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1. How difficult is it to observe a neutrino?

Already since 1934, after the first calculations by Bethe and Peierls, it is known that the neutrino interaction cross sections are very small compared to other processes. For instance, the inverse beta decay process

 $\overline{\nu}_e p \rightarrow n e^+$,

used as target reaction in reactor experiments, has a cross section of the order of 3×10^{-43} cm² for a 3 MeV neutrino.

a) Estimate the neutrino mean free path in water for the process above. The mean free path is the average distance traveled by a particle in a given medium between two successive interactions, and can be calculated through the expression

$$\lambda = (n\sigma)^{-1}$$
.

Here σ is the cross-section of the process and n is the number density of target particles per unit volume. Consider only the free protons in the water molecule as target particles.

b) The event number in a neutrino experiment is obtained (at first approximation) from the convolution of the initial neutrino flux at the detector, the cross section of the process under study, the number of target particles in the detector and the exposure time of the experiment.

For a neutrino process with cross section around 10^{-43} cm², estimate the size of the detector required to observe few neutrino events per day in a solar and a reactor neutrino experiment. Consider a solar neutrino flux of 10^{10} cm⁻²s⁻¹. For the reactor experiment, consider an antineutrino production rate of 10^{20} s⁻¹ and a detector located at 1 km from the reactor core.

2. Neutrino oscillations in vacuum and matter

- a) According to the oscillation probability expression, do neutrino oscillations preserve CPT invariance? And CP invariance? Is it preserved in 2-neutrino oscillations? Is there any difference between appearance and disappearance experiments?
- b) Neutrino oscillation experiments have measured very accurately the solar and atmospheric mass splittings. Reactor experiments have proven to be a very powerful tool, since they can be sensitive to the two splittings, with a different choice for the baseline between reactor and detector. Estimate the value of these baselines.
- c) Neutrino interactions with matter in the Sun affect solar neutrino oscillation probabilities. Using the two-neutrino survival probability given by Parke's formula

$$P(\nu_e \to \nu_e) = \frac{1}{2} \left[1 + \cos 2\theta \cos 2\theta_m \right] \,,$$

show why the oscillation probability is different for pp neutrinos (E~ 0.3 MeV) and for ⁸B neutrinos (E~ 5 MeV). In the formula, θ corresponds to the vacuum mixing angle, while θ_m is the effective mixing angle at the neutrino production point in the Sun. Consider an electron density of 100 mol/cm³ at the center of the Sun.

3. Dirac and Majorana neutrinos

- a) In the minimal seesaw model, one can explain the light neutrino masses by introducing very heavy right-handed neutrinos. From the lagrangian with Dirac neutrino mass and Majorana mass term for the right-handed neutrinos, estimate the required Majorana masses to obtain light neutrino masses, of the order of the current cosmological bounds. Assume only one neutrino flavour and consider Dirac masses of the same order of the charged lepton ones.
- b) If $0\nu\beta\beta$ is not observed by the new generation of experiments, does it necessarily mean that neutrinos are Dirac particles?