Testing GR with Gravitational Waves: Theory-agnostic or Specific Tests?

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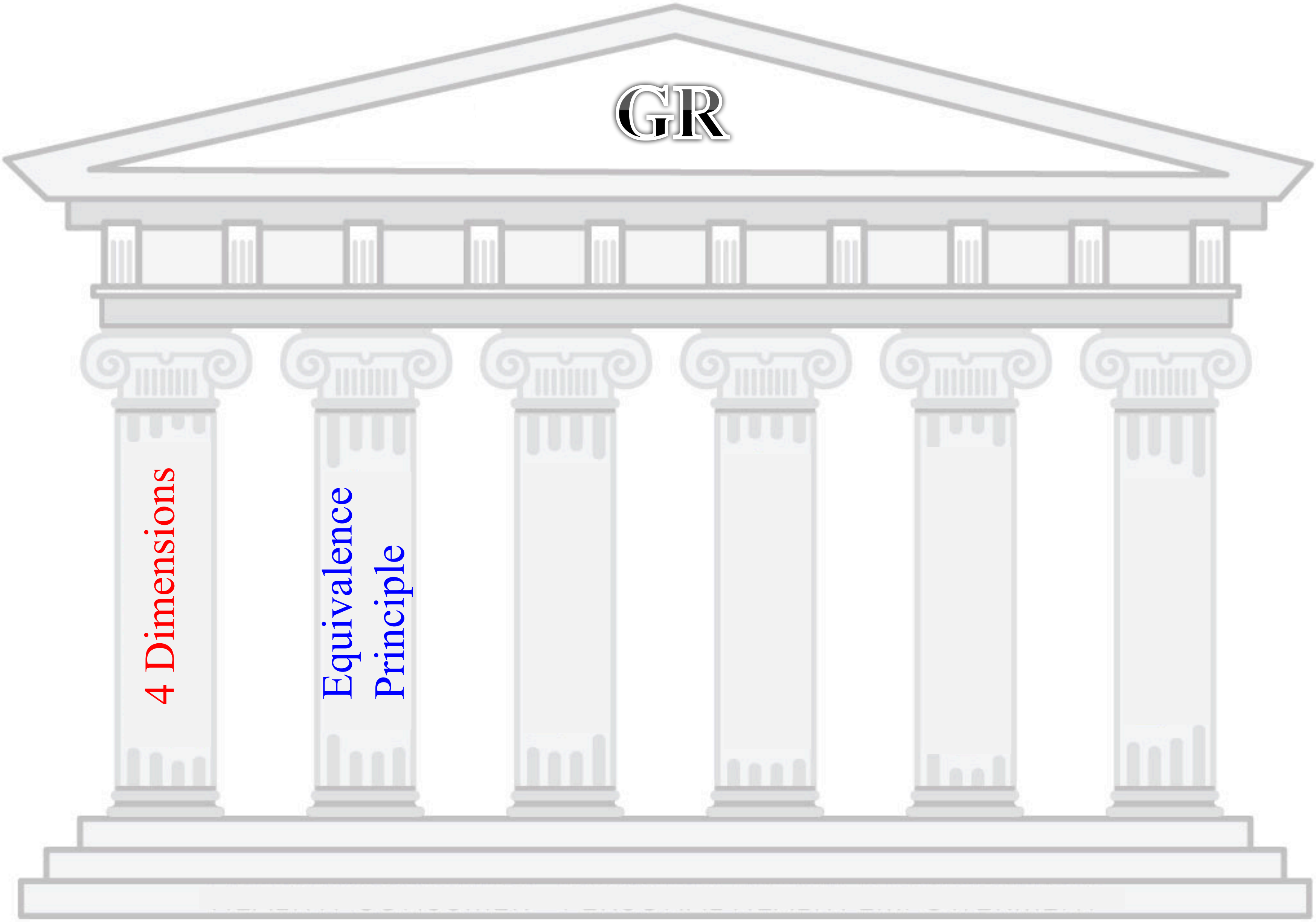
NR beyond GR
Benasque, June 4th 2018

Fundamental Pillars in General Relativity (GR)



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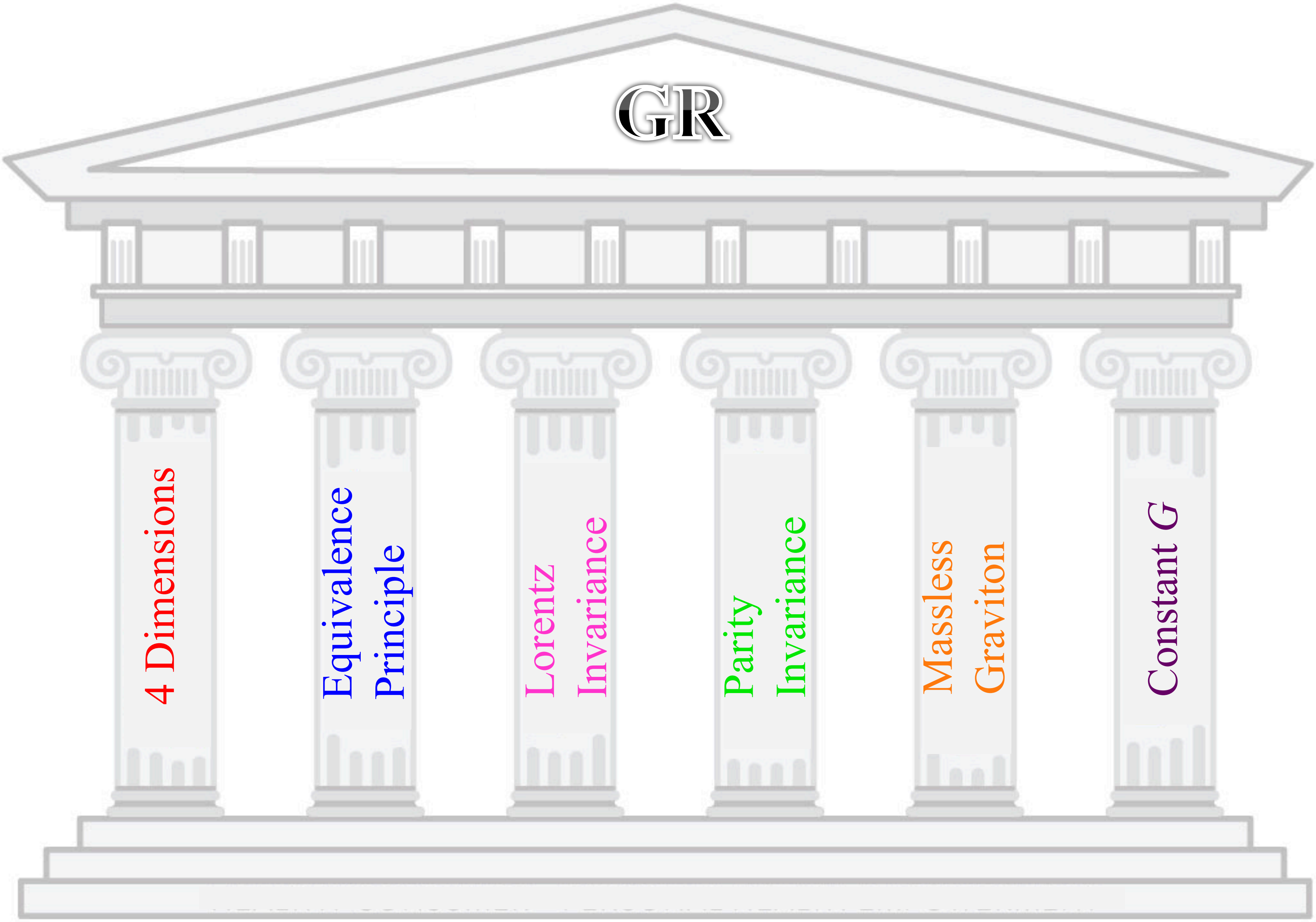
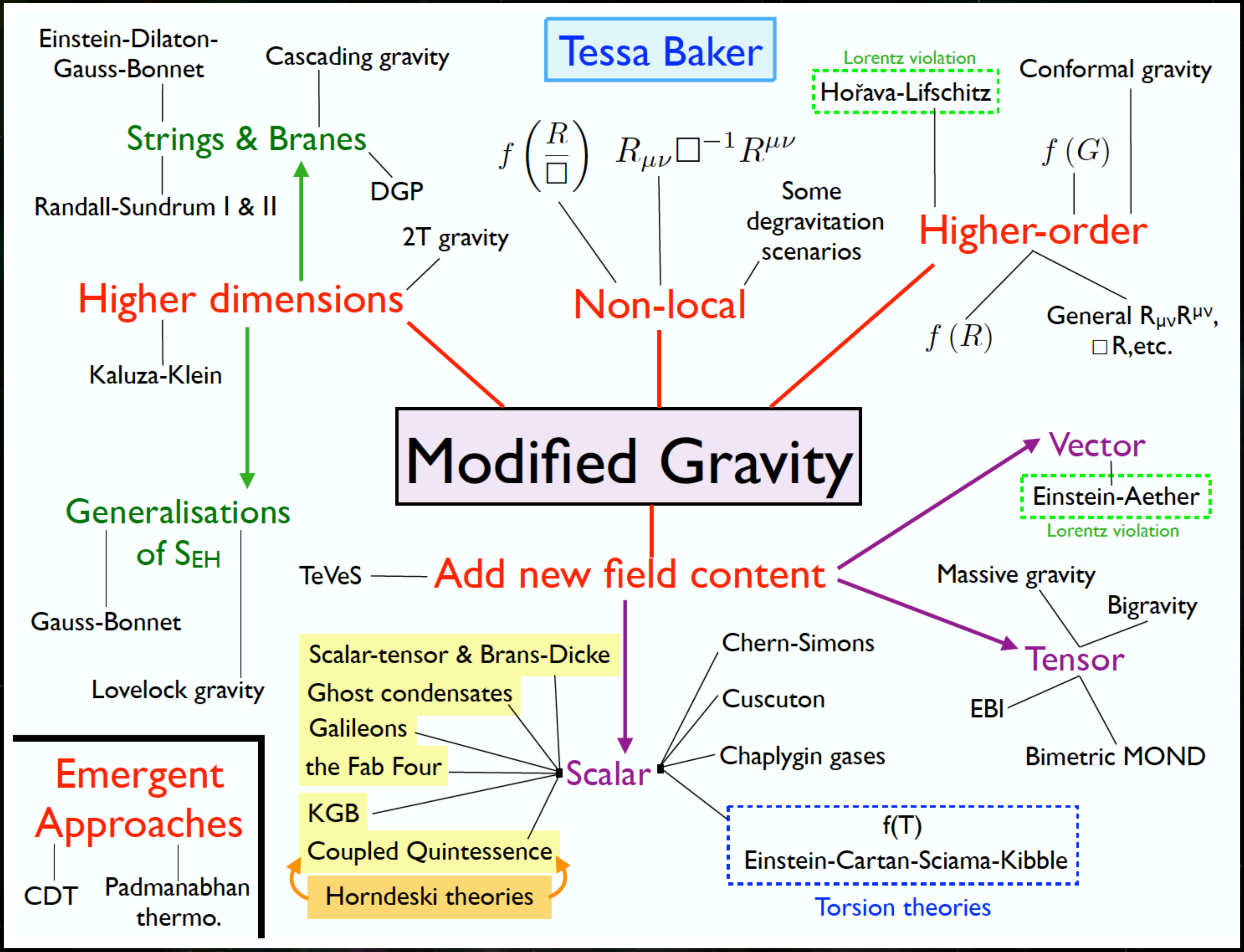


Zoo of Modified Gravity

[Bull et al. arXiv:1512.05356]



Theory-agnostic vs Specific Tests

Q: Shall we compare GW data against each theory one by one?

A: More efficient to first carry out theory-agnostic tests, and map them to specific tests.

Outline

Theory-agnostic Tests

Applications to GW150914 & GW151226

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Theory-agnostic Tests

(I) PN Tests of GR

(II) Parameterized post-Einsteinian Formalism

(III) Generalized IMRPhenom Waveform

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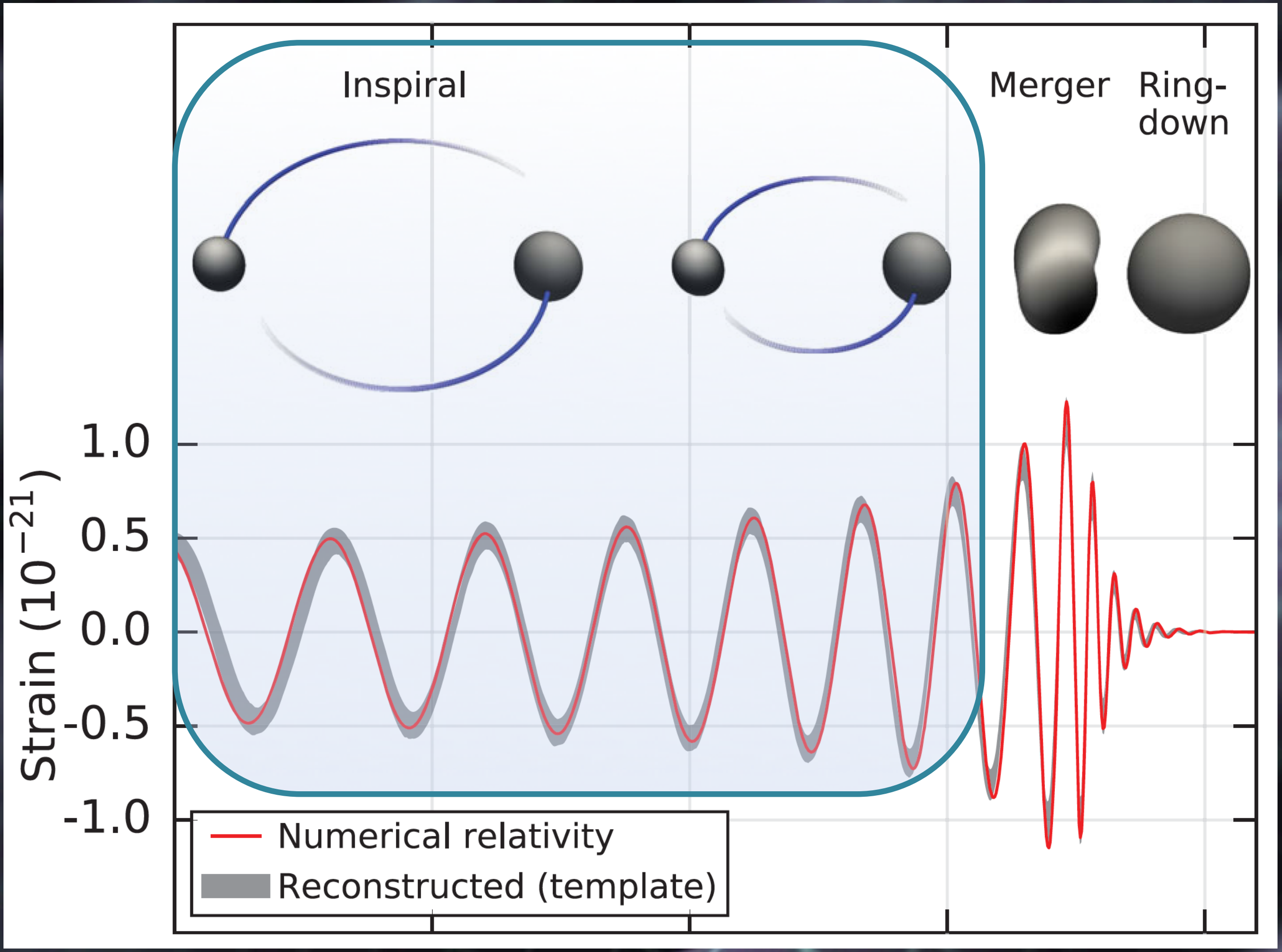
Where to start…?



[Abbott et al. PRL 116 061102 (2016)]

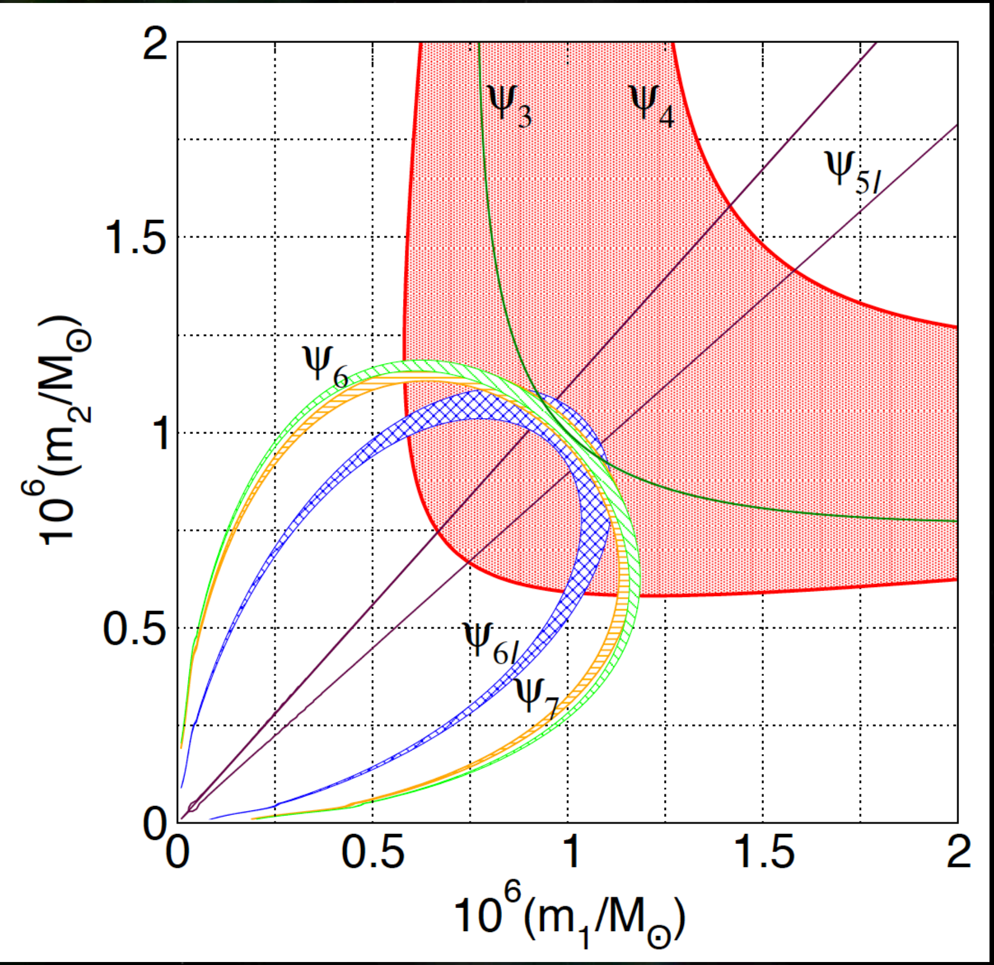
Matched filtering more sensitive to phase than amplitude

(I) PN Tests of GR

[Arun et al. (2006), Mishra et al. (2010)]

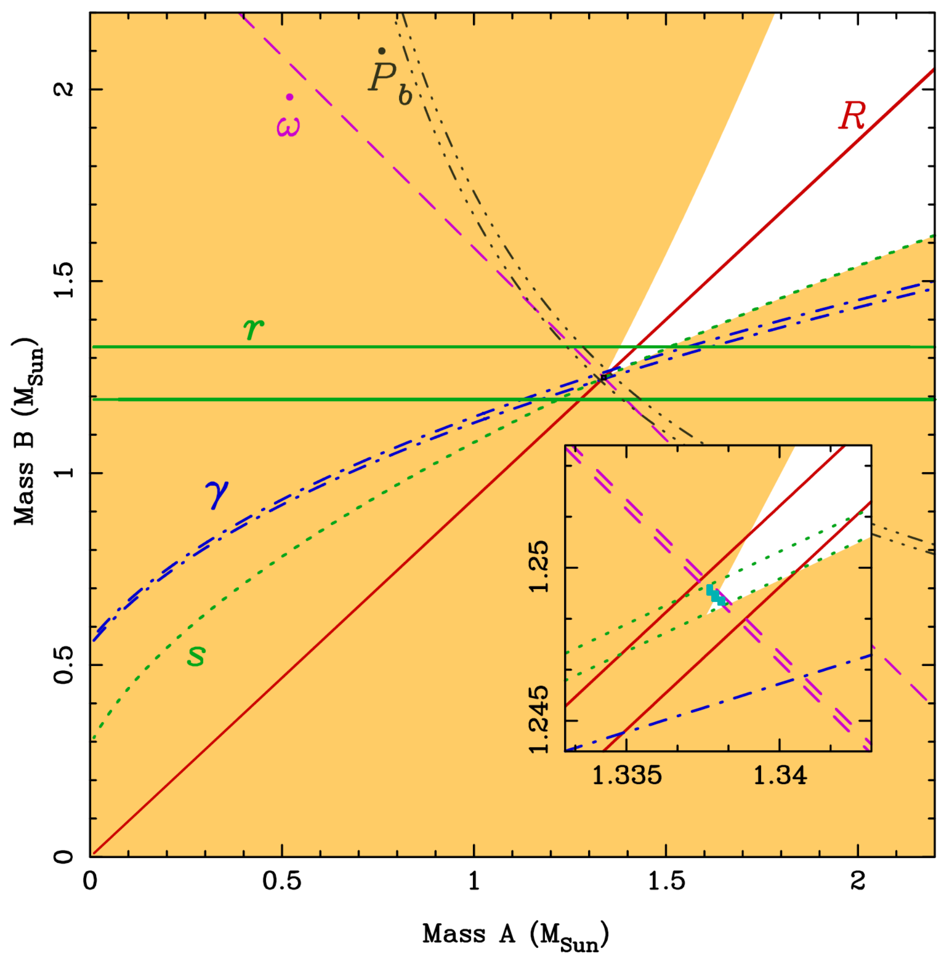
$$\Psi = \sum_{k=0}^{7} [\psi_k(m_1, m_2) + \psi_{kl}(m_1, m_2) \ln f] f^{(k-5)/3}$$

GW phase in freq. domain



Non-spinning binary BH at z=1 with LISA

Double Binary Pulsar



[Kramer & Wex (2009), Wex (2014)]

Theory-agnostic

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GW Phase in Fourier Domain in GR

$$\Psi(f) = 2\pi \int^{f} t(f')\, df' = 2\pi \int^{f} \int^{f'} \frac{dt}{df''}\, df''\, df'$$

$$= 2\pi \int^{f} \int^{f'} \frac{dt}{dE} \frac{dE}{dr} \frac{dr}{df''}\, df''\, df'$$

$$\sim (\pi \mathcal{M} f)^{-5/3}$$

chirp mass: $\mathcal{M} \equiv \left(\frac{m_1^3\, m_2^3}{m} \right)^{1/5}$

Quadrupolar Radiation:

$$\frac{dE}{dt} \sim \dddot{Q}_{ij} \dddot{Q}^{ij} \sim \mu^2 r^4 f^6$$

reduced mass (μ)

separation (r)

Binding Energy:

$$E \sim \frac{\mu\, m}{r}$$

total mass (m)

Kepler's Law:

$$f^2 \sim \frac{m}{r^3}$$

(II) PPE-modified Inspiral Waveform Phase

[Yunes & Pretorius PRD80 122003 (2009)]

$$\Psi(f) = 2\pi \int^{f} \int^{f'} \frac{dt}{dE} \frac{dE}{dr} \frac{dr}{df''} df'' df'$$

$$\sim \Psi(f)_{\rm GR} + \beta u^{b}$$

PPE parameters:

$$\beta = \beta(A, B)$$

$$b = \min(2p - 5, 2q - 5)$$

Quadrupolar Radiation:

q PN

$$\frac{dE}{dt} = \left(\frac{dE}{dt}\right)_{\rm GR} (1 + B u^{2q}) \qquad u \equiv (\pi \mathcal{M} f)^{1/3}$$

Binding Energy:

$$E = E_{\rm GR} (1 + A u^{2p})$$

⇒

Kepler's Law:

$$r(f) \sim r(f)_{\rm GR} (1 + A u^{2p})$$

PPE Mapping to Specific Theories

PPE inspiral waveform: $\tilde{h}^{(\mathrm{I})}_{\mathrm{GR}}(f)\,(1+\alpha\, u^a)\exp\left(i\,\beta\, u^b\right)$

Theories	GR Pillars	PPE a	PPE b	PN Order	PPE (α,β)
time-varying G	Constant G	-8	-13	-4 PN	$(\alpha_{\dot{G}},\beta_{\dot{G}})$
RS-II Braneworld	4D	-8	-13	-4 PN	$(\alpha_{\mathrm{ED}},\beta_{\mathrm{ED}})$
Scalar-Tensor (including Brans-Dicke)	Strong Equivalence Principle	-2	-7	-1 PN	$(\alpha_{\mathrm{ST}},\beta_{\mathrm{ST}})$
Einstein-dilaton Gauss-Bonnet	Strong Equivalence Principle	-2	-7	-1 PN	$(\alpha_{\mathrm{EdGB}},\beta_{\mathrm{EdGB}})$
dynamical Chern-Simons	Parity Invariance	$+4$	-1	$+2$ PN	$(\alpha_{\mathrm{dCS}},\beta_{\mathrm{dCS}})$
Einstein-Æther, Hořava-Lifshitz	Lorentz Invariance	-2	-7	-1 PN	$\left(\alpha^{(-1)}_{\text{Æ}},\beta^{(-1)}_{\text{Æ}}\right)$
		0	-5	0 PN	$\left(\alpha^{(0)}_{\text{Æ}},\beta^{(0)}_{\text{Æ}}\right)$
Massive Gravity	$m_g=0$	—	-3	1 PN	$(0,\beta_{\mathrm{MG}})$

$$(\alpha_{\dot{G}}\,,\beta_{\dot{G}})=\left(-\frac{5}{512}\dot{G}\mathcal{M}\,,-\frac{25}{65536}\dot{G}\mathcal{M}\right)$$

[Yunes, Pretorius & Spergel PRD80 122003 (2010)]

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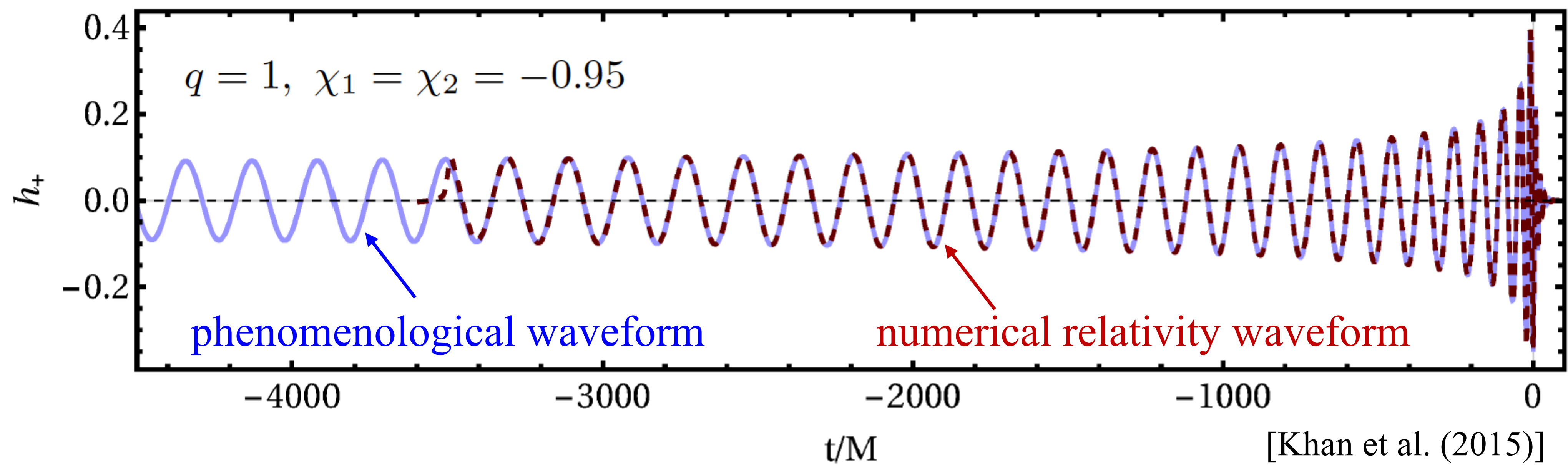
(III) Generalized IMRPhenom Waveform

Inspiral-merger-ringdown Phenomonological D (IMRPhenomD) waveform in GR



[Khan et al. (2015)]

phenomenological parameters $\vec{p}(\eta, \hat{\chi}) \longrightarrow p^i_{\rm GR}(1 + \delta p^i)$

symmetric mass ratio

effective dimensionless spin

generalized IMRPhenom (gIMR) waveform

(III) Generalized IMRPhenom Waveform

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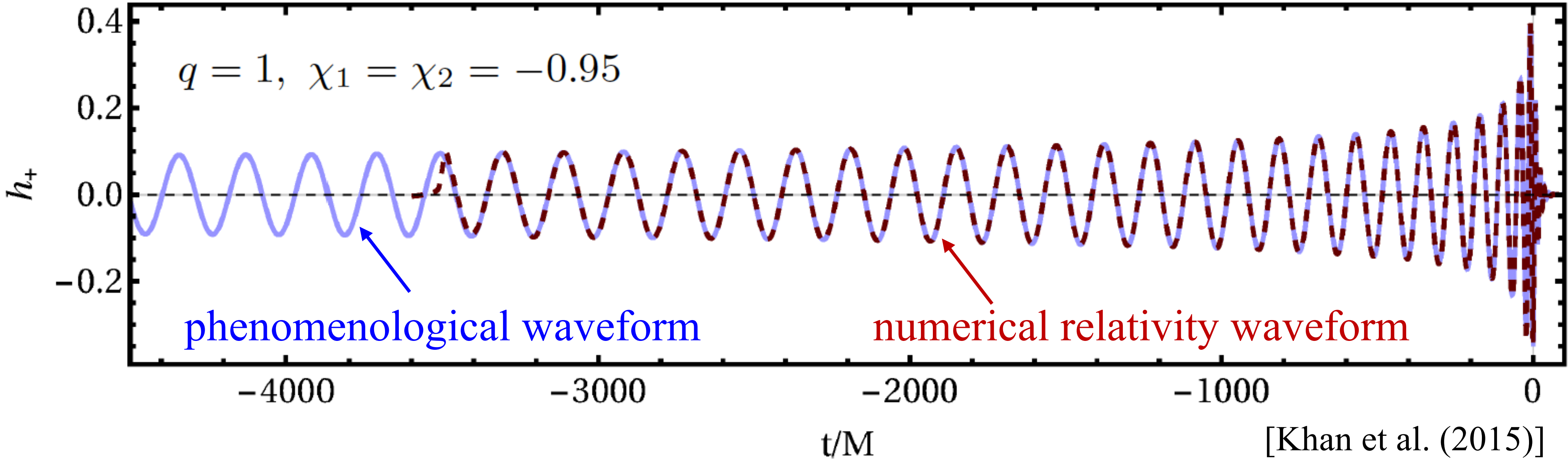

[Khan et al. (2015)]

phenomenological parameters $\vec{p}(\eta, \hat{\chi}) \longrightarrow p^i_{\rm GR}(1 + \delta p^i)$

generalized IMRPhenom (gIMR) waveform

possible to model non-GR merger & ringdown
(no known mapping to specific theories)

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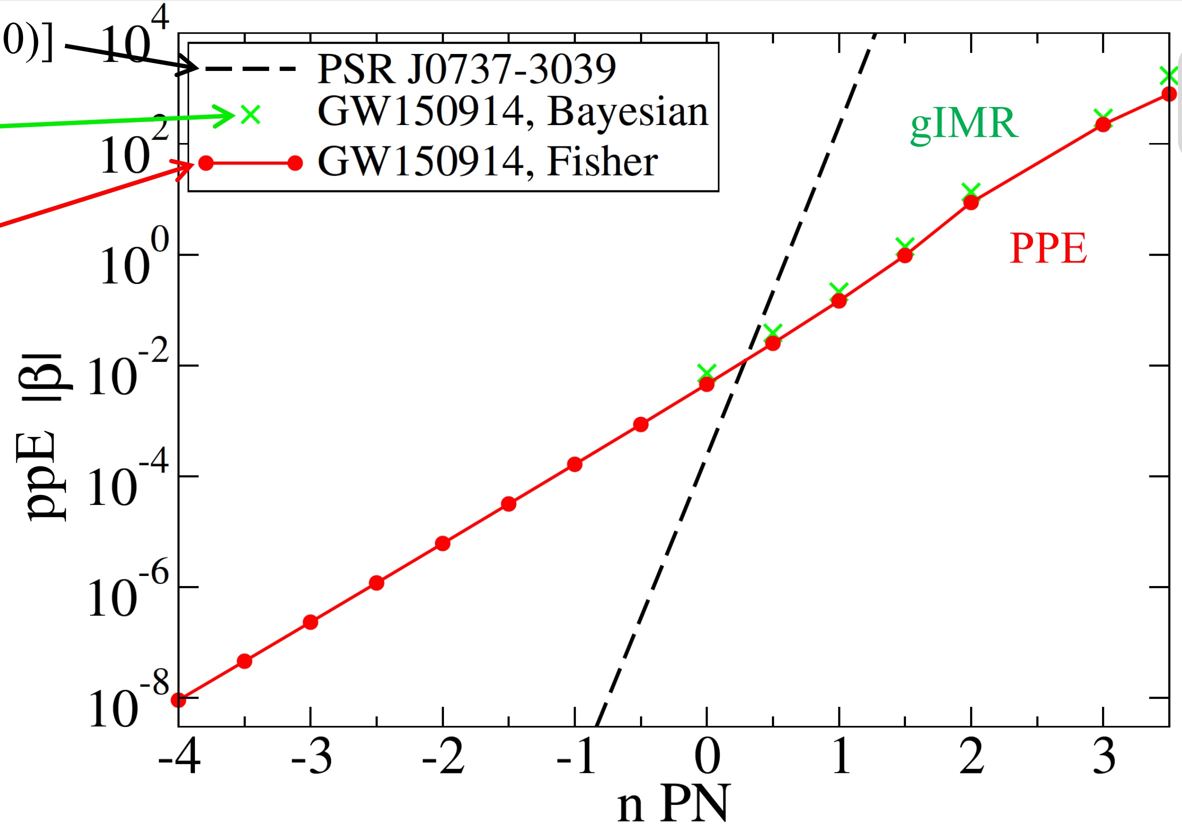
Constraining GR Fundamental Pillars

[Yunes & Hughes (2010)]

[LIGO-Virgo Collaboration (2016)]

[Yunes, KY & Pretorius (2016)]



$$\delta\Psi = \beta\, v^{2n-5}$$

Constraining GR Fundamental Pillars

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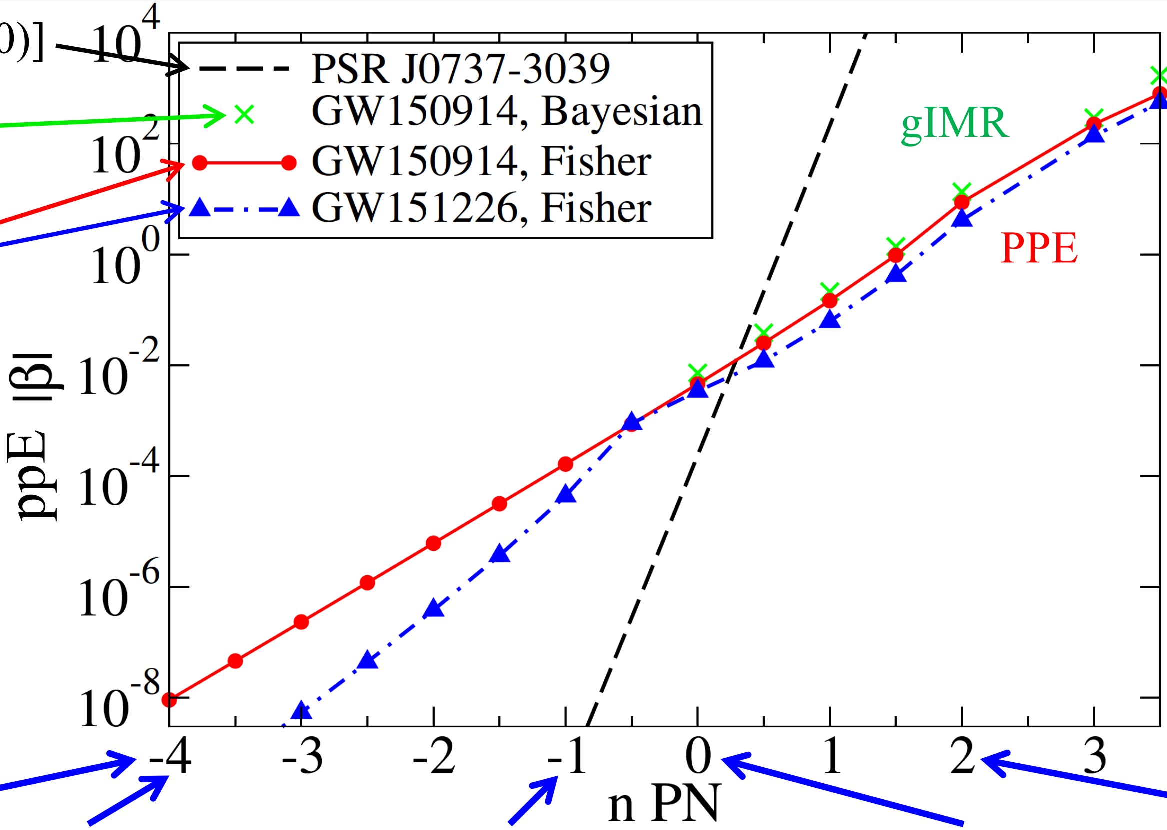

$$\delta\Psi = \beta\, v^{2n-5}$$

extra dimension	time varying G	scalar monopole activation	vector field activation	scalar dipole activation
⬇	⬇	⬇	⬇	⬇
4D	equivalence principle	equivalence principle	Lorentz invariance	parity invariance

Theoretical Constraints

[Yunes, KY & Pretorius PRD (2016)]

Example Theories (Theoretical Parameters)	GR Pillar	Example Theory Constraints	
		GW150914	Others

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Example Theories (Theoretical Parameters)	GR Pillar	Example Theory Constraints	
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Einstein-dilaton Gauss-Bonnet ($\sqrt{\lvert\alpha_{\rm EdGB}\rvert}$ [km])	Equiv. Princ.	—	10^7, 2
scalar-tensor ($\lvert\dot{\phi}\rvert$ [1/sec])	Equiv. Princ.	—	10^{-6}
dynamical Chern-Simons ($\sqrt{\lvert\alpha_{\rm dCS}\rvert}$ [km])	Parity Inv.	—	10^8

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Einstein-Æther (c_+, c_-)	Lorentz Inv.	$(\mathbf{0.9}, \mathbf{2.1})$	$(0.03, 0.003)$
RS-II Braneworld (ℓ [μm])	4D	$\mathbf{5.4 \times 10^{10}}$	10–10^3
time-varying G ($\lvert\dot{G}\rvert/G$ [10^{-12}/yr])	Equiv. Princ.	$\mathbf{5.4 \times 10^{18}}$	0.1–1

graviton dispersion relation: $E^2 = (p\ c)^2 + A\ (p\ c)^\alpha$

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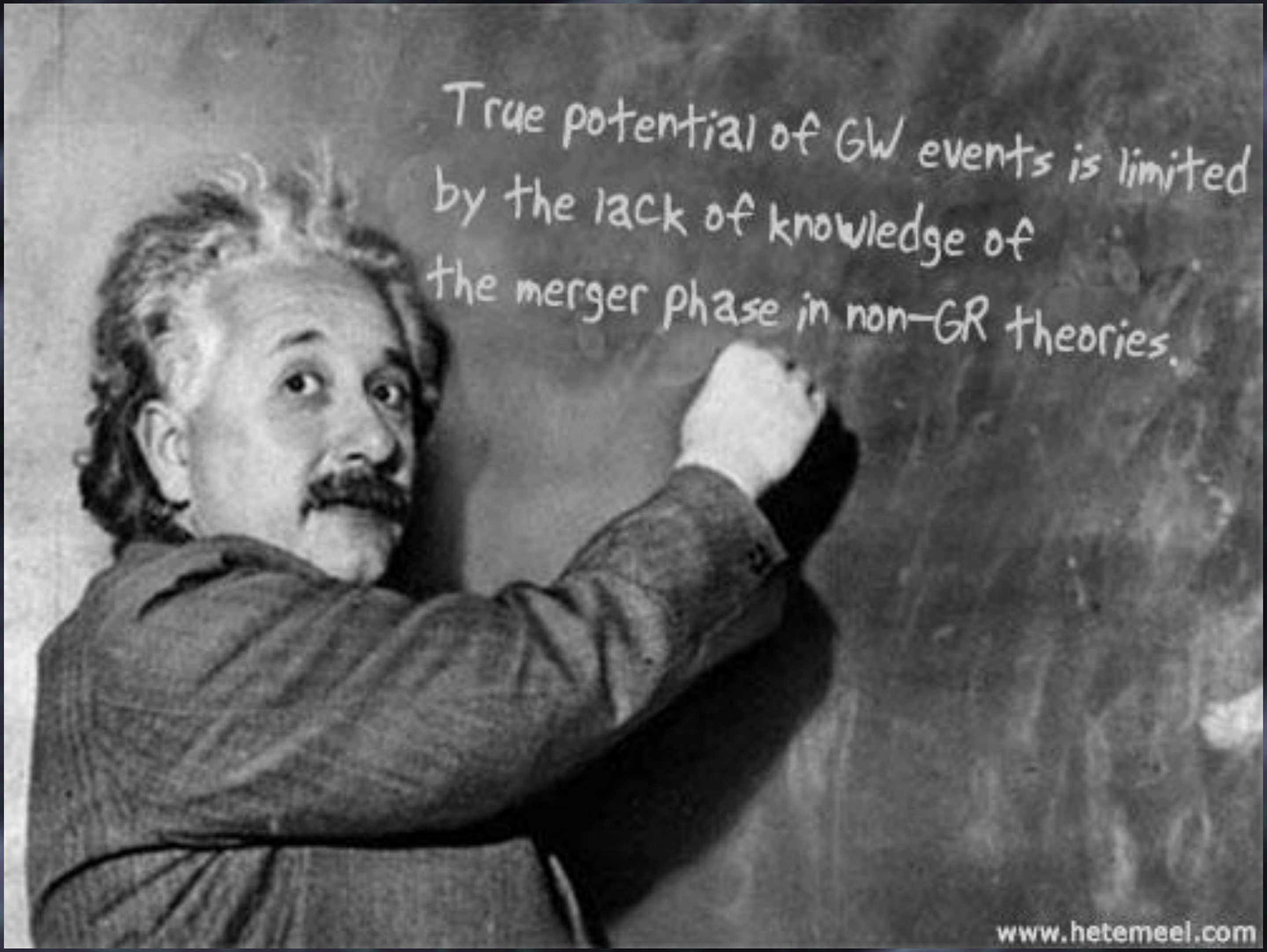
Massive Gravity (m_g [eV])	$m_g = 0$	10^{-22}	10^{-29}–10^{-18}
Modified Special Rel. ($\eta_{\rm dsrt}/L_{\rm Pl} > 0$)	Lorentz Inv.	$\mathbf{1.3 \times 10^{22}}$	—
Modified Special Rel. ($\eta_{\rm dsrt}/L_{\rm Pl} < 0$)	Lorentz Inv.	$\mathbf{1.3 \times 10^{22}}$	2.1×10^{-7}

graviton dispersion relation: $E^2 = (p\ c)^2 + A\ (p\ c)^\alpha$

Important Message



Conclusions

Takeaway & Future Work

Step 1. Carry out theory-agnostic tests

Step 2. Map to specific theories

Generic non-GR waveforms including merger & ringdown

Mapping between generic non-GR parameters to specific theoretical coupling constants

Need many merger simulations in non-GR theories

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Thank You