Quantum Duality of the Noncommutative Field Theories related by θ-exact Seiberg-Witten map and its cosmological consequence*

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- *Main collaborations with J. You, C.P. Martin, R. Horvat:*
- *Spacetime noncommutativity and ultra-high energy cosmic ray experiments*, Phys.Rev. D83 (2011) 065013, arXiv:1005, with Zagreb group.
- *Forbidden and invisible Z boson decays in a covariant θ-exact noncommutative standard model*, J.Phys. Gbf 41 (2014) 055007, arXiv:1703.3209, with Zagreb group.
- *θ-exact Seiberg-Witten maps at order e^3*, Phys. Rev. D **91** (2015) no.12, 125027, arXiv:1501.00276, with J.You,
- *Photon self-interaction on deformed spacetime*, Phys. Rev. D **92** (2015) 12, 125006, arXiv:1510.08691, with Zagreb group.
- *Super Yang-Mills and θ-exact Seiberg-Witten map: absence of quadratic noncommutative IR divergences,* JHEP **1605** (2016) 169, arXiv:1602.01333, with C. P. Martin and J. You.

- *Equivalence of quantum field theories related by the θ-exact Seiberg-Witten map,* Phys. Rev. D **94** (2016) no.4, 041703, arXiv:1606.03312, with C. P. Martin and J. You.
- *Quantum duality under the θ-exact Seiberg-Witten map,* JHEP **1609** (2016) 052, arXiv:1607.0154, with C. P. Martin and J. You.
- *UV-IR mixing in nonassociative Snyder ϕ^4 theory* Phys. Rev. D **97** (2018) no.5, 055041, arXiv:1606.03312, Zagreb group.
- *Spacetime Deformation Effects on the Early Universe and the PTOLEMY Experiment,* Phys. Lett. B **772** (2017) 130, arXiv:1703.04800, with R.Horvat, J.You.
- *Nonassociative Snyder ϕ^4 Quantum Field Theory* Phys. Rev. D **96** (2017) no.4, 045021, arXiv:1711.03312, Zagreb group.
- *Inferring type and scale of noncommutativity from the PTOLEMY experiment,* Eur.Phys.J. C**78** (2018) no.7, 572, arXiv:1711.09643, with R.Horvat, J.You.

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Noncommutative U(1) gauge field theory

Seiberg-Witten (SW) map based NCQGFT

Bubble diagrams in the Seiberg-Witten U(1)

The 2nd order in gauge field θ-exact SW maps

The 4-photon tadpole diagram

Seiberg-Witten map equivalence between QGFT?

One-loop 1PI 2-point function via background field method

Exploring the Universe using Neutrinos

UHE cosmic ray RICE experiment: $\nu + N \to \nu + X$

Ptolemy Experiment

Ptolemy experiment: $\nu_e + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-$, $(\nu_e + n \to p + e^-)$

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Scattering: $e\nu_R \to e\nu_R$

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Noncommutative quantum field theories (NCQFT)

- (1) Moyal spacetime: x^μ promoted to hermitian operators, $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ –antisymmetric matrix of dimension $(\mathit{length})^2$.
- (2) Snyder spacetime: $[x_\mu, x_\nu] = i\beta M_{\mu\nu},\ [p_\mu, x_\nu] = -i(\eta_{\mu\nu} + \beta p_\mu p_\nu),\ [p_\mu, p_\nu] = 0.$
- (3) κ-Minkowski spacetime: $[x_\mu, x_\nu] = \frac{i}{\kappa}(x^\mu \xi^\nu - x^\nu \xi^\mu)$, $[p^\mu, x_\nu] = ih\delta^\mu_\nu + \frac{i}{\kappa}(p^\mu \xi_\nu + p_\nu \xi^\mu),\ [p_\mu, p_\nu] = 0.$
- Tree level: Breaking of Lorentz invariance / angular momentum conservation (1), momentum conservations (2,3).
- 1-loop: UV/IR mixing in ϕ^4 with NC-exact in all above.
  - Exists on Moyal for constant θ-exact U(N) NCYM with/without Seiberg-Witten maps.
  - Cancelations of UV, quadratic IR and log-IR divergences in θ-exact U(N) NCYM with/without SW maps by $\mathcal{N} = 4$ SUSY.
- Constant $\theta \to$ Heisenberg algebra $\to \Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}|$.
- Generates new couplings: γ-self and contact $\nu - \gamma$. Phenomenology: $g-2$, μ_ν-EM, γ/ν-dispersions, $Z(\gamma_{pl}) \to \bar{\nu}\nu, \gamma\gamma$, scatterings,..., forbidden in SM $Z(\bar{Q}Q) \to 2\gamma$, $K \to \pi\gamma$,....

Motivation for NCQFT: $\nu + N \rightarrow \nu + X$

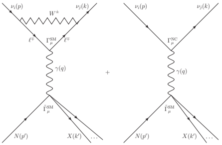

Figure: 1.

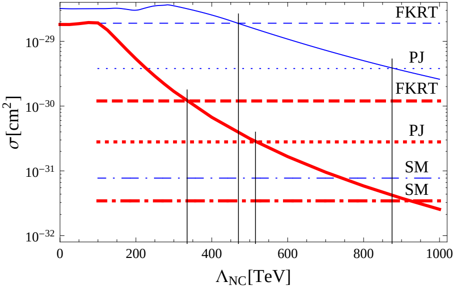

Figure: 2.

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The Moyal-Weyl star product

- Moyal-Weyl Star product realizes one simplest NC structure

$$
\begin{aligned}
f \star g =& f(x) e^{\frac{i}{2}\partial_{x^\mu}\theta^{\mu\nu}\partial_{y^\nu}} g(y)|_{y\to x} \\
=& \frac{1}{2\pi^n}\int d^n k d^n k' \tilde{f}(k)\tilde{g}(k') exp[i(k_i + k'_i)x^i \underbrace{-i\theta^{ij}k_i k'_j}_{\text{nonlocal}}]
\end{aligned}
$$

$$
\longrightarrow [x^i \overset{\star}{,} x^j] = x^i \star x^j - x^j \star x^i = i\theta^{ij}
$$

- Properties of the Moyal product
  - Associativity

$$
\begin{aligned}
f \star (g \star h) = & \int d^d k_1 dk_2 dk_3 \tilde{f}(k_1)\tilde{g}(k_2)\tilde{h}(k_3) \\
& \exp[i(k_1 + k_2 + k_3)x + i(\tfrac{k_1\theta k_2}{2} + \tfrac{k_1\theta k_3}{2} + \tfrac{k_2\theta k_3}{2})] \\
& = (f \star g) \star h = f \star g \star h
\end{aligned}
$$

  - Cyclicity in integration

$$
\int f_1 \star \cdots \star f_n = \int f_n (f_1 \star \cdots \star f_{n-1}) = \int f_n \star f_1 \star \cdots \star f_{n-1}
$$

Noncommutative quantum gauge field theory (NCQGFT)

- NCGFT on Moyal space

$$S = -\frac{1}{2}\int F_{\mu\nu}\star F^{\mu\nu}, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \stackrel{\star}{,} A_\nu]$$
$$\delta_\Lambda A_\mu = \partial_\mu \Lambda + i[\Lambda \stackrel{\star}{,} A_\mu],\ \delta_\Lambda F_{\mu\nu} = i[\Lambda \stackrel{\star}{,} F_{\mu\nu}];$$

- Introducing Moyal product turns the commutative $\mathrm{U}(1)$ theory which is a local theory into the noncommutative (NC) $\mathrm{U}_\star(1)$ gauge theory which is a nonlocally interacting theory.
- Thanking to the cyclicity field theories on Moyal space admits relatively simple pertubative quantization, at least when the metric is Euclidean, which we will presume within this talk.
- Nonlocal factor regularize part of the Schwinger parameterized loop integral, turning it into integral over modified Bessel function K_n's which is IR divergent. Such phenomenon is called *UV/IR mixing*.

NCQGFT, an example of neutral fermion

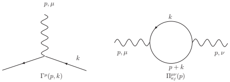

Figure: 3. Fermion loop

$$S = \int -\frac{1}{2}F^2 + i\bar{\Psi}\gamma^{\mu}(\partial_{\mu}\Psi - i[A_{\mu} \stackrel{\star}{,} \Psi]),\ \Gamma^{\mu}[p,k] = \gamma^{\mu} \underbrace{\sin\frac{p\theta k}{2}}_{\text{nonlocal}}$$

$$\Pi^{\mu\nu} = -\mathrm{tr}\ \mu^{d-D}\int \frac{d^D k}{(2\pi)^D}\sin^2\frac{k\theta p}{2}\gamma^{\mu}\frac{i(p+k)^{\rho}\gamma_{\rho}}{(p+k)^2}\gamma^{\nu}\frac{ik^{\sigma}\gamma_{\sigma}}{k^2}$$

$$= \mathrm{tr}\ \mu^{d-D}\int \frac{d^D k}{(2\pi)^D}2^{-2}(\underbrace{2}_{\text{planar}}\ \underbrace{-e^{ik\theta p} - e^{-ik\theta p}}_{\text{nonplanar}})\frac{\gamma^{\mu}(p+k)^{\rho}\gamma_{\rho}\gamma^{\nu}k^{\sigma}\gamma_{\sigma}}{(p+k)^2k^2}$$

$$\Pi\mu\nu_{D\to 4} = \pi^{-2}\Big[\underbrace{\Big(g^{\mu\nu}p^2 - p^{\mu}p^{\nu}\Big)\Big(\frac{1}{6}\Big[\frac{2}{\epsilon} + \ln \pi e^{\gamma_E} - \ln\frac{p^2}{\mu^2}\Big]}_{\text{Planar UV divergence}}$$

$$\underbrace{-2\int_0^1 dx\, x(1-x)K_0\Big[(x(1-x)p^2(\theta p)^2)^{\frac{1}{2}}\Big]\Big) - 2(\theta p)^{\mu}(\theta p)^{\nu}\int_0^1 dx\, x(1-x)p^2(\theta p)^{-2}K_2\Big[(x(1-x)p^2(\theta p)^2)^{\frac{1}{2}}\Big]}_{\text{nonplanar Bessel K}-\text{function integrals}}\Big].$$

A closer look at the K_n integrals

$$\int_0^1 dx\, x(1-x) p^2 (\theta p)^{-2} K_2\left[\left(x(1-x)p^2(\theta p)^2\right)^{\frac{1}{2}}\right]$$

$$= \frac{2}{(\theta p)^4} \longleftarrow \text{quadratic IR Divergence (UV/IR mixing)}$$

$$-\frac{1}{12}\frac{p^2}{(\theta p)^2} - \frac{p^2}{(\theta p)^2}\sum_{n=0}^{\infty}\frac{1}{n!(n+2)!}\left(\frac{p^2(\theta p)^2}{4}\right)^n$$

$$\cdot\left(\frac{1}{2}\ln p^2(\theta p)^2 - 2\ln 2 - \frac{1}{2}\Psi(n+1) - \frac{1}{2}\Psi\left(n+\frac{7}{2}\right)\right),$$

$$\int_0^1 dx\, x(1-x) K_0\left[\left(x(1-x)p^2(\theta p)^2\right)^{\frac{1}{2}}\right] = -\sum_{n=0}^{\infty}\frac{(n+1)^2}{(2n+4)!}\left(\frac{p^2(\theta p)^2}{4}\right)^n$$

$$\cdot\left(\frac{1}{2}\ln p^2(\theta p)^2 - 2\ln 2 - \Psi(n+1) - \frac{1}{2}\Psi\left(n+\frac{5}{2}\right)\right).$$

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Summary

Seiberg-Witten (SW) map based NCQGFT

- NCGF A_μ (enveloping algebra valued) and gauge transformation Λ expressed as function of commutative (Lie algebra valued) gauge field a_μ and NC parameter θ via consistency relations

$$\delta_\Lambda A_\mu \equiv \partial_\mu \Lambda + i[\Lambda \stackrel{\star}{,} A_\mu] = \delta_\lambda A_\mu[a_\mu]\,,$$
$$\delta_\Lambda F_{\mu\nu} \equiv i[\Lambda \stackrel{\star}{,} F_{\mu\nu}] = \delta_\lambda F_{\mu\nu}[a_\mu]\,,$$
$$\Lambda[[\lambda_1, \lambda_2], a_\mu] = [\Lambda[\lambda_1, a_\mu] \stackrel{\star}{,} \Lambda[\lambda_2, a_\mu]] + i\delta_{\lambda_1}\Lambda[\lambda_2, a_\mu] - i\delta_{\lambda_2}\Lambda[\lambda_1, a_\mu]\,.$$

- SW map enables defining *nonlocal* GFT invariant under *commutative* gauge transformation groups, which is considered very convenient for various applications: NCSM, etc.
- Advantage of models based on the SW mapping:
  - SW preserves quantum numbers while transferring any field: (ϕ on ordinary space) $\leftrightarrow$ (Φ on Noncommutative space)
  - Valid for any gauge groups
  - Valid for arbitrary matter representation
  - No charge quantization problem
  - Generates in principle new vertices

Seiberg-Witten map expansions

- Existence of Solutions to the SW consistency conditions can be solved by changing these conditions into (SW) ordinary differential equations, which can then be solved recursively.
- Initially an expansion over NC parameter θ was employed

$$A_\mu = a_\mu - \frac{1}{2}\theta^{\nu\rho} a_\nu(\partial_\rho a_\mu + f_{\rho\mu}) + \mathcal{O}(\theta^2)\,,$$
$$F_{\mu\nu} = f_{\mu\nu} + \theta^{\rho\tau}\Big(f_{\mu\rho}f_{\nu\tau} - a_\rho\partial_\tau f_{\mu\nu}\Big) + \mathcal{O}(\theta^2)\,.$$

  producing infinite tower of vertices. Not good due to the finite order cut n argument: $\left(\frac{E}{\Lambda_{\rm NC}}\right)^n$.
- Nonlocality is hidden on the cost of higher derivative terms (more momenta in numerator, poor power-counting) .
- For the same Feynman diagram comes an infinite sum over orders of θ, cutting off at finite order, limits the validity of the tree-level and loop-level results.
- Motivations to go beyond θ-expansion, to θ-exact SW maps.

θ-exact SW-map expansion

- Expansion over a_μ recovers the nonlocality, for example for $\mathrm{U}_\star(1)$ theory (Schupp and You 2008)

$$A_\mu = a_\mu - \frac{1}{2}\theta^{\nu\rho} a_\nu \star_2 (\partial_\rho a_\mu + f_{\rho\mu}) + \mathcal{O}(a^3)\,,$$
$$F_{\mu\nu} = f_{\mu\nu} + \theta^{\rho\tau}\Big(f_{\mu\rho} \star_2 f_{\nu\tau} - a_\rho \star_2 \partial_\tau f_{\mu\nu}\Big) + \mathcal{O}(a^3)\,.$$

- New generalized star product $\star_2$ is commutative and non-associative.

$$f \star_2 g = f(x_1)\frac{\sin\frac{\partial_1\theta\partial_2}{2}}{\frac{\partial_1\theta\partial_2}{2}} g(x_2)\Big|_{x_1=x_2=x} = g(x_2)\frac{\sin\frac{\partial_2\theta\partial_1}{2}}{\frac{\partial_2\theta\partial_1}{2}} f(x_1)\Big|_{x_1=x_2=x} = g \star_2 f\,,$$
$$(f \star_2 g) \star_2 h \neq f \star_2 (g \star_2 h), \quad \int (f \star_2 g) \star_2 h = \int f \star_2 (g \star_2 h)\,.$$

- Action expanded over formal power of fields, each vertex has contributions from all orders of θ summed up by definition.

$$S = -\frac{1}{2}\int f_{\mu\nu} f^{\mu\nu} + S^{a^3} + S^{a^4} + S^{a^5} \ldots\ldots$$

Gauge symmetry inspired freedom in SW map

- Seiberg-Witten map of $F_{\mu\nu}$ allows a further deformation

$$F_{\mu\nu}(\kappa) = f_{\mu\nu} + \theta^{\rho\tau}\left(\kappa f_{\mu\rho} \star_2 f_{\nu\tau} - a_\rho \star_2 \partial_\tau f_{\mu\nu}\right) + \mathcal{O}(a^3)\,.$$

- κ-deformation preserves the leading order consistency relation for $F_{\mu\nu}$

$$\delta_\lambda F_{\mu\nu}(\kappa) = -\theta^{ij}\partial_i\lambda \star_2 \partial_j f_{\mu\nu} + \mathcal{O}(a^2)\lambda = i[\lambda \stackrel{\star}{,} f_{\mu\nu}] + \mathcal{O}(a^2)\lambda$$

- Manifestly $\mathrm{U}(1)$ gauge invariant action up to three-photon self-coupling with κ-deformation follows after an integral by part

$$S = \int -\frac{1}{2} f_{\mu\nu} f^{\mu\nu} - f^{\mu\nu}\left(\kappa f_{i\mu} \star_2 f_{j\nu} - \frac{1}{4} f_{ij} \star_2 f_{\mu\nu}\right) + \mathcal{O}(a^4).$$

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Vertices:

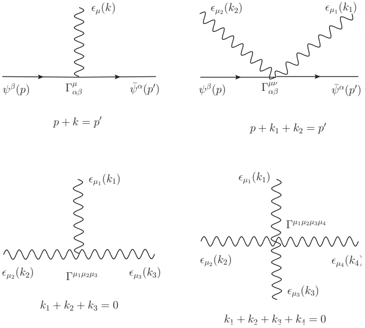

Figure: 4. Vertices

$$\Gamma^{\mu\nu\rho}_{\kappa}(p,k,q) = F_{\star_2}(k,q)V^{\mu\nu\rho}_{\kappa}(p,k,q);\; F_{\star_2}(k,q)=\frac{\sin\frac{k\theta q}{2}}{\frac{k\theta q}{2}},$$

$$V^{\mu\nu\rho}_{\kappa}(p,k,q) = \kappa\Big\{-(p\theta k)(p-k)^\rho g^{\mu\nu} - \theta^{\mu\nu}\Big[p^\rho(kq)-k^\rho(pq)\Big] + (\theta p)^\nu\Big[g^{\mu\rho}q^2 - q^\nu q^\rho\Big]$$

$$+\;(\theta p)^\rho\Big[g^{\mu\nu}k^2 - k^\mu k^\nu\Big] + \theta^{\mu\sigma}(k+q+\kappa^{-1}p)_\sigma\Big[g^{\nu\rho}(kq) - q^\nu k^\rho\Big]\Big\}$$

$$+\;\text{cyclic permutations.}$$

Bubble diagrams:

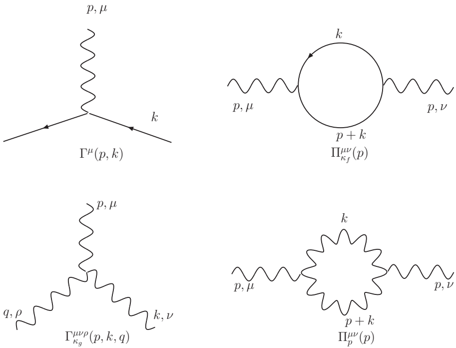

Figure: 5. Photon loop

$$\mathcal{B}^{\mu\nu}(p) = \frac{1}{2}\int \frac{d^D k}{(2\pi)^D}\Gamma^{\mu\rho\sigma}_{\kappa}(p, k, -p-k)\frac{-ig_{\rho\rho'}}{k^2}\Gamma^{\nu\rho'\sigma'}_{\kappa}(-p, -k, k+p)\frac{-ig_{\sigma\sigma'}}{(p+k)^2}.$$

Evaluate the bubble diagram

- Moving from phase factor $\sin^2 \frac{k\theta p}{2}$ to "diffraction" factor $f_{\star_2}(k,q)^2$ requires new parametrization technique, which is solved by us (using HQEFT) in a relative simple way

$$\frac{\{\text{numerator}\}}{k^2(p+k)^2} \cdot f_{\star_2}(k,q)^2 = \frac{2 - e^{ik\theta p} - e^{-ik\theta p}}{k^2(p+k)^2(k\theta p)^2} \cdot \{\text{numerator}\}$$

$$= -2\int_0^1 dx \int_0^{\frac{1}{\lambda}} dy \int_0^{\infty} d\lambda \lambda^3 \exp\left[-\lambda\left(l^2 + x(1-x)p^2 + \frac{y^2}{4}(\theta p)^2\right)\right]$$

$$\cdot \Bigg(\underbrace{y \cdot \{y \text{ even terms of the numerator}\}}_{\text{Planar and Bessel K}-\text{function integrals}}$$

$$\underbrace{-\lambda^{-1} \cdot \{y \text{ even terms of the numerator}\}}_{\text{hypergeometric integrals}}\Bigg),\; l = k + xp + \frac{i}{2}y(\theta p).$$

Tensor structure

- In QED, vacuum polarization is associated with dispersion $p^\mu p^\nu - p^2 g^{\mu\nu}$ to satisfy Ward identity.
- NCQED brings additional NC terms to the dispersion structure proportional to $(\theta p)^\mu (\theta p)^\nu$.
- The nonlocal model here has larger tensor structures in general

$$\mathcal{B}^{\mu\nu}(p)_D = \frac{1}{(4\pi)^2}\Big\{\Big[g^{\mu\nu}p^2 - p^\mu p^\nu\Big]B_1^\kappa(p) + (\theta p)^\mu(\theta p)^\nu B_2^\kappa(p)$$
$$+ \Big[g^{\mu\nu}(\theta p)^2 - (\theta\theta)^{\mu\nu}p^2 + p^{\{\mu}(\theta\theta p)^{\nu\}}\Big]B_3^\kappa(p)$$
$$+ \Big[(\theta\theta)^{\mu\nu}(\theta p)^2 + (\theta\theta p)^\mu(\theta\theta p)^\nu\Big]B_4^\kappa(p) + (\theta p)^{\{\mu}(\theta\theta\theta p)^{\nu\}}B_5^\kappa(p)\Big\}.$$

- Transversality condition $p_\mu \mathcal{B}^{\mu\nu}(p)_D = 0$ holds well since each of the 5 tensor structures satisfies it.

Lots of divergences arise when $D \to 4$

$$B_1^\kappa(p) \sim \Bigg(\frac{2}{3}(1-3\kappa)^2 + \frac{2}{3}(1+2\kappa)^2\,\frac{p^2(\mathrm{tr}\theta\theta)}{(\theta p)^2} + \frac{4}{3}(1+4\kappa+\kappa^2)\,\frac{p^2(\theta\theta p)^2}{(\theta p)^4}\Bigg)$$

$$\cdot\left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2)\right] \quad - \frac{16}{3}(1-\kappa)^2\,\frac{1}{(\theta p)^6}\Big((\mathrm{tr}\theta\theta)(\theta p)^2 + 4(\theta\theta p)^2\Big)\,,$$

$$B_2^\kappa(p) \sim \Bigg(\frac{8}{3}(1-\kappa)^2\,\frac{p^4(\theta\theta p)^2}{(\theta p)^6} + \frac{2}{3}(1-2\kappa-5\kappa^2)\,\frac{p^4(\mathrm{tr}\theta\theta)}{(\theta p)^4} + \frac{2}{3}(25-86\kappa$$

$$+73\kappa^2)\,\frac{p^2}{(\theta p)^2}\Bigg)\left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2)\right] - \frac{16}{3}(1-3\kappa)(3-\kappa)\,\frac{1}{(\theta p)^4}$$

$$+\frac{32}{3}(1-\kappa)^2\,\frac{1}{(\theta p)^8}\Big((\mathrm{tr}\theta\theta)(\theta p)^2 + 6(\theta\theta p)^2\Big),$$

$$B_3^\kappa(p) \sim -\frac{1}{3}(1-2\kappa-11\kappa^2)\,\frac{p^2}{(\theta p)^2}\left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2)\right] - \frac{8}{3(\theta p)^4}(1-10\kappa+17\kappa^2),$$

$$B_4^\kappa(p) \sim -2(1+\kappa)^2\,\frac{p^4}{(\theta p)^4}\left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2)\right] - \frac{32p^2}{3(\theta p)^6}(1-6\kappa+7\kappa^2),$$

$$B_5^\kappa(p) \sim \frac{4}{3}(1+\kappa+4\kappa^2)\,\frac{p^4}{(\theta p)^4}\left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2)\right] + \frac{64p^2}{3(\theta p)^6}(1-\kappa)(1-2\kappa).$$

Special $\theta^{\mu\nu}$

- A lot of divergences appear in our 1-loop amplitude, some simplification needed.
- Introducing a noncommutative parameter with unique arithmetic property

$$\theta^{\mu\nu}_{\sigma_2} = \theta^2 \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} = \frac{1}{\Lambda^2_{\rm NC}} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \ (\theta\theta)^{\mu\nu} = -\frac{1}{\Lambda^4_{\rm NC}} g^{\mu\nu}.$$

- Five tensor structures reduce to two

$$\begin{aligned} \mathcal{B}^{\mu\nu}(p)_4\Big|^{\theta_{\sigma_2}} &= \frac{1}{(4\pi)^2}\Big\{\Big[g^{\mu\nu}p^2 - p^\mu p^\nu\Big] B^\kappa_I(p) + (\theta p)^\mu (\theta p)^\nu \, B^\kappa_{II}(p)\Big\} \\ &= \frac{e^2}{(4\pi)^2}\Big\{\Big[g^{\mu\nu}p^2 - p^\mu p^\nu\Big]\Big(B^\kappa_1 + 2\frac{B^\kappa_3}{\Lambda^4_{\rm NC}} - \frac{B^\kappa_4}{\Lambda^8_{\rm NC}}\Big) \\ &\qquad + (\theta p)^\mu (\theta p)^\nu \Big(B^\kappa_2 - 2\frac{B^\kappa_5}{\Lambda^4_{\rm NC}}\Big)\Big\}, \end{aligned}$$

Divergence cancelation

- Summing over divergent terms according to the relations induced by θ_{σ_2}

$$B_I^\kappa(p) \sim (1-3\kappa)\left\{\frac{4(1-3\kappa)}{3}\left(\frac{2}{\epsilon}+\ln\left(\mu^2(\theta p)^2\right)\right)+\frac{16}{3}\frac{(1+\kappa)}{p^2(\theta p)^2}\right\},$$

$$B_{II}^\kappa(p) \sim (1-3\kappa)\left\{2\,p^2\frac{(7-9\kappa)}{(\theta p)^2}\left(\frac{2}{\epsilon}+\ln\left(\mu^2(\theta p)^2\right)\right)-\frac{16}{3}\frac{(7-5\kappa)}{(\theta p)^4}\right\}.$$

- All divergences, UV and IR, vanish when we set $\kappa = 1/3$.
- Some explicit computation shows that the lengthy special function integrals reduce to two finite terms when $\kappa = 1/3$, thus

$$\mathcal{B}^{\mu\nu}(p)_4\Big|_{\kappa=1/3}^{\theta_{\sigma_2}} = \frac{p^2}{\pi^2}\left\{\frac{7}{27}\left[g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right]-\frac{1}{2}\frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\right\},$$

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The 2nd order in gauge field θ-exact SW maps

The 4-photon tadpole diagram

Seiberg-Witten map equivalence between QGFT?

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Exploring the Universe using Neutrinos

UHE cosmic ray RICE experiment: $\nu + N \to \nu + X$

Ptolemy Experiment

Ptolemy experiment: $\nu_e + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-, \ (\nu_e + n \to p + e^-)$

Plasmon decay: $\gamma_{pl} \to \bar{\nu}_R \nu_R$

Scattering: $e\nu_R \to e\nu_R$

Summary

Motivation for 2nd order in gauge field θ-exact SW maps

- Obviously the action with triple photon coupling only does not cover all contributions to the photon two point function in SW mapped $\mathrm{U}_\star(1)$ model, four photon coupling is needed for completeness.
- Four photon self-interaction takes following general form

$$S^{e^2} = -\frac{1}{4e^2}\int F^{(e^2)}_{\mu\nu}F^{(e^2)\mu\nu} + 2ef^{\mu\nu}F^{(e^3)}_{\mu\nu}.$$

- To define a model with complete four photon self-interaction we need at least one explicit solution for θ-exact second order SW map expansion $F^{(2)}_{\mu\nu}$.
- We have investigated two different 2nd order SW map's.

Second order map (I)

C.P. Martin 2012

$$\Lambda^{(\mathrm{I})} = \lambda - \frac{1}{2}\theta^{ij} a_i \star_2 \partial_j\lambda - \frac{1}{8}\theta^{ij}\theta^{kl}\left((\partial_i\lambda a_k(\partial_l a_j + f_{lj}))_{\star_{3'}} - (a_i\partial_j(a_k\partial_l\lambda))_{\star_{3'}}\right),$$

$$\begin{aligned} A_\mu^{(\mathrm{I})} =& a_\mu - \frac{1}{2}\theta^{ij} a_i \star_2 (\partial_j a_\mu + f_{j\mu}) \\ & - \frac{1}{8}\theta^{ij}\theta^{kl}\big[(\partial_i a_\mu + f_{i\mu})a_k(\partial_l a_j + f_{lj}) - a_i\partial_j(a_k(\partial_l a_\mu + f_{l\mu})) \\ & + 2a_i(f_{jk}f_{\mu l} - a_k\partial_l f_{j\mu})\big]_{\star_{3'}}, \end{aligned}$$

$$\begin{aligned} [fgh]_{\star_{3'}}(x) = \int & e^{-i(p+q+k)x}\tilde{f}(p)\tilde{g}(q)\tilde{h}(k) \\ & \cdot \left[\frac{\cos(\frac{p\theta q}{2} + \frac{p\theta k}{2} - \frac{q\theta k}{2}) - 1}{(\frac{p\theta q}{2} + \frac{p\theta k}{2} - \frac{q\theta k}{2})\frac{q\theta k}{2}} - \frac{\cos(\frac{p\theta q}{2} + \frac{p\theta k}{2} + \frac{q\theta k}{2}) - 1}{(\frac{p\theta q}{2} + \frac{p\theta k}{2} + \frac{q\theta k}{2})\frac{q\theta k}{2}}\right]. \end{aligned}$$

Second order map (II)

Mehen & Wise 2001

$$\Lambda^{(\mathrm{II})} = \lambda - \frac{1}{2}\theta^{ij} a_i \star_2 \partial_j \lambda + \frac{1}{2}\theta^{ij}\theta^{kl}[\frac{1}{2}(a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j \lambda$$

$$+ \frac{1}{2} a_i \star_2 \partial_j (a_k \star_2 \partial_l \lambda)] - \frac{1}{2}\theta^{ij}\theta^{kl}[\partial_k \partial_i \lambda a_j a_l + \partial_k \lambda a_i \partial_l a_j]_{\star_3} + \mathcal{O}(a^3),$$

$$A_\mu^{(\mathrm{II})} = a_\mu - \frac{1}{2}\theta^{ij} a_i \star_2 (\partial_j a_\mu + f_{j\mu}) + \frac{1}{2}\theta^{ij}\theta^{kl}[\frac{1}{2}(a_k \star_2 (\partial_l a_i + f_{li})) \star_2 (\partial_j a_\mu$$

$$+ f_{j\mu}) + a_i \star_2 (\partial_j (a_k \star_2 (\partial_l a_\mu + f_{l\mu})) - \frac{1}{2}\partial_\mu (a_k \star_2 (\partial_l a_j + f_{lj})))]$$

$$- \frac{1}{2}\theta^{ij}\theta^{kl} a_i \star_2 (\partial_k a_j \star_2 \partial_l a_\mu) - \frac{1}{2}\theta^{ij}\theta^{kl}[-a_i \partial_k a_\mu (\partial_j a_l + f_{jl}) + \partial_k \partial_i a_\mu a_j a_l$$

$$+ 2\partial_k a_i \partial_\mu a_j a_l]_{\star_3} + \mathcal{O}(a^4),$$

$$[fgh]_{\star_3}(x) = \int e^{-i(p+q+k)x} \tilde{f}(p)\tilde{g}(q)\tilde{h}(k)$$

$$\cdot \left[\frac{\sin \frac{p\theta k}{2} \sin(\frac{p\theta q}{2} + \frac{p\theta k}{2})}{(\frac{p\theta q}{2} + \frac{p\theta k}{2})(\frac{p\theta k}{2} + \frac{q\theta k}{2})} + \frac{\sin \frac{q\theta k}{2} \sin(\frac{p\theta q}{2} - \frac{q\theta k}{2})}{(\frac{p\theta q}{2} - \frac{q\theta k}{2})(\frac{p\theta k}{2} + \frac{q\theta k}{2})} \right].$$

SW map ambiguities

- SW map is not unique if only gauge equivalence (consistency) conditions and smooth commutative limit are required.
- Ambiguities can be studied by taking the composition of one SW and one inverse SW maps

$$\partial_\mu \lambda_2\big(A_{1_\mu}(a_\mu), \Lambda_1(a_\mu, \lambda)\big) = \delta_\lambda a_{2_\mu}\big(A_{1_\mu}(a_\mu)\big).$$

- General ansatz for the ambiguities (Barnich et. al. 2003)

$$\begin{aligned} a_{2_\mu}\big(A_{1_\mu}(a_\mu)\big) &= a_\mu + X_\mu(a_\mu) + \partial_\mu Y(a_\mu), \\ \lambda_2\big(A_{1_\mu}(a_\mu), \Lambda_1(a_\mu, \lambda)\big) &= \lambda + \delta_\lambda Y(a_\mu), \\ \delta_\lambda X_\mu(a_\mu) &= 0. \end{aligned}$$

- Question: Solve X_μ and Y for ambiguity between (I)&(II)?

Solving the ambiguity between (I) and (II)

- Identical first order simplifies the composition into just a difference

$$a_{\mu_{(\mathrm{II})}}(A_{\mu_{(\mathrm{I})}}(a_\mu)) = a_\mu(x) + A^{(2)}_{\mu_{(\mathrm{I})}}(x) - A^{(2)}_{\mu_{(\mathrm{II})}}(x) + \mathcal{O}(a^4).$$

- θ^2 order difference can be put into an infinitesimal gauge transformation.

$$A'^{(2)}_{\mu_{(\mathrm{I},\mathrm{II})}}(x) = A^{(2)}_{\mu_{(\mathrm{I},\mathrm{II})}}(x) + \delta_{\Xi^{(2)}_{(\mathrm{I},\mathrm{II})}} A^{(2)}_{\mu_{(\mathrm{I},\mathrm{II})}}(x) = A^{(2)}_{\mu_{(\mathrm{I},\mathrm{II})}}(x) + \partial_\mu \Xi^{(2)}_{(\mathrm{I},\mathrm{II})}(x) + \mathcal{O}(a^4).$$

$$\Xi^{(2)}_{(\mathrm{I})}(x) = -\frac{1}{8}\theta^{ij}\theta^{kl}\Big(2\left[a_l\partial_j a_k a_i\right]_{\star_{3'}} + \left[a_i\partial_j a_k a_l\right]_{\star_{3'}}\Big) + \mathcal{O}(a^4),$$

$$\Xi^{(2)}_{(\mathrm{II})}(x) = -\frac{1}{4}\theta^{ij}\theta^{kl}\Big(2\left(a_i \star_2 \partial_j a_k\right) \star_2 a_l + a_i \star_2 \left(\partial_j a_k \star_2 a_l\right)\Big) + \mathcal{O}(a^4),$$

- Rest of the difference can be expressed in momentum space as nontrivial momentum structure in θ^4 and θ^6 order multiplying totally symmetric θ-exact structures.

Solving the ambiguity between (I) and (II),cont'd

$$
\begin{aligned}
\tilde{W}_\mu =& \tilde{A}'^{(2)}_{\mu(\mathrm{I})} - \tilde{A}'^{(2)}_{\mu(\mathrm{II})} \\
=& \Big[\tilde{a}_\mu(k) \Big((\tilde{a}(p)\theta q)(\tilde{a}(q)\theta k) \frac{p\theta k}{2} - \frac{1}{2}(\tilde{a}(p)\theta \tilde{a}(q))(q\theta k) \frac{p\theta k}{2} \\
& + (\tilde{a}(p)\theta k)(\tilde{a}(q)\theta k) \frac{q\theta p}{2} \Big) - k_\mu \Big((\tilde{a}(p)\theta \tilde{a}(q))(q\theta \tilde{a}(k)) \Big) \frac{p\theta k}{2} \Big] \\
& \cdot \left[\left(\frac{p\theta q}{2} - \frac{p\theta k}{2} + \frac{q\theta k}{2} \right) f_1\left(\frac{p\theta q}{2}, \frac{p\theta k}{2}, \frac{q\theta k}{2} \right) + \frac{p\theta q}{2} \frac{p\theta k}{2} \frac{q\theta k}{2} f_2\left(\frac{p\theta q}{2}, \frac{p\theta k}{2}, \frac{q\theta k}{2}; 0 \right) \right],
\end{aligned}
$$

$$
\begin{aligned}
f_1(a,b,c) =& \frac{1}{4} \Bigg(\frac{\cos(a+b+c)}{(a+b)(b+c)(a+c)(a+b+c)} + \frac{\cos(a+b-c)}{(a+b)(b+c)(a-c)(a+b-c)} \\
& - \frac{\cos(a-b+c)}{(a-b)(b-c)(a+c)(a-b+c)} - \frac{\cos(a-b-c)}{(a-b)(b+c)(a-c)(a-b-c)} \\
& - \frac{8}{(a+b+c)(a+b-c)(a-b+c)(a-b-c)} \Bigg), \\
f_2(a,b,c;n) =& \frac{a^{2n}\cos a \sin b \sin c}{bc(a^2-b^2)(c^2-a^2)} + \frac{b^{2n}\sin a \cos b \sin c}{ac(a^2-b^2)(b^2-c^2)} + \frac{c^{2n}\sin a \sin b \cos c}{ab(b^2-c^2)(c^2-a^2)}.
\end{aligned}
$$

- $f_1(a,b,c)$ and $f_2(a,b,c;n)$ both even and totally symmetric, in position space they induce totally commutative 3-products $\diamond_1$ & $\diamond_{2(n)}$.

SW map ambiguity between (I) and (II)

$$[fgh]_{\diamond_1}(x) = \int e^{-i(p+q+k)x}\tilde{f}(p)\tilde{g}(q)\tilde{h}(k)f_1\left(\frac{p\theta q}{2},\frac{p\theta k}{2},\frac{q\theta k}{2}\right),$$

$$[fgh]_{\diamond_2(n)}(x) = \int e^{-i(p+q+k)x}\tilde{f}(p)\tilde{g}(q)\tilde{h}(k)f_2\left(\frac{p\theta q}{2},\frac{p\theta k}{2},\frac{q\theta k}{2};n\right).$$

- Studying the permutations within $\diamond_1$ and $\diamond_{2(0)}$ products is reduced to permutations of their θ^4 and θ^6 counterparts, respectively, some arithmetics leads us to the final result

$$a_{\mu_{(\mathrm{II})}}(A_{\mu_{(\mathrm{I})}}(a_\mu)) = a_\mu(x) + A^{(2)}_{\mu_{(\mathrm{I})}}(x) - A^{(2)}_{\mu_{(\mathrm{II})}}(x) + \mathcal{O}(a^4) = a_\mu(x) + X^{(2)}_\mu(x) + \partial_\mu Y^{(2)}(x) + \mathcal{O}(a^4);$$

$$X^{(2)}_\mu(x) = -\frac{e^3}{8}\theta^{ij}\theta^{kl}\theta^{pq}\theta^{rs}\Big([\partial_r f_{ip} f_{jk}\partial_s\partial_q f_{l\mu}]_{\diamond_1} + [\partial_r f_{ip} f_{jk}\partial_s\partial_l f_{q\mu}]_{\diamond_1} + [\partial_p f_{ri}\partial_q f_{jk}\partial_s f_{l\mu}]_{\diamond_1}$$
$$+\frac{1}{4}\theta^{ab}\theta^{cd}\left[\partial_p\partial_a f_{ri}\partial_q\partial_c f_{jk}\partial_s\partial_b\partial_d f_{l\mu}\right]_{\diamond_{2(0)}}\Big),$$

$$Y^{(2)}(x) = \frac{e^3}{8}\theta^{ij}\theta^{kl}\Big(2\left[a_l\partial_j a_k a_i\right]_{\star_{3'}} + \left[a_i\partial_j a_k a_l\right]_{\star_{3'}} - 4\left(a_i\star_2\partial_j a_k\right)\star_2 a_l - 2a_i\star_2\left(\partial_j a_k\star_2 a_l\right)\Big)$$
$$-\frac{e^3}{8}\theta^{ij}\theta^{kl}\theta^{pq}\theta^{rs}\Big([\partial_p f_{ir}\partial_q f_{jk}\partial_s a_l]_{\diamond_1} + 2[\partial_p\partial_r a_i\partial_q\partial_j a_k\partial_s a_l]_{\diamond_1}$$
$$-\frac{1}{12}\theta^{ab}\theta^{cd}\left(3[\partial_p\partial_a\partial_r a_i\partial_q\partial_c\partial_j a_k\partial_s\partial_b\partial_d a_l]_{\diamond_{2(0)}} - [\partial_p\partial_a\partial_i a_r\partial_q\partial_c\partial_k a_j\partial_s\partial_b\partial_d a_l]_{\diamond_{2(0)}}\right)\Big).$$

Gauge symmetry inspired freedoms at 2nd order

- κ-deformation modifies the 2nd order consistency condition for field strength

$$\begin{aligned}\delta_\lambda F^{(2)}_{\mu\nu}(x)_\kappa =& i\left(\left[\Lambda^{(1)} \stackrel{\star}{,} f_{\mu\nu}\right] + \left[\lambda \stackrel{\star}{,} F^{(1)}_{\mu\nu}(x)_\kappa\right]\right)\\ =& i\left(\left[\Lambda^{(1)} \stackrel{\star}{,} f_{\mu\nu}\right] + \left[\lambda \stackrel{\star}{,} \theta^{ij}\left(\kappa f_{\mu i} \star_2 f_{\nu j} - a_i \star_2 \partial_j f_{\mu\nu}\right)\right]\right).\end{aligned}$$

- κ-deformed consistency can be solved by a single term

$$\delta_\lambda\left(- \theta^{ij}\theta^{kl} a_i \star_2 \partial_j \left(f_{\mu k} \star_2 f_{\nu l}\right)\right) = i\theta^{kl}\left[\lambda \stackrel{\star}{,} f_{\mu k} \star_2 f_{\nu l}\right].$$

- After inserting this term we manage to identify in the second order field strength some freedoms analogous to the κ-deformation in the first order.

$\kappa, \kappa_i\,[\kappa_i']$ deformation for the field strength (I) [II]

- The field strengths $F^{(2)}_{\mu\nu_{(\mathrm{I})}}(x)$ and $F^{(2)}_{\mu\nu_{(\mathrm{II})}}(x)$ each allow two gauge freedom (deformation) parameters, $\kappa_{1,2}$ and $\kappa'_{1,2}$, respectively.

$$F^{(2)}_{\mu\nu_{(\mathrm{I})}}(x)_{\kappa,\kappa_1,\kappa_2} = \frac{1}{2}\theta^{ij}\theta^{kl}\Big[-\kappa a_i \star_2 \partial_j \left(f_{\mu k} \star_2 f_{\nu l}\right) + \kappa_1 \left(\left[f_{\mu k} f_{\nu i} f_{lj}\right]_{\star_{3'}} + \left[f_{\nu l} f_{\mu i} f_{kj}\right]_{\star_{3'}}\right)$$
$$- \kappa_2 \left(\left[f_{\nu l} a_i \partial_j f_{\mu k}\right]_{\star_{3'}} + \left[f_{\mu k} a_i \partial_j f_{\nu l}\right]_{\star_{3'}} + \left[a_k \partial_l \left(f_{\mu i} f_{\nu j}\right)\right]_{\star_{3'}} - 2a_i \star_2 \partial_j \left(f_{\mu k} \star_2 f_{\nu l}\right)\right)$$
$$+ \left[a_i \partial_j a_k \partial_l f_{\mu\nu} + \partial_l f_{\mu\nu} a_i \partial_j a_k + a_k a_i \partial_l \partial_j f_{\mu\nu}\right]_{\star_{3'}} - \frac{1}{2}\left(\left[a_i \partial_k a_j \partial_l f_{\mu\nu} + \partial_l f_{\mu\nu} a_i \partial_k a_j\right]_{\star_{3'}} \right)\Big],$$

$$F^{(2)}_{\mu\nu_{(\mathrm{II})}}(x)_{\kappa,\kappa'_1,\kappa'_2} = \theta^{ij}\theta^{kl}\Big[-\kappa a_i \star_2 \partial_j \left(f_{\mu k} \star_2 f_{\nu l}\right)$$
$$+ \kappa'_1 \left(f_{\mu i} \star_2 \left(f_{jk} \star_2 f_{l\nu}\right) + f_{l\nu} \star_2 \left(f_{jk} \star_2 f_{\mu i}\right) - \left[f_{\mu i} f_{jk} f_{l\nu}\right]_{\star_3}\right)$$
$$- \kappa'_2 ((a_i \star_2 \partial_j f_{\mu k}) \star_2 f_{\nu l} + (a_i \star_2 \partial_j f_{\nu l}) \star_2 f_{\mu k} - [a_i \partial_j (f_{\mu k} f_{\nu l})]_{\star_3})$$
$$+ (a_i \star_2 \partial_j a_k) \star_2 \partial_l f_{\mu\nu} + a_i \star_2 (\partial_j a_k \star_2 \partial_l f_{\mu\nu}) + a_i \star_2 (a_k \star_2 \partial_j \partial_l f_{\mu\nu}) - [a_i \partial_j a_k \partial_l f_{\mu\nu}]_{\star_3}$$
$$- \frac{1}{2}\left(a_i \star_2 (\partial_k a_j \star_2 \partial_l f_{\mu\nu}) + (a_i \star_2 \partial_k a_j) \star_2 \partial_l f_{\mu\nu} - [a_i \partial_k a_j \partial_l f_{\mu\nu}]_{\star_3} + [a_i a_k \partial_j \partial_l f_{\mu\nu}]_{\star_3}\right)\Big].$$

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Four photon self-interactions (I)

- The four photon self-interactions can be shown to be explicitly gauge invariant after considerable integral-by-part operations.

$$
\begin{aligned}
S^{a^4}_{(\mathrm{I})_{\kappa,\kappa_1,\kappa_2,\kappa_3,\kappa_4}} = & -\frac{1}{2}\theta^{ij}\theta^{kl}\int \kappa^2 (f_{\mu i}\star_2 f_{\nu j})(f^{\mu}{}_{k}\star_2 f^{\nu}{}_{l}) \\
& -\kappa(f_{ij}\star_2 f_{\mu\nu})(f^{\mu}{}_{k}\star_2 f^{\nu}{}_{l}) + 2\kappa_1 f^{\mu\nu}[f_{\mu i}f_{\nu k}f_{jl}]_{\star_{3'}} \\
& + 2\kappa_2 f^{\mu\nu}\left(a_i\star_2 \partial_j(f_{\mu k}\star_2 f_{\nu l}) - [f_{\mu k}a_i\partial_j f_{\nu l}]_{\star_{3'}} - [a_i f_{\mu k}\partial_j f_{\nu l}]_{\star_{3'}}\right)) \\
& -\frac{\kappa_3}{4} f^{\mu\nu}\left[f_{\mu\nu}f_{ik}f_{jl}\right]_{\star_{3'}} + \frac{\kappa_4}{8}\left(f^{\mu\nu}\star_2 f_{ij}\right)\left(f_{kl}\star_2 f_{\mu\nu}\right) \\
& + \frac{1}{2}\theta^{pq} f_{\mu\nu}\left[\partial_i f_{jk} f_{lp}\partial_q f_{\mu\nu}\right]_{\mathcal{M}_{(\mathrm{I})}},
\end{aligned}
$$

$$
[fgh]_{\mathcal{M}_{(\mathrm{I},\mathrm{II})}}(x) = \int e^{-i(p+q+k)x}\tilde{f}(p)\tilde{g}(q)\tilde{h}(k) f_{(\mathrm{I},\mathrm{II})}(p,q,k)\,.
$$

Four photon self-interaction, model (II)

$$
\begin{aligned}
S^{a^4}_{\mathrm{(II)}_{\kappa,\kappa'_1,\kappa'_2,\kappa'_3,\kappa'_4}} = & -\frac{1}{2}\theta^{ij}\theta^{kl}\int \kappa^2(f_{\mu i}\star_2 f_{\nu j})(f^{\mu}{}_k \star_2 f^{\nu}{}_l) - \kappa(f_{ij}\star_2 f_{\mu\nu})(f^{\mu}{}_k\star_2 f^{\nu}{}_l) \\
& + 2\kappa'_1 f^{\mu\nu}\left(2f_{\mu i}\star_2 (f_{jk}\star_2 f_{l\nu}) - [f_{\mu i}f_{jk}f_{l\nu}]_{\star_3}\right) \\
& - 4\kappa'_2\left((a_i\star_2 \partial_j f_{\mu k})\star_2 f_{\nu l} - [a_i\partial_j f_{\mu k}f_{\nu l}]_{\star_3}\right) \\
& - \frac{\kappa'_3}{4}f^{\mu\nu}\left(3f_{ik}\star_2 (f_{jl}\star_2 f_{\mu\nu}) - 2\left[f_{ik}f_{jl}f_{\mu\nu}\right]_{\star_3}\right) \\
& + \frac{\kappa'_4}{8}f^{\mu\nu}\left(2f_{ij}\star_2 (f_{kl}\star_2 f_{\mu\nu}) - \left[f_{ij}f_{kl}f_{\mu\nu}\right]_{\star_3}\right) \\
& - \frac{1}{4}\theta^{pq}\theta^{rs}\left[\partial_k f_{ri}\partial_j f_{lp}\partial_q\partial_s f_{\mu\nu} + \partial_i\partial_r f_{jk}\partial_s(f_{lp}\partial_q f_{\mu\nu})\right]_{\mathcal{M}_{\mathrm{(II)}}},
\end{aligned}
$$

- New gauge invariant terms emerge after integral-by-part. Consequently gauge freedom parameters $\kappa_{3,4}$ & $\kappa'_{3,4}$ are assigned to $S^{a^4}_{\mathrm{(I,II)}}$, respectively.

Four photon self-coupling vertex:

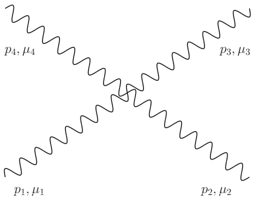

Figure: 6.

$$\Gamma_{(\mathrm{I})}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = -\frac{i}{2}\Big(\kappa^2\Gamma_{\mathrm{A}}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) + \kappa\Gamma_{\mathrm{B}}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)$$
$$+\kappa_1\Gamma_1^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) + \kappa_2\Gamma_2^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) + \kappa_3\Gamma_3^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)$$
$$+\kappa_4\Gamma_4^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) + \Gamma_5^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\Big)$$
$$+\text{ all } \mathrm{S}_4 \text{ permutations over } \{p_i\} \text{ and } \{\mu_i\} \text{ simutaneously},$$
$$\Gamma_{(\mathrm{II})}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = -\frac{i}{2}\Big(\kappa^2\Gamma_{\mathrm{A}}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) + \kappa\Gamma_{\mathrm{B}}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)$$
$$+\kappa'_1\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_1(p_1,p_2,p_3,p_4) + \kappa'_2\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_2(p_1,p_2,p_3,p_4) + \kappa'_3\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_3(p_1,p_2,p_3,p_4)$$
$$+\kappa'_4\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_4(p_1,p_2,p_3,p_4) + \Gamma'^{\mu_1\mu_2\mu_3\mu_4}_5(p_1,p_2,p_3,p_4)\Big)$$
$$+\text{ all } \mathrm{S}_4 \text{ permutations over } \{p_i\} \text{ and } \{\mu_i\} \text{ simutaneously}.$$

$$\Gamma_A^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = f_{\star_2}(p_1,p_2)\, f_{\star_2}(p_3,p_4)\, V_A^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,$$
$$\Gamma_B^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = f_{\star_2}(p_1,p_2)\, f_{\star_2}(p_3,p_4)\, V_B^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,$$

$$\begin{aligned}
\Gamma_1^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) =& 2f_{\star_{3'}}(p_2,p_3,p_4)\, V_1^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma_2^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) =& \Big(2f_{\star_2}(p_1,p_2)\, f_{\star_2}(p_3,p_4) - f_{\star_{3'}}(p_2,p_3,p_4)\\
&- f_{\star_{3'}}(p_4,p_3,p_2)\Big) V_2^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma_3^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) =& f_{\star_{3'}}(p_4,p_2,p_3) V_3^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma_4^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) =& f_{\star_2}(p_1,p_2) f_{\star_2}(p_3,p_4) V_4^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma_5^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) =& f_{(\mathrm{I})}(p_2,p_3,p_4) V_5^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,
\end{aligned}$$

$$\begin{aligned}
\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_1(p_1,p_2,p_3,p_4) =& \Big(4f_{\star_2}(p_1,p_2)\, f_{\star_2}(p_3,p_4)\\
&- 2f_{\star_3}(p_2,p_3,p_4)\Big) V_1^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_2(p_1,p_2,p_3,p_4) =& 2\Big(f_{\star_3}(p_2,p_3,p_4)\\
&- f_{\star_2}(p_2,p_3)\, f_{\star_2}(p_1,p_4)\Big) V_2^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_3(p_1,p_2,p_3,p_4) =& \Big(3f_{\star_2}(p_1,p_2) - 2f_{\star_3}(p_2,p_3,p_4)\Big) V_3^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_4(p_1,p_2,p_3,p_4) =& \Big(2f_{\star_2}(p_1,p_2) - f_{\star_3}(p_2,p_3,p_4)\Big) V_4^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)\,,\\
\Gamma'^{\mu_1\mu_2\mu_3\mu_4}_5(p_1,p_2,p_3,p_4) =& f_{(\mathrm{II})}(p_2,p_3,p_4) V'^{\mu_1\mu_2\mu_3\mu_4}_5(p_1,p_2,p_3,p_4)\,.
\end{aligned}$$

$$
\begin{aligned}
V_A^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) =& (p_1p_3)(p_2p_4)\theta^{\mu_1\mu_2}\theta^{\mu_3\mu_4} - (p_1p_3)\theta^{\mu_1\mu_2}(\theta p_4)^{\mu_3}p_2^{\mu_4} \\
&+ (p_2p_4)\theta^{\mu_1\mu_2}p_1^{\mu_3}(\theta p_3)^{\mu_4} + (p_3\theta p_4)\theta^{\mu_1\mu_2}p_1^{\mu_3}p_2^{\mu_4} \\
&- (p_1p_3)(\theta p_2)^{\mu_1}p_4^{\mu_2}\theta^{\mu_3\mu_4} + (p_1p_3)(\theta p_2)^{\mu_1}(\theta p_4)^{\mu_3}g^{\mu_2\mu_4} \\
&- (\theta p_2)^{\mu_1}p_4^{\mu_2}p_1^{\mu_3}(\theta p_3)^{\mu_4} - (p_3\theta p_4)(\theta p_2)^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4} \\
&+ (p_2p_4)p_3^{\mu_1}(\theta p_1)^{\mu_2}\theta^{\mu_3\mu_4} - p_3^{\mu_1}(\theta p_1)^{\mu_2}(\theta p_4)^{\mu_3}g^{\mu_2\mu_4} \\
&+ (p_2p_4)g^{\mu_1\mu_3}(\theta p_1)^{\mu_2}(\theta p_3)^{\mu_4} + (p_3\theta p_4)g^{\mu_1\mu_3}(\theta p_1)^{\mu_2}p_2^{\mu_4} \\
&+ (p_1\theta p_2)p_3^{\mu_1}p_4^{\mu_2}\theta^{\mu_3\mu_4} - (p_1\theta p_2)p_3^{\mu_1}(\theta p_4)^{\mu_3}g^{\mu_2\mu_4} \\
&+ (p_1\theta p_2)g^{\mu_1\mu_3}p_4^{\mu_2}(\theta p_3)^{\mu_4} + (p_1\theta p_2)(p_3\theta p_4)g^{\mu_1\mu_3}g^{\mu_2\mu_4},
\end{aligned}
$$

$$
\begin{aligned}
V_B^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) =& 2(\theta p_1)^{\mu_1}\Big((p_2p_3)p_4^{\mu_2}\theta^{\mu_3\mu_4} - (p_2p_3)(\theta p_4)^{\mu_3}g^{\mu_2\mu_4} \\
&+ p_4^{\mu_2}p_2^{\mu_3}(\theta p_3)^{\mu_4} + (p_3\theta p_4)p_2^{\mu_3}g^{\mu_2\mu_4} - (p_2p_4)p_3^{\mu_2}\theta^{\mu_3\mu_4} \\
&+ p_3^{\mu_2}(\theta p_4)^{\mu_3}p_2^{\mu_4} - (p_2p_4)g^{\mu_2\mu_3}(\theta p_3)^{\mu_4} - (p_3\theta p_4)g^{\mu_2\mu_3}p_2^{\mu_4}\Big),
\end{aligned}
$$

$$
\begin{aligned}
V_1^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = {} & 2(p_1p_2)\Big(p_3^{\mu_1}(\theta p_4)^{\mu_2}\theta^{\mu_3\mu_4} - p_3^{\mu_1}(\theta p_4)^{\mu_3}\theta^{\mu_2\mu_4} \\
& + g^{\mu_1\mu_3}(\theta p_4)^{\mu_2}(\theta p_3)^{\mu_4} + (p_3\theta p_4)g^{\mu_1\mu_3}\theta^{\mu_2\mu_4}\Big) \\
& - 2p_1^{\mu_2}\Big((p_2\theta p_4)p_3^{\mu_1}\theta^{\mu_3\mu_4} + p_3^{\mu_1}(\theta p_4)^{\mu_3}(\theta p_2)^{\mu_4} \\
& + (p_2\theta p_4)g^{\mu_1\mu_3}(\theta p_3)^{\mu_4} - (p_3\theta p_4)g^{\mu_1\mu_3}(\theta p_2)^{\mu_4}\Big) \\
& + 2(p_1p_3)\Big(p_2^{\mu_1}(\theta p_4)^{\mu_3}\theta^{\mu_2\mu_4} - p_2^{\mu_1}(\theta p_4)^{\mu_2}\theta^{\mu_3\mu_4} \\
& + g^{\mu_1\mu_2}(\theta p_4)^{\mu_3}(\theta p_2)^{\mu_4} + (p_2\theta p_4)g^{\mu_1\mu_2}\theta^{\mu_3\mu_4}\Big) \\
& - 2p_1^{\mu_3}\Big((p_3\theta p_4)p_2^{\mu_1}\theta^{\mu_2\mu_4} + p_2^{\mu_1}(\theta p_4)^{\mu_2}(\theta p_3)^{\mu_4} \\
& + (p_3\theta p_4)g^{\mu_1\mu_2}(\theta p_2)^{\mu_4} - (p_2\theta p_4)g^{\mu_1\mu_2}(\theta p_3)^{\mu_4}\Big), \\
V_2^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = {} & (\theta p_4)^{\mu_3}\Big((p_1p_2)p_4^{\mu_1}\theta^{\mu_2\mu_4} - (p_1p_2)(\theta p_4)^{\mu_2}g^{\mu_1\mu_4} + p_1^{\mu_2}(\theta p_2)^{\mu_4}p_4^{\mu_1} \\
& + p_1^{\mu_2}(p_2\theta p_4)g^{\mu_1\mu_4} - (p_1p_4)p_2^{\mu_1}\theta^{\mu_1\mu_4} + p_1^{\mu_4}p_2^{\mu_1}(\theta p_4)^{\mu_2} \\
& - (p_1p_4)(\theta p_2)^{\mu_4}g^{\mu_1\mu_2} - p_1^{\mu_4}(p_2\theta p_4)g^{\mu_1\mu_2}\Big), \\
V_3^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = {} & -\Big((p_1p_4)g^{\mu_1\mu_4} - p_1^{\mu_4}p_4^{\mu_1}\Big)\Big((\theta p_2)^{\mu_3}(\theta p_3)^{\mu_2} + \theta^{\mu_2\mu_3}(p_2\theta p_3)\Big), \\
V_4^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = {} & \Big((p_1p_4)g^{\mu_1\mu_4} - p_1^{\mu_4}p_4^{\mu_1}\Big)(\theta p_2)^{\mu_2}(\theta p_3)^{\mu_3}, \\
V_5^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4) = {} & -\Big((p_1p_4)g^{\mu_1\mu_4} - p_1^{\mu_4}p_4^{\mu_1}\Big)(\theta p_2)^{\mu_2}\cdot\Big((\theta p_2)^{\mu_3}(p_3\theta p_4) + (\theta p_4)^{\mu_3}(p_2\theta p_3)\Big), \\
V'^{\mu_1\mu_2\mu_3\mu_4}_5(p_1,p_2,p_3,p_4) = {} & \Big((p_1p_4)g^{\mu_1\mu_4} - p_1^{\mu_4}p_4^{\mu_1}\Big)\cdot\Big((\theta p_2)^{\mu_2}(p_1\theta p_2)\Big((\theta p_2)^{\mu_3}(p_3\theta p_4) + (\theta p_4)^{\mu_3}(p_2\theta p_3)\Big) \\
& - \Big((\theta p_2)^{\mu_3}(p_3\theta p_4) + (\theta p_4)^{\mu_3}(p_2\theta p_3)\Big)\cdot\Big((\theta p_3)^{\mu_2}(p_2\theta p_4) - (\theta p_4)^{\mu_2}(p_2\theta p_3)\Big)\Big).
\end{aligned}
$$

$$f_{\star_{\mathbf{2}}}(p,q) = \frac{\sin\frac{p\theta q}{2}}{\frac{p\theta q}{2}},$$

$$f_{\star_{\mathbf{3}}}(p,q,k) = \frac{\sin\frac{p\theta k}{2}\sin(\frac{p\theta q}{2}+\frac{p\theta k}{2})}{(\frac{p\theta q}{2}+\frac{p\theta k}{2})(\frac{p\theta k}{2}+\frac{q\theta k}{2})} + \frac{\sin\frac{q\theta k}{2}\sin(\frac{p\theta q}{2}-\frac{q\theta k}{2})}{(\frac{p\theta q}{2}-\frac{q\theta k}{2})(\frac{p\theta k}{2}+\frac{q\theta k}{2})},$$

$$f_{\star_{\mathbf{3}'}}(p,q,k) = \frac{\cos(\frac{p\theta q}{2}+\frac{p\theta k}{2}-\frac{q\theta k}{2})-1}{(\frac{p\theta q}{2}+\frac{p\theta k}{2}-\frac{q\theta k}{2})\frac{q\theta k}{2}} - \frac{\cos(\frac{p\theta q}{2}+\frac{p\theta k}{2}+\frac{q\theta k}{2})-1}{(\frac{p\theta q}{2}+\frac{p\theta k}{2}+\frac{q\theta k}{2})\frac{q\theta k}{2}},$$

$$\begin{aligned} f_{(\mathrm{I})}(p,q,k) =& \frac{2}{(\frac{p\theta q}{2}-\frac{p\theta k}{2}-\frac{q\theta k}{2})(\frac{p\theta q}{2}+\frac{p\theta k}{2}-\frac{q\theta k}{2})(\frac{p\theta q}{2}+\frac{p\theta k}{2}+\frac{q\theta k}{2})} \\ &+ \frac{\cos(\frac{p\theta q}{2}-\frac{p\theta k}{2}-\frac{q\theta k}{2})}{2\frac{p\theta q}{2}(\frac{p\theta q}{2}-\frac{q\theta k}{2})(\frac{p\theta q}{2}-\frac{p\theta k}{2}-\frac{q\theta k}{2})} + \frac{\cos(\frac{p\theta q}{2}+\frac{p\theta k}{2}-\frac{q\theta k}{2})}{2\frac{q\theta k}{2}(\frac{p\theta q}{2}-\frac{q\theta k}{2})(\frac{p\theta q}{2}+\frac{p\theta k}{2}-\frac{q\theta k}{2})} \\ &+ \frac{\cos(\frac{p\theta q}{2}+\frac{p\theta k}{2}+\frac{q\theta k}{2})}{2\frac{p\theta q}{2}\frac{q\theta k}{2}(\frac{p\theta q}{2}+\frac{p\theta k}{2}+\frac{q\theta k}{2})}, \end{aligned}$$

$$f_{(\mathrm{II})}(p,q,k) = \frac{\sin\left(\frac{p\theta q}{2}-\frac{q\theta k}{2}\right)\sin\frac{p\theta k}{2}}{\frac{p\theta q}{2}\frac{p\theta k}{2}\left(\frac{p\theta q}{2}-\frac{q\theta k}{2}\right)\left(\frac{p\theta k}{2}+\frac{q\theta k}{2}\right)} - \frac{\sin\left(\frac{p\theta q}{2}+\frac{p\theta k}{2}\right)\sin\frac{q\theta k}{2}}{\frac{p\theta q}{2}\frac{q\theta k}{2}\left(\frac{p\theta q}{2}+\frac{p\theta k}{2}\right)\left(\frac{p\theta k}{2}+\frac{q\theta k}{2}\right)}.$$

Tadpole integral:

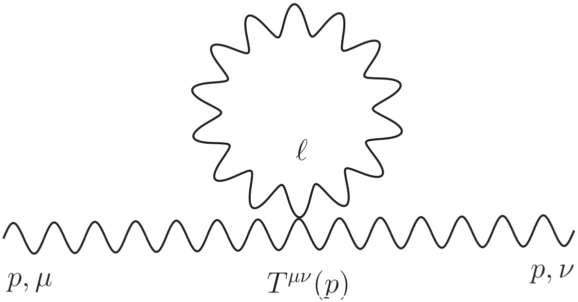

Figure: 7. Tadpole loop

$$T^{\mu\nu}_{(\mathrm{I,II})}(p) = \frac{1}{2}\mu^{d-D}\int \frac{d^D\ell}{(2\pi)^D}\frac{-ig_{\rho\sigma}}{\ell^2}\Gamma^{\mu\nu\rho\sigma}_{(\mathrm{I,II})}(p,-p,\ell,-\ell)$$

$$= \tau^{\mu\nu}_{(\mathrm{I,II})}\,\mu^{d-D}\int \frac{d^D\ell}{(2\pi)^D}\frac{\ell^2}{\ell^2} + \mathcal{T}^{\mu\nu\rho\sigma}_{(\mathrm{I,II})}\mu^{d-D}\int \frac{d^D\ell}{(2\pi)^D}\frac{\ell_\rho\ell_\sigma}{\ell^2}f^2_{\star_2}(p,\ell).$$

$$
\begin{aligned}
\tau_{(\mathrm{I})}^{\mu\nu} = &- \frac{2}{D}\Big\{ \Big[g^{\mu\nu}\boldsymbol{p}^2 - \boldsymbol{p}^{\mu}\boldsymbol{p}^{\nu}\Big] (\mathrm{tr}\theta\theta)(\kappa_3 - 1) \\
&+ \Big[g^{\mu\nu}(\theta\boldsymbol{p})^2 - (\theta\theta)^{\mu\nu}\boldsymbol{p}^2 + \boldsymbol{p}^{\{\mu}(\theta\theta\boldsymbol{p})^{\nu\}}\Big] 4(\theta\boldsymbol{p})^2(\kappa_1 - \kappa_2) \\
&+ (\theta\boldsymbol{p})^{\mu}(\theta\boldsymbol{p})^{\nu}\Big(4 + (1-\kappa_3)D(D-1) - 16\kappa + 8\kappa^2 + 8(D-1)(\kappa_1 - \kappa_2) + 4\kappa_4\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{(\mathrm{I})}^{\mu\nu\rho\sigma} = &- 2\Big\{\Big(g^{\mu\nu}(\theta\boldsymbol{p})^{\rho}(\theta\boldsymbol{p})^{\sigma} + \theta^{\mu\rho}\boldsymbol{p}^{\nu}(\theta\boldsymbol{p})^{\sigma} + \theta^{\nu\rho}\boldsymbol{p}^{\mu}(\theta\boldsymbol{p})^{\sigma} + \theta^{\mu\rho}\theta^{\nu\sigma}\boldsymbol{p}^2\Big) \\
&\quad \cdot 2\Big((D-3)\kappa^2 - 2\kappa + \kappa_1 + \kappa_2\Big) \\
&+ \Big(2g^{\mu\nu}\boldsymbol{p}^{\rho}(\theta\theta\boldsymbol{p})^{\sigma} - g^{\mu\rho}\boldsymbol{p}^{\nu}(\theta\theta\boldsymbol{p})^{\sigma} - g^{\nu\rho}\boldsymbol{p}^{\mu}(\theta\theta\boldsymbol{p})^{\sigma} \\
&- \boldsymbol{p}^{\mu}(\theta\theta)^{\nu\rho}\boldsymbol{p}^{\sigma} - \boldsymbol{p}^{\nu}(\theta\theta)^{\mu\rho}\boldsymbol{p}^{\sigma} + g^{\mu\rho}\theta^{\nu\sigma}\boldsymbol{p}^2 + g^{\nu\rho}\theta^{\mu\sigma}\boldsymbol{p}^2\Big) \cdot (2\kappa - \kappa_1 - \kappa_2) \\
&+ \Big(g^{\mu\rho}(\theta\boldsymbol{p})^{\nu}(\theta\boldsymbol{p})^{\sigma} + g^{\nu\rho}(\theta\boldsymbol{p})^{\mu}(\theta\boldsymbol{p})^{\sigma} + \theta^{\mu\rho}(\theta\boldsymbol{p})^{\nu}\boldsymbol{p}^{\sigma} + \theta^{\nu\rho}(\theta\boldsymbol{p})^{\mu}\boldsymbol{p}^{\sigma}\Big) \\
&\quad \cdot \Big(-1 - 2\kappa_3 + \kappa_4 + (2+D)\kappa_1 + (D-2)(\kappa_2 - 2\kappa) + 4\kappa^2\Big) \\
&+ \Big(g^{\mu\nu}g^{\rho\sigma}(\theta\boldsymbol{p})^2 + (\theta\theta)^{\mu\nu}(\boldsymbol{p}^{\rho}\boldsymbol{p}^{\sigma} - \boldsymbol{p}^2 g^{\rho\sigma}) + (\boldsymbol{p}^{\mu}(\theta\theta\boldsymbol{p})^{\nu} + \boldsymbol{p}^{\nu}(\theta\theta\boldsymbol{p})^{\mu})g^{\rho\sigma} \\
&- g^{\mu\rho}g^{\nu\sigma}(\theta\boldsymbol{p})^2 - g^{\mu\rho}(\theta\theta\boldsymbol{p})^{\nu}\boldsymbol{p}^{\sigma} - g^{\nu\rho}(\theta\theta\boldsymbol{p})^{\mu}\boldsymbol{p}^{\sigma}\Big) \cdot 2\kappa^2 \\
&+ (\theta\boldsymbol{p})^{\mu}(\theta\boldsymbol{p})^{\nu}g^{\rho\sigma} \cdot 4\Big(\kappa_1 + \kappa_2 - 2\kappa + (D-1)\kappa_4\Big) \\
&- \Big[g^{\mu\nu}\boldsymbol{p}^2 - \boldsymbol{p}^{\mu}\boldsymbol{p}^{\nu}\Big] (\theta\theta)^{\rho\sigma}(\kappa_4 - 1)\Big\},
\end{aligned}
$$

$$\tau^{\mu\nu}_{(\mathrm{II})} = -\frac{2}{D}\Big\{ \left[g^{\mu\nu}p^2 - p^\mu p^\nu\right](\mathrm{tr}\theta\theta)3(\kappa_3 - 1)$$
$$+ \left[g^{\mu\nu}(\theta p)^2 - (\theta\theta)^{\mu\nu}p^2 + p^{\{\mu}(\theta\theta p)^{\nu\}}\right]8(\theta p)^2(\kappa_1' - \kappa_2')$$
$$+ (\theta p)^\mu(\theta p)^\nu\Big(3(1-\kappa_3')D(D-1) - 16\kappa + 8\kappa^2 + 16(D-1)(\kappa_1' - \kappa_2') + 8\kappa_4'\Big)\Big\},$$

$$\mathcal{T}^{\mu\nu\rho\sigma}_{(\mathrm{II})} = -2\Big\{\Big(g^{\mu\nu}(\theta p)^\rho(\theta p)^\sigma + \theta^{\mu\rho}p^\nu(\theta p)^\sigma + \theta^{\nu\rho}p^\mu(\theta p)^\sigma - \theta^{\mu\rho}\theta^{\nu\sigma}p^2\Big)$$
$$\cdot 2\Big((D-3)\kappa^2 - 2\kappa + 2\kappa_2'\Big)$$
$$+\Big(2g^{\mu\nu}p^\rho(\theta\theta p)^\sigma - g^{\mu\rho}p^\nu(\theta\theta p)^\sigma - g^{\nu\rho}p^\mu(\theta\theta p)^\sigma$$
$$- p^\mu(\theta\theta)^{\nu\rho}p^\sigma - p^\nu(\theta\theta)^{\mu\rho}p^\sigma + g^{\mu\rho}\theta^{\nu\sigma}p^2 + g^{\nu\rho}\theta^{\mu\sigma}p^2\Big)\cdot 2\left(\kappa_2' - \kappa\right)$$
$$+\Big(g^{\mu\rho}(\theta p)^\nu(\theta p)^\sigma + g^{\nu\rho}(\theta p)^\mu(\theta p)^\sigma + \theta^{\mu\rho}(\theta p)^\nu p^\sigma + \theta^{\nu\rho}(\theta p)^\mu p^\sigma\Big)$$
$$\cdot\Big(-2\kappa_3' - 2(D-2)\kappa + 4\kappa^2 + 2(D-2)\kappa_2'\Big)$$
$$+\Big(g^{\mu\nu}g^{\rho\sigma}(\theta p)^2 + (\theta\theta)^{\mu\nu}(p^\rho p^\sigma - p^2g^{\rho\sigma}) + (p^\mu(\theta\theta p)^\nu + p^\nu(\theta\theta p)^\mu)g^{\rho\sigma}$$
$$- g^{\mu\rho}g^{\nu\sigma}(\theta p)^2 - g^{\mu\rho}(\theta\theta p)^\nu p^\sigma - g^{\nu\rho}(\theta\theta p)^\mu p^\sigma\Big)\cdot 2\kappa^2$$
$$+(\theta p)^\mu(\theta p)^\nu g^{\rho\sigma}\cdot\Big(8\kappa_2' - 8\kappa + (D-1)(2\kappa_3' + \kappa_4' - 3)\Big)$$
$$-\left[g^{\mu\nu}p^2 - p^\mu p^\nu\right](\theta\theta)^{\rho\sigma}(2\kappa_3' + \kappa_4' - 3)\Big\}.$$

Commutative massless tadpole

- t'Hooft and Veltman conjectured that commutative massless tadpole should vanish in the dimensional regularization (Leibbrandt 1975)
- Two methods available for proving the conjecture at $D \to 4$.
  1. Turn tadpole into bubble integral

$$\int \frac{d^D\ell}{(2\pi)^D} \frac{N(\ell,p)}{\ell^2} = \int \frac{d^D\ell}{(2\pi)^D} \frac{N(\ell,p)}{\ell^2} \frac{(\ell+p)^2}{(\ell+p)^2}.$$

  2. Use the n-nested zero regulator $f\left(\frac{D}{2}\right)$

$$\int \frac{d^D\ell}{(2\pi)^D} \exp\left[-\lambda\ell^2\right] = (4\pi\lambda)^{-\frac{D}{2}} \exp\left[-\lambda f\left(\frac{D}{2}\right)\right],$$

  with following properties
  - $f\left(\frac{D}{2}\right)$ is a nonzero analytic function;
  - $f\left(\frac{D}{2}\right) = 0$ when $D \in Z^+$;
  - $f^{(\ell)}\left(\frac{D}{2}\right) = 0$ when $D \in Z^+$ and $\ell \leq \ell_0 \in N$;
  - $\forall \mathrm{Re} D \notin Z^+,\ \exists \mathrm{Im} D,\ \mathrm{Re}\left[f\left(\frac{D}{2}\right)\right] > 0.$

Evaluate the NC massless tadpole

- *Both* methods admit reasonable extension to the NC theory.
  1. NC tadpole can be simply made into the NC bubble

$$I^{\mu\nu} = \int \frac{d^D\ell}{(2\pi)^D} \frac{\ell^\mu \ell^\nu}{\ell^2} = \int \frac{d^D\ell}{(2\pi)^D} \frac{\ell^\mu \ell^\nu}{\ell^2} \frac{(\ell+p)^2}{(\ell+p)^2} f_{\star_2}(\ell,p)^2.$$

  2. The n-nested zero regulator $f\left(\frac{D}{2}\right)$ can be imposed on a simpler parametrization to obtain the aforementioned Bessel K plus hypergeometric function combinations

$$I^{\mu\nu} = \int \frac{d^Dk}{(2\pi)^D} \frac{\ell^\mu \ell^\nu}{\ell^2} f^2_{\star_2}(p,\ell) = -2\int_0^\infty d\lambda \int_0^{1/\lambda} dy\, y \int \frac{d^D\ell}{(2\pi)^D} \left(\frac{\ell^2}{D} g^{\mu\nu} - \frac{y^2}{4}(\theta p)^\mu (\theta p)^\nu\right) \cdot \exp\left[-\lambda \ell^2 - \frac{\lambda y^2}{4}(\theta p)^2\right].$$

- General D-dimensional integral results are different between methods, however $D \to 4$ limit matches.

$I^{\mu\nu}$, method 1

$$I^{\mu\nu} = g^{\mu\nu}\frac{1}{D-1}\left(4(\theta p)^{-\frac{D}{2}}\mathcal{K}\left[\frac{D}{2}-1;0,0\right] - \frac{4p^2}{(\theta p)^2}\left((1-D)\mathcal{K}\left[\frac{D}{2}-2;0,1\right]\right.\right.$$
$$\left.\left.+2D\mathcal{K}\left[\frac{D}{2}-2;1,1\right]\right) - p^2\left((1-D)\mathcal{W}\left[\frac{D}{2}-1;0,1\right] - 2D\mathcal{W}\left[\frac{D}{2}-1;1,1\right]\right)\right)$$
$$+ p^\mu p^\nu\left(\frac{4}{(\theta p)^2}\left((D-2)\mathcal{K}\left[\frac{D}{2}-2;1,0\right] + 2(1-D)\mathcal{K}\left[\frac{D}{2}-2;1,1\right]\right)\right.$$
$$\left.-\left((1-D)\mathcal{W}\left[\frac{D}{2}-1;1,0\right] + 2D\mathcal{W}\left[\frac{D}{2}-1;1,1\right]\right)\right)$$
$$+ (\theta p)^\mu(\theta p)^\nu\frac{1}{D-1}\left(-4D(\theta p)^{-1-\frac{D}{2}}\mathcal{K}\left[\frac{D}{2}-1;0,0\right] + \frac{4p^2}{(\theta p)^4}\left((1-D)\mathcal{K}\left[\frac{D}{2}-2;0,1\right]\right.\right.$$
$$\left.\left.+(2D-1)\mathcal{K}\left[\frac{D}{2}-2;1,1\right]\right) + \frac{p^2}{(\theta p)^2}\left((1-D)\mathcal{W}\left[\frac{D}{2}-1;0,1\right] + 2D\mathcal{W}\left[\frac{D}{2}-1;1,1\right]\right)\right),$$

$$\mathcal{K}[\nu;a,b] = 2^\nu(\theta p)^{-\nu}\int_0^1 dx\, x^a(1-x)^b X^\nu K_\nu[X],$$
$$\mathcal{W}[\nu;a,b] = \int_0^1 dx\, x^a(1-x)^b W_\nu[X].$$

4-photon tadpole at $D \to 4$

$$T^{\mu\nu}_{(\mathrm{I})}(p) = \frac{1}{3\pi^2}\Bigg\{\Big[g^{\mu\nu}p^2 - p^\mu p^\nu\Big]\left(\frac{tr\theta\theta}{(\theta p)^4} + 4\frac{(\theta\theta p)^2}{(\theta p)^6}\right)(1-\kappa_4)$$
$$+ (\theta p)^\mu(\theta p)^\nu \frac{4}{(\theta p)^4}\left(2\kappa^2 - 4\kappa + 6\kappa_1 + 2\kappa_2 - 2\kappa_3 + \kappa_4 - 1\right)$$
$$+ \Big[g^{\mu\nu}(\theta p)^2 - (\theta\theta)^{\mu\nu}p^2 + p^{\{\mu}(\theta\theta p)^{\nu\}}\Big]\frac{4}{(\theta p)^4}(2\kappa^2 - 2\kappa + \kappa_1 + \kappa_2)$$
$$+ \Big[(\theta\theta)^{\mu\nu}(\theta p)^2 + (\theta\theta p)^\mu(\theta\theta p)^\nu\Big]\frac{8p^2}{(\theta p)^6}(\kappa^2 - 2\kappa + \kappa_1 + \kappa_2)$$
$$+ (\theta p)^{\{\mu}(\theta\theta\theta p)^{\nu\}}\frac{4p^2}{(\theta p)^6}(2\kappa - \kappa_1 - \kappa_2)\Bigg\},$$

$$T^{\mu\nu}_{(\mathrm{II})}(p) = \frac{1}{3\pi^2}\Bigg\{\Big[g^{\mu\nu}p^2 - p^\mu p^\nu\Big]\left(\frac{tr\theta\theta}{(\theta p)^4} + 4\frac{(\theta\theta p)^2}{(\theta p)^6}\right)(3 - 2\kappa'_3 - \kappa'_4)$$
$$+ (\theta p)^\mu(\theta p)^\nu \frac{8}{(\theta p)^4}\left(\kappa^2 - 2\kappa + 2\kappa'_1 + 2\kappa'_2 - \kappa'_3\right)$$
$$+ \Big[g^{\mu\nu}(\theta p)^2 - (\theta\theta)^{\mu\nu}p^2 + p^{\{\mu}(\theta\theta p)^{\nu\}}\Big]\frac{4}{(\theta p)^4}(2\kappa^2 - 2\kappa + 2\kappa'_2)$$
$$+ \Big[(\theta\theta)^{\mu\nu}(\theta p)^2 + (\theta\theta p)^\mu(\theta\theta p)^\nu\Big]\frac{8p^2}{(\theta p)^6}(\kappa^2 - 2\kappa + 2\kappa'_2)$$
$$+ (\theta p)^{\{\mu}(\theta\theta\theta p)^{\nu\}}\frac{8p^2}{(\theta p)^6}(\kappa - \kappa'_2)\Bigg\}.$$

$I^{\mu\nu}$, method 2

$$I^{\mu\nu} = I \cdot \left(-\frac{g^{\mu\nu}}{D} + \frac{(\theta p)^\mu (\theta p)^\nu}{(\theta p)^2} \right),$$

$$I = (4\pi)^{-\frac{D}{2}} \left(\frac{(\theta p)^2}{4} \right)^{-1} \left\{ \left(f\left(\frac{D}{2}\right) \right)^{\frac{D}{2}-1} \Gamma\left(1-\frac{D}{2}\right) - W_{\frac{D}{2}} \left[\left(f\left(\frac{D}{2}\right)(\theta p)^2 \right)^{\frac{1}{2}} \right] \right.$$
$$\left. -2 \left(f\left(\frac{D}{2}\right) \right)^{\frac{D}{4}-\frac{1}{2}} \left(\frac{(\theta p)^2}{4} \right)^{-\frac{1}{2}-\frac{D}{4}} K_{\frac{D}{2}-1} \left[\left(f\left(\frac{D}{2}\right)(\theta p)^2 \right)^{\frac{1}{2}} \right] \right\},$$

$$W_\nu[X] = (\theta p)^{-2\nu} \left[X^{2\nu} \Gamma[-\nu]\, {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \nu+1; \frac{X^2}{4} \right) \right.$$
$$\left. - \frac{2^{2\nu}}{1-2\nu} \Gamma[\nu]\, {}_1F_2\left(\frac{1-2\nu}{2}; 1-\nu, \frac{3-2\nu}{2}; \frac{X^2}{4} \right) \right].$$

$$\lim_{D\to 4} I(D) = -\frac{2}{3\pi^2} (\theta p)^{-4}$$

$\Rightarrow$ NC tadpole is quadratic IR divergent only when $D \to 4$.

Summing up the quadratic IR divergence from both tadpole and bubble diagrams when $\theta = \theta_{\sigma_2}$

- Both bubble and tadpole diagrams contain quadratic IR divergence, summing up both makes more sense.
- $\theta = \theta_{\sigma_2}$ simplifies the structure.

$$
\Pi^{\mu\nu}_{(\mathrm{I},\mathrm{II})}(p)\Big|^{\theta_{\sigma_2}}_{\mathrm{IR}} \sim \mathcal{B}^{\mu\nu}(p)\Big|^{\theta_{\sigma_2}}_{\mathrm{IR}} + T^{\mu\nu}_{(\mathrm{I},\mathrm{II})}(p)\Big|^{\theta_{\sigma_2}}_{\mathrm{IR}}
$$

$$
\sim \frac{1}{(4\pi)^2}\Big\{\Big(g^{\mu\nu}p^2 - p^\mu p^\nu\Big) B_{\mathcal{I}_{\mathrm{sum}\,(\mathrm{I},\mathrm{II})}}(p) + (\theta p)^\mu(\theta p)^\nu\, B_{\mathcal{I}\mathcal{I}_{\mathrm{sum}\,(\mathrm{I},\mathrm{II})}}(p)\Big\},
$$

$$
B_{\mathcal{I}_{\mathrm{sum}\,(\mathrm{I})}}(p) \sim +\frac{16}{3}\frac{1}{p^2(\theta p)^2}\Big(5\kappa^2 - 2\kappa + 1\Big),
$$

$$
B_{\mathcal{I}\mathcal{I}_{\mathrm{sum}\,(\mathrm{I})}}(p) \sim -\frac{16}{3}\frac{1}{(\theta p)^4}\Big(7\kappa^2 - 10\kappa + 7 - 32\kappa_1 - 16\kappa_2 + 8\kappa_3 - 4\kappa_4 + 4\Big),
$$

$$
B_{\mathcal{I}_{\mathrm{sum}\,(\mathrm{II})}}(p) \sim +\frac{16}{3}\frac{1}{p^2(\theta p)^2}\Big(5\kappa^2 - 2\kappa + 1\Big),
$$

$$
B_{\mathcal{I}\mathcal{I}_{\mathrm{sum}\,(\mathrm{II})}}(p) \sim -\frac{16}{3}\frac{1}{(\theta p)^4}\Big(7\kappa^2 - 10\kappa + 7 - 16\kappa_1' - 32\kappa_2' + 8\kappa_3'\Big).
$$

- $B_{\mathcal{I}}$ depends only on κ and does not vanish for any real κ......

$\theta = \theta_{\sigma_2}$, $\kappa = 1/3$

- Relatively concise full $\Pi^{\mu\nu}_{(\mathrm{I,II})}(p)$ is available when $\theta = \theta_{\sigma_2}$, $\kappa = 1/3$

$$\Pi^{\mu\nu}_{(\mathrm{I})}(p)\Big|^{\theta_{\sigma_2}}_{\kappa=1/3} = \frac{p^2}{\pi^2}\left\{\frac{7}{27}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\left(1 + \frac{8}{7}\frac{1}{p^2(\theta p)^2}\right) - \frac{1}{2}\frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\left(1 + \frac{8}{27}\frac{1}{p^2(\theta p)^2}(19 - 72\kappa_1 - 36\kappa_2 + 18\kappa_3 - 9\kappa_4)\right)\right\},$$

$$\Pi^{\mu\nu}_{(\mathrm{II})}(p)\Big|^{\theta_{\sigma_2}}_{\kappa=1/3} = \frac{p^2}{\pi^2}\left\{\frac{7}{27}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\left(1 + \frac{8}{7}\frac{1}{p^2(\theta p)^2}\right) - \frac{1}{2}\frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\left(1 + \frac{8}{27}\frac{1}{p^2(\theta p)^2}(10 - 36\kappa'_1 - 72\kappa'_2 + 18\kappa'_3)\right)\right\}.$$

- One can further set $\kappa_1 = \kappa_2 = \kappa_3 = \kappa'_1 = \kappa'_2 = \kappa'_3 = 1/9$, $\kappa_4 = 1$ and make

$$\Pi^{\mu\nu}_{(\mathrm{I})}(p) = \Pi^{\mu\nu}_{(\mathrm{II})}(p) = \frac{p^2}{\pi^2}\left\{\frac{7}{27}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\left(1 + \frac{8}{7}\frac{1}{p^2(\theta p)^2}\right) - \frac{1}{2}\frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\right\}.$$

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Ptolemy experiment: $\nu_e + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-, \ (\nu_e + n \to p + e^-)$

Plasmon decay: $\gamma_{pl} \to \bar{\nu}_R \nu_R$

Scattering: $e\nu_R \to e\nu_R$

Summary

DeWitt effective action, with C.P. Martin and J. You

- Is the QGFT defined in terms of NC fields the same as QGFT defined in terms of comm. fields and obtained from NC action by θ-exact SW maps, i.e. is for example $\hat{\Pi}^{\mu\nu}(p) = \Pi^{\mu\nu}(p)$?
- Path integral DeWitt effective action in terms of NC fields

$$e^{\frac{i}{\hbar}\hat{\Gamma}_{\rm DeW}\left[\hat{B}_\mu\right]} = \int d\hat{Q}^a_\mu d\hat{C}^a d\hat{\bar{C}}^a d\hat{F}^a \cdot e^{\frac{i}{\hbar}S_{\rm NCYM}\left[\hat{B}_\mu + \hbar^{\frac{1}{2}}\hat{Q}_\mu\right] + iS_{\rm gf}\left[\hat{B}_\mu, \hat{Q}_\mu, \hat{F}, \hat{\bar{C}}, \hat{C}\right]},$$

- DeWitt eff. action in terms of commutative fields via SW map

$$e^{\frac{i}{\hbar}\Gamma_{\rm DeW}\left[B_\mu\right]} = \int dQ^a_\mu dC^a d\hat{\bar{C}}^a d\hat{F}^a \cdot e^{\frac{i}{\hbar}S_{\rm NCYM}\left[B_\mu + \hbar^{\frac{1}{2}}Q_\mu\right] + iS_{\rm gf}\left[B_\mu, Q_\mu, \hat{F}, \hat{\bar{C}}, C\right]}.$$

- $\hat{\bar{C}}$ and $\hat{F}$ are the same before and after SW map. $S_{\rm gf}$ is the gauge fixing action in the BRST language.

θ-exact Seiberg-Witten map expansion

$$\hat{A}_\mu\left[A_\mu,\theta\right](x) = A_\mu(x) + \sum_{n=2}^{\infty} \mathcal{A}_\mu^{(n)}(x),$$

$$\hat{C}\left[A_\mu, C,\theta\right](x) = C(x) + \sum_{n=1}^{\infty} \mathcal{C}^{(n)}(x).$$

$$\mathcal{A}_\mu^{(n)}(x) = \int \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4} e^{i\left(\sum_{i=1}^{n} p_i\right)x}$$

$$\cdot\, \mathfrak{A}_\mu^{(n)}\left[(a_1,\mu_1,p_1),\ldots\ldots,(a_n,\mu_n,p_n);\theta\right] \tilde{A}_{\mu_1}^{a_1}(p_1)\ldots\ldots\tilde{A}_{\mu_n}^{a_n}(p_n),$$

$$\mathcal{C}^{(n)}(x) = \int \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4} e^{i\left(p+\sum_{i=1}^{n} p_i\right)x}$$

$$\cdot\, \mathfrak{C}^{(n)}\left[(a_1,\mu_1,p_1),\ldots\ldots,(a_n,\mu_n,p_n);(a,p);\theta\right] \tilde{A}_{\mu_1}^{a_1}(p_1)\ldots\ldots\tilde{A}_{\mu_n}^{a_n}(p_n)C^a(p).$$

Background field splitting and SW map

- Background field splitting can be SW mapped in a straightforward way.
  - SW maps: $\hat{A}_\mu = \hat{A}_\mu [A_\mu; \theta]$, $\hat{C} = \hat{C} [A_\mu, C; \theta]$
  - background field splitting of the ordinary field $A_\mu = B_\mu + Q_\mu$

$$\begin{aligned} \Longrightarrow \hat{A}_\mu [B_\mu + Q_\mu; \theta] =& \hat{A}_\mu [B_\mu; \theta] + \hat{Q}_\mu [B_\mu, Q_\mu; \theta] \\ =& \hat{B}_\mu [B_\mu; \theta] + \hat{Q}_\mu [B_\mu, Q_\mu; \theta] \end{aligned}$$

- NC BRS transformations are correctly induced
  - Usual SW map consistency.

$$\hat{\delta}_{\mathrm{BRS}} \hat{A}_\mu = \delta_{\mathrm{BRS}} \hat{A}_\mu [A_\mu; \theta] = \hat{\mathcal{D}}_\mu \hat{C}.$$

  - BRS transformation of the ordinary fields after splitting

$$\delta_{\mathrm{BRS}} B_\mu = 0, \delta_{\mathrm{BRS}} Q_\mu = \delta_{\mathrm{BRS}} A_\mu = D_\mu [B_\mu + Q_\mu] C$$

$$\begin{aligned} \Longrightarrow & \hat{\delta}_{\mathrm{BRS}} \hat{B}_\mu = \delta_{\mathrm{BRS}} B_\mu [b_\mu, \theta] = 0, \\ & \hat{\delta}_{\mathrm{BRS}} \hat{Q}_\mu = \delta_{\mathrm{BRS}} \hat{A}_\mu [B_\mu + Q_\mu, \theta] = \hat{\mathcal{D}}_\mu [B_\mu + Q_\mu] \hat{C}. \end{aligned}$$

SW map changing variable in the path integral

- If SW map is invertible, then

$$e^{\frac{i}{\hbar}\Gamma_{\mathrm{DeW}}[B_\mu]} = \int d\hat{Q}^a_\mu d\hat{C}^a d\hat{\bar{C}}^a d\hat{F}^a \; J_1^{-1}[B,Q]\, J_2[B,Q] \cdot e^{\frac{i}{\hbar}S_{\mathrm{NCYM}}\left[\hat{B}_\mu + \hbar^{\frac{1}{2}}\hat{Q}_\mu\right] + iS_{\mathrm{gf}}\left[\hat{B}_\mu, \hat{Q}_\mu, \hat{F}, \hat{\bar{C}}, \hat{C}\right]},$$

$$J_1[B^a,Q^a] = \det \frac{\delta \hat{Q}^a_\mu(x)}{\delta Q^b_\nu(y)} = \exp \,\mathrm{Tr}\, \ln \left(\frac{\delta \hat{Q}^a_\mu(x)}{\delta Q^b_\nu(y)}\right),$$

$$J_2[B^a,Q^a] = \det \frac{\delta \hat{C}^a(x)}{\delta C^b(y)} = \exp \,\mathrm{Tr}\, \ln \left(\frac{\delta \hat{C}^a(x)}{\delta C^b(y)}\right).$$

- Consequently

$$J_1[B^a,Q^a] = J_2[B^a,Q^a] = 1 \Longrightarrow \Gamma_{\mathrm{DeW}}\left[B_\mu\right] = \hat{\Gamma}_{\mathrm{DeW}}\left[\hat{B}_\mu[B]\right].$$

Evaluate $J_1[B^a, Q^a]$

- Perturbative expansion of the SW map changing variable

$$\frac{\delta \hat{Q}^a_\mu(x)}{\delta Q^b_\nu(y)} = \frac{1}{\hbar^{\frac{1}{2}}} \frac{\delta \hat{A}^a_\mu(x)}{\delta Q^b_\nu(y)} = \delta^a_b \delta^\nu_\mu \, \delta(x-y) + \sum_{n=2}^{\infty} \int \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4}$$

$$\cdot e^{i\left(\sum_{i=1}^{n-1} p_i\right)x} e^{ip_n(x-y)} \mathcal{M}^{(n)\,a\,\nu}_{\ \ \ \ b\,\mu}(p_1, p_2, \ldots . p_{n-1}; p_n; \theta),$$

$$\mathcal{M}^{(n)\,a\,\nu}_{\ \ \ \ b\,\mu}(p_1, p_2, \ldots . p_{n-1}; p_n; \theta) = n \, \mathrm{tr}\Big(T^a \mathfrak{A}^{(n)}_\mu \big[(a_1, \mu_1, p_1),$$

$$\ldots, (a_{n-1}, \mu_{n-1}, p_{n-1}), (b, \nu, p_n); \theta\big]\Big) \tilde{A}^{a_1}_{\mu_1}(p_1) \ldots \tilde{A}^{a_{n-1}}_{\mu_{n-1}}(p_{n-1}).$$

- How to solve that expression? By isolating one momentum dependence and integrate it out. So we get:

$$\ln\, J_1[B,Q] = \mathrm{Tr}\ln\left(\frac{\delta \hat{Q}^a_\mu(x)}{\delta Q^b_\nu(y)}\right) = \sum_{n=2}^{\infty}\int \prod_{i=1}^{n-1} \frac{d^4 p_i}{(2\pi)^4}\,\delta(\sum_{i=1}^{n-1} p_i)$$

$$\cdot \int \frac{d^4 q}{(2\pi)^4}\,\mathcal{M}^{(n)\,a\,\mu}{}_{a\,\mu}(p_1, p_2, ..., p_{n-1}; q; \theta) + \sum_{m=1}^{\infty} \frac{(-1)^m}{m+1}$$

$$\cdot \sum_{n_1=2}^{\infty} \cdots \sum_{n_{m+1}=2}^{\infty} \int \prod_{i_1=1}^{n_1-1} \frac{d^4 p_{1,i_1}}{(2\pi)^4} \cdots \int \prod_{i_{m+1}=1}^{n_{m+1}-1} \frac{d^4 p_{m+1,i_{m+1}}}{(2\pi)^4}$$

$$\cdot \delta\left(\sum_{i=1}^{m+1} l_i\right) \int \frac{d^4 q}{(2\pi)^4}\Big[\mathcal{M}^{(n_1)\,a\,\mu_1}{}_{a_1\,\mu}(p_{1,1}, p_{1,2}, ..., p_{1,n_1-1}; q; \theta)$$

$$\cdot \mathcal{M}^{(n_2)\,a_1\,\mu_2}{}_{a_2\,\mu_1}(p_{2,1}, p_{2,2}, ..., p_{2,n_2-1}; q - l_2; \theta)$$

$$\cdots\cdots\cdots$$

$$\cdot \mathcal{M}^{(n_{m+1})\,a_m\,\mu}{}_{a\;\;\mu_m}\left(p_{m+1,1}, p_{m+1,2}, ..., p_{m+1,n_{m+1}-1}, q - \sum_{i=2}^{m+1} l_i; \theta\right)\Big].$$

$$l_1 = \sum_{i_1=1}^{n_1-1} p_{1,i_1},\; ...., l_{m+1} = \sum_{i_{m+1}=1}^{n_{m+1}} p_{m+1,i_{m+1}}.$$

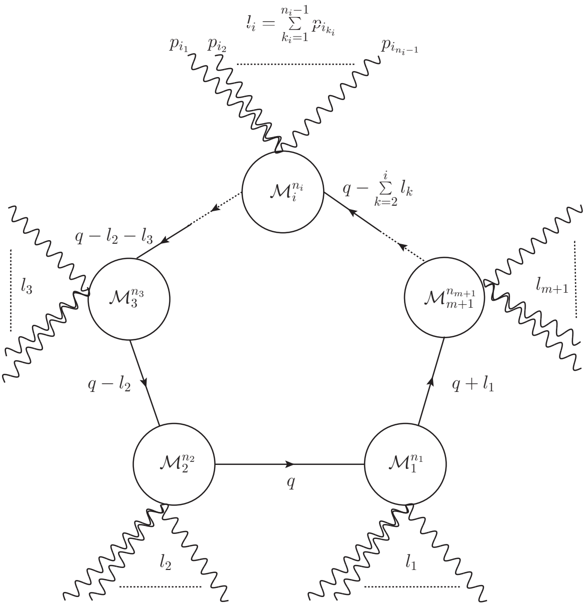

Figure: 8. The one-loop diagram interpretation/ilustration of $\ln J_1[B, Q]$.

Relevant loop integral

- Integral over the loop momenta q factorizes

$$\mathfrak{V} = \int \frac{d^D q}{(2\pi)^D} \Big\{ \mathrm{tr}\Big(T^a \mathfrak{A}^{(n_1)}_\mu \big[(b_{1,1}, \nu_{1,1}, p_{1,1}),$$

$$....., (b_{1,n_1-1}, \nu_{1,n_1-1}, p_{1,n_1-1}), (a_1, \mu_1, q); \theta\big]\Big)$$

$$\cdots\cdots\cdots$$

$$\cdot \mathrm{tr}\Big(T^{a_m} \mathfrak{A}^{(n_{m+1})}{}_{\mu_m} \big[(b_{m+1,1}, \nu_{m+1,1}, p_{m+1,1}), ...,$$

$$(b_{m+1,n_{m+1}-1}, \nu_{m+1,n_{m+1}-1}, p_{m+1,n_{m+1}-1}),$$

$$(a, \mu, q - \sum_{i=2}^{m+1} l_i); \theta\big]\Big)\Big\} = \sum \mathfrak{I} = \sum \int \frac{d^D q}{(2\pi)^D} \mathbb{Q}(q)\, \mathbb{I}(q\theta k_i, k_i \theta k_j).$$

- $\mathbb{Q}(q)$ is polynomial with respect to loop momenta q.

$\mathfrak{I} = 0$

- Space-like $\theta^{\mu\nu}$ can serve as a projector

$$\begin{aligned} \theta^{\mu\nu} &= \theta, \quad \text{if} \quad \mu = 2, \nu = 3 \\ \theta^{\mu\nu} &= -\theta, \quad \text{if} \quad \mu = 3, \nu = 2 \\ \theta^{\mu\nu} &= 0, \quad \text{otherwise.} \end{aligned} \implies \begin{aligned} q^{\mu} &= q_{\perp}^{\mu} + q_{||}^{\mu}, \\ q_{||} &= (0, 0, l^2, l^3), \\ \theta q_{\perp} &= 0. \end{aligned}$$

$$\begin{aligned} \mathfrak{I} = &\int \frac{d^D q}{(2\pi)^D} \mathbb{Q}(q) \mathbb{I}(q\theta k_i, k_i \theta k_j) \\ = &\frac{1}{(2\pi)^D} \int dq_{||} \left\{ \int d^{D-2} q_{\perp} \mathbb{Q}(q) \mathbb{I}(q_{||}\theta k_i, k_i \theta k_j) \right\}, \\ &\frac{1}{(2\pi)^D} \int dq_{||} \mathbb{I}(q_{||}\theta k_i, k_i \theta k_j) \left\{ \int d^{D-2} q_{\perp} \mathbb{Q}(q) \right\}. \end{aligned}$$

- Integral over $q_{\perp}$ vanishes under Dimensional regularization.

$$\int d^{D-2} q_{\perp} \mathbb{Q}(q) = 0.$$

- Our "chain of evidence"

$$
\begin{aligned}
\int d^{D-2}q_\perp \, \mathbb{Q}(q) = 0 \\
\Longrightarrow \mathfrak{I} = 0 \\
\Longrightarrow \mathfrak{V} = 0 \\
\Longrightarrow \ln J_1[B, Q] = 0 \\
\Longrightarrow J_1[B, Q] = 1.
\end{aligned}
$$

- The same argument hold for J_2, thus

$$J_2[B, Q] = 1.$$

- In the end, indeed

$$J_1[B^a, Q^a] = J_2[B^a, Q^a] = 1 \Longrightarrow \Gamma_{\text{DeW}}\left[B_\mu\right] = \hat{\Gamma}_{\text{DeW}}\left[\hat{B}_\mu[B]\right].$$

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Summary

Perturbative computation in Background field method

- Gauge fixing action

$$S_{\rm BFG} = \int {\rm tr} \left(\alpha \hat{F}^2 + \hat{F} \hat{\mathcal{D}}_\mu \left[\hat{B} \right] \hat{Q}^\mu - \hat{\bar{C}} \hat{\mathcal{D}}_\mu \left[\hat{B} \right] \hat{\mathcal{D}}^\mu \left[\hat{B} + \hat{Q} \right] \hat{C} \right),$$

- Classical action

$$S_{\rm NCYM} \left[\hat{B} + \hat{Q} \right] = -\frac{1}{4} \int {\rm tr} \left(\hat{F}_{\mu\nu} \left(\hat{B} + \hat{Q} \right) \hat{F}^{\mu\nu} \left(\hat{B} + \hat{Q} \right) \right)$$

$$= -\frac{1}{4} \int {\rm tr} \left(\hat{F}_{\mu\nu} \left(\hat{B} \right) \hat{F}^{\mu\nu} \left(\hat{B} \right) \right) - \frac{1}{2} \int {\rm tr} \left(\hat{\mathcal{D}}^\mu \hat{F}_{\mu\nu} \left(\hat{B} \right) \hat{Q}^\nu \right)$$

$$- \frac{1}{4} \int {\rm tr} \left(\hat{\mathcal{D}}_\mu \left(\hat{B} \right) \hat{Q}_\nu - \hat{\mathcal{D}}_\nu \left(\hat{B} \right) \hat{Q}_\mu \right)^2$$

$$- \frac{1}{2} \int {\rm tr} F^{\mu\nu} \left(\hat{B} \right) \left[\hat{Q}_\mu \stackrel{\star}{,} \hat{Q}_\nu \right] + \mathcal{O} \left(\hat{Q}^3 \right).$$

- Total input action

$$\begin{aligned}
\hat{S}_{\mathrm{loop}} =& S_{\mathrm{BFG}} + S_{\mathrm{NCYM}}\left[\hat{B}+\hat{Q}\right] \\
& - S_{\mathrm{NCYM}}\left[\hat{B}\right] - \int \frac{\delta S_{\mathrm{NCYM}}\left[\hat{B}_\mu\right]}{\delta \hat{B}_\mu}\hat{Q}_\mu \\
=& -\frac{1}{4}\int d^4x\, \mathrm{tr}\left(\hat{\mathcal{D}}_\mu\left(\hat{B}\right)\hat{Q}_\nu - \hat{\mathcal{D}}_\nu\left(\hat{B}\right)\hat{Q}_\mu\right)^2 \\
& - \frac{1}{2}\int d^4x\, \mathrm{tr} F^{\mu\nu}(B)\left[Q_\mu \stackrel{\star}{,} Q_\nu\right] + \mathcal{O}\left(Q^3\right) \\
& - \int \mathrm{tr}\left(\frac{1}{2}\left(\hat{\mathcal{D}}_\mu\left[\hat{B}\right]\hat{Q}^\mu\right)^2 + \hat{\bar{C}}\hat{\mathcal{D}}_\mu\left[\hat{B}\right]\hat{\mathcal{D}}^\mu\left[\hat{B}+\hat{Q}\right]\hat{C}\right).
\end{aligned}$$

Classical equations of motion

- Subtract $\int \frac{\delta S_{\rm NCYM}[\hat{B}_\mu]}{\delta \hat{B}_\mu}[B] \cdot \hat{Q}_\mu[B,Q]$ or $\int \frac{\delta S_{\rm NCYM}[\hat{B}_\mu[B_\nu]]}{\delta B_\mu} Q_\mu$?
- The ordinary and NC E.O.M. are equivalent if SW map is invertible, but not identical because of the chain rule.

$$0 = \frac{\delta S_{\rm NCYM}}{\delta B^a_\mu(x)} = \int d^4y \frac{\delta S_{\rm NCYM}}{\delta \hat{B}^b_\nu(y)} \frac{\delta \hat{B}^b_\nu(y)}{\delta B^a_\mu(x)}\Bigg|_{\hat{B}^a_\mu = \hat{B}^a_\mu[B^b_\nu]}$$

$$\Leftrightarrow \frac{\delta S_{\rm NCYM}}{\delta \hat{B}^a_\mu}\Bigg|_{\hat{B}^a_\mu[B^b_\nu]} = \hat{\mathcal{D}}^\mu \left[\hat{B}_\mu\left[B_\mu\right]\right] \hat{F}_{\mu\nu}\left[\hat{B}_\mu\left[B_\mu\right]\right] = 0.$$

- We choose: $\hat{S}_{\rm loop} \overset{\rm SWmap}{\longrightarrow} S_{\rm loop}$, i.e. subtract NC E.O.M. and have:

$$\begin{aligned} S_{\rm loop} =& S_{\rm BFG}\left[\hat{C}[B,Q,C;\theta], \hat{B}[B;\theta], \hat{Q}[B,Q;\theta]\right] \\ &+ S_{\rm NCYM}\left[\hat{B}[B;\theta] + \hat{Q}[B,Q;\theta]\right] \\ &- S_{\rm NCYM}\left[\hat{B}[B;\theta]\right] - \int \frac{\delta S_{\rm NCYM}\left[\hat{B}\right]}{\delta \hat{B}_\mu}[B;\theta] \cdot \hat{Q}_\mu[B,Q;\theta]\,. \end{aligned}$$

Photon 1PI two point function

- After hugh computation we find

$$\Pi^{\mu\nu}_{\mathrm{BFG}} = \mathcal{B}^{\mu\nu}_{1_{\mathrm{BFG}}} + \mathcal{T}^{\mu\nu}_{1_{\mathrm{BFG}}}.$$

$$\begin{aligned}\mathcal{B}^{\mu\nu}_{1_{\mathrm{BFG}}} =&(4\pi)^{-2}\Bigg((g^{\mu\nu}p^2 - p^\mu p^\nu)\Big((4\pi\mu^2)^{2-\frac{D}{2}}(p^2)^{\frac{D}{2}-2}2(6-7D)\\ &\cdot\Gamma\left(1-\frac{D}{2}\right)\mathrm{B}\left(\frac{D}{2},\frac{D}{2}\right)\Big|_{D\to 4-\epsilon} - 12I_{K_0} - 16I_{K_1}\Big)\\ &- g^{\mu\nu}p^2(\theta p)^2T_{-2} - \frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\left(\frac{16}{3}T_0 + 8p^2I^0_K - 48p^2I^1_K\right)\Bigg)\end{aligned}$$

$$\mathcal{T}^{\mu\nu}_{1_{\mathrm{BFG}}} = (4\pi)^{-2}\left(g^{\mu\nu}p^2(\theta p)^2T_{-2} - \frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\frac{32}{3}T_0\right)$$

$$\begin{aligned}\Longrightarrow \Pi^{\mu\nu}_{\mathrm{BFG}} =&(4\pi)^{-2}\Bigg((g^{\mu\nu}p^2 - p^\mu p^\nu)\Big((4\pi\mu^2)^{2-\frac{D}{2}}(p^2)^{\frac{D}{2}-2}2(6-7D)\\ &\cdot\Gamma\left(1-\frac{D}{2}\right)\mathrm{B}\left(\frac{D}{2},\frac{D}{2}\right)\Big|_{D\to 4-\epsilon} - 12I_{K_0}\\ &- 16I_{K_1}\Big) - \frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\left(16T_0 + 8p^2I^0_K - 48p^2I^1_K\right)\Bigg) = \hat{\Gamma}^{\mu\nu}_{\mathrm{BFG}}.\end{aligned}$$

- By turn on SUZY, IR and UV cancellation results found prior to SW map now hold precisely after the SW map.
- The divergent parts with super partners now read

$$\begin{aligned}\mathcal{N} &= 1 : n_f = 1, n_s = 0,\\ \mathcal{N} &= 2 : n_f = 2, n_s = 2,\\ \mathcal{N} &= 4 : n_f = 4, n_s = 6,\end{aligned}$$

$$\begin{aligned}\Pi^{\mu\nu}_{\mathrm{BFG-SUSY}} \sim & \frac{1}{(4\pi)^2}(g^{\mu\nu}p^2 - p^\mu p^\nu)\\ & \cdot\left(\frac{22}{3} - \frac{4}{3}\mathrm{n_f} - \frac{1}{3}\mathrm{n_s}\right)\left(\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2)\right)\\ & + \frac{1}{(4\pi)^2}(32 - 32n_f + 16n_s)\frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^4},\end{aligned}$$

fully gauge fixing independent, shown explicitly via Feynman diagrams in our SuperSWNCGFT paper [arXiv:1602.01333].

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Cosmological Application of Neutrinos in NCQGFT, with R. Horvat and J. You

- Key new elements of NCQGFT on Moyal: photon-self and direct couplings of neutral (even sterile!) matter fields, promoted by hybrid SW to NC fields coupled directly to A_μ

$$
\begin{aligned}
S &= \int -\frac{1}{4} F^{\mu\nu} \star F_{\mu\nu} + i\bar{\Psi} \star (\gamma_\mu D^\mu - m)\,\Psi, \ \Psi \equiv \Psi_{\binom{L}{R}}, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ie\kappa [A_\mu \stackrel{\star}{,} A_\nu], \ D_\mu \Psi = \partial_\mu \Psi - ie\kappa [A_\mu \stackrel{\star}{,} \Psi].
\end{aligned}
$$

  In view of the NC covariant derivative D^μ one may think of NC ν-field Ψ as having left charge $+e\kappa$, right charge $-e\kappa$ and total charge zero. From the perspective of non Abelian gauge theory, one could also say that ν-field is charged in NC analogue of the adjoint representation with the matrix multiplication replaced by the $\star$-product. From a geometric point of view, the interaction is seen as a modified photon-θ background throughout which νs tend to propagate. Coupling $e\kappa$ corresponds to positive multiple (fraction) κ of charge $|e|$.
- Phenomenologically very promising!

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UHE cosmic ray RICE experiment: $\nu + N \to \nu + X$

- Our NCQFT is capable to push the neutrino-nucleon inelastic cross section three orders of magnitude beyond the SM prediction. UHE neutrinos have already (IceCube) been observed at neutrino observatories. We use such a constraint to reveal information on the NC scale $\Lambda_{\rm NC}$ in NCGFT where neutrinos possess a tree-level coupling to photons in a generation-independent manner. In the energy range of interest (10^{10} to 10^{11} GeV) the θ-expansion ($|\theta| \sim 1/\Lambda^2_{\rm NC}$) and therefore the perturbative expansion in terms of $\Lambda_{\rm NC}$ retains no longer its meaningful character.

$\nu + N \to \nu + X$

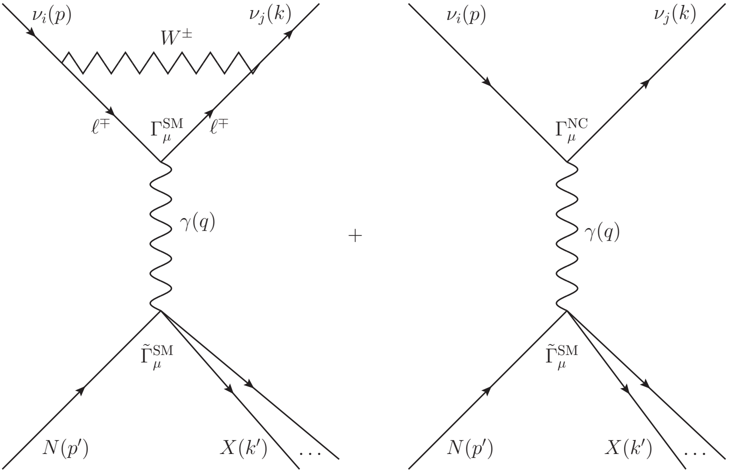

Figure: 9.

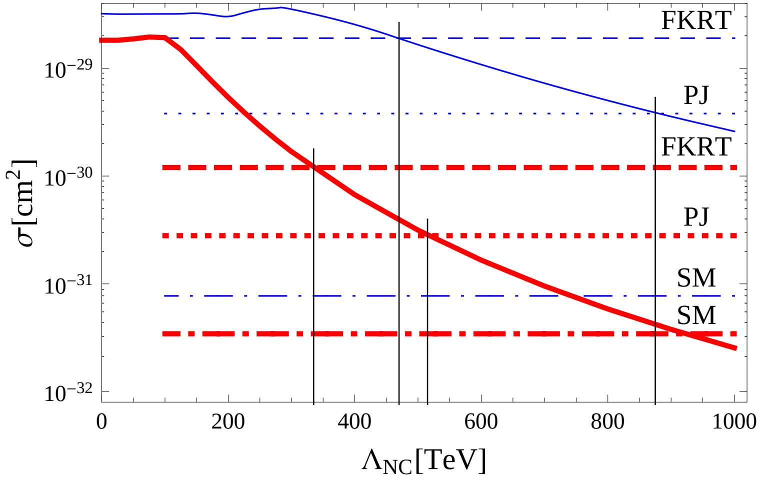

Figure: 10. $E_\nu = 10^{10}$ GeV (thick lines) and $E_\nu = 10^{11}$ GeV (thin lines). Upper bounds on νN cosmogenic neutrino fluxes obtained from RICE collaboration experiments in papers: FKRT (Fodor et al., JCAP 0311, 015 (2003)) / PJ (Protheroe-Johnson, Astro.Phys. 4, 253 (1996)).

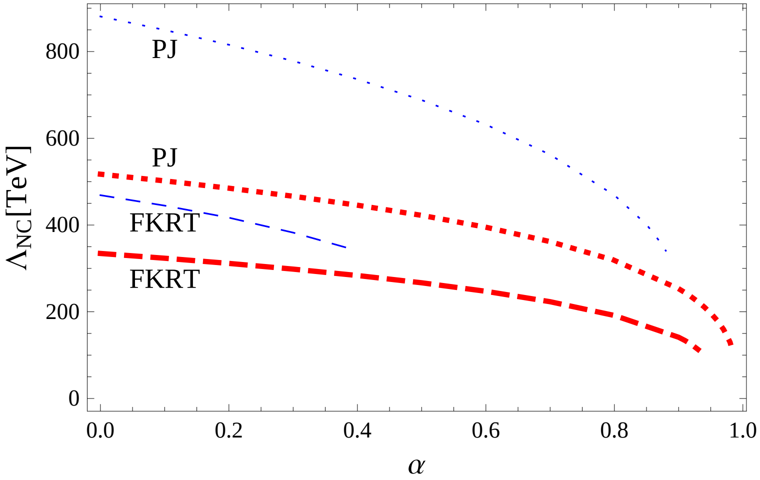

Figure: 11. The intersections of our curves with the RICE results (cf. Fig.10) as a function of the fraction of iron (Fe) nuclei in the UHE cosmic rays. The terminal point on each curve represents the highest fraction (α) of Fe nuclei above which no useful information on $\Lambda_{\rm NC}$ can be inferred with our method.

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Princeton Tritium Observatory for Light, Early-Univese, Massive-Neutrino Yield (PTOLEMY)

- Early Univiverse: ν's in hot, dens, plasma maintained thermal equilibrium with $e^{\pm}, \gamma$ via: $e\nu \leftrightarrow e\nu$ and $e^+e^- \leftrightarrow \nu\bar{\nu}$. Scattering rate $\Gamma \simeq G_F^2 T^5$ –strong T dependence, ($T \simeq 3E$).
- Cosmic Neutrino Background (CνB) key ingredients from primordial synthesis of elements to anisotropies of cosmic microwave background (CMB). Strong bounds from Planck satellite:
  - $N_\nu^{\mathrm{eff}} = 3.30 \pm 0.27$,
  - $\sum m_\nu \leq 0.23 (\rightarrow 0.05 \textit{ in near future}) \; eV \; 95\% CL$.
- Important:
  - Distinction between helicity and chirality
  - Distinction between ν-Dirac and ν-Majorana

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Ptolemy experiment: $\nu_e + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-$

- Cosmic ν's, whose relic background is today in the form of a non-relativistic, cold gas of particles, have been directly related to the Big Bang Nucleosynthesis (BBN), and some % of dark matter. Detection: by proposed PTOLEMY experiment.
- A first pertinent proposal to detect such a cold sea of ν's at the present day temperature of $T_\nu \approx 2$ K by ν-capture on tritium $\nu_e + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-$ was proposed by Weinberg 1962.
- Total ν-capture rate estimated by Long et al.-2014:

$$\Gamma = \bar{\sigma}[n(\nu_l) + n(\nu_r)]N_{trit}, \; \bar{\sigma} \approx 4 \times 10^{-45}\ \mathrm{cm}^2,$$

  N_{trit} is the number of tritium nuclei and $n(\nu_l)$ and $n(\nu_r)$ are the number densities of l- and r-helical ν's per degree of freedom (d.o.g.). In SM, both active d.o.g. for the massive Majorana case equally contribute to the process, while in the Dirac case only 1 active (out of 4) doffs does, so capture rate for Majorana case is 2 x Dirac case: $\Gamma^D \simeq 4$, $\Gamma^M \simeq 8$ per year (for 100 g ^{3}H target) for unclustered by gravity ν^D and ν^M's.
- Signature for ν-capture: peak in electron spectrum.

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Plasmon decay: $\gamma_{pl} \to \bar{\nu}_R \nu_R$

- We propose: Cosmic neutrino background dominated by the NC spacetime. Than ν_R decouple at T_{dec} when condition $\Gamma^{\rm rate}_{\rm NC} \simeq H(T_{dec})$ is satisfied (considering sterile ν_R), with $H(T)$ being the Hubble expansion rate of the Universe. We use 2 processes in our NCQFT, plasmon decay ($\gamma_{pl} \to \bar{\nu}_R \nu_R$) and elastic scattering ($e\nu_R \to e\nu_R$) to compute $\Gamma^{\rm rate}$. FR is

$$\Gamma^\mu = ie\kappa\Big[(\gamma_\mu p^\mu - m)(\theta q)^\mu - (p\theta q)\gamma^\mu - (\theta p)^\mu \gamma_\mu q^\mu\Big] F(q,p),$$

$$\Gamma^{\rm rate}_{\rm NC}(\gamma_{pl.} \to \bar{\nu}_R \nu_R) \simeq H(T_{dec}) = \left(\frac{8\pi^3}{90} g_*(T_{dec})\right)^{1/2} \frac{T^2_{dec}}{M_{Pl}}\,,$$

$$T_{dec} \simeq \frac{\kappa^2}{2\pi}\sqrt{5\alpha^3 \frac{g^{\rm ch}_*}{g_*}} M_{pl}\left(1 - \frac{\sin X}{X}\right), X = \frac{2\pi\alpha g^{ch}_* T^2_{dec}}{9\Lambda^2_{\rm NC}}\,.$$

- For the light-like NC preserving unitarity the full NC effect will be still exhibited through $X = \omega^2_{pl}/(2\Lambda^2_{\rm NC})$, $\omega_{pl} = \frac{e}{3} T_{dec}\sqrt{g^{\rm ch}_*}$.

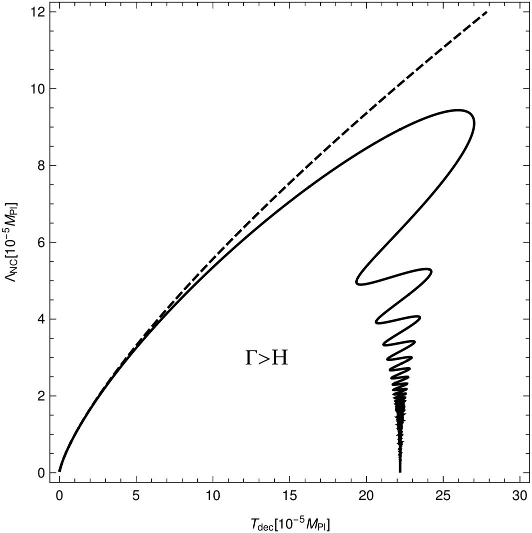

Figure: 12. Numerical plot of the NC scale $\Lambda_{\rm NC}$ versus decoupling temperature T_{dec} according to (solid curve), and its θ-first order approximation (dashed curve). In this plot we are using $\kappa = 1$, and the SM numbers $g_* = g_*^{\rm ch} = 100$, respectively.

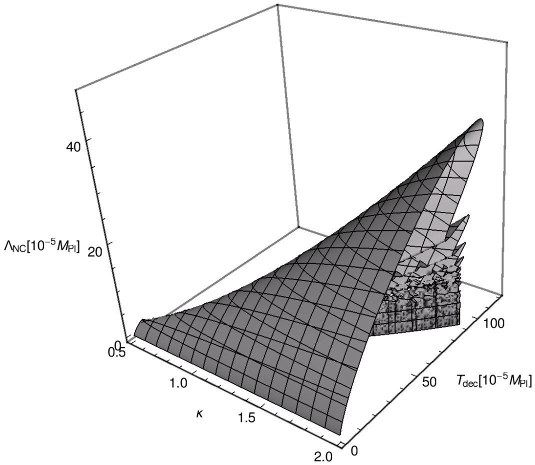

Figure: 13. 3D plot of the decoupling relation with respect to temperature, NC scale $\Lambda_{\rm NC}$, and coupling ratio κ range $[0.5, 2]$ for degrees of freedoms $g_* = g_*^{\rm ch} = 100$. The high temperature boundary of the plot decays very quickly due to the κ^2 dependence in T_0.

- The existence of T_{couple}, bounded from above by $T^{max}|_{\kappa=1} \simeq 2.7 \times 10^{-4} M_{Pl} (= 1.221 \times 10^{16} \mathrm{TeV})$ and an upper bound on the NC scale $\Lambda_{\mathrm{NC}}^{max} \simeq 0.95 \times 10^{-4} M_{Pl}$ for RH neutrino to reach thermal equilibrium via NC coupling to photon from Figs.12,13 represent additional results of our work. Now we note that decoupling of the production rate for two branches of T_{dec} (T_{dec} and T_{couple}) exhibits certain similarity to the UV/IR mixing in the radiative corrections of the NC theories. Both phenomena share the same origin from the exact (nonperturbative) treatment of the NC parameter θ at the quantum loops of the theory as well.
- In total we have shown that the PTOLEMY total capture rate in the Dirac ν case may be enhanced in the present scenario up to 20% (10%) if the NC scale $\Lambda_{\mathrm{NC}} \gtrsim \mathcal{O}(1)$ TeV ($\gtrsim \mathcal{O}(100)$) TeV.

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Summary

Scattering: $e\nu_R \to e\nu_R$

- In the lab frame total cross-section is very-very small

$$\sigma_{\rm SM}(e\nu \to e\nu) = 17.2 \times 10^{-45}\ {\rm cm}^2 \times E_\nu/{\rm MeV}. \qquad (1)$$

- Scattering $e(p)\nu_R(k) \to e(p')\nu_R(k')$ is t-channel process. This entails that the scattering proceeds well for space-/light-like NC.
- FR's give $\sigma_{\rm NC}^{e\nu_R \to e\nu_R}$ from lowest order NC photon exchange where $\bar\nu_R \nu_R \gamma$ vertex is from the NC model, while $\bar e e \gamma$ vertex is the SM one:

$$\begin{aligned}
\sigma_{\rm NC}^{e\nu_R \to e\nu_R} &= \frac{\kappa^2\alpha^2}{16E^2}\int \sin\vartheta d\vartheta \frac{4+(1+\cos\vartheta)^2}{(1-\cos\vartheta)^2}\,\mathcal{I},\\
\mathcal{I} &= \int d\varphi\, 4\sin^2\frac{p\theta p'}{2} = 2\Big(1-\cos(\xi)J_0(\zeta)\Big),\ E = 3T,\\
\xi &= \frac{E^2}{\Lambda_{\rm NC}^2}c_{03}\big(\cos\vartheta - 1\big),\ p = (E,0,0,E), k = (E,0,0,-E)\\
\zeta &= \frac{E^2}{\Lambda_{\rm NC}^2}\sin\vartheta\Big({\rm sign}(c_{01}-c_{03})\Big)\sqrt{(c_{01}-c_{13})^2+(c_{02}-c_{23})^2}.
\end{aligned}$$

- A presence of all, the time-like ($\theta^{0i} \neq 0$), the space-like ($\theta^{ij} \neq 0$), and consequently the light-like ($\theta^{01} = -\theta^{1i}$) components of $\theta^{\mu\nu}$, respectively.

- For space-like NC $\theta^{0i} = 0$, factor in ζ boils down to $\sqrt{c_{13}^2 + c_{23}^2}$, for light-like NC ζ is reduced to $\sqrt{c_{02}^2 + c_{03}^2}(= \sqrt{c_{12}^2 + c_{13}^2})$ giving:

$$\sigma_{\rm NC}^{e\nu_R \to e\nu_R} = \frac{\kappa^2\alpha^2}{8E^2}\int_{-1}^{1} dx \frac{4+(1+x)^2}{(1-x+\frac{\omega_{pl}^2}{E^2})^2} \cdot \left(1 - J_0\left[\frac{E^2}{\Lambda_{\rm NC}^2}(\sqrt{1-x^2})\sqrt{c_{13}^2 + c_{23}^2}\right]\right).$$

In order to secure the integration over ϑ, which goes from $\epsilon + 0$ to π where ϵ is a cut-off to avoid the forward scattering singularity we use a Debye mass $m_D = \sqrt{g_*^{ch}}T/3$ as a regulator, $E = 3T$.

- Now we compare the rate $\Gamma_{\rm NC}^{\rm rate}(e\nu_R \to e\nu_R) \simeq 0.18\ T_{dec}^3\, \sigma_{\rm NC}^{e\nu_R \to e\nu_R}$, with the Hubble expansion rate $H(T_{dec})$

$$0.18\ T_{dec}^3\, \sigma_{\rm NC}^{e\nu_R \to e\nu_R} = \left(\frac{8\pi^3}{90} g_*(T_{dec})\right)^{1/2} \frac{T_{dec}^2}{M_{Pl}},$$

where $g_*(T_{dec})$ counts the total number of effectively massless degrees of freedom.

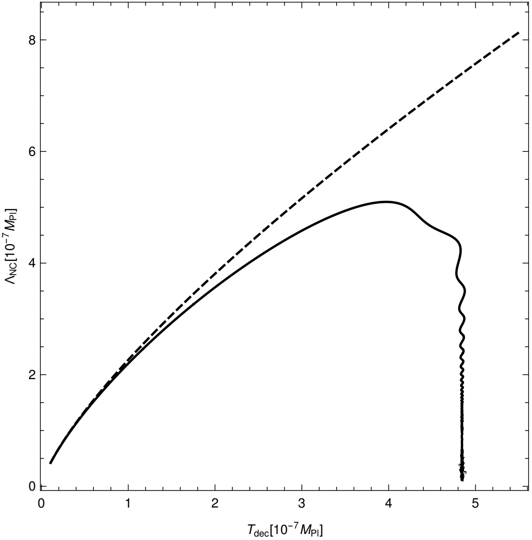

Figure: 14. Numerical plot of the NC scale $\Lambda_{\rm NC}$ versus "decoupling temperature" T_{dec} evaluated from for the full-θ contribution to the elastic $e\nu_R \to e\nu_R$ scattering amplitude (solid curve) and its first-order in θ (dashed curve), for $g_* \simeq g_*^{ch} \simeq 100$ and $\kappa = 1$.

- Sensitivity to PTOLEMY requires small T_{dec}, which one can only achieve for $T_{dec}^2/\Lambda_{\rm NC}^2 \ll 1$. In this limit we can use the leading order term in the Bessel function expansion: $1 - J_0\left[\frac{9T_{dec}^2}{\Lambda_{\rm NC}^2}\sqrt{1-x^2}\right] = \frac{1}{4}\frac{81T_{dec}^4}{\Lambda_{\rm NC}^4}(1-x^2)$, and obtain a lower bounds on $\Lambda_{\rm NC}$ for two phase transitions:

$$\Lambda_{\rm NC}\Bigg|_{\substack{T_{dec}\gtrsim 200 \text{ MeV (quark-hadron)} \\ \\ T_{dec}\gtrsim 200 \text{ GeV (EW)}}} \gtrsim \Bigg|_{\substack{0.77\sqrt{\kappa} \\ \\ 137\sqrt{\kappa}}} \text{TeV}, \qquad (2)$$

  a somewhat lower values to what we have obtained in the case of plasmon decay dominating Hubble expansion rate.

- Having solved numerically we plot the solution in Fig.14 for $g_* \simeq g_*^{ch} \simeq 100$. Fig.14 shows how the nonlocality of these field theories (featuring an explicit UV/IR mixing) may also have an important consequences for cosmology. Within the region surrounded by a solid curve, the Hubble expansion rate is always surpassed by the ν_R scattering rate.

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Summary

- Our verdict:
  The θ-exact Seiberg-Witten map on Moyal space, if it is invertible, is an equivalence/duality relation between perturbative –in coupling constant QFT– defined with respect to NC and commutative QFTs.
- Our test:
  (A) One-loop 1PI two point functions (for scalar, fermion and gauge fields) from quantizing NC U(1) gauge theory with and without SW map and with and without SUZY are equivalent to each other on-shell, $\rightarrow \hat{\Gamma}_{s,f} = \Gamma_{s,f}$ and $\hat{\Pi}^{\mu\nu} = \Pi^{\mu\nu}$.
- (B) Cancelations of UV, quadratic IR and log-IR divergences in θ-exact U(N) NCYM with and without SW maps by $\mathcal{N} = 4$ SUSY.
- Due to the above duality/equivalence our model shows manifestation of the UV/IR correspondence on the tree level when applied to: Particle Physics, Astrophysics and Cosmology

- Applications to Neutrino experiments in Cosmology:
  - RICE experiments: UHE ν's scattered on N given in Figs.9-11.
  - Ptolemy experiment: There appears a new physical quantity called *the* ν_R *coupling temperature* T_{couple} instead of ν_R decoupling temperature T_{dec} only. Closed forms in Figs.12,14 show how the nonlocality of these field theories (explicit UV/IR mixing) may also have an important consequences for cosmology.
  - Ptolemy experiment: Signature for ν-capture is a peak in electron spectrum $2m_\nu$ above β-decay endpoint.
- Usual T_{dec} is in Figs.12,14 splited into two branches, the usual decoupling temperature (a lower one) and the coupling temperature (a higher one). Within the solid curve region, the Hubble expansion rate is always surpassed by the ν_R decay/scattering rate. Splitting of original T_{dec} into two branches, the usual T_{dec} (lower) and new T_{couple} (higher) is a direct consequence of UV/IR correspondence, unfolding nicely from our full-θ NC model. Above $\Lambda_{\rm NC}^{max}$ ν_Rs can never attain thermal equilibrium via the NC coupling to photons and thus would have no impact on the PTOLEMY capture rate.

- Important to stress again: Higher temperature solution(s) at each given $\Lambda_{\rm NC}$, sitting on the right hand side of the solid curve, may be interpreted as the *coupling temperature* T_{couple} i.e. the temperature where NC plasmon decay rate first time catches (or it may be the reheating temperature, whichever is lower) the Hubble rate during cooling.
- With the use of θ^1-expanded model (dashed curve in Figs.12,14), the T_{couple} is missing since the absence of the sine term destroys the UV/IR connection. It is just the switch in the behavior of the decay/scattering rate, from T^5 at low to T at very high temperatures, which is responsible for the closed contour in the T_{dec}–$\Lambda_{\rm NC}$ plane as depicted in Fig.12,14.

- As the temperature decreases further, the decay rate once again drops below the Hubble rate, starting at the lower temperature/left-hand side solution on the solid curve. The equation actually allows us to estimate the T_{couple}^{max} analytically:

$$\mathrm{d} \rightarrow \mathrm{Fig.12}: \quad T_{couple}^{max} \simeq 2.22 \times 10^{-4} M_{pl} = 2.71 \times 10^{12}\ \mathrm{TeV}.$$

$$\Lambda_{\mathrm{NC}}^{max} \simeq 2.02 \times 10^{-4} M_{pl} = 2.47 \times 10^{12}\ \mathrm{TeV}.$$

$$\mathrm{s} \rightarrow \mathrm{Fig.14}: \quad T_{couple}^{max} \simeq 4.84 \times 10^{-7} M_{Pl} = 5.91 \times 10^{9}\ \mathrm{TeV}.$$

$$\Lambda_{\mathrm{NC}}^{max} \simeq 5.26 \times 10^{-7} M_{Pl} = 6.42 \times 10^{9}\ \mathrm{TeV}.$$

- Bounds almost three orders of magnitude below those obtained from the plasmon decay. They are certainly closer to the experimental accessibility.
- Summing up, we have shown that if the PTOLEMY experiment would register an enhanced capture rate, than this could have far reaching consequences for the scale and the type of noncommutativity inferred from cosmology.