DMRG investigation of Quantum Dimer Ladders

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in collaboration with Frédéric Mila

University of California, Irvine

28 February 2018

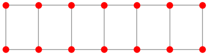
Scope

- Introduction
  - QDM constraint
  - Hilbert space
- QDM constraint in DMRG
  - Labelling the states
  - Trivial tensors in QDM
  - Filter the MPO
- Spin-1/2 Quantum dimer ladder
  - QDM ladder
  - Hard bosons and Fendley-Sengupta-Sachdev's prediction
  - Critical incommensurate phase
  - Tricritical Ising point
- Generalization to spin-S
  - Ising transition
- Conclusion

Introduction to Quantum Dimer Model (QDM)

Spin model

Spin d.o.f. are located in the nodes of the lattice



$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

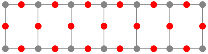
$$S^z_{tot} = 0$$

Each spin belongs to only one VBS

Number of particles: $2N_r$
Hilbert space: 2^{2N_r}

Quantum dimer model

Dimer d.o.f. are assiciated with the bonds of original lattice



$$H_{QDM} = \sum_{\text{Plaquettes}} -J\,(|{\parallel}\rangle\langle{=}| + \text{h.c.}) + v\,(|{=}\rangle\langle{=}| + |{\parallel}\rangle\langle{\parallel}|)$$

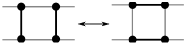
QDM constraints:
No free nodes
Dimers do not touch

Number of particles: $3N_r - 2$
Hilbert space: $\mathcal{F}(N_r)$

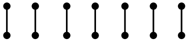
QDM Hamiltonian

$$H_{QDM} = \sum_{\text{Plaquettes}} -J\left(|\,\mathrm{|\;|}\,\rangle\langle\,\mathrm{=}\,| + \text{h.c.}\right) + v\left(|\,\mathrm{=}\,\rangle\langle\,\mathrm{=}\,| + |\,\mathrm{|\;|}\,\rangle\langle\,\mathrm{|\;|}\,|\right)$$

- J-term flips dimers on the plaquetts

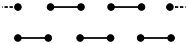


- v-term counts the number of flippable plaquetts
  - $v \to -\infty$ maximally flippable state

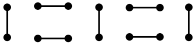


  - $v \to +\infty$ least flippable state

PBC



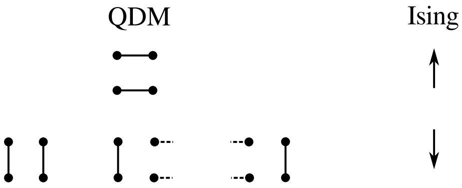
OBC



- $J = v$ Rokhsar-Kivelson point: Zero-energy ground state is an equal weight superposition of **all possible dimer coverings**

Rokhsar, Kivelson, PRL **61**,2376'88

Mapping to Ising model



$$H_{Ising} = \sum_i v \left(S^z_{i-1} + \frac{1}{2}\right)\left(S^z_{i+1} + \frac{1}{2}\right) - 2vS^z_i - 2JS^x_i + U\left(S^z_i + \frac{1}{2}\right)\left(S^z_{i+1} + \frac{1}{2}\right)$$

The mapping is exact for $U \to \infty$

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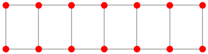

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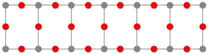

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QDM constraints:

No free nodes

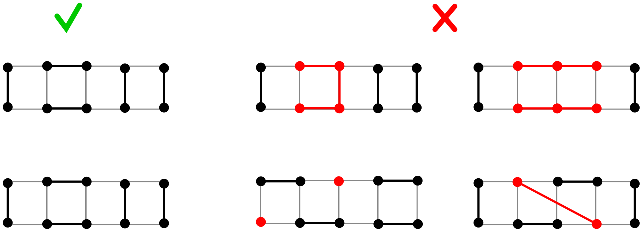
Dimers do not touch

Number of particles: $3N_r - 2$

Hilbert space: $\mathcal{F}(N_r)$

QDM constraint

1. **Every lattice node belongs to one and only one dimer**
   - Dimers do not touch
   - No free nodes
2. **Dimers are only between nearest neighbors**



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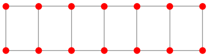

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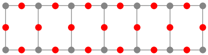

$$H_{QDM} = \sum_{\text{Plaquettes}} -J\left(|\,\|\,\rangle\langle\,=\,| + \text{h.c.}\right) + v\left(|\,=\,\rangle\langle\,=\,| + |\,\|\,\rangle\langle\,\|\,|\right)$$

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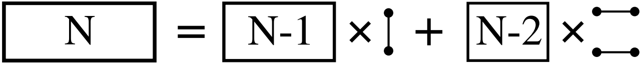
Hilbert space

Size of the Hilbert space = number of dimer coverings

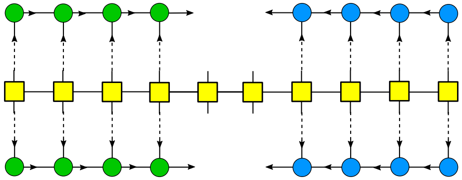


$$\mathcal{H}(N) = \mathcal{H}(N-1) + \mathcal{H}(N-2)$$

$$\mathcal{H}(N) \equiv \mathcal{F}(N)$$

Martin-Delgado, Sierra, PRL **56**, '97

DMRG for QDM

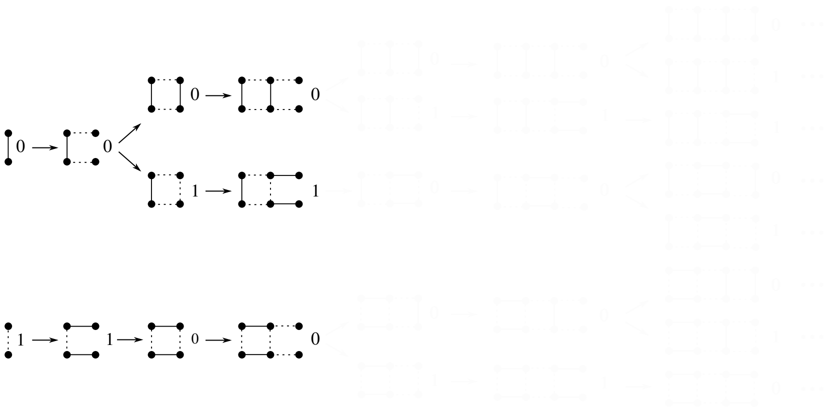


Left environment

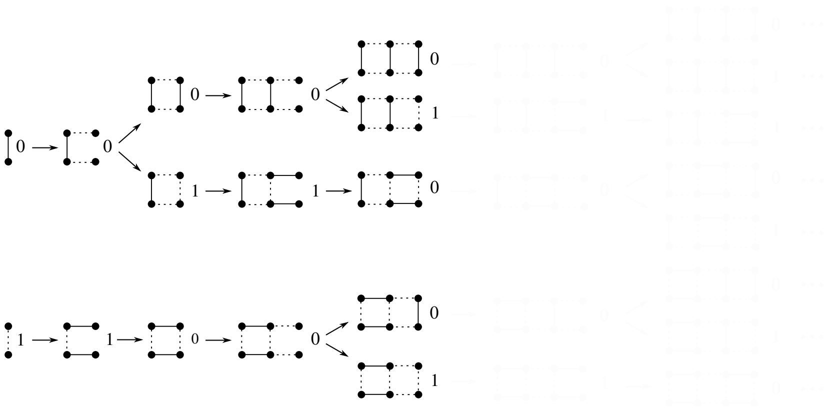
Left environment

Left environment

Left environment



Left environment



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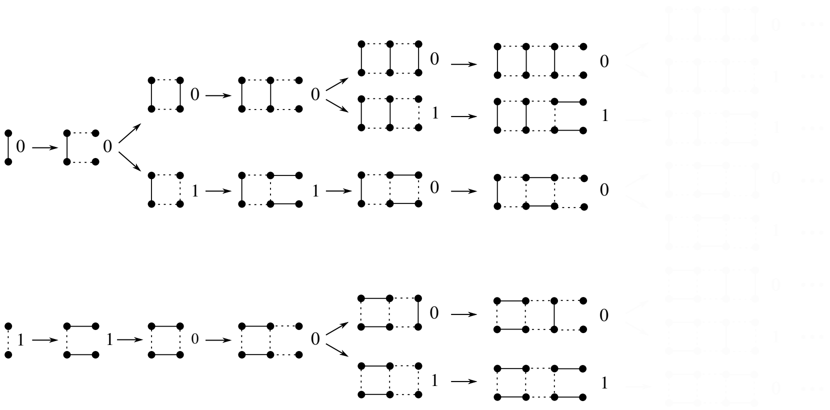

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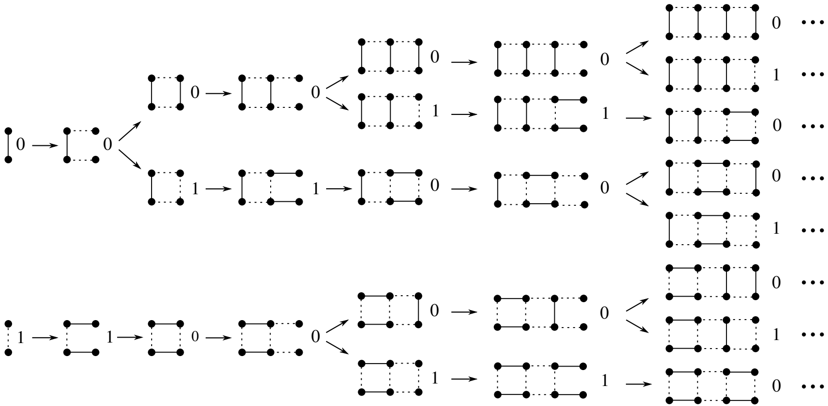
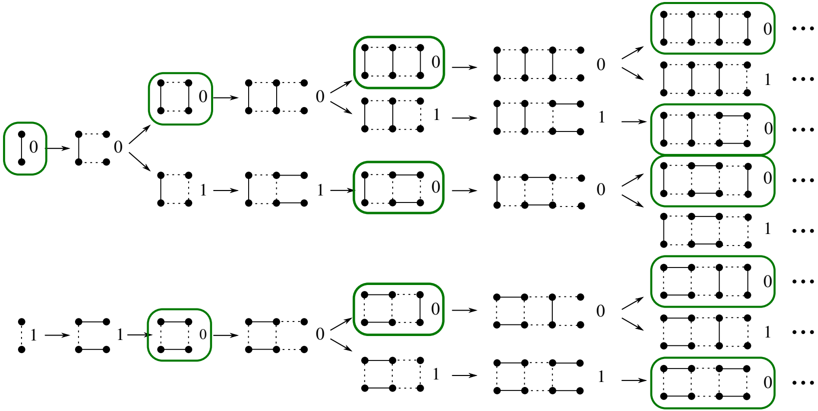
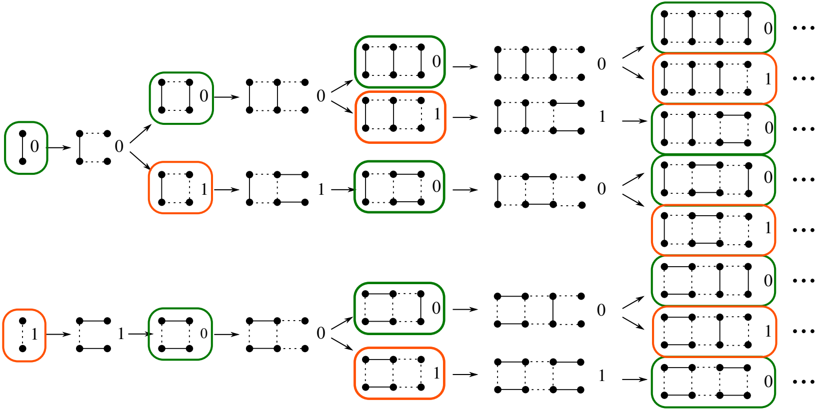


Hilbert space



$\mathcal{F}(1) = 1$ $\mathcal{F}(2) = 2$ $\mathcal{F}(3) = 3$ $\mathcal{F}(n_r)$

Hilbert space

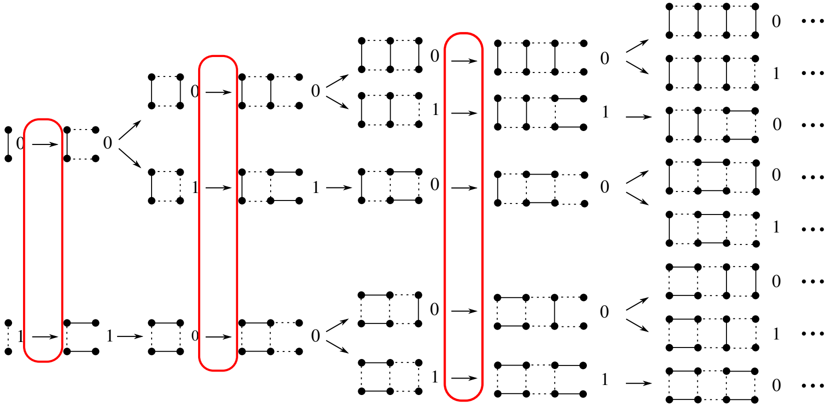


$\mathcal{F}(1) = 1$ $\mathcal{F}(2) = 2$ $\mathcal{F}(3) = 3$ $\mathcal{F}(n_r)$

$\mathcal{F}(0) = 1$ $\mathcal{F}(1) = 1$ $\mathcal{F}(2) = 2$ $\mathcal{F}(n_r - 1)$

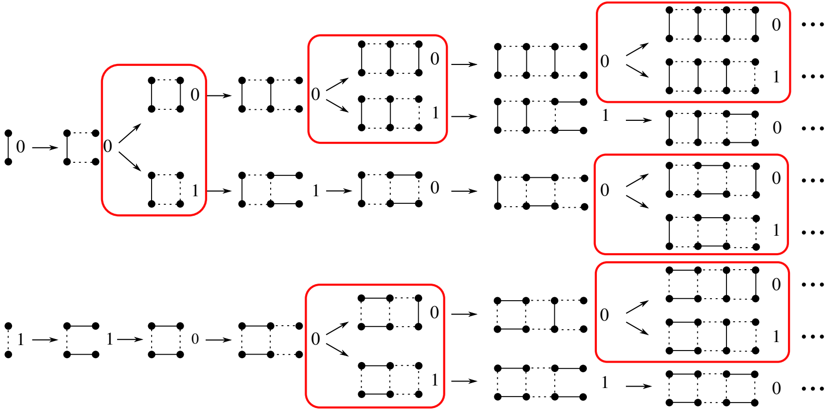
$\mathcal{F}(2) = 2$ $\mathcal{F}(3) = 3$ $\mathcal{F}(4) = 5$ $\mathcal{F}(n_r + 1)$

Triviality of the leg tensors

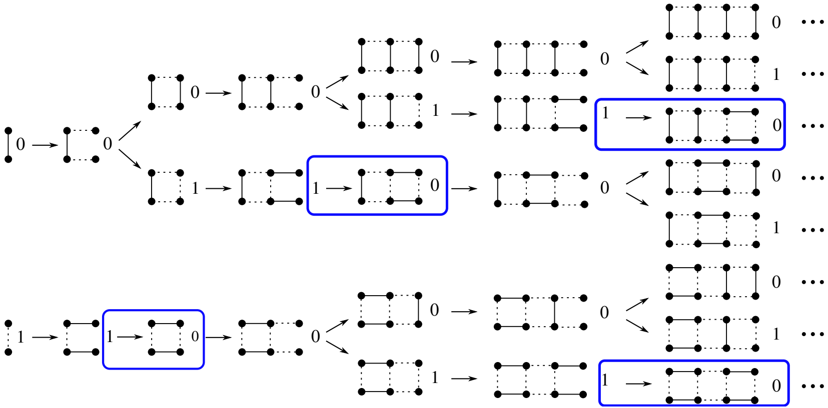


Exists a basis, in which leg tensors are the **identity** matrix

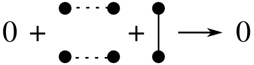
Fusion rules

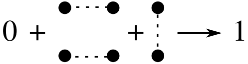


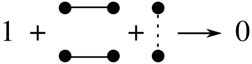
Fusion rules



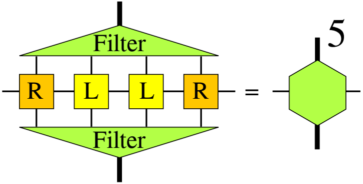
Fusion rules



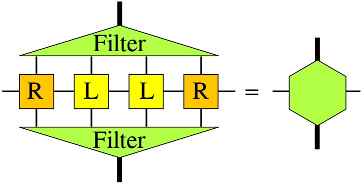




Filter



Filter



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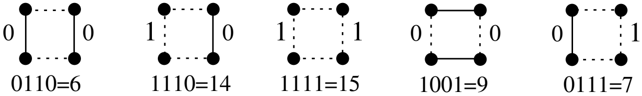


0
0
0110=6
1
0
1110=14
1
1
1111=15
0
0
1001=9
0
1
0111=7

Spin-1/2 Quantum dimer ladder

$$H_{QDM} = \sum_{\text{Plaquettes}} -J \left(|\,\|\ \|\,\rangle\langle\,=\,| + \text{h.c.} \right) + v \left(|\,=\,\rangle\langle\,=\,| + |\,\|\ \|\,\rangle\langle\,\|\ \|\,| \right)$$

Trimerization

$$H_{QDM} = \sum_{\text{Plaquettes}} -J\left(|\,\mathrm{I}\;\mathrm{I}\,\rangle\langle\,=\,| + \text{h.c.}\right) + v\left(|\,=\,\rangle\langle\,=\,| + |\,\mathrm{I}\;\mathrm{I}\,\rangle\langle\,\mathrm{I}\;\mathrm{I}\,|\right)$$

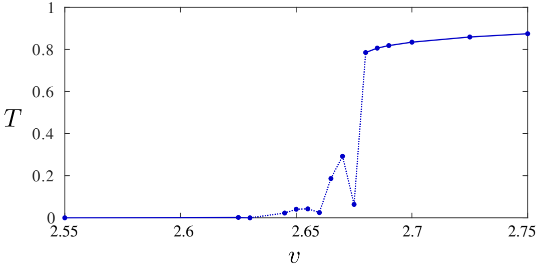
- $v \to -\infty$

  rung-dimer phase

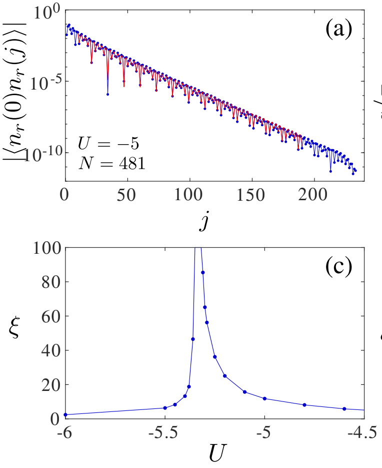
- $v \to +\infty$

  period-three phase

$$T = \max(|n_r(j) - n_r(j-1)|, |n_r(j) - n_r(j+1)|)|_{j=\frac{N_r}{2}}$$

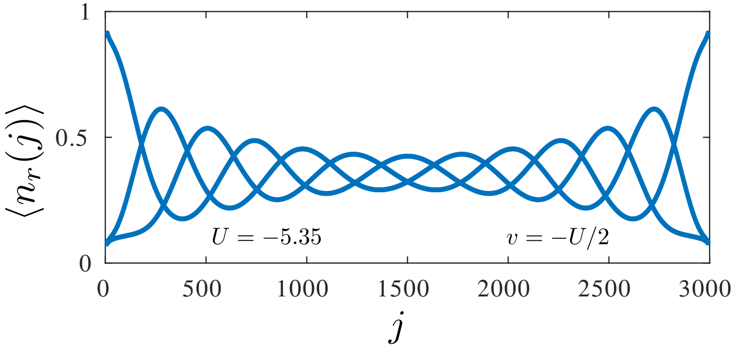


Incommensurate short range order



Notation: $U = -2v$

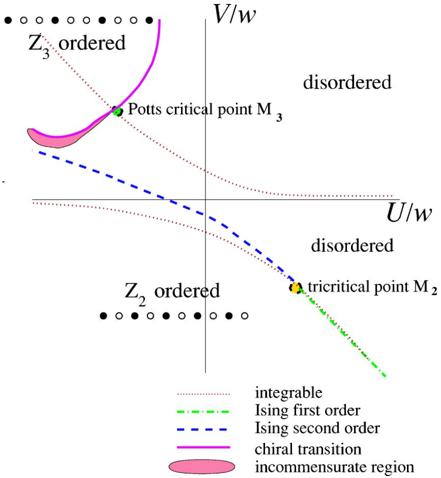
Incommensurate critical phase



Incommensurate algebraic order?

Hard bosons

$$H_{HB} = \sum_j -w(d_j^\dagger + d_j) + U n_j + V n_j n_j.$$



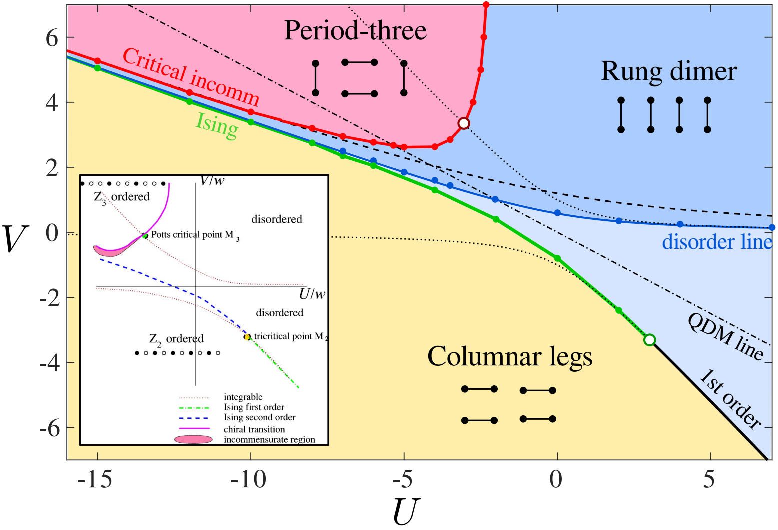
Fendley, Sengupta, Sachdev, PRB **69**, 075106'04

Mapping between the hard bosons and QDM

$$H_{HB} = \sum_j -w(d_j^\dagger + d_j) + Un_j + Vn_jn_{j+2},$$

$$\begin{aligned} H_{HB} = & \sum_{\text{Plaquettes}} -w\left(|\vert\,\vert\rangle\langle=| + \text{h.c.}\right) - \frac{U}{2}\left(|=\rangle\langle=| + |\vert\,\vert\rangle\langle\vert\,\vert|\right) \\ & + \left(V + \frac{U}{2}\right)|=\;=\rangle\langle=\;=| \end{aligned}$$

Phase diagram



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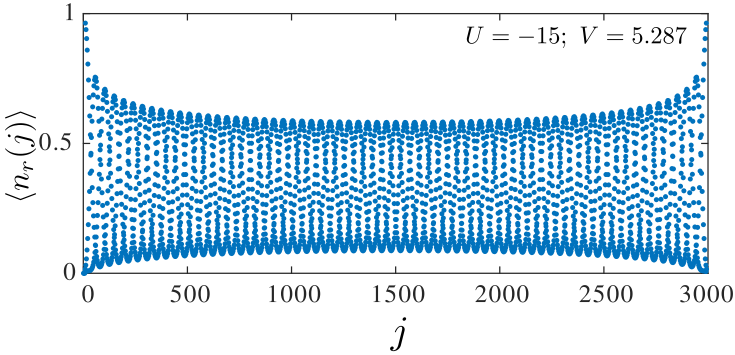
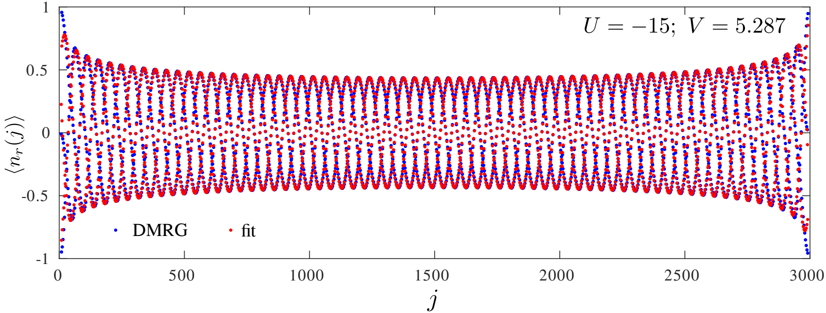


Incommensurate critical phase

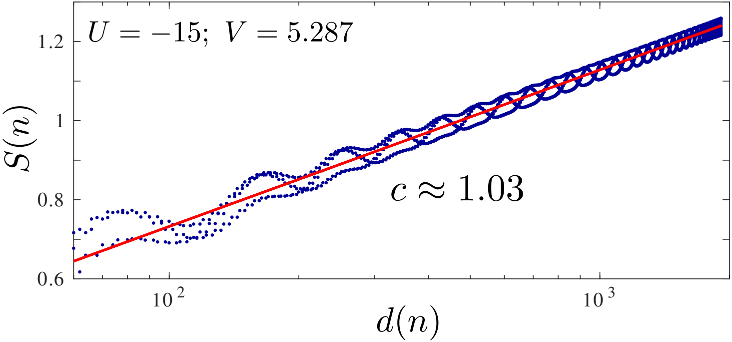


Algebraic decay of the Friedel oscillations:

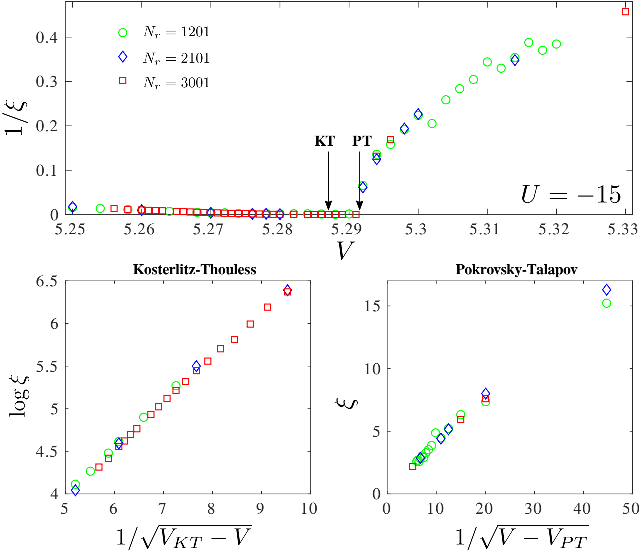
$$\langle n_r(j) \rangle \propto \frac{\cos(qj + \varphi_0)}{[N_r \sin(\pi j / N_r)]^d} \tag{1}$$

Numerical results: $d \approx 0.15$, $q \approx 0.68\pi$

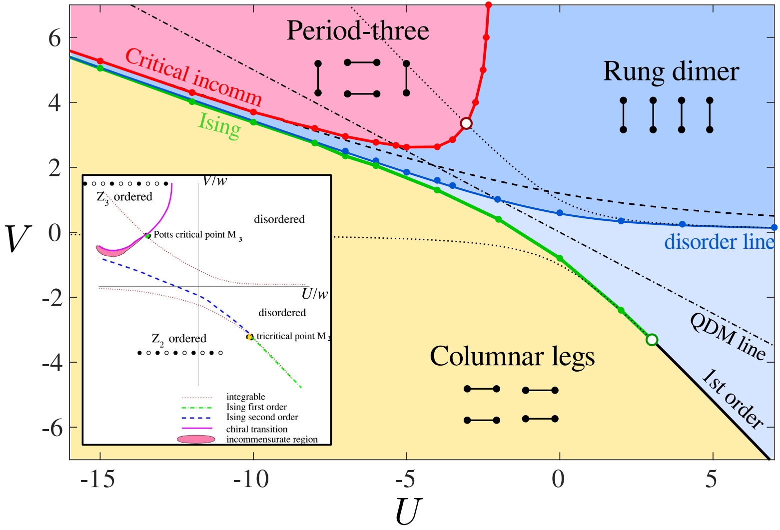
Incommensurate critical phase. Central charge



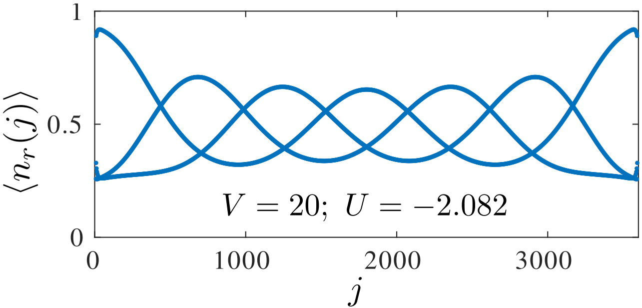
Incommensurate critical phase. Phase boundaries



Phase diagram



Incommensurate critical phase **above** Potts point



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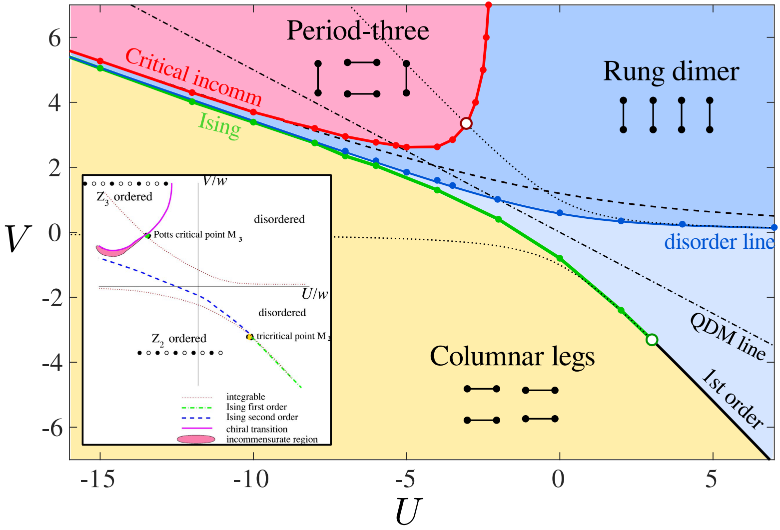

Step aside:

Excitation spectrum with DMRG

Excitation spectrum with DMRG/MPS

1. The excited state is the 'ground-state' of the different symmetry sector

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2. Conventional DMRG: Mixed states
   - The ground-state is spoilt
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   - Time consuming
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2. Conventional DMRG: Mixed states
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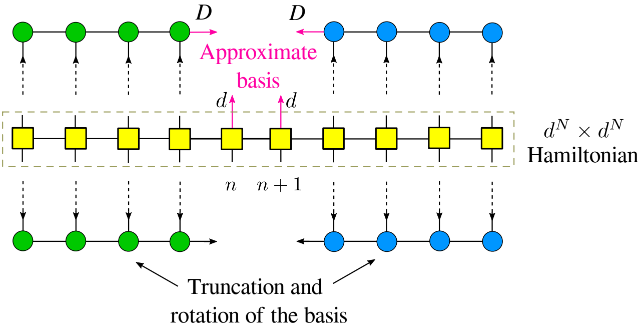
There is a cheaper option:

Sometimes it is sufficient to target multiple eigenstates of the effective Hamiltonian and keep track of the energies as a function of iterations [NC, Mila, Phys.Rev.B **96**, 054425 (2017)]

Effective Hamiltonian

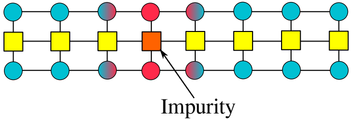


Hamiltonian is written in a truncated and rotated basis selected for the ground state

When does it work?

Local impurities

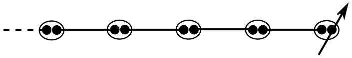
- Localized excitations
- MPS is the same except for a few sites



When does it work?

Edge states

- Edge spins are entangled through the entire network
- All edge states are in the basis



$$\left(\frac{B}{2} \otimes \uparrow\right) \oplus \left(\frac{B}{2} \otimes \downarrow\right)$$

Local impurities

- Localized excitations
- MPS is the same except for a few sites

When does it work?

Critical systems

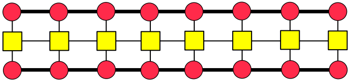
- Divergent correlation length
- Slow decay of Schmidt values
- Special structure of spectrum

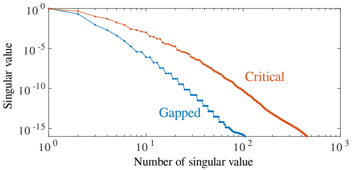
Edge states

- Edge spins are entangled through the entire network
- All edge states are in the basis

Local impurities

- Localized excitations
- MPS is the same except for a few sites



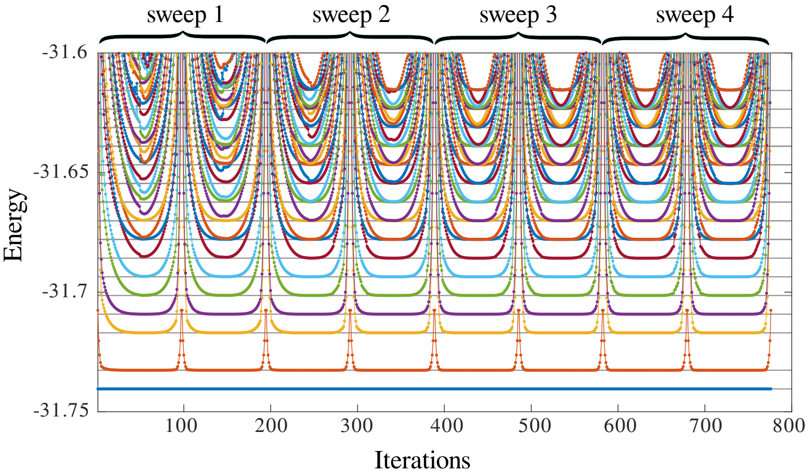


Transverse field Ising model

$$H = \sum_i J S_i^x S_{i+1}^x + h S_i^z$$

- Critical at $h = J/2$
- Solved by Jordan-Wigner transformation
- Corresponds to the minimal model (4,3) in CFT

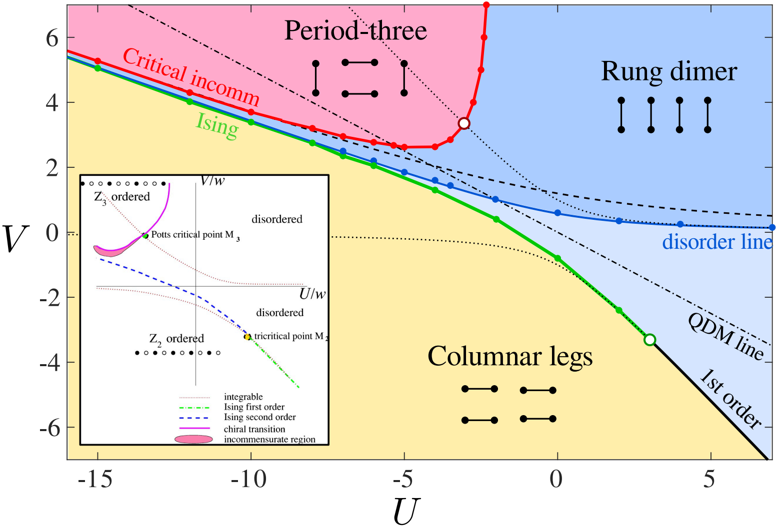
Transverse field Ising model. Excitation spectrum



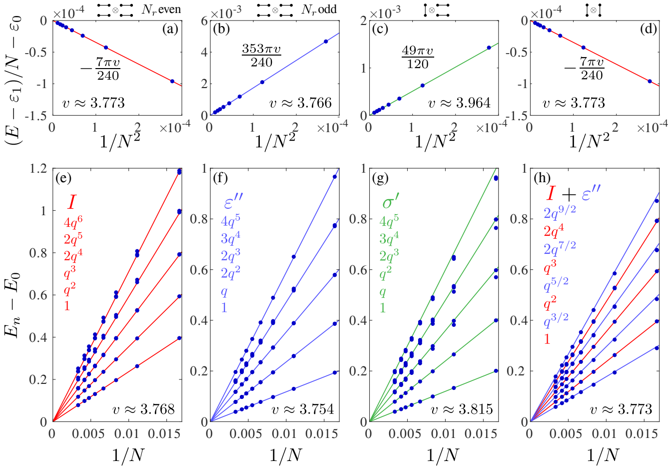
- 30 states within a single run!
- Flat modes signal convergence

NC, F. Mila, Phys. Rev. B 96, 054425'17

Phase diagram

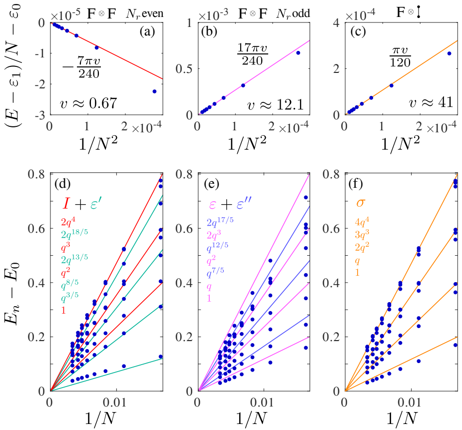


Tricritical Ising point



- N_r even
  $|\uparrow\rangle \otimes |\uparrow\rangle = |I\rangle$
- N_r odd
  $|\uparrow\rangle \otimes |\downarrow\rangle = |\varepsilon''\rangle$
- $\forall N_r$
  $|\uparrow\rangle \otimes |0\rangle = |\sigma'\rangle$
- $\forall N_r$
  $|0\rangle \otimes |0\rangle = |I + \varepsilon''\rangle$

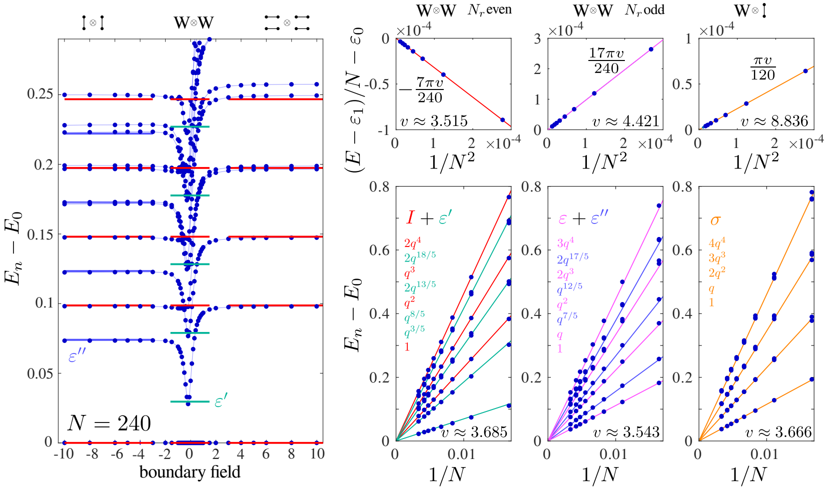
Tricritical Ising point



- $F \otimes F$, N_r even
  $|0\uparrow\rangle \otimes |0\uparrow\rangle = I + \varepsilon'$
- $F \otimes F$, N_r odd
  $|0\uparrow\rangle \otimes |0\downarrow\rangle = \varepsilon + \varepsilon''$
- $\otimes F$, $\forall N_r$
  $|0\uparrow\rangle \otimes |0\rangle = \sigma$

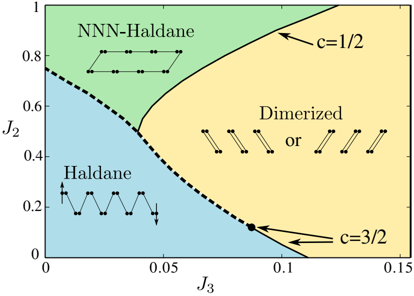
Boundary-field correspondance is almost correct

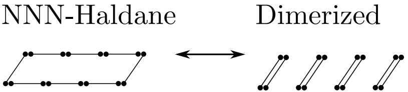
Tricritical Ising point



Spin-1 zig-zag chain

$J_1 - J_2 - J_3$ model

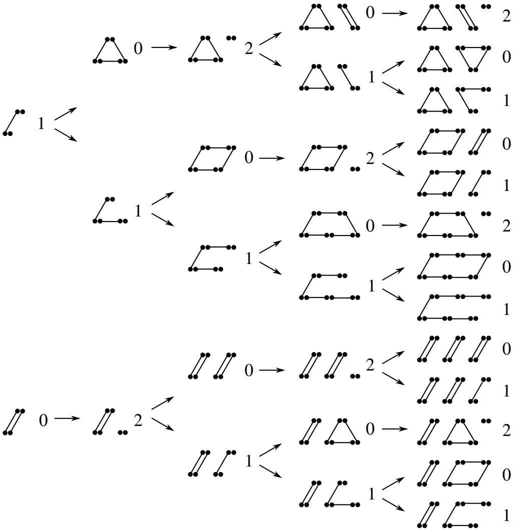




Non-magnetic domain wall

$$H = \sum_{\text{Plaquettes}} -J\left(|\ldots\rangle\langle\ldots| + \text{h.c.}\right) + v\left(|\ldots\rangle\langle\ldots|\right)$$

Fusion rules

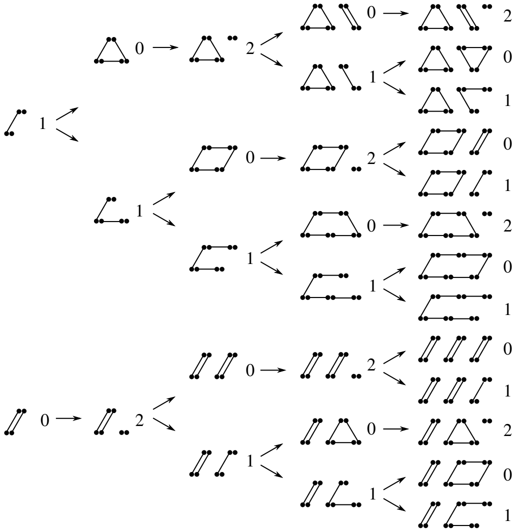


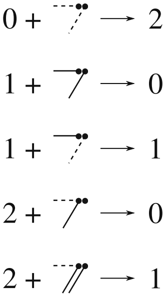
Hilbert space:

- 0-sector: $\mathcal{F}(n_r - 1)$
- 1-sector: $\mathcal{F}(n_r - 1)$
- 2-sector: $\mathcal{F}(n_r - 2)$

Total: $\mathcal{F}(n_r + 1)$

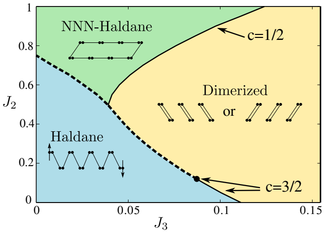
Fusion rules

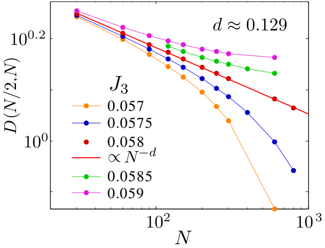




Spin-1 zig-zag chain

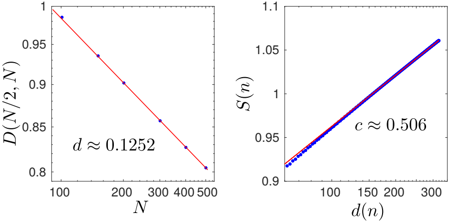
$J_1 - J_2 - J_3$ model





$$H = \sum_{\text{Plaquettes}} -J\,(|\ldots\rangle\langle\ldots| + \text{h.c.}) + v\,(|\ldots\rangle\langle\ldots|)$$

At the critical point $v/J \approx -0.8385$



Ising critical theory:

- $c = 1/2$
- $d = 1/8$

NC, Affleck, Mila, PRB **93**, 241108'16

Conclusions

- We show how to **implement QDM constraint** into DMRG
- Powerful toy model to simulate **non-magnetic** quantum phase transitions
- **Easy to generalize** to any kind of strong local constraints

- In quantum dimer ladder transition between rung dimer and period three phase is through an intermediate **incommensurate critical phase**
- In hard-bosons model critical incommensurate phase appears **below and above** Potts critical point
- Boundary-field correspondence for **tri-critical Ising** point