Signatures of the Many-body Localized Regime in Two Dimensions

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Benasque, 28 February 2018

European Commission

Marie Skłodowska-Curie Actions

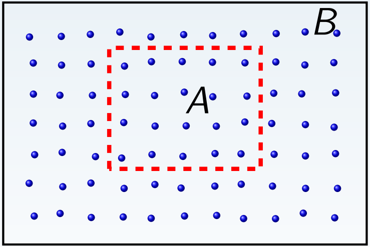
T. B. Wahl, A. Pal, and S. H. Simon, arXiv:1711.02678

Thermalization in classical systems

$\longrightarrow$

Ergodicity in quantum systems

$$|\psi(t)\rangle = e^{iHt}|\psi(0)\rangle$$
$$H = H_A + H_B + H_{AB}$$



$$\rho_A \propto e^{-\beta H_A}$$

Eigenstate Thermalization Hypothesis (ETH)

J. M. Deutsch, Phys. Rev. A. **43**, 2046 (1991)
M. Srednicki, Phys. Rev. E **50**, 888 (1994)

Many-body localization in one dimension

Sufficiently strong disorder in 1D $\Rightarrow$ ergodicity breaking:

Many-body localization (MBL)

D. Basko, I. Aleiner, and B. Altshuler, Ann. Phys. **321**, 1126 (2006).

I. Gornyi, A. Mirlin, and D. Polyakov, Phys. Rev. Lett. **95**, 206603 (2005).

Rigorous proof: J. Z. Imbrie, J. Stat. Phys. **163**, 998 (2016)

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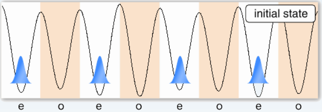
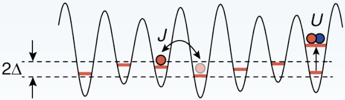
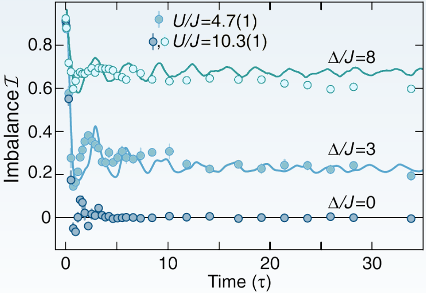






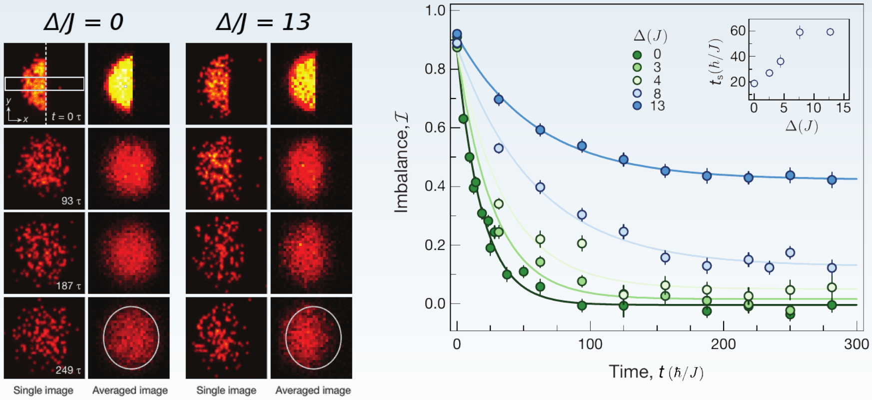
taken from: M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Science **349**, 842 (2015)

Many-body localization in higher dimensions?

Theralizing behavior in higher dimensions: $\rho_A \xrightarrow[t\to\infty]{} e^{-\beta H_A}$

W. De Roeck, J. Z. Imbrie, Phil. Trans. R. Soc. A 375, 20160422 (2017).

But:



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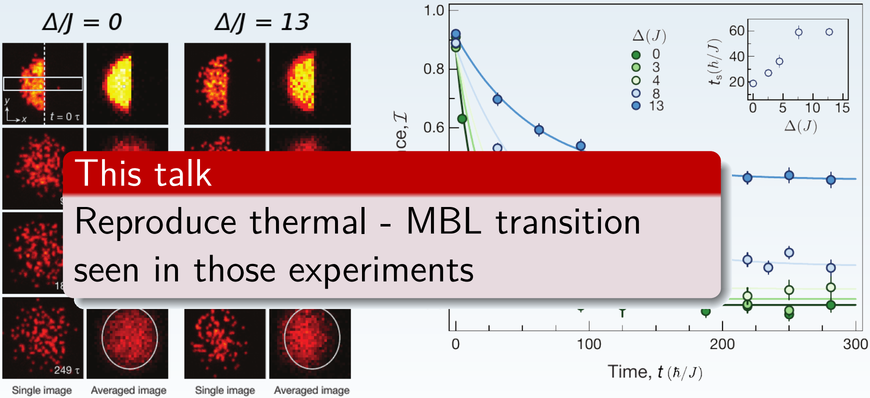

This talk

Reproduce thermal - MBL transition seen in those experiments

taken from: J.-y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, Science **352**, 1547 (2016).

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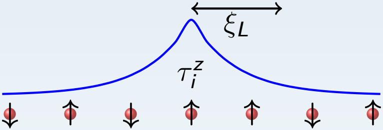
Many-body localization (MBL)

Disordered Heisenberg antiferromagnet: MBL for $W > W_c \approx 3.5$

$$H = \sum_{i=1}^{N} (J\,\mathbf{S}_i \cdot \mathbf{S}_{i+1} + h_i S_i^z), \qquad h_i \in [-W, W]$$



Local integrals of motion (LIOM):

$$\tau_i^z = U \sigma_i^z U^\dagger$$

$$[H, \tau_i^z] = [\tau_i^z, \tau_j^z] = 0$$

M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. **110**, 260601 (2013)

D. A. Huse, and V. Oganesyan, Phys. Rev. B **90**, 174202 (2014)

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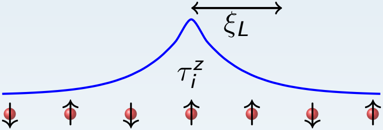

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$$H|\psi_{i_1 \dots i_N}\rangle = E_{i_1 \dots i_N} |\psi_{i_1 \dots i_N}\rangle$$

$$\tau_1^z |\psi_{\uparrow i_2 \dots i_N}\rangle = |\psi_{\uparrow i_2 \dots i_N}\rangle$$

$$\tau_1^z |\psi_{\downarrow i_2 \dots i_N}\rangle = -|\psi_{\downarrow i_2 \dots i_N}\rangle \quad \text{etc.}$$

M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. **110**, 260601 (2013)

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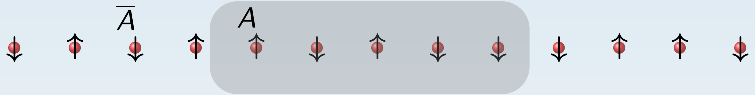
Area-law entangled eigenstates

$|\psi_{i_1 \ldots i_N}\rangle$:



$$\rho_A = \mathrm{tr}_{\overline{A}}(|\psi_{i_1 \ldots i_N}\rangle\langle\psi_{i_1 \ldots i_N}|), \quad \text{entanglement entropy } S(\rho_A) \leq \text{const.}$$

M. Friesdorf, A. H. Werner, W. Brown, V. B. Scholz, and J. Eisert, Phys. Rev. Lett. **114**, 170505 (2015).

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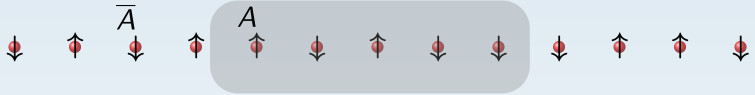

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M. Friesdorf, A. H. Werner, W. Brown, V. B. Scholz, and J. Eisert, Phys. Rev. Lett. **114**, 170505 (2015).

Approximation by Tensor Network States

- DMRG-X

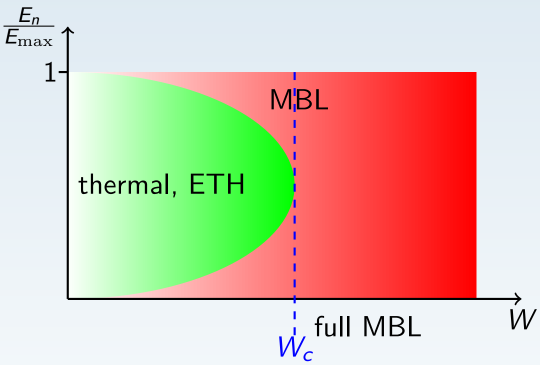
  V. Khemani, F. Pollmann, and S. L. Sondhi, Phys. Rev. Lett. **116**, 247204 (2016)

- spectral tensor networks

  F. Pollmann, V. Khemani, J. I. Cirac, and S. L. Sondhi, Phys. Rev. B **94**, 041116(R) (2016)

  T. B. Wahl, A. Pal, and S. H. Simon, Phys. Rev. X **7**, 021018 (2017)

Mobility edge



D. J. Luitz, N. Laflorencie, and F. Alet, Phys. Rev. B **91**, 081103 (2015)

However:

W. De Roeck, F. Huveneers, M. Müller, and M. Schiulaz, Phys. Rev. B **93**, 014203 (2016)

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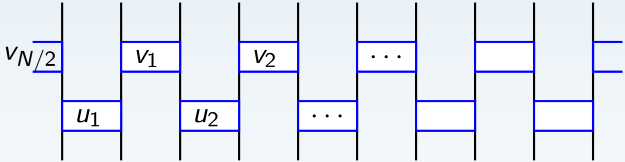
Spectral Tensor Networks

Goal

$\tilde{U} H \tilde{U}^\dagger \approx$ diagonal matrix

$\tilde{U} =$



Spectral Tensor Networks

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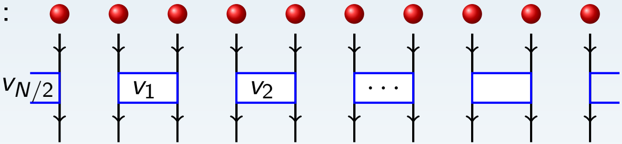
$\tilde{U} H \tilde{U}^\dagger \approx$ diagonal matrix

$|\psi_{i_1,i_2,\ldots,i_N}\rangle$:



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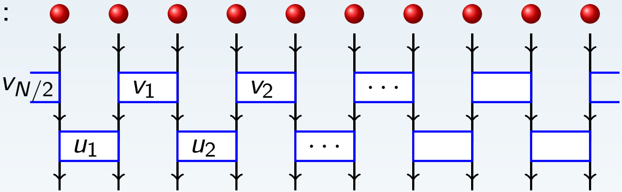
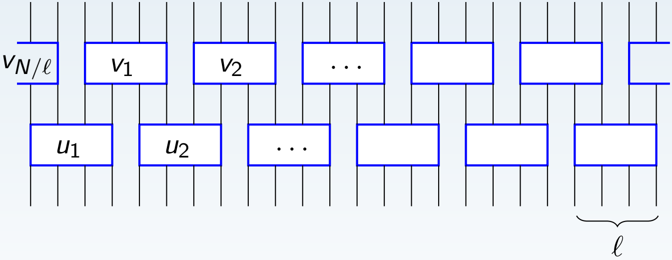


Spectral Tensor Networks

Goal

$\tilde{U} H \tilde{U}^\dagger \approx$ diagonal matrix

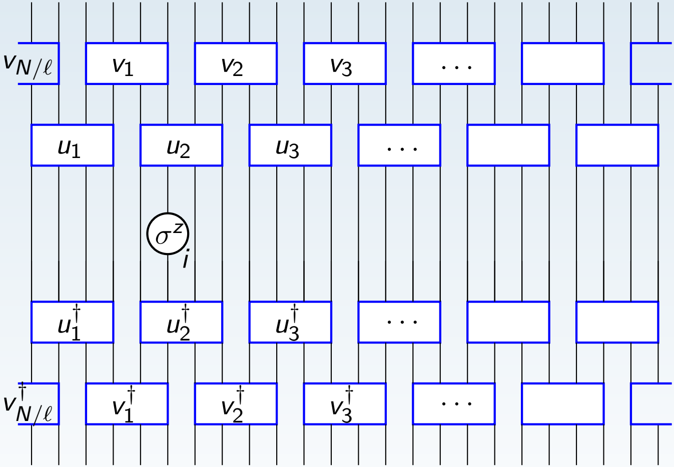
$\tilde{U} =$



$$\tilde{\tau}_i^z = \tilde{U} \sigma_i^z \tilde{U}^\dagger \quad \Rightarrow \quad [\tilde{\tau}_i^z, \tilde{\tau}_j^z] = 0$$

Approximate local integrals of motion

$$\tilde{\tau}_i^z = \tilde{U}\sigma_i^z\tilde{U}^\dagger =$$



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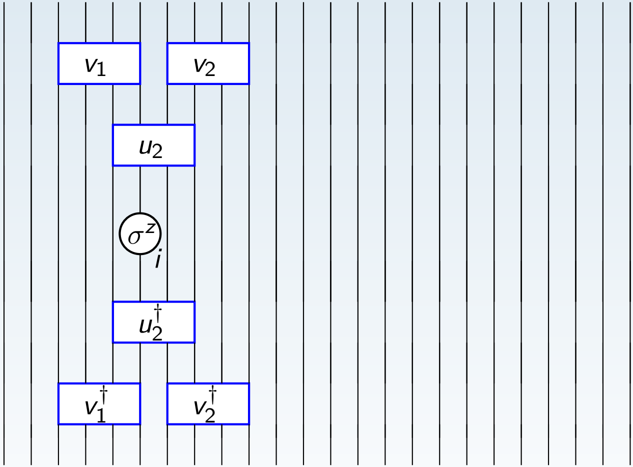


$$\tilde{U}^\dagger H \tilde{U} = \text{diagonal} + \mathcal{O}(e^{-\frac{\ell}{\xi_L}})$$

Figure of merit

Minimize $[H, \tilde{\tau}_i^z]$:

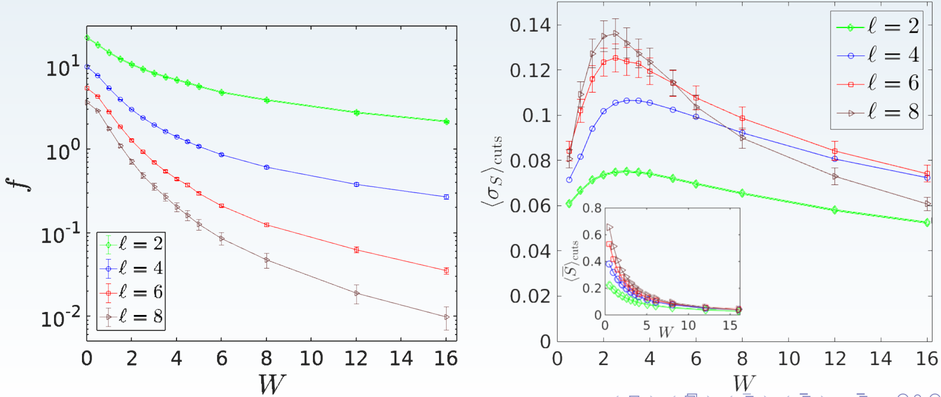
$$f = \frac{1}{2^N} \sum_{i=1}^{N} \mathrm{tr}\left([H, \tilde{\tau}_i^z] [H, \tilde{\tau}_i^z]^\dagger \right) = \sum_i f_i$$

Figure of merit

Minimize $[H, \tilde{\tau}_i^z]$:

$$f = \frac{1}{2^N} \sum_{i=1}^{N} \mathrm{tr}\left([H, \tilde{\tau}_i^z] \, [H, \tilde{\tau}_i^z]^\dagger \right) = \sum_i f_i$$

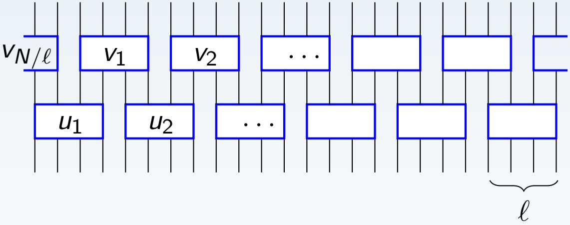
Heisenberg model, $N = 72$:



Summary (one dimension)

Full MBL regime

- local integrals of motion: τ_i^z
- all eigenstates fulfill the area law $\rightarrow$ spectral tensor networks: error $\propto exp\left(-\frac{\ell}{\xi_L}\right)$

$\tilde{U} =$ 

Numerical analysis

- minimize $[H, \tilde{\tau}_i^z]$
- phase transition: maximum of σ_S

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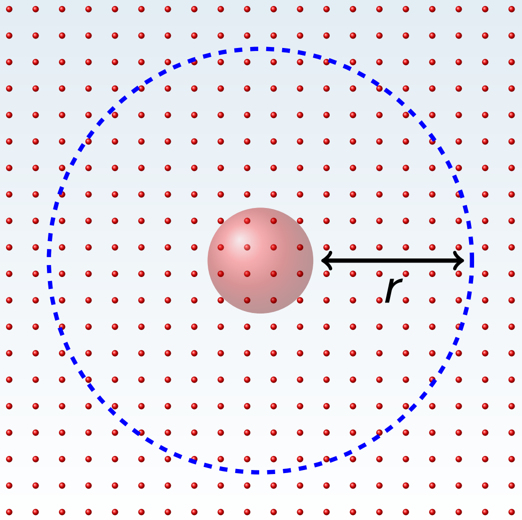
Delocalization in two dimensions

For any set of local τ_i^z: $[H, \tau_i^z] \neq 0$ for some i

W. De Roeck, and J. Z. Imbrie, Phil. Trans. R. Soc. A **375**, 20160422 (2017)



$$\frac{\text{interactions}}{\text{level spacing}} \sim \frac{\gamma^r}{2^{-r^d}} \to 0$$

only for $d = 1$

See however:

K. Agarwal, E. Altman, E. Demler, S. Gopalakrishnan, D. A. Huse, and M. Knap, Ann. d. Phys. 2017, 1600326

Delocalization in two dimensions

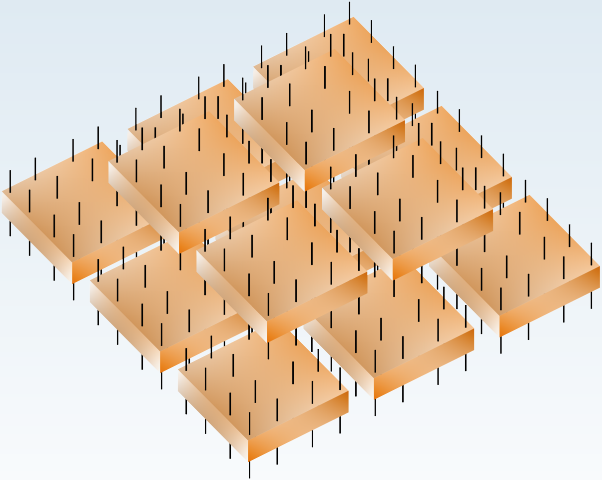
For any set of local τ_i^z: $[H, \tau_i^z] \neq 0$ for some i

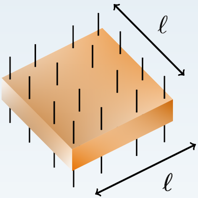
However: $\|[H, \tau_i^z]\|_{\mathrm{op}} \lll 1$

Relaxation time: $\tau \geq \frac{1}{\max_i \|[H, \tau_i^z]\|_{\mathrm{op}}} \ggg 1$

A. Chandran, A. Pal, C.R. Laumann, and A. Scardicchio, Phys. Rev. B **94**, 144203 (2016)

2D quantum circuits



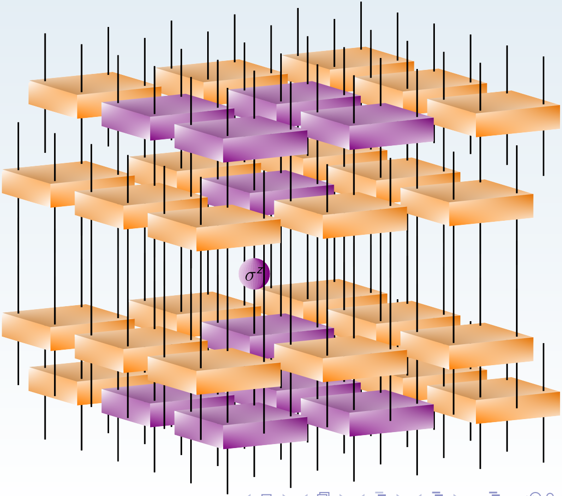


$N \times N$ lattice:

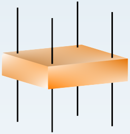
$$f = \frac{1}{2^{N^2}} \sum_{i=1}^{N^2} \mathrm{tr}\left([H, \tilde{\tau}_i^z] \, [H, \tilde{\tau}_i^z]^\dagger \right) = \sum_i f_i$$

Approximate local integrals of motion in 2D

$$\tilde{\tau}_i^z = \tilde{U}\sigma_i^z\tilde{U}^\dagger =$$



$u_{x,y,z} =$



$\sigma^z =$

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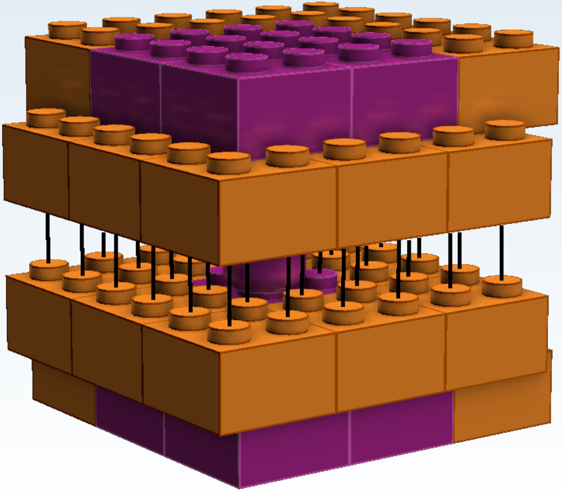

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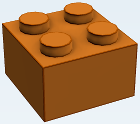

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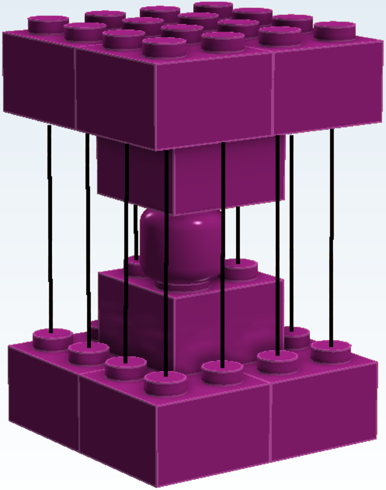

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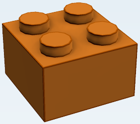

$\sigma^z =$

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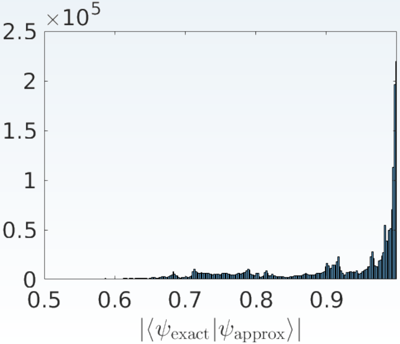
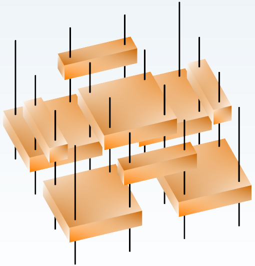
Bose-Hubbard model in 2D

$$H = -\sum_{\langle i,j \rangle} \left(a_i^\dagger a_j + a_i a_j^\dagger \right) + \frac{U'}{2} \sum_i n_i(n_i - 1) + \delta_i n_i$$

- $U' = 24.4$, δ_i from Gaussian distribution (half maximum width Δ)
- $n_{\max} = 1$ (spin 1/2), $n_{\max} = 2$ (spin 1)

$\Delta = 40$, 4×4 lattice

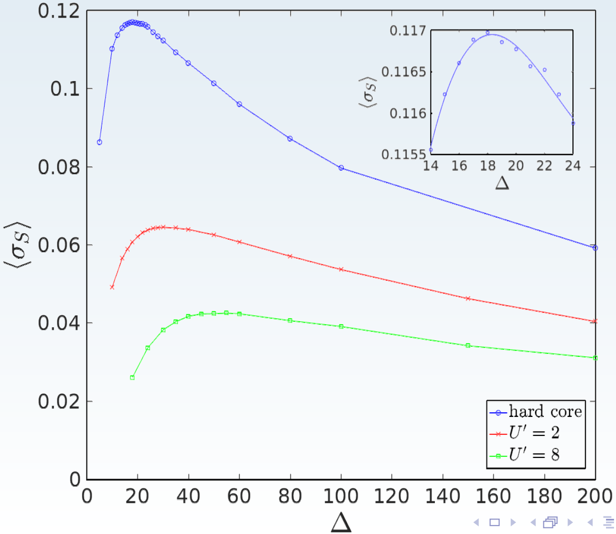




Main Result

10×10 lattice $\ell \times \ell = 2 \times 2$

ρ_A: one-site reduced density matrix $\rightarrow$ $S(\rho_A)$



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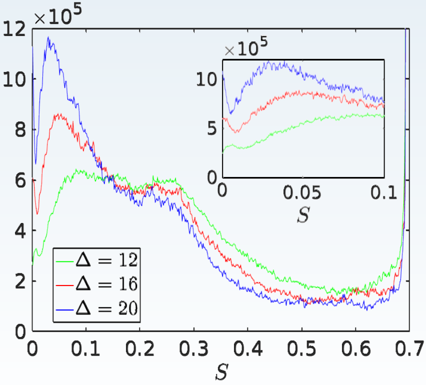
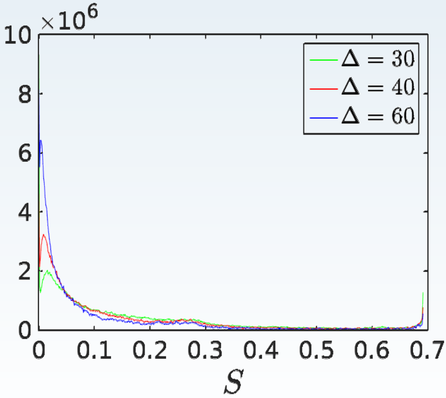
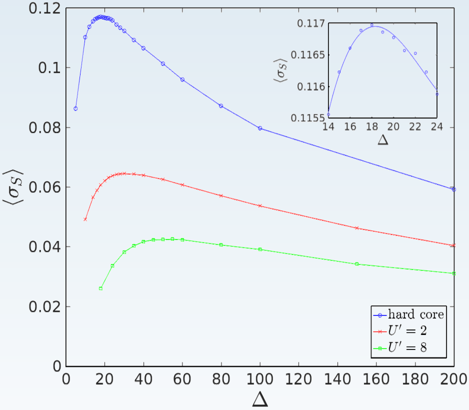
for $n_{\max} = 1$:



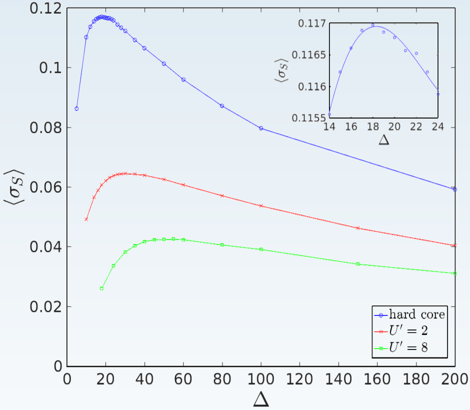


Comparison to the experiment



- $n_{\max} = 1$: $\Delta_c = 18.3$
- $n_{\max} = 2$:
  $\Delta_c(U' = 2) \approx 30$
  $\Delta_c(U' = 8) \approx 50$

Comparison to the experiment



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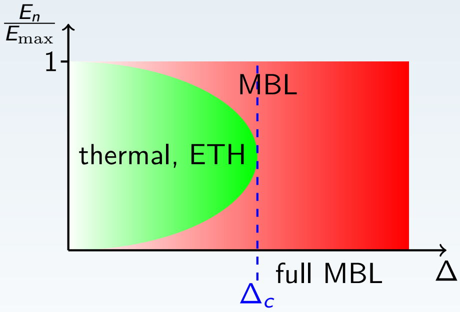


Experimentally: $\Delta_c(U' = 24.4) \approx 5.3$
however: only $\sim 7\%$ doublons

J.-y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, Science **352**, 1547 (2016).

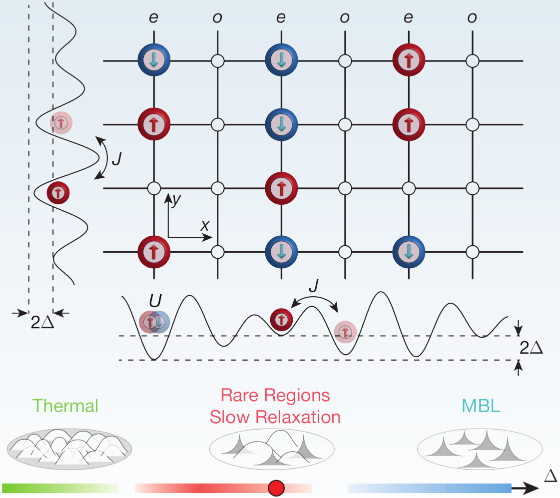
Experiment

$\Delta_c = 5.3$

Theory

$\Delta_c = 18.3$

But:



^{40}K gas (fermions)

3 regimes:

1. $0 \leq \Delta \lessapprox 2$
2. $2 \lessapprox \Delta \lessapprox 9$
3. $\Delta \gtrapprox \Delta_c = 9$

taken from: P. Bordia, H. Lüschen, S. Scherg, S. Gopalakrishnan, M. Knap, U. Schneider, and I. Bloch, Phys. Rev. X **7**, 041047 (2017)

Summary and Outlook

Summary:

- experimentally observed MBL in 2D is "short"-time phenomenon
- approximately conserved local integrals of motion
- 2D quantum circuits
- experiment: $\Delta_c = 5.3$, theory (2×2): $\Delta_c = 18.3$

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- approximately conserved local integrals of motion
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- experiment: $\Delta_c = 5.3$, theory (2×2): $\Delta_c = 18.3$

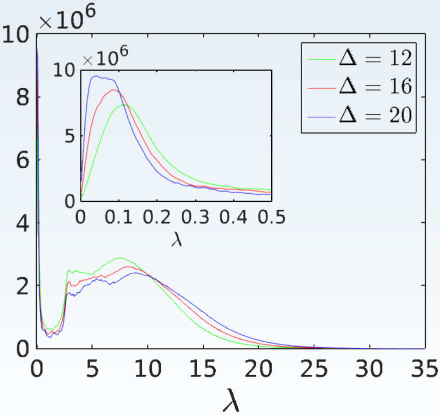
Outlook:

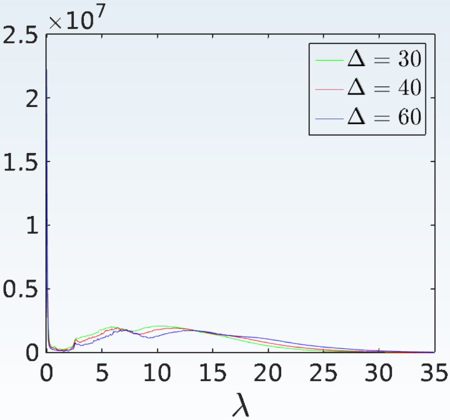
- experiment: charge density wave / larger filling
- theory: $\ell \times \ell = 3 \times 3$ simulations
- analytical tool to classify 2D topological and symmetry protected MBL phases

T. B. Wahl, arXiv:1712.07238

Entanglement energies

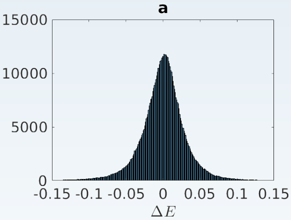
2×2 reduced density matrix: $\rho_A = e^{-H_{\mathrm{ent}}}$



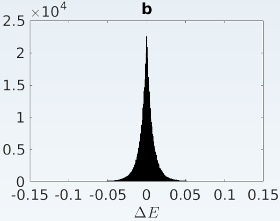


Overlaps in 1D for $N = 16$

$\ell = 2$



$\ell = 4$



$\ell = 8$