Composite SPT Order and Effective Models

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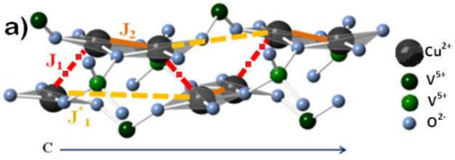
Storyline:

- Effective Models
- Main Result (informal)
- Technicalities (SPT, algebra)
- Result (more formal)

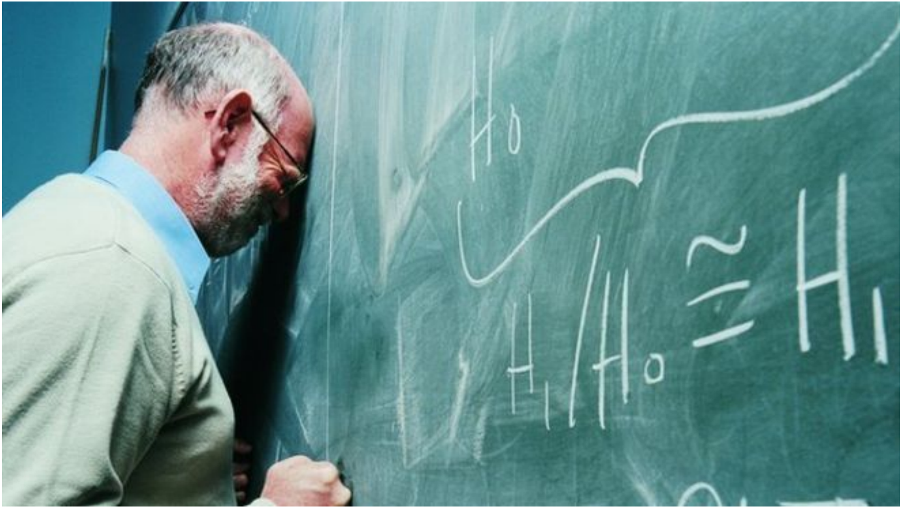
Effective Models

Effective Models



E.S. Klyushina: Phys. Rev. B 93, 241109 (2016)

Effective Models



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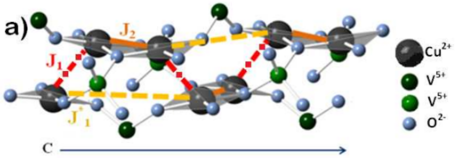
Effective Models



$$H = \sum_i J_{\text{intra}} \mathbf{S}_{i,1} \cdot \mathbf{S}_{i,2} + J_{\text{inter}} \mathbf{S}_{i,2} \cdot \mathbf{S}_{i+1,1}. \quad (1)$$

E.S. Klyushina: Phys. Rev. B 93, 241109 (2016)

Effective Models

- Cond. Mat.: connections to experiment
- Stat. Phys.: ladders interpolating between low and high spin physics
- Science. in general: toy models
- Phys. in general as search for effective models / descriptions

$\rightarrow$ How to estimate the validity?

Main Result (informal)

Main Result (The Paradigm)

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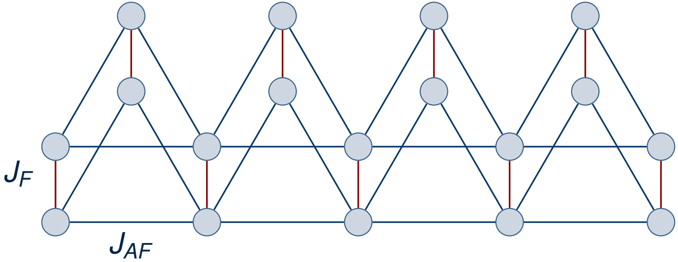
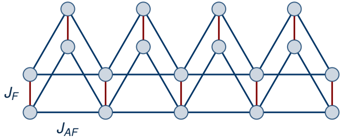
- When 'zooming out' sufficiently, an effective model should be indistinguishable from the original model.
- Minimal requirement: if one model S_1 effectively describes another model S_2 both are required to be in the same phase '$[S_1] = [S_2]$'.

Main Result (Concrete Example)

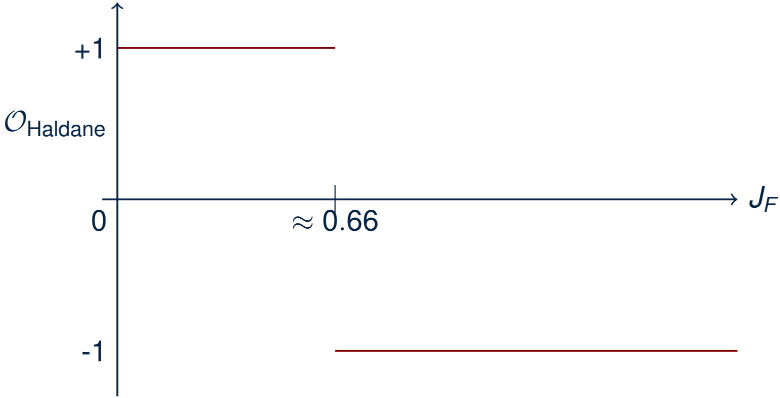


$$H(J_{AF}, J_F) = J_{AF} \overbrace{\sum_{<ij>_{AF}} s_i \cdot s_j}^{\equiv H_{AF}} - J_F \overbrace{\sum_{<ij>_F} s_i \cdot s_j}^{\equiv -H_F}$$

Main Result (Concrete Example)



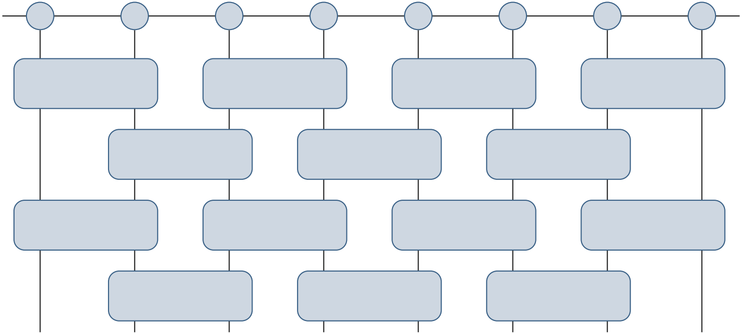
$$H(J_F) = H_{AF} + J_F \cdot H_F$$



A.N.: Phys. Rev. B 96, 235138 (2017)

Technicalities

Local Unitary Circuits



SPT Order

- Quantum Phases $\sim$ equivalence relations $[\psi]_{LUC}$:

  $$|\psi\rangle \sim |\phi\rangle \Leftrightarrow \exists \text{ constant depth LUC } U : U|\phi\rangle = |\psi\rangle$$

- With symmetries: Quantum Phases $\sim$ equivalence relations $[(|\psi\rangle, R)]_{LUC}$ of pairs of states and symmetries (group representations R):

  $$(|\psi\rangle, R_1) \sim (|\phi\rangle, R_2) \Leftrightarrow \exists \text{ constant depth LUC } U : U|\phi\rangle = |\psi\rangle \text{ and } UR_2U^\dagger = R_1$$

SPT Order

- Quantum Phases $\sim$ equivalence relations $[\psi]_{LUC}$:

  $|\psi\rangle \sim |\phi\rangle \Leftrightarrow \exists$ constant depth LUC $U : U|\phi\rangle = |\psi\rangle$

- Quantum Phases $\rightarrow$ Topological Order
- Symmetric Quantum Phases $\rightarrow$ Symmetry Protected Topological (SPT) Order

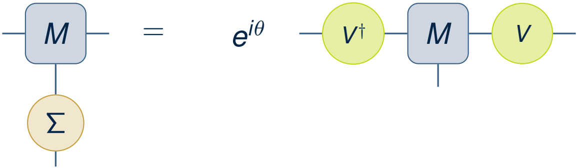
In d dimensions:

$$\text{SPT Order wrt. group } G \Leftrightarrow H^{d+1}(G, U(1))$$

N. Schuch: Phys. Rev. B 84, 165139 (2011)
X. Chen: Phys. Rev. B 84, 235128 (2011)
F. Pollmann: Phys. Rev. B 81, 064439 (2010)

Cohomology and Phases in 1d

Given a uMPS $|\psi(M)\rangle$ symmetric wrt. Σ, then there exists a phase θ and a unitary V such that



Garcia et al: Phys. Rev. Lett. 100, 167202 (2008)

Cohomology and Phases in 1d

Given a uMPS $|\psi(M)\rangle$ symmetric wrt. Σ, then there exists a phase θ and a unitary V such that

Directly obtain $\omega \in H^2(G, U(1))$ via

$$\omega(g,h)\mathbb{1} = V_g V_h V_g^\dagger V_h^\dagger$$

Pollmann Phys. Rev. B. 81, 064439 (2010)

Garcia et al: Phys. Rev. Lett. 100, 167202 (2008)

$SU(2)$ and $SO(3)$

Standard definitions and faithful representations on $\mathbb{C}^n$:

$$\begin{aligned} SO(3) &\equiv \{g \in \mathbb{R}^{3\times 3} : g^T g = \mathbb{1}\} \\ g_1(\mathbf{v}) &= \exp(i\mathbf{v} \cdot \mathbf{s}_1) \end{aligned}$$

$$\begin{aligned} SU(2) &\equiv \{g \in \mathbb{C}^{2\times 2} : g^\dagger g = \mathbb{1}\} \\ g_{\frac{1}{2}}(\mathbf{v}) &= \exp(i\mathbf{v} \cdot \mathbf{s}_{\frac{1}{2}}) \end{aligned}$$

with $\mathbf{v} = \theta\mathbf{n}$ for an angle θ and a unit vector $\mathbf{n}$

$SU(2)$ and $SO(3)$

Group algebras, and why we should care:

$$\mathcal{K}(SU(2)) \simeq M(1,1) \oplus M(2,2) \oplus M(3,3) \oplus \dots$$

$$\mathcal{K}(SO(3)) \simeq M(1,1) \oplus M(3,3) \oplus M(5,5) \oplus \dots$$

Moreover it is equivalent:

- V is a faithful group representation of G
- for any irreducible representation W of G there exists an n such that $\underbrace{V \otimes \cdots \otimes V}_{n \text{ times}} \simeq W \oplus \dots$.

$SU(2)$ and $SO(3)$

Local (on site) description:

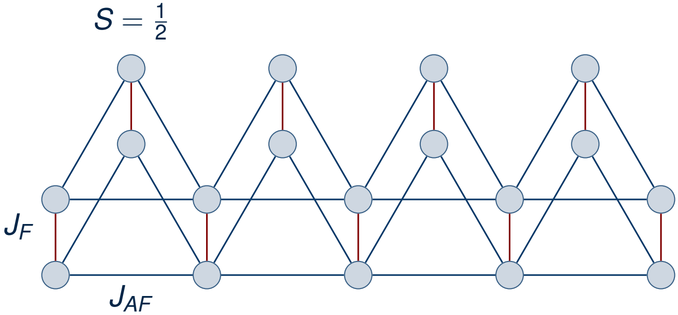
$\mathcal{G} \equiv \{g \otimes g : g \in SU(2)\}$
$\simeq SO(3)$

$\mathbb{Z}_2 \lhd SU(2)$ trivial
under $g \mapsto g \otimes g$

$SU(2) \simeq$ Double-Cover($SO(3)$)

Results

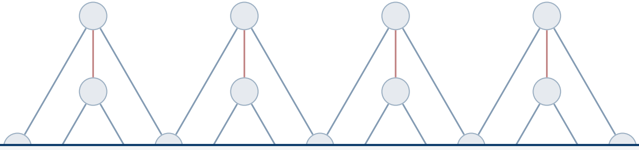
The Model



$$H(J_{AF}, J_F) = J_{AF} \overbrace{\sum_{<ij>_{AF}} s_i \cdot s_j}^{\equiv H_{AF}} - J_F \overbrace{\sum_{<ij>_F} s_i \cdot s_j}^{\equiv -H_F}$$

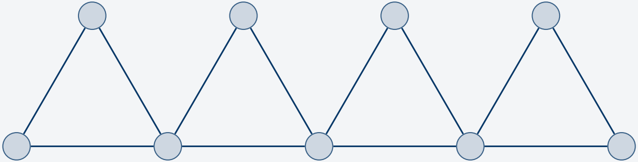
The Model

$S = \frac{1}{2}$



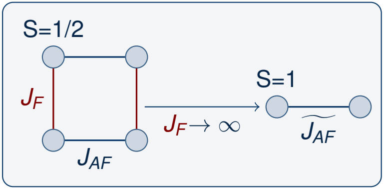
Effective description:

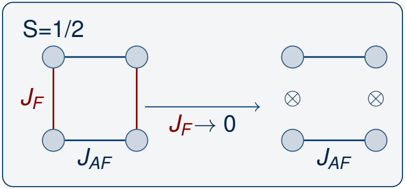
$S = 1$



$$H(J_{AF}, J_F) = J_{AF} \sum_{<ij>_{AF}} s_i \cdot s_j - J_F \sum_{<ij>_F} s_i \cdot s_j$$

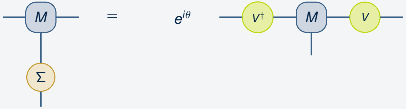
Model vs. Effective Model





T. Giamarchi: Quantum physics in one dimension (2003)

Idea

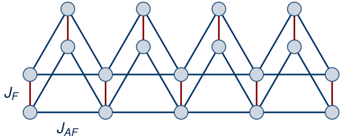


$\mathcal{G} \simeq SO(3)$

Combine!

Ladder order parameter measuring validity of the effective $S = 1$ model!

Result



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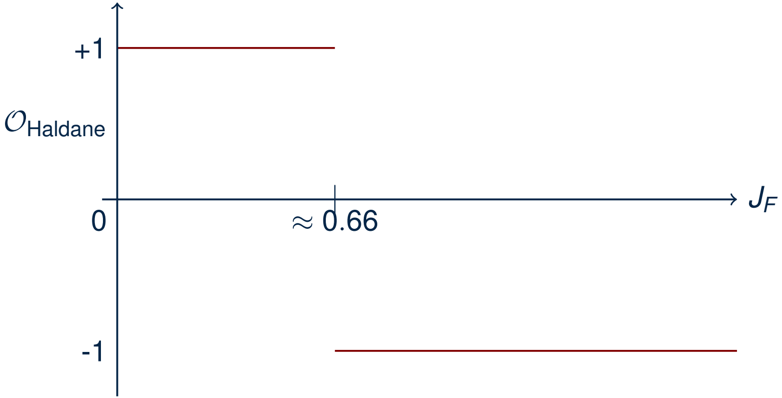
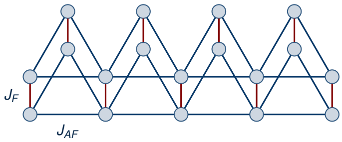


A.N.: Phys. Rev. B 96, 235138 (2017)

Thank you for your attention!

Small J_F limit



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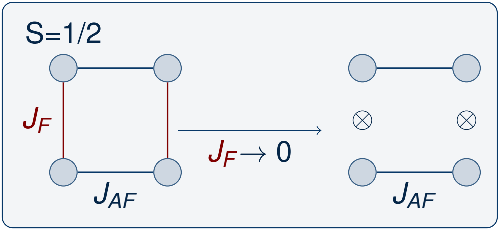

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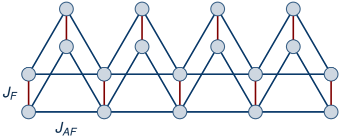

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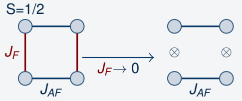
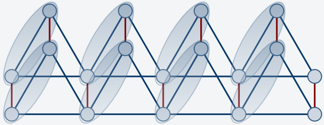
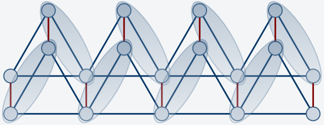


Ground State Space





Small J_F limit

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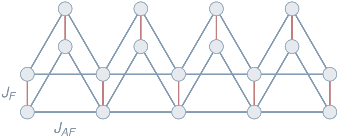

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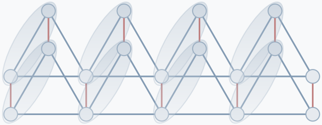
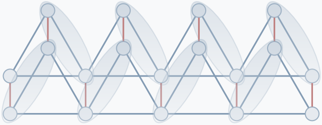
S=1/2

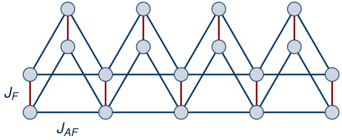
⇒ Perturbation theory favors the parallel configurations !

Ground State Space





Large J_F limit



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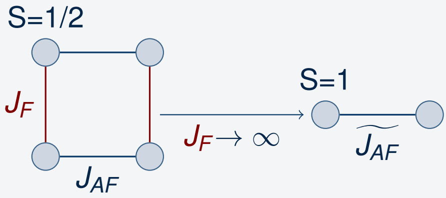

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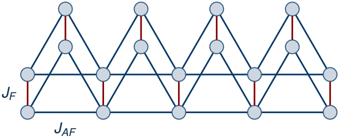

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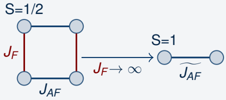

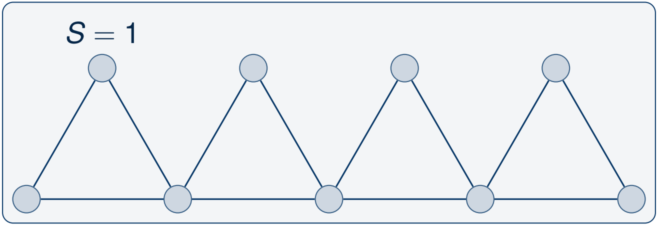

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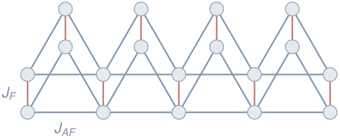
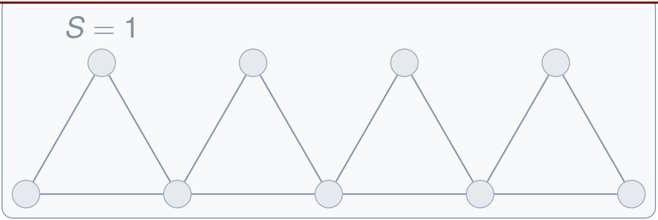


$$H(J_F) = H_{AF} + J_F \cdot H_F$$

⇒ Is numerically found to be in a SPT **Haldane** phase. In particular, we find it to be in the SPT non-trivial phase wrt. $SO(3)$.



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