Toward a unified approach to particle production in high energy collisions

Jamal Jalilian-Marian

Baruch College and CUNY Graduate Center

New York, NY

Collectivity and correlations in high energy hadron and nuclear collisions
August 2018, Benasque, Spain

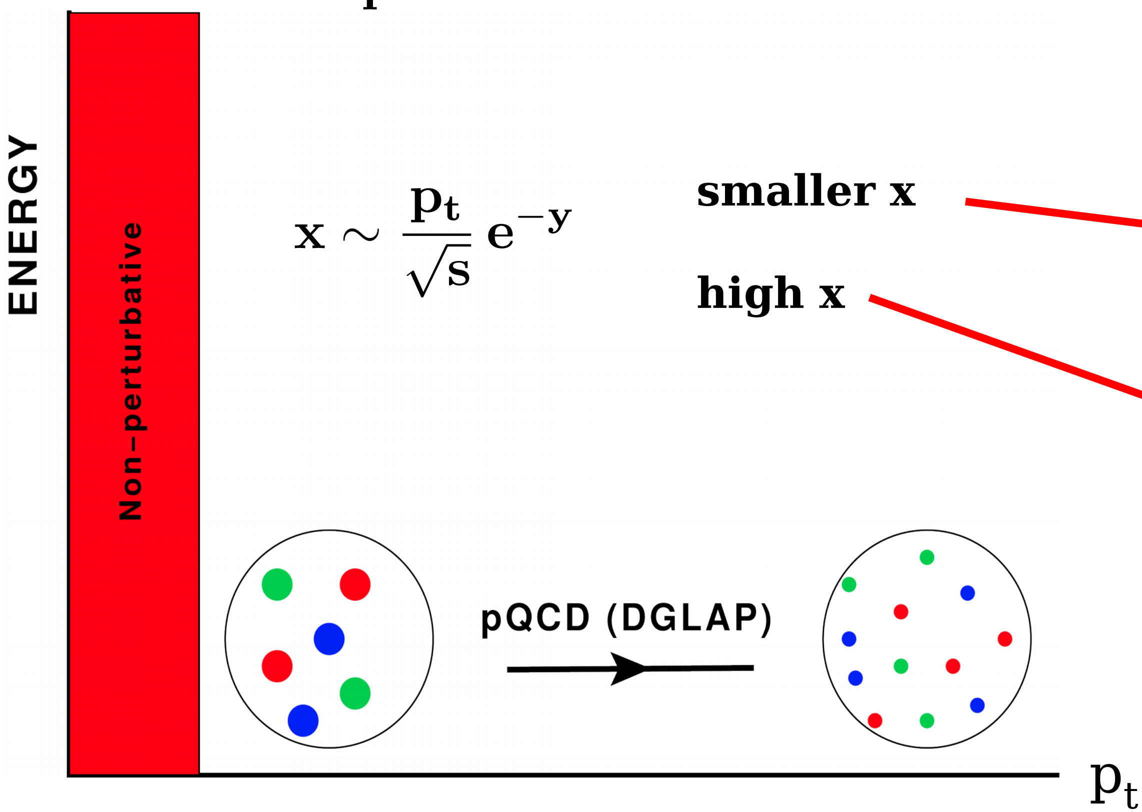
pQCD: the standard paradigm

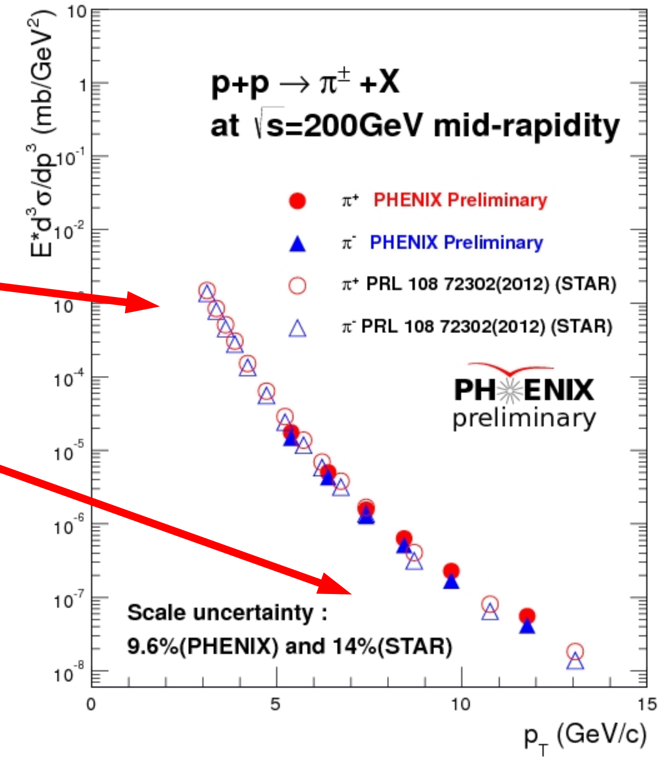
$$E\,\frac{d\sigma}{d^3p} \sim f_1(x) \otimes f_2(x) \otimes \frac{d\sigma}{dt} \otimes D(z)$$

$$x \sim \frac{p_t}{\sqrt{s}}\, e^{-y}$$

smaller x

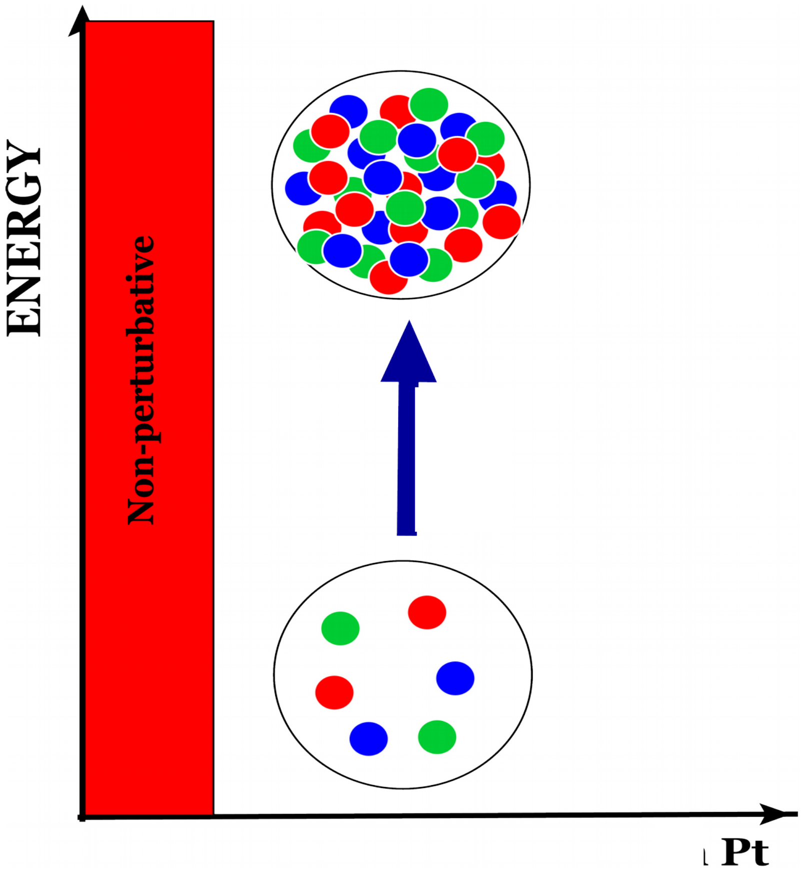
high x





bulk of QCD phenomena happens at low p_t (small x)

A hadron/nucleus at high energy: gluon saturation



$$\frac{\partial \mathbf{W_E}[\mathbf{A}]}{\partial \mathbf{E}} = \mathcal{H}\, \mathbf{W_E}[\mathbf{A}]$$

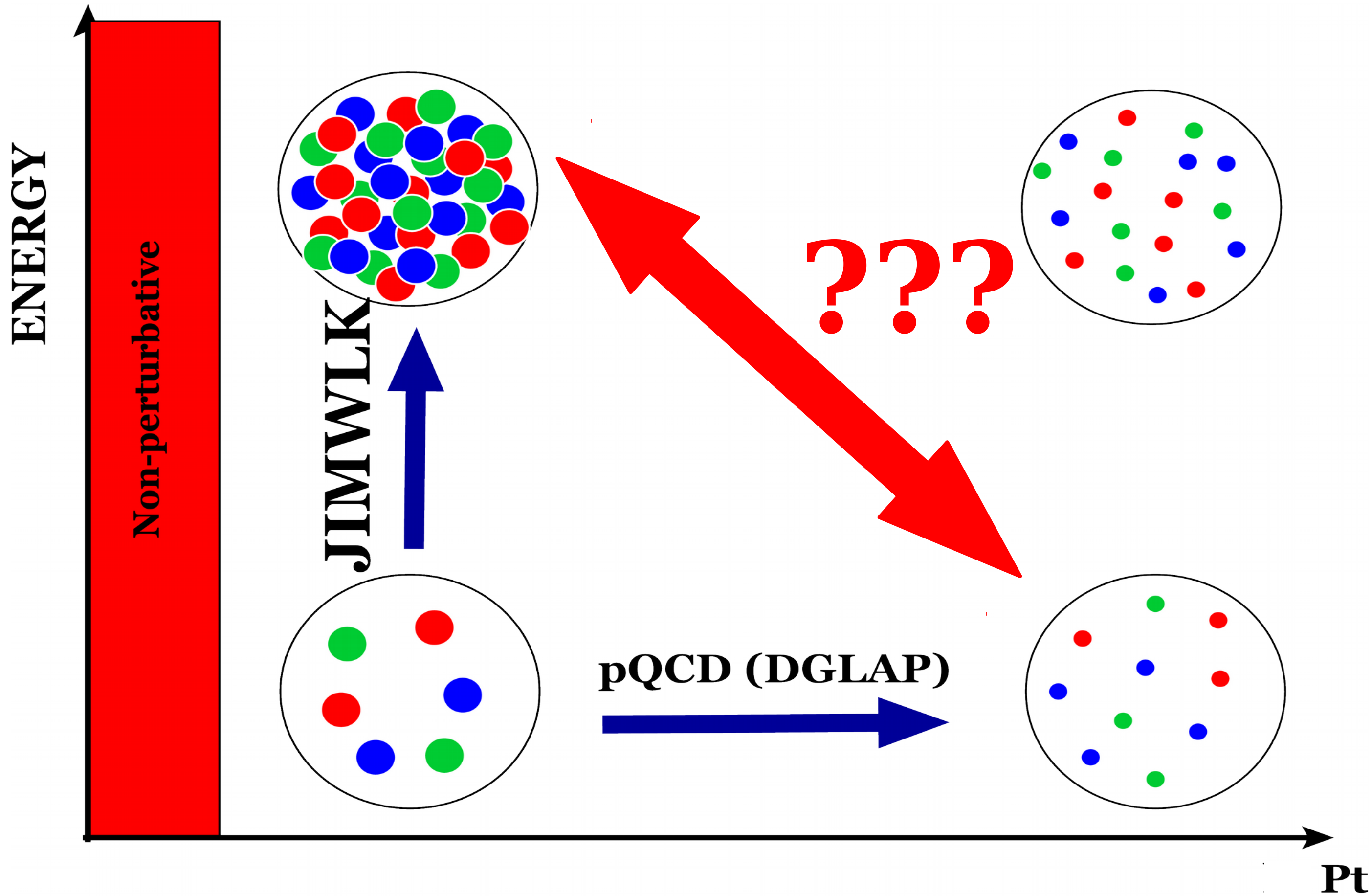
JIMWLK eq.

dynamics of universal gluonic matter

a framework for multi-particle production in QCD

not applicable at high x/high p_t

QCD kinematic phase space



connecting JIMWLK with pQCD?

jet physics
partially/fully coherent energy loss
interactions of UHE neutrinos, ...

how to tackle this problem?

what should be the *starting point/expression/operator*?

pQCD: quark and gluon operators

$$\overline{\Psi}(y^-, 0_t)\gamma^+\Psi(0^-, 0_t)$$

renormalization lead to DGLAP evolution eq.

CGC: correlators of Wilson lines (DIS, Hybrid,....)

$$F_2 \sim Tr\, V(x_t)\, V^\dagger(y_t)$$

renormalization leads to JIMWLK/BK evolution eq.

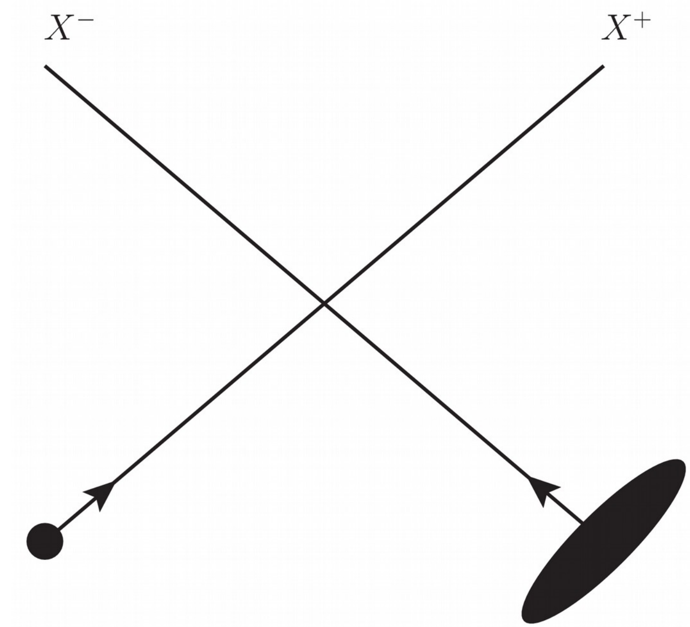
dense target (proton/nucleus) as a background color field

$$J^\mu_a \simeq \delta^{\mu -}\rho_a$$

$$D_\mu\, J^\mu \quad = \quad D_-\, J^- = 0$$

$$\partial_- J^- \quad = \quad 0$$ (in A+ = 0 gauge)

does not depend on x-



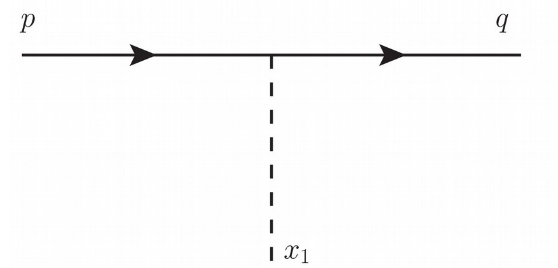
solution to classical EOM: $$A^-_a(x^+, x_t) \equiv n^-\, S_a(x^+, x_t)$$

scattering of a quark from background color field $A^-_a(x^+, x_t)$

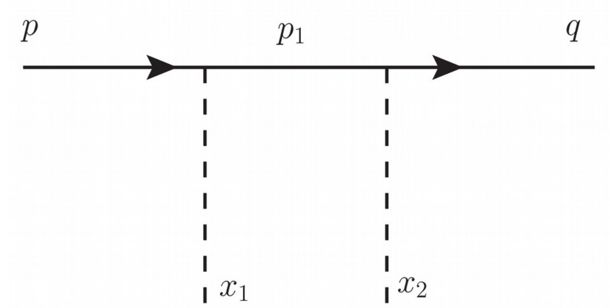
recall (eikonal limit): $$\bar{u}(q)\gamma^\mu u(p) \rightarrow \bar{u}(p)\gamma^\mu\, u(p) \sim p^\mu$$

$$\bar{u}(q)\not{A}\, u(p) \rightarrow p\cdot A \sim p^+\, A^-$$

$$
\begin{aligned}
i\mathcal{M}_1 &= (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p) \\
&= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}} \\
&\quad \bar{u}(q)\left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)
\end{aligned}
$$



$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2 \int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1 - p)x_1}\, e^{i(q-p_1)x_2} \\
&\quad \bar{u}(q)\, \left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right]\, u(p)
\end{aligned}
$$

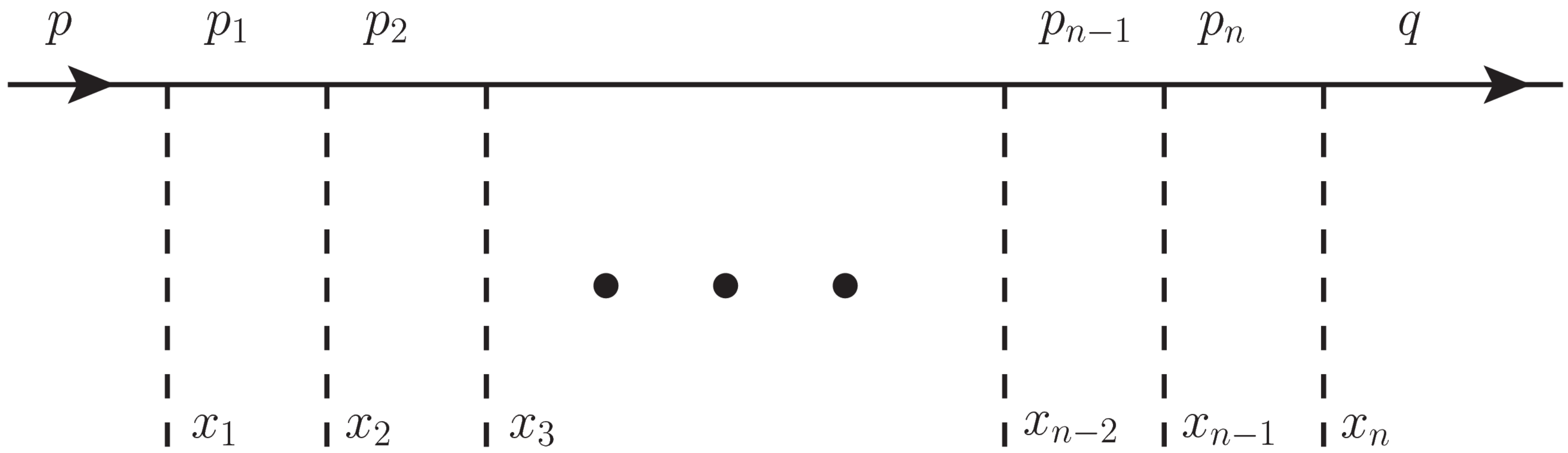


$$
\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\, \theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}
$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\, \frac{\not{p}_1}{2n\cdot p}\, \not{n} = \not{n}$

$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2\, (-i)(i)\, 2\pi\delta(p^+ - q^+) \int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+) \int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}} \\
&\quad \bar{u}(q)\, \left[S(x_2^+, x_{1t})\, \not{n}\, S(x_1^+, x_{1t})\right]\, u(p)
\end{aligned}
$$



$$
\begin{aligned}
i\mathcal{M}_n &= 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t} \\
&\quad \Big\{ (ig)^n\, (-i)^n (i)^n \int dx_1^+\, dx_2^+ \cdots dx_n^+\, \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \\
&\quad \left[S(x_n^+, x_t)\, S(x_{n-1}^+, x_t) \cdots S(x_2^+, x_t) S(x_1^+, x_t) \right] \Big\}\, u(p)
\end{aligned}
$$

sum over all scatterings gives (Eikonal approximation)

$$
i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\, \left[V(x_t) - 1 \right]\, u(p)
$$

with $V(x_t) \equiv \hat{P}\, \exp\left\{ ig \int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a \right\}$

propagator: including propagation “backward” gives $V^\dagger$

toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$
\begin{aligned}
p^\mu &= (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0) \\
S &= S(p^+ \sim 0, p^-/P^- \ll 1, p_t)
\end{aligned}
$$

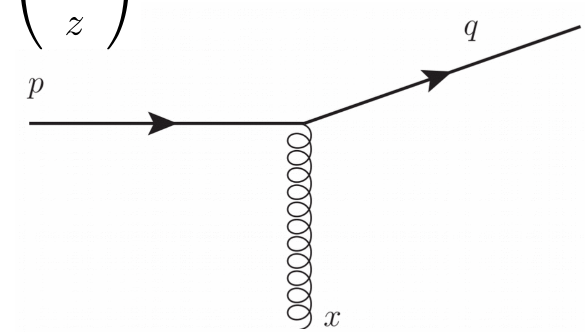
allow hard scattering by including one all x field $A_a^\mu(x^+, x^-, x_t)$ during which there is large momenta exchanged and quark can get deflected by a large angle.

include eikonal multiple scattering before and after (along a different direction) the hard scattering

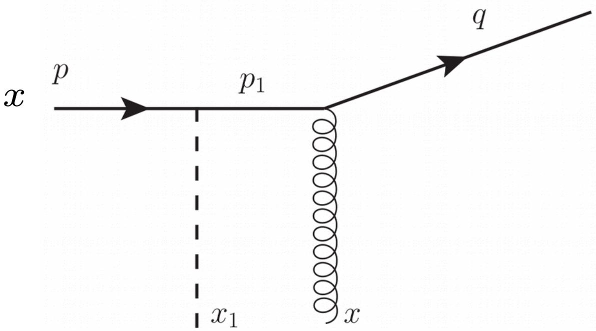
hard scattering: large deflection
scattered quark travels in the new “z” direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

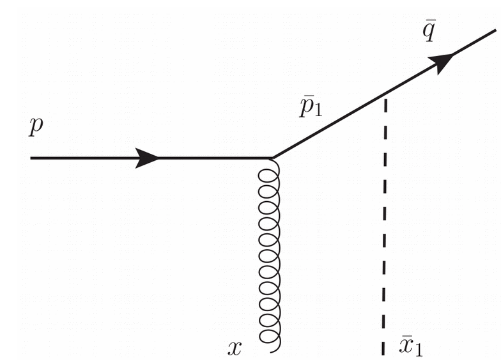
$$i\mathcal{M}_1 = (ig) \int d^4x \, e^{i(\bar{q}-p)x} \, \bar{u}(\bar{q}) \, \left[\not{A}(x)\right] \, u(p)$$



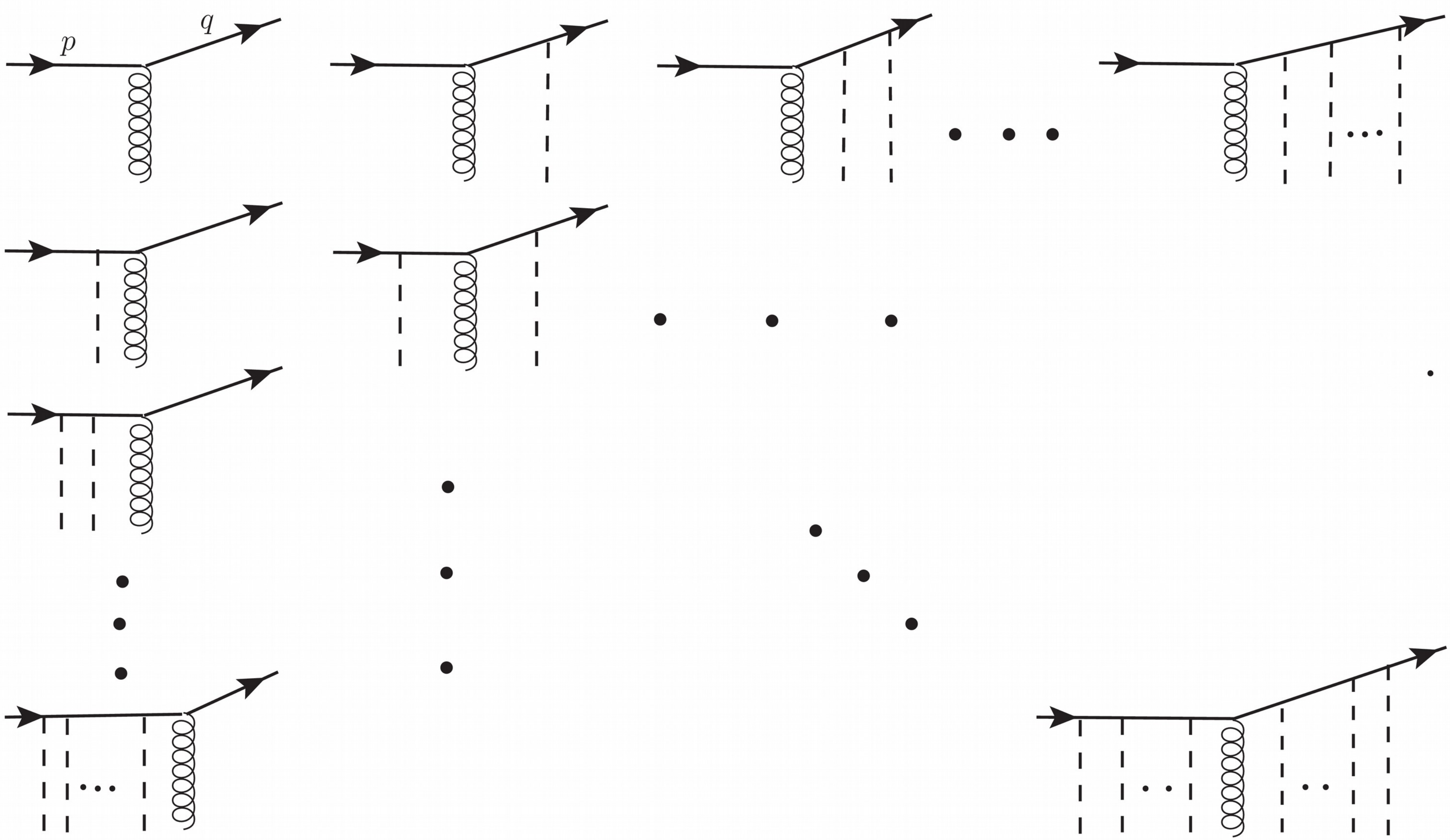
$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x \, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(\bar{q}-p_1)x} \\ & \qquad \bar{u}(\bar{q}) \left[\not{A}(x) \, \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1)\right] u(p) \end{aligned}$$



$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x \, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} \, e^{i(\bar{p}_1-p)x} \, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \\ & \qquad \bar{u}(\bar{q}) \left[\bar{\not{n}} \, \bar{S}(\bar{x}_1) \, \frac{i\bar{\not{p}}_1}{\bar{p}_1^2 + i\epsilon} \not{A}(x)\right] u(p) \end{aligned}$$



with $\quad \bar{v}^{\mu} = \Lambda^{\mu}_{\nu} \, v^{\nu}$



define: $\quad i\mathcal{M}(p,\bar{q}) = \bar{u}(\bar{q})\,\tau_F\,u(p)$

$$S_F(p,\bar{q}) = (2\pi)^4 \delta^4(p-\bar{q})\, S_F^0(p) + S_F^0(p)\, \tau_{hard}(p,\bar{q})\, S_F^0(\bar{q})$$

$$\begin{aligned}
\tau_{hard}(p,\bar{q}) \;\; &\equiv \;\; (ig) \int d^4x \int \frac{d^2k_t}{(2\pi)^2} \frac{d^2\bar{k}_t}{(2\pi)^2}\, d^2z_t\, d^2\bar{z}_t\, e^{i(\bar{k}-k)x}\; e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\; e^{-i(k_t-p_t)\cdot z_t} \\
&\Big\{\theta(p^+)\,\theta(\bar{q}^+)\, V(z_t,x^+)\, \not{n}\, \frac{\not{k}}{2k^+} \not{A}(x)\, \frac{\not{\bar{k}}}{2\bar{k}^+}\, \not{\bar{n}}\, \overline{V}(x^+,\bar{z}_t) - \\
&\theta(-p^+)\,\theta(-\bar{q}^+)\, V^\dagger(z_t,x^+)\, \not{n}\, \frac{\not{k}}{2k^+} \not{A}(x)\, \frac{\not{\bar{k}}}{2\bar{k}^+}\, \not{\bar{n}}\, \overline{V}^\dagger(x^+,\bar{z}_t)\Big\}
\end{aligned}$$

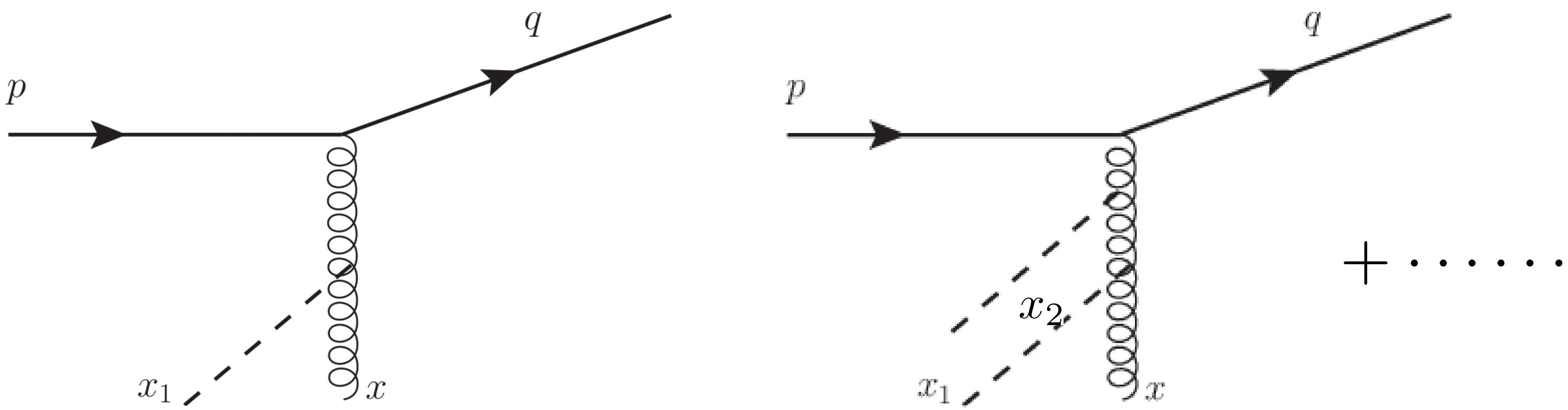
with

$$\overline{V}(x^+,\bar{z}_t) \equiv \hat{P}\,\exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\, \bar{S}_a(\bar{z}^+,\bar{z}_t)\, t_a\right\}$$

all “bar-ed” quantities are in a rotated frame where quark’s new direction of propagation (after a hard scattering) is $\bar{z}$

this quark propagator is the building block for DIS structure functions, single inclusive particle production in pA,….

but there is more to do:
interactions of large and small x modes



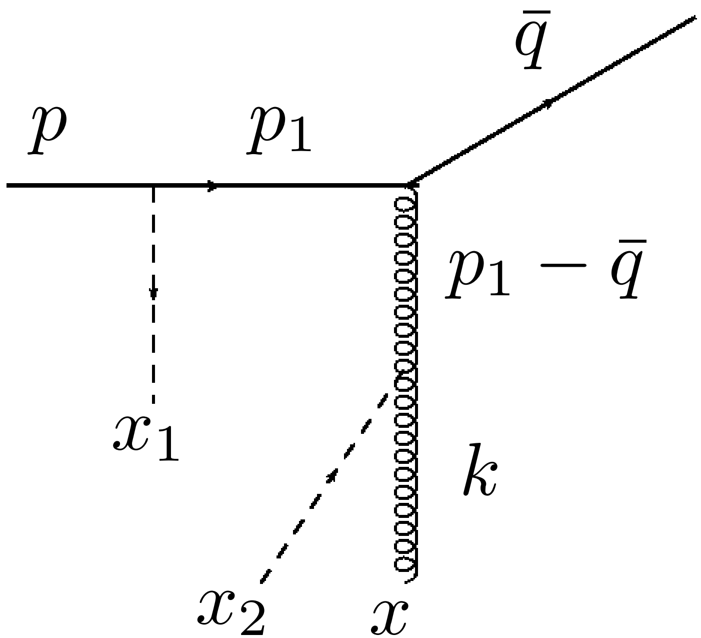
these re-sum to

$$
\begin{aligned}
\tau_{hard}(p,\bar{q}) &= 2g\,\frac{\theta(p^+)\,\theta(\bar{q}^+)}{(p-\bar{q})^2+i\epsilon}\int d^4x\,e^{i(\bar{q}-p)\cdot x} \\
& \quad t^a\,\left[\partial_+^{x^+}\,U(x^+,x_t)\right]^{ab}\,\left[n\cdot(\bar{q}-p)\not{A}^b(x)-(\bar{q}-p)\cdot A^b(x)\not{n}\right] \\
&+ \quad \theta(-p^+)\,\theta(-\bar{q}^+)\cdots
\end{aligned}
$$

with

$$
U(x^+,x_t)=\hat{P}\,exp\left\{ig\int_{-\infty}^{x^+}dz^+\,S_a(z^+,x_t)\,T_a\right\}
$$

but there is more!



both initial state quark and hard gluon interacting:

integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\,[(p_1 - \bar{q})^2 + i\epsilon]}$$

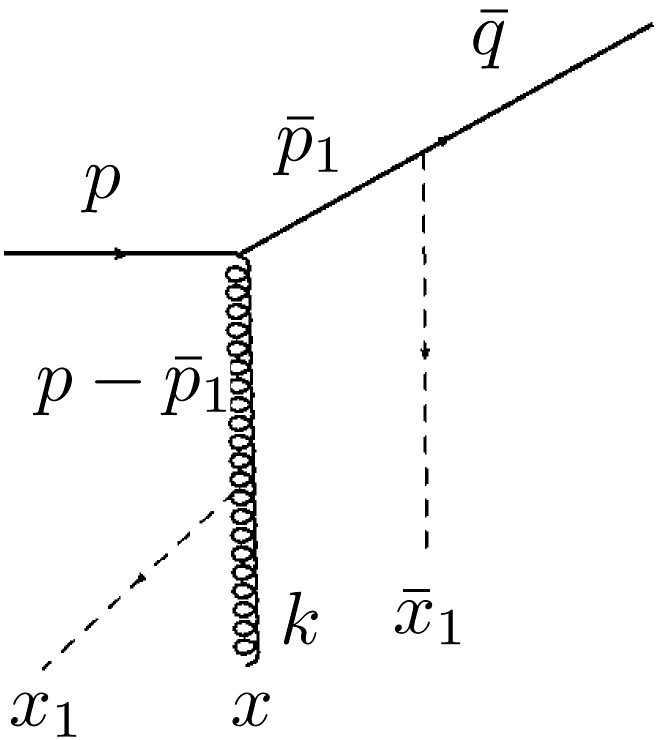
both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

ignoring phases we get a cancellation!

can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

how about the final state quark interactions?



integration over $\bar{p}_1^-$

$$\int \frac{d\bar{p}_1^-}{2\pi} \frac{e^{i\bar{p}_1^- (\bar{x}_1^+ - x^+)}}{[\bar{p}_1^2 + i\epsilon]\ [(p_1 - \bar{p}_1)^2 + i\epsilon]}$$

now the poles are on the opposite side of the real axis, we get both ordering

$$\theta(x^+ - \bar{x}_1^+) \text{ and } \theta(\bar{x}_1^+ - x^+)$$

ignoring the phases the contribution of the two poles add!

path ordering is lost!

however further rescatterings are still path-ordered

before/after $\mathbf{x_1^+}, \bar{\mathbf{x}}_1^+$

again the combination of a Wilson line and the soft field can be written as a derivative

$$
\begin{aligned}
\tau_F(p,\bar q) \quad = \quad & -2g\theta(p^+)\theta(\bar q)\int d^4x\, d^2\bar x_{1t}\, d\bar x_1^+\, \frac{d^2\bar p_{1t}}{(2\pi)^2} \\
& e^{i(\bar q^+ - p^+)x^- - i(\bar p_{1t}-p_t)\cdot x_t - i(\bar q_t - \bar p_{1t})\cdot \bar x_{1t}} \\
& t^a\, \left[\partial_+^{x^+}\, U(x^+,x_t)\right]^{ad} \left[\not n\,(p-\bar p_1)\cdot A^d(x) - n\cdot(p-\bar q)\not A^d(x)\right] \\
& i\not\bar p_1\, \left[\partial_+^{\bar x_1^+}\, \overline{V}(\bar x_1^+,\bar x_{1t})\right] \not n \\
& \frac{1}{[2n\cdot\bar q\, 2n\cdot(\bar q - p)\, p^- + 2n\cdot\bar q\,(\bar p_{1t}-p_t)^2 - 2n\cdot(\bar q - p)\,\bar p_{1t}^2]}
\end{aligned}
$$

so it seems all order soft scattering + one hard one can be re-summed!

soft (eikonal) limit: $A^\mu(x) \to n^-\, S(x^+,x_t) \qquad n\cdot\bar q \to n\cdot p$

the propagator can be symbolically written as

SUMMARY

CGC is a systematic approach to high energy collisions

CGC breaks down at large x (high pt)

Toward a unified formalism:

- ***quark propagator in the background of small and large x fields***
- ***QCD structure functions at both small and large x***
- ***particle production in pp, pA in both small and large p_t regions***