# 3. Meson Mixing & C

OUR PANSIES

#### A. Pich IFIC, U. Valencia - CSIC

TAE 2018, Centro de Física de Benasque "Pedro Pascual", Benasque, Spain, 2-15 September, 2018

## $B^0 - \overline{B}^0$ MIXING



very small

• 
$$\Delta M_{B_d^0} / \Gamma_{B_d^0} = 0.770 \pm 0.004$$

• 
$$\Delta M_{B_s^0} = (17.757 \pm 0.021) \text{ ps}^{-1}$$

• 
$$\Delta \Gamma_{B^0} / \Delta M_{B^0} \sim m_b^2 / m_t^2 \ll 1$$

• 
$$\operatorname{Re}\left(\overline{\varepsilon}_{B_d^0}\right) = -0.0005 \pm 0.0004$$

$$\Delta M_{B_s^0} / \Gamma_{B_s^0} = 26.72 \pm 0.09$$
$$|V_{ts}|^2 \gg |V_{td}|^2$$
$$\Delta \Gamma_{B_s^0} / \Gamma_{B_s^0} = -0.130 \pm 0.009$$
$$\operatorname{Re}\left(\overline{\varepsilon}_{B_s^0}\right) = -0.0002 \pm 0.0007$$
$$|q/p| - 1 \sim m_c^2 / m_t^2$$

$$\begin{split} \mathbf{q} & \mathbf{u}, \mathbf{c}, \mathbf{t} \quad \mathbf{b} \\ \mathbf{w} & \mathbf{c}, \mathbf{t} \quad \mathbf{b} \\ \mathbf{w} & \mathbf{c}, \mathbf{t} \quad \mathbf{c}, \mathbf{t} \quad \mathbf{c}, \mathbf{t} \\ \mathbf{b} & \mathbf{u}, \mathbf{c}, \mathbf{t} \\ \mathbf{b} & \mathbf{u}, \mathbf{c}, \mathbf{t} \\ \mathbf{b} & \mathbf{c}, \mathbf{t} \\ \mathbf{c}, \mathbf{t} & \mathbf{c}, \mathbf{t} \\ \mathbf{c}, \mathbf{c}, \mathbf{c}, \mathbf{t} \\ \mathbf{c}, \mathbf{c},$$

### Widths & Mass Differences



## **Time Scales:**

**Oscillation** ~  $\sin\left[\left(x - iy\right)\Gamma t/2\right]$ 



**D**<sup>0</sup>:  $x \sim y \sim 0.01$  Slow oscillation (decays faster)

•  $B_s$ : x ~ 25 , y ~ 0.05 Fast oscillation (averages out to 0)

## **Time Scales:**

**Oscillation** ~ 
$$\sin[(x-iy)\Gamma t/2]$$

$$x \equiv \Delta M / \Gamma$$
 ,  $y \equiv \Delta \Gamma / 2 \Gamma$ 



- **D**<sup>0</sup>:  $x \sim y \sim 0.01$  Slow oscillation (decays faster)
- $B_d$ :  $x \sim 1$  ,  $y \sim 0.01$
- $B_s$ : x ~ 25 , y ~ 0.05 Fast oscillation (averages out to 0)

# $B^0 - \overline{B}^0$ MIXING AND DIRECT CP



$$\begin{split} T_{\rm f} &\equiv T[B^0 \to f] \quad ; \quad \overline{T}_{\rm f} \equiv -T[\overline{B}^0 \to f] \quad ; \quad \overline{\rho}_{\rm f} \equiv \overline{T}_{\rm f} \,/\, T_{\rm f} \\ T_{\overline{\rm f}} &\equiv T[B^0 \to \overline{\rm f}] \quad ; \quad \overline{T}_{\overline{\rm f}} \equiv -T[\overline{B}^0 \to \overline{\rm f}] \quad ; \quad \rho_{\overline{\rm f}} \equiv T_{\overline{\rm f}} \,/\, \overline{T}_{\overline{\rm f}} \end{split}$$

$$\mathcal{CP} B^0 = -\overline{B}^0$$
 ;  $\mathcal{CP} f = \overline{f}$ 

$$\Gamma[B^{0}(t) \to \mathbf{f}] \sim \frac{1}{2} e^{-\Gamma t} \left( |\mathbf{T}_{\mathbf{f}}|^{2} + |\mathbf{\overline{T}}_{\mathbf{f}}|^{2} \right) \left\{ 1 + \mathbf{C}_{\mathbf{f}} \cos(\Delta M t) - \mathbf{S}_{\mathbf{f}} \sin(\Delta M t) \right\}$$
  
$$\Gamma[\overline{B}^{0}(t) \to \overline{\mathbf{f}}] \sim \frac{1}{2} e^{-\Gamma t} \left( |\overline{\mathbf{T}}_{\mathbf{f}}|^{2} + |\mathbf{T}_{\mathbf{f}}|^{2} \right) \left\{ 1 - \mathbf{C}_{\mathbf{f}} \cos(\Delta M t) + \mathbf{S}_{\mathbf{f}} \sin(\Delta M t) \right\}$$

$$C_{\mathbf{f}} = \frac{1 - |\overline{\rho}_{\mathbf{f}}|^{2}}{1 + |\overline{\rho}_{\mathbf{f}}|^{2}} \quad ; \quad S_{\mathbf{f}} = \frac{2 \operatorname{Im} \left(\frac{q}{p} \overline{\rho}_{\mathbf{f}}\right)}{1 + |\overline{\rho}_{\mathbf{f}}|^{2}} \quad ; \quad C_{\overline{\mathbf{f}}} = -\frac{1 - |\rho_{\overline{\mathbf{f}}}|^{2}}{1 + |\rho_{\overline{\mathbf{f}}}|^{2}} \quad ; \quad S_{\overline{\mathbf{f}}} = \frac{-2 \operatorname{Im} \left(\frac{p}{q} \rho_{\overline{\mathbf{f}}}\right)}{1 + |\rho_{\overline{\mathbf{f}}}|^{2}}$$

$$\Delta \Gamma \ll \Delta M \qquad \longrightarrow \qquad \frac{q}{p} \approx \sqrt{\frac{M_{12}^{*}}{M_{12}}} \approx \frac{\mathbf{V}_{tb}^{*} \mathbf{V}_{tq}}{\mathbf{V}_{tb} \mathbf{V}_{tq}^{*}} = e^{-2i\phi_{M}} \quad ; \qquad \phi_{M} \approx \begin{cases} \beta & (B_{d}^{0}) \\ -\beta_{s} \approx -\lambda^{2}\eta & (B_{s}^{0}) \end{cases}$$
Flavour Physics
$$A. \operatorname{Pich} - \operatorname{TAE} 2018 \qquad 7 \end{cases}$$

# $B^{0} - \overline{B}^{0}$ MIXING AND DIRECT CP



$$\begin{split} \mathbf{T}_{\mathbf{f}} &\equiv \mathbf{T}[B^0 \to \mathbf{f}] \quad ; \quad \overline{\mathbf{T}}_{\mathbf{f}} \equiv -\mathbf{T}[\overline{B}^0 \to \mathbf{f}] \quad ; \quad \overline{\rho}_{\mathbf{f}} \equiv \overline{\mathbf{T}}_{\mathbf{f}} / \mathbf{T}_{\mathbf{f}} \\ \mathbf{T}_{\overline{\mathbf{f}}} &\equiv \mathbf{T}[B^0 \to \overline{\mathbf{f}}] \quad ; \quad \overline{\mathbf{T}}_{\overline{\mathbf{f}}} \equiv -\mathbf{T}[\overline{B}^0 \to \overline{\mathbf{f}}] \quad ; \quad \rho_{\overline{\mathbf{f}}} \equiv \mathbf{T}_{\overline{\mathbf{f}}} / \overline{\mathbf{T}}_{\overline{\mathbf{f}}} \end{split}$$

$$\mathcal{CP} B^0 = -\overline{B}^0$$
 ;  $\mathcal{CP} f = \overline{f}$ 

$$\Gamma[B^{0}(t) \to \mathbf{f}] \sim \frac{1}{2} e^{-\Gamma t} \left( |\mathbf{T}_{\mathbf{f}}|^{2} + |\mathbf{\overline{T}}_{\mathbf{f}}|^{2} \right) \left\{ 1 + \mathbf{C}_{\mathbf{f}} \cos(\Delta M t) - \mathbf{S}_{\mathbf{f}} \sin(\Delta M t) \right\}$$
  
$$\Gamma[\overline{B}^{0}(t) \to \overline{\mathbf{f}}] \sim \frac{1}{2} e^{-\Gamma t} \left( |\overline{\mathbf{T}}_{\mathbf{f}}|^{2} + |\mathbf{T}_{\mathbf{f}}|^{2} \right) \left\{ 1 - \mathbf{C}_{\mathbf{f}} \cos(\Delta M t) + \mathbf{S}_{\mathbf{f}} \sin(\Delta M t) \right\}$$

$$\mathbf{C}_{\mathbf{f}} = \frac{1 - |\bar{\rho}_{\mathbf{f}}|^{2}}{1 + |\bar{\rho}_{\mathbf{f}}|^{2}} \quad ; \quad \mathbf{S}_{\mathbf{f}} = \frac{2 \operatorname{Im}\left(\frac{q}{p} \bar{\rho}_{\mathbf{f}}\right)}{1 + |\bar{\rho}_{\mathbf{f}}|^{2}} \quad ; \quad \mathbf{C}_{\mathbf{\bar{f}}} = -\frac{1 - |\rho_{\mathbf{\bar{f}}}|^{2}}{1 + |\rho_{\mathbf{\bar{f}}}|^{2}} \quad ; \quad \mathbf{S}_{\mathbf{\bar{f}}} = \frac{-2 \operatorname{Im}\left(\frac{p}{q} \rho_{\mathbf{\bar{f}}}\right)}{1 + |\rho_{\mathbf{\bar{f}}}|^{2}}$$

 $\begin{array}{c} \text{CP self-conjugate: } \overline{f} = \eta_{f} \ f \quad \Longrightarrow \quad T_{\overline{f}} = \eta_{f} \ T_{f} \ ; \quad \overline{T}_{\overline{f}} = \eta_{f} \ \overline{T}_{f} \ ; \quad \rho_{\overline{f}} \equiv 1/\overline{\rho}_{f} \\ C_{\overline{f}} = C_{f} \ ; \quad S_{\overline{f}} = S_{f} \end{array}$ 

## $B^{0} - \overline{B}^{0}$ MIXING AND DIRECT CP



**CP self-conjugate:** 
$$\overline{f} = \eta_f f$$

$$\frac{q}{p} \approx \frac{\mathbf{V}_{tb}^* \mathbf{V}_{tq}}{\mathbf{V}_{tb} \mathbf{V}_{tq}^*} = e^{-2i\phi_M} \quad ; \qquad \phi_M \approx \begin{cases} \beta & (B_d^0) \\ -\beta_s \approx -\lambda^2 \eta & (B_s^0) \end{cases}$$



qAssumption:Only 1 decay amplitudeq' $A_{b \to q\bar{q}q'}$  $V_{qb}^* V_{qq'}^* = e^{-2i\phi_D}$  $\rho_{\bar{f}} = \bar{\rho}_f^* = \eta_f e^{2i\phi_D}$  $\bar{q}$  $A_{\bar{b} \to \bar{q}q\bar{q}'}$  $V_{qb}^* V_{qq'} = e^{-2i\phi_D}$  $\rho_{\bar{f}} = \bar{\rho}_f^* = \eta_f e^{2i\phi_D}$ 

$$\frac{\Gamma\left(\overline{B}^{0} \to \overline{f}\right) - \Gamma\left(B^{0} \to f\right)}{\Gamma\left(\overline{B}^{0} \to \overline{f}\right) + \Gamma\left(B^{0} \to f\right)} = -\eta_{f} \sin(2\phi) \sin(\Delta M t) \qquad ; \qquad \phi = \phi_{M} + \phi_{D}$$

#### **Direct information on the CKM matrix**

Flavour Physics

A. Pich – TAE 2018



A. Pich – TAE 2018

$$\frac{\Gamma(\overline{B}^{0} \to J/\psi K_{S}) - \Gamma(B^{0} \to J/\psi K_{S})}{\Gamma(\overline{B}^{0} \to J/\psi K_{S}) + \Gamma(B^{0} \to J/\psi K_{S})} = -\eta_{f} \sin(2\beta) \sin(\Delta M t)$$



**HFAG:** 

 $sin(2\beta) = 0.691 \pm 0.017$ 

 $B^0 \rightarrow J/\psi K_{S,L}, \psi(2S) K_S, \chi_c K_S, \eta_c K_S$ 



					-	 -
la١	/ou	ır	Ph	ysi	CS	

1.0	1.4	-1.2	-0.0

b→ccs	World Averag	je			0.69 ± 0.02
φ Κ <sup>0</sup>	Average			+++	0.74 +0.11
η΄ Κ <sup>0</sup>	Average		1	*	$0.63 \pm 0.06$
K <sub>S</sub> K <sub>S</sub> K <sub>S</sub>	Average		F	-	$0.72 \pm 0.19$
$\pi^0 K^0$	Average		<b>⊢</b> →	-	$0.57 \pm 0.17$
ρ <sup>0</sup>	Average		⊢+		0.54 +0.18
ωK <sub>S</sub>	Average		F	-	0.71 ± 0.21
f <sub>0</sub> K <sub>S</sub>	Average		ŀ		0.69 +0.10
f <sub>2</sub> K <sub>S</sub>	Average	F	*		$0.48 \pm 0.53$
f <sub>x</sub> K <sub>s</sub>	Average	<b></b>	*	-	$0.20 \pm 0.53$
π <sup>0</sup> π <sup>0</sup> K <sub>S</sub>	Average				$-0.72 \pm 0.71$
$\phi \pi^0 K_S$	Average				0.97 +0.03
$\pi^+ \pi^- K_S$	NAverage	·	<b>*</b> i		0.01 ± 0.33
$K^+ K^- K^0$	Average			-	0.68 +0.09

 $b \rightarrow qqs$ 

		$C_f = -A_f$	Summer 2016
φ K <sup>0</sup>	Average		0.01 ± 0.14
η΄ Κ <sup>0</sup>	Average	H	-0.05 ± 0.04
K <sub>s</sub> K <sub>s</sub> K <sub>s</sub>	Average	<b>⊢★</b> -1	-0.24 ± 0.14
π <sup>0</sup> K <sup>0</sup>	Average	<b>H</b>	0.01 ± 0.10
ρ <sup>0</sup> K <sub>S</sub>	Average	<b>⊢</b> ★ <u>−</u> 1	-0.06 ± 0.20
ωK <sub>s</sub>	Average	<b>⊢</b> <u>→</u> 1	-0.04 ± 0.14
f <sub>o</sub> K <sub>S</sub>	Average	<b>⊢★</b> -1	0.14 ± 0.12
f <sub>2</sub> K <sub>S</sub>	Average	<b>⊢</b> ★ → 1	0.28 +0.3
f <sub>x</sub> K <sub>s</sub>	Average	<b>⊢</b>	0.13 +0.3
$\pi^0 \pi^0 K_S$	Average	► <u>+</u>	$0.23 \pm 0.54$
$\phi \pi^0 K_S$	Average	<b>⊢★</b> -1	-0.20 ± 0.15
π <sup>+</sup> π <sup>-</sup> K <sub>S</sub> I	<b>NR</b> verage		0.01 ± 0.26
K <sup>+</sup> K <sup>-</sup> K <sup>0</sup>	Average		$0.06 \pm 0.08$
-1.6 -1.4 -	1.2 -1 -0.8 -0	0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8	1 1.2 1.4 1

(Reff	$= \sin(2\phi_{t}^{\text{eff}})$ HFAG	

# **6666666**

$$q = d, s$$

$$V_{tb} V_{ts}^* \sim -A\lambda^2$$

Sensitive to New Physics in Penguin diagram



Agreement with  $B^0 \to J/\Psi K_S \quad (b \to c \overline{c} s)$ 

No signal of direct C/P





### **MEASURING HADRONIC CONTAMINATIONS**

- Time Evolution
- Transversity Analysis:  $B \rightarrow V V$
- Isospin Relations (Gronau-London)
- $\mathbf{D}^{0} \overline{\mathbf{D}}^{0}$  **Mixing** (Gronau-London-Wyler, Atwood-Dunietz-Soni)

 $\sqrt{2} \operatorname{T}(B^+ \to D^0_+ K^+) = \operatorname{T}(B^+ \to D^0 K^+) + \operatorname{T}(B^+ \to \overline{D}^0 K^+)$ 

 $\sqrt{2} \operatorname{T}(B^0_d \to D^0_+ K_S) = \operatorname{T}(B^0_d \to D^0 K_S) + \operatorname{T}(B^0_d \to \overline{D}^0 K_S)$ 

- Dalitz Analysis
- SU(3) Relations:  $B \rightarrow \pi K, \pi \pi, ...$



Flavour Physics

A. Pich – TAE 2018



#### $\mathbf{D}^{0}$ - $\overline{\mathbf{D}}^{0}$ Mixing

Gronau-London-Wyler Atwood-Dunietz-Soni Giri-Grossman-Soffer-Zupan

 $\sqrt{2} \operatorname{T}(B^{+} \to D^{0}_{+} K^{+}) = \operatorname{T}(B^{+} \to D^{0} K^{+}) + \operatorname{T}(B^{+} \to \overline{D}^{0} K^{+})$  $\sqrt{2} \operatorname{T}(B^{0}_{d} \to D^{0}_{+} K_{S}) = \operatorname{T}(B^{0}_{d} \to D^{0} K_{S}) + \operatorname{T}(B^{0}_{d} \to \overline{D}^{0} K_{S})$ 



$$\gamma \equiv \arg \left[ -\frac{\mathbf{V}_{ud} \, \mathbf{V}_{ub}^*}{\mathbf{V}_{cd} \, \mathbf{V}_{cb}^*} \right] = \left( 74.0 \, {}^{+5.8}_{-6.4} \right)^\circ$$

## UNITARITY TRIANGLES

 $\mathbf{V}_{ui} \; \mathbf{V}_{uj}^{*} + \mathbf{V}_{ci} \; \mathbf{V}_{cj}^{*} + \mathbf{V}_{ti} \; \mathbf{V}_{tj}^{*} \; = \; \mathbf{0}$  $(i \neq j)$ 



$$\mathbf{V} \approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

 $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$ 





$$\mathbf{\overline{\eta}} \equiv \eta \left(1 - \frac{1}{2}\lambda^2\right) = 0.343 \pm 0.011$$
$$\mathbf{\overline{\rho}} \equiv \rho \left(1 - \frac{1}{2}\lambda^2\right) = 0.153 \pm 0.013$$
$$\alpha = 91.0 \pm 2.5^\circ \ ; \ \beta = 23.2 \pm 1.2^\circ \ ; \ \gamma = 65.3 \pm 2.0^\circ$$

#### **Tree-level determinations**

#### Loop processes





**CP** Conserving







## Backup



## $P^0 - \overline{P}^0$ MIXING

Phase convention:  $\mathcal{CP} | P^0 \rangle = - | \overline{P}^0 \rangle$ 

$$i\frac{d}{dt}|\psi(t)\rangle = \mathbf{M}|\psi(t)\rangle$$

$$\mathcal{CPT}: \mathbf{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

 $\left|\psi(t)\right\rangle = a(t)\left|P^{0}\right\rangle + b(t)\left|\overline{P}^{0}\right\rangle$ 

$$CP: M_{12} = M_{12}^*, \Gamma_{12} = \Gamma_{12}^*$$

• **Dispersive:**  $M_{12} = \frac{1}{2M} \left\{ \left\langle P^0 \middle| H_{\Delta P=2} \middle| \overline{P}^0 \right\rangle + PP \int \frac{ds}{M^2 - s} \sum_X dQ_X \left\langle P^0 \middle| H_{\Delta P=1} \middle| X \right\rangle \left\langle X \middle| H_{\Delta P=1} \middle| \overline{P}^0 \right\rangle \right\}$ 

• **Absorptive:**  $\Gamma_{12} = \frac{\pi}{M} \int ds \sum_{X} dQ_X \, \delta(s - M^2) \left\langle P^0 \left| H_{\Delta P=1} \right| X \right\rangle \left\langle X \left| H_{\Delta P=1} \right| \overline{P}^0 \right\rangle$ 

• Eigenvalues:  $|P_{\mp}^{0}\rangle = \frac{p |P^{0}\rangle \mp q |\overline{P}^{0}\rangle}{\sqrt{|p|^{2} + |q|^{2}}}$ ,  $\frac{q}{p} \equiv \frac{1 - \overline{\varepsilon}}{1 + \overline{\varepsilon}} = \left(\frac{M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*}}{M_{12} - \frac{i}{2}\Gamma_{12}}\right)^{1/2}$  $\langle P_{-} | P_{+}\rangle = \frac{|p|^{2} - |q|^{2}}{|p|^{2} + |q|^{2}} = \frac{2\operatorname{Re}(\overline{\varepsilon})}{1 + |\overline{\varepsilon}|^{2}}$ 

$$\mathcal{CP} \implies q / p = 1 \implies |P_{1,2}\rangle = \frac{1}{\sqrt{2}} \left( |P^0\rangle \mp |\overline{P}^0\rangle \right) , \quad \mathcal{CP} |P_{1,2}\rangle = \pm |P_{1,2}\rangle$$
Flavour Physics A. Pich – TAE 2018 21

#### Asymmetries B<sub>s</sub>







$$\overline{\rho}_{sb} + i \,\overline{\eta}_{sb} = -V_{us} \, V_{ub}^* \, \big/ V_{cs} \, V_{cb}^*$$

## $D^0 - \overline{D}^0$ MIXING









## $D^0 - \overline{D}^0$ MIXING: No evidence of C/P

