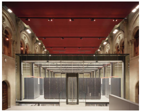



The spinodal instability near a critical point in a droplet

Maximilian Attems (IGFAE)

arXiv:1603.01254 **arXiv:1703.09681** **arXiv:1807.05175**

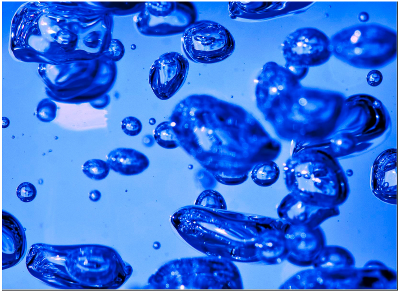
arXiv:1809.XXXXX

Collaborators:
Yago Bea (UB), Jorge Casalderrey-Solana (Oxford, UB),
David Mateos (UB), Miguel Zilhao (CENTRA)

Benasque COST workshop on collectivity in small systems

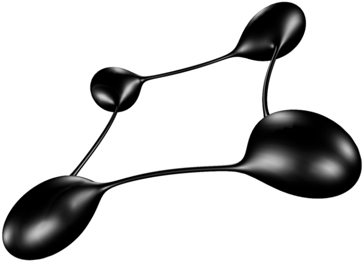
Motivation I

Phase transition:



Vaporization

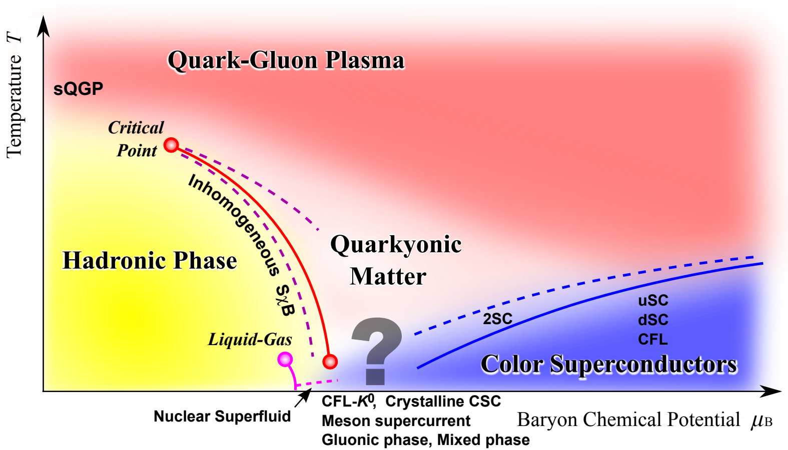
Gregory-Laflamme Instability:



Black ring pinching off

gauge/gravity correspondence:
bridge between physical phenomena in gauge theories and gravity.

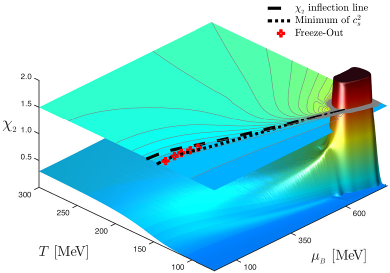
Motivation II

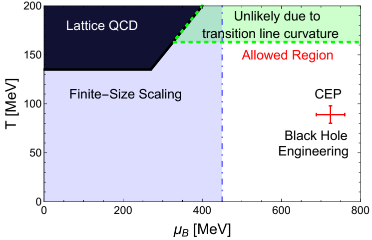


QCD phase diagram [Fukushima, Hatsudo 2010]

Motivation III

Black-hole engineered critical point w QCD lattice match [Critelli, Noronha, Noronha-Hostler, Portillo, Ratti, Rougemont 2017]





Baryon susceptibility χ_2 extended in the full $T - \mu_B$ plane and critical point located at $T_{CEP} = 89$ MeV and $\mu_B^{CEP} = 724$ MeV

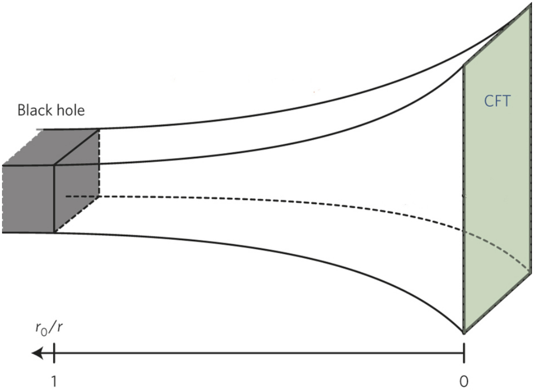
Outline

Excitation and saturation of the spinodal instability

- Non-conformal General Relativity model
- Non-conformal thermodynamics
- Spinodal instability
- Inhomogeneous Horizon
- Hydrostatic + Hydrodynamic evolution
- Phase separation
- Unseeded spinodal instability
- Pressure evolution
- Preferred final state
- Phase mergers

Introduction gauge gravity duality

Quantum gravity in $d+1$ dimension AdS $\leftrightarrow$ QFT in d dimension

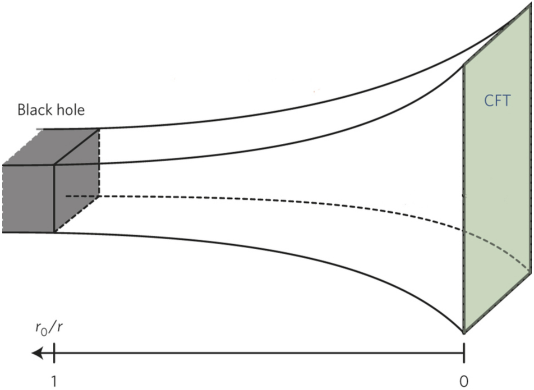


IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}_5 \leftrightarrow \mathcal{N} = 4$ Super-Yang-Mills
[Maldacena 1998, Witten 1998]

shear visocity over entropy density ratio $\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$
[Policastro, Son, Starinets 2001; Kovtun, Son, Starinets 2005]

Introduction gauge gravity duality

Quantum gravity in $d+1$ dimension AdS $\leftrightarrow$ QFT in d dimension

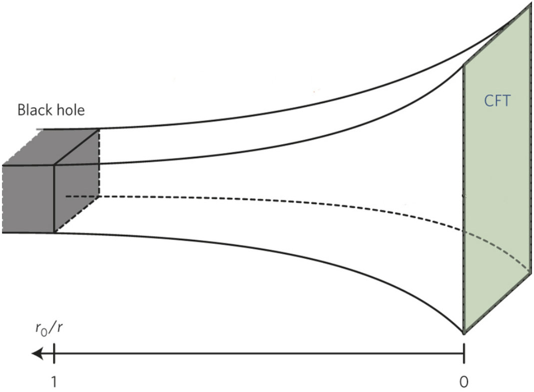


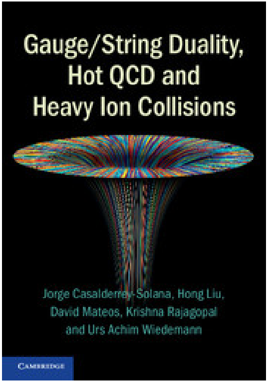
Holographic dictionary relates:

Black hole	$\Longleftrightarrow$	Equilibrium state with temperature
$g_{\mu\nu}$		$T_{\mu\nu}$

Introduction gauge gravity duality

Quantum gravity in $d+1$ dimension AdS $\leftrightarrow$ QFT in d dimension





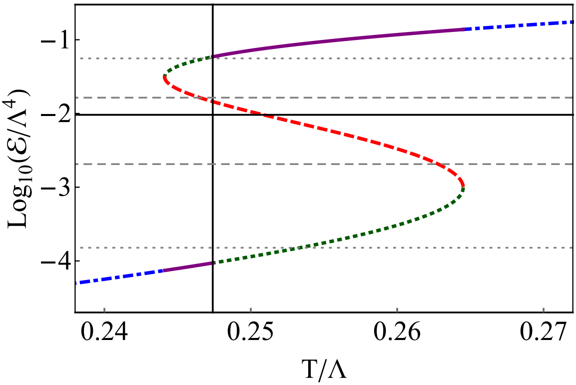
Use of the duality:
To solve complicated dynamical problems in non-abelian theories.
As a source of new modeling ideas for strongly coupled QGP.

Non-conformal General Relativity model

Dual field theory: 'mimics'a deformation of N=4 SYM with a dimension 3 operator O and Λ as 'mass'

$$S_{\mathrm{GaugeTheory}} = S_{\mathrm{conformal}} + \int d^4x \Lambda O$$

First order phase transition at the critical temperature seen in the the multivalued plot of the energy density in function of the temperature



Non-conformal thermodynamics

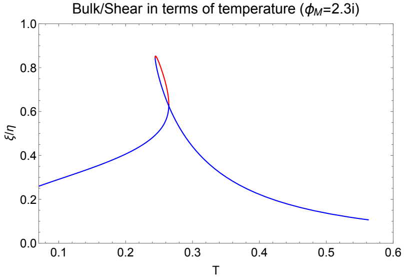
Einstein-Hilbert action coupled to a scalar with non-trivial potential (single parameter ϕ_M) in five-dimensional bottom-up model:

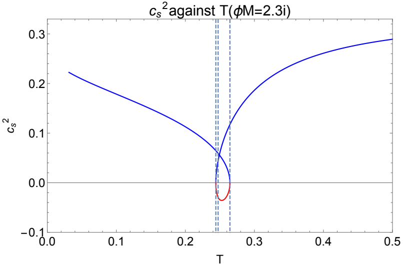
$$S = \frac{2}{\kappa_5^2}\int d^5x\sqrt{-g}\left[\frac{1}{4}\mathcal{R} - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

Holographic renormalization [Bianchi, Freedman, Skenderis 2002]

$$V(\phi) = -\frac{1}{12\phi_M^4}\phi^8 + \left(\frac{1}{2\phi_M^4} \mp \frac{1}{3\phi_M^2}\right)\phi^6 - \frac{1}{3}\phi^3 - \frac{3}{2}\phi^2 - 3\,.$$

Small IR modification of the model leads to rich phase structure

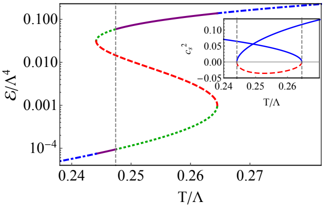




Deconfined strong non-conformality with 1st order phase transition

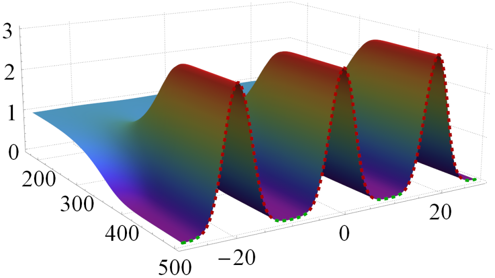
Homogeneous periodic box in thermodynamical unstable region:

Energy density versus temperature for the gauge theory:



The dashed red curve is locally unstable, the dotted green curve metastable.

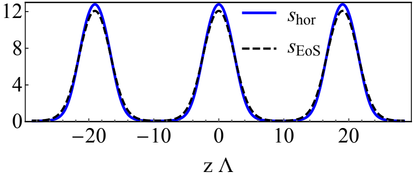
Energy density evolution of black branes afflicted by the Gregory-Laflamme instability:



excited unstable mode growth until non-linear saturation

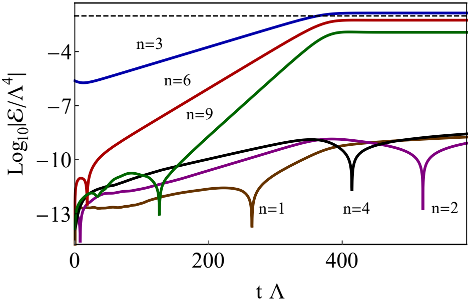
Inhomogeneous Horizon

Final entropy density



Final entropy density extracted from the area of the horizon and estimated from the equation of state

Evolution of Fourier modes of the local energy density

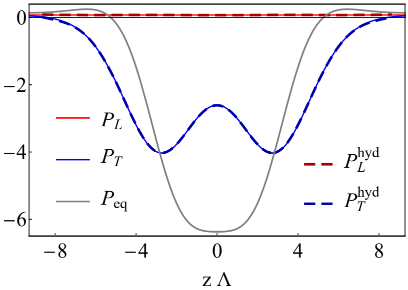


momentum dependent growth rate

$$\Gamma(k) \simeq |c_s|\, k - \frac{1}{2T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right) k^2 \, .$$

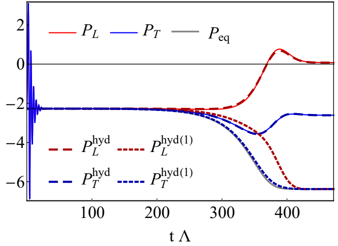
Hydrostatic + Hydrodynamic evolution

Hydrodynamics description with transport coefficients $c_{\mathrm{L/T}}, f_{\mathrm{L/T}}$:
$$P_{L/T}^{\mathrm{hyd}} = P_{\mathrm{eq}}(\mathcal{E}) + c_{\mathrm{L/T}}(\mathcal{E})(\partial_z \mathcal{E})^2 + f_{\mathrm{L/T}}(\mathcal{E})(\partial_z^2 \mathcal{E})$$



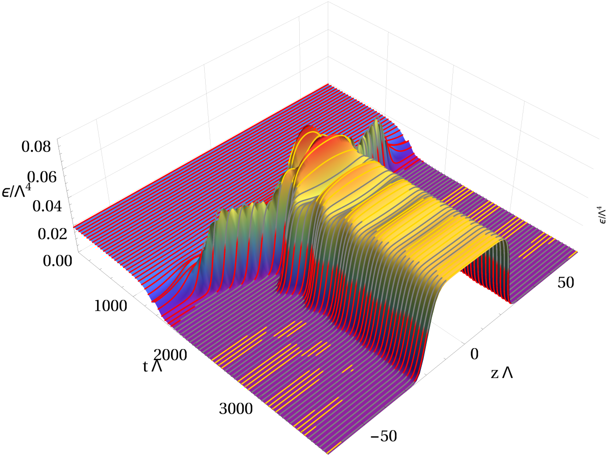
Pressures agree with hydrodynamic prediction for a different state

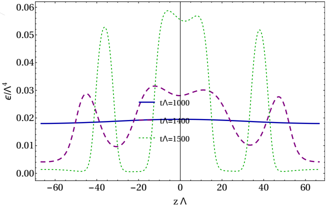
Pressures predicted by hydro match:



Early time behaviour with exponential decay of quasi-normal modes

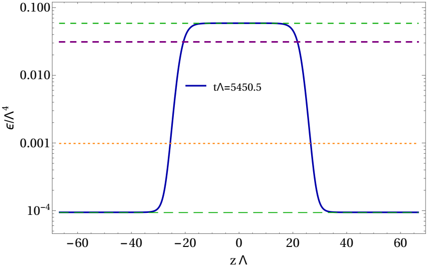
Bigger periodic box with single excited mode:





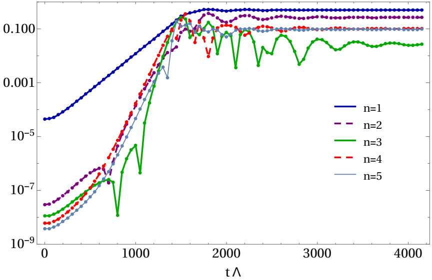
Cooled and hot regions are on the respective stable phase [Janik, Jankowski, Soltanpanahi 2017]

Final static state:



Phase separated final state

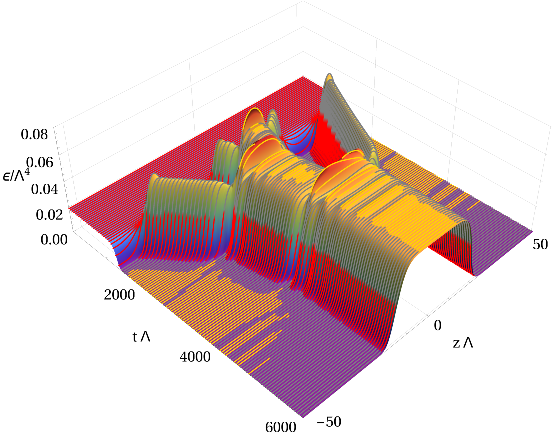
mode analysis



Initial $n = 1$ excited

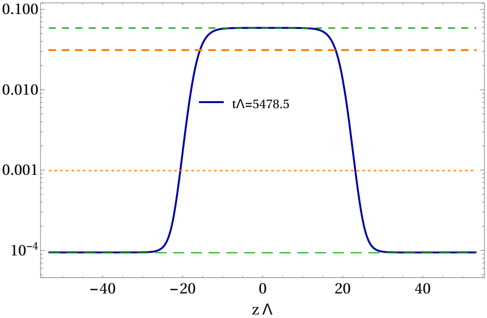
Spinodal instability triggered by numerical noise I

Unexcited initial state:



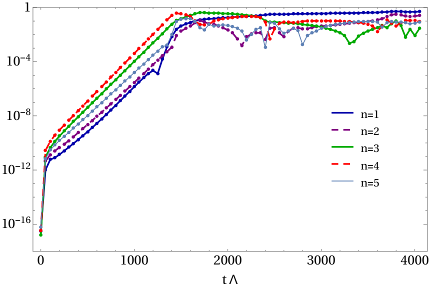
Minimizing free energy to have only a single blob

Final static state:



Phase separated final state

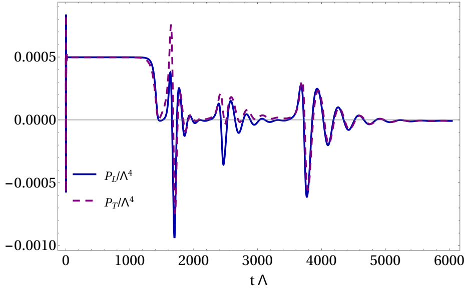
Unstable mode growth



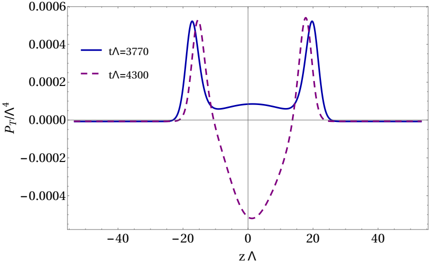
$n = 4$ unstable mode pushing four initial peaks

Pressure evolution

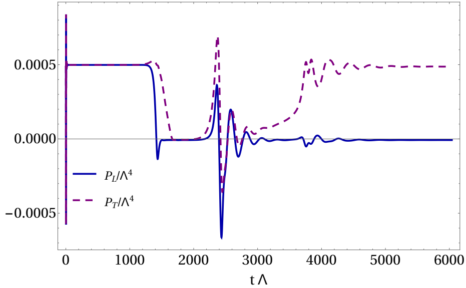
Pressures at midpoint:



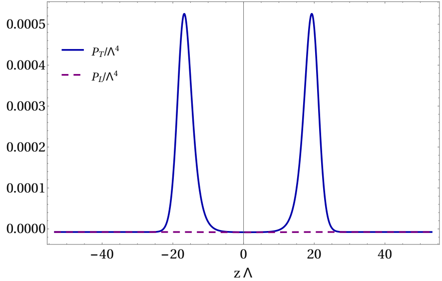
Longitudinal merger snapshots:



Pressures at the "wall":

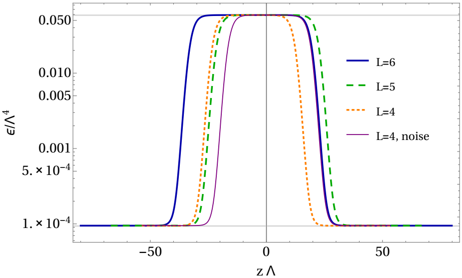


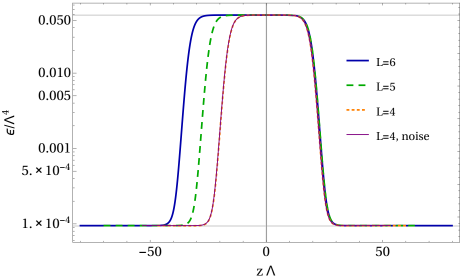
Final equilibrated state:



Preferred final state

Large boxes with different extent and initial conditions:

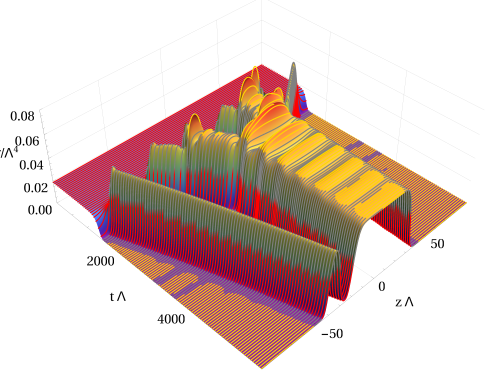




Universal "domain-wall" type phase separation.

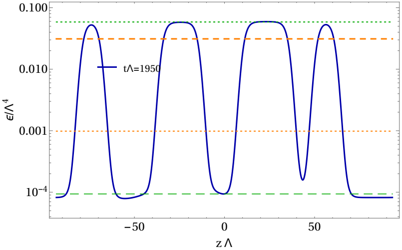
Phase mergers I

Different phase separated region:

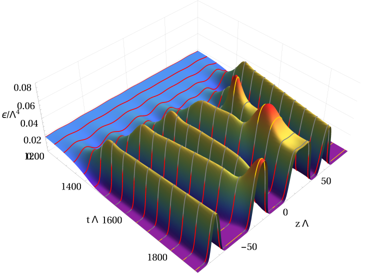


Merger to preferred state.

Snapshot at phase separation:

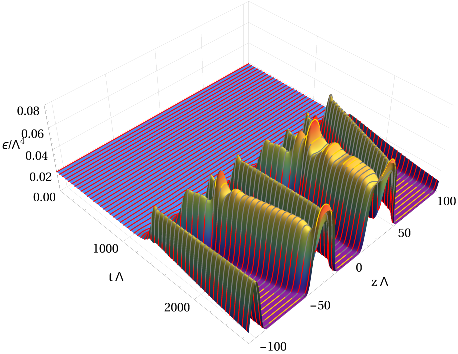


Early time evolution:

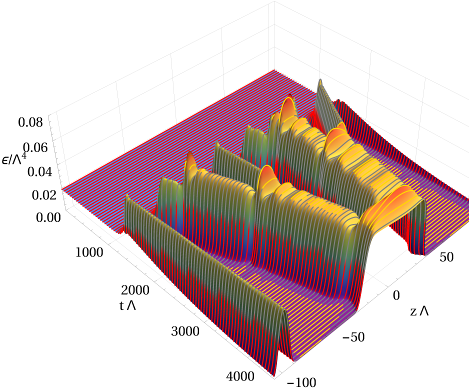


Phase mergers II

Premerger time snapshot:



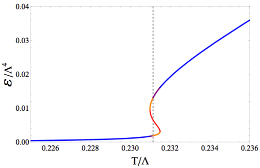
Ongoing evolution:



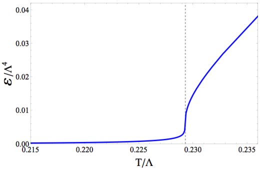
Separated phases over longer times merge to preferred state

One more thing: Collisions near a critical point

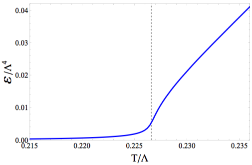
1st order:

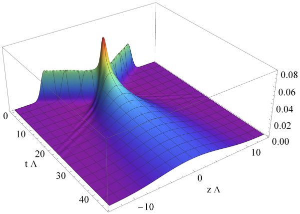


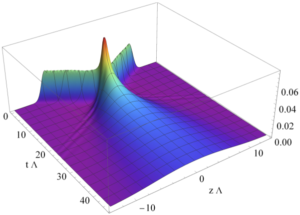
2nd order:

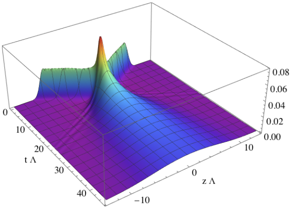


crossover:





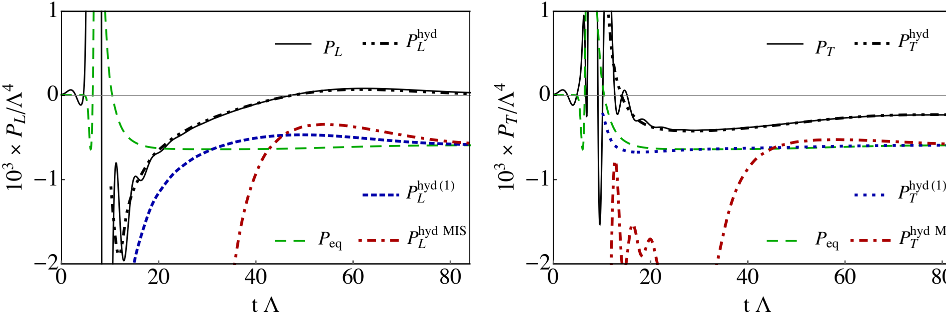




Different thermodynamics with similar off-equilibrium blob formation

One more thing: Collisions near a critical point

Müller-Israel-Stewart-type hydrodynamics fails to describe the pressure evolution at mid rapidity in the formed blob:



Well described by the constitutive relations of second-order hydrodynamics that include all spatial second-order gradients.

Conclusions

- First simulation of a holographic spinodal instability
- Excitation of the Gregory-Laflamme instability
- Final set of static inhomogeneous black branes
- Spinodal instabilities lead to phase separation and mergers
  - The system settles (!?) into a preferred inhomogenous state (!)
  - The final state is also described by hydrodynamics!
- New example of the applicability of hydrodynamics to systems with large gradients in energy densities - even in non-trivial phase structure - both for the time evolution of the spinodal instability and the static final states
- More studies are on the way

Benasque Cost workshop on Small Systems