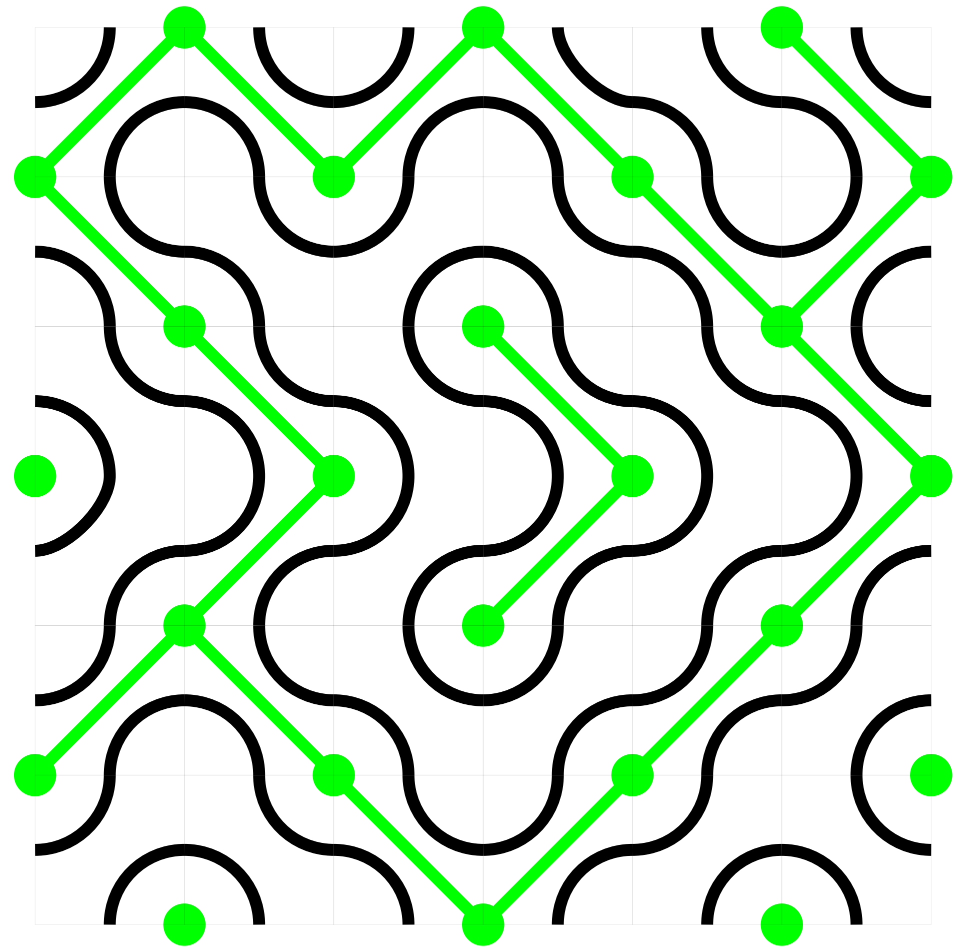
PEPS with continuous virtual symmetries



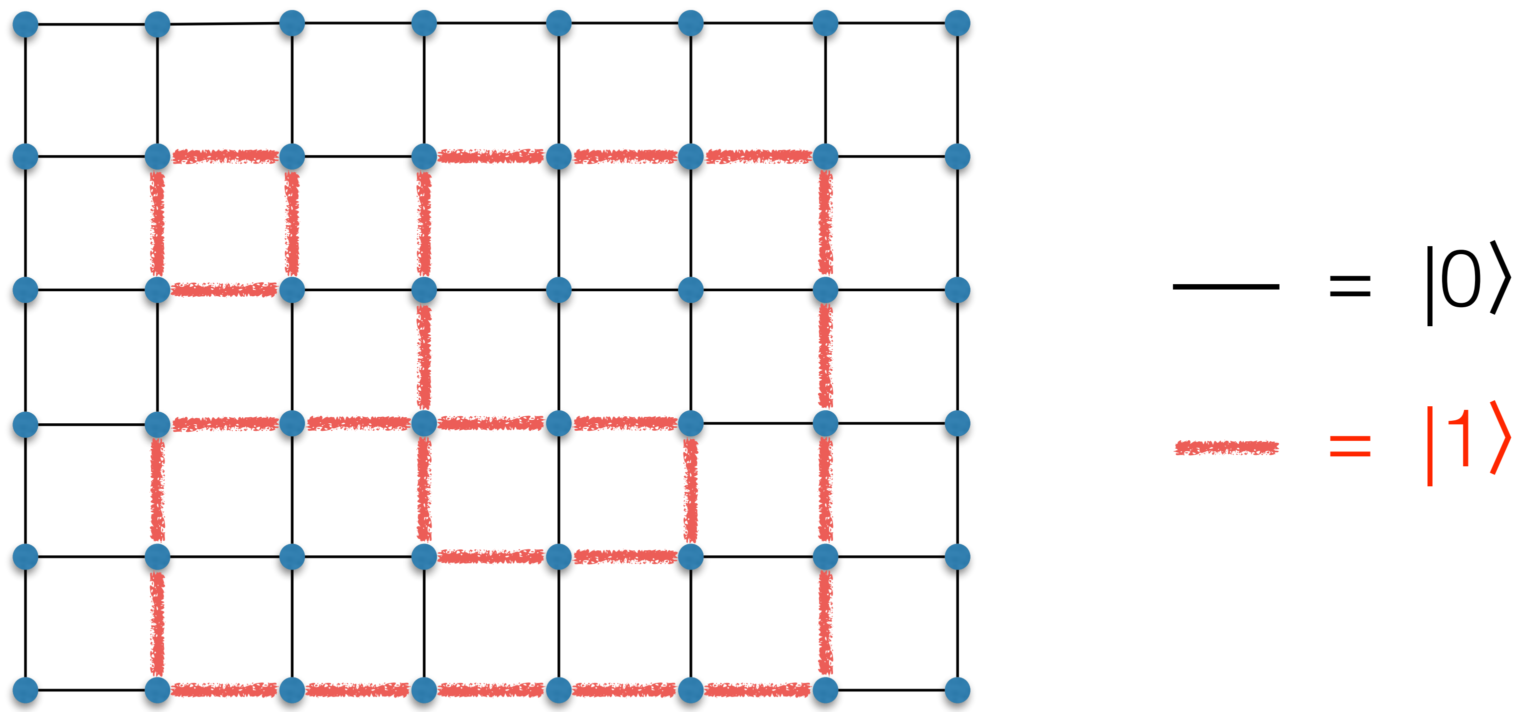
Henrik Dreyer, Ignacio Cirac and Norbert Schuch
Max-Planck Institute for Quantum Optics

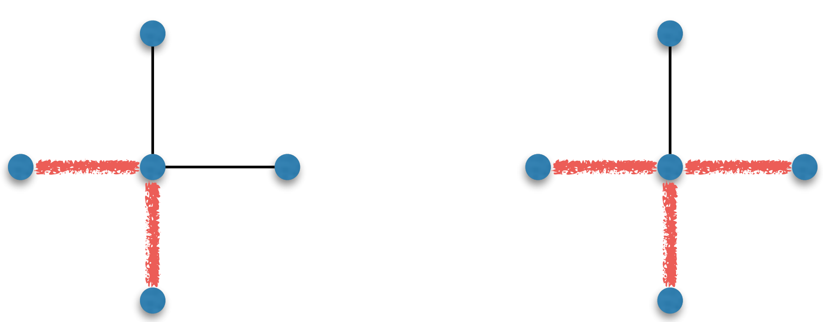
Benasque, 01.03.2018

Part I

…in which I recapitulate three properties of the Toric Code and then claim that they are superficially unrelated.

Part I - The Toric Code





even parity   odd parity

$$|\psi\rangle = \sum_{\substack{L \text{ has} \\ \text{even parity} \\ \text{everywhere}}} |L\rangle$$

This wavefunction has 3 interesting properties (for the purpose of this talk).

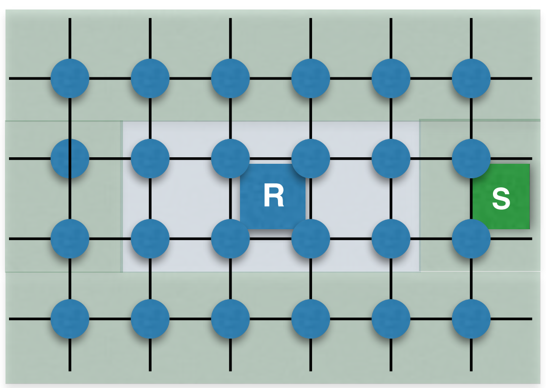
Part I - The Toric Code

Local parent Hamiltonian

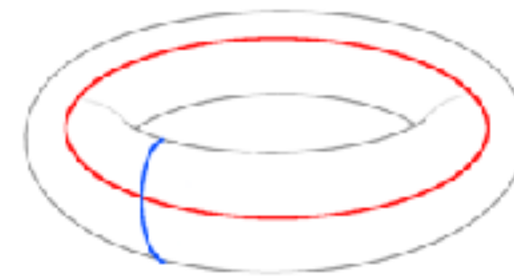
P1

$$H = -\sum_p A_p - \sum_v B_v$$

P2



P3



Four-fold degeneracy

$$|\psi_{ij}\rangle = (Z^{\otimes N_h})^i (Z^{\otimes N_v})^j |\psi\rangle$$

$$i, j \in \{0, 1\}$$

$$\langle \psi_{ij} | \psi_{kl} \rangle = \delta_{ik}\delta_{jl}$$

$$H |\psi_{ij}\rangle = E_{\min} |\psi_{ij}\rangle$$

Constant correction to the area law

$$S_0(\mathrm{Tr}_S |\psi\rangle \langle\psi|) = |\partial R| - \gamma$$

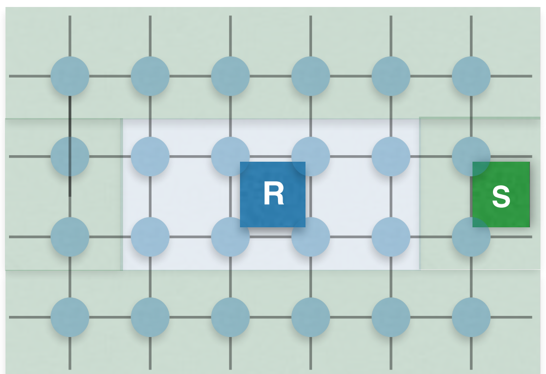
$$\gamma = \log 2$$

Local parent Hamiltonian

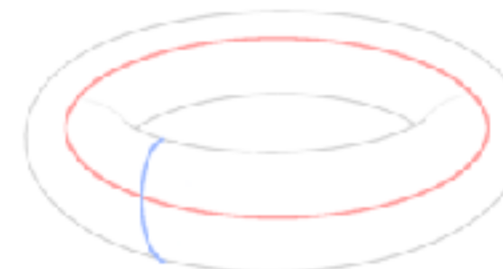
P1

$$H = -\sum A_p - \sum B_v$$

P2



P3



Is there a unifying picture that explains all of these properties?

Four-fold degeneracy

$$|\psi_{ij}\rangle = (Z^{\otimes N_h})^i (Z^{\otimes N_v})^j |\psi\rangle$$

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Constant correction to the area law

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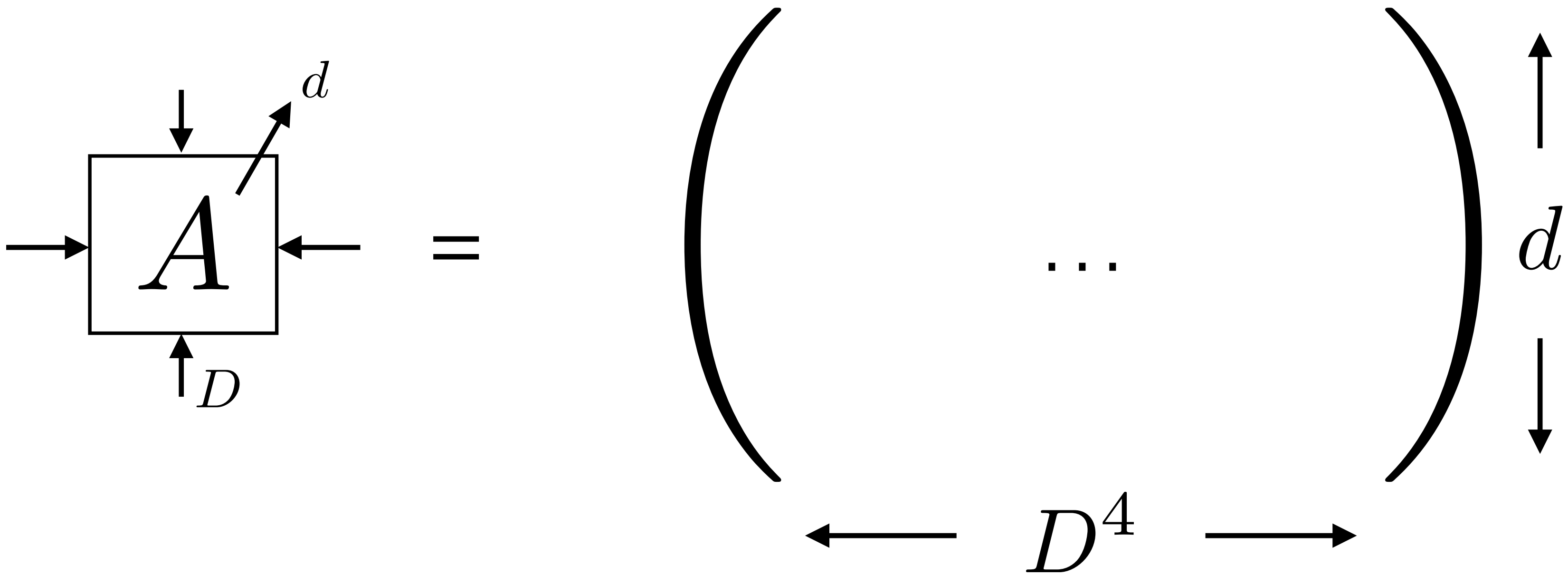
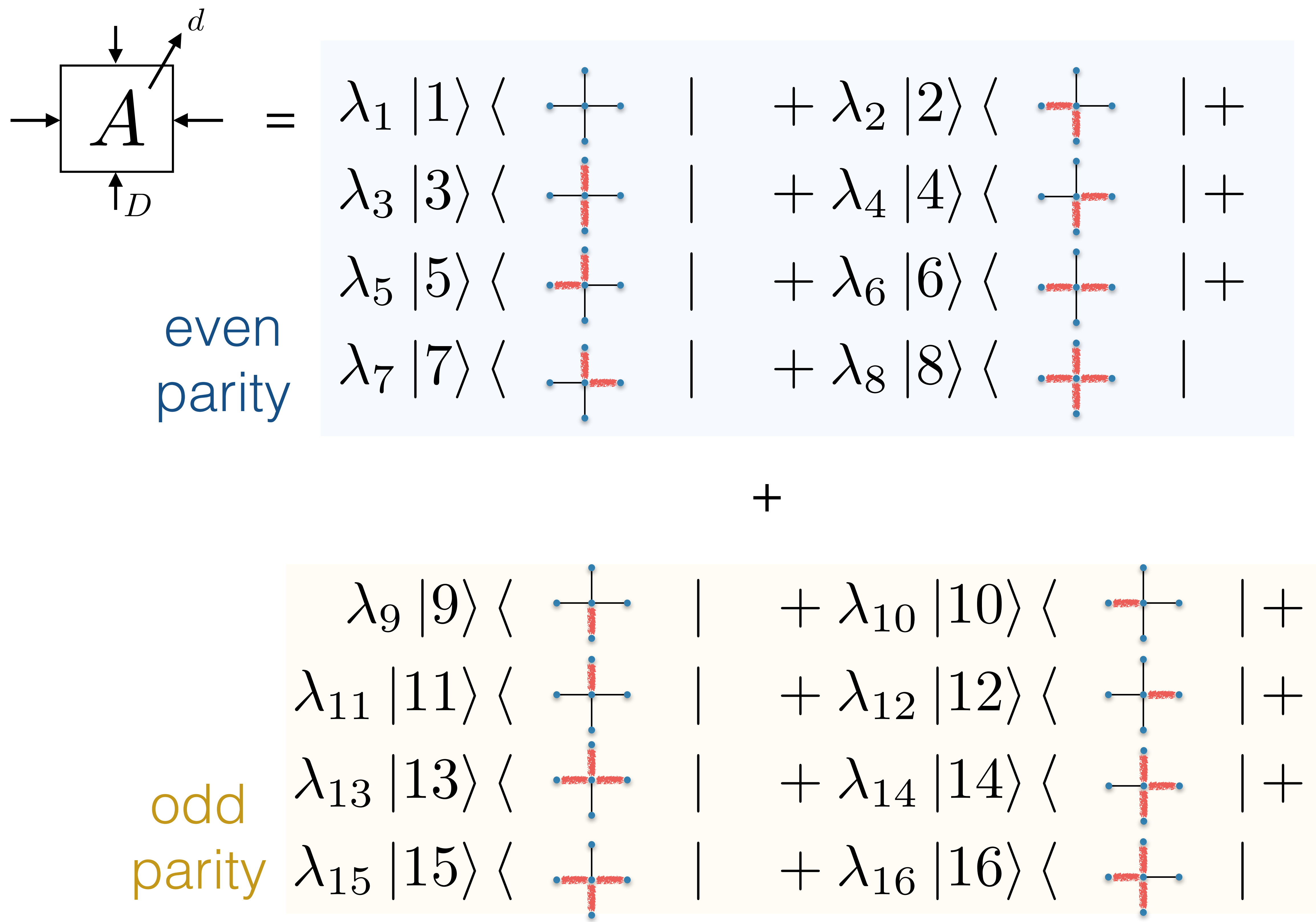
Part II

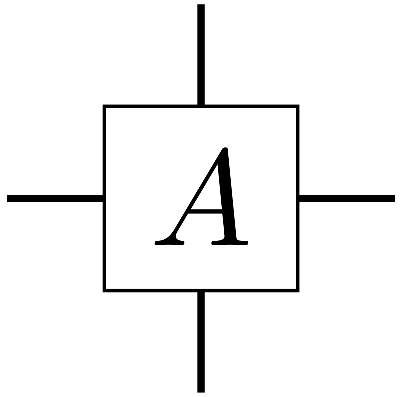
…in which I show that P1-P3 arise from a single equation, if we write the Toric Code as a G-invariant PEPS, thereby motivating that G-invariant PEPS generically have interesting behaviour.


d
A
D
=
...
d
D^4

Most general PEPS tensor with D = 2:



Part II - G-invariant PEPS

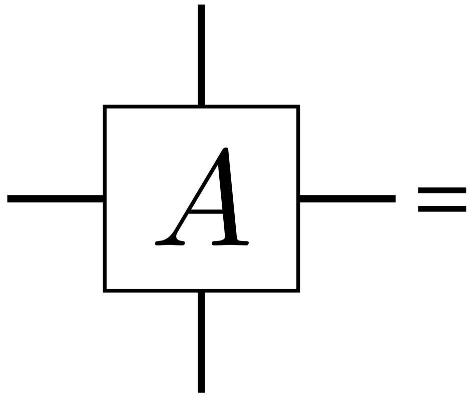


$$= \underbrace{\sum_{i=1}^{8} \lambda_i |i\rangle \langle v_i|}_{\text{even parity}} + \underbrace{\sum_{i=9}^{16} \lambda_i |i\rangle \langle v_i|}_{\text{odd parity}}$$

- Assume $\langle i \,|\, j\rangle = \delta_{ij}$
- If *all* $\lambda_i \neq 0$, we call this tensor *injective*
- If $\lambda_i = 0$ for $i = 9,\ldots,16$, the tensor is *Z_2-invariant*
  - If also $\lambda_i \neq 0$ for $i=1,\ldots,8$, the tensor is *Z_2-injective*
    - If also $\lambda_i = \lambda_j$ for $i,j = 1,\ldots, 8$, the tensor is *Z_2-isometric*

$$|\psi_{\text{Z2-isometric}}\rangle = |\psi_{\text{Toric Code}}\rangle$$

Part II - G-invariant PEPS



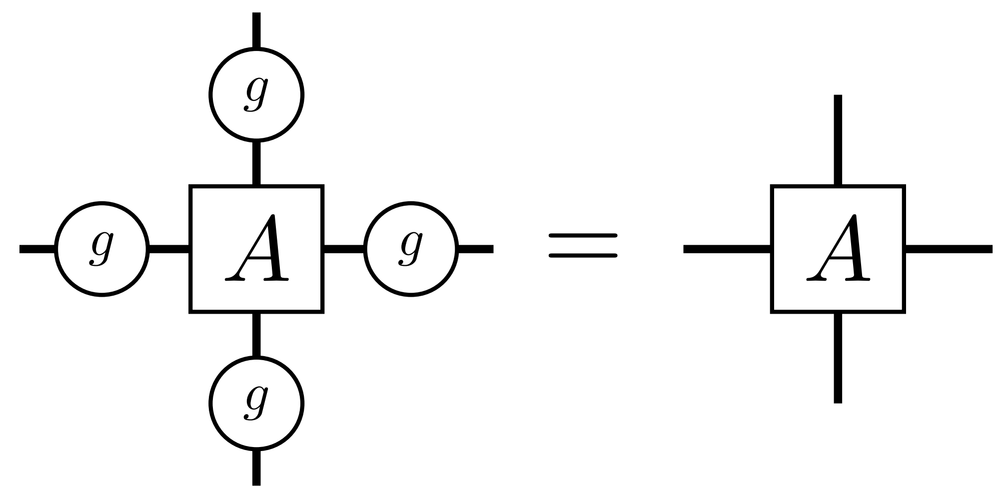
$$= \sum_{i=1}^{8} \lambda_i |i\rangle \langle v_i| + \sum_{i=9}^{16} \lambda_i |i\rangle \langle v_i|$$

even parity — odd parity

Z_2-invariance

$\lambda_i = 0$ for $i = 9, \ldots, 16$

↔



$g \in \{1, Z\}$

This graphical equation enables us to prove P1-3.

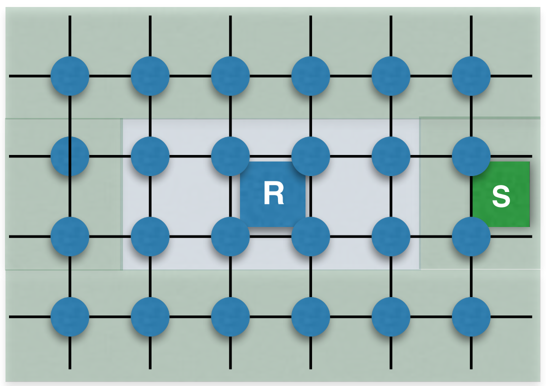
Part II - G-invariant PEPS

P1

Local parent Hamiltonian

$$H = -\sum_p A_p - \sum_v B_v$$

P2

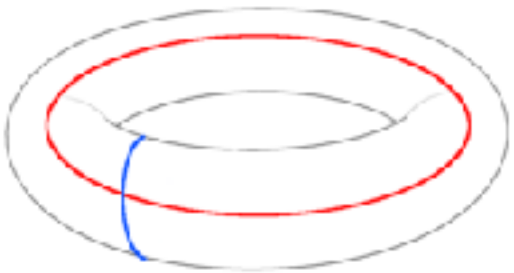


Constant correction to the area law

$$S_0(\mathrm{Tr}_S |\psi\rangle\langle\psi|) = |\partial R| - \gamma$$

$$\gamma = \log 2$$

P3



Four-fold degeneracy

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Part II - G-invariant PEPS

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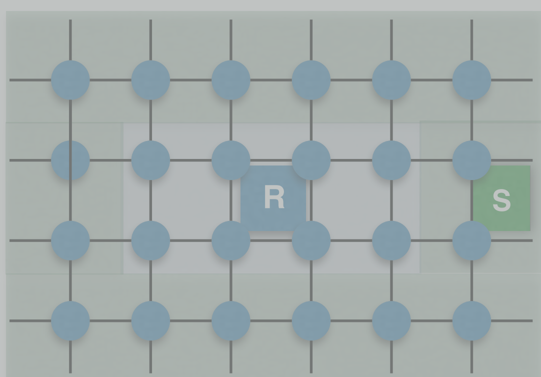

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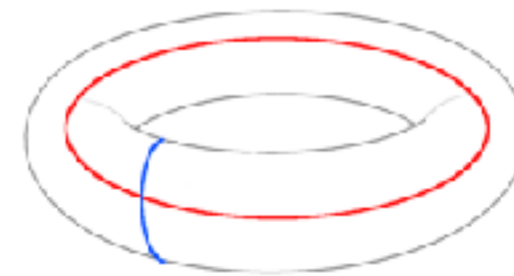

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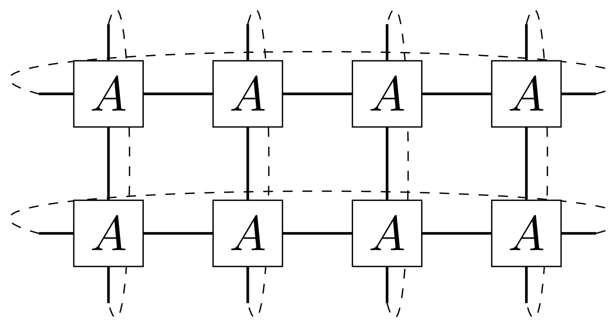
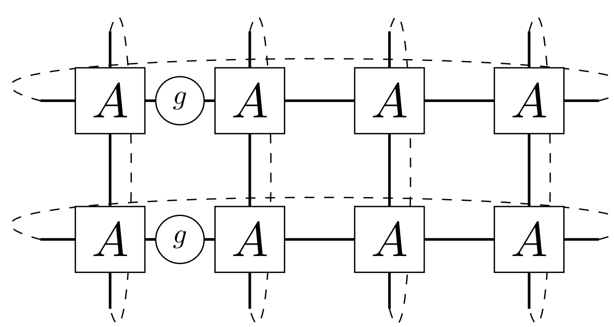
$$H |\psi_{ij}\rangle = E_{\min} |\psi_{ij}\rangle$$

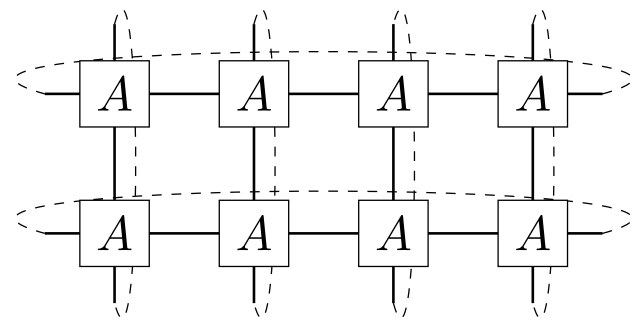
- Let $|\psi_1\rangle :=$ 

  be the ground state of some local Hamiltonian.

- Define $|\psi_2\rangle :=$ 

- Then $|\psi_2\rangle$ is also a ground state.

Part II - G-invariant PEPS | Proof of P3 - Outline

- Let $|\psi_1\rangle :=$ 

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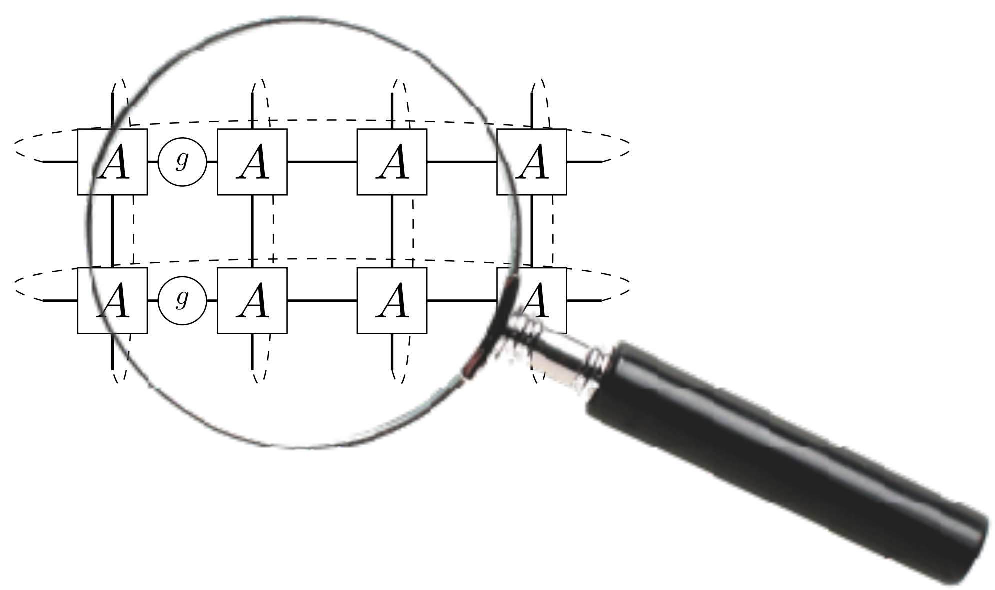
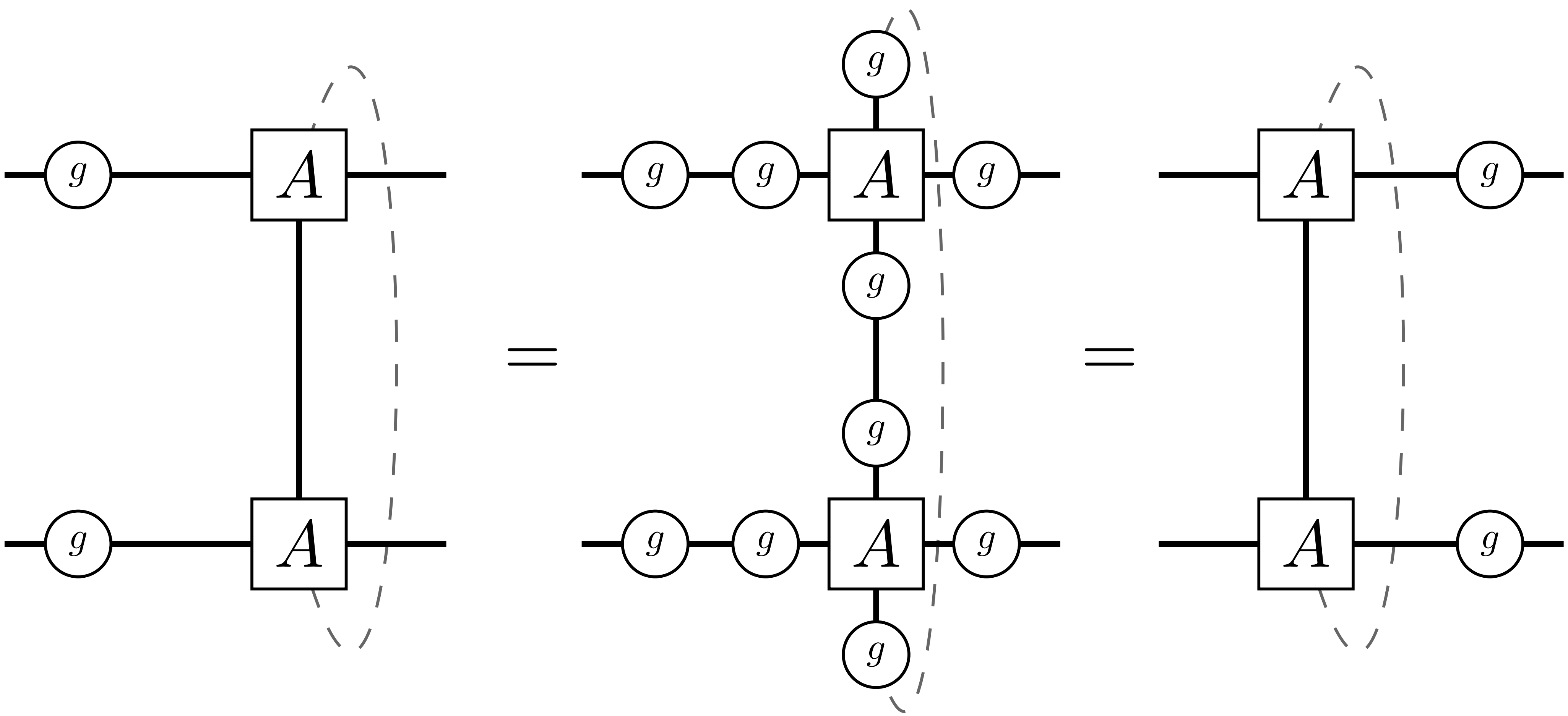
- Define $|\psi_2\rangle :=$ 

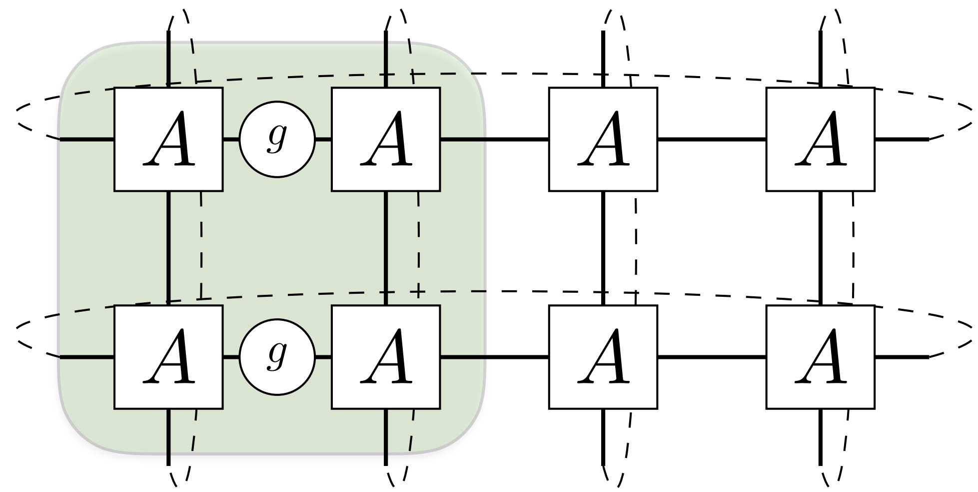
- Then $|\psi_2\rangle$ is also a ground state.



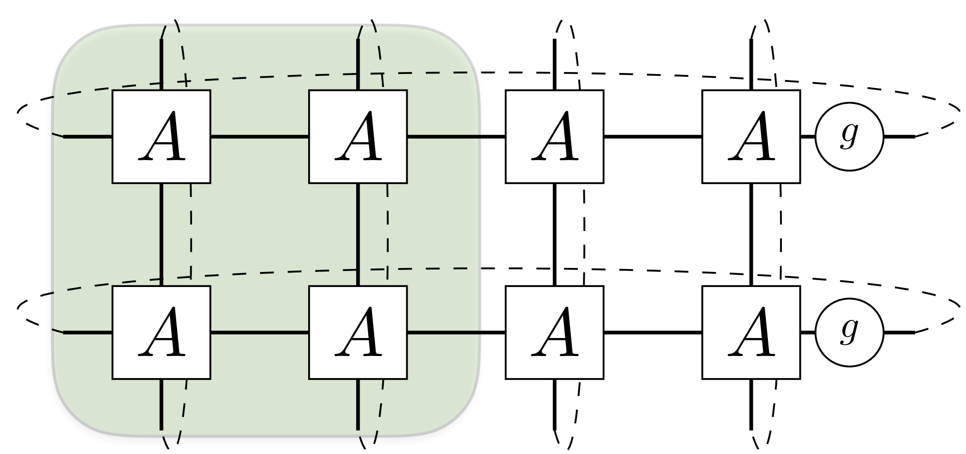
Strings wrapping around the cylinder have
no location.

Part II - G-invariant PEPS | Proof of P3 - Step 2

$$\langle\psi_2|h|\psi_2\rangle =$$



$$=$$



$$= \langle\psi_1|h|\psi_1\rangle$$

In particular, for any local Hamiltonian

$$\langle\psi_1|H|\psi_1\rangle = \langle\psi_2|H|\psi_2\rangle$$

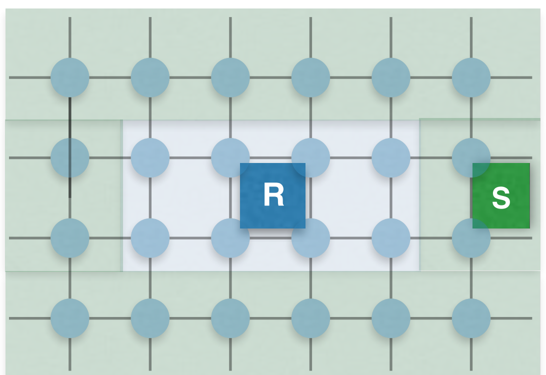
and if $|\psi_1\rangle$ is a ground state, so is $|\psi_2\rangle$.

N. Schuch, J.I. Cirac, D. Perez-Garcia,
Annals of Physics 325, 2153 (2010),
[arXiv:1001.3807]

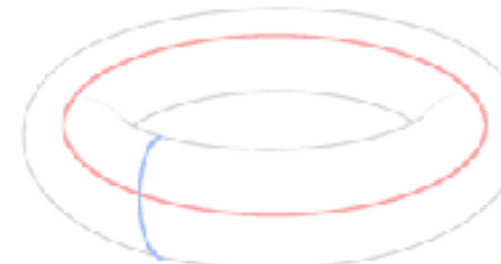
Part II - G-invariant PEPS

There exists a local Hamiltonian H that has $|\psi>$ as a ground state.

$$H = -\sum A_p - \sum B_v$$



Is there a unifying picture that explains all of these properties?



hree extra ground states on the torus.

$|\psi>$ exhibits a constant correction to the area law of its entanglement entropy.

$$|\psi_{ij}\rangle = (Z^{\otimes N_h})^i (Z^{\otimes N_v})^j |\psi\rangle$$

$i, j \in \{0, 1\}$

$$S_0(\mathrm{Tr}_S |\psi\rangle\langle\psi|) = |\partial R| - \gamma$$

$$\langle\psi_{ij}|\psi_{kl}\rangle = \delta_{ik}\delta_{jl}$$

$$\gamma = \log 2$$

$$H|\psi_{ij}\rangle = E_{\min}|\psi_{ij}\rangle$$

Part II - G-invariant PEPS | Perspective

Discrete
Virtual
Symmetry

[meat grinder courtesy of
David T. Stephen]

Quantum
Double Models

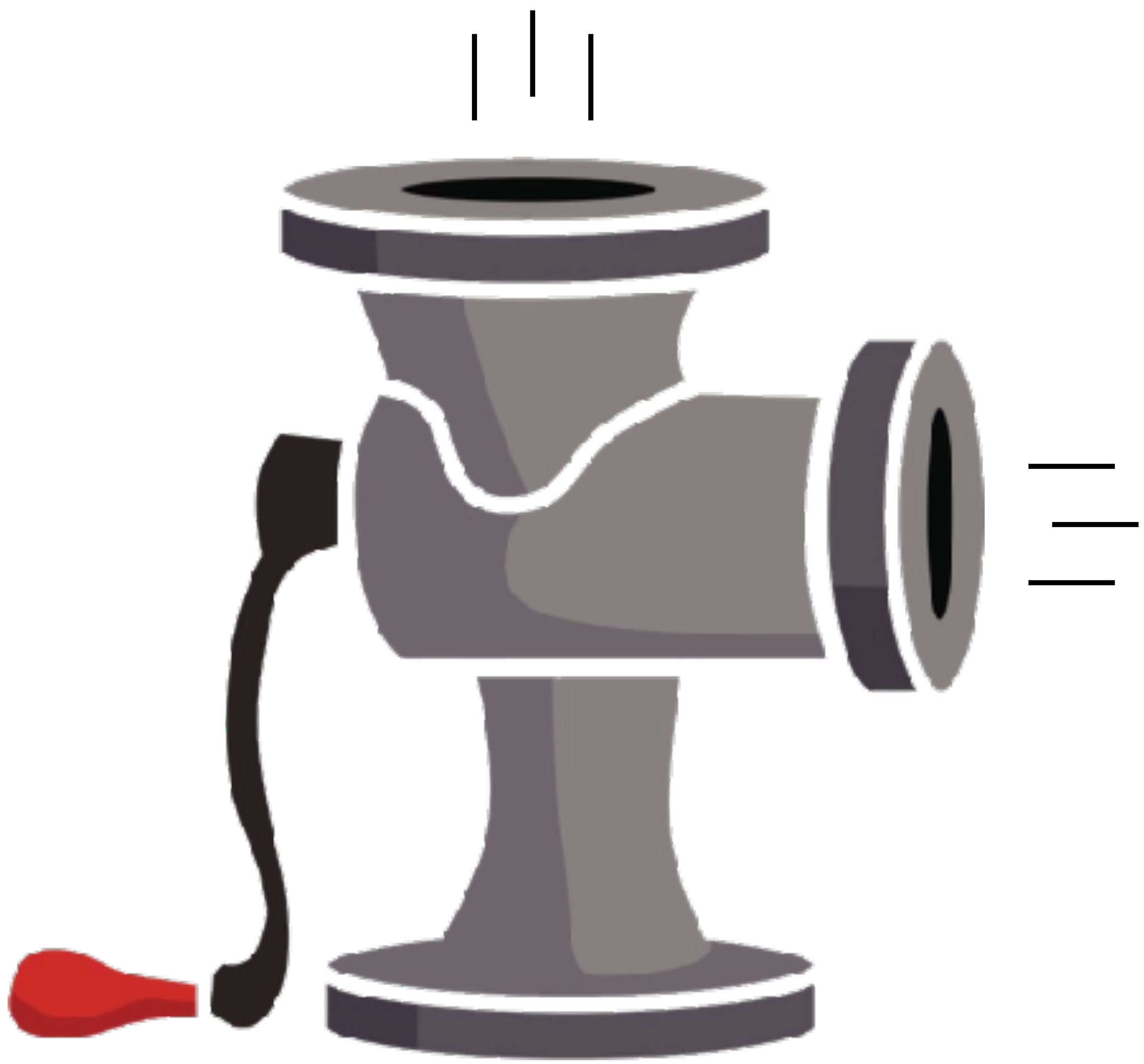
Parent
Hamiltonian

Finite
ground state
degeneracy

Entanglement
Entropy

Part II - G-invariant PEPS | Perspective

Continuous
Virtual
Symmetry



[meat grinder courtesy of
David T. Stephen]

<u>Unknown class
of states?</u>

Hamiltonian?

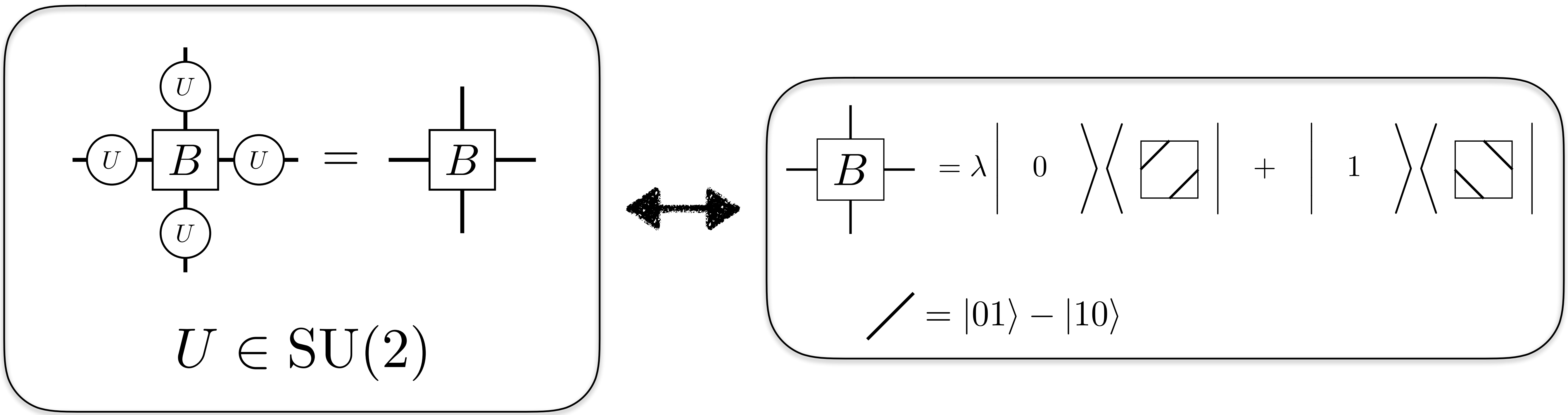
Ground
states?

Entropy?

Part III

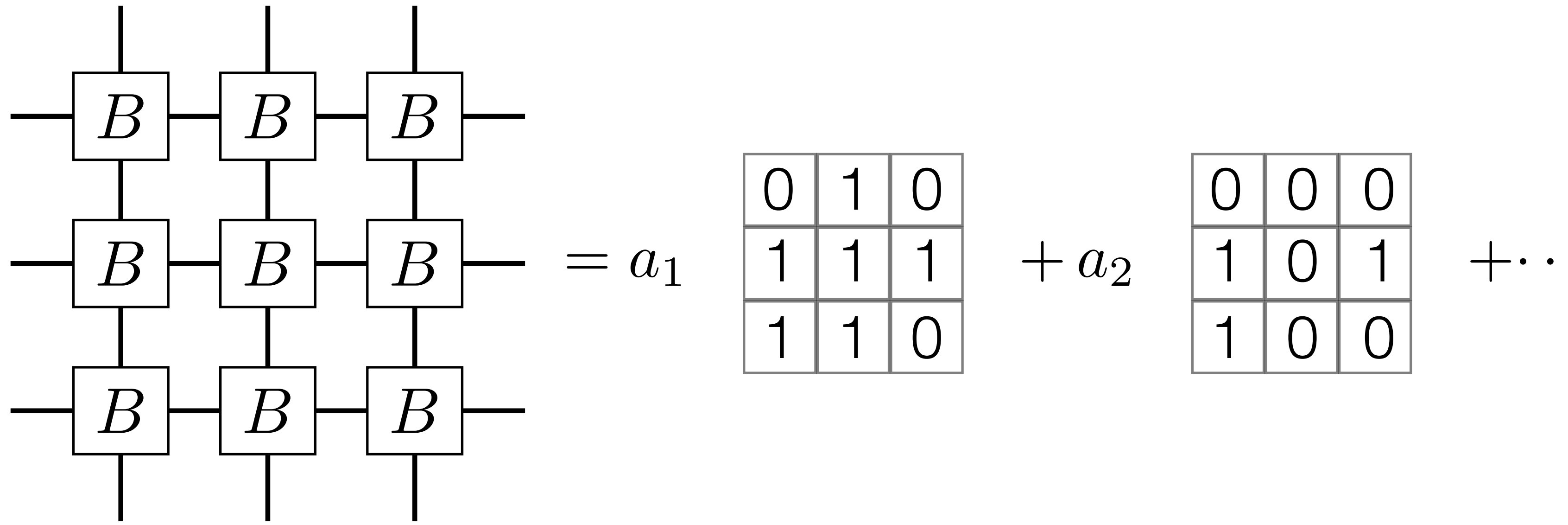
…where I define a class of G-invariant PEPS with G=SU(2) and show four results that demonstrate that these new states behave quite differently from the case of G discrete.

Part III - SU(2)-invariant PEPS | The Wavefunction

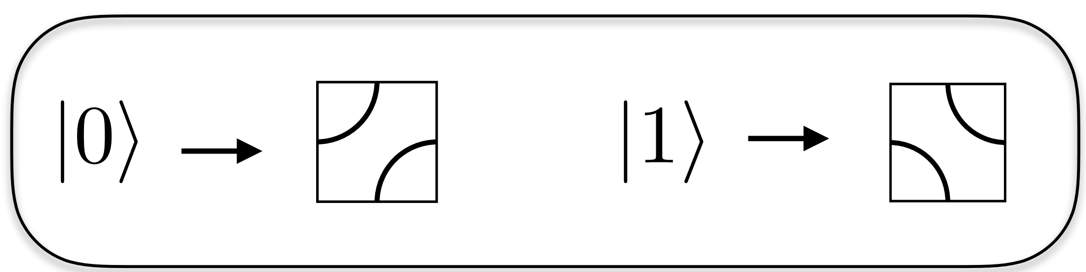


Part III - SU(2)-invariant PEPS | The Wavefunction

- Contracting this tensor network…



- …and identify the physical basis vectors with pictures



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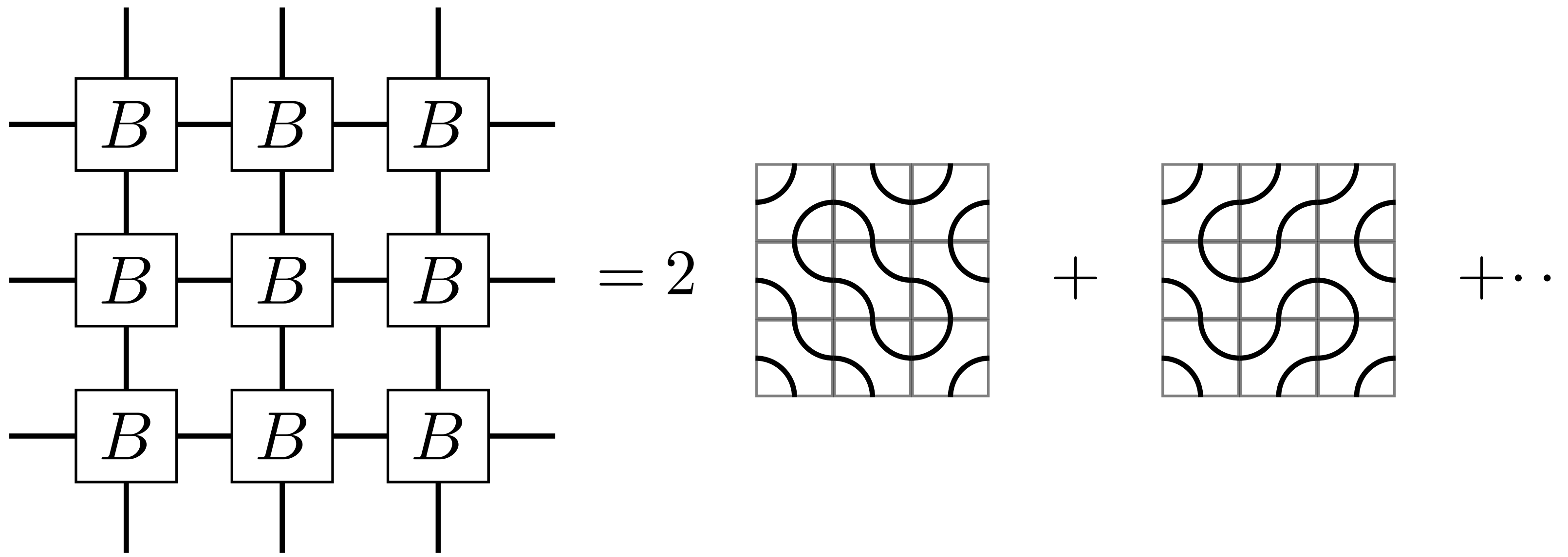
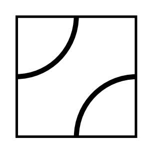
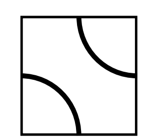


- …and identify the physical basis vectors with pictures

$|0\rangle$ →  $|1\rangle$ → 

- …then the wavefunction becomes a *quantum loop model*:

$$|\psi\rangle = \sum_L D^{n_L} \, |L\rangle$$

n_L = number of closed loops in L

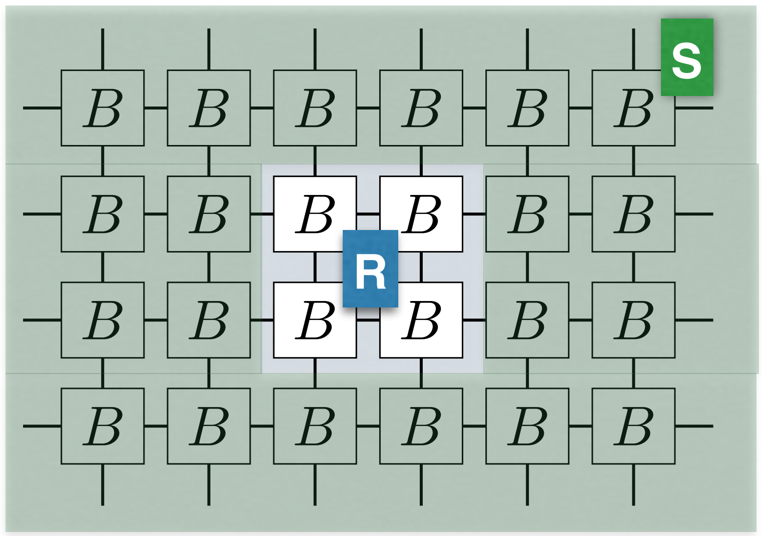
- Plenty of Quantum Loop Model literature for $D = -A^2 - A^{-2} < 2$.
- In our case, $D = 2$ is given by the bond dimension.

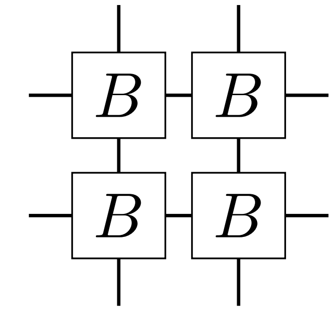
- P. Fendley, arXiv:cond-mat.stat-mech/0711.0014
- M. Troyer, S. Trebst, K. Shtengel and C. Nayak, Phys. Rev. Lett. 101, 230401 (2008)
- P. Fendley, Annals of Physics 323 (2008) 3113
- P. Fendley and J. L. Jacobsen, J.Phys. A41:215001 (2008)
- P. Fendley, S. V. Isakov and M. Troyer, Phys. Rev. Lett. 110, 260408 (2013)

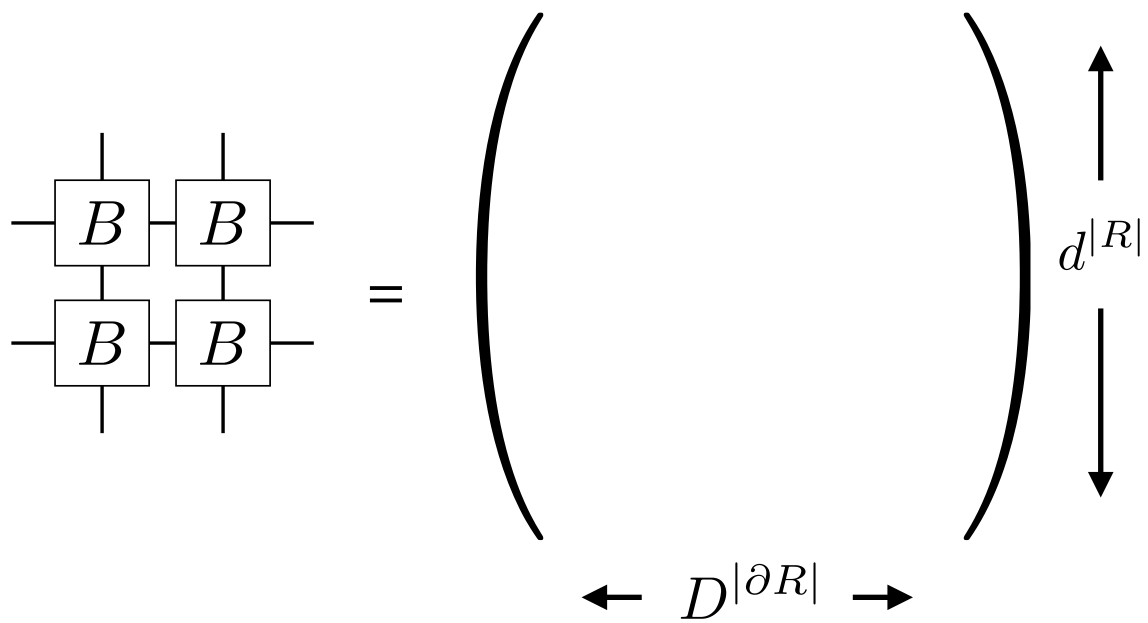
$$|\psi\rangle = \sum_L D^{n_L} \, |L\rangle$$

n_L = number of closed loops in L

Part III - SU(2)-invariant PEPS | Result #1 - Entanglement Entropy



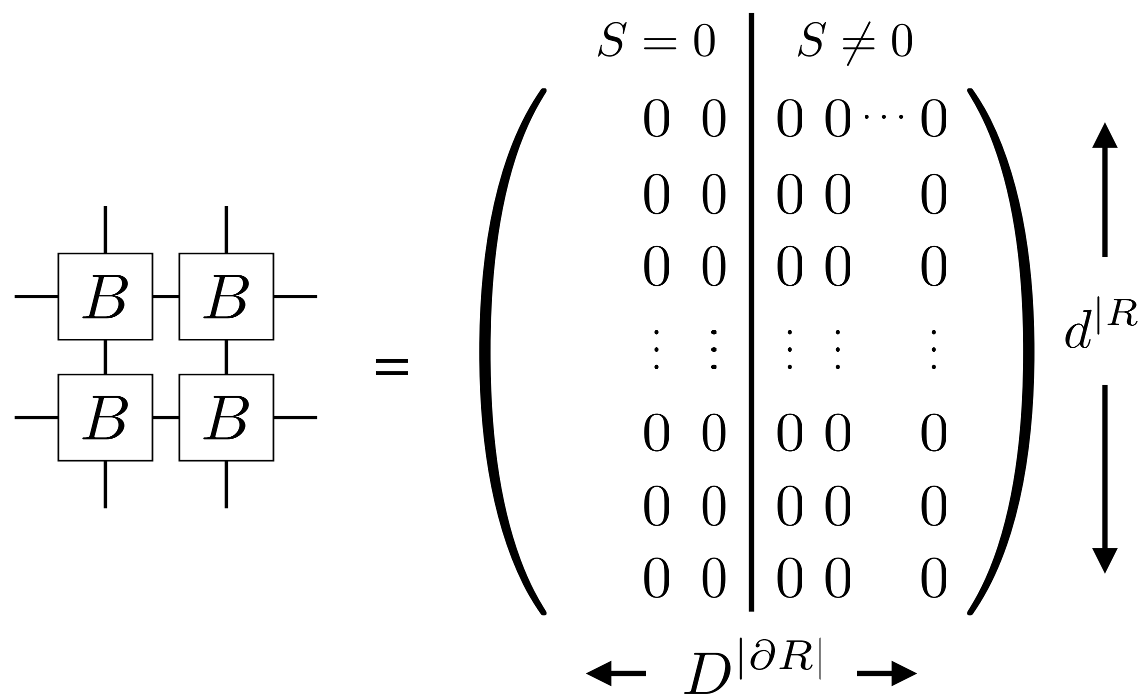
$$S_0(\rho_R) = \log \operatorname{rank}$$ 



$$S_0(\rho_R) = \log D^{|\partial R|} = |\partial R| \log D$$

Part III - SU(2)-invariant PEPS | Result #1 - Entanglement Entropy

- SU(2)-symmetry imposes constraints.
- Rank = *height-restricted Dyck paths* on $|\partial R|$



aspect ratio

$$S_0(\rho_R) = |\partial R| \log 2 - 3/2 \log |\partial R| + \log f(\alpha)$$

area law

correction

- Examples of models with logarithmic correction terms

- E. Ardonne, P. Fendley, and E. Fradkin, Ann. Phys. 310, 493 (2004)
- E. Fradkin and J. E. Moore, Phys.Rev.Lett.97:050404 (2006)

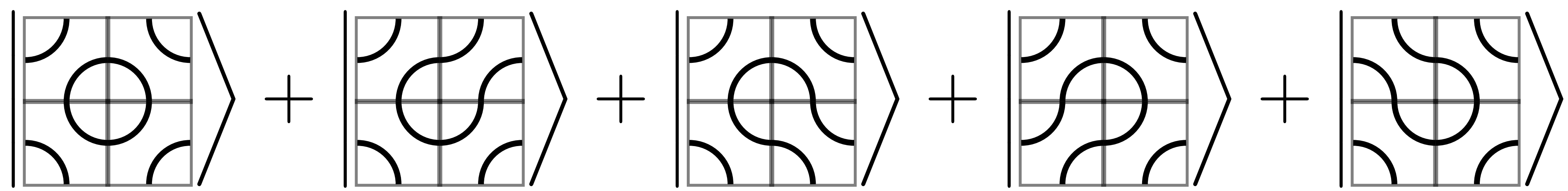
aspect ratio

$$S_0(\rho_R) = |\partial R| \log 2 - 3/2 \log |\partial R| + \log f(\alpha)$$

area law

correction

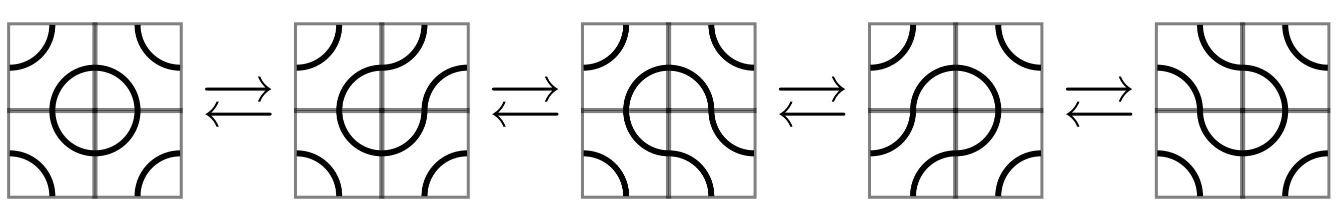
- Define a loop state on a 2 x 2 plaquette

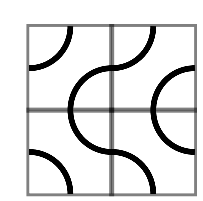
$|\phi\rangle = 2$ 

- Hamiltonian:

$$h = \mathbb{1} - |\phi\rangle\langle\phi| - \Pi_{\substack{\text{no bubbles}\\ \text{or tadpoles}}} \qquad H = \sum h \geq 0$$

- Action of H:

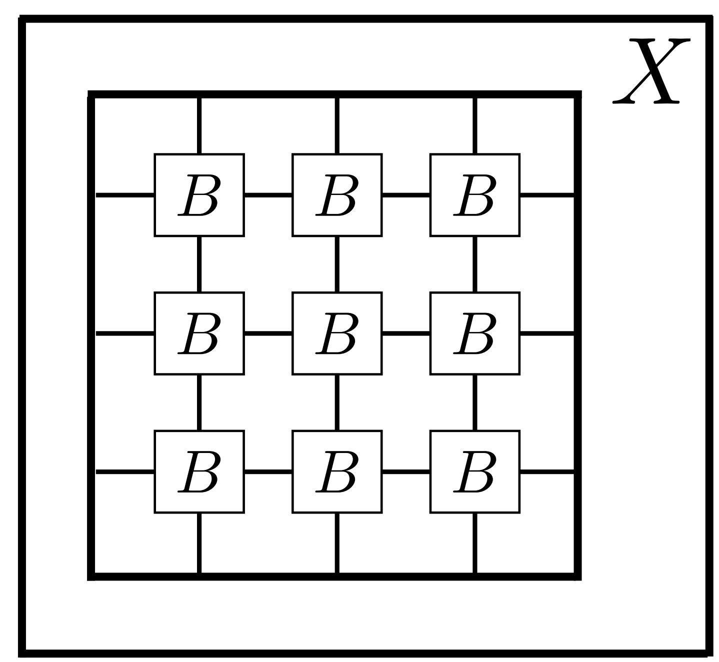
2 

 $\to 0$

On open boundaries, the ground space of H is exactly spanned by the PEPS, i.e., for every state which fulfils

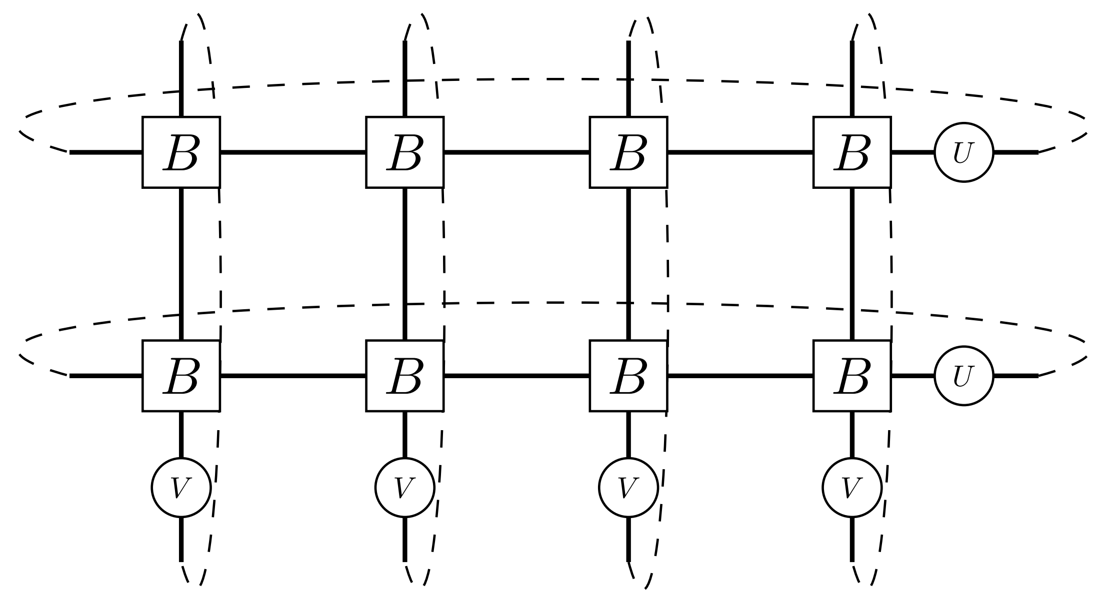
$$H\,|\psi\rangle = 0$$

there exists a boundary vector X, such that

$|\psi\rangle =$ 

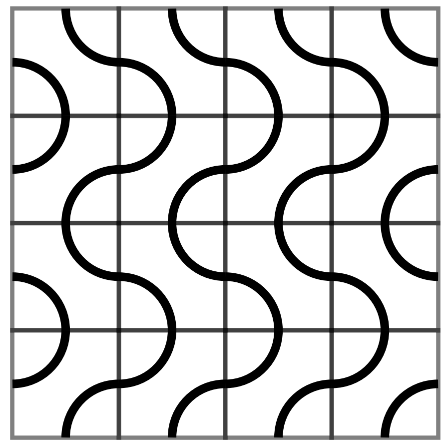
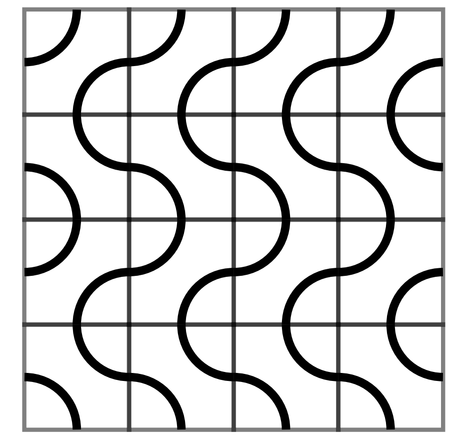
Part III - SU(2)-invariant PEPS | Result #3 - String-inserted states

dim span $\left\{ |\psi\rangle = \right.$



$\left. \middle| \; [U,V] = 0 \right\}$

$\leq (N+1)^2$ (on an N x N torus)

$|\psi_1\rangle =$  and $|\psi_2\rangle =$ 

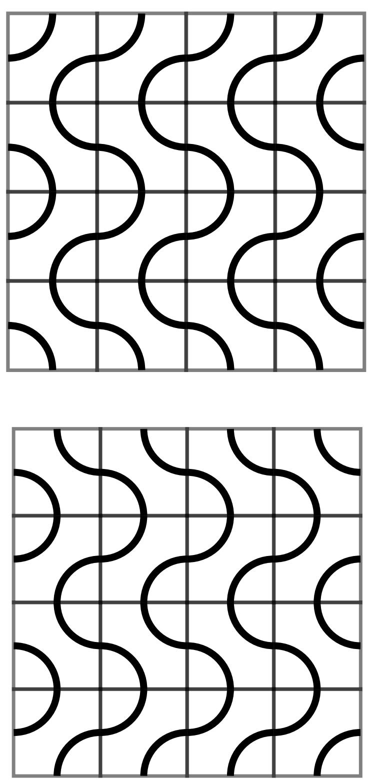
$$H\,|\psi_i\rangle = 0$$

Number of such *isolated states* = $2(2^N - 1)$ on N x N torus.

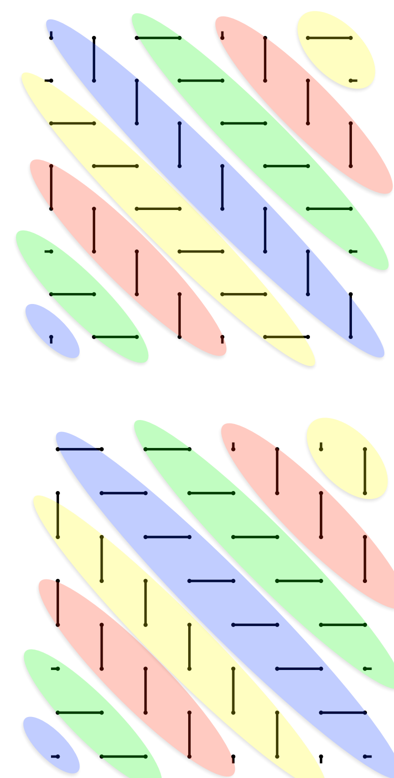
There is a second mechanism that contributes an exponential number of ground states to the degeneracy.

Part III - SU(2)-invariant PEPS | Result #4 - Isolated states

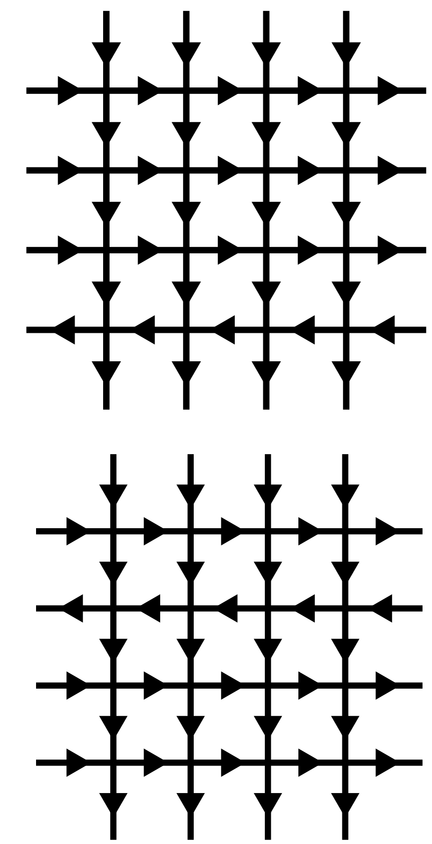
SU(2) PEPS



Quantum Dimer Model



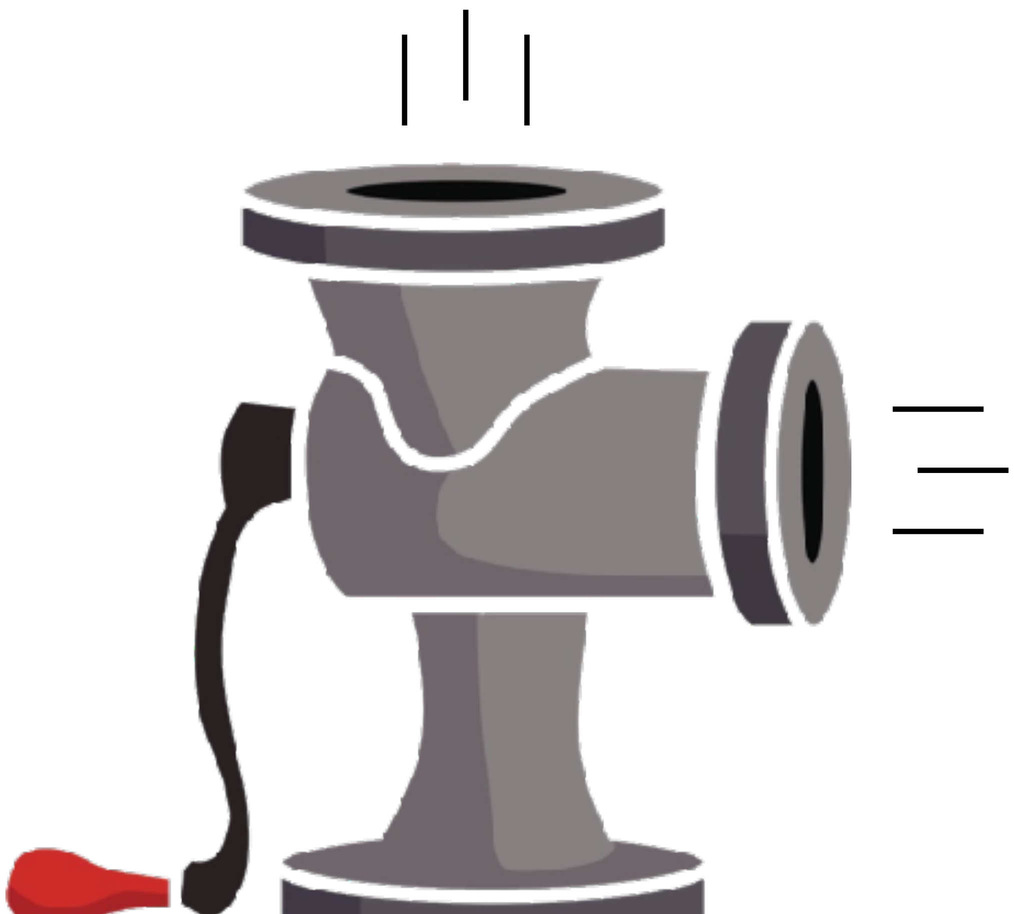
Quantum 6-Vertex Model

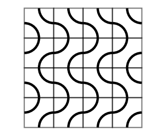


- D. S. Rokhsar and S. A. Kivelson, Phys. Rev. Lett. 61, 2376 (1988)
- R. Moessner and K. S. Raman arXiv:cond-mat.str-el/0809.3051

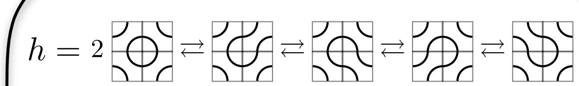
Part III - Conclusion

Virtual
SU(2)
Symmetry

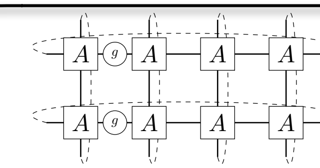




Exponential number of isolated ground states



Existence of a local *parent* Hamiltonian



Polynomial number of string-inserted ground states

$$|\partial R| - \log|\partial R|$$

Logarithmic correction to the area law

Part III - Outlook

- Proof without explicit representation?
- Relationships with quantum loop, dimer and six-vertex models & conformal critical points?