mathbf{b}_2,\mathbf{y}_2) + 1 \Big]
$$

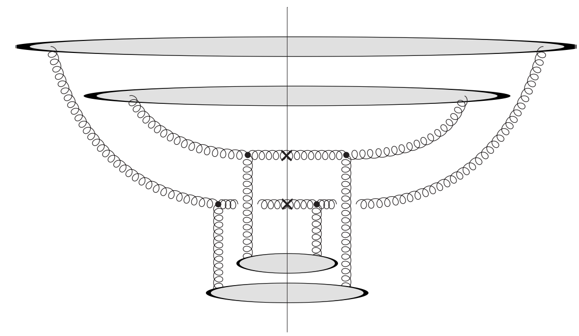
- We introduced the adjoint color-dipole and color quadrupole amplitudes:

$$
S_G(\mathbf{x}_1,\mathbf{x}_2,y) \equiv \frac{1}{N_c^2-1}\,\left\langle Tr[U_{\mathbf{x}_1} U^\dagger_{\mathbf{x}_2}] \right\rangle
$$

$$
Q(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4) \equiv \frac{1}{N_c^2-1}\,\left\langle Tr[U_{\mathbf{x}_1} U^\dagger_{\mathbf{x}_2} U_{\mathbf{x}_3} U^\dagger_{\mathbf{x}_4}] \right\rangle
$$

Two-gluon production: properties

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2\, C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$
$$\times\, \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\,\frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$
$$\times\,\Big\langle\Big(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{y}_1}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{b}_1}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_1}U^\dagger_{\mathbf{y}_1}] + 1\Big)$$
$$\times\,\Big(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{y}_2}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{b}_2}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_2}U^\dagger_{\mathbf{y}_2}] + 1\Big)\Big\rangle$$



- If we expand the interaction with the target to the lowest non-trivial order, one reproduced the contribution of the 'glasma' graphs:

$$\left.\frac{d\sigma_{corr}}{d^2k_1 dy_1 d^2k_2 dy_2}\right|_{LO} = \frac{\alpha_s^2}{4\,\pi^4}\int d^2B\, d^2b\, [T_1(\mathbf{B}-\mathbf{b})]^2\,\frac{Q_{s0}^4(\mathbf{b})}{\mathbf{k}_1^2\,\mathbf{k}_2^2}\int_\Lambda \frac{d^2l}{(\mathbf{l}^2)^2}\left[\frac{1}{(\mathbf{k}_1-\mathbf{l})^2\,(\mathbf{k}_2+\mathbf{l})^2}+\frac{1}{(\mathbf{k}_1-\mathbf{l})^2\,(\mathbf{k}_2-\mathbf{l})^2}\right]$$

away-side correlations

$$\sim\frac{1}{(\mathbf{k}_1+\mathbf{k}_2)^2}$$

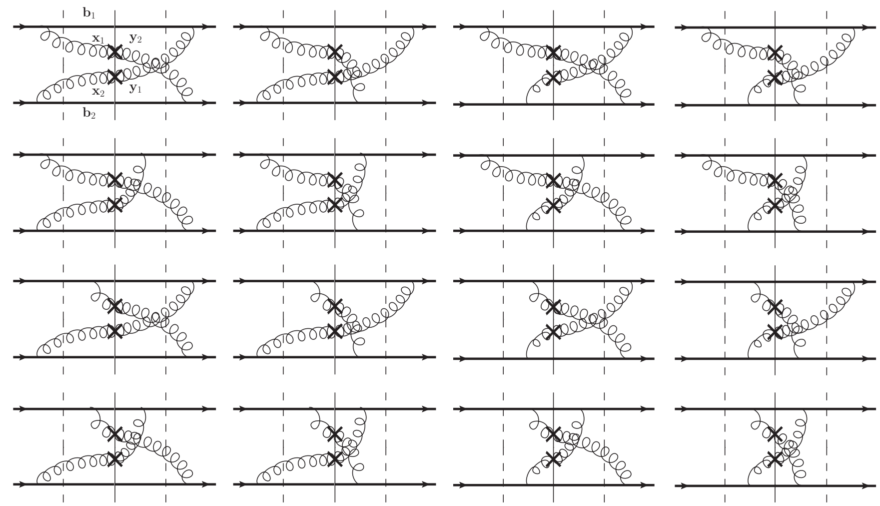
near-side correlations

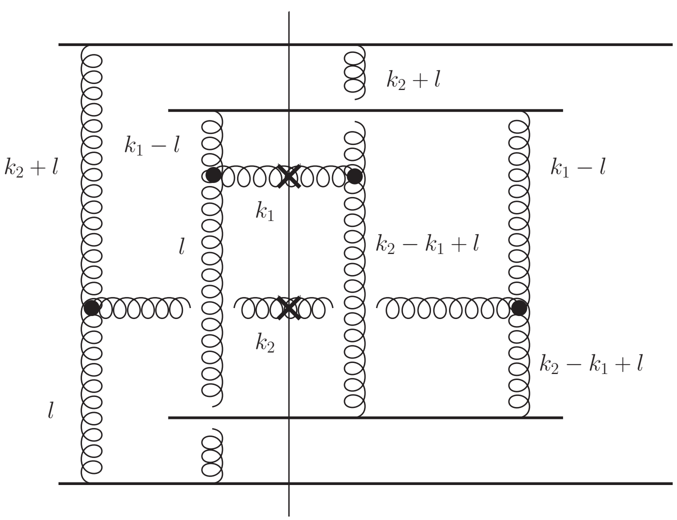
$$\sim\frac{1}{(\mathbf{k}_1-\mathbf{k}_2)^2}$$

(cf. Dumitru, Gelis, McLerran, Venugopalan '08)

Two-gluon production: properties

- Crossed diagrams at lowest nontrivial order



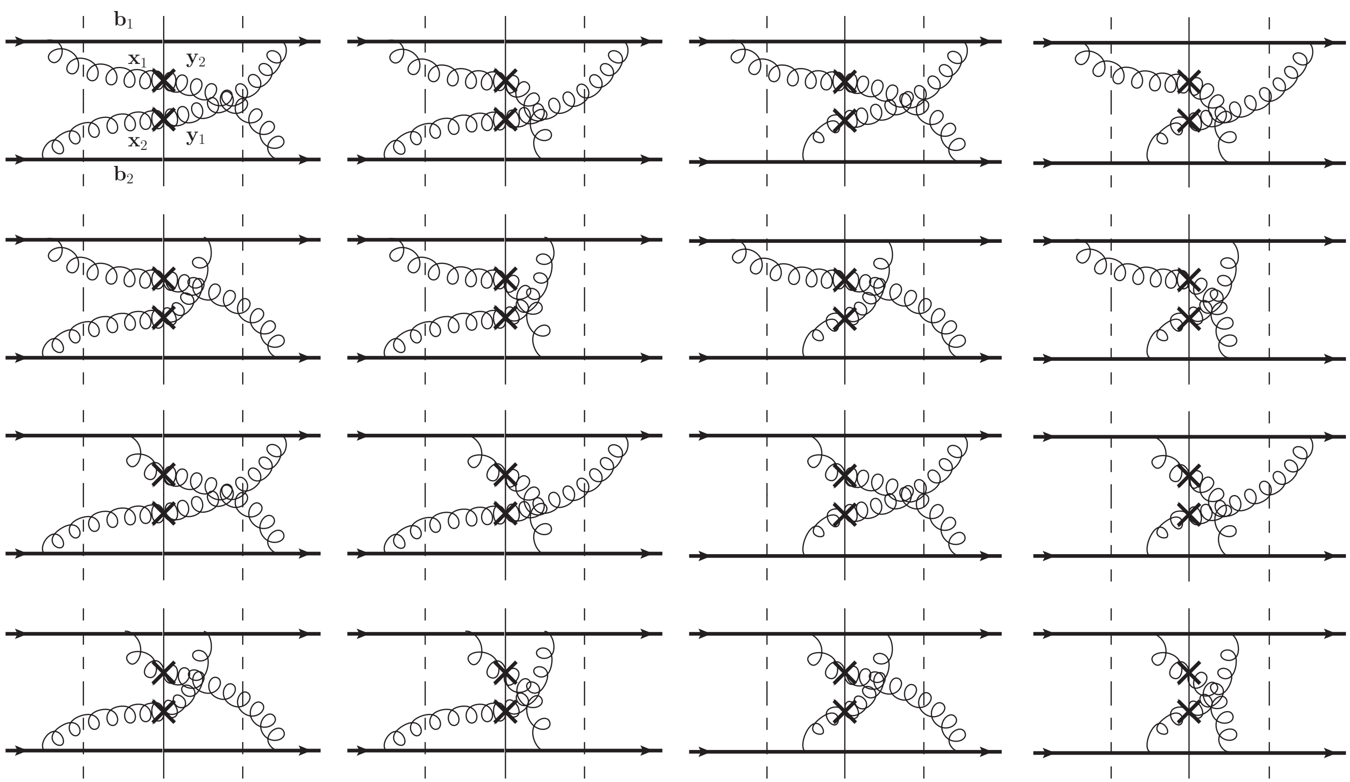


$$\sim \cos 2\Delta\phi \left[\frac{1}{(\mathbf{k}_1-\mathbf{k}_2)^2}+\frac{1}{(\mathbf{k}_1+\mathbf{k}_2)^2}\right]$$

Stronger correlations!

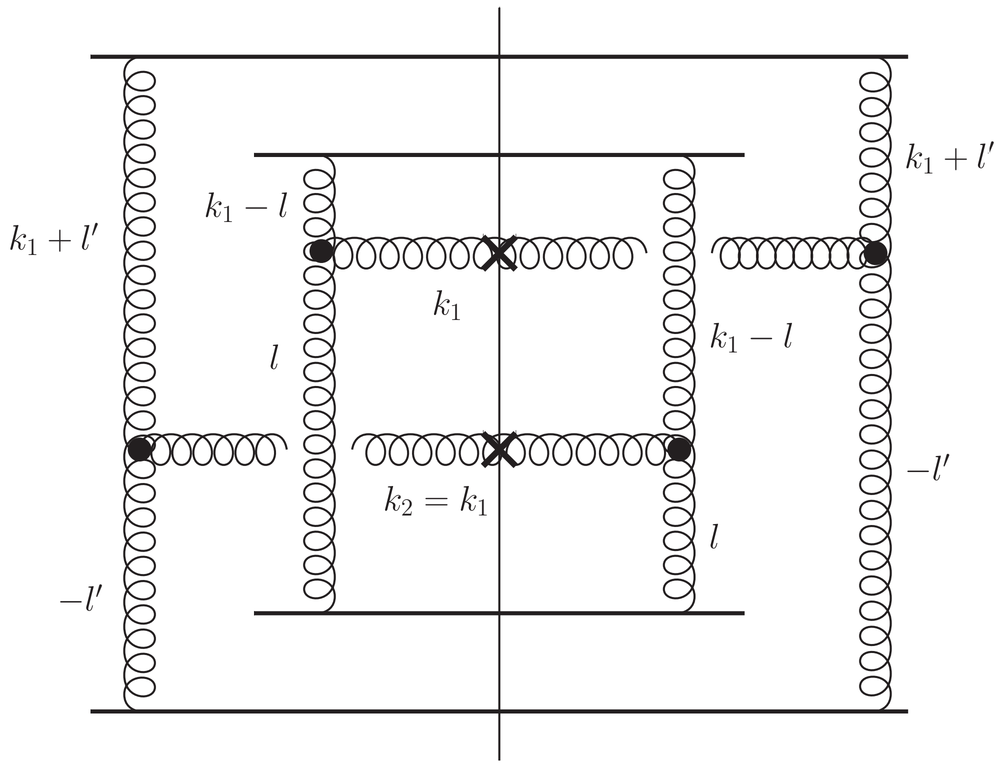
HBT diagrams

- There is another contribution coming from the "crossed" diagrams



HBT diagrams

- They give HBT correlations (with R_{long} =0 due to Lorentz contraction)



$$\sim \delta^2(\mathbf{k}_1 - \mathbf{k}_2)$$

- Just like the standard HBT correlations

$$|\Psi_1(\mathbf{k}_1)\,\Psi_2(\mathbf{k}_2) + \Psi_1(\mathbf{k}_2)\,\Psi_2(\mathbf{k}_1)|^2 \to \Psi_1(\mathbf{k}_1)\,\Psi_2(\mathbf{k}_2)\,\Psi_1^*(\mathbf{k}_2)\,\Psi_2^*(\mathbf{k}_1) + c.c. + \ldots$$

- Possibly fragmentation would break phase coherence making these perturbative HBT correlations not observable.

Back-to-back HBT?

- Note that all our formulas are symmetric under

$$\mathbf{k}_2 \to -\mathbf{k}_2$$

- Therefore, the HBT correlation is accompanied by the identical back-to-back HBT correlation

$$\sim \delta^2(\mathbf{k}_1 + \mathbf{k}_2)$$

- Note again that this correlation may be destroyed in hadronization.

Two-gluon production: properties

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2\, C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$
$$\times \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\;\frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$
$$\times \left\langle \left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{y}_1}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_1}U^\dagger_{\mathbf{b}_1}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_1}U^\dagger_{\mathbf{y}_1}] + 1\right)\right.$$
$$\left.\times \left(\frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{y}_2}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{x}_2}U^\dagger_{\mathbf{b}_2}] - \frac{1}{N_c^2-1}\,Tr[U_{\mathbf{b}_2}U^\dagger_{\mathbf{y}_2}] + 1\right)\right\rangle$$

- The cross section is symmetric under (ditto for the “crossed” term)

$\mathbf{k}_1 \leftrightarrow \mathbf{k}_2$ (just coordinate relabeling)

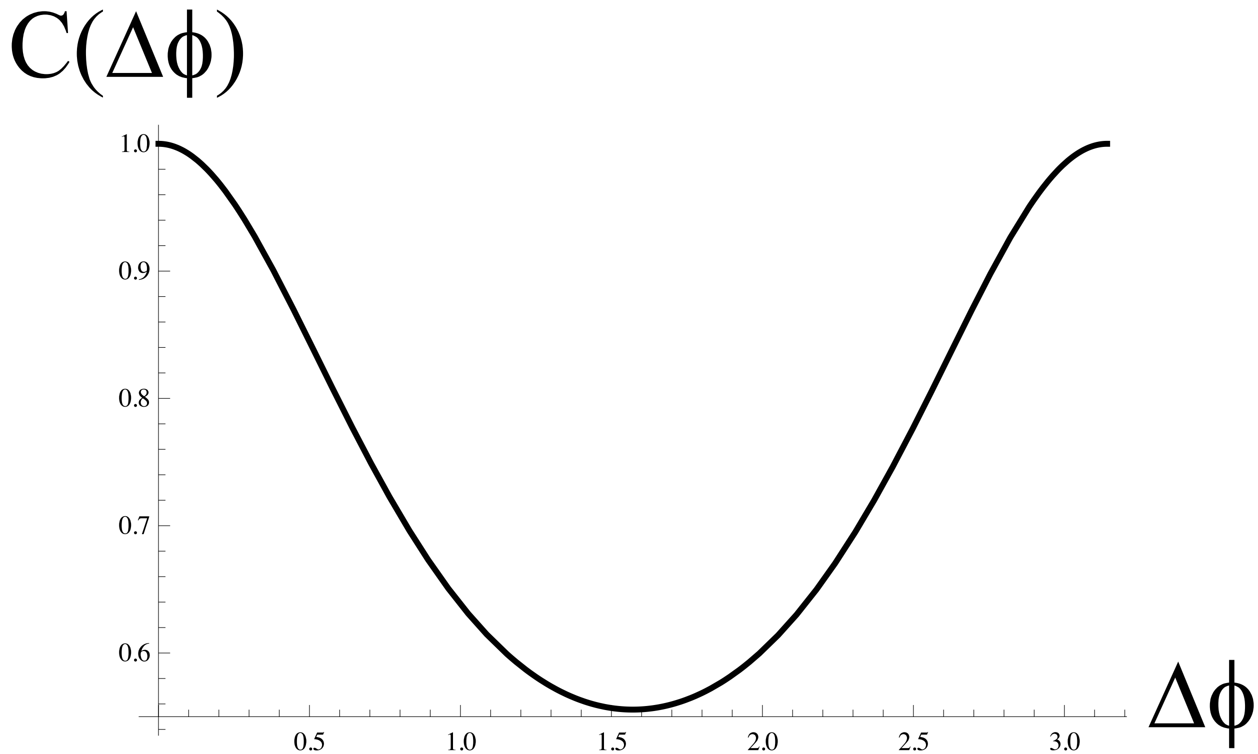
$\mathbf{k}_2 \to -\mathbf{k}_2$ as $Tr\left[U_{\mathbf{x}}\,U^\dagger_{\mathbf{y}}\right] = Tr\left[U_{\mathbf{y}}\,U^\dagger_{\mathbf{x}}\right]$

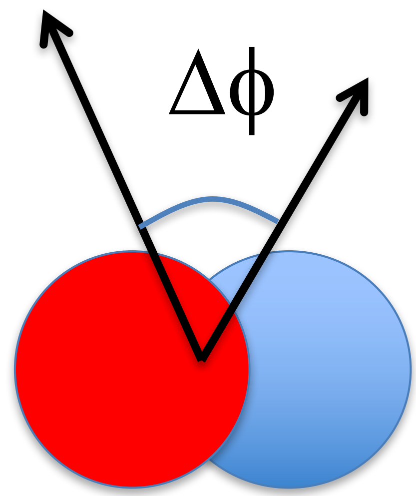
- Hence the correlations generate only even azimuthal harmonics

$$\sim \cos 2\,n\,(\phi_1-\phi_2)$$

Correlation function

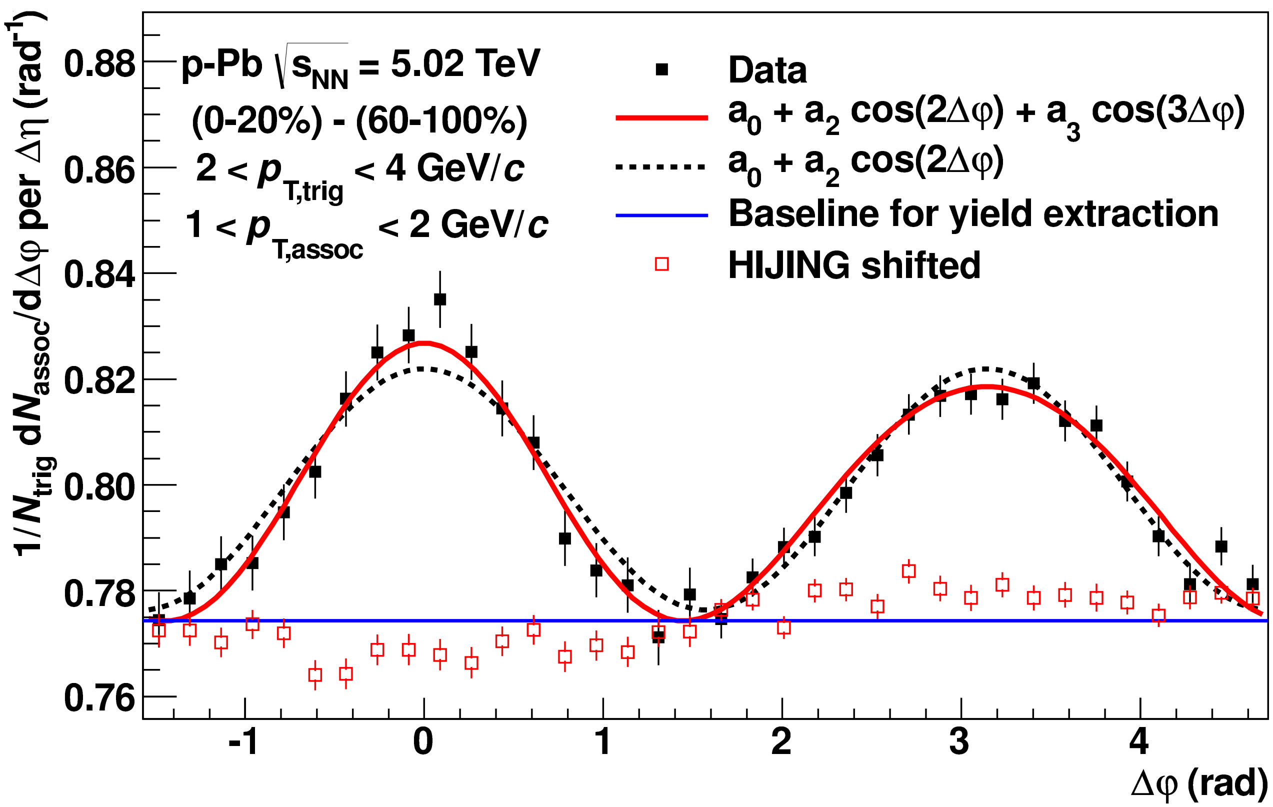
May look like this (a toy model; two particles far separated in rapidity, jets subtracted, pA and AA):





Dumitru, Gelis, McLerran, Venugopalan ’08; Kovner, Lublinsky ‘10; Yu.K., D. Wertepny ‘12; Lappi, Srednyak, and Venugopalan ‘09

LHC p+Pb data from ALICE



- These are high-multiplicity collisions: it is possible that quark-gluon plasma is created in those collisions, with the hydrodynamics contributing to these correlations.

- Saturation approach is lacking the odd harmonics, like cos (3 $\Delta\phi$) , etc. Can they be generated by corrections to the leading-order CGC calculation?

The Problem:
Can we obtain odd harmonics in the saturation approach?

The Problem

- Can we get odd harmonics in the saturation approach?

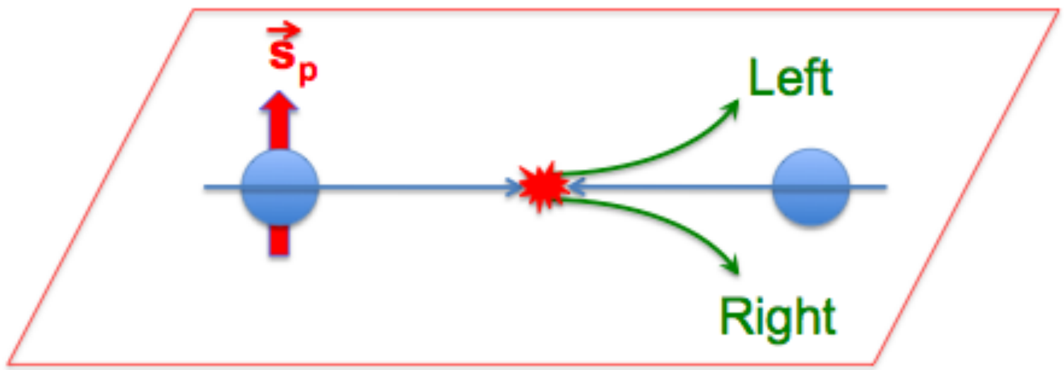
- Several people believed a phase difference between the amplitude and the cc amplitude is needed for odd harmonics – more on this later. (e.g. Kovner, Lublinsky).

- I thought the solution was in calculating higher-order corrections to the classical gluon production picture.
- This is how one gets the single transverse spin asymmetry.

Single Transverse Spin Asymmetry

- Consider transversely polarized proton scattering on an unpolarized proton or nucleus.

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, ...) + X$$

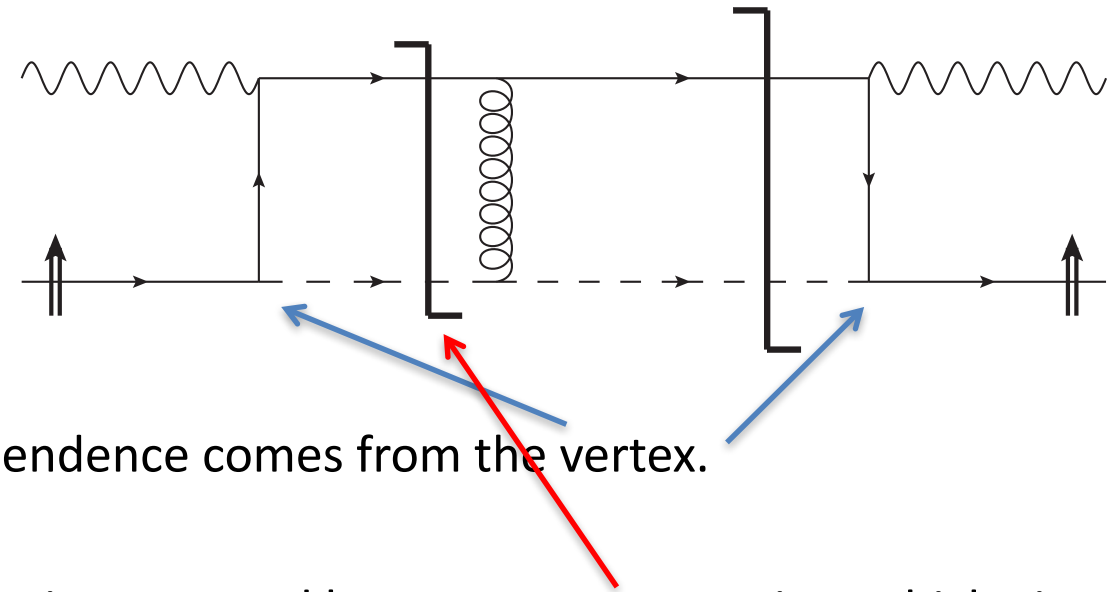


- Single Transverse Spin Asymmetry (STSA) is defined by

$$A_N(\mathbf{k}) \equiv \frac{\frac{d\sigma^\uparrow}{d^2k\,dy} - \frac{d\sigma^\downarrow}{d^2k\,dy}}{\frac{d\sigma^\uparrow}{d^2k\,dy} + \frac{d\sigma^\downarrow}{d^2k\,dy}} = \frac{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) - \frac{\mathbf{d}\sigma^\uparrow}{\mathbf{d^2k\,dy}}(-\mathbf{k})}{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) + \frac{\mathbf{d}\sigma^\uparrow}{\mathbf{d^2k\,dy}}(-\mathbf{k})} \equiv \frac{d(\Delta\sigma)}{2\,d\sigma_{unp}}$$

STSA in SIDIS

- STSA arises from the interference diagrams between Born-level and the one-rescattering graphs:



- Spin-dependence comes from the vertex.

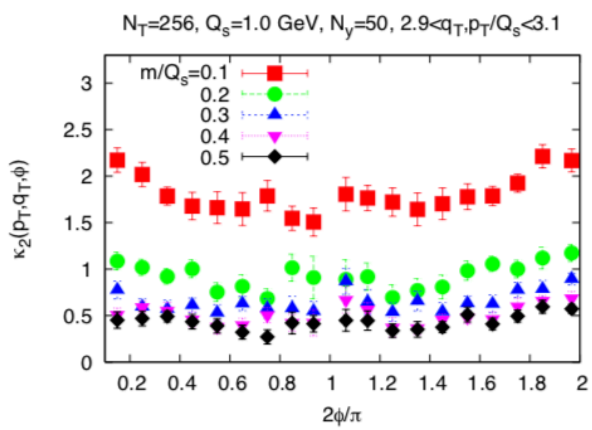
- The phase is generated by an extra rescattering, which gives the amplitude an Im part represented by the second "cut".

Brodsky, Hwang, YK, Schmidt, Sievert '13

The Problem

- <u>Can we get odd harmonics in the saturation approach?</u>

- However, numerical simulations of classical gluon fields by T. Lappi, S. Srednyak and R. Venugopalan '09 appeared to give odd harmonics.



- L. McLerran and V. Skokov, '16: odd harmonics may originate in the classical MV model from higher-order interactions with the projectile. One has to go beyond the "dilute projectile" approximation.

Solution Outline

- Even harmonics arise due to the following symmetries of the above calculation:

$$\mathbf{k}_1 \leftrightarrow \mathbf{k}_2 \qquad\qquad \mathbf{k}_2 \to -\mathbf{k}_2$$

- For two gluons originating from two identical classical fields the

$$\mathbf{k}_1 \leftrightarrow \mathbf{k}_2$$

symmetry appears impossible to break (have cosines only).

- Hence, for the classical MV-model 2-gluon production, the only way to generate odd harmonics is to violate the

$$\mathbf{k}_2 \to -\mathbf{k}_2$$

symmetry.

Solution Outline

- Imagine single-gluon production which is part of a larger 2-gluon production process. The cross section is

$$\frac{d\sigma}{d^2k} \sim |M(\underline{k})|^2 = \int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\, M(\underline{x})\, M^*(\underline{y})$$

- Suppose the amplitude can be expanded in powers of the coupling,

$$M(\underline{x}) = M_1(\underline{x}) + M_3(\underline{x}) + \ldots$$

- If M_1 is the LO result, it is often purely real (or purely imaginary). Hence

$$\int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\, M_1(\underline{x})\, M_1^*(\underline{y})$$

is even under

$$\underline{k} \to -\underline{k}$$

as we saw above.

Solution Outline

- To get odd harmonics need to look at the interference between M_1 and M_3:

$$\int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\, \left[M_1(\underline{x})\, M_3^*(\underline{y}) + M_3(\underline{x})\, M_1^*(\underline{y})\right]$$

- If this term is odd under $\underline{k} \to -\underline{k}$, need

$$M_1(\underline{x})\, M_3^*(\underline{y}) + M_3(\underline{x})\, M_1^*(\underline{y}) = -M_1(\underline{y})\, M_3^*(\underline{x}) - M_3(\underline{y})\, M_1^*(\underline{x})$$

- This is easiest to satisfy by

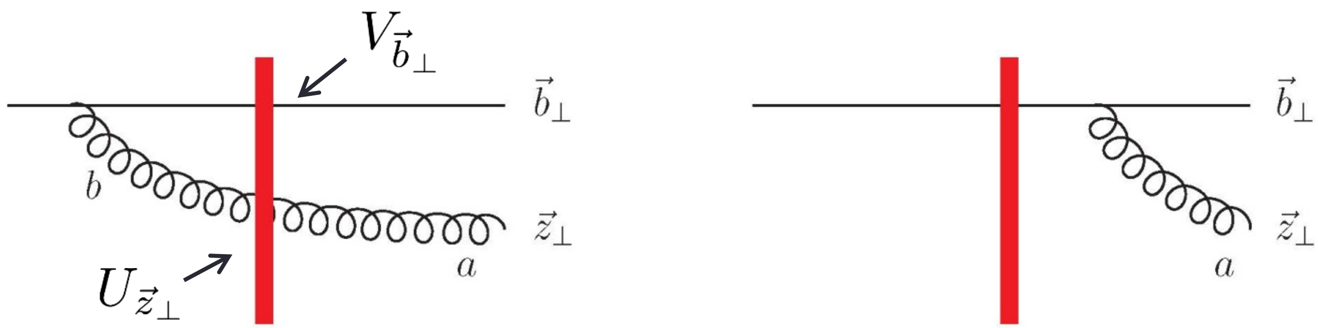
$$M_1(\underline{x})\, M_3^*(\underline{y}) = -M_3(\underline{y})\, M_1^*(\underline{x})$$

- The phases of M_1 and M_3 have to be off by π !
- All we have to do is find M_3 with a phase difference compared to M_1. This is easier said than done.

Classical Single Gluon Production in Heavy-Light Ion Collisions

G. Chirilli , YK, D. Wertepny, arXiv:1501.03106 [hep-ph]

M_1: Gluon Production Amplitude for pA Collisions



- High energy scattering between the projectile and the target is an instantaneous interaction (shockwave, red bar) at $x^+ = 0$.
- Gluon emission can happen before or after, not during.
- The projectile interacting with the target results in a power counting of

$$|M|^2 \sim \frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})(\alpha_s^2 A_T^{\frac{1}{3}})^N \qquad A_P^{\frac{1}{3}} = 1 \quad \alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

- In total the amplitude is

$$M(\vec{z}_\perp, \vec{b}_\perp) = \frac{i\,g}{\pi} \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_\perp)}{|\vec{z}_\perp - \vec{b}_\perp|^2} \left[U_{\vec{z}_\perp}^{ab} - U_{\vec{b}_\perp}^{ab} \right] \left(V_{\vec{b}_\perp} t^b \right)$$

- Used the relation: $\left(t^a V_{\vec{b}_\perp} \right) = \left(V_{\vec{b}_\perp} t^b \right) U_{\vec{b}_\perp}^{ab}$

Heavy-Light Collision Case

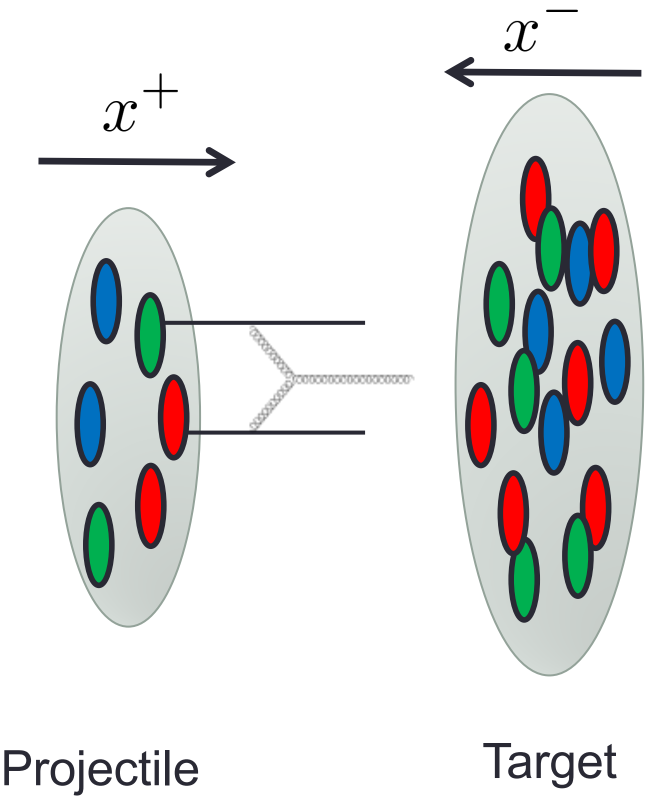
- Target nucleus has same power counting as before.
- Projectile has many nucleons, but not too many such that

$$A_P \gg 1$$

$$\alpha_s \ll \alpha_s^2 A_P^{\frac{1}{3}} \lesssim 1 \qquad \alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

- Two nucleons from projectile.
- Power counting for the cross section:

$$\sim \frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})^2(\alpha_s^2 A_T^{\frac{1}{3}})^N \sim \alpha_s^3\, A_P^{1/3}$$



Power Counting

- For the single-inclusive gluon production in AA collisions in the classical MV model we have

$$\frac{d\sigma}{d^2k\, d^2b\, d^2B} = \frac{1}{\alpha_s}\, f\left(\alpha_s^2\, A_1^{1/3}, \alpha_s^2\, A_2^{1/3}\right)$$

- The function f is known only numerically.

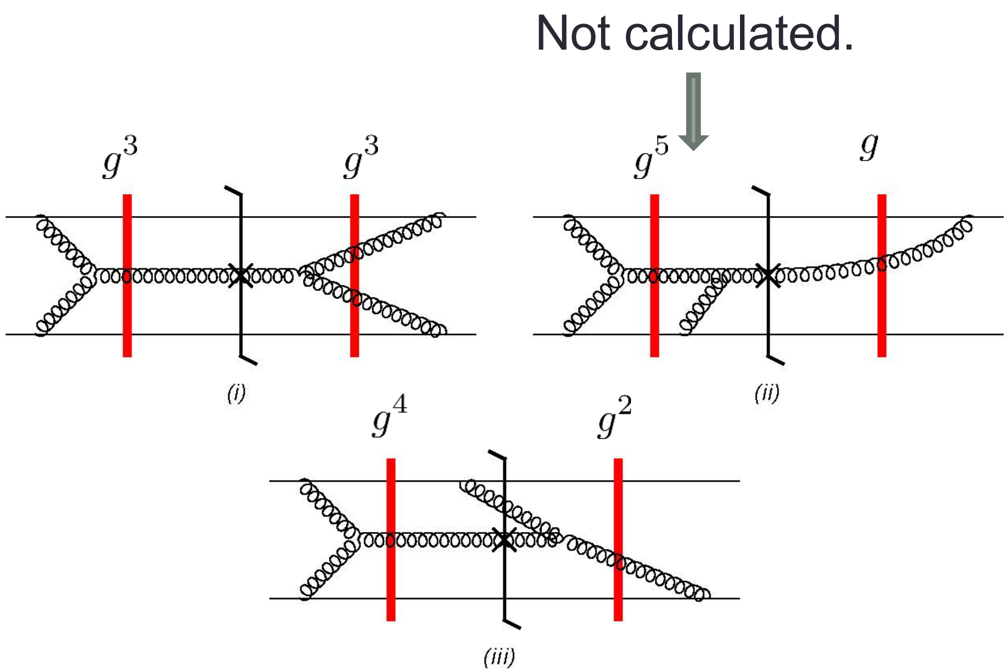
- If we expand in the interaction with one of the nuclei (the projectile) we get

$$\frac{d\sigma}{d^2k\, d^2b\, d^2B} = \frac{1}{\alpha_s}\left[\alpha_s^2\, A_1^{1/3}\ f_1\left(\alpha_s^2\, A_2^{1/3}\right) + \left(\alpha_s^2\, A_1^{1/3}\right)^2\ f_2\left(\alpha_s^2\, A_2^{1/3}\right) + \ldots\right]$$

This is pA.
(A. Mueller, YK, '97)

This is what we are trying to calculate here.

Types of Diagrams



- Diagrams have two quarks from the projectile and are order g^6.
- Huge number of diagrams.
- Diagrams can be separated into three classes:

i) Square of order-g^3 amplitudes

ii) Interference between order-g^5 and order-g amplitudes

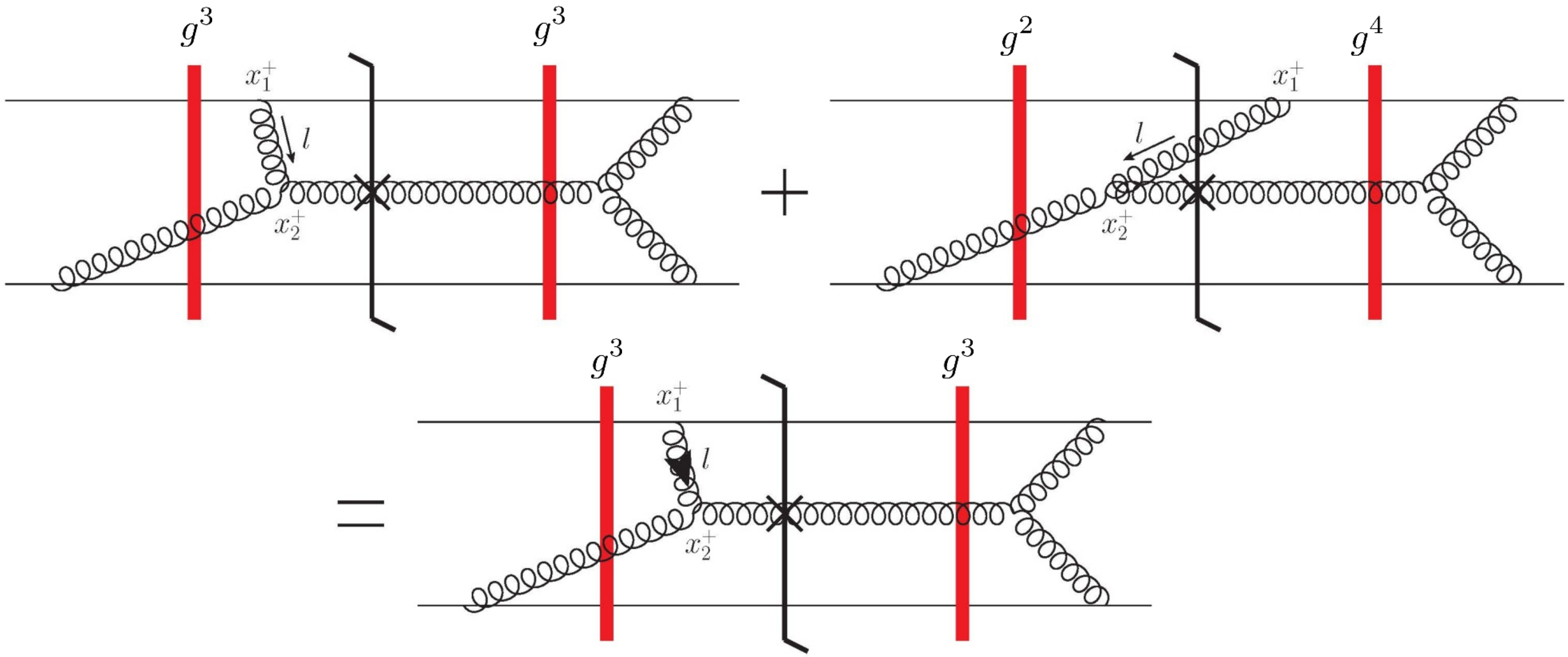
iii) Interference between order-g^4 and order-g^2 amplitudes

- These can be combined together in various ways to reduce the number of diagrams.
- Light-cone gauge,

$$\eta_\mu A^\mu = A^+ = 0$$

$$\frac{-i\, D_{\mu\nu}(l)}{l^2 + i\,\epsilon} \qquad \text{where} \qquad D_{\mu\nu}(l) = g_{\mu\nu} - \frac{1}{\eta \cdot l}(\eta_\mu\, l_\nu + \eta_\nu\, l_\mu)$$

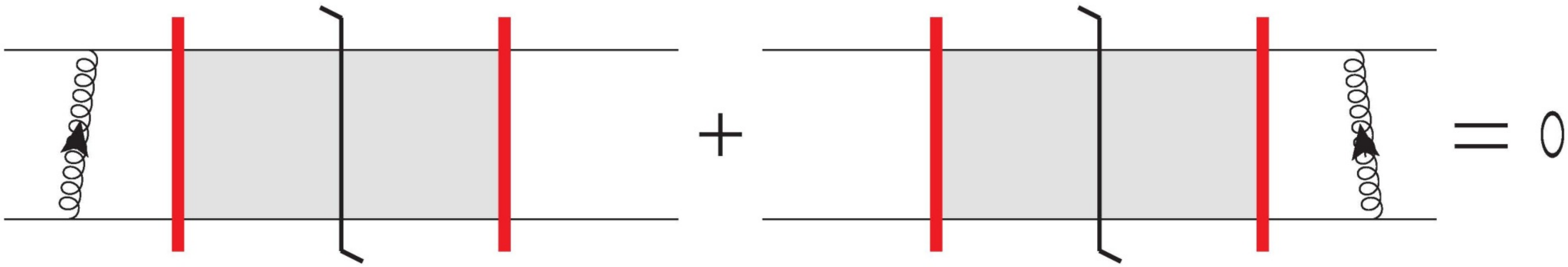
Retarded Green Function



- Adding the top two diagrams turns the propagator into a retarded propagator, represented by the arrow.

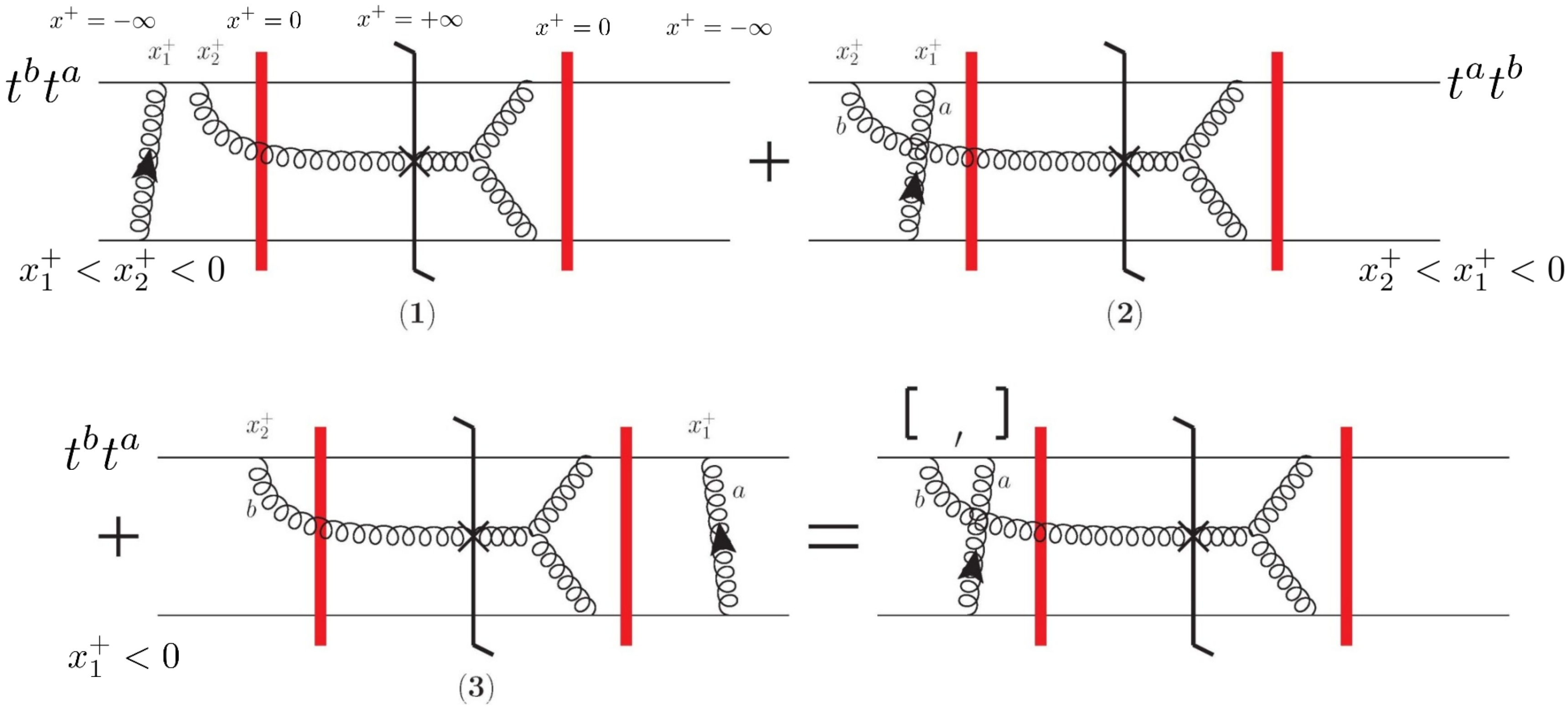
$$\frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon} + 2\pi\,\theta(-l^+)\,\delta(l^2)\,D_{\mu\nu}(l) = \frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon\,l^+}$$

Cancellations



- Shaded region represents any late-time interaction.
- Moving the retarded gluon propagator across the cut gives rise to a minus sign.
- The sum of the diagrams is zero.

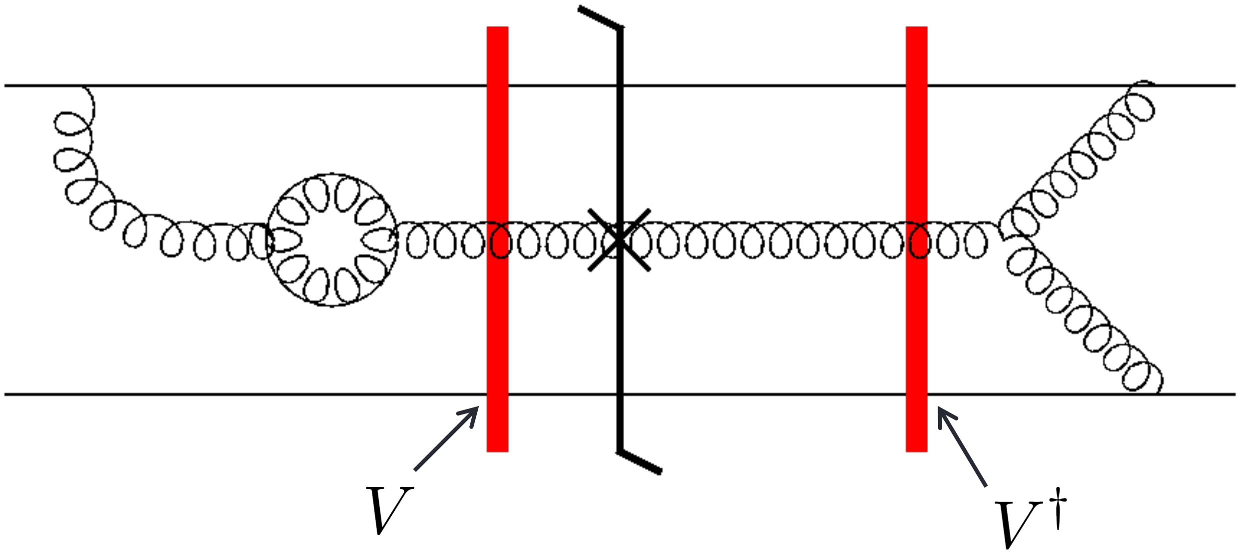
Commutators



- Using the cancellation shown previously diagrams (1), (2), and (3) can be combined into a single diagram, diagram (2), with the color factor on the quark line replaced by a commutator, denoted by the square brackets:

$$t^a t^b \rightarrow [t^a, t^b]$$

No Quantum Contributions

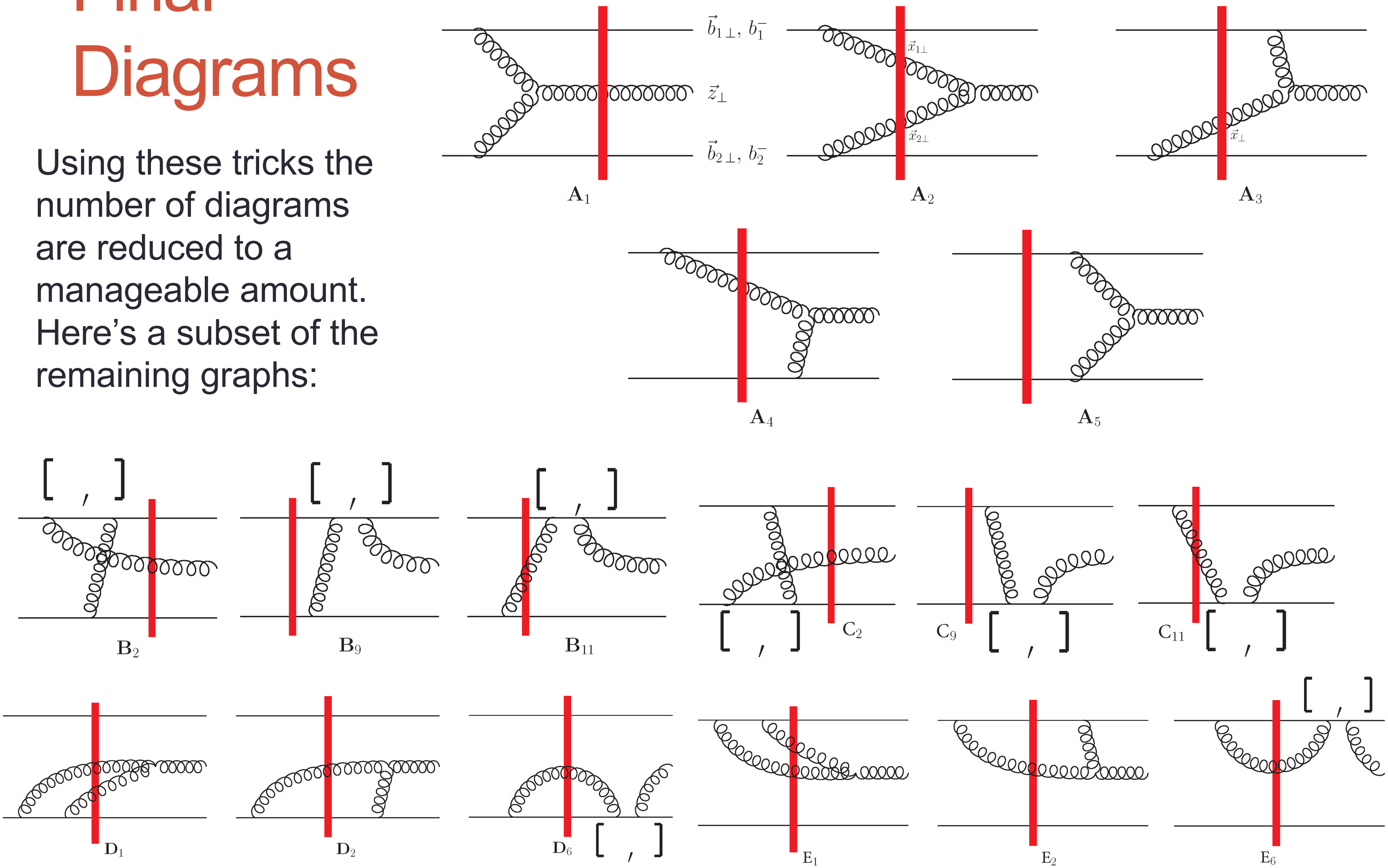


- Quantum corrections go away at this order.
- Left with classical fields.
- Zero due to color averaging of quark two.

$$tr[t^a V^\dagger V] = tr[t^a] = 0$$

Final Diagrams

Using these tricks the number of diagrams are reduced to a manageable amount. Here's a subset of the remaining graphs:



Results: Amplitude – A, B, and C graphs

phase difference!

M_3

$$
\sum_i A_i + \sum_i B_i + \sum_i C_i
$$

$$
= -\frac{g^3}{4\pi^4} \int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{1\perp} - \vec{b}_{1\perp})}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right.
$$

$$
\left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right] f^{abc} \left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{1\perp}} \right] \left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2
$$

$$
+ \frac{i\, g^3}{4\pi^3} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \int d^2x \left[U^{bd}_{\vec{b}_{1\perp}} \left(U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right) \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \right.
$$

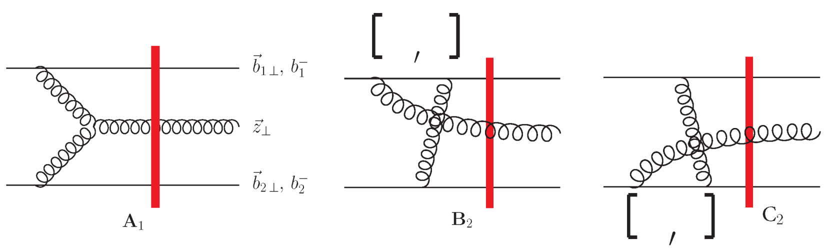
$$
\left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right)
$$

$$
- \left(U^{bd}_{\vec{x}_\perp} - U^{bd}_{\vec{b}_{1\perp}} \right) U^{ce}_{\vec{b}_{2\perp}} \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right.
$$

$$
\left. \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \right]
$$

$$
- \frac{i\, g^3}{4\pi^2} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \left[\left(U^{bd}_{\vec{z}_\perp} - U^{bd}_{\vec{b}_{1\perp}} \right) U^{ce}_{\vec{b}_{2\perp}} \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}|\, \Lambda} - U^{bd}_{\vec{b}_{1\perp}} \left(U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right) \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{1\perp}|\, \Lambda} \right]
$$

$$
- \frac{i\, g^3}{4\pi^3} \int d^2x \left[U^{ab}_{x_\perp} - U^{ab}_{z_\perp} \right] f^{bde} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \operatorname{Sign}(b_2^- - b_1^-)
$$



see also I. Balitsky, '04

$$
\vec{x}_\perp \times \vec{y}_\perp = x_1\, y_2 - x_2\, y_1
$$

$$
\Lambda = \text{IR cutoff}
$$

Results: Amplitude – D graphs

$$\sum_i D_i = -\frac{g^3}{8\,\pi^4}\int d^2x_1\, d^2x_2\, \delta[(\vec z_\perp - \vec x_{1\perp})\times(\vec z_\perp - \vec x_{2\perp})]\left[\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec x_{2\perp}-\vec x_{1\perp})}{|\vec x_{2\perp}-\vec x_{1\perp}|^2}\,\frac{\vec x_{1\perp}-\vec b_{2\perp}}{|\vec x_{1\perp}-\vec b_{2\perp}|^2}\cdot\frac{\vec x_{2\perp}-\vec b_{2\perp}}{|\vec x_{2\perp}-\vec b_{2\perp}|^2}\right.$$

$$\left.-\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec x_{1\perp}-\vec b_{2\perp})}{|\vec x_{1\perp}-\vec b_{2\perp}|^2}\,\frac{\vec z_\perp-\vec x_{1\perp}}{|\vec z_\perp-\vec x_{1\perp}|^2}\cdot\frac{\vec x_{2\perp}-\vec b_{2\perp}}{|\vec x_{2\perp}-\vec b_{2\perp}|^2}+\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec x_{2\perp}-\vec b_{2\perp})}{|\vec x_{2\perp}-\vec b_{2\perp}|^2}\,\frac{\vec x_{1\perp}-\vec b_{2\perp}}{|\vec x_{1\perp}-\vec b_{2\perp}|^2}\cdot\frac{\vec z_\perp-\vec x_{2\perp}}{|\vec z_\perp-\vec x_{2\perp}|^2}\right]$$

$$\times\, f^{abc}\left[U^{bd}_{\vec x_{1\perp}}-U^{bd}_{\vec b_{2\perp}}\right]\left[U^{ce}_{\vec x_{2\perp}}-U^{ce}_{\vec b_{2\perp}}\right]\left(V_{\vec b_{1\perp}}\right)_1\left(V_{\vec b_{2\perp}}\,t^e\,t^d\right)_2$$

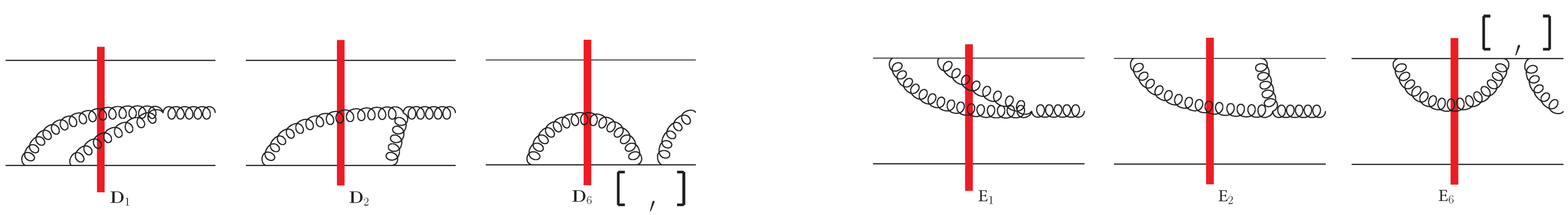
$$+\frac{i\,g^3}{4\,\pi^3}\int d^2x\, f^{abc}\,U^{bd}_{\vec b_{2\perp}}\left[U^{ce}_{\vec x_\perp}-U^{ce}_{\vec b_{2\perp}}\right]\left(V_{\vec b_{1\perp}}\right)_1\left(V_{\vec b_{2\perp}}\,t^e\,t^d\right)_2\left(\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec x_\perp)}{|\vec z_\perp-\vec x_\perp|^2}\,\frac{1}{|\vec x_\perp-\vec b_{2\perp}|^2}\right.$$

$$\left.-\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{2\perp})}{|\vec z_\perp-\vec b_{2\perp}|^2}\,\frac{\vec z_\perp-\vec x_\perp}{|\vec z_\perp-\vec x_\perp|^2}\cdot\frac{\vec x_\perp-\vec b_{2\perp}}{|\vec x_\perp-\vec b_{2\perp}|^2}-\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{2\perp})}{|\vec z_\perp-\vec b_{2\perp}|^2}\,\frac{1}{|\vec x_\perp-\vec b_{2\perp}|^2}\right)$$

$$+\frac{i\,g^3}{4\,\pi^2}\,f^{abc}\,U^{bd}_{\vec b_{2\perp}}\left[U^{ce}_{\vec z_\perp}-U^{ce}_{\vec b_{2\perp}}\right]\left(V_{\vec b_{1\perp}}\right)_1\left(V_{\vec b_{2\perp}}\,t^e\,t^d\right)_2\,\frac{\vec\epsilon_\perp^{\,\lambda *}\cdot(\vec z_\perp-\vec b_{2\perp})}{|\vec z_\perp-\vec b_{2\perp}|^2}\,\ln\frac{1}{|\vec z_\perp-\vec b_{2\perp}|\,\Lambda}$$

D_3

- To get the E graph results switch quark 1 with quark 2 (1 ↔2)



Compare this with the dilute projectile (pA) amplitude!

$$M(\vec{z}_\perp, \vec{b}_\perp) = \frac{i\, g}{\pi} \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_\perp)}{|\vec{z}_\perp - \vec{b}_\perp|^2} \left[U^{ab}_{\vec{z}_\perp} - U^{ab}_{\vec{b}_\perp} \right] \left(V_{\vec{b}_\perp} t^b \right)$$

Odd Harmonics

YK, V. Skokov, arXiv:1802.08166 [hep-ph]

Power Counting

- For the double-inclusive gluon production in AA collisions in the classical MV model we have

$$\frac{d\sigma}{d^2k_1\, d^2b_1\, d^2k_2\, d^2b_2\, d^2B} = \frac{1}{\alpha_s^2}\, h\left(\alpha_s^2\, A_1^{1/3}, \alpha_s^2\, A_2^{1/3}\right)$$

- The function h is not known numerically.

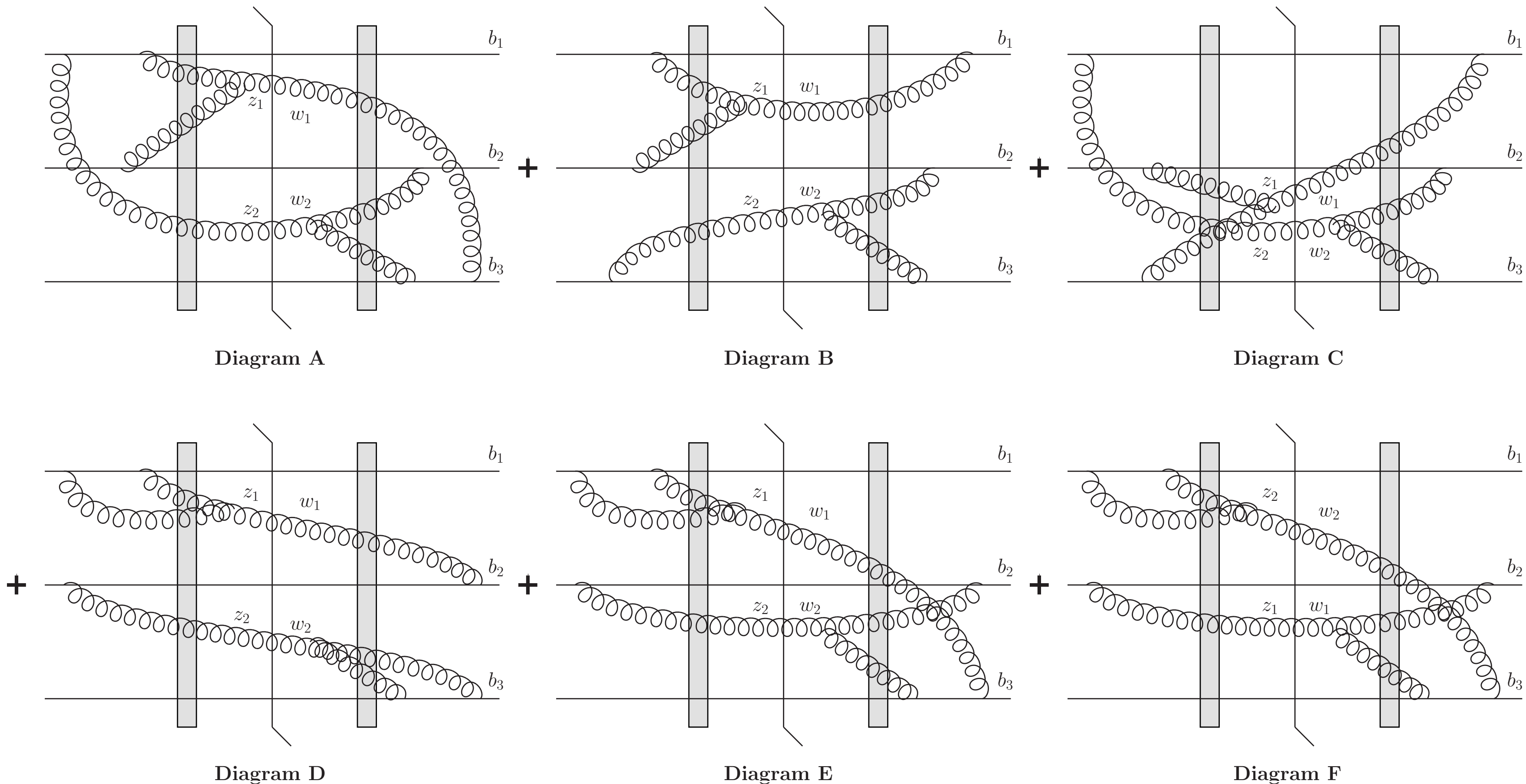
- If we expand in the interaction with one of the nuclei (the projectile) we get

$$\frac{d\sigma}{d^2k_1\, d^2b_1\, d^2k_2\, d^2b_2\, d^2B} = \frac{1}{\alpha_s^2}\, \left[\left(\alpha_s^2\, A_1^{1/3}\right)^2\, h_1\left(\alpha_s^2\, A_2^{1/3}\right) + \left(\alpha_s^2\, A_1^{1/3}\right)^3\, h_2\left(\alpha_s^2\, A_2^{1/3}\right) + \ldots\right]$$

Kovner, Lublinsky, ‘12; Wertepny, YK, ’12 (above); even harmonics only

Here we are trying to calculate the odd harmonic part of this term.

The diagrams



remember: we need $\int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})} \left[M_1(\underline{x})\, M_3^*(\underline{y}) + M_3(\underline{x})\, M_1^*(\underline{y})\right]$

repeated twice (for 2 produced gluons)

The answer

- The part of the cross section giving odd harmonics can be calculated this way to give

$$\frac{d\sigma_{odd}}{d^2k_1\, dy_1\, d^2k_2\, dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, d^2b_3\, T_1(\underline{B} - \underline{b}_1)\, T_1(\underline{B} - \underline{b}_2)\, T_1(\underline{B} - \underline{b}_3)$$
$$\times\, d^2z_1\, d^2w_1\, d^2z_2\, d^2w_2\, e^{-i\underline{k}_1\cdot(\underline{z}_1-\underline{w}_1)-i\underline{k}_2\cdot(\underline{z}_2-\underline{w}_2)}\, \langle \hat{A} \rangle_{\rho_T}$$

where

$$\begin{aligned}
\hat{A} = {} & \underline{M}_3(\underline{z}_1, \underline{b}_1, \underline{b}_2) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_3)\, \underline{M}_1(\underline{z}_2, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_1, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_1, \underline{b}_2)\, \underline{M}_1(\underline{z}_2, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \\
& + \underline{M}_3(\underline{z}_1, \underline{b}_1, \underline{b}_2) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_3)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_1) + \underline{M}_1(\underline{z}_1, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_1, \underline{b}_2)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \\
& + \underline{M}_3(\underline{z}_1, \underline{b}_1, \underline{b}_2) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_1)\, \underline{M}_1(\underline{z}_2, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_1, \underline{b}_2)\, \underline{M}_1(\underline{z}_2, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \\
& + \underline{M}_3(\underline{z}_1, \underline{b}_1, \underline{b}_2) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_1)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_1, \underline{b}_2)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \\
& + \underline{M}_3(\underline{z}_1, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_1)\, \underline{M}_1(\underline{z}_2, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_2, \underline{b}_3)\, \underline{M}_1(\underline{z}_2, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \\
& + \underline{M}_3(\underline{z}_1, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_1)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_1) + \underline{M}_1(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_2, \underline{b}_3)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \\
& + \underline{D}_3(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_2)\, \underline{M}_1(\underline{z}_2, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_1, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_1, \underline{b}_1)\, \underline{M}_1(\underline{z}_2, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_2, \underline{b}_3) \\
& + \underline{D}_3(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_2)\, \underline{D}_3(\underline{z}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_2) + \underline{M}_1(\underline{z}_1, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_1, \underline{b}_1)\, \underline{D}_3(\underline{z}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_2) \\
& + \underline{D}_3(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_2)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_1, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_1, \underline{b}_1)\, \underline{M}_3(\underline{z}_2, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_3) \\
& + \underline{D}_3(\underline{z}_1, \underline{b}_1) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_2)\, \underline{M}_1(\underline{z}_2, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_1, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_1, \underline{b}_1)\, \underline{M}_1(\underline{z}_2, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_2, \underline{b}_2, \underline{b}_3) \\
& + \underline{D}_3(\underline{z}_2, \underline{b}_1) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_2)\, \underline{M}_3(\underline{z}_1, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_3) + \underline{M}_1(\underline{z}_2, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_2, \underline{b}_1)\, \underline{M}_3(\underline{z}_1, \underline{b}_2, \underline{b}_3) \cdot \underline{M}_1^*(\underline{w}_1, \underline{b}_3) \\
& + \underline{D}_3(\underline{z}_2, \underline{b}_1) \cdot \underline{M}_1^*(\underline{w}_2, \underline{b}_2)\, \underline{M}_1(\underline{z}_1, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_2, \underline{b}_3) + \underline{M}_1(\underline{z}_2, \underline{b}_2) \cdot \underline{D}_3^*(\underline{w}_2, \underline{b}_1)\, \underline{M}_1(\underline{z}_1, \underline{b}_3) \cdot \underline{M}_3^*(\underline{w}_1, \underline{b}_2, \underline{b}_3).
\end{aligned}$$

The answer (continued)

- Here $$\underline{\epsilon}_\lambda^* \cdot \underline{M}_1(\underline{z}, \underline{b}) = \frac{i\, g}{\pi} \, \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{z} - \underline{b})}{|\underline{z} - \underline{b}|^2} \, \left[U_{\underline{z}}^{ab} - U_{\underline{b}}^{ab} \right] \, (V_{\underline{b}} \, t^b)$$

$$\begin{aligned} \underline{\epsilon}_\lambda^* \cdot \underline{M}_3(\underline{z}, \underline{b}_1, \underline{b}_2) = - \frac{g^3}{4\pi^4} \int d^2 x_1 \, d^2 x_2 \; \delta \left[(\underline{z} - \underline{x}_1) \times (\underline{z} - \underline{x}_2) \right] \times \\ \left[\frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_2 - \underline{x}_1)}{|\underline{x}_2 - \underline{x}_1|^2} \, \frac{\underline{x}_1 - \underline{b}_1}{|\underline{x}_1 - \underline{b}_1|^2} \cdot \frac{\underline{x}_2 - \underline{b}_2}{|\underline{x}_2 - \underline{b}_2|^2} \right. \\ \left. - \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_1 - \underline{b}_1)}{|\underline{x}_1 - \underline{b}_1|^2} \, \frac{\underline{z} - \underline{x}_1}{|\underline{z} - \underline{x}_1|^2} \cdot \frac{\underline{x}_2 - \underline{b}_2}{|\underline{x}_2 - \underline{b}_2|^2} + \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_2 - \underline{b}_2)}{|\underline{x}_2 - \underline{b}_2|^2} \, \frac{\underline{x}_1 - \underline{b}_1}{|\underline{x}_1 - \underline{b}_1|^2} \cdot \frac{\underline{z} - \underline{x}_2}{|\underline{z} - \underline{x}_2|^2} \right] \\ \times f^{abc} \left[U_{\underline{x}_1}^{bd} - U_{\underline{b}_1}^{bd} \right] \left[U_{\underline{x}_2}^{ce} - U_{\underline{b}_2}^{ce} \right] \, (V_{\underline{b}_1} \, t^d)_1 \, (V_{\underline{b}_2} \, t^e)_2 \end{aligned}$$

$$\begin{aligned} \underline{\epsilon}_\lambda^* \cdot \underline{D}_3(\underline{z}, \underline{b}_2) = - \frac{g^3}{8\pi^4} \int d^2 x_1 \, d^2 x_2 \; \delta \left[(\underline{z} - \underline{x}_1) \times (\underline{z} - \underline{x}_2) \right] \left[\frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_2 - \underline{x}_1)}{|\underline{x}_2 - \underline{x}_1|^2} \, \frac{\underline{x}_1 - \underline{b}_2}{|\underline{x}_1 - \underline{b}_2|^2} \cdot \frac{\underline{x}_2 - \underline{b}_2}{|\underline{x}_2 - \underline{b}_2|^2} \right. \\ \left. - \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_1 - \underline{b}_2)}{|\underline{x}_1 - \underline{b}_2|^2} \, \frac{\underline{z} - \underline{x}_1}{|\underline{z} - \underline{x}_1|^2} \cdot \frac{\underline{x}_2 - \underline{b}_2}{|\underline{x}_2 - \underline{b}_2|^2} + \frac{\underline{\epsilon}_\lambda^* \cdot (\underline{x}_2 - \underline{b}_2)}{|\underline{x}_2 - \underline{b}_2|^2} \, \frac{\underline{x}_1 - \underline{b}_2}{|\underline{x}_1 - \underline{b}_2|^2} \cdot \frac{\underline{z} - \underline{x}_2}{|\underline{z} - \underline{x}_2|^2} \right] \\ \times f^{abc} \left[U_{\underline{x}_1}^{bd} - U_{\underline{b}_2}^{bd} \right] \left[U_{\underline{x}_2}^{ce} - U_{\underline{b}_2}^{ce} \right] \, (V_{\underline{b}_1}) \, (V_{\underline{b}_2} \, t^e \, t^d) \end{aligned}$$

Can we get a non-zero analytic expression?

- Diagrams A alone bring in 64 terms with 3 quardupoles each, QQQ…

- At the lowest order in the interaction with the target (= 3 gluon exchanges) we can get a 'finite' & **non-zero** expression (Λ is an IR cutoff)

$$\frac{d\sigma_{odd}}{d^2k_1\, dy_1\, d^2k_2\, dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b\, [T_1(\underline{B}-\underline{b})]^3\; g^8\, Q_{s0}^6(b)\, \frac{1}{\underline{k}_1^6\, \underline{k}_2^6}$$

$$\times \left\{ \left[\frac{(\underline{k}_1^2 + \underline{k}_2^2 + \underline{k}_1 \cdot \underline{k}_2)^2}{(\underline{k}_1 + \underline{k}_2)^6} - \frac{(\underline{k}_1^2 + \underline{k}_2^2 - \underline{k}_1 \cdot \underline{k}_2)^2}{(\underline{k}_1 - \underline{k}_2)^6} \right] + \frac{10\, c^2}{(2\pi)^2}\, \frac{1}{\Lambda^2}\, \frac{\underline{k}_1 \cdot \underline{k}_2}{k_1\, k_2} + \frac{1}{4\pi}\, \frac{k_1^4}{\Lambda^4}\, \left[\delta^2(\underline{k}_1 - \underline{k}_2) - \delta^2(\underline{k}_1 + \underline{k}_2)\right] \right\}.$$

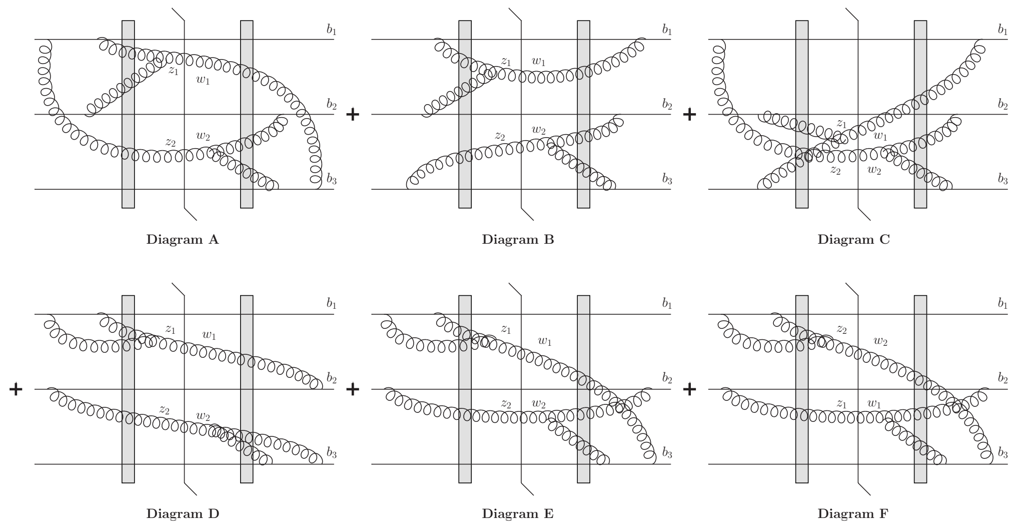
- Seems like HBT diagrams dominate due to their IR divergence.

- In arriving at the above we have also used the Golec-Biernat—Wusthoff approximation (dropped the logs),

$$N(x_\perp) = 1 - e^{-\frac{1}{4}\, x_\perp^2\, Q_s^2\, \ln\left(\frac{1}{x_\perp \Lambda}\right)} \approx 1 - e^{-\frac{1}{4}\, x_\perp^2\, Q_s^2}$$

To get odd harmonics need (at least):

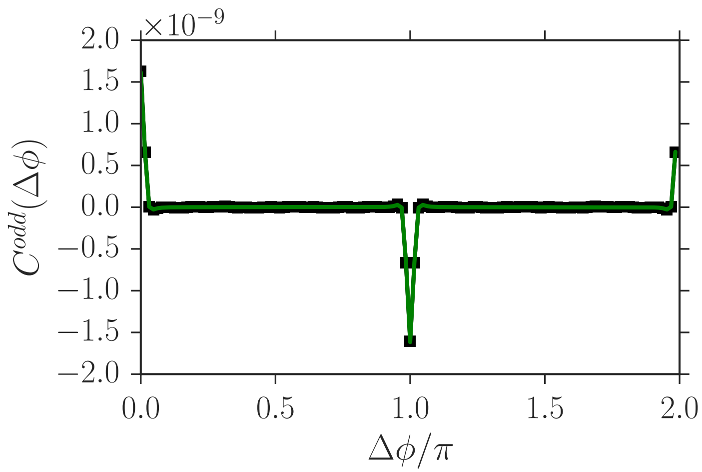
- Six gluon emissions from three sources (nucleons) in the projectile:

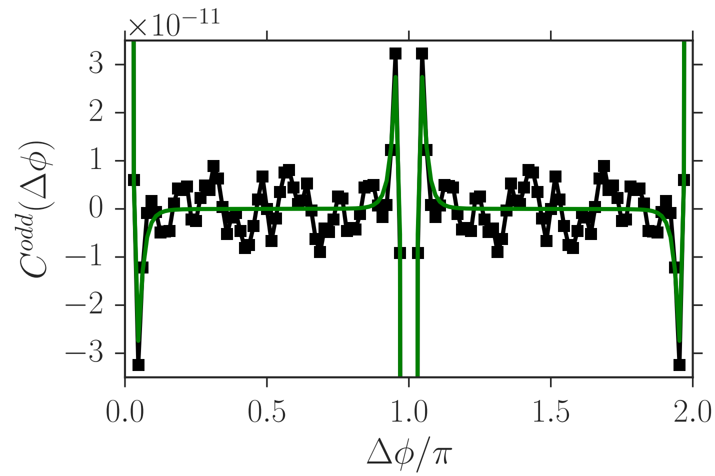


- Six gluon exchanges with the target (each M_3 or D_3 has at least 2 gluon exchanges, each M_1 has 1 gluon exchange).

Numerical Evaluation

- The same leading-order odd harmonics contribution can be evaluated numerically.
- Here we plot the anti-symmetrized correlation function C^{odd}:





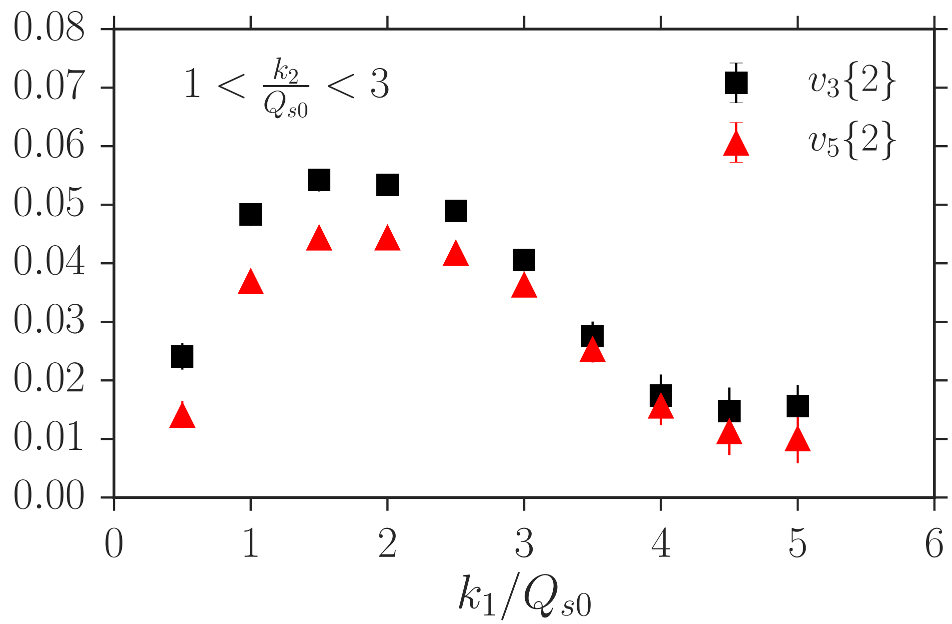
- Note that the HBT peaks at 0 and π dominate!

$$C^{\mathrm{odd}}(|\underline{k}_1|, |\underline{k}_2|, \Delta\phi)$$

$$= \frac{1}{4} E_1 E_2 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_0^{2\pi} \frac{d\phi_2}{2\pi} \, \delta(\Delta\phi - \phi_1 + \phi_2) \left(\frac{d^2 N(\underline{k}_1, \underline{k}_2)}{d^3 k_1 d^3 k_2} - \frac{d^2 N(\underline{k}_1, -\underline{k}_2)}{d^3 k_1 d^3 k_2} - \frac{d^2 N(-\underline{k}_1, \underline{k}_2)}{d^3 k_1 d^3 k_2} + \frac{d^2 N(-\underline{k}_1, -\underline{k}_2)}{d^3 k_1 d^3 k_2} \right)$$

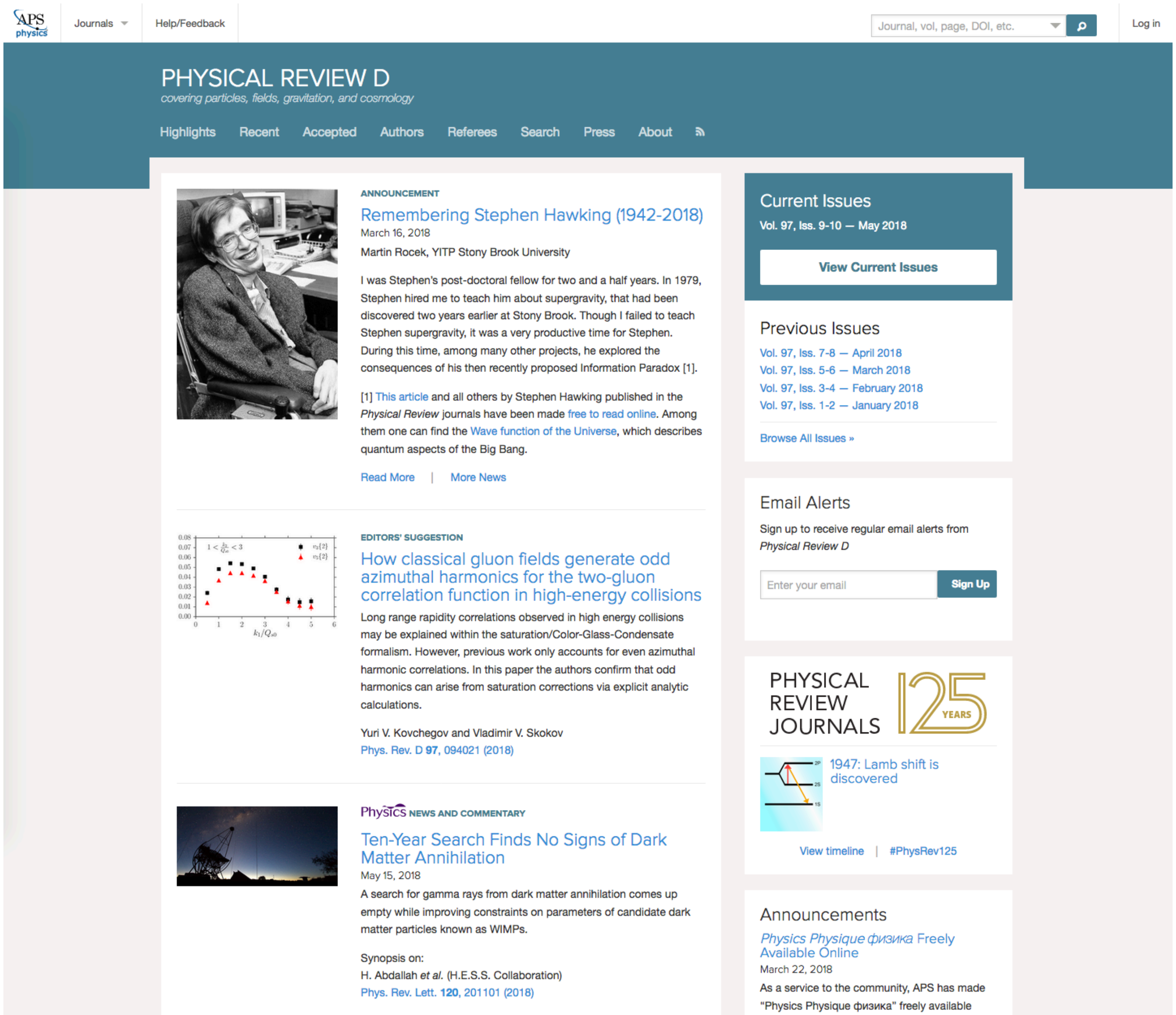
Numerical evaluation of harmonics

- One can extract the odd harmonics from the correlation functions:



- They are clearly non-zero.

PRD Editor's Suggestion…



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ANNOUNCEMENT

Remembering Stephen Hawking (1942-2018)

March 16, 2018

Martin Rocek, YITP Stony Brook University

I was Stephen's post-doctoral fellow for two and a half years. In 1979, Stephen hired me to teach him about supergravity, that had been discovered two years earlier at Stony Brook. Though I failed to teach Stephen supergravity, it was a very productive time for Stephen. During this time, among many other projects, he explored the consequences of his then recently proposed Information Paradox [1].

[1] This article and all others by Stephen Hawking published in the *Physical Review* journals have been made free to read online. Among them one can find the Wave function of the Universe, which describes quantum aspects of the Big Bang.

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EDITORS' SUGGESTION

How classical gluon fields generate odd azimuthal harmonics for the two-gluon correlation function in high-energy collisions

Long range rapidity correlations observed in high energy collisions may be explained within the saturation/Color-Glass-Condensate formalism. However, previous work only accounts for even azimuthal harmonic correlations. In this paper the authors confirm that odd harmonics can arise from saturation corrections via explicit analytic calculations.

Yuri V. Kovchegov and Vladimir V. Skokov

Phys. Rev. D **97**, 094021 (2018)

Physics NEWS AND COMMENTARY

Ten-Year Search Finds No Signs of Dark Matter Annihilation

May 15, 2018

A search for gamma rays from dark matter annihilation comes up empty while improving constraints on parameters of candidate dark matter particles known as WIMPs.

Synopsis on:

H. Abdallah *et al.* (H.E.S.S. Collaboration)

Phys. Rev. Lett. **120**, 201101 (2018)

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Announcements

Physics Physique физика Freely Available Online

March 22, 2018

As a service to the community, APS has made "Physics Physique физика" freely available online. This small journal published fewer than

Conclusions

- Odd azimuthal harmonics are an inherent property of particle production in the saturation framework.
- In the saturation formalism they arise from including saturation corrections in the projectile (not from the standard dilute projectile approximation) – need at least 3 sources in the projectile and 3 in the target.
- Odd (and even) harmonics appear to be dominated by 'HBT peaks', the narrow maxima at $\Delta\phi = 0$ and $\Delta\phi = \pi$.
- Odd harmonics = evidence for saturation? Certainly need a lot of scatterings.

Backup Slides

A Phase from the Classical Fields

- But wait, classical fields are real… What phase?
- Take a single gluon field contribution to the LSZ formula

$$\int d^4x\, e^{ik\cdot x}\, \Box A_\mu = -\int d^4x\, \partial_0 \left[A_\mu \overleftrightarrow{\partial_0}\, e^{ik\cdot x} \right] = \int d^3x\, \left[\left(\partial_0 - iE_k \right) A_\mu \right] e^{ik\cdot x} \Big|_{t\to+\infty}$$

and do an ultra-boost. One gets in light-cone coordinates

$$\int d^4x\, e^{ik\cdot x}\, \Box A_\mu^\perp = \int d^2x_\perp\, dx^- \, \left[\left(\partial_- - ik^+ \right) A_\mu^\perp \right] e^{ik\cdot x} \Big|_{x^+\to+\infty}$$

- Defining

$$A_\mu^\perp(x^+, k^+, \underline{k}) = \int d^2x_\perp\, dx^-\, e^{ik^+x^- - i\underline{k}\cdot\underline{x}}\, A_\mu^\perp(x)$$

we obtain

$$\int d^4x\, e^{ik\cdot x}\, \Box A_\mu^\perp = -2ik^+\, e^{ik^-x^+}\, A_\mu^\perp(x^+, k^+, \underline{k}) \Big|_{x^+\to+\infty}$$

- Gluon field is real in coordinate space. This gives

$$A_\mu^\perp(x^+, k^+, \underline{k})^* = A_\mu^\perp(x^+, -k^+, -\underline{k})$$

A Phase from the Classical Fields

- Two-gluon production cross section is then

$$\frac{dN}{d^2 k_1 \, dy_1 \, d^2 k_2 \, dy_2} \sim \left\langle A_\mu^\perp (x^+, k_1^+, \underline{k}_1) \, A_\perp^\mu (x^+, -k_1^+, -\underline{k}_1) \, A_\nu^\perp (x^+, k_2^+, \underline{k}_2) \, A_\perp^\nu (x^+, -k_2^+, -\underline{k}_2) \right\rangle \Big|_{x^+ \to +\infty}$$

- Once again, we have $A_\mu^\perp (x^+, k^+, \underline{k})^* = A_\mu^\perp (x^+, -k^+, -\underline{k})$

- Classical fields are k⁺-independent: then we have $\underline{k}_i \to -\underline{k}_i$ symmetry and there are <u>no odd harmoncs</u>!?

- Resolution: classical fields may depend on k⁺ via theta-functions!
- One may (and does) have

$$A_\mu^\perp (x^+, k^+, \underline{k}) = A_\mu^{(1)\perp} (x^+, \underline{k}) + i \, \mathrm{Sign}(k^+) \, A_\mu^{(2)\perp} (x^+, \underline{k})$$

A Phase from the Classical Fieldss

- With $A^{\perp}_{\mu}(x^+, k^+, \underline{k}) = A^{(1)\perp}_{\mu}(x^+, \underline{k}) + i\,\text{Sign}(k^+)\, A^{(2)\perp}_{\mu}(x^+, \underline{k})$

  the two-gluon distribution has a non-zero odd-harmonics part

$$\frac{dN_{\text{odd}}}{d^2k_1\, dy_1\, d^2k_2\, dy_2} \sim \text{Sign}(k_1^+)\,\text{Sign}(k_2^+)$$
$$\times \left\langle \left[A^{(1)\perp}_{\mu}(x^+, k_1^+, \underline{k}_1)\, A^{(2)\mu}_{\perp}(x^+, -k_1^+, -\underline{k}_1) - A^{(2)\perp}_{\mu}(x^+, k_1^+, \underline{k}_1)\, A^{(1)\mu}_{\perp}(x^+, -k_1^+, -\underline{k}_1) \right] \right.$$
$$\left. \times \left[A^{(1)\perp}_{\nu}(x^+, k_2^+, \underline{k}_2)\, A^{(2)\nu}_{\perp}(x^+, -k_2^+, -\underline{k}_2) - A^{(2)\perp}_{\nu}(x^+, k_2^+, \underline{k}_2)\, A^{(1)\nu}_{\perp}(x^+, -k_2^+, -\underline{k}_2) \right] \right\rangle \Big|_{x^+ \to +\infty}$$

- One may speculate that the origin of this phase difference between the LO and part of the NLO gluon production amplitudes is due to oddness of the latter with respect to flipping the gluon from the final state to the initial state…

Multiplicity dependence: scaling argument

(Vladi's slide)

- Physical two-particle anisotropy coefficients can be simply expressed as

$$v_n^2\{2\}(N_{\rm ch}) = \int \mathcal{D}\rho_p \mathcal{D}\rho_t \; W[\rho_p]\, W[\rho_t]\, |Q_n\left[\rho_p,\rho_t\right]|^2\, \delta\left(\frac{dN}{dy}\left[\rho_p,\rho_t\right] - N_{\rm ch}\right)$$

with

$$Q_{2n}\left[\rho_p,\rho_t\right] = \frac{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i2n\phi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]},\; Q_{2n+1}\left[\rho_p,\rho_t\right] = \frac{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{i(2n+1)\phi} \frac{dN^{\rm odd}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right]}$$

- High multiplicity is driven by fluctuations in ρ_p
- To study multplicity dependence, rescale $\rho_p \to \; c\,\rho_p$
- Under this rescaling:

$$\frac{dN}{dy} \to c^2 \frac{dN}{dy}; \qquad v_{2n}^2\{2\} \to v_{2n}^2\{2\}; \qquad v_{2n+1}^2\{2\} \to c^2\; v_{2n+1}^2\{2\}$$

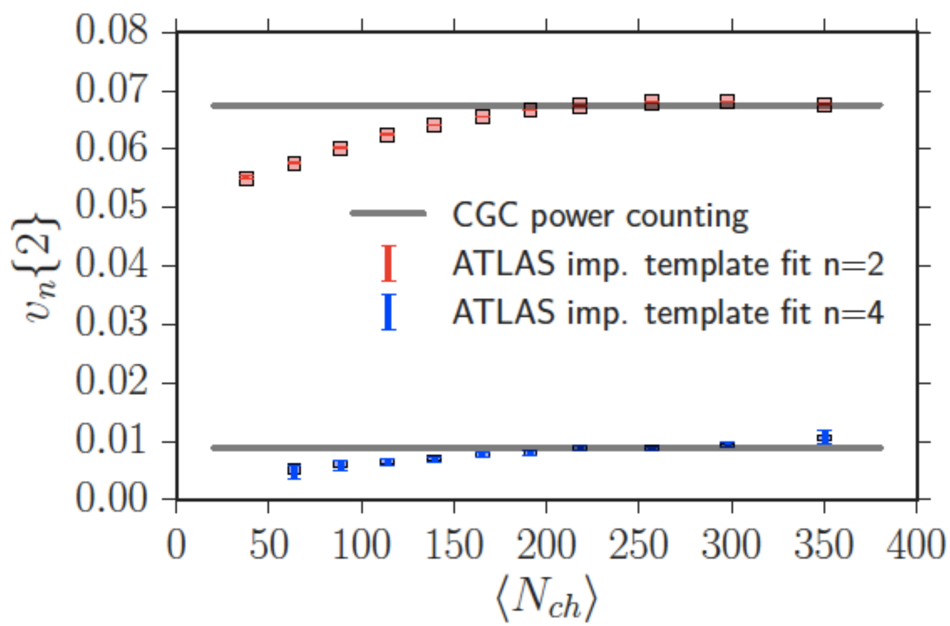
- Therefore in the first approximation: $v_{2n}\{2\}$ is independent of multiplicity

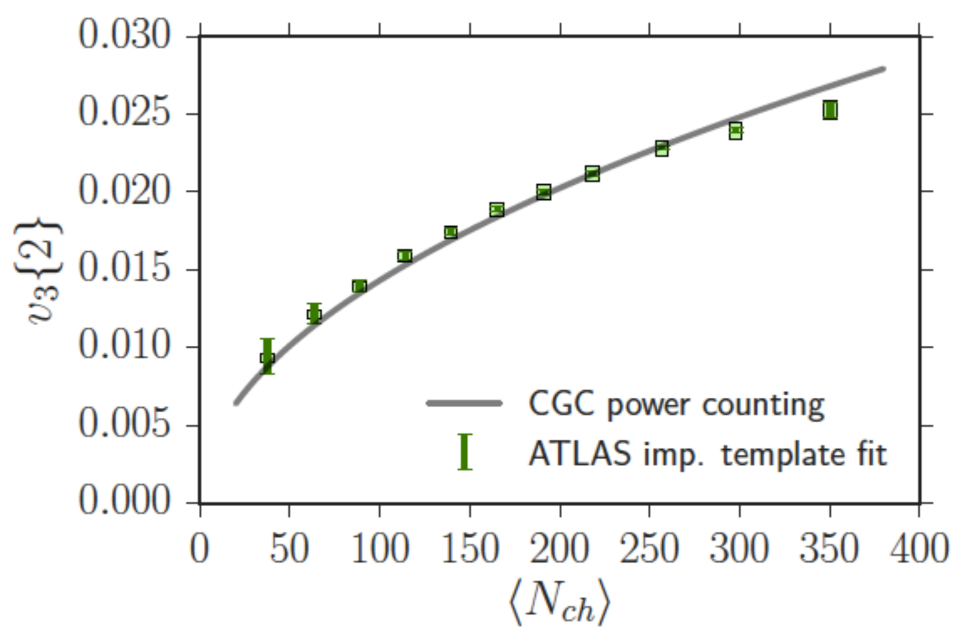
$$v_{2n+1}\{2\} \propto \sqrt{\frac{dN}{dy}}$$

Multiplicity dependence: scaling argument

(Vladi's slide)

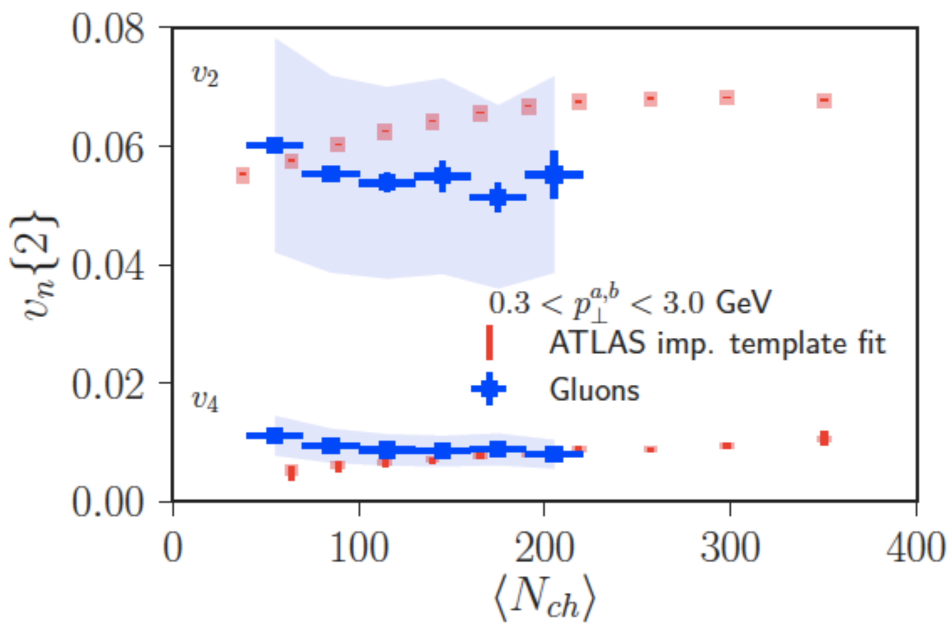
M. Mace , V. S., P. Tribedy, & R. Venugopalan, arXiv:1807.00825





MULTIPLICITY DEPENDENCE: NUMERICAL RESULT

(Vladi's slide)



M. Mace , V. S., P. Tribedy, & R. Venugopalan, arXiv:1807.00825

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- Multiplicity dependence of integrated v_3 is beyond our computational resources