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State Estimation and Feedback Cooling of Levitated Nanoparticles

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NGCM

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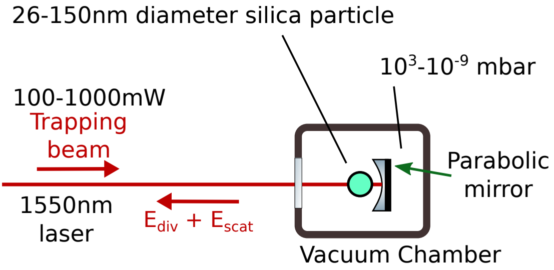
EPSRC
Engineering and Physical Sciences Research Council

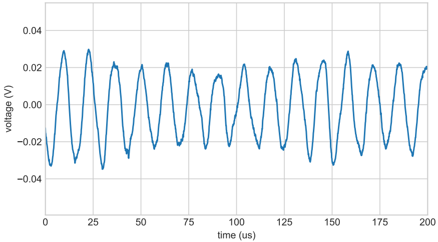
- Focus of my work is on implementing estimation and feedback to cool particles
- Goal is cooling a levitated nanoparticle to ground state of harmonic potential
- Need an accurate estimation method in order to implement the correct feedback
- Need an accurate model of the system against which to evaluate estimation methods

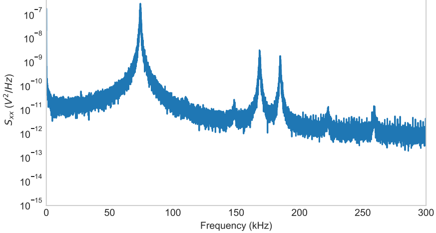
Talk Outline

- Classical Simulation
- Kalman filtering for tracking and cooling
- Quantum Simulation
- Quantum tracking and cooling
- Summary

Experimental Setup







[0]Vovrosh et al. 2017, J. Opt. Soc. Am. B (1364/JOSAB.34.001421).

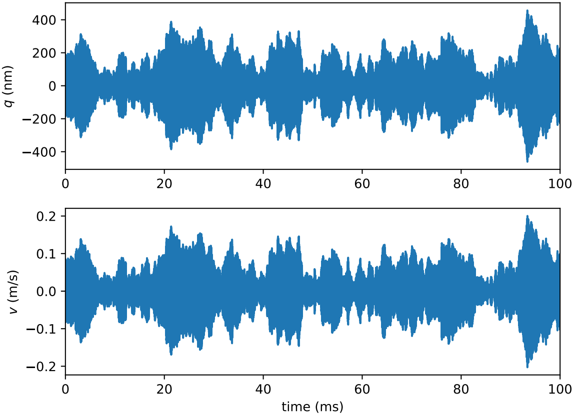
The classical equation of motion for a levitated nanoparticle without feedback control. Includes the effect of gas collisions.[1]

$$\begin{bmatrix} dq \\ dv \end{bmatrix} = \begin{bmatrix} v \\ -\Gamma_0 v + -\omega_0^2 q \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sqrt{\frac{2\Gamma_0 k_B T_0}{m}} \end{bmatrix} dW,$$

dW is the time increment of a Wiener process, $dW \sim \mathcal{N}(0, \sqrt{dt})$, ω_0 is the frequency of the harmonic trap, Γ_0 is the damping due to the gas, T_0 is the temperature of the gas, and m is the mass of the nanoparticle.

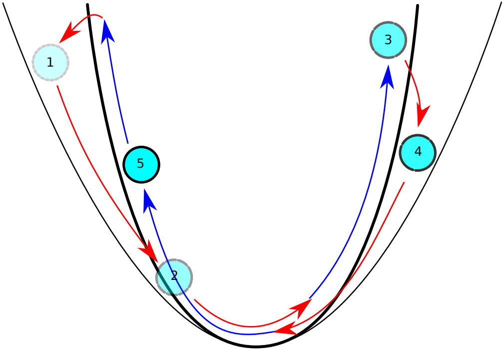
[1] Gieseler, Novotny, and Quidant 2013, Nat Phys (10.1038/nphys2798).

An example time trace of the output position and velocity from simulating this equation. The variation in amplitude and phase is due to stochastic gas collisions.



Feedback cooling works to cool the centre of mass motion of the trapped particle.

- When particle is moving away from the centre of trap $\rightarrow$ potential is stiffened.
- When particle is moving towards the centre of trap $\rightarrow$ potential is shallowed.

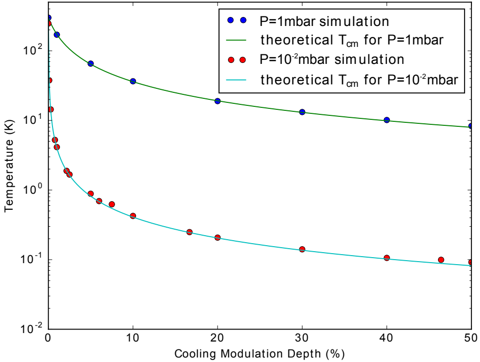


The classical equation with feedback cooling.

$$\begin{bmatrix} dq \\ dv \end{bmatrix} = \begin{bmatrix} v \\ -\Gamma_0 v + [1 + A sin(2\phi(t))](-\omega_0^2 q) \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sqrt{\frac{2\Gamma_0 k_B T_0}{m}} \end{bmatrix} dW,$$

$$\phi(t) = tan^{-1}\left(\frac{v}{q\Omega_0}\right)$$

Plot showing the time-averaged temperature reached in simulation when cooling the particle with different damping rates and modulation depths and it's agreement with the theoretical expression.

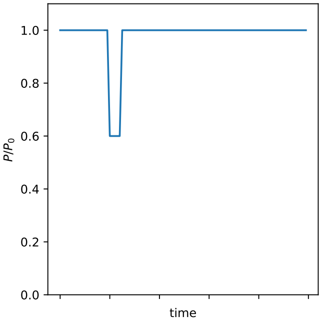


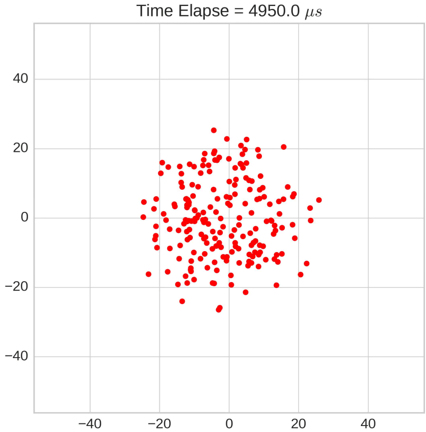
Theoretical expression [2]

$$T_{cm} = \frac{T_0}{1 + \frac{\eta\omega_0}{2\Gamma_0}}$$

[2]Vovrosh et al. 2017, J. Opt. Soc. Am. B (1364/JOSAB.34.001421).

A reduction in the trapping frequency by a large amount for $\frac{1}{4}$ of a period causes squashing. [3]





[3] Rashid et al. 2016, Phys. Rev. Lett. (10.1103/PhysRevLett.117.273601).

The classical equation with squeezing operations applied.

$$\begin{bmatrix} dq \\ dv \end{bmatrix} = \begin{bmatrix} v \\ -\Gamma_0 v + [S_q + A sin(2\phi(t))](-\omega_0^2 q) \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sqrt{\frac{2\Gamma_0 k_B T_0}{m}} \end{bmatrix} dW,$$

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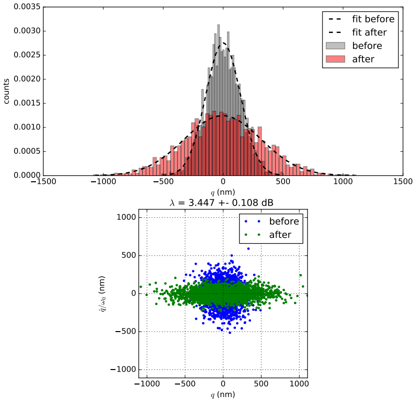
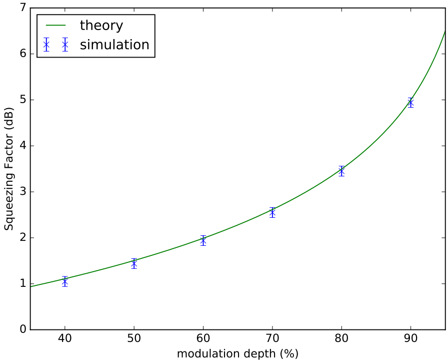
Resulting phase space just after a squeezing operation has been applied. The squeezing parameter λ is calculated as $10log_{10}(\frac{\sigma_{x-after}}{\sigma_{x-before}})$ [4]



[4]Rashid et al. 2016, Phys. Rev. Lett. (10.1103/PhysRevLett.117.273601).

Plot of the squeezing parameter reached in simulations of different modulation depths along with the theoretical value which should be reached.



Theoretical expression [5]

$$\lambda = -5 log_{10} \left(\frac{\mu_{min}(\tau)}{2N_1 + 1} \right)$$

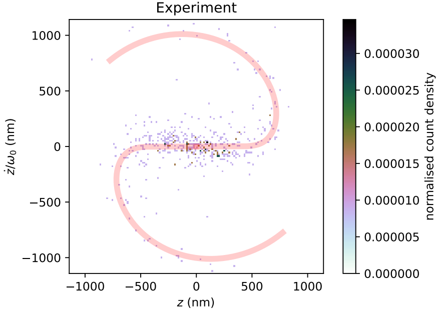
[5] Rashid et al. 2016, Phys. Rev. Lett. (10.1103/PhysRevLett.117.273601).

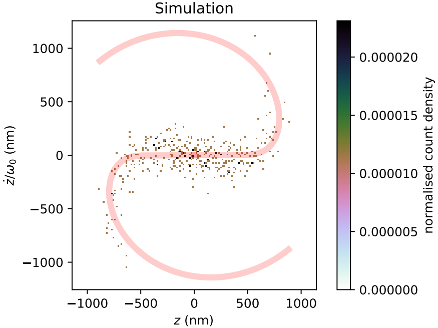
The classical equation with an added Duffing non-linearity.[6]

$$\begin{bmatrix} dq \\ dv \end{bmatrix} = \begin{bmatrix} v \\ -\Gamma_0 v + [S_q + A sin(2\phi(t))](-\omega_0^2 q + \alpha^3 q^3) \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sqrt{\frac{2\Gamma_0 k_B T_0}{m}} \end{bmatrix} dW,$$

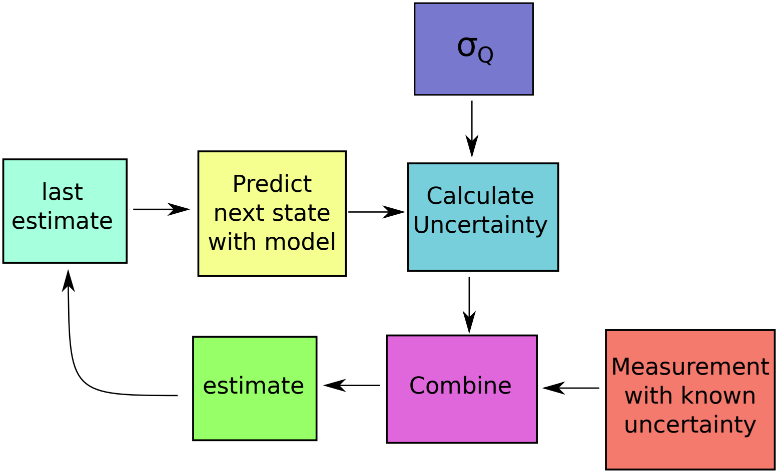
[6] Gieseler, Novotny, and Quidant 2013, Nat Phys (10.1038/nphys2798).

The experimental phase-space plot of performing squeezing pulses with a large modulation depth along with the simulated phase-space plot of squeezing in a Duffing potential.

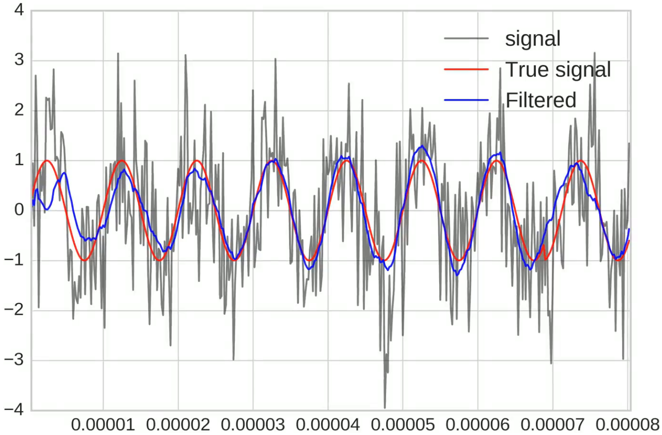




Flow-diagram schematic of how a Kalman filter operates.

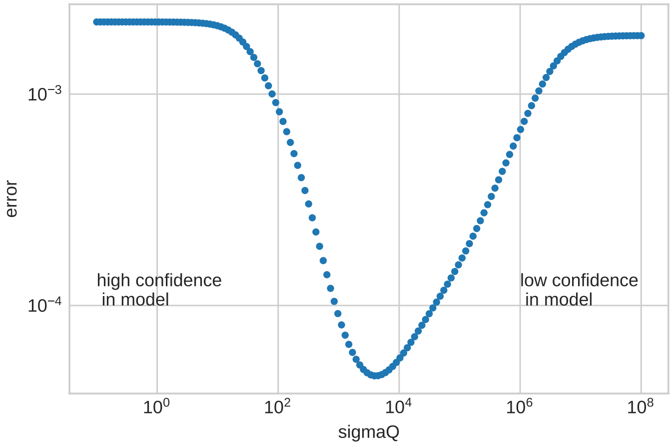


Example of applying a Kalman filter to recover a sine wave from noisy data.



Tuning σ_Q

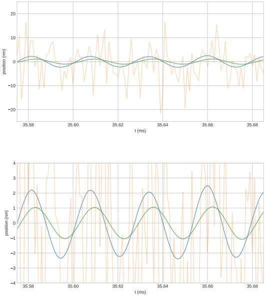
How changing σ_Q effects the error in your estimated signal from the original, un-corrupted, signal.

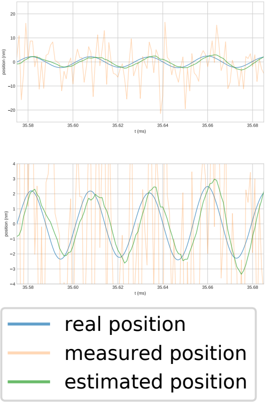


Tuning σ_Q

optimal σ_Q

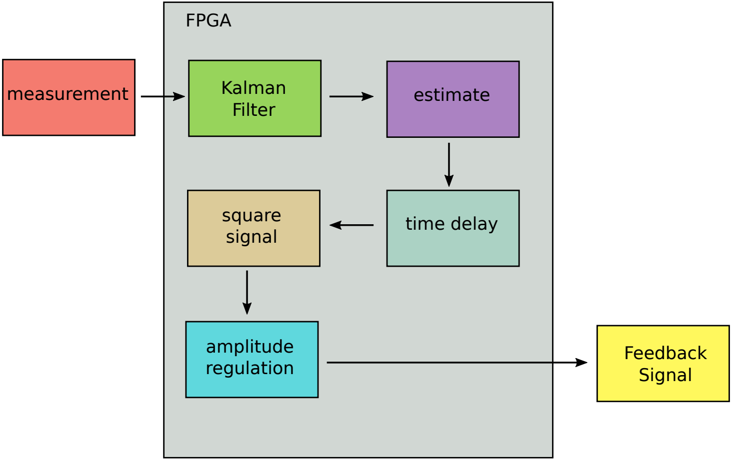
low σ_Q - high confidence in model





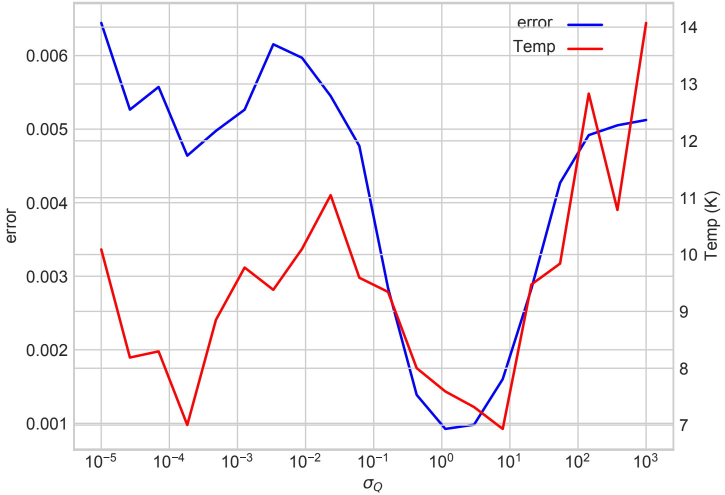
high σ_Q - low confidence in model

Flow-diagram of how we use the Kalman filter to cool the particle.



Tuning σ_Q for Optimal Cooling

$$P = 5 \times 10^{-3} mbar$$



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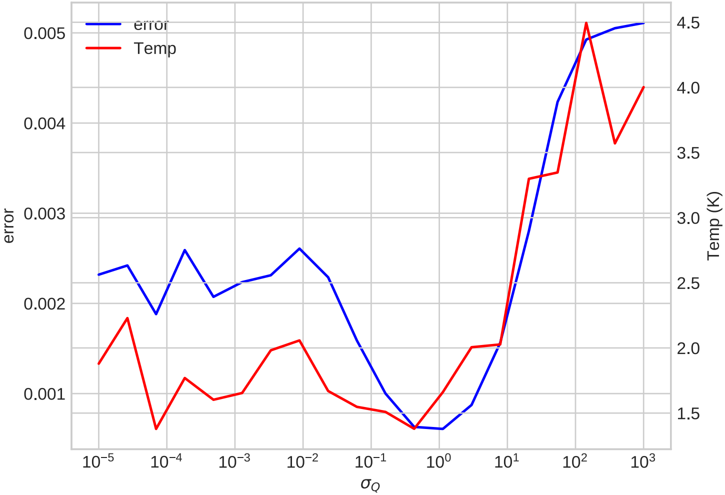
$$P = 1 \times 10^{-3} mbar$$



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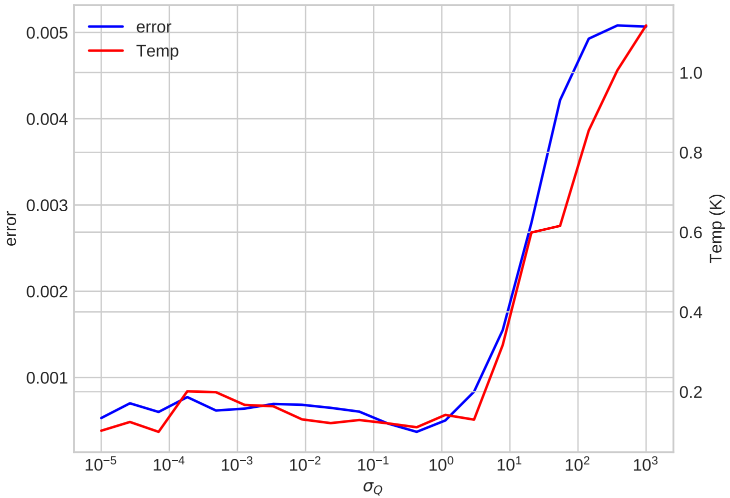
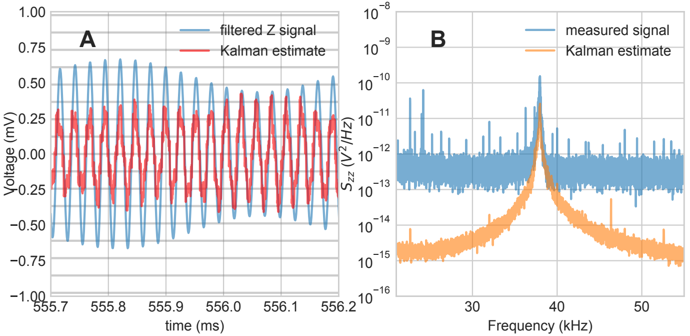
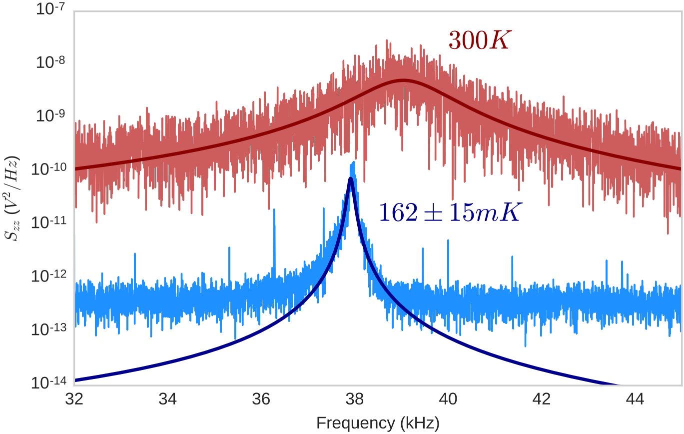
$$P = 5 \times 10^{-5} mbar$$



Real-time tracked signal and post-processed band-pass filtered signal.



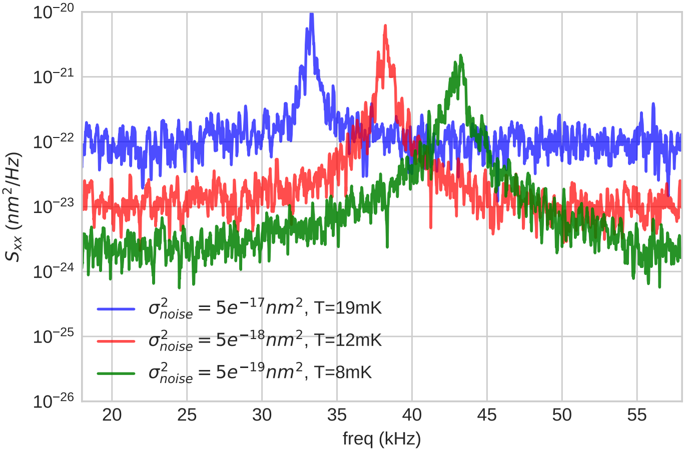
Experimental results of applying a real-time FPGA based implementation of this Kalman filter to feedback cool the motion of a particle at 5.7×10^{-5} mbar.[7]



[7] Setter et al. 2018, Phys. Rev. A (10.1103/PhysRevA.97.033822).

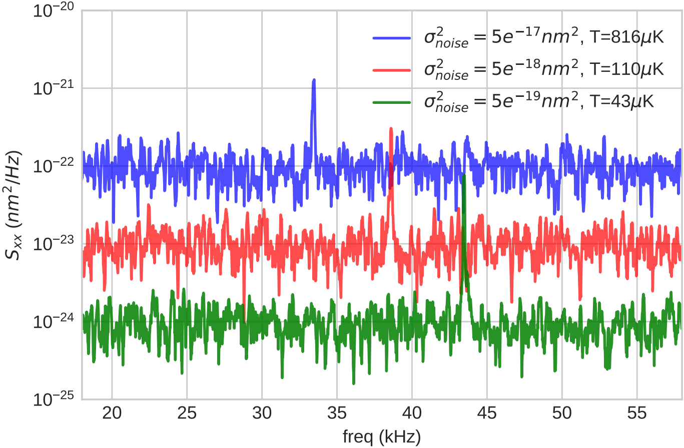
Detection noise from the detector and oscilloscope limits how low in temperature you can track the system with the Kalman filter.

$P = 5.7 \times 10^{-5}$ mbar



Detection noise from the detector and oscilloscope limits how low in temperature you can track the system with the Kalman filter.

$P = 1 \times 10^{-9}$ mbar



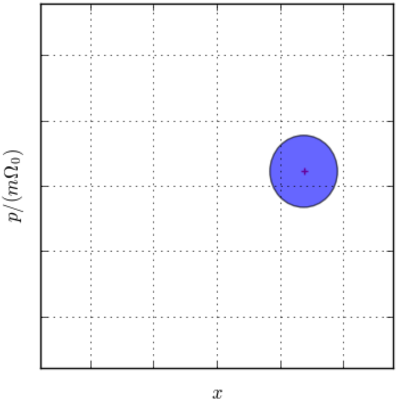
- We have an accurate classical model of the system which agrees with theory and experimental results
- We have an accurate estimation algorithm in the Kalman filter which has been implemented in real time
- We have used this to experimentally perform real-time feedback cooling down to a temperature of $162mK$.
- What about a quantum model of the system? As we get towards the ground state this classical model will break down.

Stochastic Master Equation describing the evolution of the density matrix for a Gaussian state can be reduced to 5 coupled Stochastic Differential Equations for the expectation values and variances of the position and momentum.[8],[9]

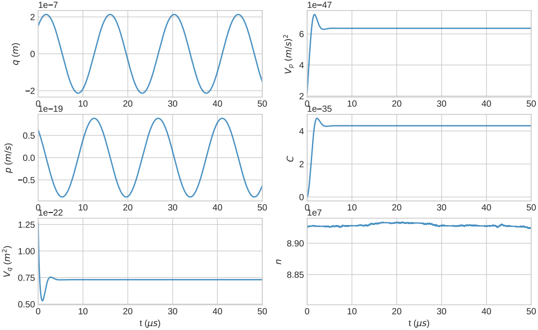
- This model includes photon recoil and gas collisions
- Includes effect of continuous weak measurement
- Includes measurement efficiency $\eta = 0.3\%$
- This model doesn't include any electronics noise

[8] Doherty and Jacobs 1999, Phys. Rev. A (10.1103/PhysRevA.60.2700).
[9] Toroš, Rashid, and Ulbricht 2018, arXiv (1804.01150).

Example output of the quantum simulation in phase-space, a Gaussian state oscillating about the origin.

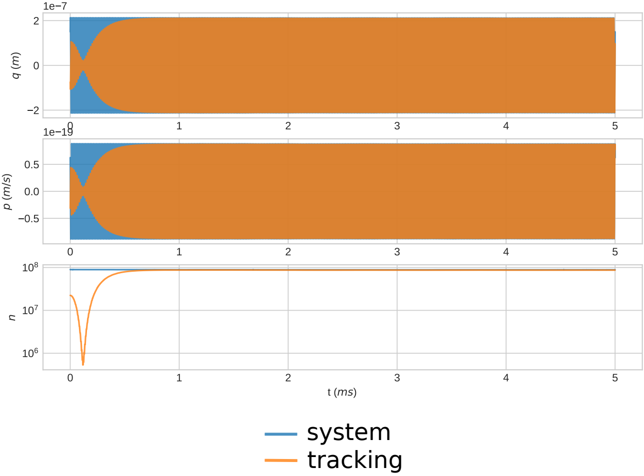


Time trace showing how simulated variables in time. The particles expectation values oscillate as in the classical simulation and the variances quickly damp down to the values predicted by the Heisenburg uncertainty principle.

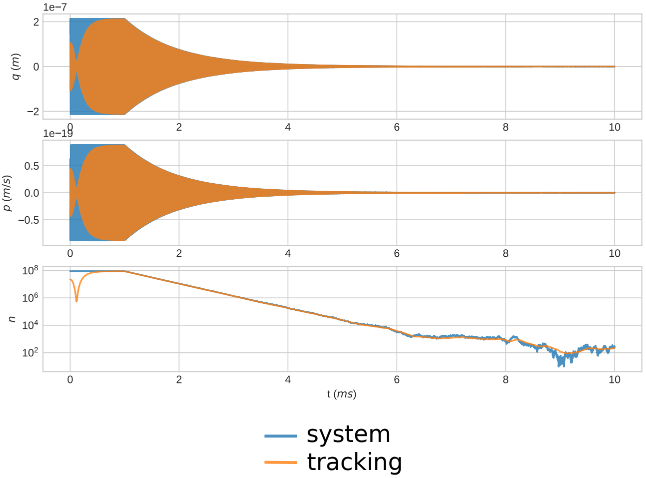


$$\sigma_x \sigma_p \geq \frac{\hbar}{2}, \quad C = \frac{1}{2}\langle xp + px \rangle - \langle x \rangle \langle p \rangle$$

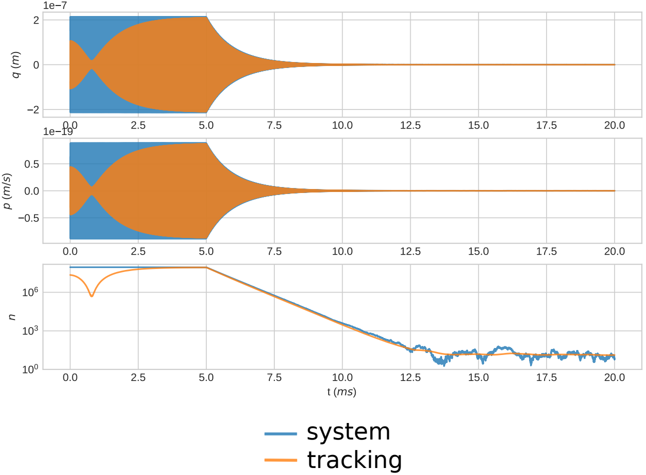
Tracking without feedback



Tracking with feedback $P = 1 \times 10^{-6}$ mbar, $A_{cool} = 0.01$

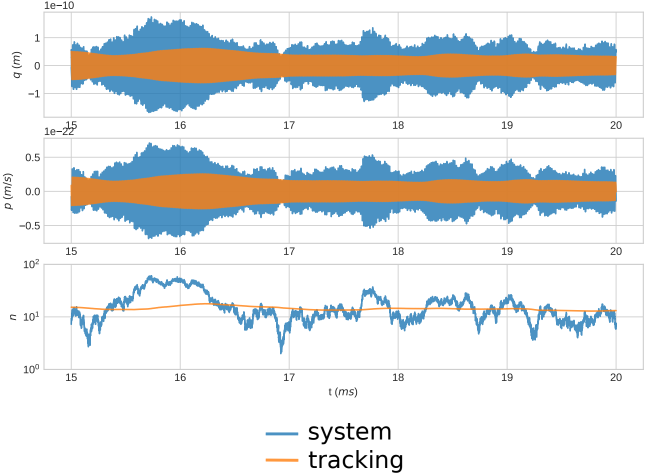


Tracking with feedback at lower pressures. $P = 1 \times 10^{-9}$ mbar, $A_{cool} = 0.01$

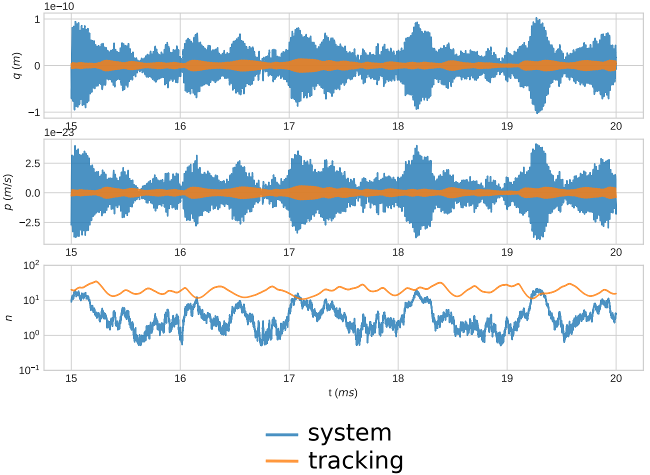


Zoom in of the previous plot after cooling has converged.

$$P = 1 \times 10^{-9} \text{ mbar}, \;\; A_{cool} = 0.01$$



Zoom in of simulation with higher modulation depth after cooling has converged. $P = 1 \times 10^{-9}$ mbar, $A_{cool} = 0.05$



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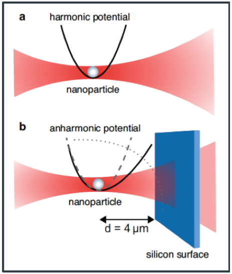
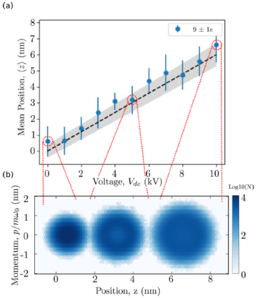
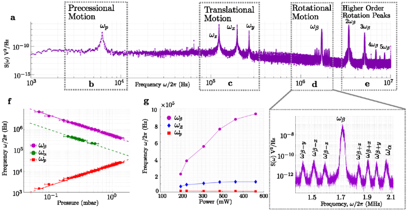
- We have an accurate classical model of the system
- We have developed a real-time Kalman filter on an FPGA using this model
- We have used this Kalman filter to cool the centre of mass motion to $162mK$
- We have a quantum model of the system while it remains in a Gaussian state
- The quantum model shows that cooling to single phonon energies should be feasible at very low pressures with sufficiently good tracking and high enough modulation

For more details on our other work, see our group's poster presented by Chris Timberlake on Wednesday.







The Group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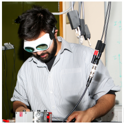
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