Courant algebroids, Poisson-Lie T-duality and supergravity (of type II)

Fridrich Valach

University of Geneva

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Joint work with Pavol Ševera

- Construct a generalised Ricci tensor defined for any Courant algebroid in a way which does not require any auxiliary data
- Construct a generalised scalar curvature and a (generalised) string effective action for any Courant algebroid
- Prove compatibility of P-L T-duality and (generalised)
 SUGRA equations of motion
- Find new solutions to (generalised) SUGRA equations of motion

Courant algebroid: [Liu, Weinstein, Xu 1997] a vector bundle $E \to M$ with an inner product $\langle \cdot, \cdot \rangle$, anchor map $\rho: E \to TM$ and a bracket $[\cdot, \cdot]: \Gamma(E) \times \Gamma(E) \to \Gamma(E)$ such that for all $u, v, w \in \Gamma(E)$ and $f \in C^\infty(M)$

$$[u, [v, w]] = [[u, v], w] + [v, [u, w]]$$
$$[u, fv] = f[u, v] + (\rho(u)f)v$$
$$\rho(u)\langle v, w \rangle = \langle [u, v], w \rangle + \langle v, [u, w] \rangle$$
$$\langle u, [v, v] \rangle = \langle [u, v], v \rangle$$

Generalised metric [Gualtieri 2014]: a subbundle $V_+ \subset E$ for which $\langle \cdot, \cdot \rangle|_{V_+}$ is non-degenerate (define $V_- = V_+^{\perp}$)

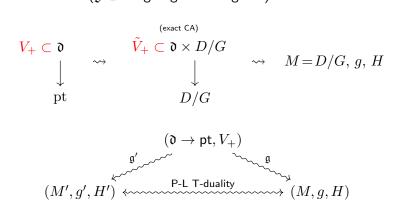
Courant algebroids - examples

 \bullet $\mathfrak{d} \to \mathrm{pt}$, \mathfrak{d} a Lie algebra with an invariant inner product

•
$$TM \oplus T^*M \to M$$

 $\rho(u+\alpha) = u$
 $\langle u+\alpha, v+\beta \rangle = \alpha(v) + \beta(u)$
 $[u+\alpha, v+\beta] = [u,v] + \mathcal{L}_u\beta - \iota_v\alpha + H(u,v,\cdot) \quad (dH=0)$
(exact Courant algebroid)

Construction: $(\mathfrak{g} \subset \mathfrak{d} \text{ lagrangian subalgebra})$



Generalised metric: $V_+ \subset E$

 $ightarrow V_+$ on an exact CA encodes g and H

Generalised divergence: [Alexeev, Xu; Garcia-Fernandez 2014] an \mathbb{R} -linear map $\operatorname{div}\colon \Gamma(E) \to C^\infty(M)$ satisfying $\operatorname{div}(fu) = f\operatorname{div} u + \rho(u)f$ **Example**: if σ is a half-density on M, set $\operatorname{div} u = \sigma^{-2}\mathcal{L}_{\rho(u)}\sigma^2 \to \sigma$ encodes the dilaton; general div gives generalised SUGRA

Spinor (w.r.t. E): \mathcal{F} section of the spinor bundle S_E \rightarrow on an exact CA the form $\mathcal{F}\sigma \in \wedge^{\bullet}T^*M$ encodes RR fields

[Coimbra, Strickland-Constable, Waldram 2011; Garcia-Fernandez 2014; Jurčo, Vysoký 2015; Ševera, V. 2017; Ševera, V. in prep.]

Generalised Ricci tensor: $GRic_{V_+,div} : \Gamma(V_+) \times \Gamma(V_-) \to C^{\infty}(M)$

$$GRic_{V_+,div}(u,v) = div[v,u]_+ - \rho(v) div u - Tr_{V_+}([[\cdot,v]_-,u]_+),$$

where $(\cdot)_{\pm}: E \to V_{\pm}$ are orthogonal projections

Generalised scalar curvature:

$$\mathcal{R}_{V_+,\text{div}} = (\text{div } e^a)(\text{div } e_a) + 2\rho(e^a) \cdot \text{div } e_a$$
$$-\frac{1}{2}\langle [e^a, e^b], [e_a, e_b] \rangle + \frac{1}{3}\langle [e^a, e^b], e^c \rangle \langle [e_a, e_b], e_c \rangle,$$

where e_a is a (local) orthonormal frame of V_+

(Bosonic part of) the type II SUGRA action and equations of motion

Action:

$$S(V_+,\sigma,\mathcal{F}) = \int_M \sigma^2(\mathcal{R}_{V_+,\operatorname{div}_\sigma} - \frac{1}{2}(\mathcal{F},\mathcal{F})), \quad \mathcal{F} \text{ self-dual}$$

Equations of motion:

$$\operatorname{GRic}_{V_+,\operatorname{div}}(u,v) \propto (u \cdot \mathcal{F}, v \cdot \mathcal{F}), \quad \mathcal{R}_{V_+,\operatorname{div}} = 0, \quad \mathcal{D}_{\operatorname{div}}\mathcal{F} = 0$$

Here $\mathcal{D}_{
m div}$ is the generating Dirac operator on spinors w.r.t. E [Alexeev, Xu; Ševera 1998].

- Poisson-Lie T-duality is compatible with GRic and $\mathcal R$ and thus also with the equations of motion.
- This includes the case of spectators and CA reductions (gauged σ -models).
- New solutions to (generalised) supergravity equations, in particular non-isometric backgrounds for
 - $AdS_d \times S^{10-d}$, $AdS_d \times \mathbb{C}P^{10-d}$, $AdS_2 \times G_2/SO(4)$, $AdS_2 \times SU(4)/S(U(2) \times U(2))$, $AdS_4 \times SO(5)/U(2)$, . . .
- Recover known λ , η deformations.

Outlook

 Try to generalise the framework for the case of M-theory and exceptional field theory.

• Include fermions and study supersymmetric variations in the case of $\mathfrak{d} \to \mathrm{pt}.$

• Find a physical interpretation of GRic.