

Neutrino Physics (I)

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International Summer Workshop on High Energy Physics 2018

TAE 2018 – Benasque (Spain), Sep 02 – Sep 15



Outline

- Historical introduction to neutrino physics
- Neutrinos in the Standard Model
- Neutrino masses beyond the Standard Model
- Neutrino oscillations in vacuum and matter
- Three-flavour neutrino oscillations
- Beyond three-neutrino flavours: sterile neutrinos
- The absolute scale of neutrino mass
- Future prospects in neutrino oscillations
- Neutrino physics beyond the Standard Model

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What is a neutrino?

- spin 1/2 particle
- massless particle (almost)
- neutral
- 3 flavors (mixing)

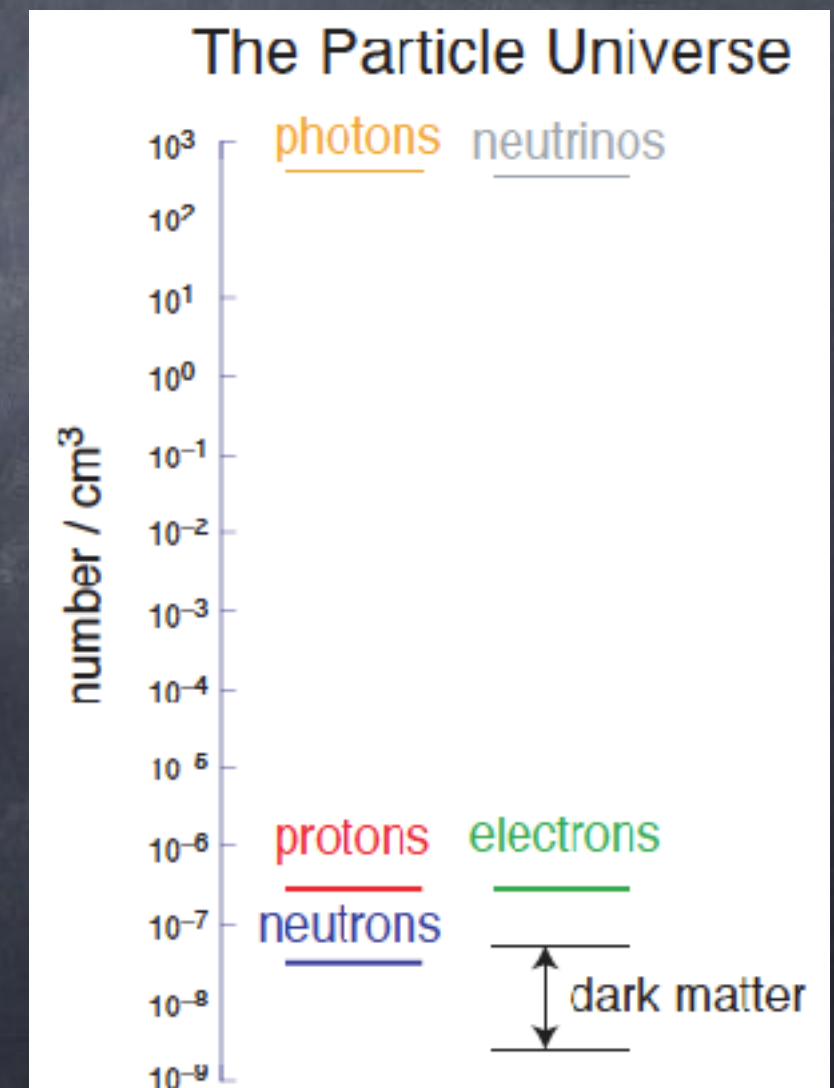
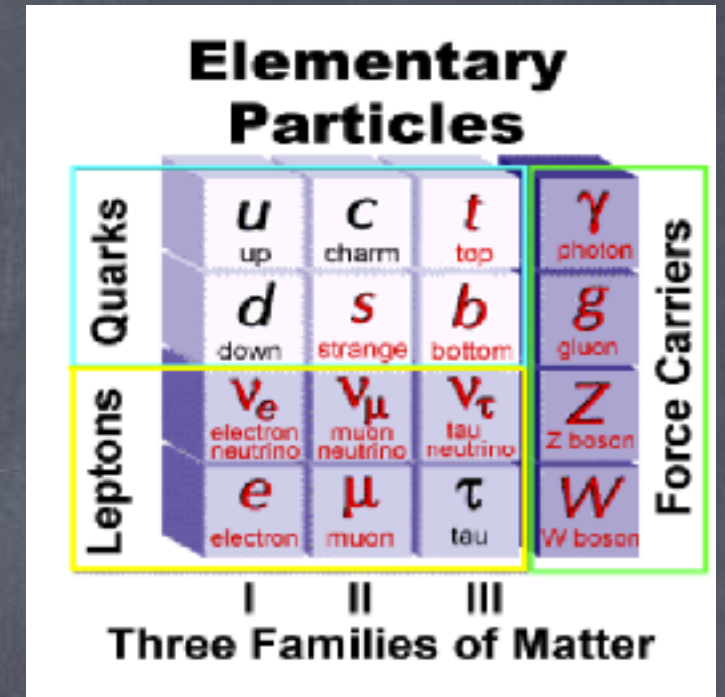
Anything else?

Every second we are traversed by:

- 400×10^{12} neutrinos from the Sun
- 50×10^9 neutrinos from natural radioactivity
- 10×10^9 neutrinos from nuclear power plants

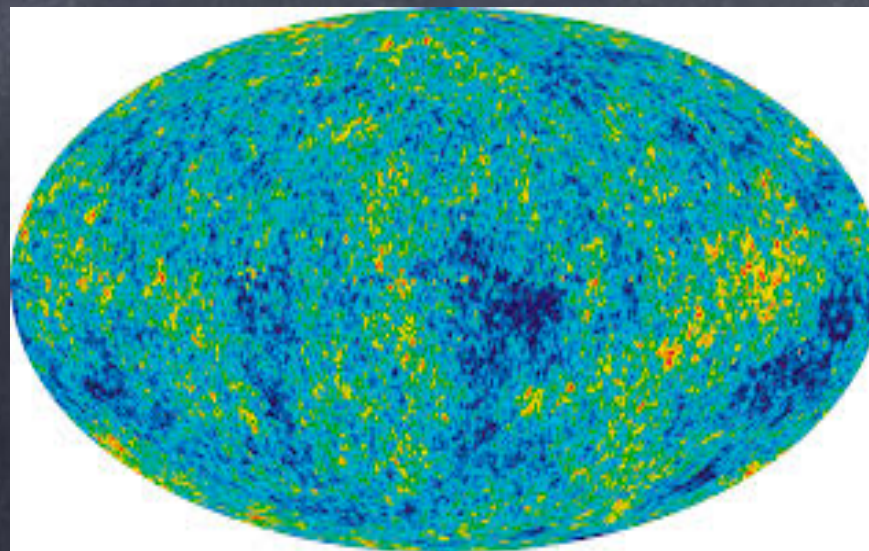
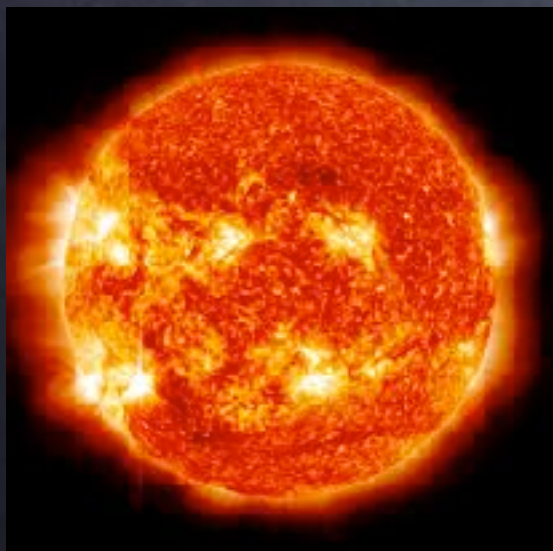
Moreover:

- our body emits 400 neutrinos/s (^{40}K decay)
- the Universe contains ~ 330 neutrinos/cm³



Why neutrinos are so important?

- they can probe environments that other techniques cannot: SN explosions, core of the Sun,...
- their role is crucial for the evolution of the universe (Big Bang Nucleosynthesis, structure formation)
- they could help explaining the matter-antimatter asymmetry of the Universe (leptogenesis mechanism)
- they could be a component of the dark matter of the universe.
- they provide the first evidence for physics beyond the SM!!!



Why neutrinos are so important?

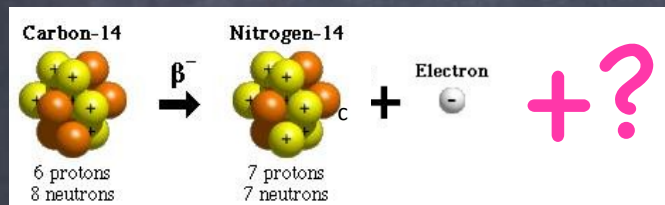
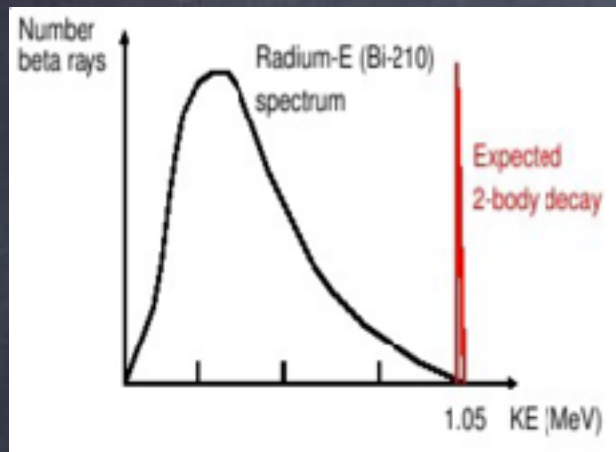
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- they provide the first evidence for physics beyond the SM!!!

However: there are still many open questions in neutrino physics

Historical introduction to neutrino physics

The proposal of the neutrino

- 1930: Pauli introduced the **neutrino** to explain continuous electron spectrum in nuclear beta decay.



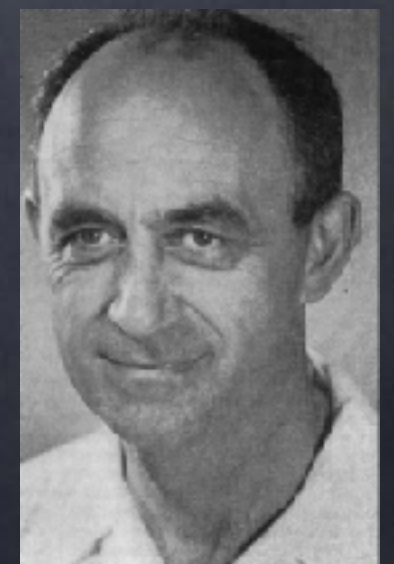
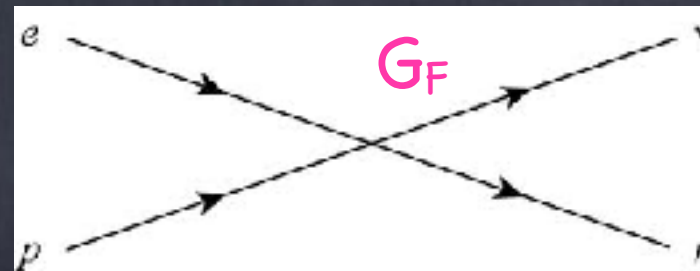
"Dear radioactive ladies and gentlemen,
I have come upon a desperate way out regarding ... [some fairly obscure data], as well as to the continuous β -spectrum, in order to save ... The energy law. To wit, the possibility that there could exist in the nucleus **electrically neutral particles** which I shall call **neutrons**, which have spin 1/2 and satisfy the exclusion principle and which are further distinct from light-quanta in that they do not move with light velocity. ... The continuous β -spectrum would then become understandable from the assumption that in β -decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant."



- 1933: Fermi postulated the first **theory of nuclear beta decay**, the theory of weak interactions

$$n \rightarrow p + e^- + \bar{\nu}_e$$

→ new name for particle: **neutrino**



But, where was the neutrino?

Pauli, 1930

"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."

►1934: Bethe and Peierls calculated the cross section σ for the processes:

$$\nu + n \rightarrow p + e^{-}$$

$$\bar{\nu} + p \rightarrow n + e^{+}$$



According to Fermi theory, they obtained (for antineutrino + proton):

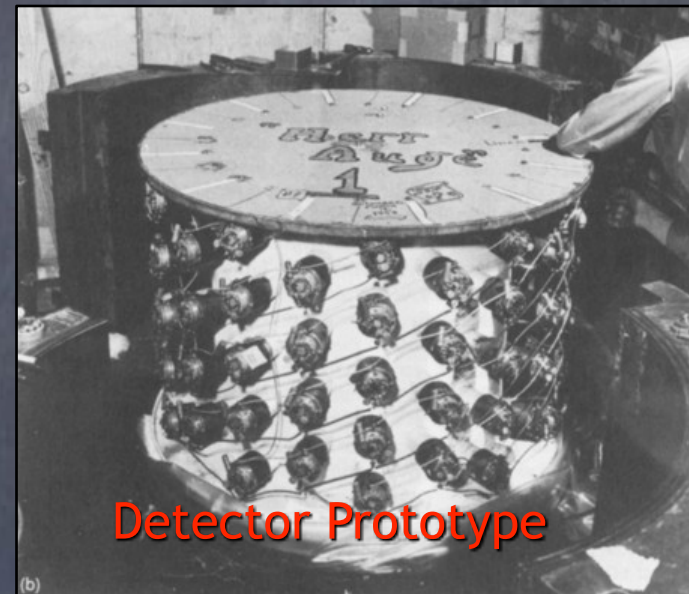
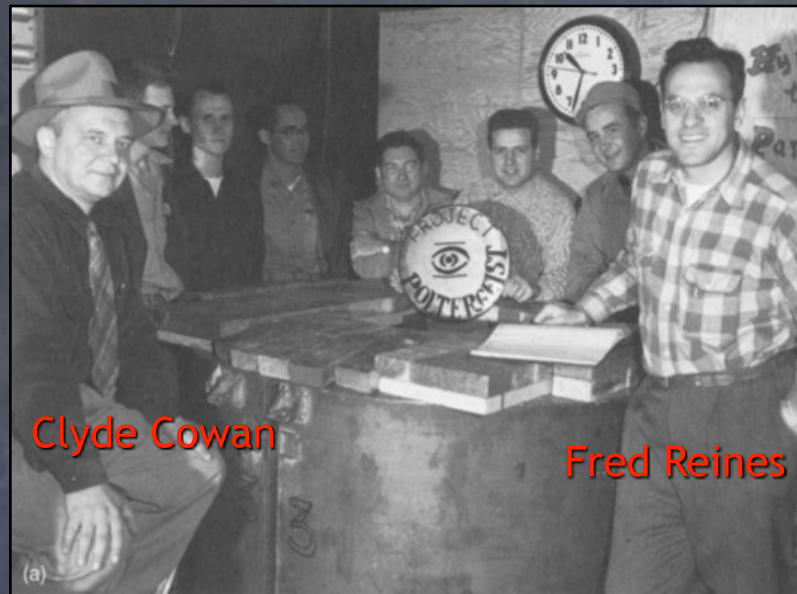
$\sigma \sim 5 \times 10^{-44} \text{ cm}^2$ for a 2 MeV neutrino **factor 2 missing!**

(to be compared with $\sigma_{\gamma p} \sim 10^{-25} \text{ cm}^2$) !!!

Difficult but not impossible!

Discovery of the neutrino

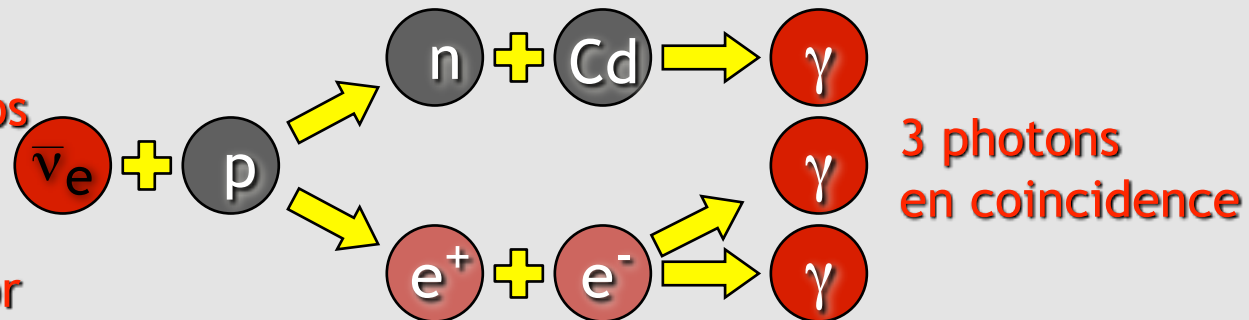
►1956: First observation of reactor $\bar{\nu}_e$ by Reines and Cowan.



2 tanks with
200 liters H_2O
+
40 kg CdCl_2

3 scintillator
layers with PMTs

Electron
Antineutrinos
from
Savannah
River reactor



1995 Nobel Prize
in Physics to
Reines

Telegram to Pauli on 12/06/1956

"We are happy to inform you that we have definitely detected neutrinos from fission fragments by observing inverse beta decay of protons. Observed cross section agrees well with expected six times ten to minus forty-four square centimeters"

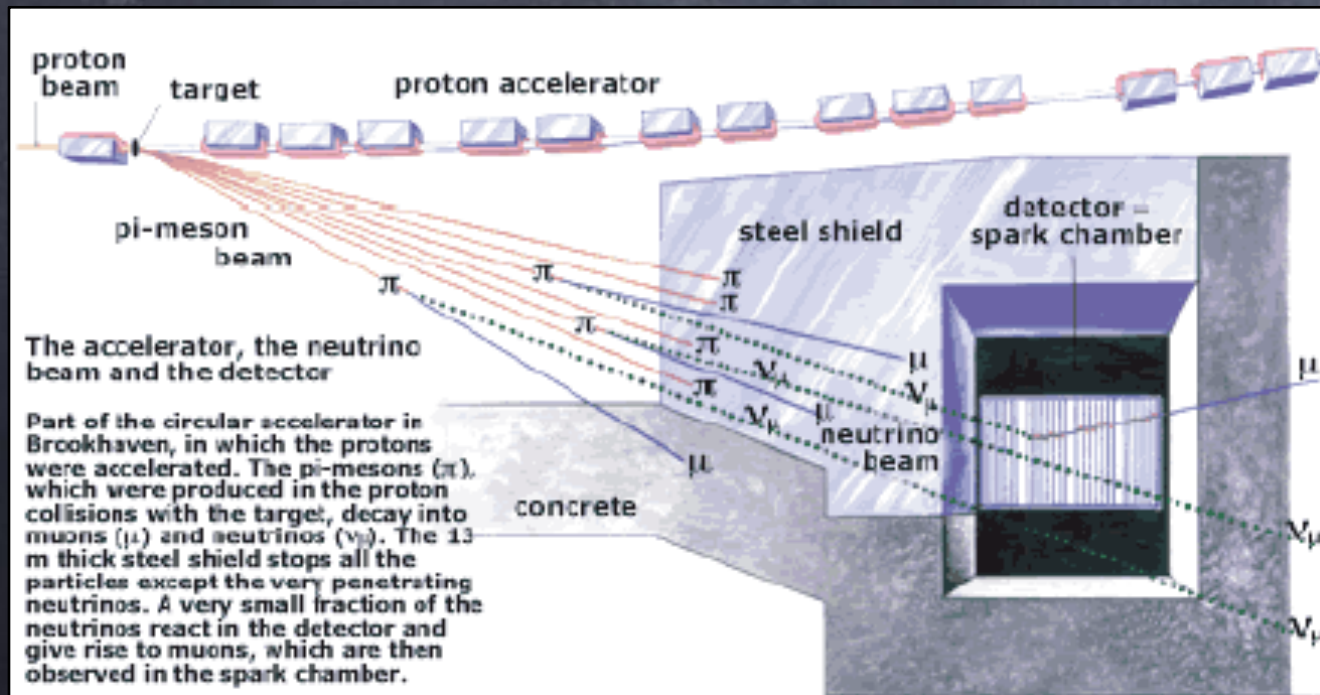


More than one neutrino flavour?

- 1959: Pontecorvo suggested the existence of a different neutrino, associated to muon decay and proposed an experiment to check it.

$$\nu_{\text{acc}} + n \rightarrow p + (e^- \text{ or } \mu^- ?)$$

- 1962: Discovery of ν_{μ} by Lederman, Schwartz and Steinberger



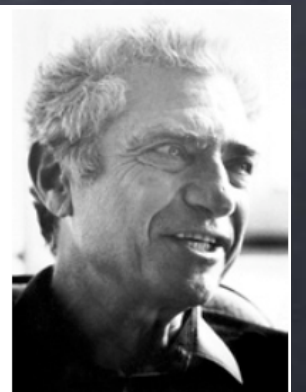
$$\pi^+ \rightarrow \mu^+ + \nu_{\mu} \quad \text{not } e^-$$
$$\nu_{\mu} + n \rightarrow p + \mu^-$$



Leon M. Lederman



Melvin Schwartz



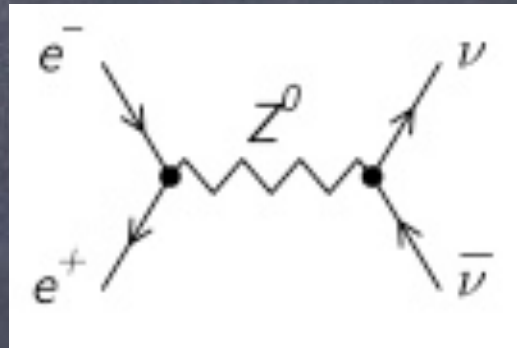
Jack Steinberger

1988 Nobel Prize in Physics

More than two neutrino flavours?

►1978: Discovery of τ at SLAC $\rightarrow$ imbalance of energy in τ decay suggests existence of a third neutrino.

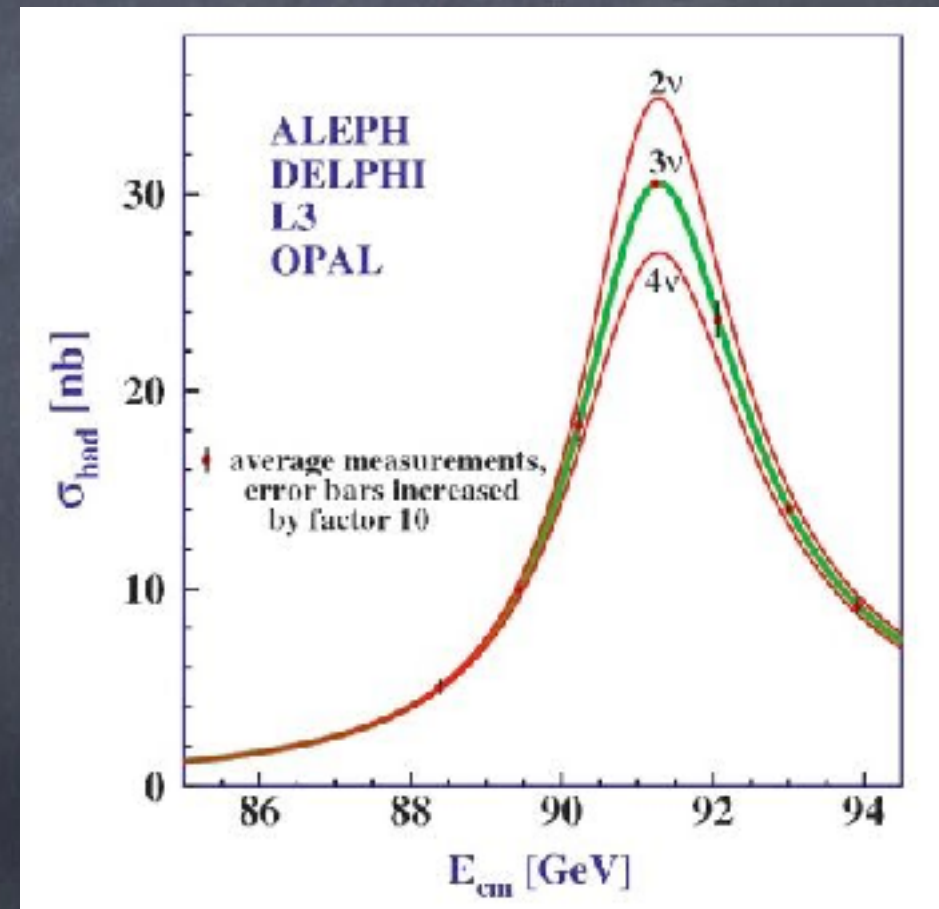
►1989: LEP measurements of the invisible decay width of Z boson



$$\Gamma_{\text{inv}} \equiv \Gamma_Z - \Gamma_{\text{had}} - 3\Gamma_{\text{lep}}$$

$$N_\nu = \Gamma_{\text{inv}} / \Gamma_{\text{SM}}(Z \rightarrow \nu_i \bar{\nu}_i)$$

$$\rightarrow N_\nu = 2.984 \pm 0.008$$



►2000: Discovery of ν_τ by the DONUT Collaboration.

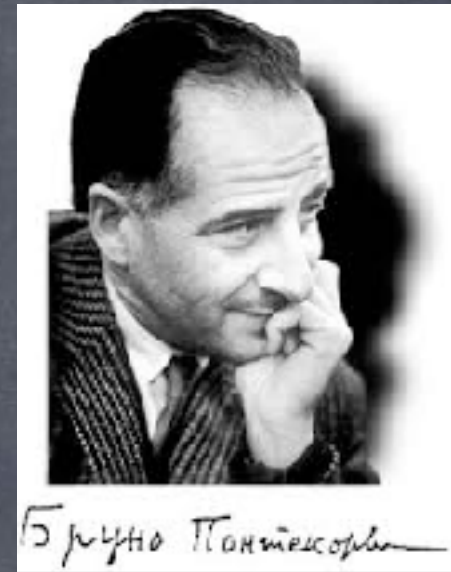
800 GeV $p \rightarrow D_s$ meson ($\equiv cs$) $\rightarrow \nu_\tau^-$ beam $\rightarrow \tau$ detected

Neutrino oscillations

- 1957: Pontecorvo suggests oscillations between neutrinos & antineutrinos (only ν_e).

B. Pontecorvo, J. Exp. Theor. Phys. 33 (1957)549.

B. Pontecorvo, J. Exp. Theor. Phys. 34 (1958) 247.



- 1962: Maki, Nakagawa and Sakata proposed flavor neutrino oscillations.

$$\begin{aligned}\nu_1 &= \nu_e \cos \delta + \nu_\mu \sin \delta, \\ \nu_2 &= -\nu_e \sin \delta + \nu_\mu \cos \delta.\end{aligned}$$

true
neutrinos

weak
neutrinos

2 ν mixing

Z. Maki, M. Nakagawa, S. Sakata,
Prog. Theor. Phys. 28 (1962) 870.

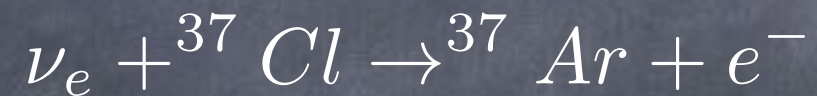


- 1969: Gribov & Pontecorvo calculated the neutrino oscillation probability (in vacuum) for the first time

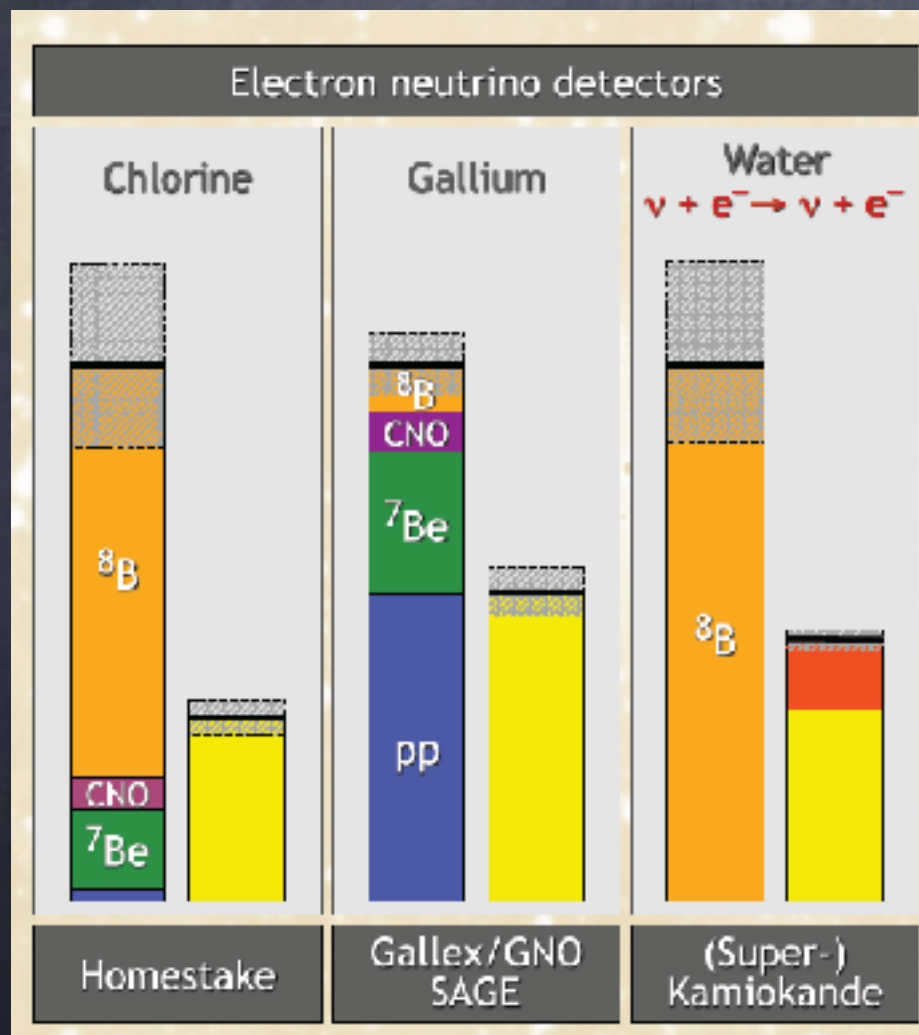
V. Gribov, B. Pontecorvo, Phys. Lett. B28 (1969) 493.

First indication of ν oscillations

►1968: First observation of **solar neutrinos** by R. Davis in Homestake.



→ 1/3 of the Standard Solar Model prediction !!



→ confirmed by the following experiments

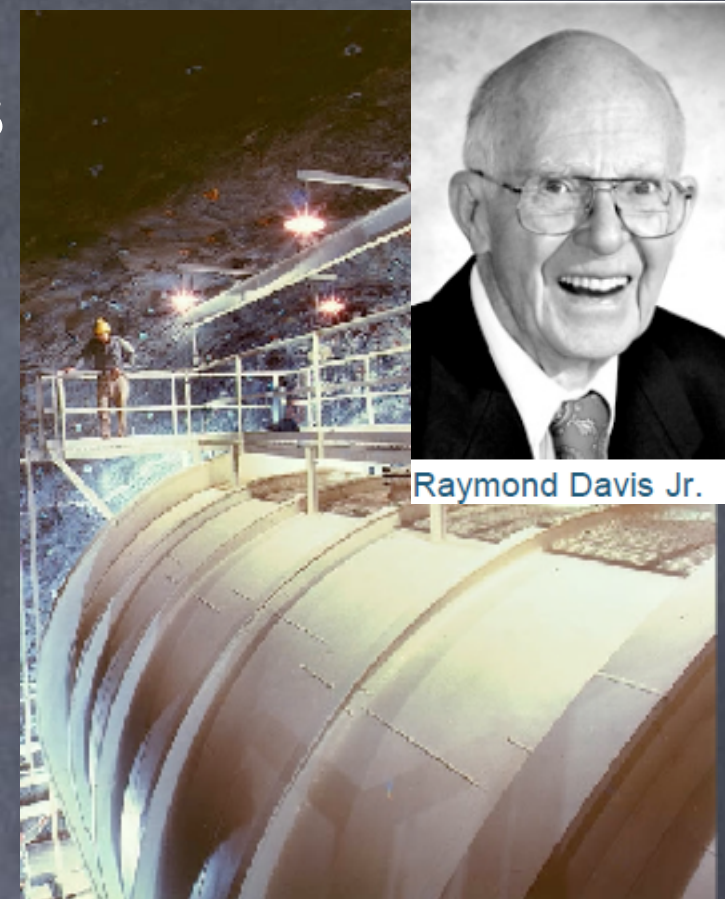
2002 Nobel Prize in Physics

Explanation?

→ theory (SM, SSM) was wrong

→ experiments were wrong (all of them?)

→ something was happening to neutrinos

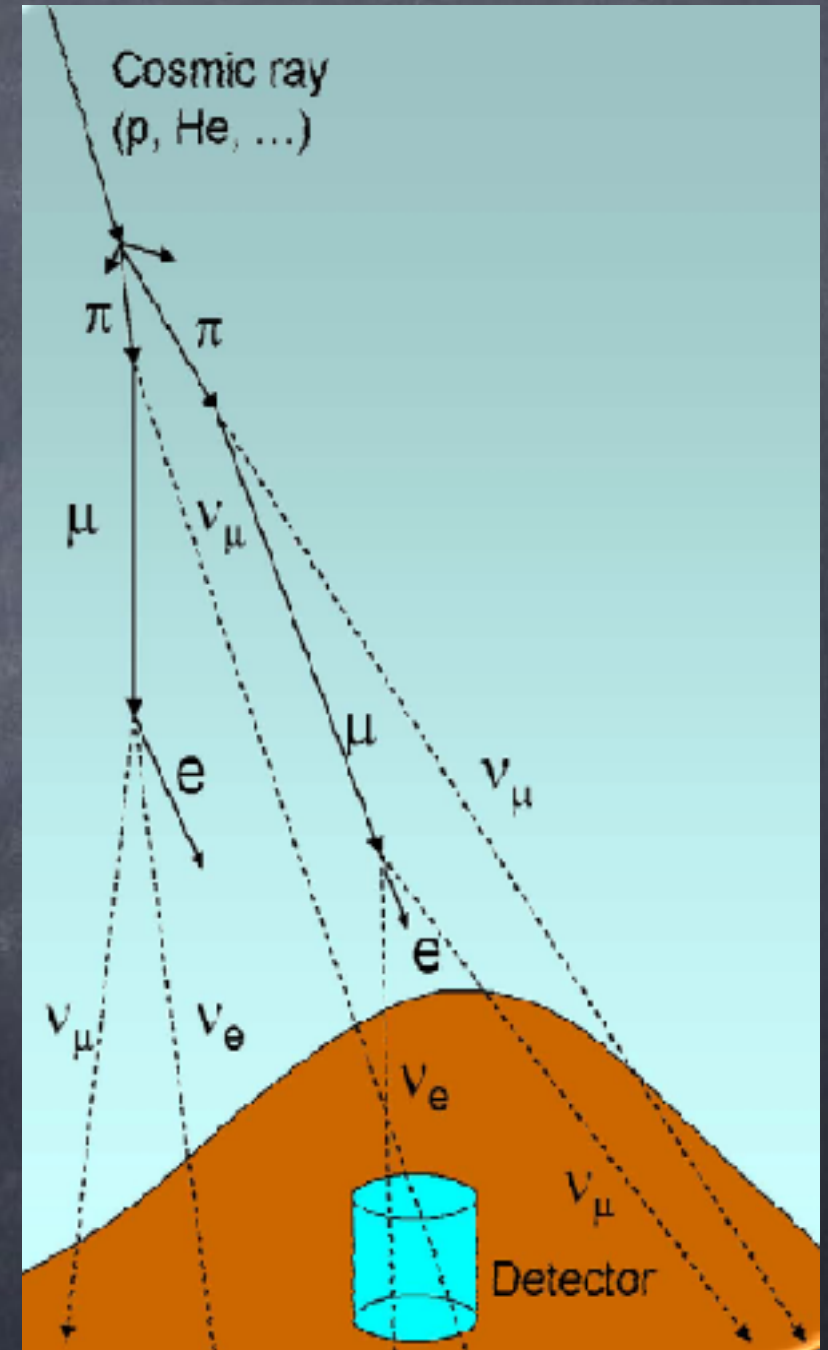


Raymond Davis Jr.

The atmospheric ν anomaly

- ▶1985: First indications of a deficit in the observed number of atmospheric ν_μ at the IMB experiment.
- ▶1994: Kamiokande finds the ν_μ deficit depends on the distance travelled by the neutrino.
- ▶1998: Discovery of **atmospheric neutrino oscillations** in Super-Kamiokande.
 - ⇒ first evidence for non-zero **neutrino masses**.

oscillation channel $\nu_\mu \rightarrow \nu_\tau$



Other important dates

M. Koshihara

- ▶1987: Supernova neutrino detection from supernova 1987A in Kamiokande & IMB.



2002 Nobel Prize in Physics

- ▶2001: Sudbury Neutrino Observatory (SNO) confirms a change of flavor in solar ν_e flux.
- ▶2002: KamLAND experiment confirms solar neutrino oscillations using neutrinos from nuclear reactors
- ▶2011–2012: neutrino oscillations observed in solar, atmospheric, reactor and accelerator neutrino experiments.

Neutrinos in the Standard Model

Neutrinos in the Standard Model

Elementary Particles						
Quarks	u up	c charm	t top	Force Carriers	γ photon	
	d down	s strange	b bottom		g gluon	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino		Z Z boson	
	e electron	μ muon	τ tau		W W boson	
I			II	III		
Three Families of Matter						

- neutrinos come in 3 flavours, corresponding to the charged lepton associated

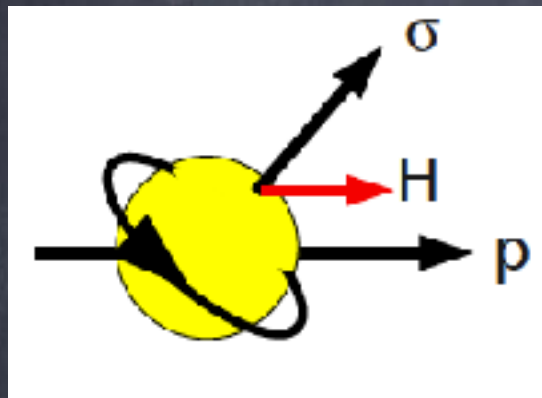
- they belong to $SU(2)$ lepton doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

- In the SM, there are no $SU(2)$ neutrino singlets (alike e_R, μ_R, τ_R)
- neutrinos are left handed and antineutrinos right handed

Helicity and Chirality (handedness)

► **Helicity** is the projection of spin along the momentum direction



$$\hat{H} = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

- Lorentz-invariant only for massless particles
- conserved in time

Massless particles: Helicity = Chirality

Massive particles: Chiral states contain contributions from both helicity states

Ultra-relativistic particles: LH (RH) chiral projection dominated by a - (+) helicity state

► **Chirality** is an asymmetry property: a chiral object is not identical to its mirror image, cannot be superimposed on it.

$$P_{L,R} = \frac{1 \mp \gamma_5}{2}, \psi_{L,R} = P_{L,R} \psi$$

- Lorentz-invariant although not directly measurable
- not conserved: mass terms mix LH and RH chiral states

Neutrino interactions in the SM

- ▶ neutrinos interact only through the weak force

$$W^- \rightarrow l_\alpha^- + \bar{\nu}_\alpha$$

$$\alpha = e, \mu, \tau$$

Charged Current (CC):

$$W^+ \rightarrow l_\alpha^+ + \nu_\alpha$$

$$\mathcal{L}_{\text{int}}^{\text{CC}} = -\frac{g}{\sqrt{2}} \left(\sum_\alpha \bar{\nu}_{\alpha L} \gamma_\rho l_{\alpha L} W^\rho + \text{h.c.} \right)$$

in the SM, only LH
neutrinos and RH
antineutrinos
participate in weak
interactions

Neutral Current (NC): $Z^0 \rightarrow \nu_\alpha + \bar{\nu}_\alpha$

$$\mathcal{L}_{\text{int}}^{\text{NC}} = -\frac{g}{4 \cos \theta_W} \left(\sum_\alpha \bar{\nu}_\alpha \gamma_\rho (1 - \gamma_5) \nu_\alpha Z^\rho + \text{h.c.} \right)$$

- ▶ interactions conserve total **Lepton Number** L
- ▶ family lepton numbers L_e, L_μ, L_τ are also conserved (1998: nu oscill !!)

Neutrino mass in the Standard Model

- ▶ In the SM, fermion masses appears in the lagrangian in the term:

$$m\bar{\psi}\psi \quad \rightarrow \text{Dirac mass term}$$

decomposing into its chiral states: $\psi = \nu \equiv \nu_L + N_R$

$$-\mathcal{L}_D = m_D \bar{\nu}\nu = m_D (\bar{\nu}_L + \bar{N}_R)(\nu_L + N_R) = m_D (\bar{\nu}_L N_R + \bar{N}_R \nu_L)$$

- mass couples L and R chiral states of a particle: flips chirality
- OK for most of particles but SM neutrino has only a L-chiral state
- But in the SM there are no R-chiral states for neutrinos, N_R
- ▶ Therefore, neutrinos are massless in the SM

Neutrino masses: Majorana neutrinos

► Other option: try to make a mass term from ν_L alone Majorana, ~1930

→ a R-chiral field from a L-chiral field by charge conjugation:

$$\psi_R \equiv \psi_L^C = \hat{C} \overline{\psi_L}^T \quad \hat{C} = i\gamma^2\gamma^0$$

→ the total neutrino field is: $\psi = \psi_L + \psi_R = \psi_L + \psi_L^C$ 2 degrees of freedom

→ taking the charge conjugate $\psi^C = (\psi_L + \psi_L^C)^C = \psi_L^C + \psi_L = \psi$

$$\psi = \nu = \nu_L + \nu_L^C$$

neutrino = antineutrino

► Majorana mass term:

$$-\mathcal{L}_M = \frac{1}{2}m(\overline{\nu_L^c}\nu_L + \overline{\nu_L}\nu_L^c)$$

However: this mass term not invariant under weak isospin

Dirac mass term

$$-\mathcal{L}_D = m_D(\overline{\nu}_L N_R + \overline{N}_R \nu_L)$$

under U(1) transformation:

invariant

$$\psi \rightarrow e^{i\alpha} \psi, \quad \overline{\psi} \rightarrow \overline{\psi} e^{-i\alpha}$$

→ conserves all charges (Q, L, B)

Majorana mass term

$$-\mathcal{L}_M = \frac{1}{2} m(\overline{\nu}_L^c \nu_L + \overline{\nu}_L \nu_L^c)$$

not invariant

→ breaks all charges in 2 units

1) charged particles must be Dirac **OR** only neutral particles can be Majorana

→ neutrino, with $Q(\nu) = 0$, can be Majorana

2) if neutrinos are Majorana, total **lepton number** is not conserved

3) if neutrinos are Dirac, L conservation has to be imposed by hand

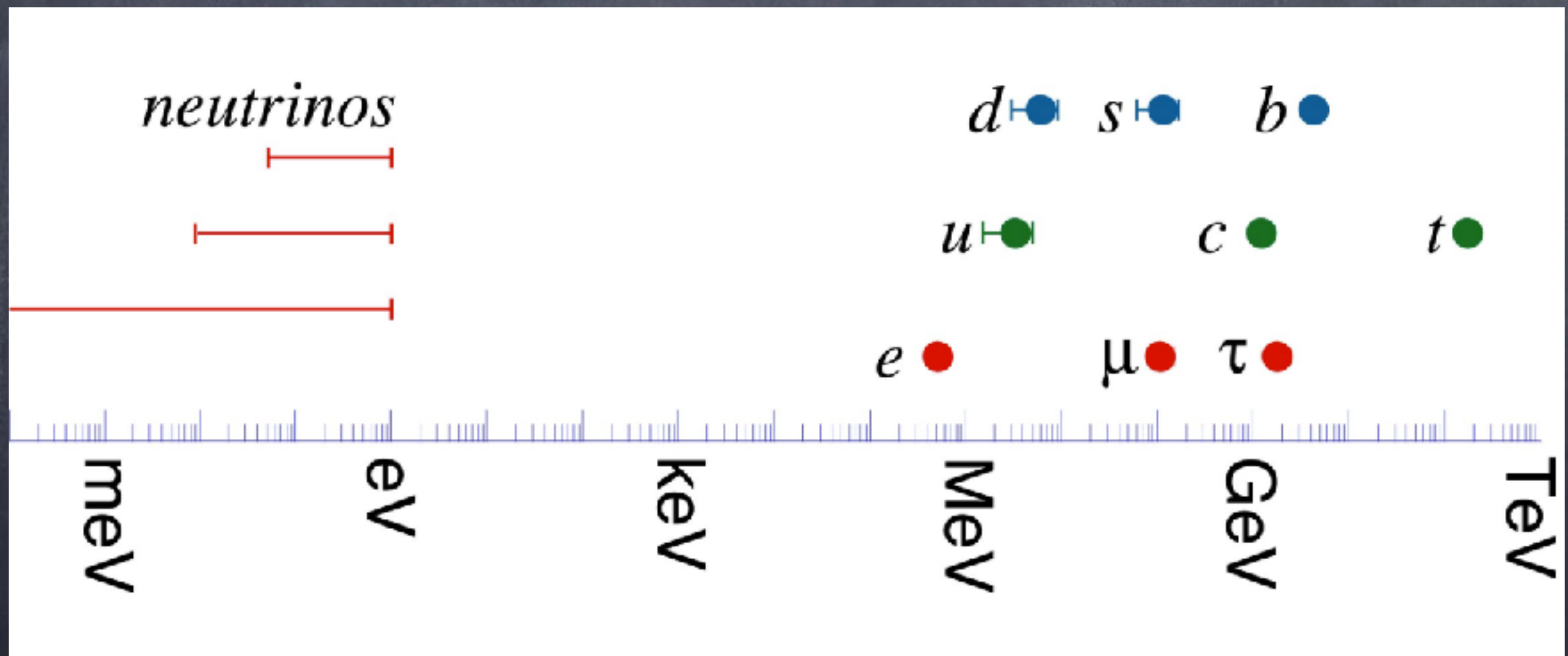
However: none of the terms can be constructed in SM

→ no N_R in SM

→ $\overline{\nu}_L^c \nu_L$ forbidden by weak isospin

Neutrinos are massless within the SM

But from oscillations we know neutrinos
do have mass!!



$$m_\nu \sim 0 - 1 \text{ eV}$$

Neutrino masses beyond the Standard Model

Dirac mass term

$$-\mathcal{L}_D = m_D(\overline{\nu}_L N_R + \overline{N}_R \nu_L)$$

Majorana mass term

$$-\mathcal{L}_M = \frac{1}{2}m(\overline{\nu}_L^c \nu_L + \overline{\nu}_L \nu_L^c)$$

under U(1) transformation:

invariant

$$\psi \rightarrow e^{i\alpha} \psi,$$

$$\overline{\psi} \rightarrow \overline{\psi} e^{-i\alpha}$$

not invariant

→ conserves all charges (Q, L, B)

→ breaks all charges in 2 units

1) charged particles must be Dirac **OR** only neutral particles can be Majorana

→ neutrino, with $Q(\nu) = 0$, can be Majorana

2) if neutrinos are Majorana, total **lepton number** is not conserved

3) if neutrinos are Dirac, L conservation has to be imposed by hand

However: none of the terms can be constructed in SM

→ no N_R in SM

→ $\overline{\nu}_L^c \nu_L$ forbidden by weak isospin

Neutrinos are massless within the SM

add N_R

add Higgs triplet

Dirac mass term

Minimal extension SM: add N_R → "sterile" neutrino

► 4 components Dirac neutrino: ν_L , $\bar{\nu}_L$, N_R , $\bar{N}_R$ 4 degrees of freedom

→ decomposing into its chiral states: $\psi = \nu \equiv \nu_L + N_R$

$$-\mathcal{L}_D = m_D \bar{\nu} \nu = m_D (\bar{\nu}_L + \bar{N}_R) (\nu_L + N_R) = m_D (\bar{\nu}_L N_R + \bar{N}_R \nu_L)$$

► From ν oscill: $m_\nu \geq \sqrt{\Delta m_{31}^2} = 0.05 \text{ eV}$

$$\mathcal{L}_{\text{Yukawa}} = Y_\nu (\bar{\nu}_e \bar{e})_L \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} N_R + h.c.$$

→ after SSB: $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$ $m_D = Y_\nu \frac{v}{\sqrt{2}} \rightarrow Y_\nu \simeq 10^{-13}$

much smaller than other Yukawas: $Y_e \simeq 10^{-5}$

Seesaw mechanism for neutrino mass

- ▶ Add a right handed neutrino singlet under SU(2)xU(1): $N = N_R + N_R^C$

SU(2) forbidden

$$\nu = \nu_L + \nu_L^C$$

- ▶ Most general mass term:

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_M = \frac{1}{2} (\overline{\nu}_L \quad \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}$$

→ diagonalization:

$$\frac{1}{2} (\overline{\nu} \quad \overline{N}) \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}$$

not mass eigenstates

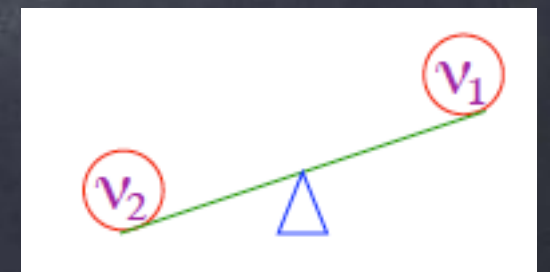
$$(m_D \simeq v Y_\nu)$$

for $M_R \gg m_D$:

$$M_1 \simeq \frac{m_D^2}{M_R}$$

$$M_2 \simeq M_R$$

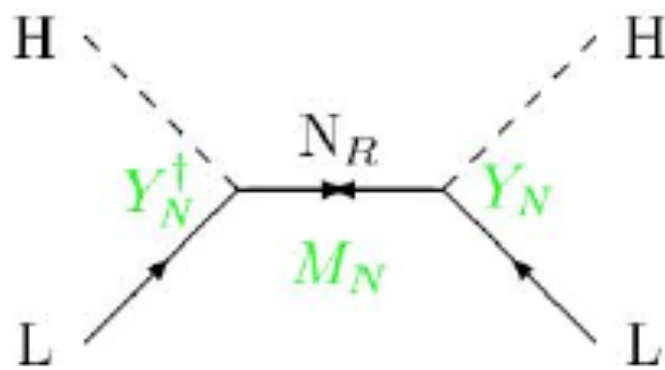
→ seesaw mechanism



Seesaw mass models

⇒ ν masses are generated through mixing with heavy particles

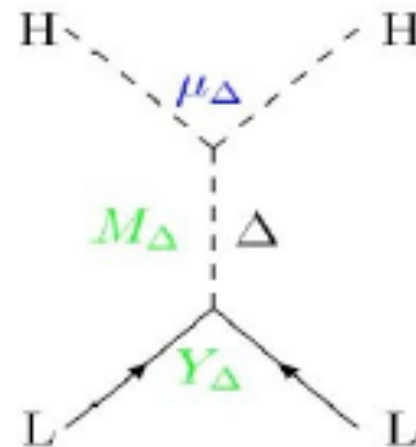
Right-handed singlet:
(type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky;
Yanagida; Glashow; Mohapatra, Senjanovic

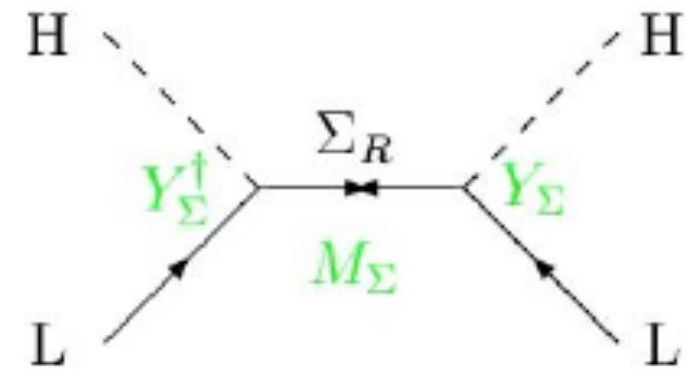
Scalar triplet:
(type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi;
Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet:
(type-III seesaw)



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy; T.H., Lin,
Notari, Papucci, Strumia; Bajc, Nemevsek,
Senjanovic; Dorsner, Fileviez-Perez;

Low energy seesaw models

Inverse seesaw model

Mohapatra and Valle, PRD 34 (1986) 1642

Extended lepton content: (ν, ν^c, S) $L=(+1,-1,+1)$

ν, ν^c are $SU(2)$ singlets

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix} \rightarrow m_\nu = M_D (M^T)^{-1} \mu M^{-1} M_D^T$$

- μ breaks L and generates neutrino mass (massless for $\mu=0$)
- m_ν can be very light even if M is far below GUT scale:

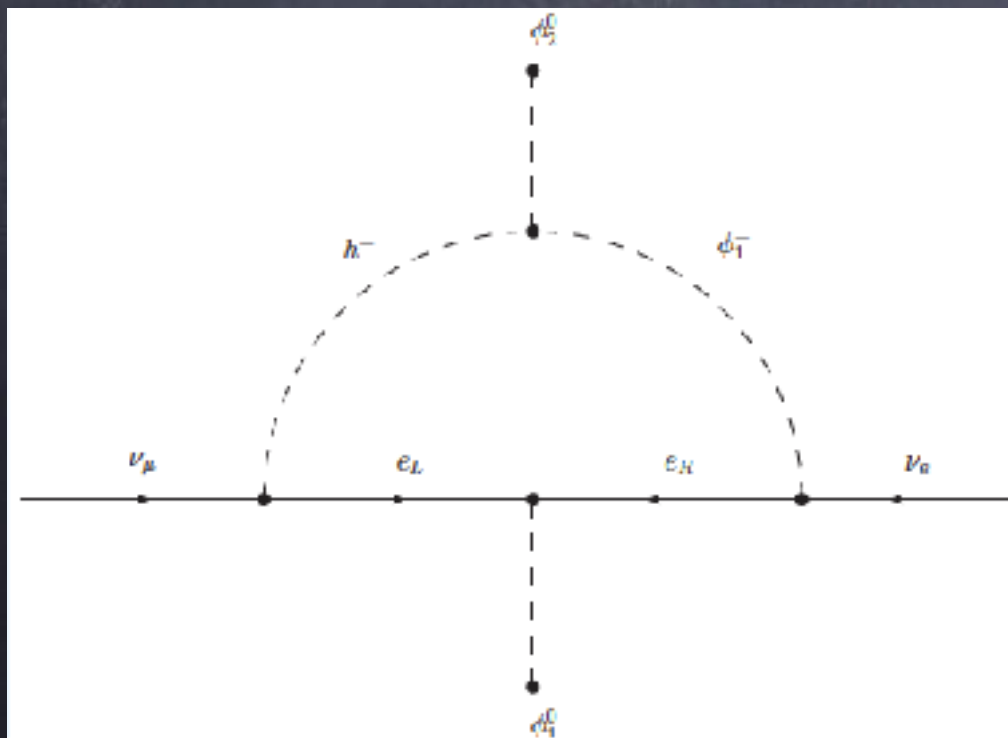
$$\text{with } \mu \sim \text{keV} \text{ and } M \sim 10^3 \text{ GeV} \rightarrow m_\nu \sim \text{eV}$$

Radiative models of neutrino masses

- * extension of scalar sector of the SM
- * neutrino masses can be generated through loops
 - ⇒ loop suppression accounts for the smallness of m_ν

Zee model

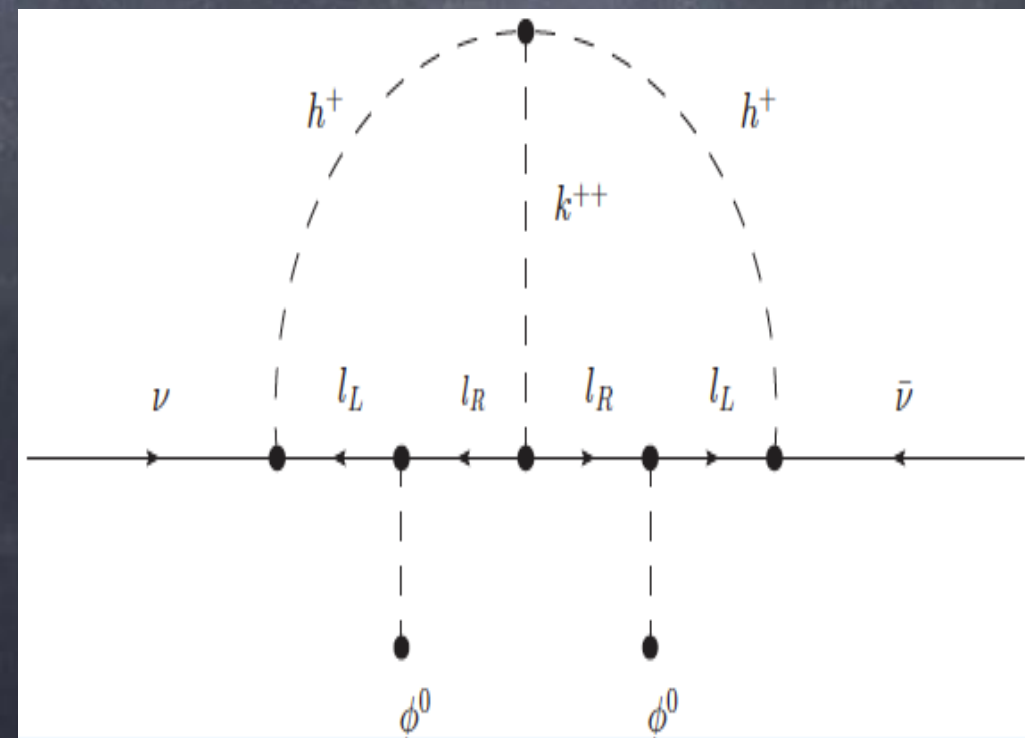
- + singlet scalar $h^\pm$
- + extra Higgs doublet H



Zee, PLB 93 (1980) 389

Zee-Babu model

- + singlet scalar $h^\pm$
- + singlet scalar k^{++}



Zee, NPB 264 (1986) 99; Babu, PLB 203 (1988) 132

The flavour problem

- ▶ seesaw models explain the smallness of neutrino masses

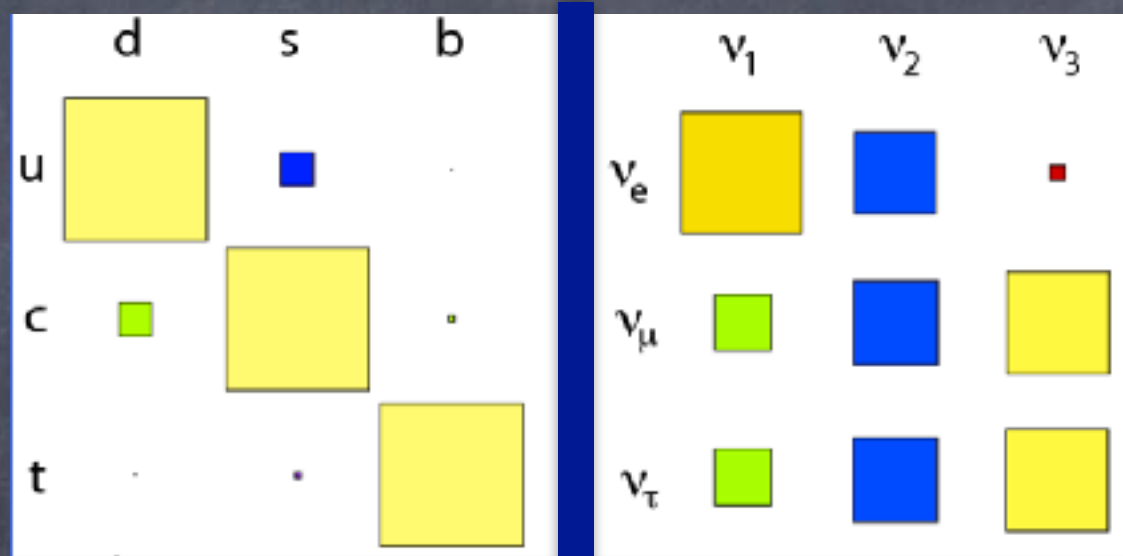
However, they can not explain:

- ▶ Why quark and lepton mixings are so different?

$$\theta_{12} \simeq 13^\circ$$

$$\theta_{13} \simeq 0.2^\circ$$

$$\theta_{23} \simeq 2.4^\circ$$



$$\theta_{12} \simeq 34^\circ$$

$$\theta_{13} \simeq 9^\circ$$

$$\theta_{23} \simeq 49^\circ$$

- ▶ Why do fermion masses show these hierarchical relations?

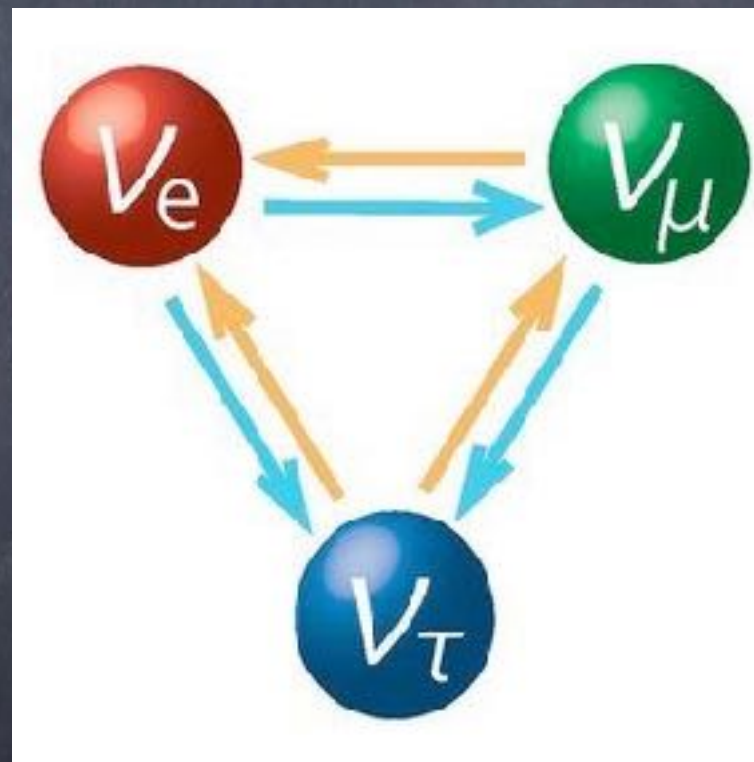
$$m_e \ll m_\mu \ll m_\tau$$

$$m_u, m_d \ll m_c, m_s \ll m_t, m_b$$

⇒ One can add new symmetries of leptons to the Standard Model

$$SU_c(3) \times SU_L(2) \times U_Y(1) \times G_f$$

Neutrino oscillations



Neutrino mixing

- ▶ Mixing is described by the **Maki-Nakagawa-Sakata** (MNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$$

- ▶ leptonic weak charged current:

$$j_{\rho}^{\text{CC}\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\alpha}_L \gamma_{\rho} \nu_{\alpha L} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{\alpha}_L \gamma_{\rho} U_{\alpha k} \nu_{k L}$$

$$U = U_l^{\dagger} U_{\nu}$$

- ▶ NxN unitary matrix: NxN mixing parameters

$$\rightarrow N(N-1)/2 \text{ mixing angles} + N(N+1)/2 \text{ phases}$$

- ▶ Lagrangian invariant under global phase transformations of **Dirac** fields:

$$\alpha \rightarrow e^{i\theta_{\alpha}} \alpha, \nu_k \rightarrow e^{i\phi_k} \nu_k$$

$$j_{\rho}^{\text{CC}\dagger} \rightarrow 2 \sum_{\alpha,k} \overline{\alpha}_L e^{-i(\theta_e - \phi_1)} e^{-i(\theta_{\alpha} - \theta_e)} \gamma_{\rho} U_{\alpha k} e^{i(\phi_k - \phi_1)} \nu_{k L}$$

$\underset{1}{\phantom{e^{-i(\theta_e - \phi_1)}}}$
 $\underset{N-1}{\phantom{e^{-i(\theta_{\alpha} - \theta_e)}}}$
 $\underset{N-1}{\phantom{e^{i(\phi_k - \phi_1)}}}$

$$\rightarrow 2N-1 \text{ phases can be eliminated: } (N-1)(N-2)/2 \text{ physical phases}$$

Neutrino mixing

► For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \qquad \nu_{kL}^T C^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T C^\dagger \nu_{kL}$$

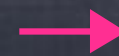
→ only N phases can be eliminated by rephasing charged lepton fields:

$$j_\rho^{CC^\dagger} \rightarrow 2 \sum_{\alpha, k} \overline{\alpha}_L e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} \nu_{kL}$$

N

→ $N(N-1)/2$ **physical phases**:

$(N-1)(N-2)/2$ Dirac phases



effect in ν oscil.

$(N-1)$ Majorana phases



relevant for $0\nu\beta\beta$

Neutrino mixing

- ▶ 2-neutrino mixing depends on 1 angle only (+1 Majorana phase)

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

- ▶ 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL
measurements

reactor disapp + LBL
appearance searches

solar + KamLAND
measurements

Neutrino oscillations

► flavour states are admixtures of mass eigenstates: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$

► Neutrino evolution equation: $-i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$

in the neutrino mass eigenstates basis ν_j :

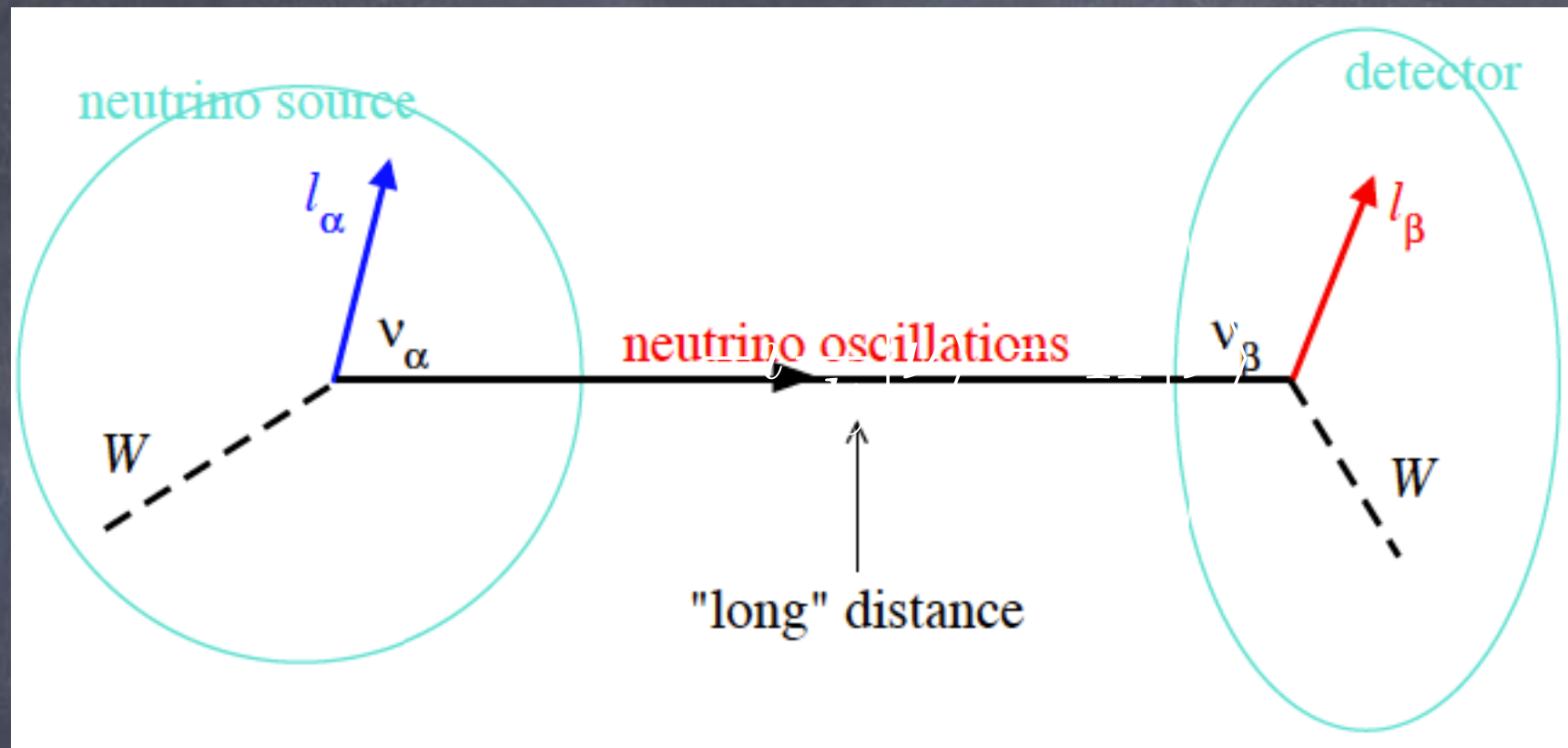
$$H = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \quad \longrightarrow \quad |\nu_j\rangle \rightarrow e^{-iE_j t} |\nu_j\rangle$$

equal momentum approx: $E_j \simeq p + \frac{m_j^2}{2p} \simeq p + \frac{m_j^2}{2E}$

for relativistic neutrinos: $t = L$

$$\longrightarrow \quad |\nu_j\rangle \rightarrow e^{-ipL} e^{-i \frac{m_j^2 L}{2E}} |\nu_j\rangle \rightarrow e^{-i \frac{m_j^2 L}{2E}} |\nu_j\rangle$$

Neutrino oscillations picture



Production

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j}^* |\nu_j\rangle$$

coherent superposition
of massive states

Propagation

$$\nu_j : e^{-i \frac{m_j^2 L}{2E}}$$

different propagation
phases change ν_j
composition

Detection

$$\langle \nu_\beta | = \sum_j \langle \nu_j | U_{\beta j}$$

projection over flavour
eigenstates

Neutrino oscillation probability

Neutrino oscillation amplitude:

$$\begin{aligned}
 A_{\nu_\alpha \rightarrow \nu_\beta} &= \langle \nu_\beta(t) | \nu_\alpha(0) \rangle = \sum_j \langle \nu_\beta | \nu_j(t) \rangle \langle \nu_j(t) | \nu_j(0) \rangle \langle \nu_j(0) | \nu_\alpha \rangle \\
 &= \sum_j U_{\beta j} e^{-i \frac{m_j^2 L}{2E}} U_{\alpha j}^*
 \end{aligned}$$

detection production
propagation

Neutrino oscillation probability:

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_j U_{\beta j} e^{-i \frac{m_j^2 L}{2E}} U_{\alpha j}^* \right|^2$$

$$\begin{aligned}
 P_{\alpha\beta} &= \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) + \\
 &\quad + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right)
 \end{aligned}$$

General properties of neutrino oscillations

► Conservation of probability: $\sum_{\beta} P(\nu_{\alpha} \rightarrow \nu_{\beta}) = 1$

► For antineutrinos: $U \rightarrow U^*$

► Neutrino oscillations violate flavour lepton number conservation (expected from mixing) but conserve **total lepton number**

► Complex phases in the mixing matrix induce **CP violation**:

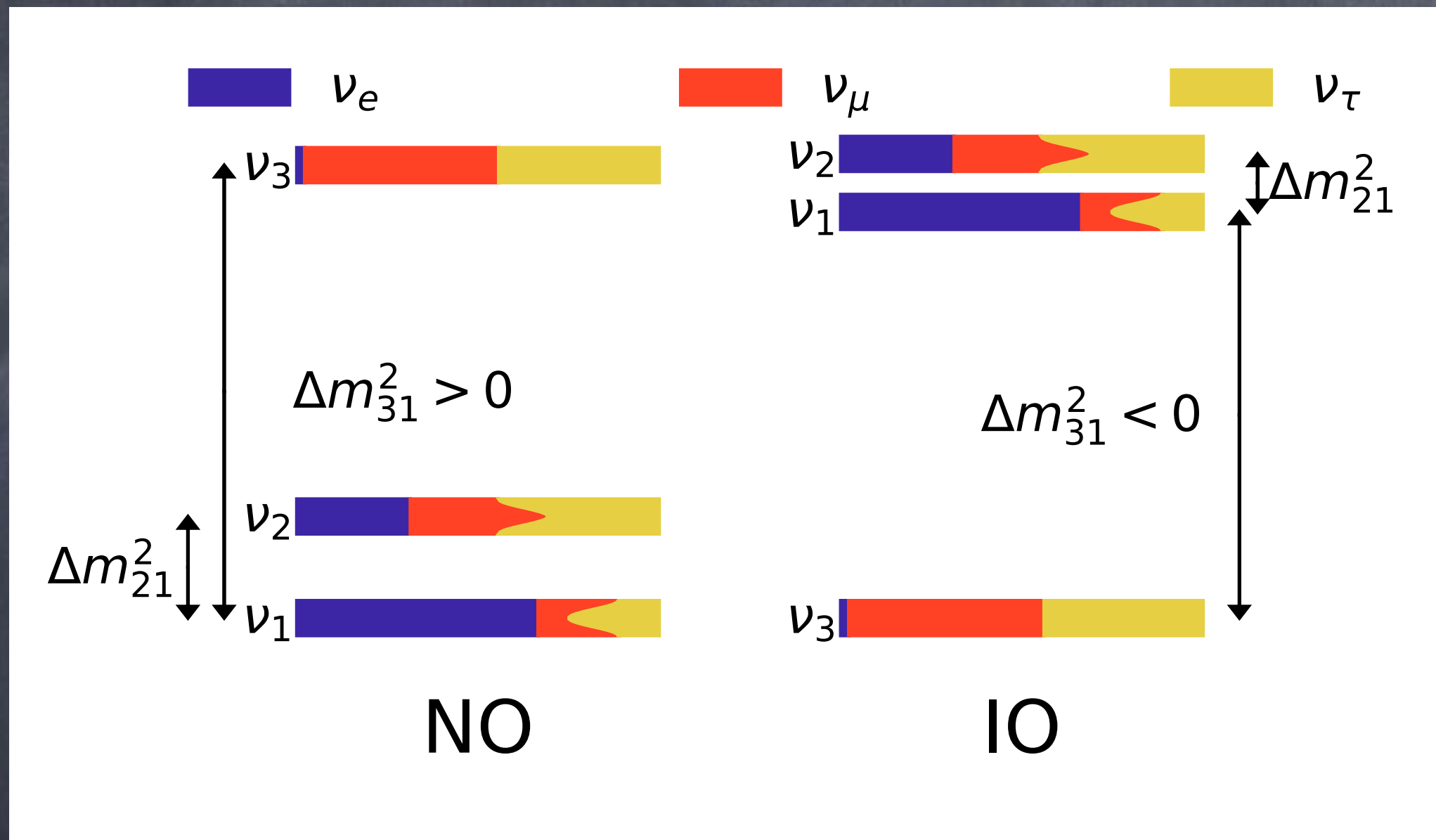
$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) \neq P(\overline{\nu}_{\alpha} \rightarrow \overline{\nu}_{\beta})$$

► Neutrino oscillations do not depend on the absolute neutrino mass scale and Majorana phases.

► Neutrino oscillations are sensitive only to **mass squared differences**:

$$\Delta m_{kj}^2 = m_k^2 - m_j^2$$

Two possible mass orderings:



- Δm_{31}^2 : atmospheric + long-baseline
- Δm_{21}^2 : solar + KamLAND (we know it is positive)

Oscillation experiments favor NO with $\sim 3\sigma$ (2018)

2-neutrino oscillations

► 2-neutrino mixing matrix:
$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

► 2-neutrino oscillation probability ($\alpha \neq \beta$):

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} \right|^2$$
$$= \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

► The oscillation phase:

$$\phi = \frac{\Delta m_{21}^2 L}{4E} = 1.27 \frac{\Delta m_{21}^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]}$$

→ short distances, $\phi \ll 1$: oscillations do not develop, $P_{\alpha\beta} = 0$

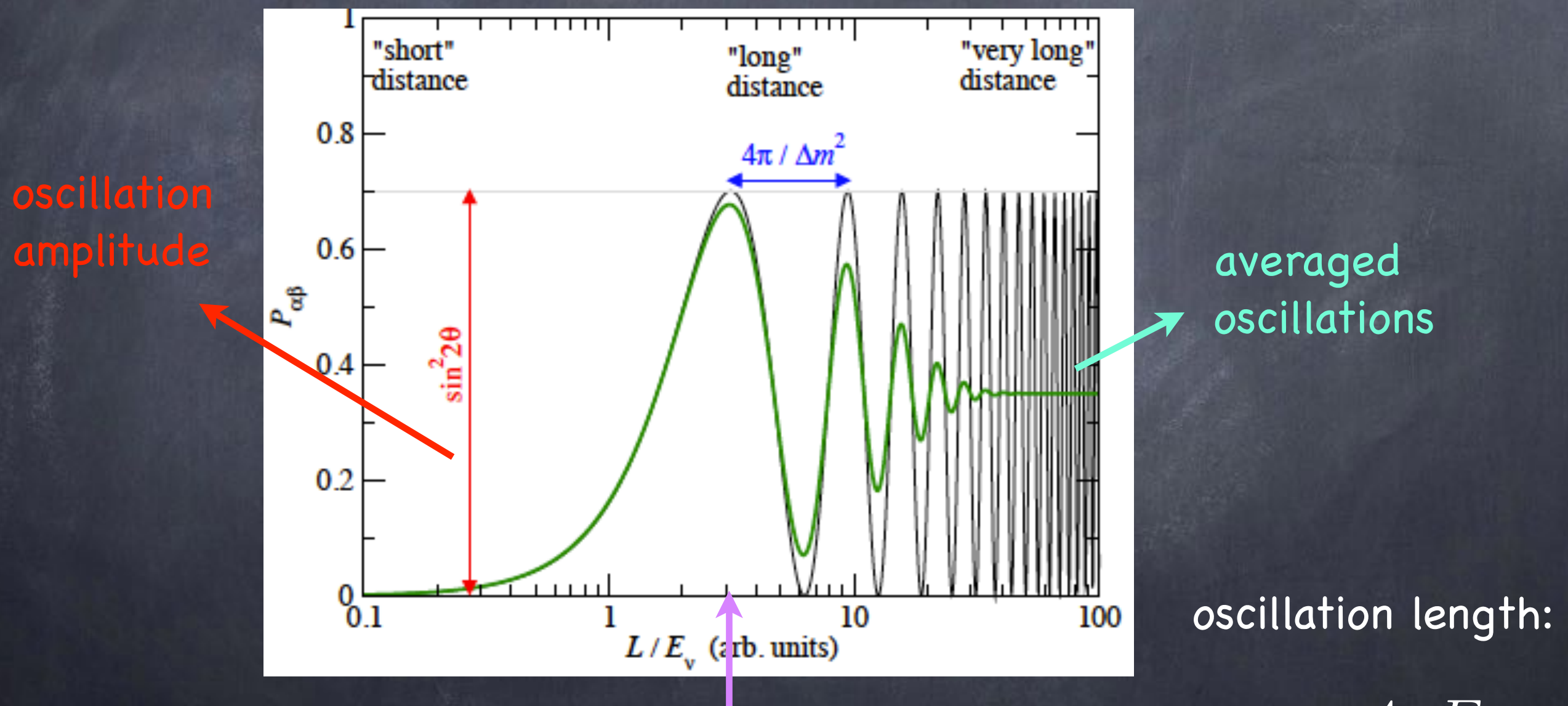
→ long distance, $\phi \sim 1$: oscillations are observable

→ very long distances, $\phi \gg 1$: oscillations are averaged out:

$$P_{\alpha\beta} \simeq \frac{1}{2} \sin^2(2\theta)$$

2-neutrino oscillation probability

$$P_{\alpha\beta} = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$



oscillation
amplitude

averaged
oscillations

oscillation length:

first oscillation maximum:

$$L_{osc} = \frac{4\pi E}{\Delta m^2}$$

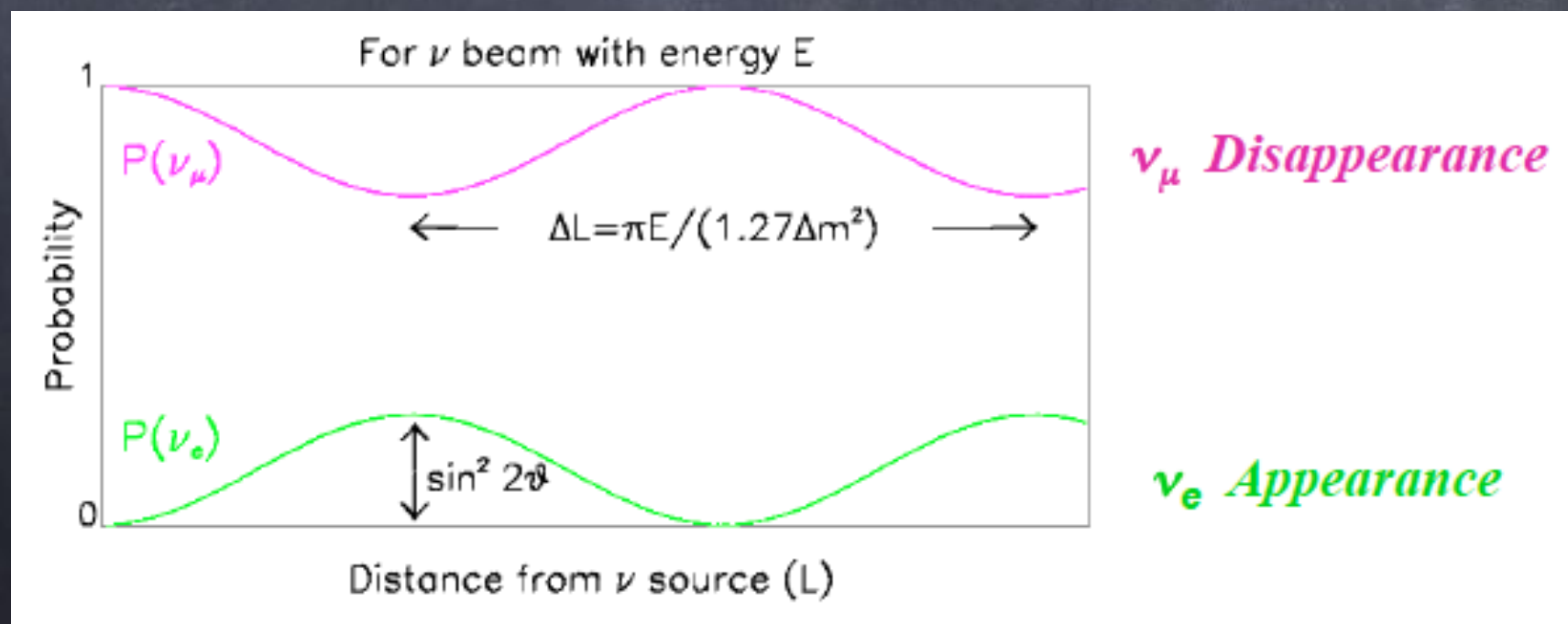
Appearance vs disappearance experiments

► appearance experiments: $\alpha \neq \beta$
$$P_{\alpha\beta} = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

→ appearance of a neutrino of a new flavour β in a beam of ν_α

► disappearance experiments: $P_{\alpha\alpha} = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

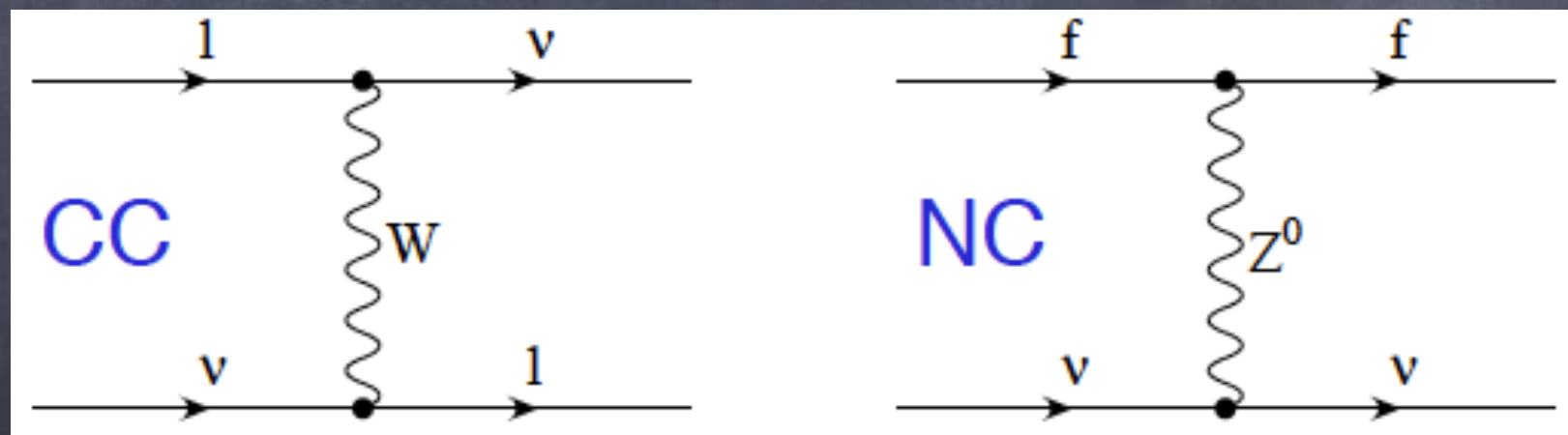
→ measurement of the survival probability of a neutrino of given flavour



Matter effects on neutrino oscillations

► When neutrinos pass through matter, the interactions with the particles in the medium induce an **effective potential** for the neutrinos.

[→ the coherent forward scattering amplitude leads to an index of refraction for neutrinos. L. Wolfenstein, 1978]



→ modifies the mixing between flavor states and propagation states as well as the eigenvalues of the Hamiltonian, leading to a different oscillation probability with respect to vacuum oscillations.

Effective matter potential

- Effective four-fermion interaction Hamiltonian (CC+NC)

$$H_{\text{int}}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\alpha \gamma_\mu (1 - \gamma_5) \nu_\alpha \sum_j \bar{f} \gamma^\mu \underbrace{(g_V^{\alpha,f} - g_A^{\alpha,f} \gamma_5)}_{J_{\text{matt}}^{\mu\alpha}} f$$

in ordinary matter: $f=e-,p,n$

To obtain the matter-induced potential we integrate over f -variables:

for a	non-relativistic	medium:	$\langle \bar{f} \gamma^\mu f \rangle = \frac{1}{2} N_f \delta_{\mu,0}$
	unpolarised		$\langle f \gamma_5 \gamma^\mu f \rangle = 0$
	neutral		$N_e = N_p$

$$J_{\text{matt}}^{\mu\alpha} = \frac{1}{2} [N_e (g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}]$$

Effective matter potential

$$J_{\text{matt}}^{\mu\alpha} = \frac{1}{2} [N_e (g_V^{\alpha,e} + g_V^{\alpha,p}) + N_n g_V^{\alpha,n}]$$

g_V	e^-	p	n
ν_e	$2 \sin^2 \Theta_W + \frac{1}{2}$	$-2 \sin^2 \Theta_W + \frac{1}{2}$	$-\frac{1}{2}$
$\nu_{\mu,\tau}$	$2 \sin^2 \Theta_W - \frac{1}{2}$	$-2 \sin^2 \Theta_W + \frac{1}{2}$	$-\frac{1}{2}$

$$J_{\text{matt}}^{\mu\alpha} = (N_e - \frac{1}{2}N_n, -\frac{1}{2}N_n, -\frac{1}{2}N_n)$$

➡ $V_{\text{matt}} = \sqrt{2}G_F \text{diag}(N_e - \frac{1}{2}N_n, -\frac{1}{2}N_n, -\frac{1}{2}N_n)$

- ▶ only ν_e are sensitive to CC (no μ, τ in ordinary matter)
- ▶ NC has the same effect for all flavours → it has no effect on evolution
(however it can be important in presence of sterile neutrinos)
- ▶ for antineutrinos the potential has opposite sign

2-neutrino oscillations in matter

- ▶ Hamiltonian in **vacuum** in the flavour basis:

$$H_f^{vac} = U H_m U^\dagger = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- ▶ Effective hamiltonian in **matter**

$$H_f^{matt} = H_f^{vac} + V_{eff} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + V_{CC} & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix}$$

$$V_{CC} = \sqrt{2} G_F N_e$$

Diagonalizing the Hamiltonian, we identify the mixing angle and mass splitting in matter:

$$H_f^{matt} = \frac{\Delta M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

In general: $N_e = N_e(x)$, so θ_M and ΔM^2 will be function of x as well

→ however, in some cases analytical solutions can be obtained

2- ν oscillations in constant matter

► If N_e is constant (good approximation for oscillations in the Earth crust):

→ θ_M and ΔM^2 are constant as well

→ we can use vacuum expression for oscillation probability, replacing “vacuum” parameters by “matter” parameters:

$$P_{\alpha\beta} = \sin^2(2\theta_M) \sin^2\left(\frac{\Delta M^2 L}{4E}\right)$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \quad A = \frac{2EV}{\Delta m^2}$$

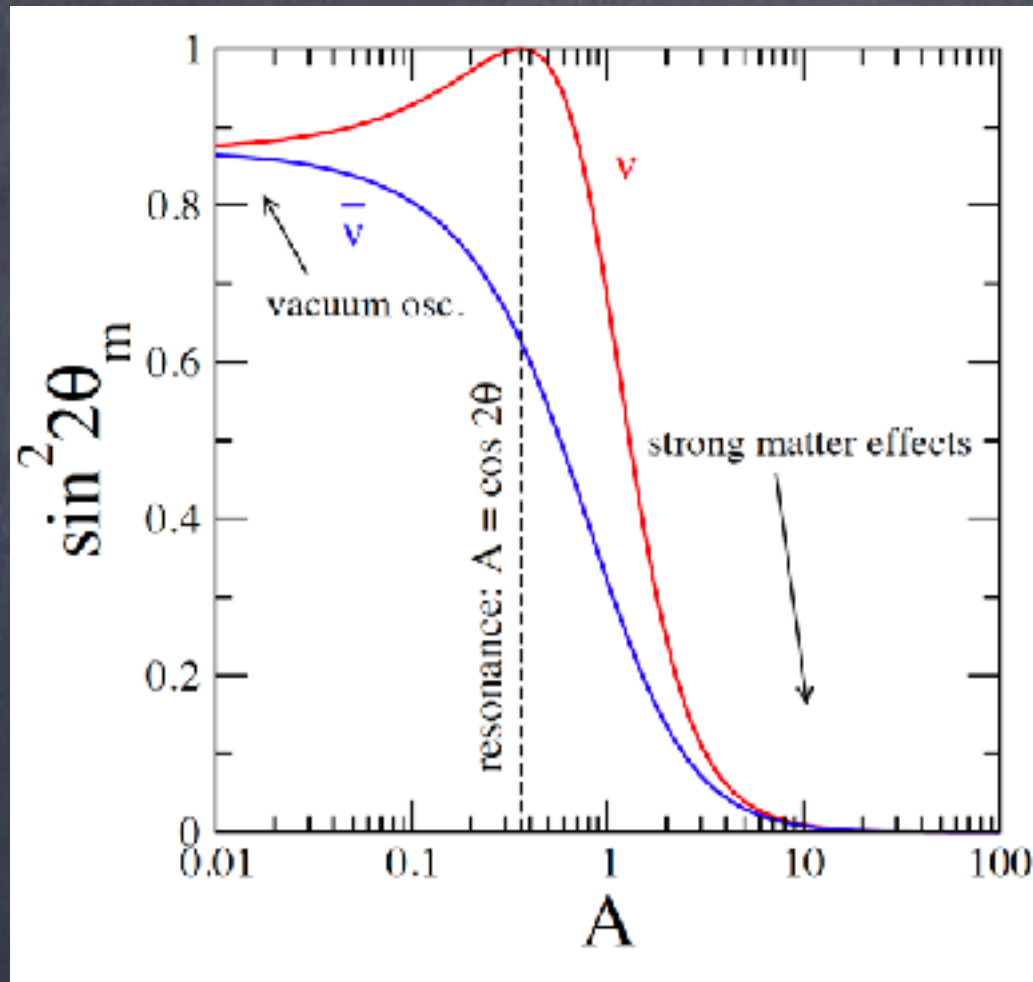
$$\Delta M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

There is a **resonance** effect for $A = \cos 2\theta \rightarrow$ **MSW effect**

Wolfenstein, 1978

Mikheyev & Smirnov, 1986

2- ν oscillations in constant matter



mixing angle in matter:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

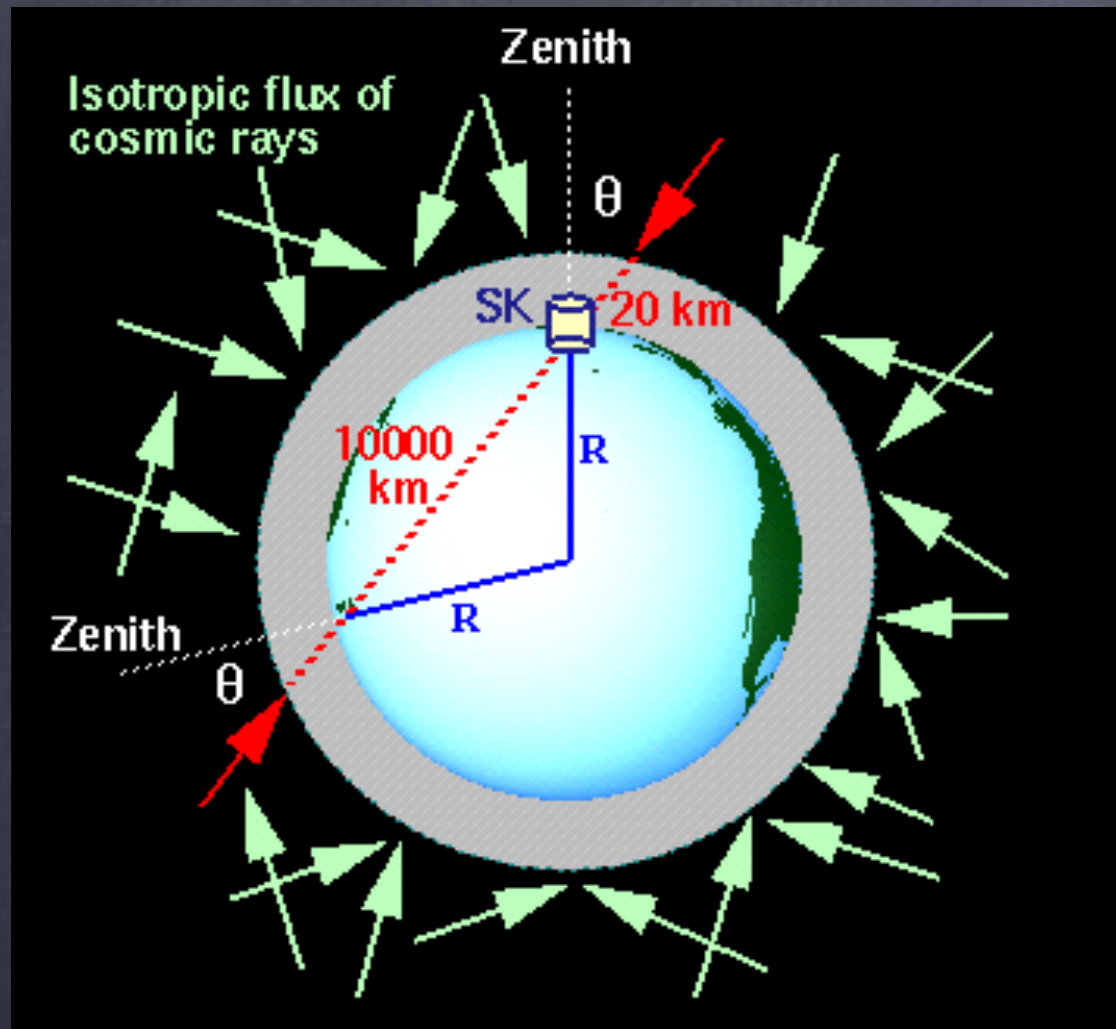
$$A = \frac{2EV}{\Delta m^2}$$

Tutorials: matter effects in solar neutrinos

- ▶ $A \ll \cos 2\theta$, small matter effect $\rightarrow$ vacuum oscillations: $\theta_M = \theta$
- ▶ $A \gg \cos 2\theta$, matter effects dominate $\rightarrow$ oscillations suppressed: $\theta_M \approx \pi/2$
- ▶ $A = \cos 2\theta$, resonance takes place $\rightarrow$ maximal mixing $\theta_M \approx \pi/4$

$\rightarrow$ **resonance condition** is satisfied for neutrinos for $\Delta m^2 > 0$
for antineutrinos for $\Delta m^2 < 0$

Matter effects in atmospheric ν 's



- atmospheric neutrinos interact with the Earth mantle and core
- ✓ no matter effects in $\nu_\mu \rightarrow \nu_\tau$ channel
- ✓ MSW resonance in $\nu_\mu \rightarrow \nu_e$ channel

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{4E} \sin 2\theta}{\frac{\Delta m^2}{4E} \cos 2\theta \mp \sqrt{2}G_F N_e}$$

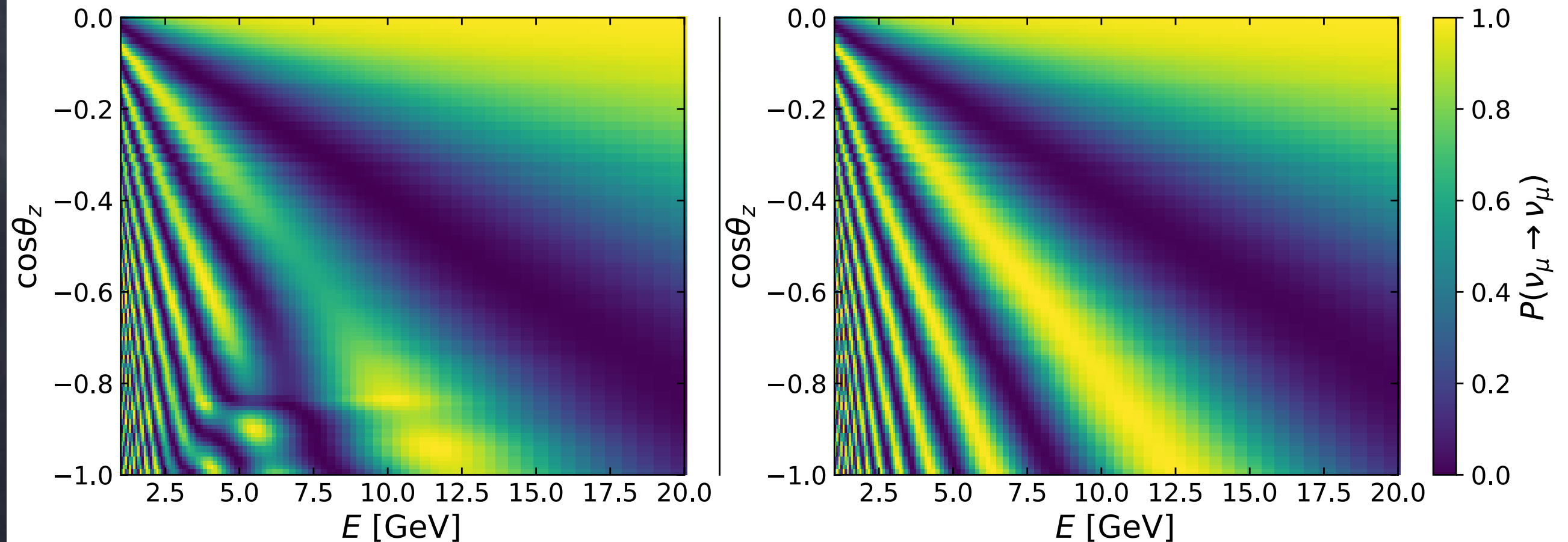
(-) neutrinos (+)antineutrinos

➔ Matter effects on the atmospheric neutrino flux are sensitive to the **mass ordering**.

Matter effects in atmospheric ν 's

NO

IO



de Salas et al, [arXiv:1806.11051](https://arxiv.org/abs/1806.11051)

At $E \sim 3-8$ GeV: MSW resonance for neutrinos and NO mass spectrum.

If IO $\Rightarrow$ resonance for antineutrinos