# Numerical Relativity with Scalar Gauss-Bonnet Gravity

**Leonardo Gualtieri**

**Sapienza Università di Roma**

*(in collaboration with H. Witek, P. Pani, T. Sotiriou)*

# Why NR in modified gravity theories?

- Conclusion of Kento's talk: "True potential of GWs is limited by the lack of knowledge of the merger phase in non-GR theories"

- NR is the only tool we have to study the merger phase of compact binaries

- Early-inspiral phase carries information,on negative-PN contributions (such as dipole radiation), late-inspiral and merger carry information on the strong-field, large curvature regime of gravity:
  complementary information

- PN approaches can not be accurate enough to test GR deviation even in late inspiral: phenomenological waveforms & EOB require calibration of parameters from NR simulations. Even few NR simulations of binary BH (BBH) mergers may be sufficient for this task!

- As discussed in Thomas' , Frans' , Luis' talks, this is a very challenging task (well posedness, gauge choices, difficult numerical implementation). Very few results up to now:
  - most results in scalar-tensor theory *(Palenzuela et al., Healy et al., Barausse et al.)* but no-hair theorems!
  - dynamical Chern-Simons: well-posed only with EFT approach *(Delsate et al. '15)*
    binary BH evolution with EFT approach in *(Okounkova et al.'17)*, see Okounkova's talk

# Why sGB gravity?

## SCALAR-TENSOR GRAVITY (one scalar field)

### Horndeski

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Big\{ & K(\phi,X) - G_3(\phi,X)\Box\phi \\
& + G_4(\phi,X)R + G_{4,X}(\phi,X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] \\
& + G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}(\phi,X)}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right. \\
& \left. + 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla_\sigma\phi)(\nabla^\nu\nabla^\sigma\phi)\right]\Big\} + S_{\mathrm{mat}}\left[\Psi,\, \gamma(\phi)g_{\mu\nu}\right]
\end{aligned}
$$

### quadratic curvature terms

$$
\begin{aligned}
S = \frac{1}{16\pi}\int \sqrt{-g}d^4x\Big[ & R - 2\nabla_a\phi\nabla^a\phi - V(\phi) + f_1(\phi)R^2 \\
& + f_2(\phi)R_{\mu\nu}R^{\mu\nu} + f_3(\phi)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + f_4(\phi){}^*RR\Big] \\
& + S_{\mathrm{mat}}\left[\Psi,\, \gamma(\phi)g_{\mu\nu}\right] ,
\end{aligned}
$$

### scalar Gauss-Bonnet

$$
S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathscr{G}\right] + S_{\mathrm{m}}[g_{\mu\nu},\psi]
$$

# Why sGB gravity?

- $\mathcal{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ Gauss-Bonnet invariant (total derivative)
- second-order field equations => no Ostrogradski instability, could exist beyond small coupling limit
- BHs have scalar hair
- GR deviations appear at large curvature => no constraints from binary pulsars, need GW
- fundamental physics motivation: low-energy effective string theory *(Gross & Sloan '87 ) ,*
- first terms in polynomial curvature expansion of a possibly renormalizable theory *(Stelle '77 )*

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathcal{G}\right] + S_m[g_{\mu\nu},\psi]$$

The simplest, well-behaved modification of the strong-field limit of gravity!

Different possible choices of the coupling function f(φ):

- exponential: $f=e^{\alpha\varphi}$ Einstein-dilaton Gauss-Bonnet (EdGB) gravity [string-inspired]
  *(Mignemi & Stewart '93, Kanti et al. '96, Pani & Cardoso '09, Yunes & Stein '11, etc.)*
- linear: $f=\alpha\varphi$ shift-symmetric Gauss-Bonnet gravity
  *(Sotiriou & Zhou '14a, '14b, Barausse & Yagi '15, Benkel et al. '16, '17)*
- scalarized: $f=\alpha\varphi^2$ Gauss-Bonnet gravity with scalarization
  $f=\alpha(1-\exp(-6\varphi^2))$ *(Silva et al. '17, Doneva & Yazadjiev '17)*

# Why sGB gravity?

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathscr{G}\right] + S_m[g_{\mu\nu},\psi]$$

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- $\zeta = \alpha/M^2$ dimensionless coupling

- stationary BH solutions *(Kanti et al. '96)*: $\zeta < \zeta_{max} \sim 1$ we do not have to require $\zeta << 1$

- scalar field profile $\varphi \sim Q/r$ Q scalar charge: BHs have hair! $Q/M \sim \zeta$

- PN description of BH binary inspiral *(Yagi et al. '12 )*

$$\delta\dot{E}^{(\varphi)} = \dot{E}^{GR}\frac{5}{96}\frac{\zeta}{\eta^4}\frac{\delta m^2}{m^2}v^{-2}$$

# Is sGB gravity ruled out?

## 1) Is sGB gravity well-posed? **Maybe!**

*Well-posedness*: existence of a unique solution which depends continuously on initial data.
It requires that $||\delta u(t,\cdot)|| \leq F(t)||\delta u(0,\cdot)||$

*Strong hyperbolicity* is a condition for well-posedness: the principal part is diagonalisable.

For instance, DCS gravity (as a full theory) seems to be not strongly hyperbolic *(Delsate et al.,'15)* so it should be considered as a truncation of a more fundamental theory (EFT approach)

Recently, it has been shown that sGB gravity is not strongly hyperbolic *(Papallo & Reall '17, Papallo '17)* …
but this was only shown in generalised harmonic gauge $0 = g^{\nu\rho}\nabla_\nu\nabla_\rho x^\mu = \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\mu\nu}\right)$

*there is no reason to believe that the same applies in a BSSN-like formulation!*

## 2) Is sGB gravity ruled out by GW observations? **No!**

GW170817 has shown that GWs and light travel with the same speed.
This sets bounds on several modified gravity theories, inclunding sGB gravity
*(Ezquiaga & Zumalacarregui '17, Baker et al. '17, Sakstein & Jain '17, Creminelli & Vernizzi '17)*

$$|\alpha_T| = \left|\frac{c^2 - c_T^2}{c^2}\right| \lesssim 10^{-15}$$

*"models such as Einstein-dilaton-Gauss-Bonnet that do not lead an accelerating universe, of interest for example for deviations detectable via black hole tests, are still allowed (Sakstein & Jain, '17)"*

$$\alpha_T \propto c_{GB}\left(\frac{\dot{\varphi}}{H_0}\right)^n$$

sGB parameter in Horndeski Lagrangian

cosmological parameter

# BH binaries in sGB gravity - first step: EFT approach

As a first step, we perform NR simulations of BBH coalescences in an EFT framework (analogue to that followed in *Okounkova et al.'17* for DCS gravity, see Okounkova's talk)

We expand the action, and then the field equations, in powers of $\zeta = \alpha_{GB}/M^2 \ll 1$ (note that sGB viable also for $\zeta \sim 1$, see later). We also assume $\phi^{(0)}=0$ (no-hair)

$$\Phi = \sum_{k=0}^{\infty} \frac{1}{k!}\zeta^k \Phi^{(k)} = \zeta\Phi^{(1)} + \ldots \qquad g_{ab} = g_{ab}^{(0)} + \sum_{k=1}^{\infty} \frac{1}{k!}\zeta^k h_{ab}^{(k)} = g_{ab}^{(0)} + \frac{\zeta^2}{2} h_{ab}^{(2)}$$

The field equations

$$\Box\Phi = -\frac{\alpha_{\rm GB}}{4} f'(\Phi)\mathcal{R}_{\rm GB}\,, \qquad G_{ab} = -\alpha_{\rm GB}\mathcal{G}_{ab} + \frac{1}{2}T_{ab}^{(\Phi)}\,,$$

with

$$T_{ab}^{(\Phi)} = \nabla_a\Phi\nabla_b\Phi - \frac{1}{2}g_{ab}\nabla^c\Phi\nabla_c\Phi$$

$$\mathcal{G}_{ab}^{\rm GB} = \frac{\delta\mathcal{R}_{\rm GB}}{\delta g^{ab}} = 2g_{c(a}g_{b)d}\epsilon^{edfg}\nabla_h\left[{}^*R^{ch}{}_{fg}\nabla_e f\right] = 2g_{c(a}g_{b)d}\epsilon^{edfg}\nabla_h\left[{}^*R^{ch}{}_{fg} f'\nabla_e\Phi\right]\,,$$

are expanded as: $\quad G_{ab}^{(0)} = 0\,, \quad \Box^{(0)}\Phi^{(1)} = -\mathcal{R}_{\rm GB}^{(0)}\,.$

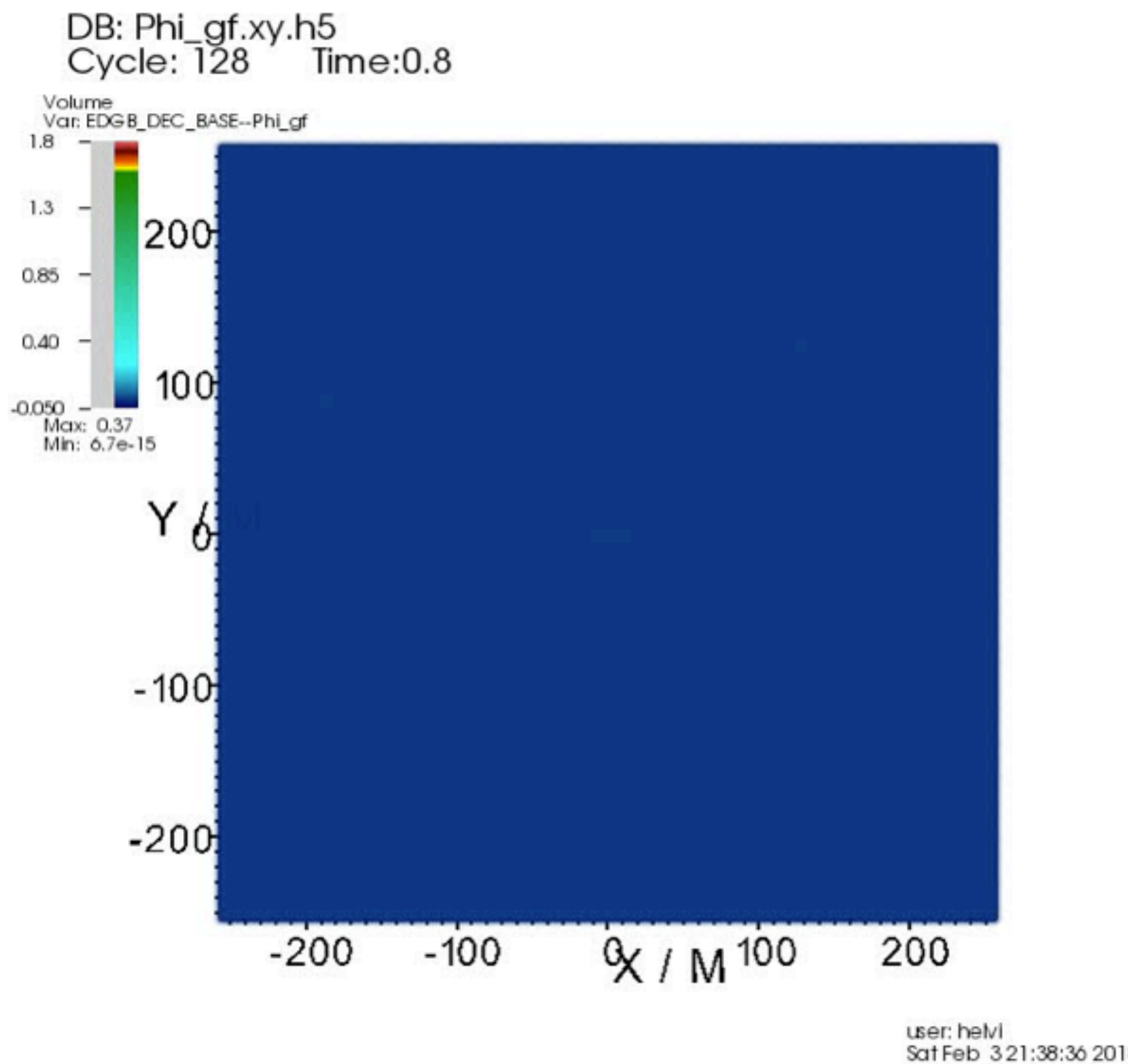
We chose EdGB gravity, $f' = e^{\varphi} = 1 + \ldots$

Scalar field in a BBH background sourced by $R_{GB}$, without backreaction.

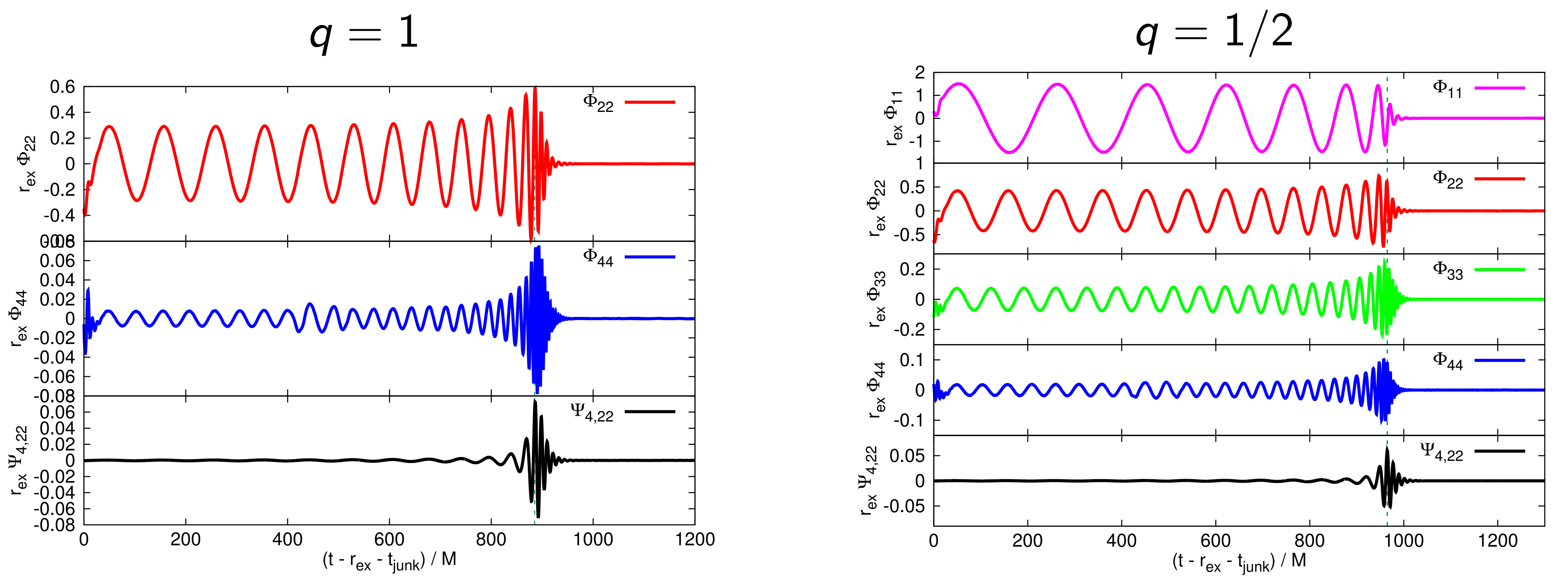
## BH binaries in sGB gravity - first step: EFT approach

With this approach, well-posedness automatically guaranteed
(at any order, it is inherited from well-posedness of equations at 0-th order)

We evolve the scalar field
simultaneously
with the BBH background
in the same BSSN framework.



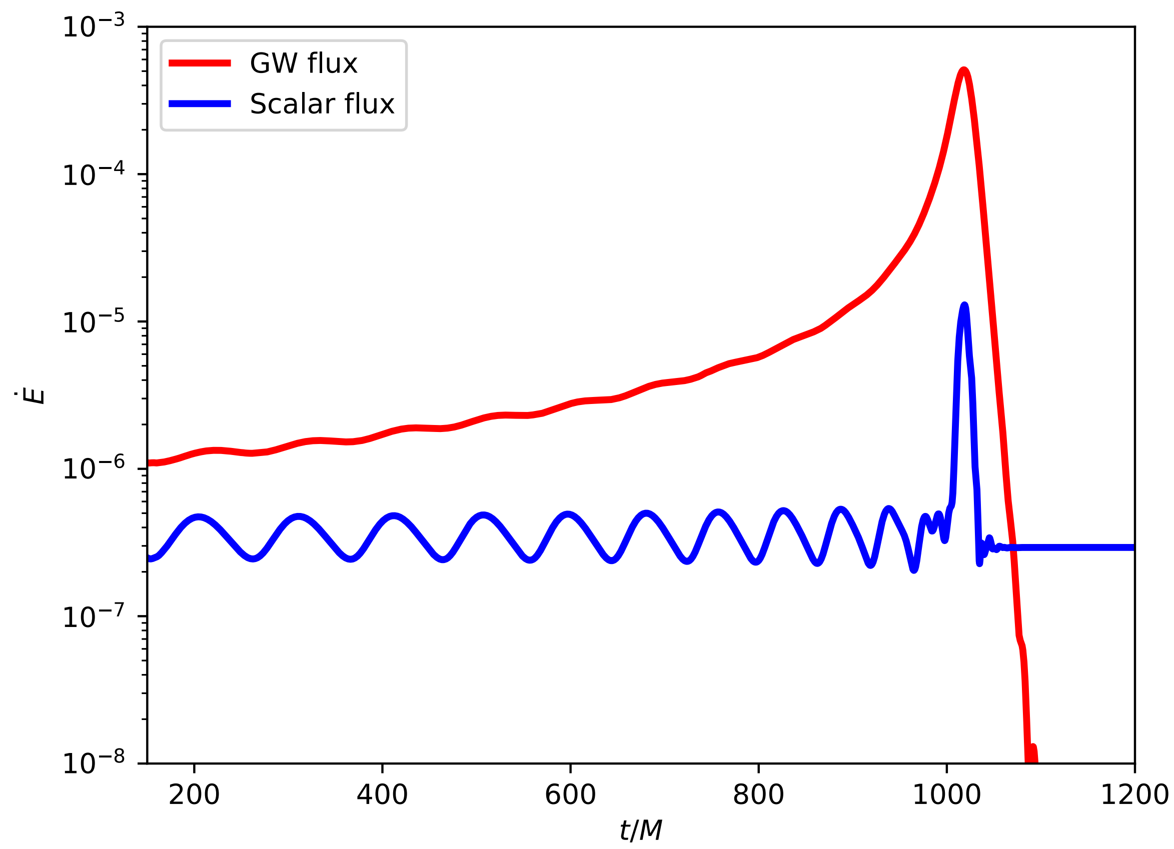
Scalar radiation extracted at r=40M:




- excitation of scalar radiation sourced by curvature / orbital dynamics

- post-merger ringdown: as expected, the scalar field oscillates with a combination of GR quasi-normal modes of scalar and gravitational radiation
(preliminary: l=m=1 has the frequency of the scalar l=m=1 QNM,
l=m=2 has both the frequencies of scalar and gravitational l=m=2 QNMs
but caution: frequency extraction difficult due to dependency on initial ringdown time)

Energy flux: preliminary results - work in progress




It should allow to estimate, even in this first-order computation, how much the GR deviation affects the orbital motion and thus the magnitude of the effects on the GW phase, and the detectability from interferometric detectors!

# BH binaries in sGB gravity - second step: fully coupled equations

This is a long-term project, just at the beginning!

- We do not assume small coupling, we do not expand metric/scalar field, and evolve the fully coupled equations of metric + scalar field
- 3+1 decomposition: $\gamma_{ab} = g_{ab} + n_a n_b \qquad ds^2 = -(\alpha^2 - \beta^a \beta_a) dt^2 + \beta_a dt dx^a + \gamma_{ab} dx^a dx^b$

$$\mathcal{L}_n \gamma_{ab} = -2K_{ab} = -2\left(A_{ab} - \frac{1}{3}\gamma_{ab} K\right)$$

$$K_\Phi = -\mathcal{L}_n \Phi$$

variables: $\Phi, \gamma_{ab}, K_\Phi, K, A_{ab}$

- Constraints (energy and momentum) involve metric and scalar field
- Dynamical evolution equations: by defining auxiliary quantities

$$\mathcal{H}^{GR} = R - K_{ab}K^{ab} + K^2$$

$$E^{GR}_{ab} = R_{<ab>} - A_{ac}A^c{}_b + \frac{1}{3}(K A_{ab} + \gamma_{ab} A^2)$$

$$\mathcal{C}_{ab} = D_a D_b f(\Phi) - K_{ab}\mathcal{L}_n f(\Phi) \;\; ; \;\; \mathcal{C} = \gamma^{ab}\mathcal{C}_{ab}$$

$$\begin{pmatrix} 1 & -2f'\mathcal{H}^{GR} & 8\alpha_{GB} f' E^{GR\,ab} \\ 2\alpha_{GB}\mathcal{H}^{GR} & 6 - 16\alpha_{GB}\mathcal{C} & 16\alpha_{GB}\mathcal{C}^{<ab>} \\ 8\alpha_{GB} f' E^{GR}_{cd} & -16\alpha_{GB}\mathcal{C}_{<cd>} & \delta^a_{<c}\delta^b_{d>}(1 - 8\alpha_{GB}\mathcal{C}) - 16\delta^a_{<c}\mathcal{C}^b{}_{d>} \end{pmatrix} \begin{pmatrix} \mathcal{L}_n K_\Phi \\ \frac{1}{6}\mathcal{L}_n K \\ \frac{1}{8}\mathcal{L}_n A_{<ab>} \end{pmatrix} = \begin{pmatrix} S_\Phi \\ S_K \\ S_{<cd>} \end{pmatrix}$$

This matrix has to be inverted to get a first-order in time formulation (work in progress…)

- Future steps: BSSN-like formulation, gauge choice, numerical implementation…

## BH binaries in sGB gravity - second step: fully coupled equations

A brief comment on the well-posedness

$$\begin{pmatrix} 1 & -2f'\mathcal{H}^{GR} & 8\alpha_{GB}f'E^{GR\,ab} \\ 2\alpha_{GB}\mathcal{H}^{GR} & 6-16\alpha_{GB}\mathcal{C} & 16\alpha_{GB}\mathcal{C}^{<ab>} \\ 8\alpha_{GB}f'E^{GR}_{cd} & -16\alpha_{GB}\mathcal{C}_{<cd>} & \delta^a_{<c}\delta^b_{d>}(1-8\alpha_{GB}\mathcal{C})-16\delta^a_{<c}\mathcal{C}^b{}_{d>} \end{pmatrix} \begin{pmatrix} \mathcal{L}_n K_\Phi \\ \frac{1}{6}\mathcal{L}_n K \\ \frac{1}{8}\mathcal{L}_n A_{<ab>} \end{pmatrix} = \begin{pmatrix} S_\Phi \\ S_K \\ S_{<cd>} \end{pmatrix}$$

The result in *Delsate et al., 15* is based on the fact that in the DCS case the matrix (more precisely, the part involving the time derivative of the traceless extrinsic curvature) is degenerate:

DCS: $$\cdots + \alpha_{CS}\left(\delta^a_{<c}\epsilon_{d>}{}^{be}D_e\Phi\right)\mathcal{L}_n X_{<ab>} = S_{<cd>}$$

sGB: $$\cdots + \left(\delta^a_{<c}\delta^b_{d>}(1-8\alpha_{GB}\mathcal{C})-16\alpha_{GB}\delta^a_{<c}\mathcal{C}^b{}_{d>}\right)\mathcal{L}_n A_{<ab>} = S_{<cd>}$$

- The former is obviously degenerate due to the presence of the Levi-Civita tensor:

$$\left(\delta^a_{<c}\epsilon_{d>}{}^{be}D_e\Phi\right)D^c\Phi D^d\Phi \equiv 0$$

- The second is non-degenerate (at least, for small enough coupling constant)

This is an indication that (non-EFT) sGB gravity has not an evident ill-posedness such as DCS gravity

but of course all this is very, very preliminar!

# Conclusions

- Developing NR techniques and performing NR simulations of modified gravity theories such as quadratic gravity theories is challenging but rewarding

- sGB gravity is a natural candidate to perform these computations. Recent claims of it ill-posedness and of tension with GW observational data only refer to a specific gauge/formulation, and to specific applications of the theory. sGB still stands as a viable and promising GR modification

- With the easiest approach one simply evolves a scalar field equation in a GR BBH background. This is a necessary first step before addressing the fully coupled problem. Still, this approach gives order-of-magnitude estimates of the orbital motion modifications and thus of the detectability.

- For the fully coupled problem we are just at the beginning. Some preliminary indication suggest that the mechnisms immediately leading to ill-posedness in DCS gravity are not present in sGB gravity. These are just preliminary and partical indications: a systematic study of well-posedness of sGB gravity is still to be done