Courant algebroids, Poisson-Lie T-duality and supergravity (of type II)

Fridrich Valach

University of Geneva

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Joint work with Pavol Ševera

- Construct a generalised Ricci tensor defined for any Courant algebroid in a way which does not require any auxiliary data
- Construct a generalised scalar curvature and a (generalised) string effective action for any Courant algebroid
- Prove compatibility of P-L T-duality and (generalised) SUGRA equations of motion
- Find new solutions to (generalised) SUGRA equations of motion

Courant algebroid: [Liu, Weinstein, Xu 1997] a vector bundle $E \to M$ with an inner product $\langle \cdot, \cdot \rangle$, anchor map $\rho : E \to TM$ and a bracket $[\cdot, \cdot] : \Gamma(E) \times \Gamma(E) \to \Gamma(E)$ such that for all $u, v, w \in \Gamma(E)$ and $f \in C^\infty(M)$

$$
\begin{aligned}
[u, [v, w]] &= [[u, v], w] + [v, [u, w]] \\
[u, fv] &= f[u, v] + (\rho(u)f)v \\
\rho(u)\langle v, w \rangle &= \langle [u, v], w \rangle + \langle v, [u, w] \rangle \\
\langle u, [v, v] \rangle &= \langle [u, v], v \rangle
\end{aligned}
$$

Generalised metric [Gualtieri 2014]: a subbundle $V_+ \subset E$ for which $\langle \cdot, \cdot \rangle|_{V_+}$ is non-degenerate (define $V_- = V_+^\perp$)

- $\mathfrak{d} \to \mathrm{pt}$, $\mathfrak{d}$ a Lie algebra with an invariant inner product

- $TM \oplus T^*M \to M$

  $\rho(u + \alpha) = u$

  $\langle u + \alpha, v + \beta \rangle = \alpha(v) + \beta(u)$

  $[u + \alpha, v + \beta] = [u, v] + \mathcal{L}_u\beta - \iota_v\alpha + H(u, v, \cdot) \quad (dH = 0)$

  (exact Courant algebroid)

Construction: ($\mathfrak{g} \subset \mathfrak{d}$ lagrangian subalgebra)

$$
\begin{array}{ccccc}
V_+ \subset \mathfrak{d} & & \overset{\text{(exact CA)}}{\tilde{V}_+ \subset \mathfrak{d} \times D/G} & & \\
\downarrow & \rightsquigarrow & \downarrow & \rightsquigarrow & M = D/G,\ g,\ H \\
\mathrm{pt} & & D/G & &
\end{array}
$$

$$
\begin{array}{ccc}
 & (\mathfrak{d} \to \mathrm{pt}, V_+) & \\
\overset{\mathfrak{g}'}{\swarrow} & & \overset{\mathfrak{g}}{\searrow} \\
(M', g', H') & \xleftrightarrow{\text{P-L T-duality}} & (M, g, H)
\end{array}
$$

Generalised metric: $V_+ \subset E$
$\rightarrow V_+$ on an exact CA encodes g and H

Generalised divergence: [Alexeev, Xu; Garcia-Fernandez 2014] an $\mathbb{R}$-linear map $\mathrm{div}\colon \Gamma(E) \rightarrow C^\infty(M)$ satisfying $\mathrm{div}(fu) = f\,\mathrm{div}\,u + \rho(u)f$
Example: if σ is a half-density on M, set $\mathrm{div}\,u = \sigma^{-2}\mathcal{L}_{\rho(u)}\sigma^2$
$\rightarrow \sigma$ encodes the dilaton; general div gives generalised SUGRA

Spinor (w.r.t. E): $\mathcal{F}$ section of the spinor bundle S_E
$\rightarrow$ on an exact CA the form $\mathcal{F}\sigma \in \wedge^\bullet T^*M$ encodes RR fields

[Coimbra, Strickland-Constable, Waldram 2011; Garcia-Fernandez 2014; Jurčo, Vysoký 2015; Ševera, V. 2017; Ševera, V. in prep.]

Generalised Ricci tensor: $\mathrm{GRic}_{V_+,\mathrm{div}}\colon \Gamma(V_+)\times\Gamma(V_-) \to C^\infty(M)$

$$\boxed{\mathrm{GRic}_{V_+,\mathrm{div}}(u,v) = \mathrm{div}[v,u]_+ - \rho(v)\,\mathrm{div}\,u - \mathrm{Tr}_{V_+}([[\cdot,v]_-,u]_+),}$$

where $(\cdot)_\pm : E \to V_\pm$ are orthogonal projections

Generalised scalar curvature:

$$\boxed{\begin{aligned}\mathcal{R}_{V_+,\mathrm{div}} &= (\mathrm{div}\,e^a)(\mathrm{div}\,e_a) + 2\rho(e^a)\cdot\mathrm{div}\,e_a \\ &\quad - \tfrac{1}{2}\langle[e^a,e^b],[e_a,e_b]\rangle + \tfrac{1}{3}\langle[e^a,e^b],e^c\rangle\langle[e_a,e_b],e_c\rangle,\end{aligned}}$$

where e_a is a (local) orthonormal frame of V_+

Action:

$$S(V_+,\sigma,\mathcal{F})=\int_M \sigma^2(\mathcal{R}_{V_+,\mathrm{div}_\sigma}-\tfrac{1}{2}(\mathcal{F},\mathcal{F})),\quad \mathcal{F}\text{ self-dual}$$

Equations of motion:

$$\mathrm{GRic}_{V_+,\mathrm{div}}(u,v)\propto(u\cdot\mathcal{F},v\cdot\mathcal{F}),\quad \mathcal{R}_{V_+,\mathrm{div}}=0,\quad \not{D}_{\mathrm{div}}\mathcal{F}=0$$

Here $\not{D}_{\mathrm{div}}$ is the generating Dirac operator on spinors w.r.t. E [Alexeev, Xu; Ševera 1998].

- Poisson-Lie T-duality is compatible with GRic and $\mathcal{R}$ and thus also with the equations of motion.
- This includes the case of spectators and CA reductions (gauged σ-models).
- New solutions to (generalised) supergravity equations, in particular non-isometric backgrounds for
  - $AdS_d \times S^{10-d}$, $AdS_d \times \mathbb{C}P^{10-d}$, $AdS_2 \times G_2/SO(4)$, $AdS_2 \times SU(4)/S(U(2) \times U(2))$, $AdS_4 \times SO(5)/U(2)$, ...
- Recover known λ, η deformations.

- Try to generalise the framework for the case of M-theory and exceptional field theory.
- Include fermions and study supersymmetric variations in the case of $\mathfrak{d} \to \mathrm{pt}$.
- Find a physical interpretation of GRic.