LIGO-G1801141-v1

Charged black holes in GR and beyond

N. K. Johnson-McDaniel

Benasque meeting: NR beyond GR

04.06.2018

Overview of charged BH solutions: Standard electrically charged solutions

- **Reissner-Nordström:** Nonspinning, discovered in 1916, the same year as the Schwarzschild solution. Simple generalization of Schwarzschild:

Electric potential

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad \Phi = \frac{Q}{r}$$

  However, introducing charge gives some radically different properties, similar to the introduction of angular momentum, e.g., a Cauchy horizon and the possibility of extremal black holes with zero temperature.

- **Kerr-Newman:** Spinning, obtained by an application of the Newman-Janis procedure to Reissner-Nordström by Newman et al. in 1965, two years after the Kerr solution was discovered. By the uniqueness theorems, the most general black hole in Einstein-Maxwell theory.

- [Note that one can obtain magnetically charged versions of these solutions with a duality transformation.]

Overview of charged BH solutions: Beyond Einstein-Maxwell

- **String theory-inspired charged black hole** with rotation (and both electric charge and magnetic dipole) constructed by Sen, PRL, 1992. Satisfies most general low-energy string theory action with at most two derivatives, i.e., Einstein-Maxwell + dilaton and (vector) Chern-Simons terms.
- No stationary Proca black holes with stationary Proca field, due to theorem by Bekenstein, PRD 1972, though there are stationary Proca black hole solutions with a Proca field with a harmonic time dependence, e.g., Herdeiro, Radu and Rúnarsson, CQG 2016.
- **Yang-Mills charged black holes:** Can be static and not spherically symmetric (e.g., Kleihaus and Kunz, PRL 1997), generally have to be constructed numerically, and are also generally unstable. There are also results in the rotating case, e.g., Kleihaus and Kunz, PRL 2001.
- **Higher-dimensional charged black holes (and rings, Saturns):** Can be constructed analytically, even with rotation, as in Caldarelli, Emparan, and Van Pol, JHEP 2011; for rings and Saturns, see Grunau, PRD 2014 for solutions in Einstein-Maxwell-dilaton theory.

Comparison of causal structure (eternal BHs)

Schwarzschild

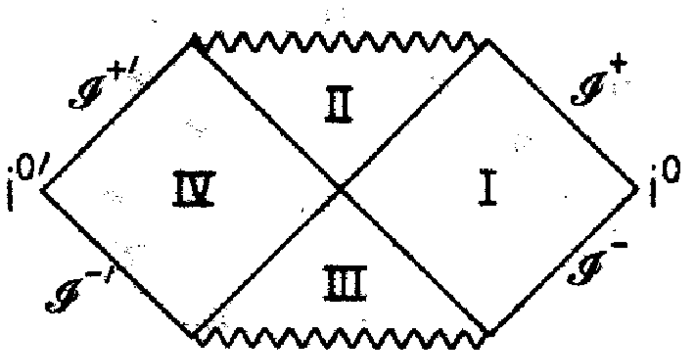


Figure from Wald: General Relativity

Reissner-Nordström (nonzero q)

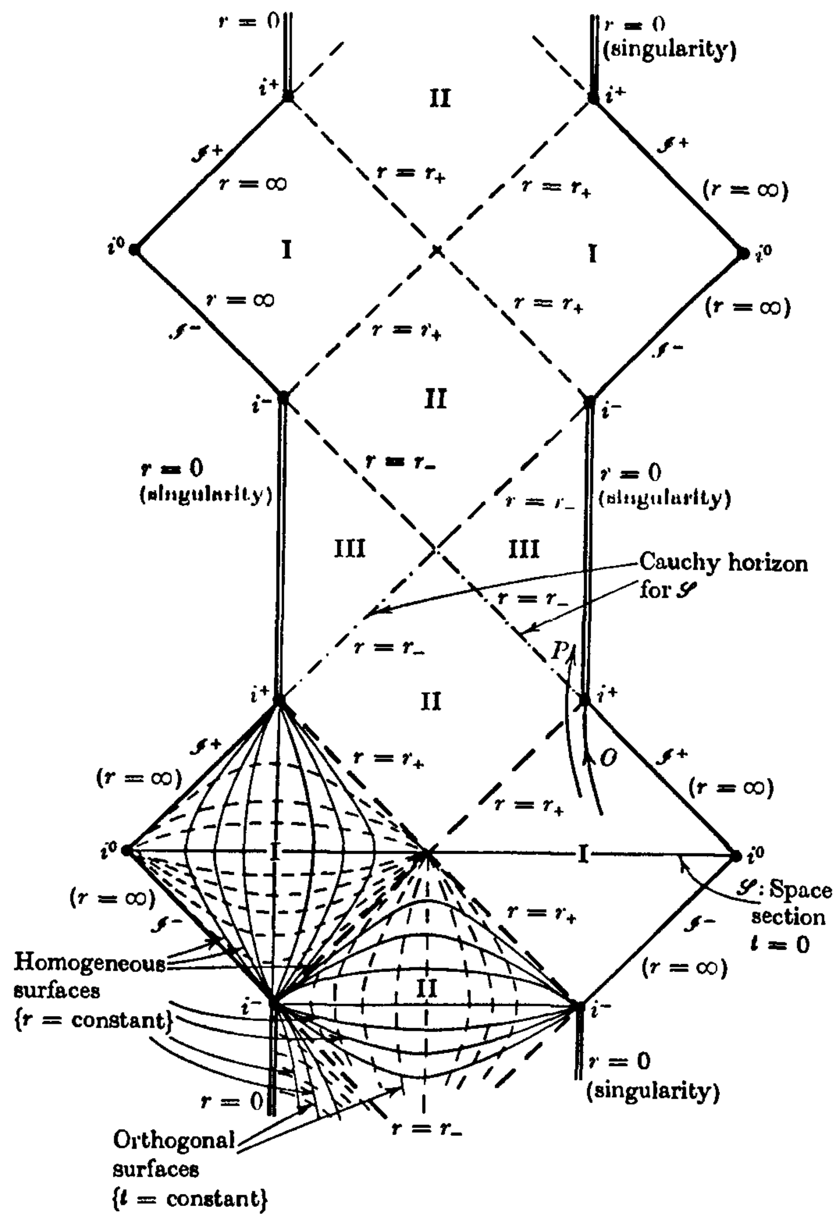


Figure from Hawking and Ellis: Large-scale structure of spacetime

Kerr(-Newman) nonzero a

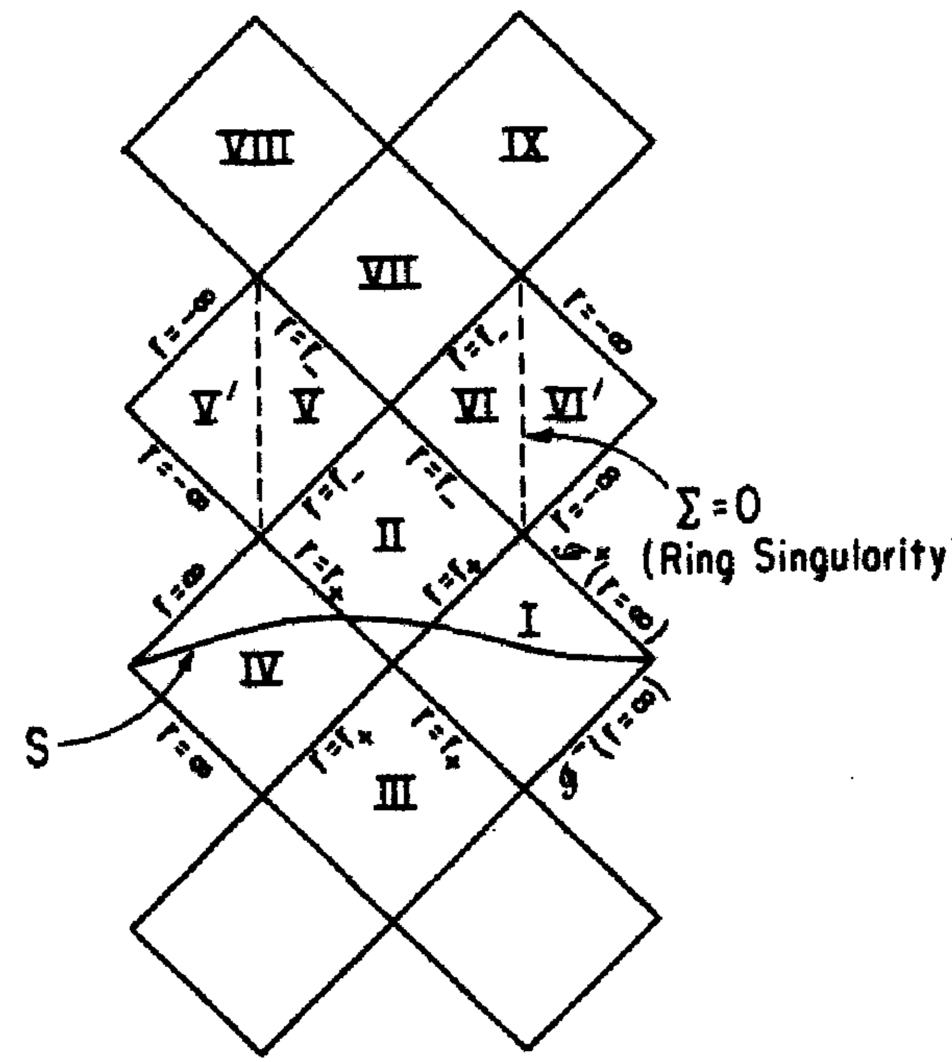


Figure from Wald: General Relativity

Comparison of ISCO velocities

- While charge mimics some of the effects of spin, in at least one sense the effects of spin are more extreme.

  Considering the PN velocity parameter (from the angular velocity at infinity), $v = (M\omega)^{1/3}$ for a neutral particle at the ISCO, we have:

  - Schwarzschild: ~0.41
  - extreme Reissner-Nordström: ~0.48
  - extreme Kerr (prograde orbit): ~0.79

Applications: Astrophysics?

- For standard electric charge, likely no astrophysical applications: There is enough plasma in the universe that even if one manages to form a charged black hole, it will discharge to a dimensionless charge of $\sim 10^{-21}$ or $\sim 10^{-18}$ (the inverse of the electron or proton's charge-to-mass ratio) on a timescale of ~1 μs or ~1 ms. [See, e.g., Cardoso et al. JCAP 2016 for a nice review.]

- People have tried various ways of circumventing these results over the years, to allow for the black holes to retain significant charge for a nonnegligible amount of time, but none of them are particularly compelling.

- If one wants to consider magnetic monopoles [as in Liebling and Palenzuela, PRD 2016], then magnetically charged black holes could be a possibility.

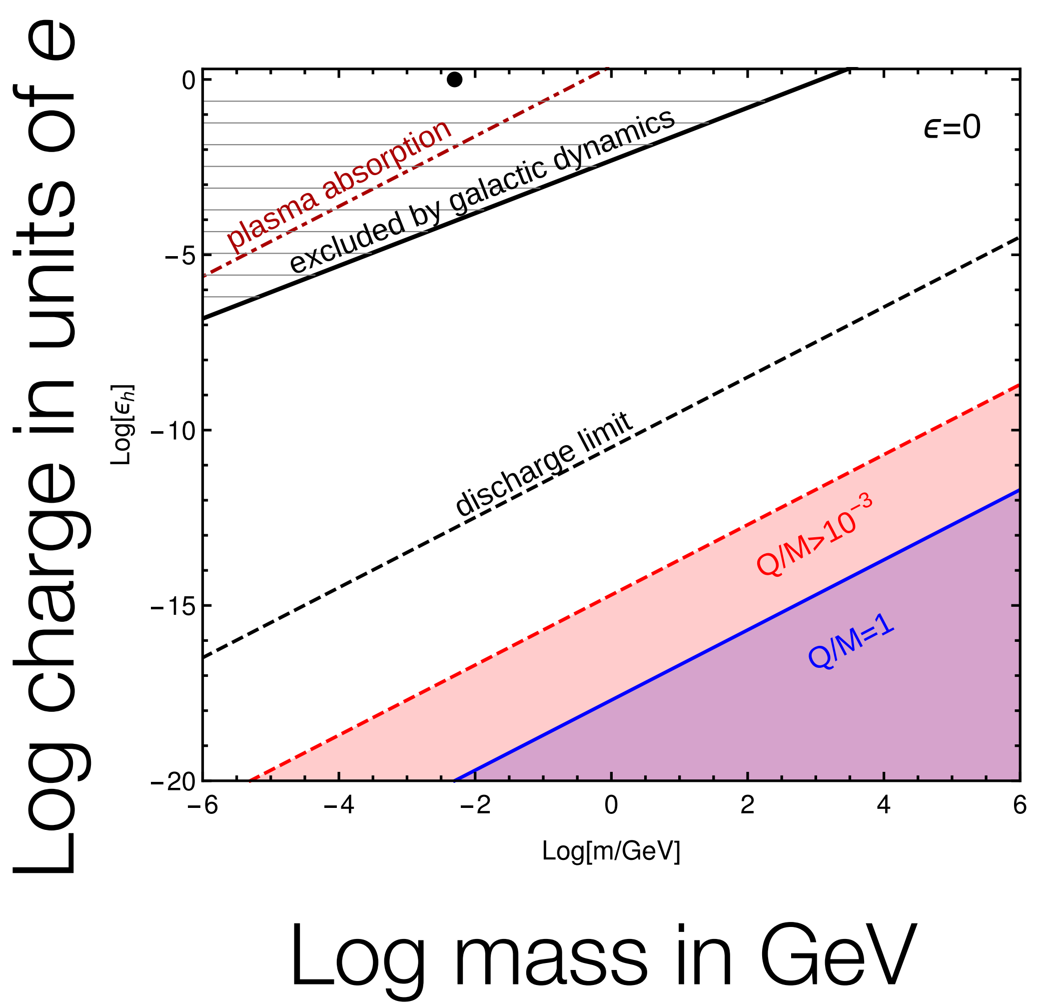
Applications: Astrophysics?

- If black holes are charged under something other than standard electromagnetism (e.g., a "dark electromagnetic field"), then there is still the possibility that this could be astrophysically relevant.

- One still has to worry that there would be associated "dark electrons" that would neutralize the black holes similarly quickly as in the standard electromagnetic field case.

- However, as pointed out by Cardoso et al. (JCAP 2016), if the particles that are charged under the dark EM field have much smaller charge-to-mass ratios than the electron, as in the millicharged dark matter scenario, then one can avoid the standard arguments about discharging black holes quickly.

Figure from
Cardoso et al. (JCAP 2016)



Applications: Mathematical/theoretical physics

- Another way of approaching extremality, and thus to test cosmic censorship [e.g., the study of self-force effects on the original overcharging scenario from Hubeny PRD 1999 by Zimmerman et al. PRD 2013].

- Lack of separability of perturbations for Kerr-Newman black holes.

- Existence of hair from non-EM charges.

- Additional tests of isolated and dynamical horizon frameworks, and calculations of black hole entropy.

Applications: Toy models

- Charged black holes are excellent toy models for certain effects that it is difficult to calculate otherwise.

- In particular, since Reissner-Nordström BHs exhibit several of the effects of rotation (e.g., extremal black holes and a Cauchy horizon), while retaining the technical simplicity of spherical symmetry, they have been used as a toy model for Kerr black holes

- My primary interest in charged black holes is in using the charge as a proxy for possible beyond-GR effects in binary black holes.

- In particular, generic charged black hole binaries will emit dipole radiation. And even in the case of equal charge-to-mass ratio, where there is no dipole radiation, the binary will have PN phase and amplitude coefficients that differ from those of vacuum BBH systems, in addition to changes to the quasinormal mode spectrum of the final black hole, except in the special case where they form an uncharged final black hole.

- Thus, one can use charged BBH waveforms to check how sensitive current LIGO tests of GR are to completely consistent parameterized deviations from GR (and whether the parameterized test can recover the correct changes to the post-Newtonian coefficients).

Studies of charged black holes: Perturbation theory

- **Post-Newtonian** (actually charged point particles, but useful for studying binaries):

  - ***Conservative dynamics:*** 1PN [O(c^{-2})] N-body Lagrangian and Hamiltonian calculated [Bażański Acta Phys. Pol. 1956 and 1957 and Barker and O'Connell, JMP 1977]. [2PN 2-body Lagrangian EM-gravity terms have been calculated in Gaida and Tretyak, Sov. Phys. J., 1991, but using a method that seems a bit suspect.]

  - ***Dissipative dynamics:*** Fluxes have just been computed to Newtonian order, as far as I can tell.

    Newtonian-order gravitational waveforms are calculated in Cardoso et al. JCAP 2016.

- **Quasinormal modes:** Difficult to compute for Kerr-Newman, as the equations are not separable, but finally computed in Dias, Godazgar, and Santos, PRL 2015 (plus calculations in the slow-rotation [Pani, Berti, and Gualtieri, PRL and PRD 2013] or weakly charged [Mark et al., PRD 2015] limits).

- **Self-force:** The electromagnetic self-force is a standard toy model, and there are now self-force calculations for Reissner-Nordström black holes, both of the scalar self-force for circular orbits [Castillo, Vega, and Wardell, arXiv 2018] and the electromagnetic self-force for radial trajectories [Zimmerman et al., PRD 2013]. There are also calculations of inspirals of uncharged point particles around a RN BH in the adiabatic approximation [Zhu and Osburn, PRD 2018].

Studies of charged black holes: Numerical relativity

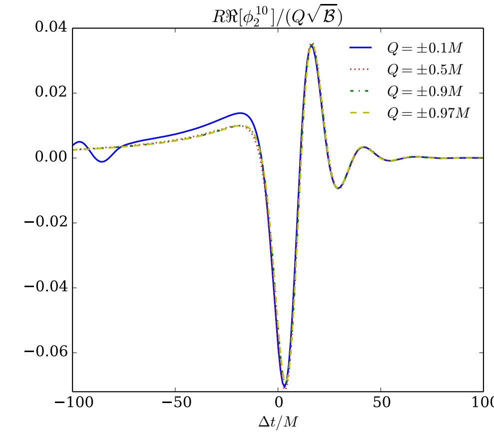
- **Zilhão et al., PRD 2012 and 2014:** Evolutions of head-on collisions of nonspinning charged black holes from rest, using either analytic equal charge-to-mass ratio initial data, or a simple numerical initial data construction to obtain opposite charge-to-mass ratios.

- **Zilhão et al., PRD 2014:** Evolutions of perturbed Kerr-Newman black holes to study stability.

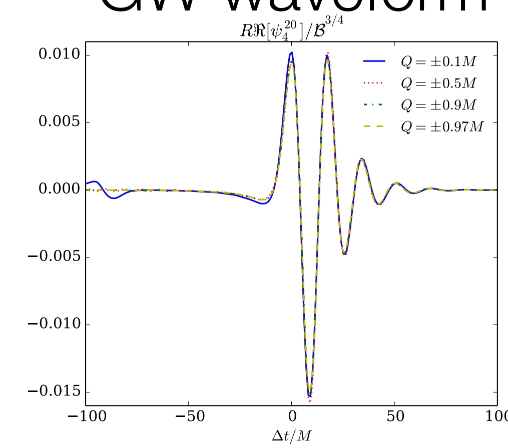
- **Liebling and Palenzuela, PRD 2016:** Evolutions of (weakly) electric and magnetically charged orbiting black holes, including force-free plasma, using metric initial data for uncharged black holes + boosted charges for the EM initial data.

- **Hirschmann et al., PRD 2018:** Evolutions of orbiting binaries of weakly charged black holes in Einstein-Maxwell-dilaton theory, with the same initial data construction as above (and a constant dilaton).

EM waveform

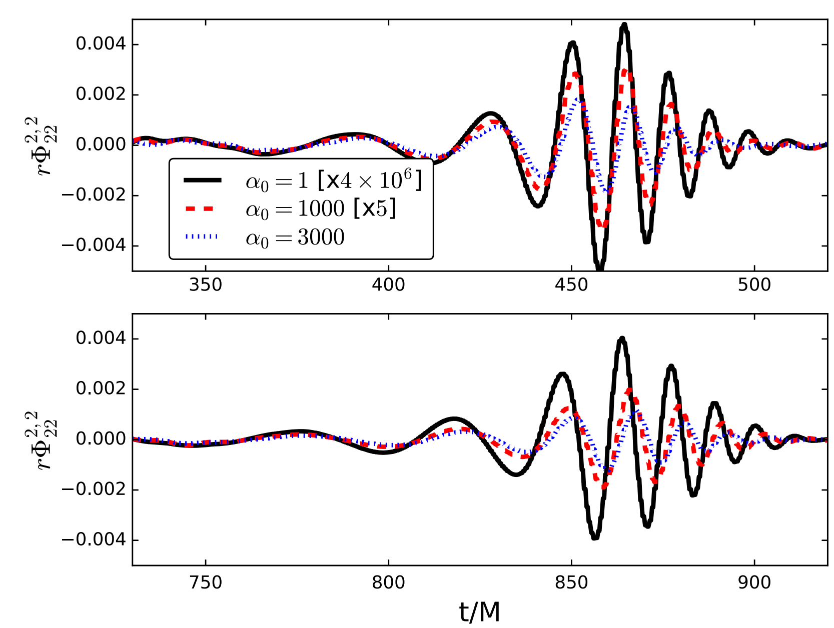


GW waveform



Oppositely charged head-on collision waveforms from Zilhão et al.

Scalar radiation from Hirschmann et al.



Initial data for orbiting charged black holes including spin and eccentricity reduction

Work with Soham Mukherjee and Wolfgang Tichy

- Generalize the superposed Kerr-Schild construction from Lovelace et al. PRD 2009 to include charge, by superposing Kerr-Newman black holes (weighted by attenuating functions) and also solving for a correction to the superposed electric field to satisfy the divergence constraint; the magnetic divergence constraint is satisfied automatically due to superposing the vector potentials.

- Specifically, solve the XCTS equations (with EM sources) for the geometry +

physical metric Laplacian

$$\nabla^2 \phi = -\nabla_j (E_{\rm sp})^j,$$

superposed electric field

and compute electric and magnetic fields using:

$$E_j = (E_{\rm sp})_j + \nabla_j \phi,$$

determinant of physical metric (computed at each iteration)

$$B^i = \frac{1}{\sqrt{g}} \epsilon_F^{ikl} \partial_k (A_{\rm sp})_l,$$

superposed vector potential

- Have implemented this method in Wolfgang's SGRID spectral code, and are currently improving boundary conditions and implementing computations of ADM quantities.

Boundary conditions

- Currently just using superposed fields to set boundary conditions for initial tests.

- Will use proper isolated horizon (really non-expanding horizon) boundary conditions for the geometry (as in Lovelace et al.), and use a Neumann boundary condition for the scalar potential to fix the charge on the black holes. The current proposal is just to scale the superposed electric field to obtain the desired charge.

Normal to horizon (= excision surface)

Charge of black hole used in superposition

$$s^a \partial_a \phi \triangleq (\zeta - 1) s^a (E_{\rm sp})_a, \qquad \zeta := 4\pi Q_{\rm H} \left[\oint_{S_{\rm exc}} (E_{\rm sp})^a dS_a \right]^{-1}$$

However, we're trying to think of better ways (where the boundary condition involves the horizon's geometry).

- [N.B.: The isolated horizon boundary conditions imply that

$$E_a^{\parallel} \triangleq (s \times B)_a$$

It doesn't seem simple to enforce this with a boundary condition, since it involves transverse derivatives of ϕ. We are waiting to see how well this is satisfied in our data with the above boundary conditions before worrying too much about how to enforce this.]

Conclusions

- There are a wide variety of charged binary black hole solutions involving GR coupled to other fields (which could also be thought of as modifications to GR itself).
- Even concentrating on just the Kerr-Newman family of Einstein-Maxwell theory, there is a possibility that these could still be astrophysically relevant if they are charged under some dark EM field or magnetically charged.
- Regardless, charged black holes make excellent toy models for various effects, including spin and modified gravity effects.
- There have been evolutions of charged black hole binaries with two codes (Lean and HAD), but only with very simple approximate initial data in the orbiting case.
- We have implemented a generalization of the Lovelace et al. superposed Kerr-Schild construction to Kerr-Newman black holes in Wolfgang Tichy's SGRID code and are currently testing the data with evolutions in the vacuum case and improving the boundary conditions.

Extra slides

Technical details about SGRID implementation

- Use Ansorg's ABφ coordinates in their black hole excision version (to give a compactified grid with more resolution close to the black holes) with a Chebyshev-Chebyshev-Fourier spectral expansion.

- Solve nonlinear equations using Newton's method, and linear equations using dctemplates (+ BLAS & LAPACK) GMRES with block Jacobi preconditioner + UMFPACK (or just UMFPACK if you have enough memory).

- OpenMP parallelized (for, e.g., setting the elements of the matrix to solve).

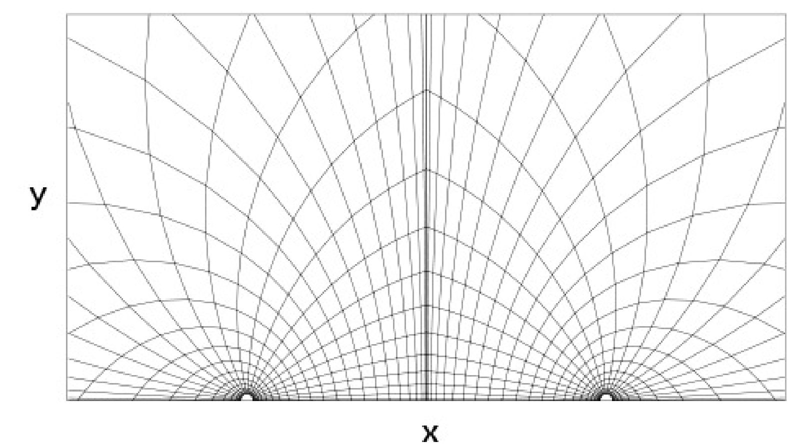


Illustration of the ABφ coordinates from George Reifenberger's thesis