Optimal position reconstruction of a thermally driven harmonic oscillator for feedback cooling

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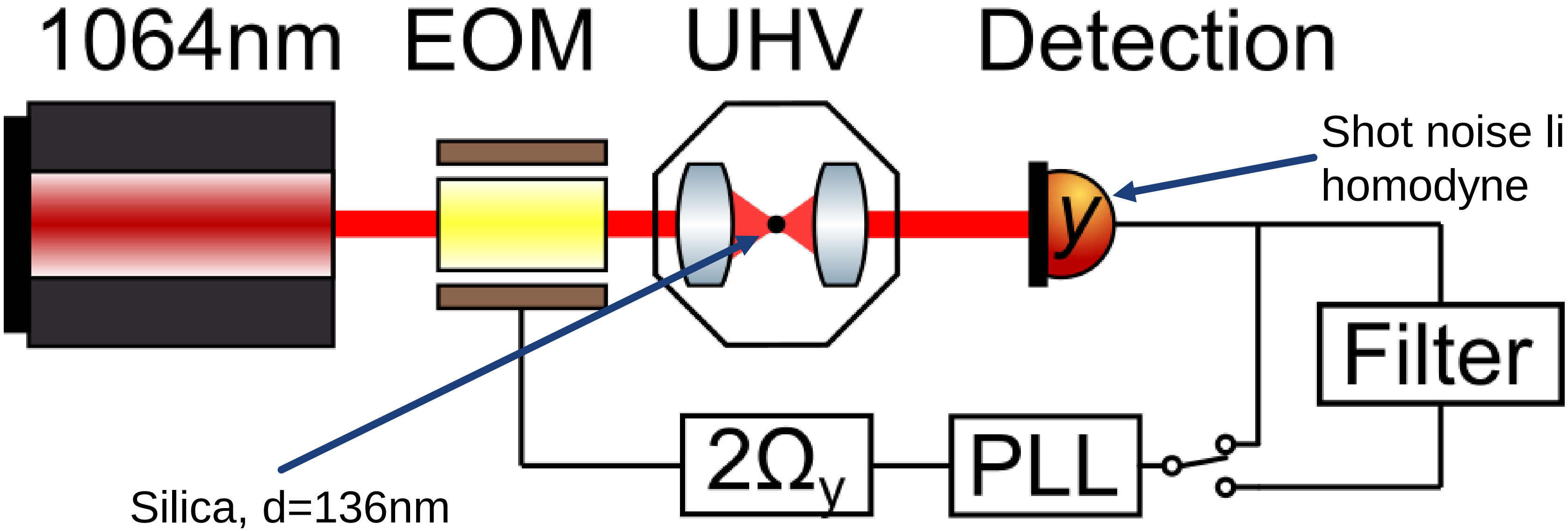
What is the ideal detection and feedback scheme in parametric feedback cooling?

1) PLL in detail
2) Let's introduce a filter to improve the PLL input signal
3) Homodyne detection: Less (photons) can be more (SNR)



Shot noise limited, homodyne

Silica, d=136nm
Pr: < 10e-7 mBar

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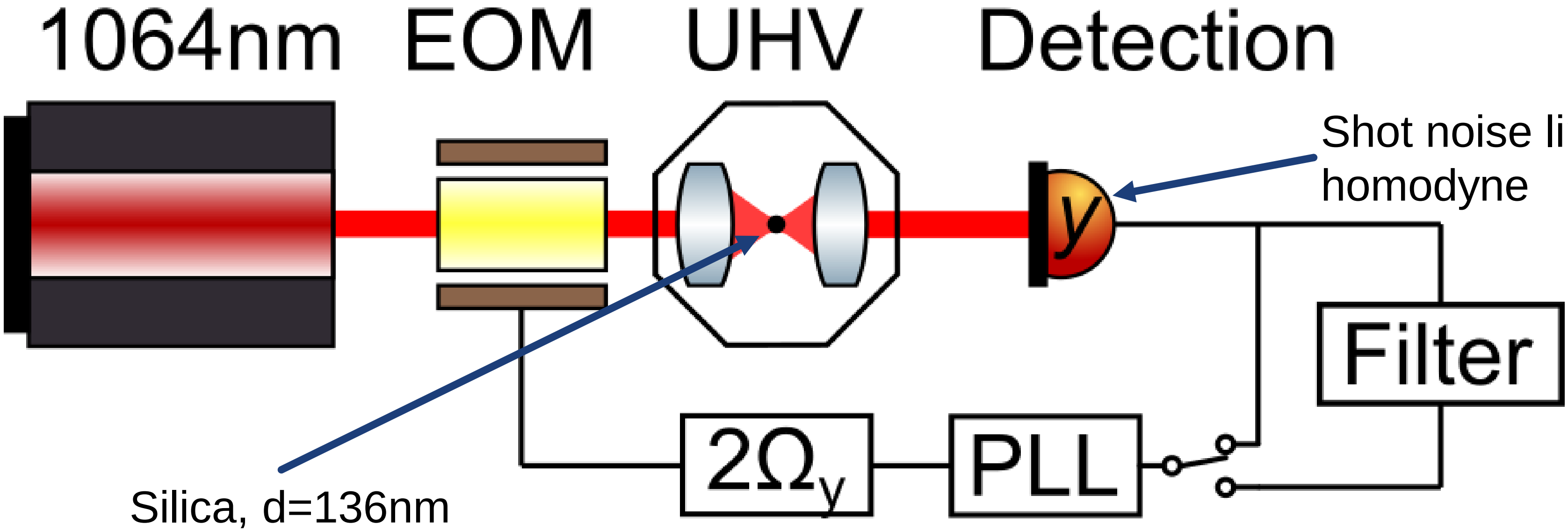

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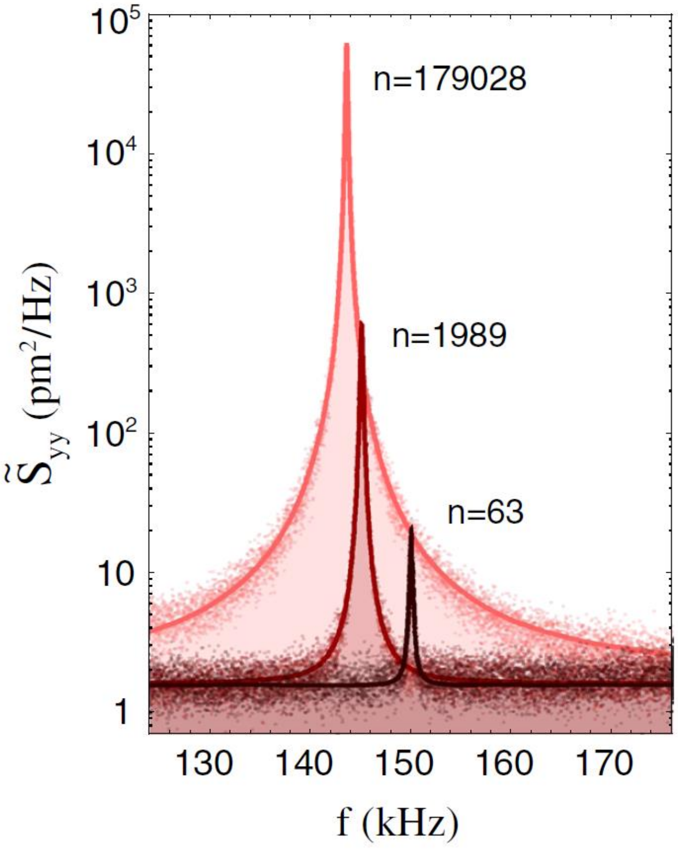
Limit to feedback cooling so far

- Occupation number

$$n = \frac{m\Omega_0^2\langle y^2\rangle}{\hbar\Omega_0}$$

- We cannot cool well below $n{\sim}50$
- Conjecture: Feeding back on a noisy signal results in heating



Jain et al. 2016

Parametric feedback cooling

- $\ddot{y} + \gamma \dot{y} + \Omega_0^2 \big(1 + \eta(t)\big) y = \frac{F_{th}}{m}$
- $E = \frac{1}{2} m \left(\Omega_0^2 y^2 + \dot{y}^2\right)$
- $\langle \dot{E} \rangle = m \langle \Omega_0^2 y \dot{y} + \dot{y} \ddot{y} \rangle = -\gamma \langle m \dot{y}^2 \rangle + \langle \dot{y} F_{th} \rangle - m \langle \eta(t) y \dot{y} \rangle$

cooling

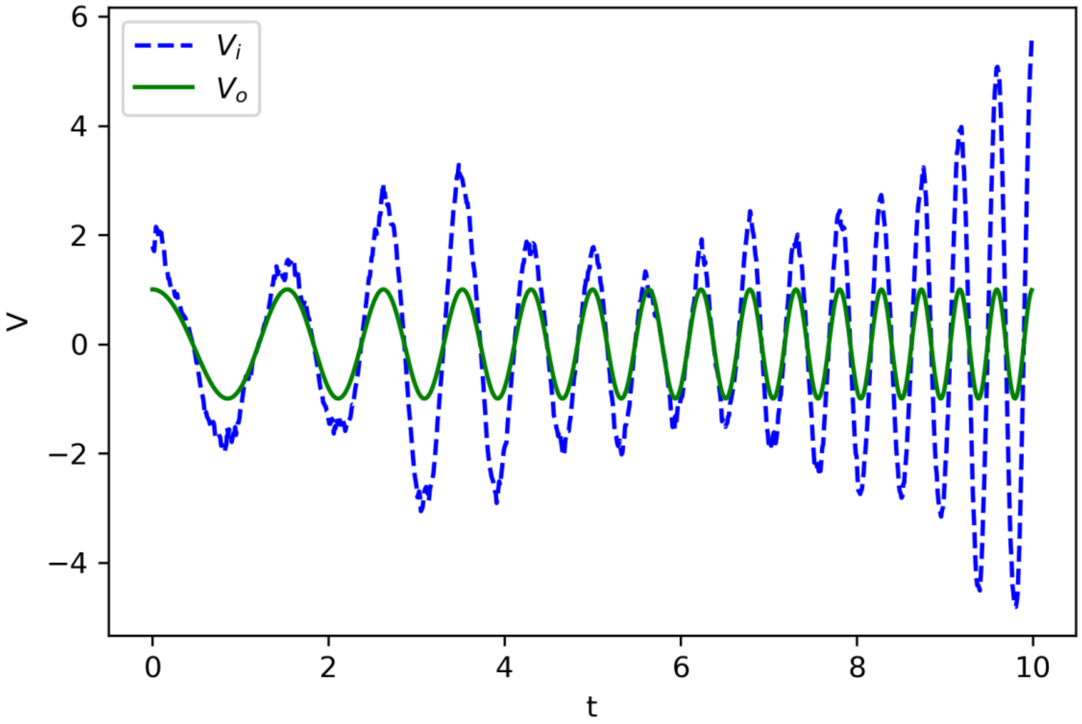
heating

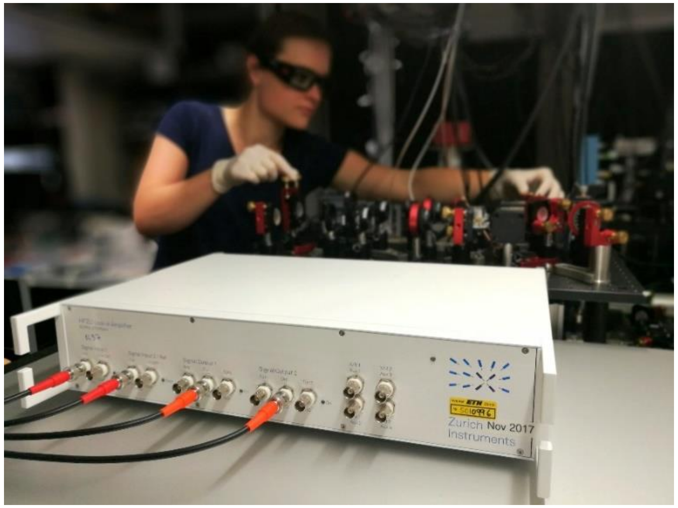
>0: cooling
<0: heating

- $\Longrightarrow$ sign $\eta(t) =$ sign $y\dot{y}$
  - $\eta(t) \propto y\dot{y}$ (*Gieseler et al. 2012*)
  - $\eta(t) = \eta_0 \cos \phi(t)$ *phase-locked* to $y\dot{y}$ (*Jain et al. 2016*)

This can be done by a PLL

Phase-locked loops (PLL)

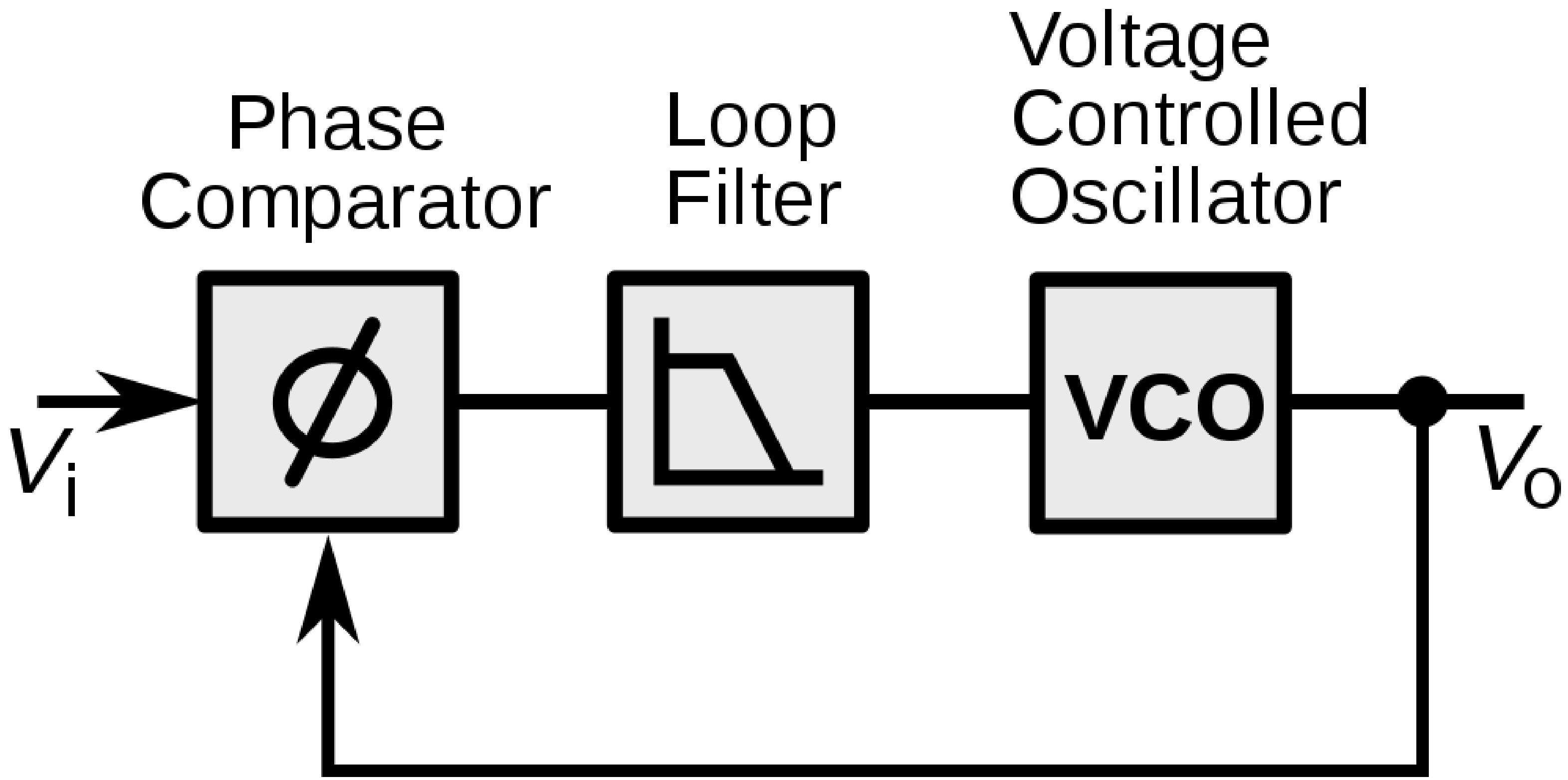




Zurich instruments, HF2LI

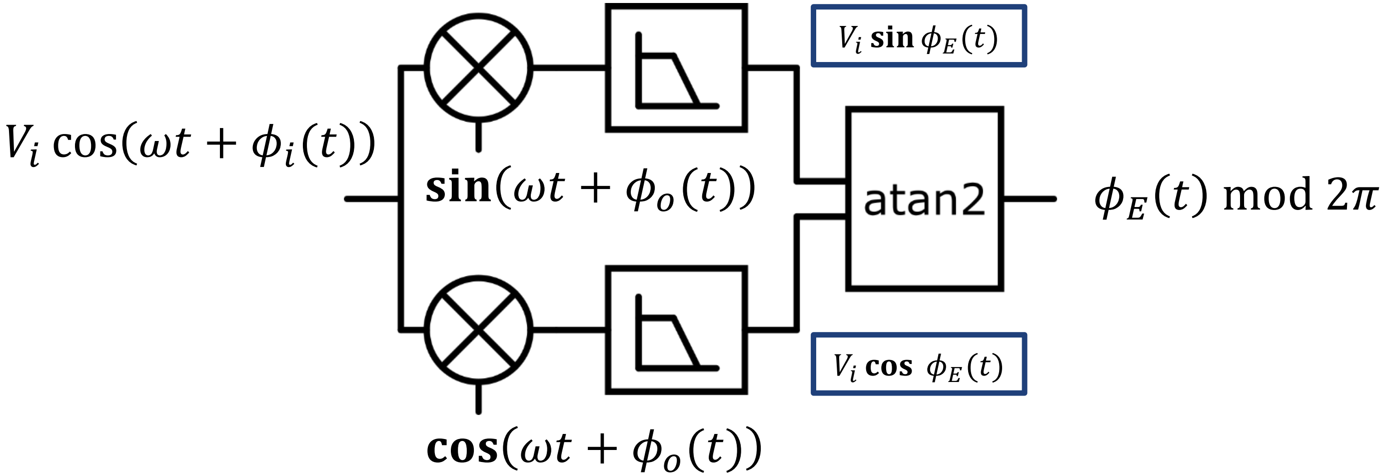
- A PLL follows a noisy oscillation only in phase.
- Amplitude is constant

Schematic of a PLL



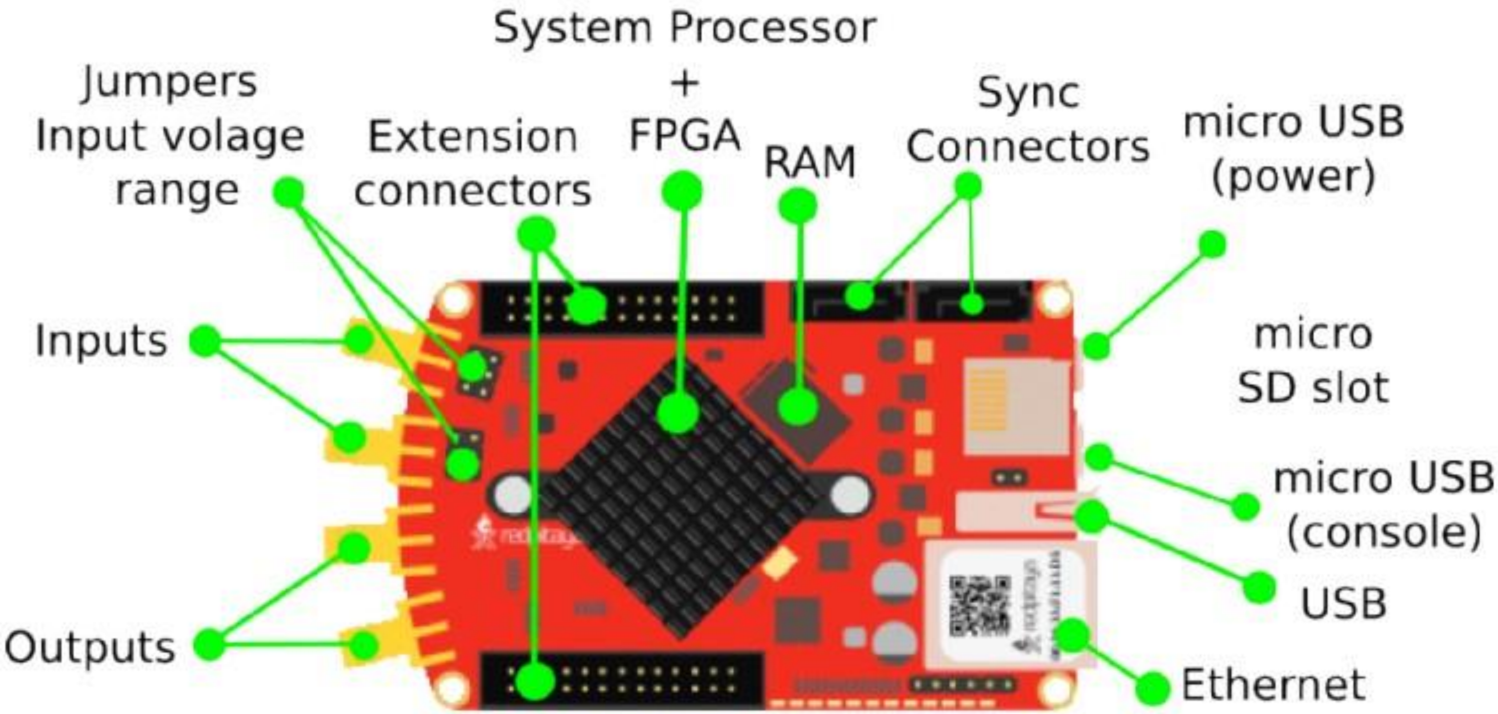
From Wikipedia: *https://en.wikipedia.org/wiki/Phase-locked_loop*

Implementation of a Phase detector using two Quadratures



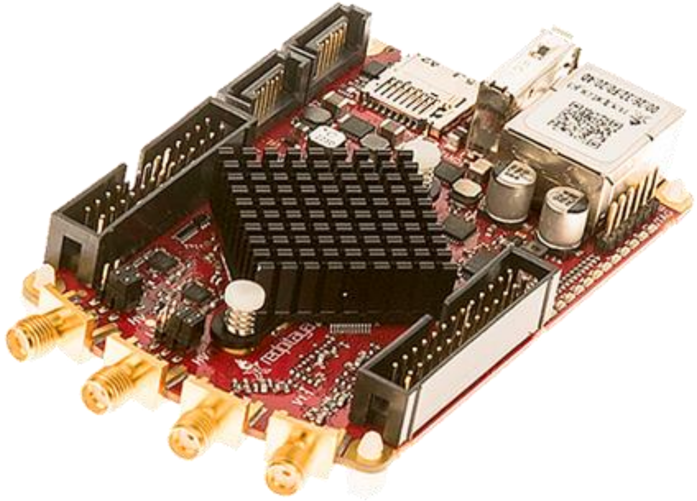
- It’s a lock-in amplifier
- atan2 can be implemented efficiently in digital electronics using the *CORDIC* algorithm

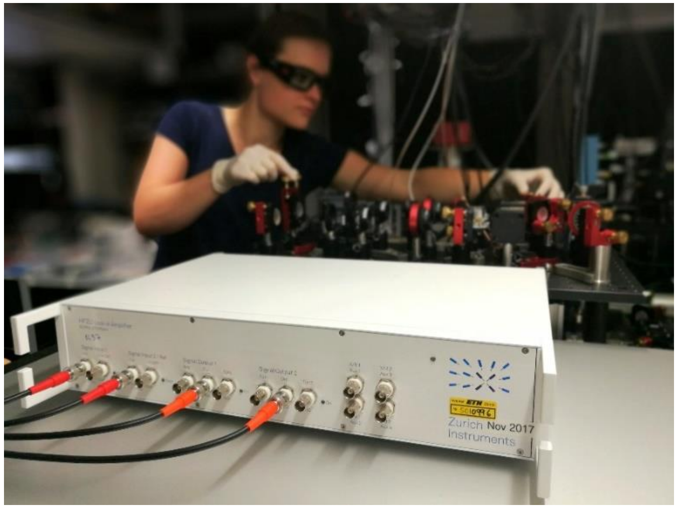
Let’s implement it on *redpitaya!*



From *http://redpitaya.readthedocs.io/en/latest/developerGuide/125-10/vs.html*

We can cool using redpitaya-based PLLs, freeing Zurich instruments channels.





- 2 PLLs
- Configurable from PC
- High input noise < $1 \frac{\mu V}{\sqrt{Hz}}$
- $

- Many other functionalities
- Ultra low noise $5 \frac{nV}{\sqrt{Hz}}$
- $$$

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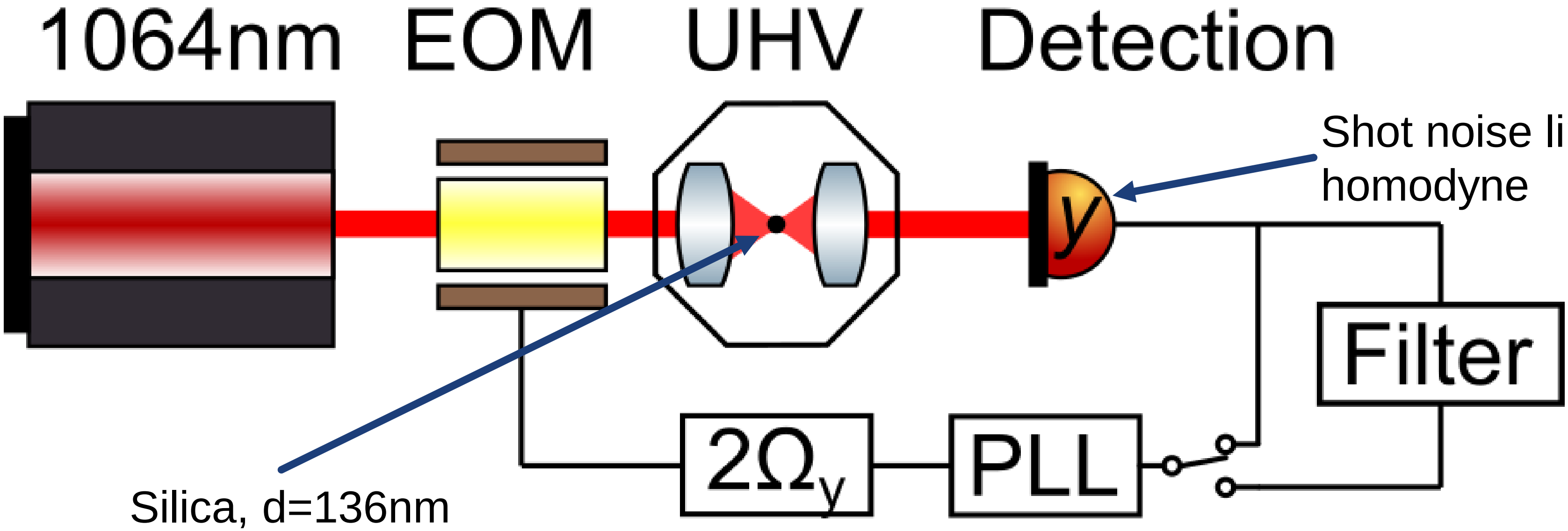

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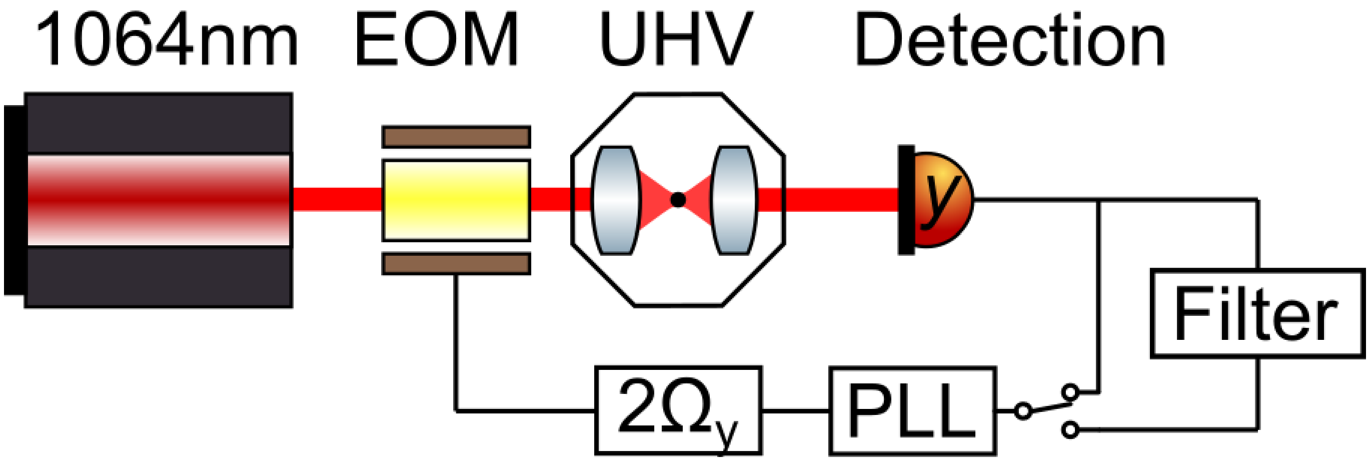
Can we improve the PLL input signal in order to cool further?

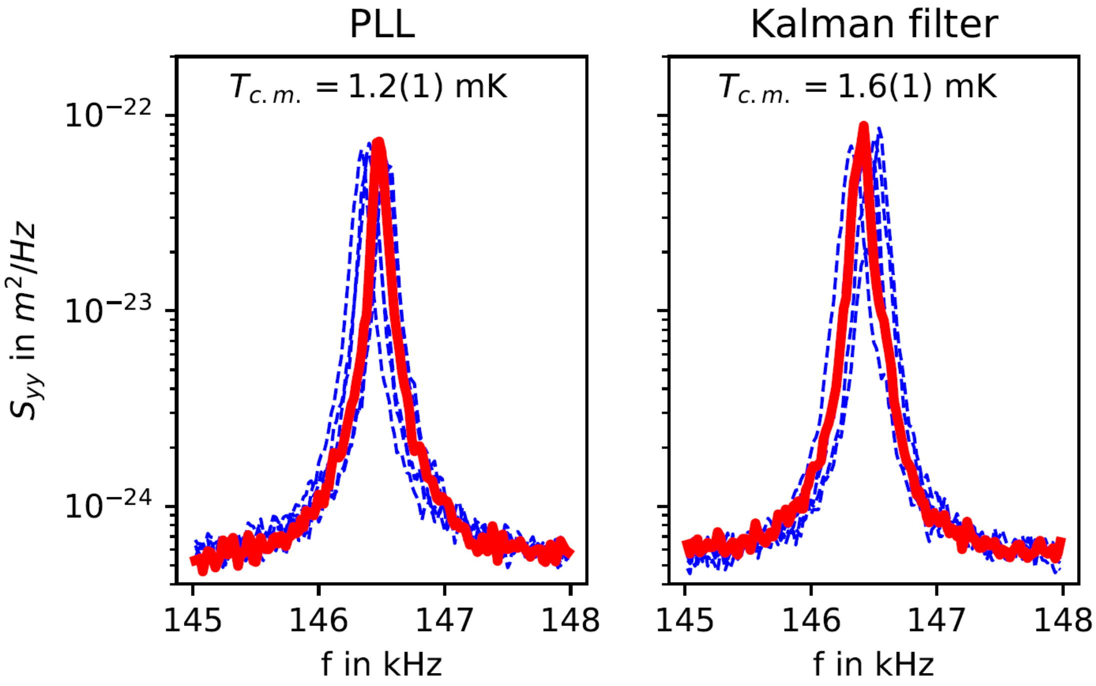


- Detector signal is obscured by shot noise: $y_{det} = y + n$
- Some filter estimates the position signal $\hat{y} = F(y_{det})$
- → The phase error made by the PLL is smaller

M. Jost et al. 2018
A. Setter et al. 2018

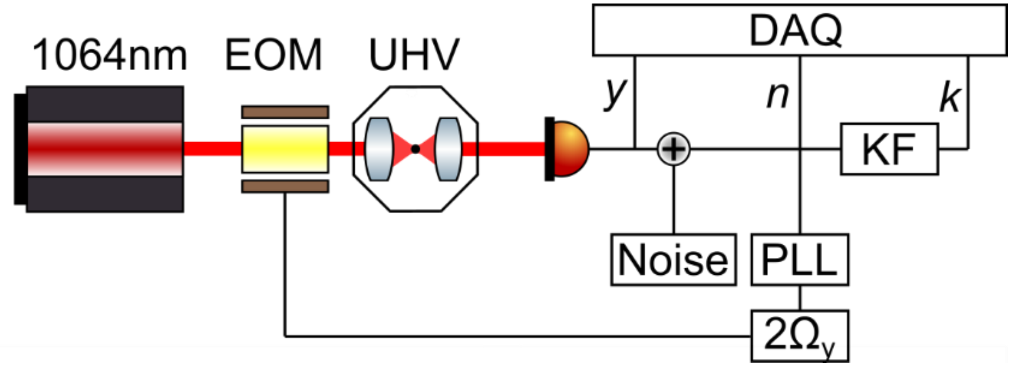
FPGA Implementation of a Kalman-based motion Estimator for Levitated Nanoparticles

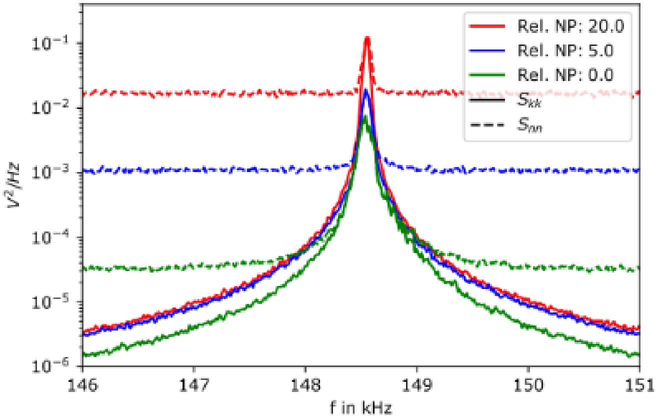
J. Liao et al. (2018), *under review;* In collaboration with the integrated systems Lab, ETH

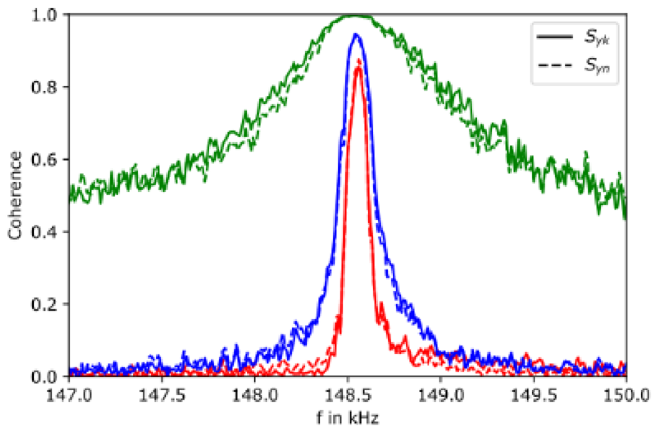


Kalman filter seems **not** to be improving the cooling

Is the Kalman filter doing its job? - Yes!



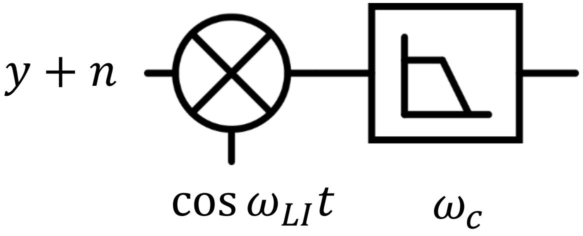




- out-of-band noise strongly suppressed
- information content maximal, since coherence cannot increase (for linear filters)
- → SNR is increased as much as possible

$$C_{xy} = \frac{\left|S_{xy}\right|^2}{S_{xx}S_{yy}}$$

So, why is cooling not improved? Let’s look at the input filter of the PLL



- It’s basically a lock-in amplifier
- Signal is shifted in frequency and low-pass filtered
- The effective input filter (1st order):

$$|\mathrm{H}|^2 = \frac{\omega_c^2}{\omega_c^2+(\omega-\omega_{LI})^2}$$

- Compare to $S_{yy}(\omega)$:

$$S_{yy}(\omega) = \frac{\Omega_0^2\gamma^2}{\left(\omega^2-\Omega_0^2\right)^2+\gamma^2\omega^2} \approx \frac{\gamma^2/4}{(\omega-\Omega_0)^2+\gamma^2/4}$$

A Kalman filter before the PLL cannot increase the cooling efficiency much

- Only the difference in the filter transfer function matters, and it’s negligible
- Could another pre-filter do?
  - Probably not. Since for Gaussian, additive noise, the Kalman filter is optimal
- Where could it be useful at all?
  - $y\dot{y}$ - type feedback
  - In non-stationary experiments (free-fall)
  - …

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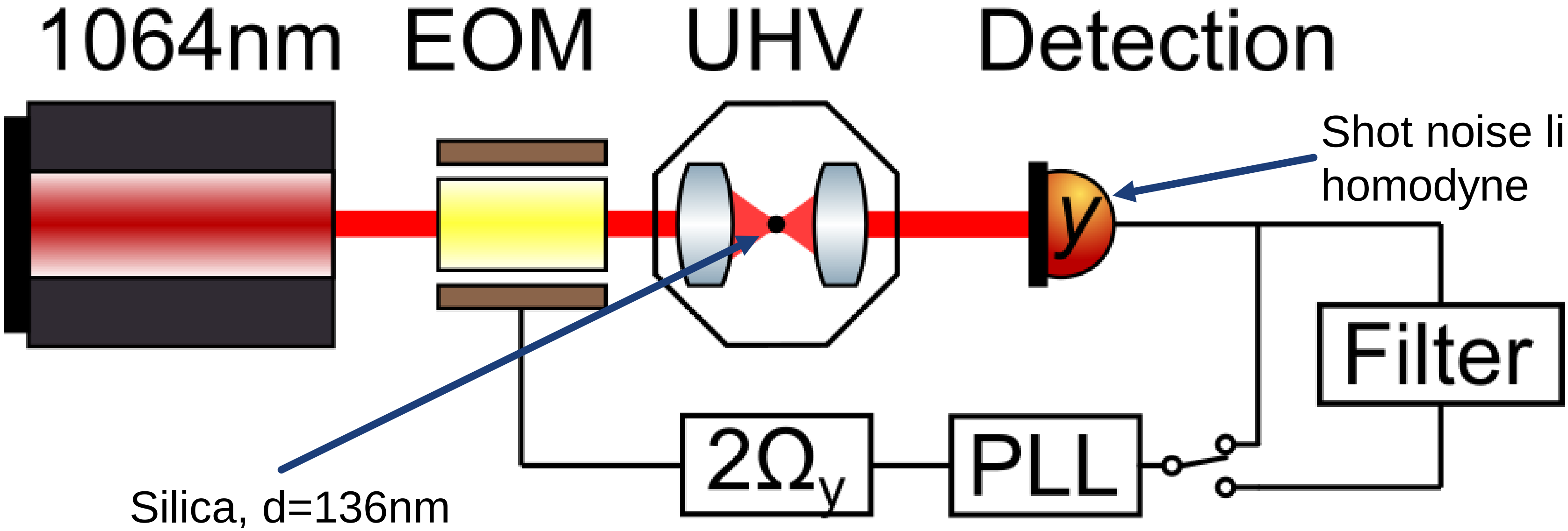

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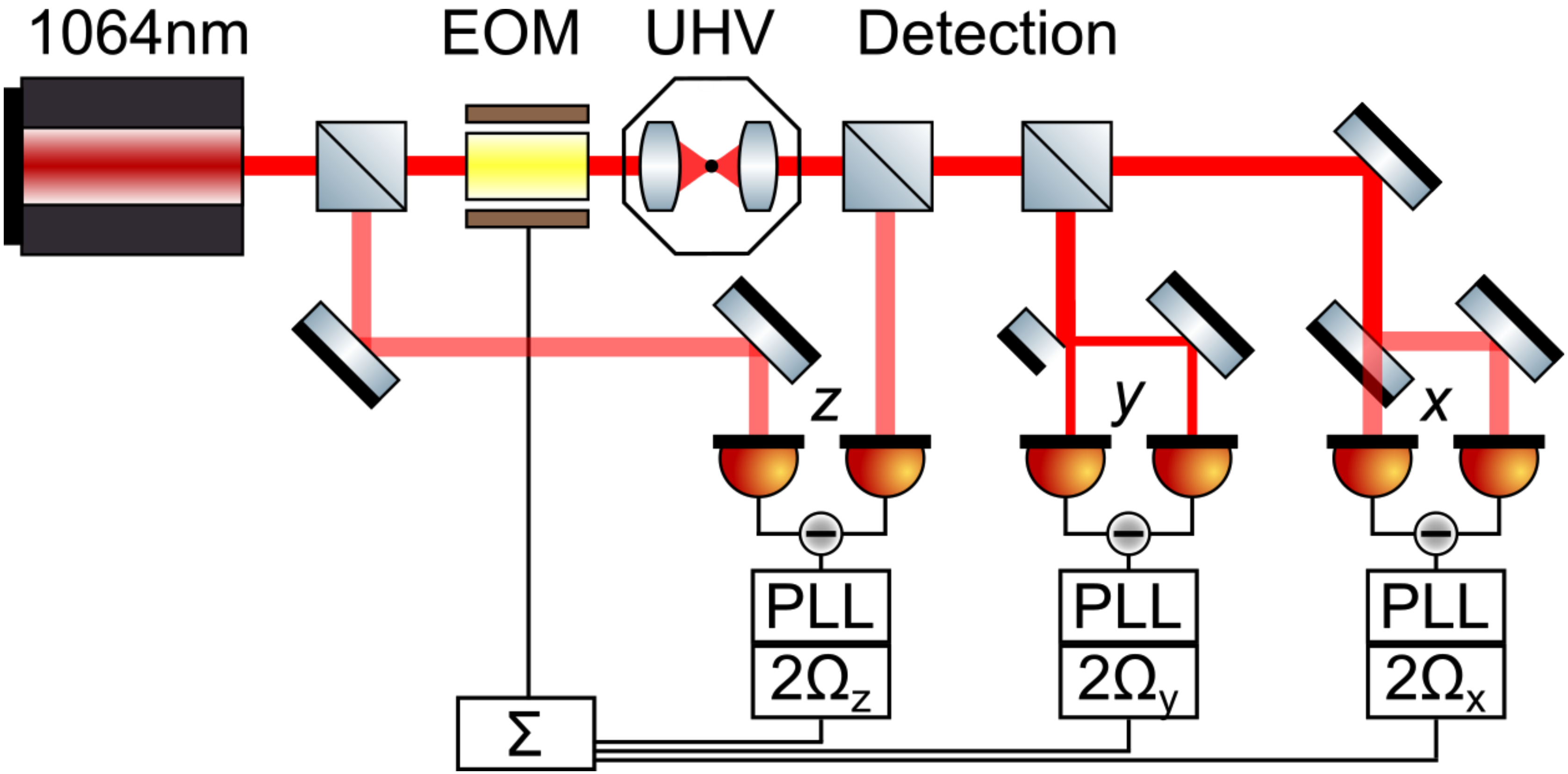
Blocking the middle of the detection beam to enhance SNR: Setup



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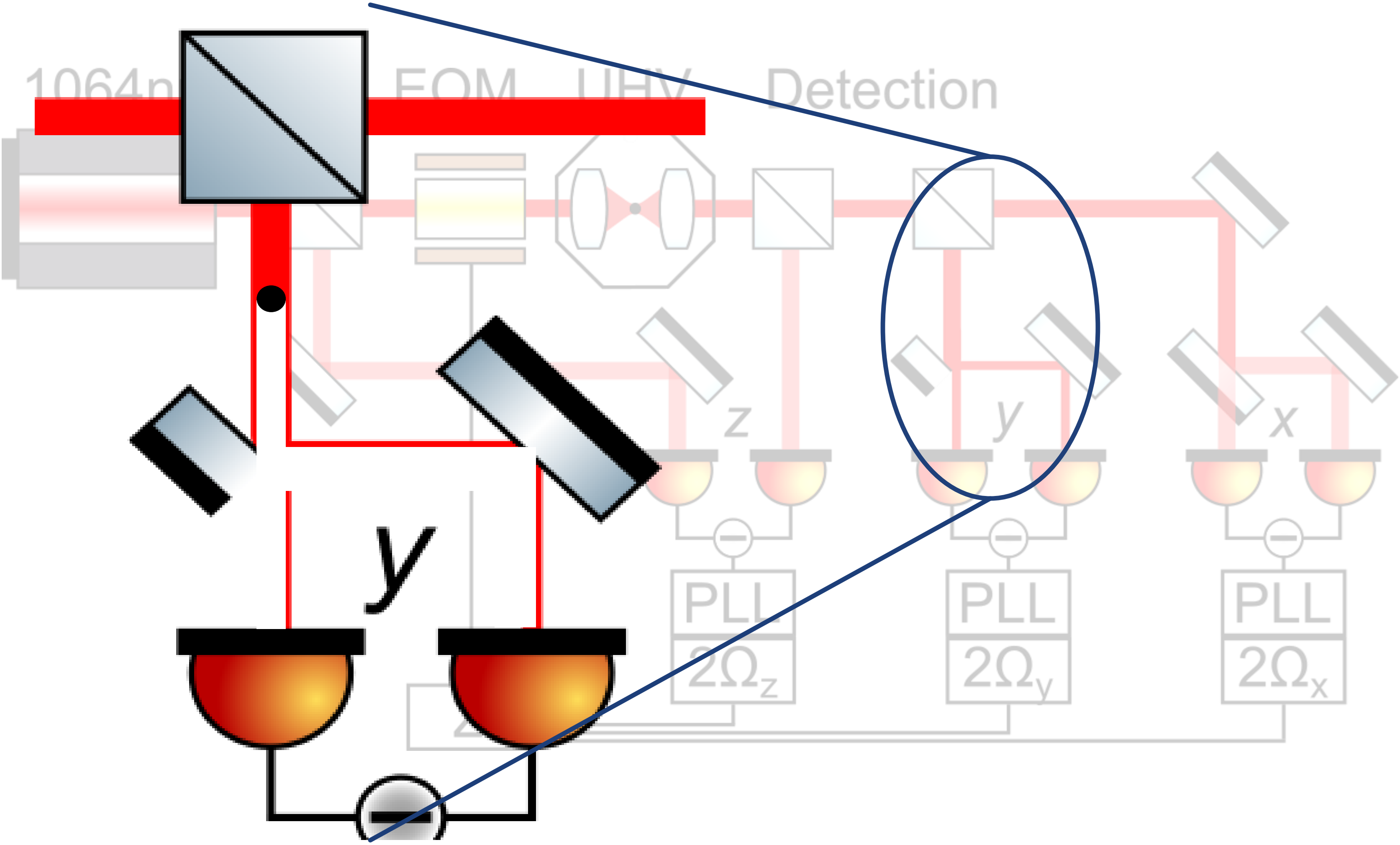
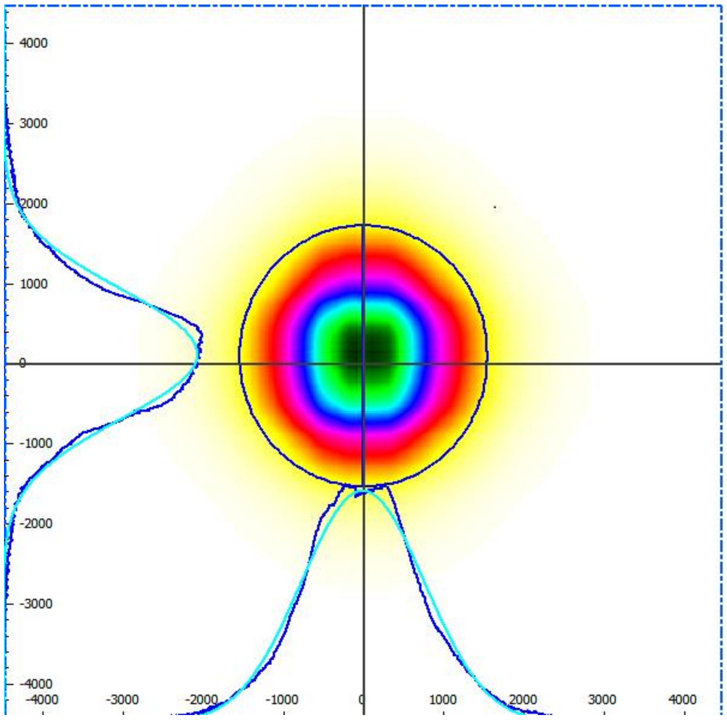
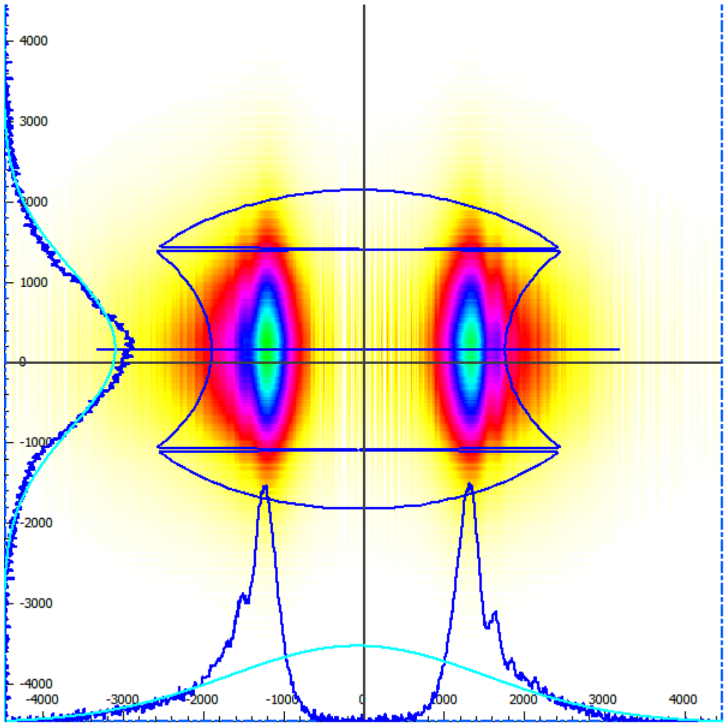


Measurements: Beam profile at detector

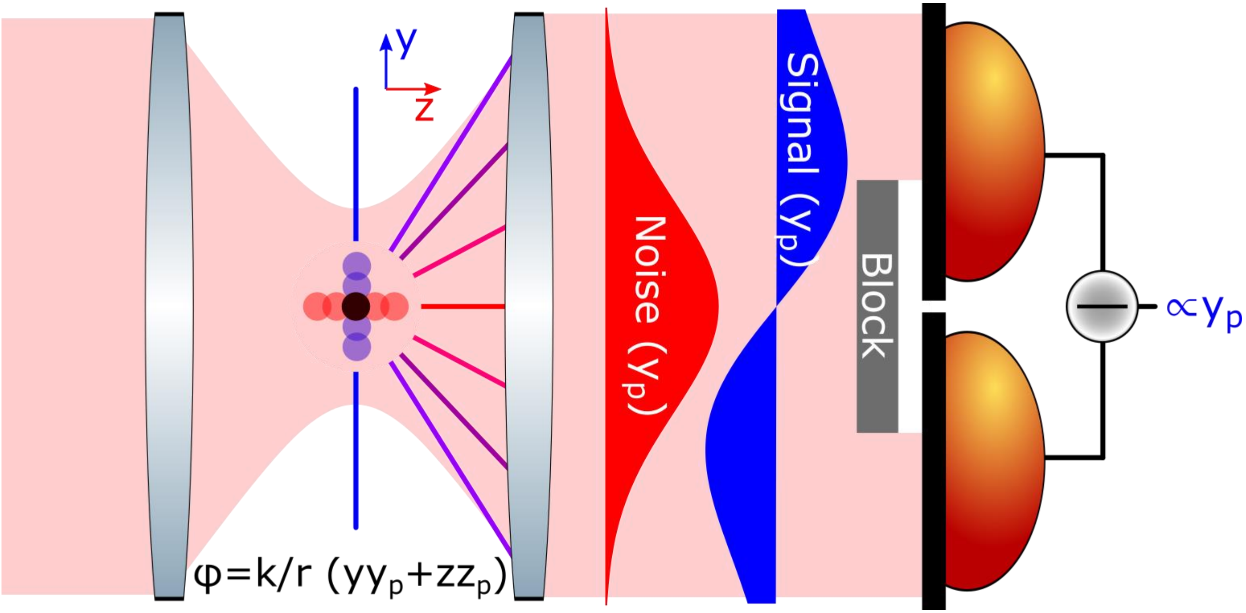


$2w_y \approx 3.4mm$



$t \approx 1.5mm$

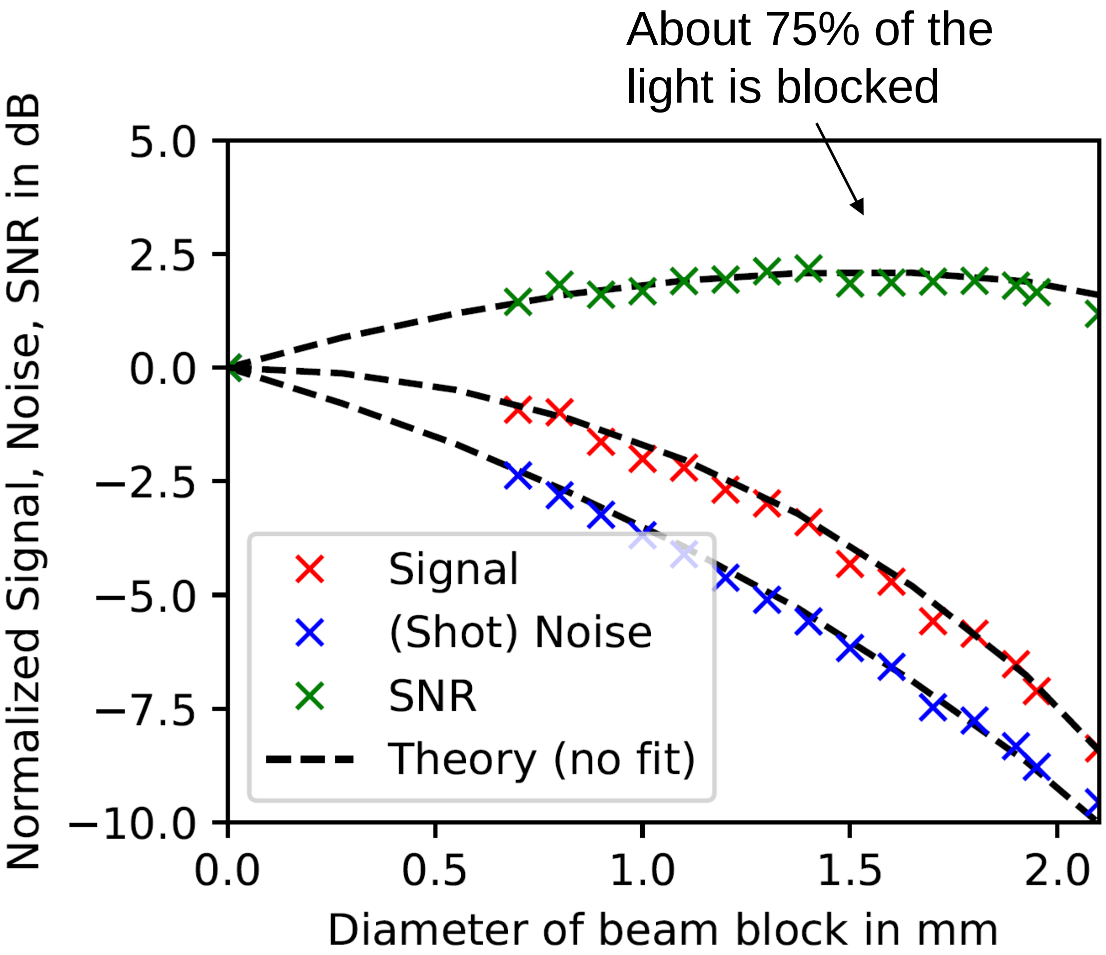
SNR is spatially dependent in our homodyne measurement



- The particle position is modulated onto only one component of the field
- Not every photon carries the same information, but the same noise

SNR increases when inserting the beam block

- Fit includes
  - dipole radiation pattern
  - focal length, NA
  - beam waist
- There's an ideal reference (LO) field, which is not a Gaussian



Conclusion and Outlook

- Filering:
  - We built a redpitaya-based PLL that can replace expensive Lab equipment for parametric feedback cooling
  - As long as we use a constant amplitude parametric feedback, a well tuned PLL is ideal and we don’t need to improve the signal quality by filters before the PLL
  - The open question remains, what is limiting the cooling performance of a PLL?

- Detecting:
  - Ideally, the reference field is mode-matched to the dipole-field for each direction