

This slide was left  
intentionally dark

## The Standard Model





All the rest



# Contents

## 1) Motivation for dark matter

DM production: Weakly-Interacting Massive Particles (WIMPs)

## 2) DM (WIMP) detection

- Indirect searches
- direct searches
  - Searches in SuperCDMS)
  - reconstruction of DM parameters
- collider searches

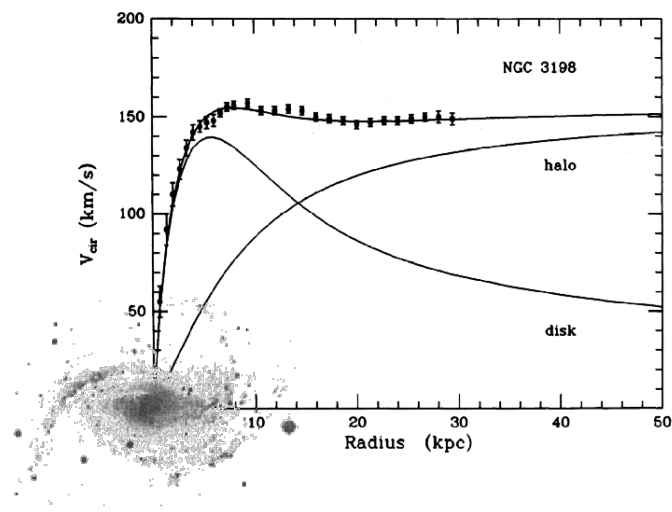
## 3) (some) DM models



# Dark Matter is a necessary (and abundant) ingredient in the Universe

## Galaxies

- Rotation curves of spiral galaxies
- Gas temperature in elliptical galaxies



It is one of the clearest hints of  
Physics Beyond the SM

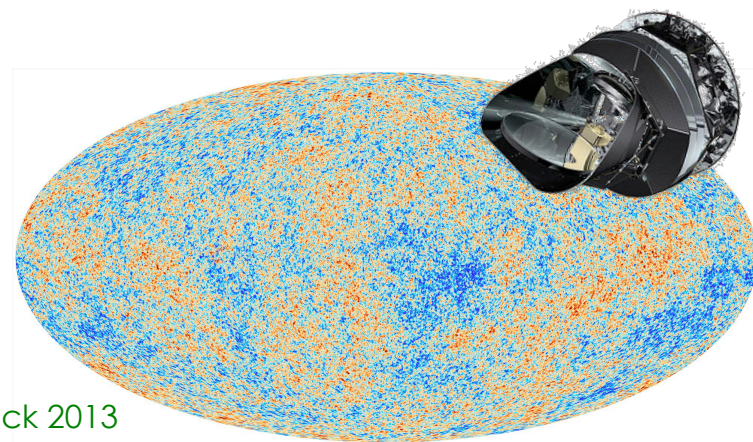
## Clusters of galaxies

- Peculiar velocities and gas temperature
- Weak lensing
- Dynamics of cluster collision
- Filaments between galaxy clusters

## Cosmological scales

Anisotropies in the Cosmic Microwave Background

$$\Omega_{\text{CDM}} h^2 = 0.1196 \pm 0.003$$



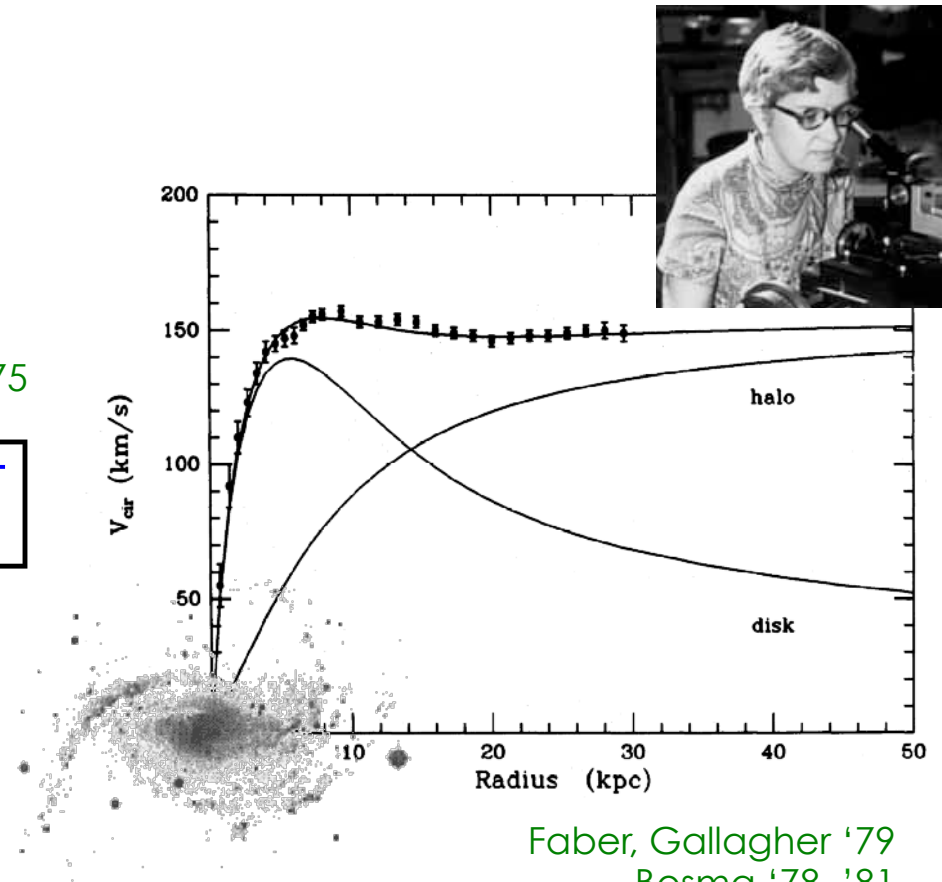
# Rotation curves of spiral galaxies become flat for large distances

From the luminous matter of the disc one would expect a decrease in the velocity that is not observed

Rubin '75

$$\frac{v_{\text{rot}}^2}{r} = \frac{G M(r)}{r^2} \rightarrow v_{\text{rot}} = \sqrt{\frac{G M(r)}{r}}$$

$$M(r) = cte \rightarrow v_{\text{rot}} \propto \frac{1}{\sqrt{r}}$$



Faber, Gallagher '79

Bosma '78, '81

van Albada, Bahcall, Begeman, Sancisi '84

Galaxies contain vast amounts of non-luminous matter

$$M \gg M_*$$



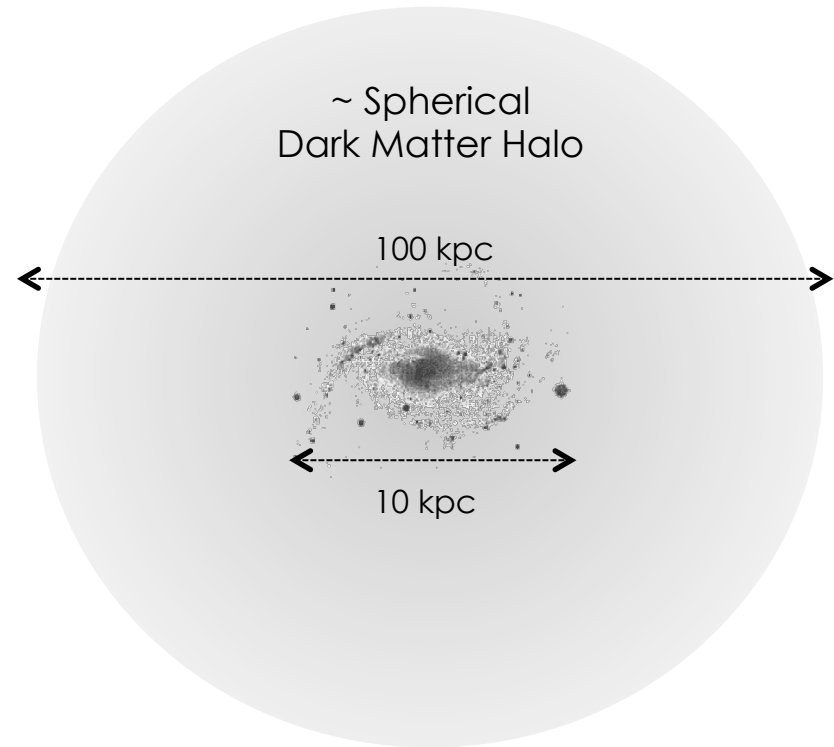
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~~Isothermal Spherical Cow Halo~~ (a.k.a. Standard Halo Model)

Isotropic

density distribution  $\rho(r) \propto r^{-2}$

it has reached a steady state (Maxwell-Boltzmann distribution of velocities)



## Rotation curves have also been measured for a large number of spiral galaxies

The mismatch in the shape cannot be compensated by modifying the contribution from luminous components (disk and bulge)

Faber, Gallagher '79  
Bosma '78, '81  
van Albada, Bahcall, Begeman, Sancisi '84

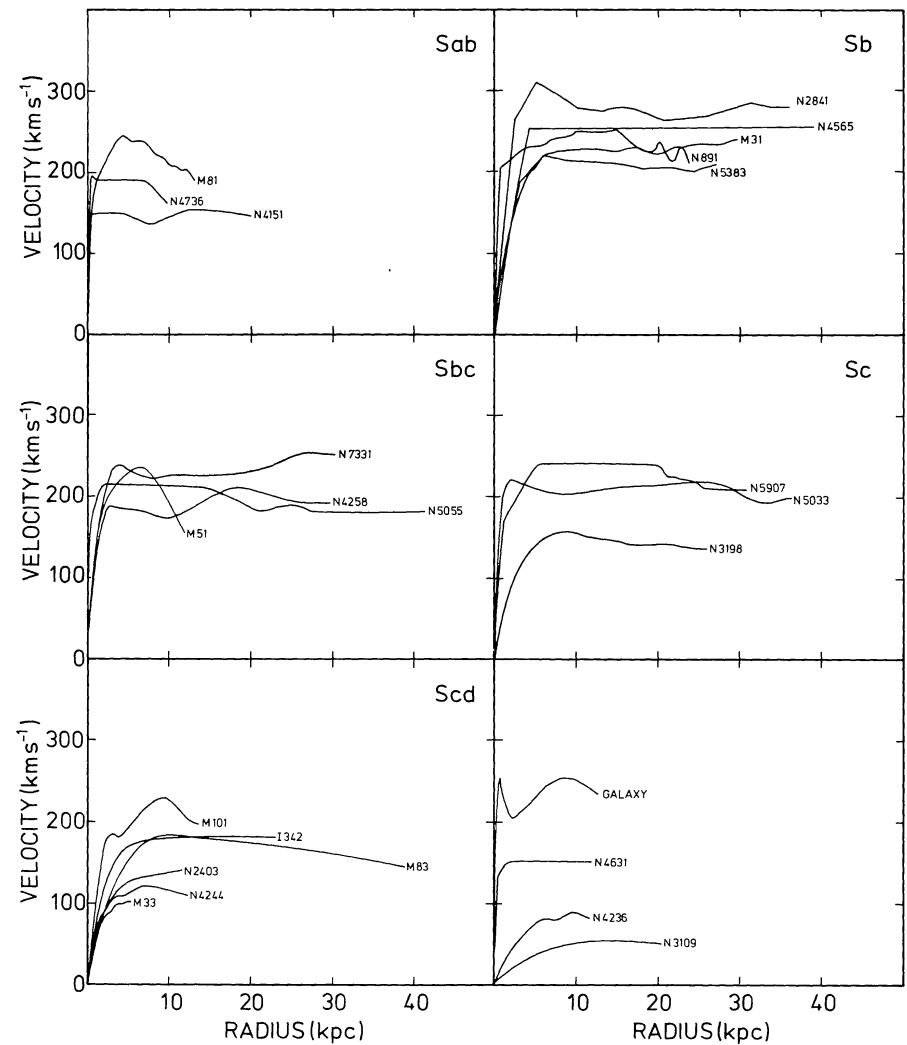
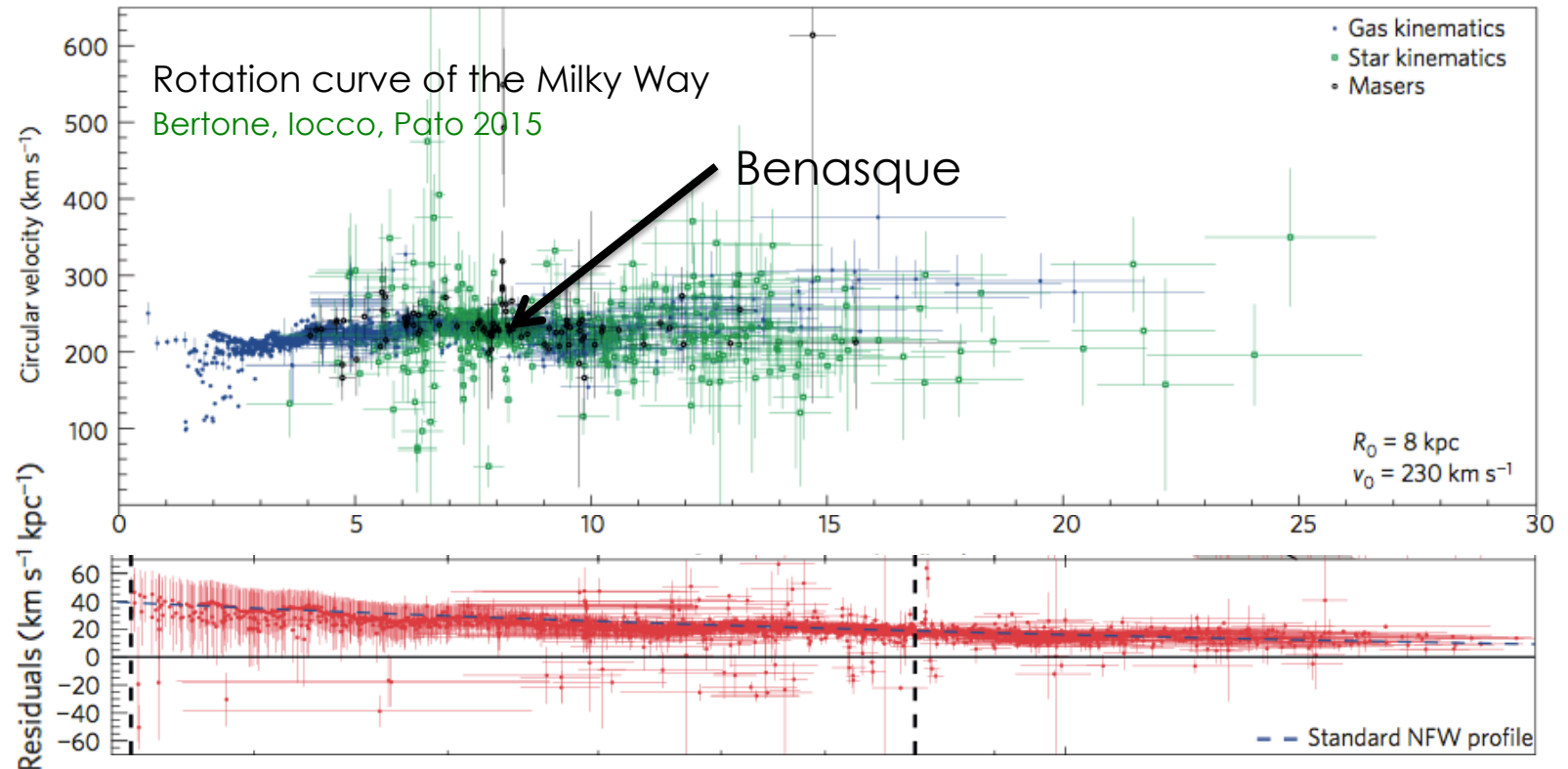


Figure 2 Rotation curves of 25 galaxies of various morphological types from Bosma (1978).

## The effect of DM has also been observed in the Milky Way...

- There is DM in the centre of our Galaxy



- Observations also show that there is need for DM in the solar neighbourhood

Bovy, Tremaine 2012



# There are substantial uncertainties in the description of our DM halo

- local DM density

$$\rho_{DM}(R_0) = 0.43(0.11)(0.10) \text{ GeV/cm}^3$$

Nesti, Salucci 2012

$$\rho_{DM}(R_0) = 0.32 \pm 0.07 \text{ GeV/cm}^3$$

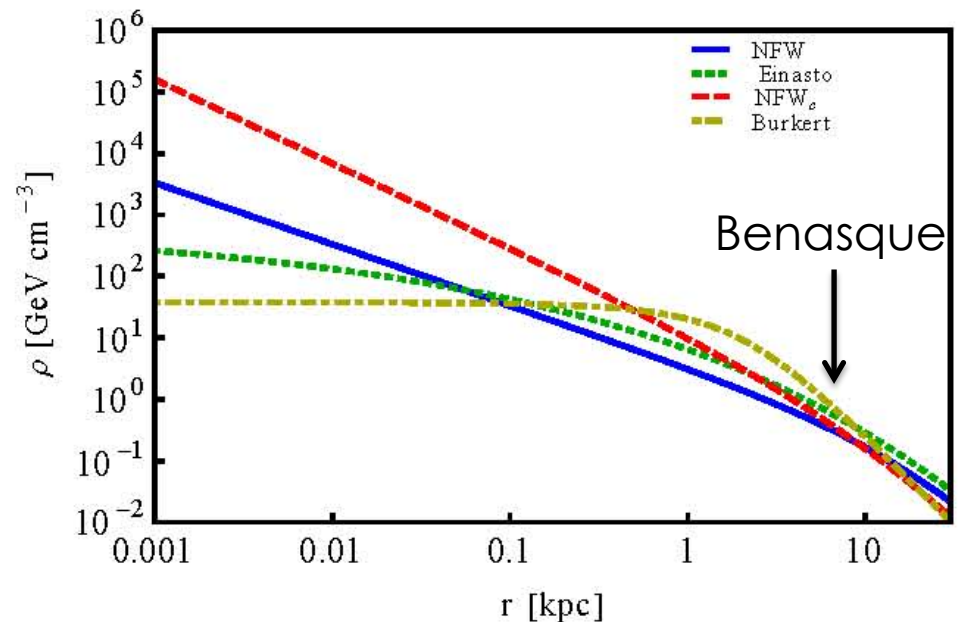
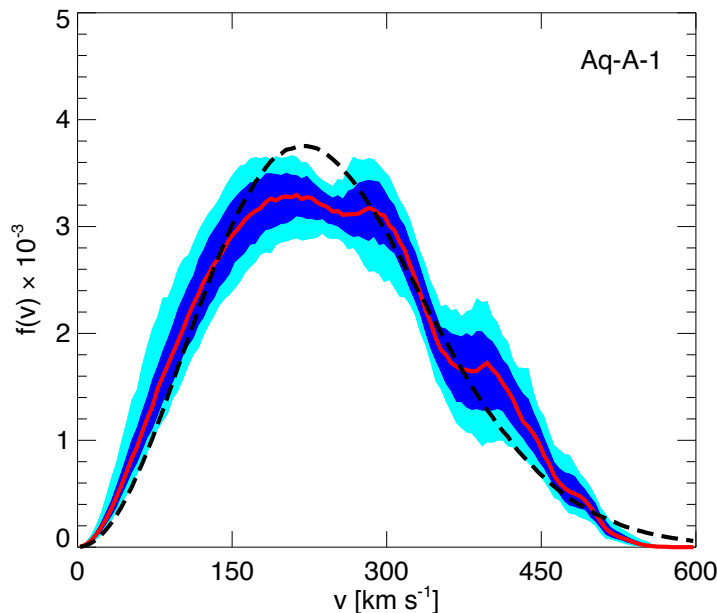
Strigari, Trotta 2009

$$\rho_{DM}(R_0) = 1.3 \pm 0.3 \text{ GeV/cm}^3$$

De Boer, Webber 2011

- DM density profile

(DM density at the galactic centre)



- Velocity distribution of DM particles

Central and escape velocities

Deviations from Maxwellian distribution







Andromeda (M31)



Sample Dark Matter  
halo from the  
Aquarius DM simulation

Andromeda (M31)



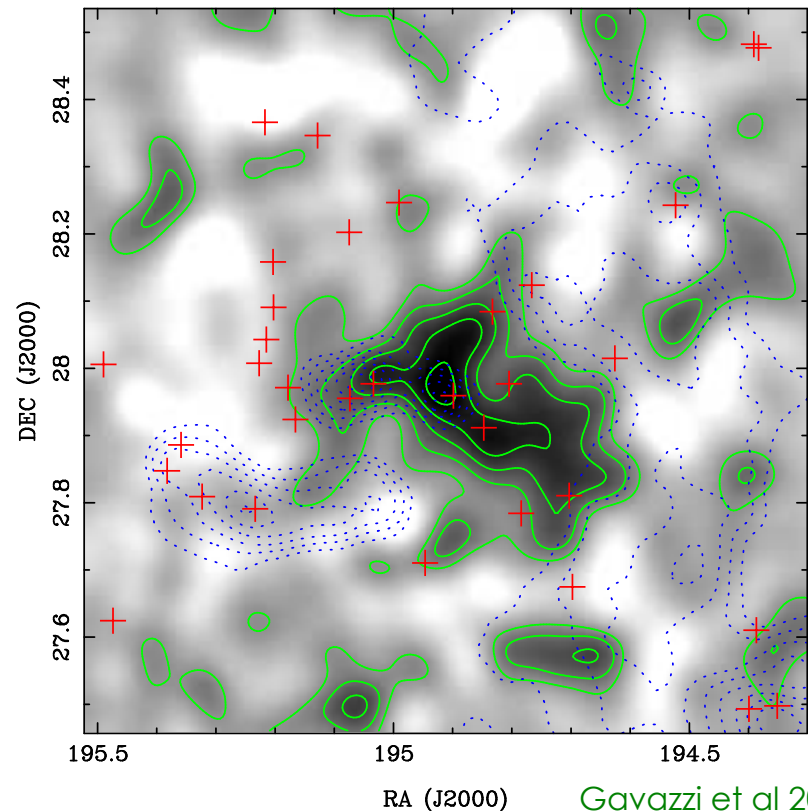
## Galaxy clusters also contain large amounts of non-luminous matter



Peculiar motions of galaxies in the Coma cluster show that the total mass is much larger than the luminous one

Zwicky 1933, 1937

Weak lensing techniques also allow to “weigh” galaxy clusters by measuring the distortion (shear) of distant galaxies behind the cluster.



Gavazzi et al 2009  
Kubo et al. 2007

- The mass of a galaxy (nebulae) cluster can be determined by different methods

(Zwicky 1933)

(Zwicky "On the masses of nebulae and of clusters of nebulae" 1937)

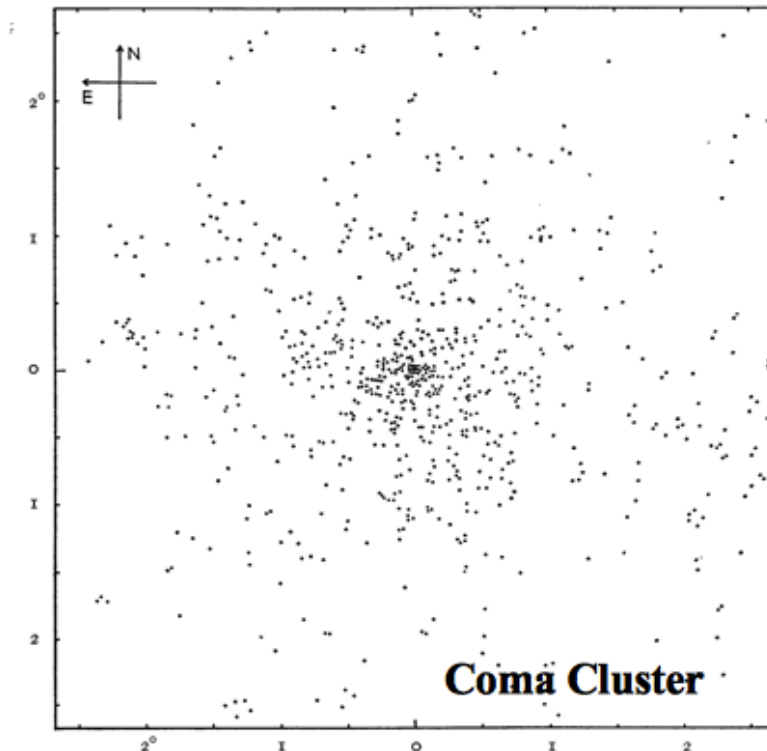


FIG. 3.—The Coma cluster of nebulae

- Luminosity of galaxies (nebulae)

This can only be used to determine the mass of the luminous component

- Peculiar motions of component galaxies (virial theorem)

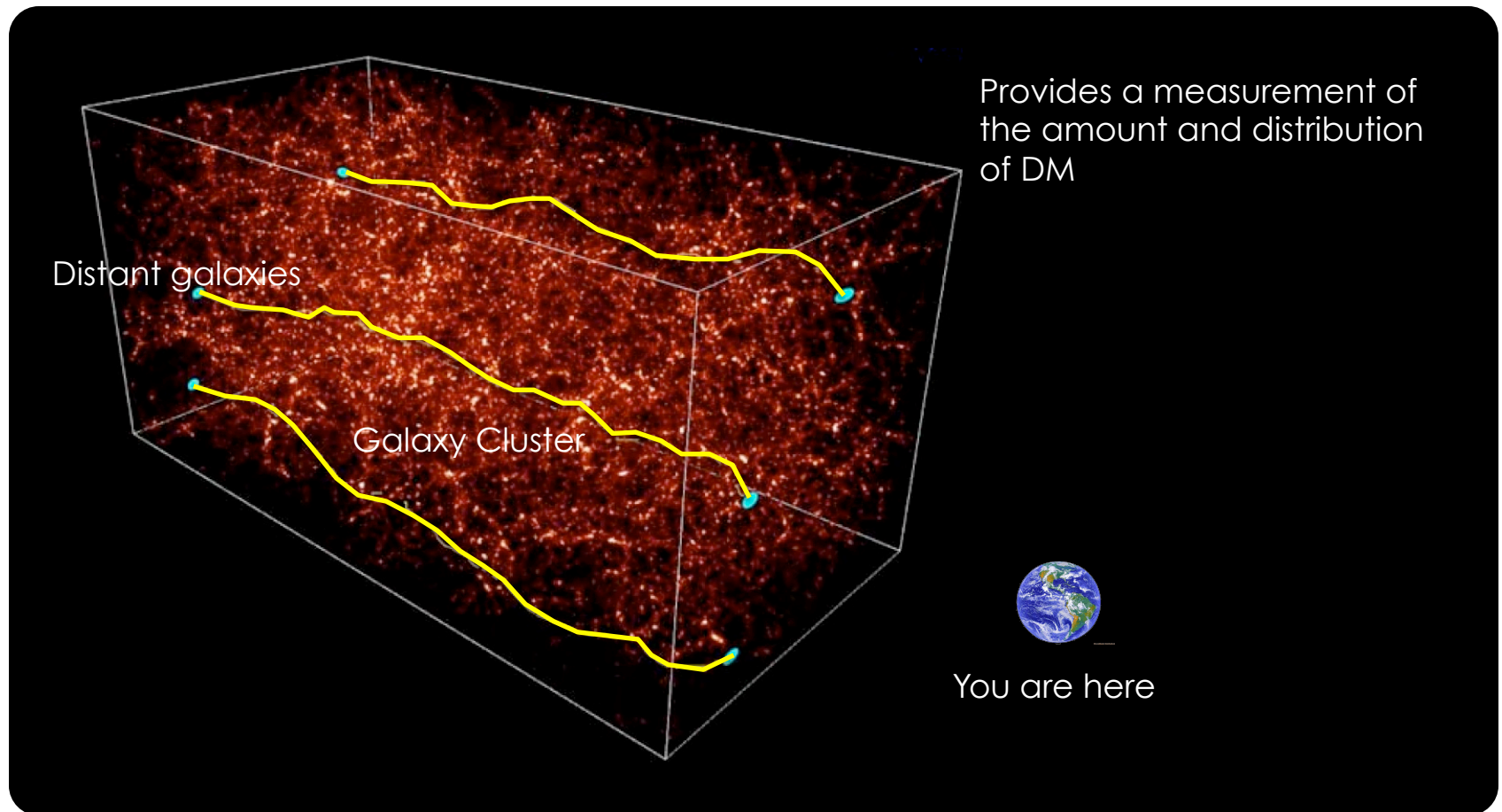
Isolated self-gravitating system

$$2K + U = 0$$

$$K = \frac{1}{2} M \langle v^2 \rangle \quad U = -\frac{\alpha G M^2}{\mathcal{R}}$$

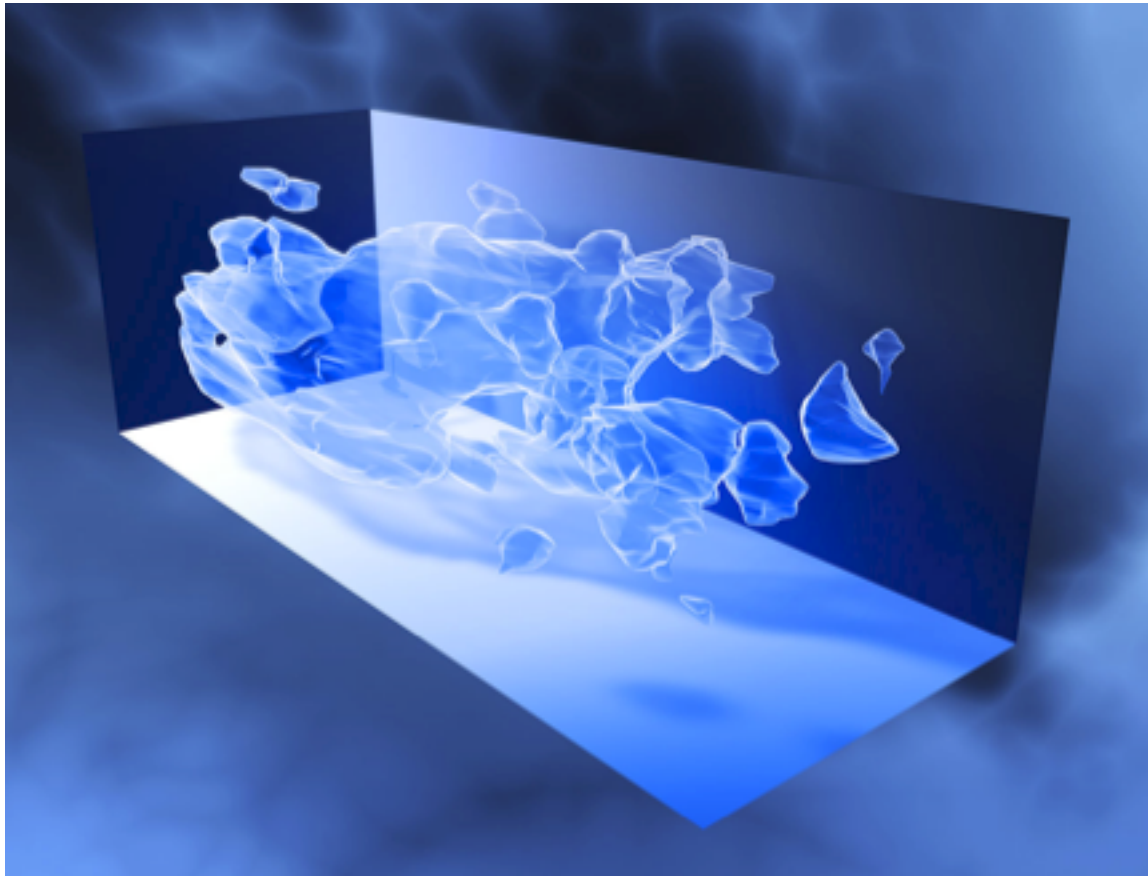
The DM in Galaxy clusters can also be observed through weak gravitational lensing

Observe collective distortions in the shape of distant galaxies whose light has crossed a heavy object ( such as a galaxy cluster)



The DM in Galaxy clusters can also be observed through weak gravitational lensing

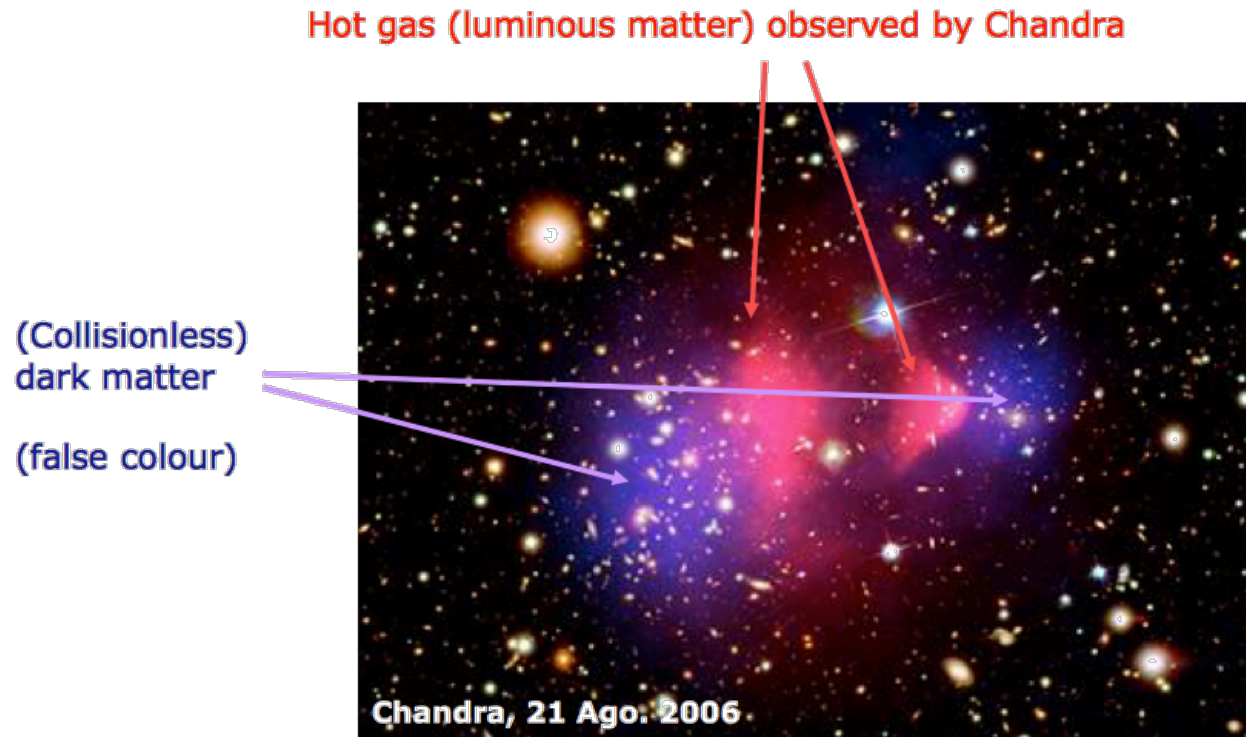
Observe collective distortions in the shape of distant galaxies whose light has crossed a heavy object ( such as a galaxy cluster)



E.g., reconstruction of the DM distribution using Hubble observations.



## The bullet cluster (a.k.a. merging galaxy cluster 1E0657-56)

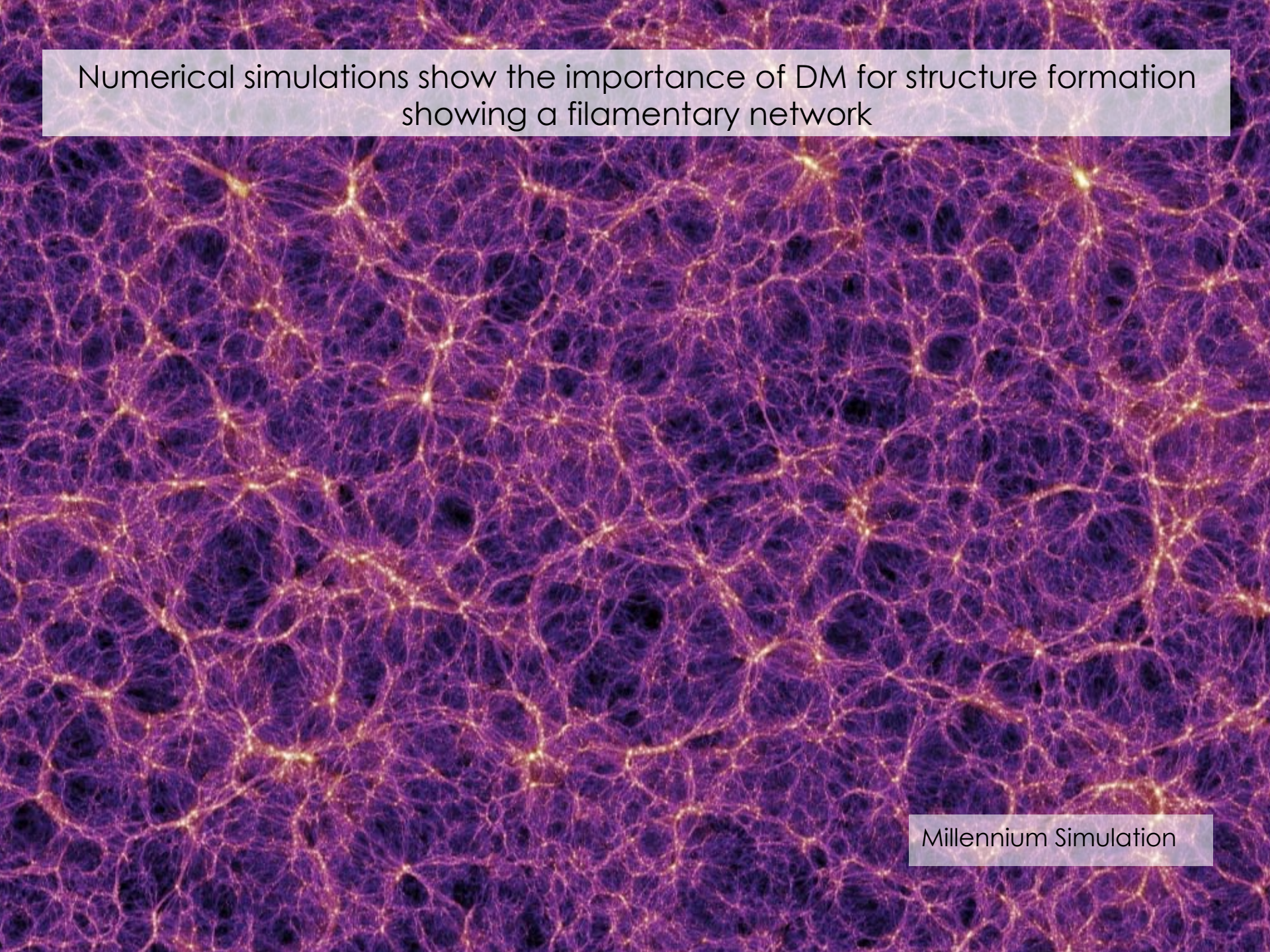


Clowe, González, Markevitch 2003  
Clowe et al. 2006  
Bradac et al. 2006

The observed displacement between the bulk of the baryons and the gravitational potential favours the dark matter hypothesis versus modifications of gravity.



Numerical simulations show the importance of DM for structure formation showing a filamentary network



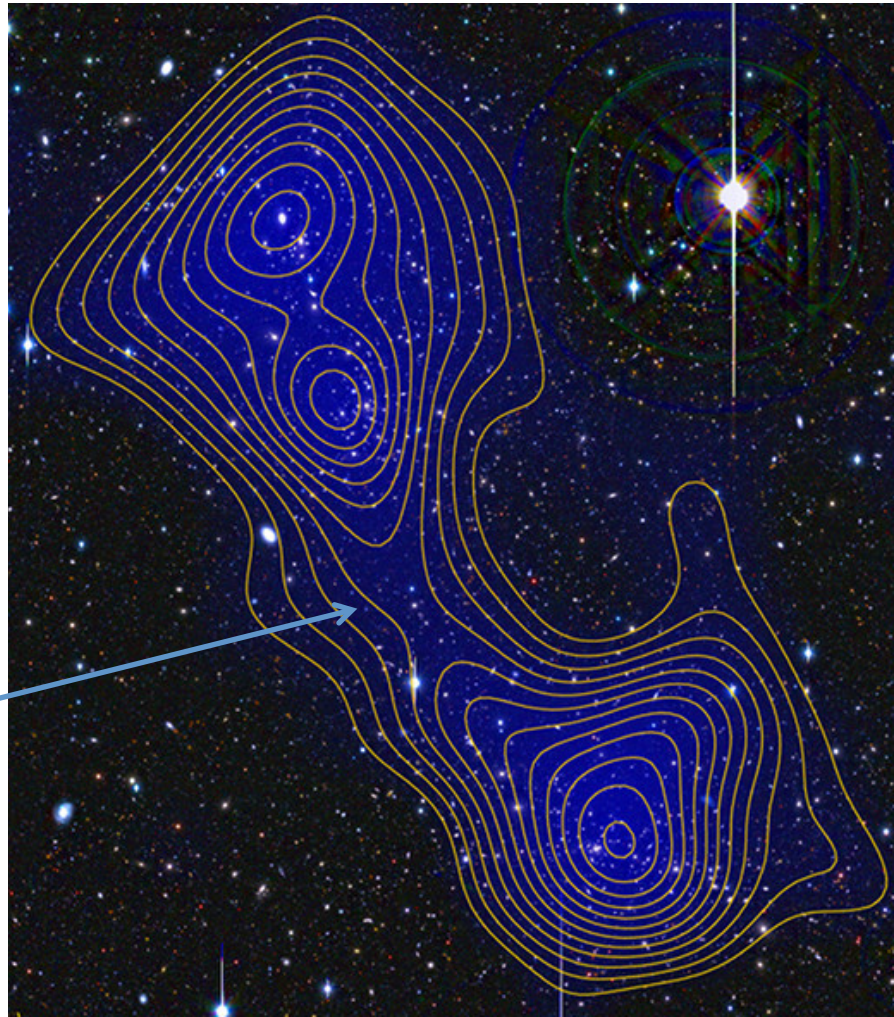
Millennium Simulation



... and in fact dark matter filaments might have been recently observed

Using weak-  
lensing  
techniques

Dark matter  
filament  
between two  
galaxy  
clusters

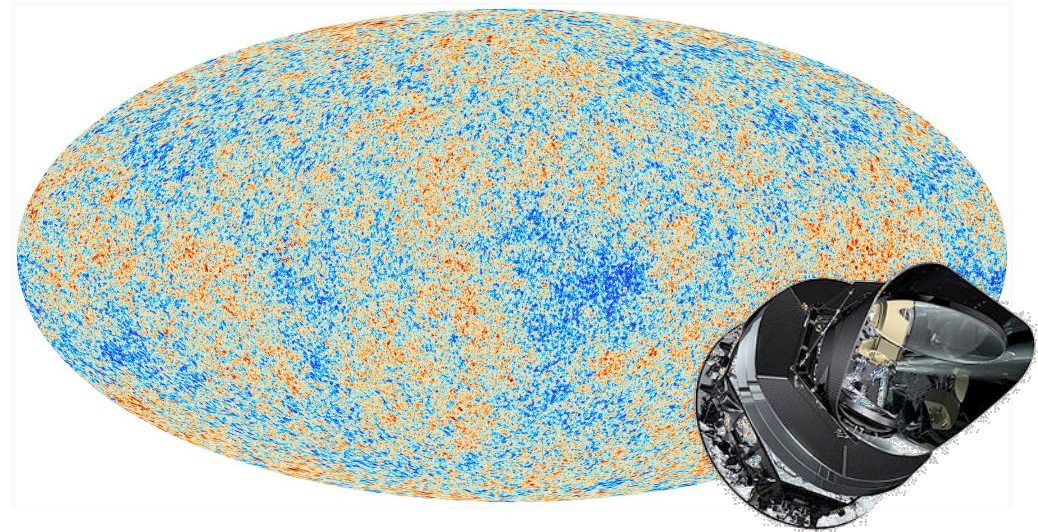


Dietrich et al. 2012

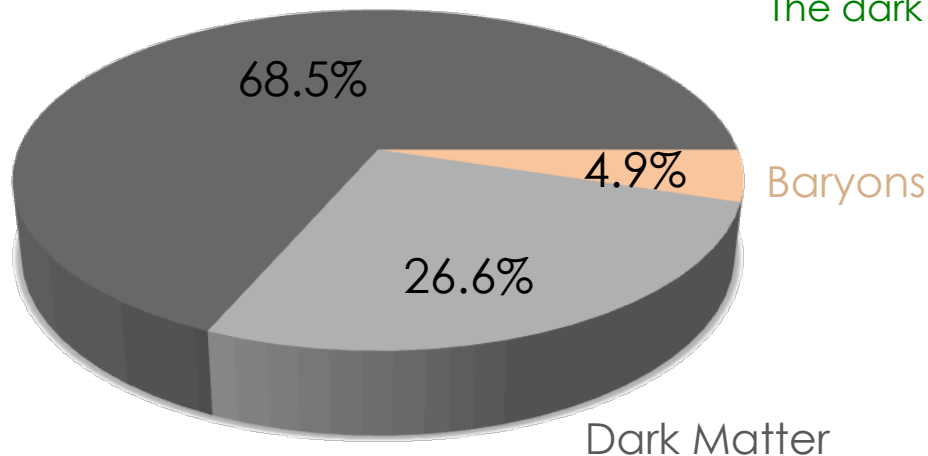
# Observations of the Cosmic microwave Background can be used to determine the components of our Universe

WMAP and Planck precision data of the CMB anisotropies allow the determination of cosmological parameters

COBE, WMAP, Planck



Dark Energy



The dark matter abundance is measured accurately

$$\Omega_{\Lambda} h^2 = 0.3116 \pm 0.009$$

$$\Omega_c h^2 = 0.1196 \pm 0.003$$

$$\Omega_b h^2 = 0.02207 \pm 0.00033$$

Planck 2013



## Challenges for **DARK MATTER** in the 80's

The main questions concerning dark matter are whether it is really present in the first place and, if so, how much is there, where is it and what does it consist of.

How much. In general one wants to know the amount of dark matter relative to luminous matter. For cosmology the main issue is whether there is enough dark matter to close the universe. Is the density parameter  $\Omega$  equal to 1?

Where. The problem of the distribution of dark matter with respect to luminous matter is fundamental for understanding its origin and composition. Is it associated with individual galaxies or is it spread out in intergalactic and intracluster space? If associated with galaxies how is it distributed with respect to the stars?

What. What is the nature of dark matter? Is it baryonic or non-baryonic or is it both?

van Albada, Sancisi '87

## Current challenges for **DARK MATTER**

- **Experimental detection:**

Does DM feel other interactions apart from Gravity?

Is the Electro-Weak scale related somehow related to DM?

How is DM distributed?

- **Determination of the DM particle parameters:**

Mass, interaction cross section, etc...

- **What is the theory for Physics beyond the SM:**

DM as a window for new Physics

Can we identify the DM candidate?

# We don't know yet what DM is... but we do know many of its **properties**

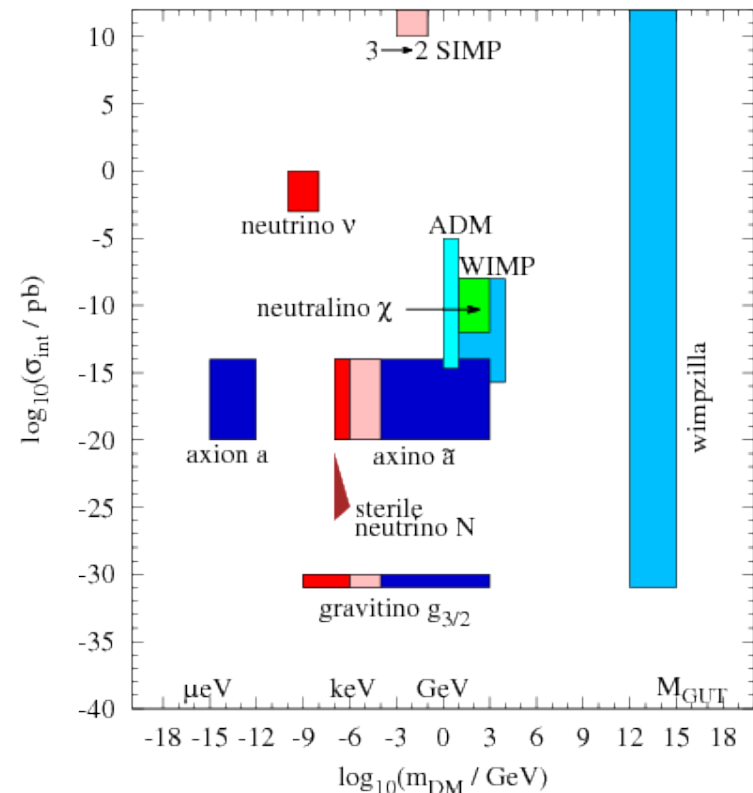
It is a NEW particle (not contained in the Standard Model)

Baer et al. 2014

- Neutral
- Stable on cosmological scales
- Reproduce the correct relic abundance
- Not excluded by current searches
- No conflicts with BBN or stellar evolution

## Many candidates in Particle Physics

- Axions
- **Weakly Interacting Massive Particles (WIMPs)**
- SuperWIMPs and Decaying DM
- WIMPzillas
- Asymmetric DM
- SIMPs, CHAMPs, SIDMs, ETCs...



... they have very **different** properties

The Standard Model does not contain any viable candidate for DM

|         | Fermions                        |                               |                               | Bosons             |                   |
|---------|---------------------------------|-------------------------------|-------------------------------|--------------------|-------------------|
| Quarks  | $u$<br>up                       | $c$<br>charm                  | $t$<br>top                    | $\gamma$<br>photon | Force<br>carriers |
|         | $d$<br>down                     | $s$<br>strange                | $b$<br>bottom                 | $Z$<br>Z boson     |                   |
| Leptons | $\nu_e$<br>electron<br>neutrino | $\nu_\mu$<br>muon<br>neutrino | $\nu_\tau$<br>tau<br>neutrino | $W$<br>W boson     |                   |
|         | $e$<br>electron                 | $\mu$<br>muon                 | $\tau$<br>tau                 | $g$<br>gluon       |                   |
|         |                                 |                               |                               |                    |                   |
|         |                                 |                               |                               | Higgs<br>boson     |                   |

Source: AAAS

Neutrinos constitute a tiny part of (Hot) dark matter

$$\Omega_\nu h^2 = \frac{\sum_i m_{\nu_i}}{91.5\text{eV}} \lesssim 0.003$$

Hot dark matter not consistent with observations on structure formation.

Dark Matter is one of the clearest hints of Physics Beyond the SM



## Some basics on Dark Matter Production

Dark matter was present in the Early Universe and it is present now, however, there are many different mechanisms to account for its correct abundance

- Thermal production (freeze-out)
- Out of equilibrium production (freeze-in)
- Late decays of unstable exotics
- Vacuum misalignment (axions)
- Asymmetry



# Cosmology 101

Friedmann-Lemaître-Robertson-Walker (FLRW) metric for a homogeneous and isotropic universe that is expanding (or contracting)

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) = g_{\mu\nu} dx^\mu dx^\nu$$

$k$  = curvature

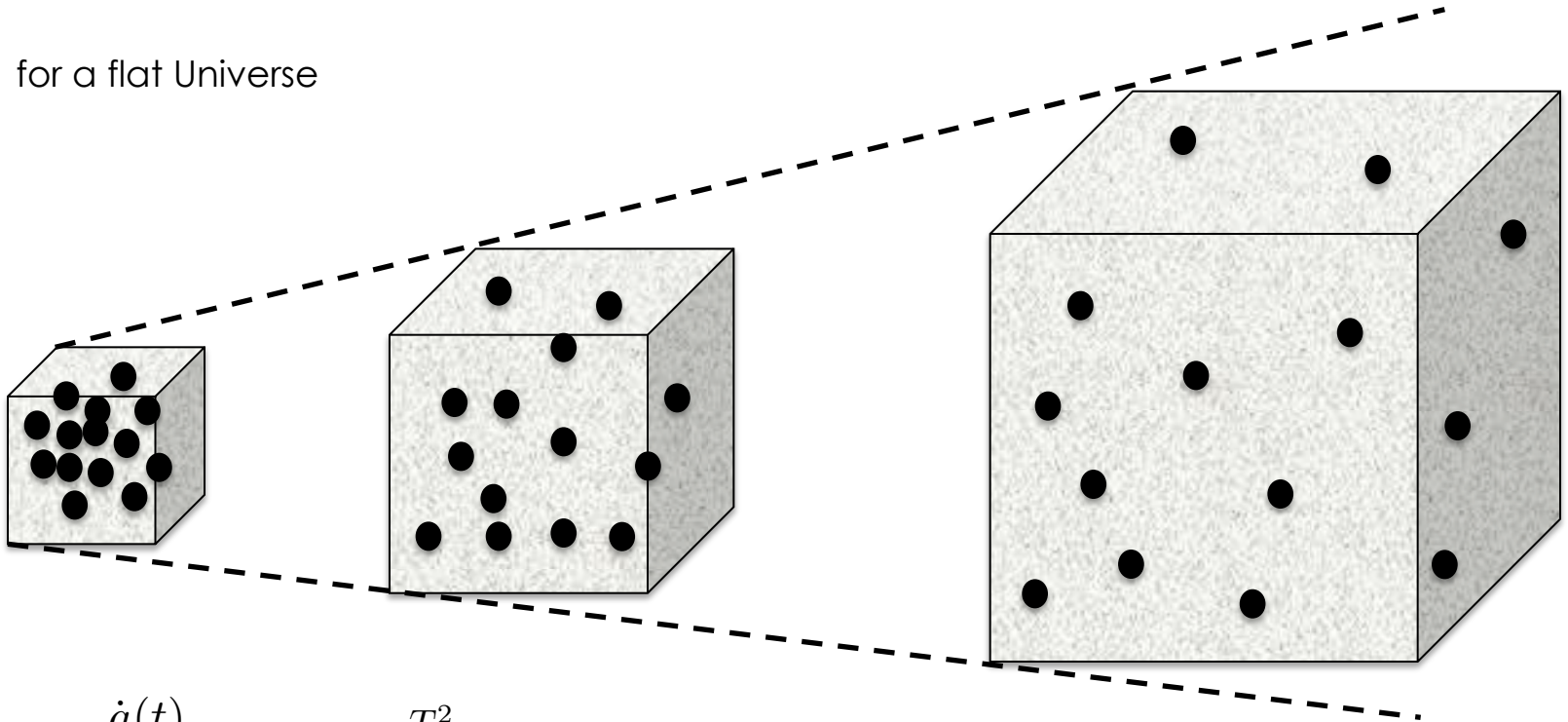
|                          |          |     |                             |
|--------------------------|----------|-----|-----------------------------|
| Components of the metric | $g_{00}$ | $=$ | $1$                         |
|                          | $g_{11}$ | $=$ | $\frac{-a(t)^2}{1 - kr^2}$  |
|                          | $g_{22}$ | $=$ | $-r^2 a(t)^2$               |
|                          | $g_{33}$ | $=$ | $-r^2 \sin^2 \theta a(t)^2$ |

$a(t)$  is the scale parameter

## WIMP dilution

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right)$$

$k=0$  for a flat Universe



$$H = \frac{\dot{a}(t)}{a(t)} = 1.66 g_*^{1/2} \frac{T^2}{M_P}$$

| Event                          | time $t$         | redshift $z$    | temperature $T$ |
|--------------------------------|------------------|-----------------|-----------------|
| Inflation                      | $10^{-34}$ s (?) | —               | —               |
| Baryogenesis                   | ?                | ?               | ?               |
| EW phase transition            | 20 ps            | $10^{15}$       | 100 GeV         |
| QCD phase transition           | 20 $\mu$ s       | $10^{12}$       | 150 MeV         |
| Dark matter freeze-out         | ?                | ?               | ?               |
| Neutrino decoupling            | 1 s              | $6 \times 10^9$ | 1 MeV           |
| Electron-positron annihilation | 6 s              | $2 \times 10^9$ | 500 keV         |
| Big Bang nucleosynthesis       | 3 min            | $4 \times 10^8$ | 100 keV         |
| Matter-radiation equality      | 60 kyr           | 3400            | 0.75 eV         |
| Recombination                  | 260–380 kyr      | 1100–1400       | 0.26–0.33 eV    |
| Photon decoupling              | 380 kyr          | 1000–1200       | 0.23–0.28 eV    |
| Reionization                   | 100–400 Myr      | 11–30           | 2.6–7.0 meV     |
| Dark energy-matter equality    | 9 Gyr            | 0.4             | 0.33 meV        |
| Present                        | 13.8 Gyr         | 0               | 0.24 meV        |



A system of particles in kinetic equilibrium has a phase space occupancy  $f$  given by the Bose-Einstein or Fermi-Dirac distributions at temperature  $T$ :

$$f(\mathbf{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1}$$

The phase space distribution allows one to compute the associated number density  $n$ , energy density  $\rho$  and pressure  $p$  for a dilute and weakly-interacting gas of particles with  $g$  internal degrees of freedom:

$$n = g \int \frac{d^3p}{(2\pi)^3} f(\mathbf{p}) ,$$

$$\rho = g \int \frac{d^3p}{(2\pi)^3} E(\mathbf{p}) f(\mathbf{p}) ,$$

$$p = g \int \frac{d^3p}{(2\pi)^3} \frac{|\mathbf{p}|^2}{3E(\mathbf{p})} f(\mathbf{p}) .$$

#### Relativistic particles

$$T \gg m \quad E \sim |\mathbf{p}|$$

$$\begin{aligned} n_b &= \frac{g}{\pi^2} \zeta(3) T^3 & \rho_b &= \frac{\pi^2}{30} g T^4 \\ n_f &= \frac{3}{4} \frac{g}{\pi^2} \zeta(3) T^3 & \rho_f &= \frac{7}{8} \frac{\pi^2}{30} g T^4 \end{aligned}$$

#### Non-Relativistic particles

$$T \ll m, \quad E = (|\mathbf{p}|^2 + m^2)^{1/2} = m \left( 1 + \frac{|\mathbf{p}|^2}{m^2} \right)^{1/2} \simeq m + \frac{|\mathbf{p}|^2}{2m}$$

$$n \simeq g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$

It is customary to define the Yield (equivalent to the number density but in a comoving volume) in terms of the entropy density (which scales as  $a^3(t)$ )

$$Y = \frac{n}{s} \quad s = \frac{2\pi^2}{45} g_{*s} T^3$$

For relativistic particles, we have

$$n = \frac{g_{eff}}{\pi^2} \zeta(3) T^3 \longrightarrow Y_{eq} = \frac{45}{2\pi^4} \zeta(3) \frac{g_{eff}}{g_{*s}}$$

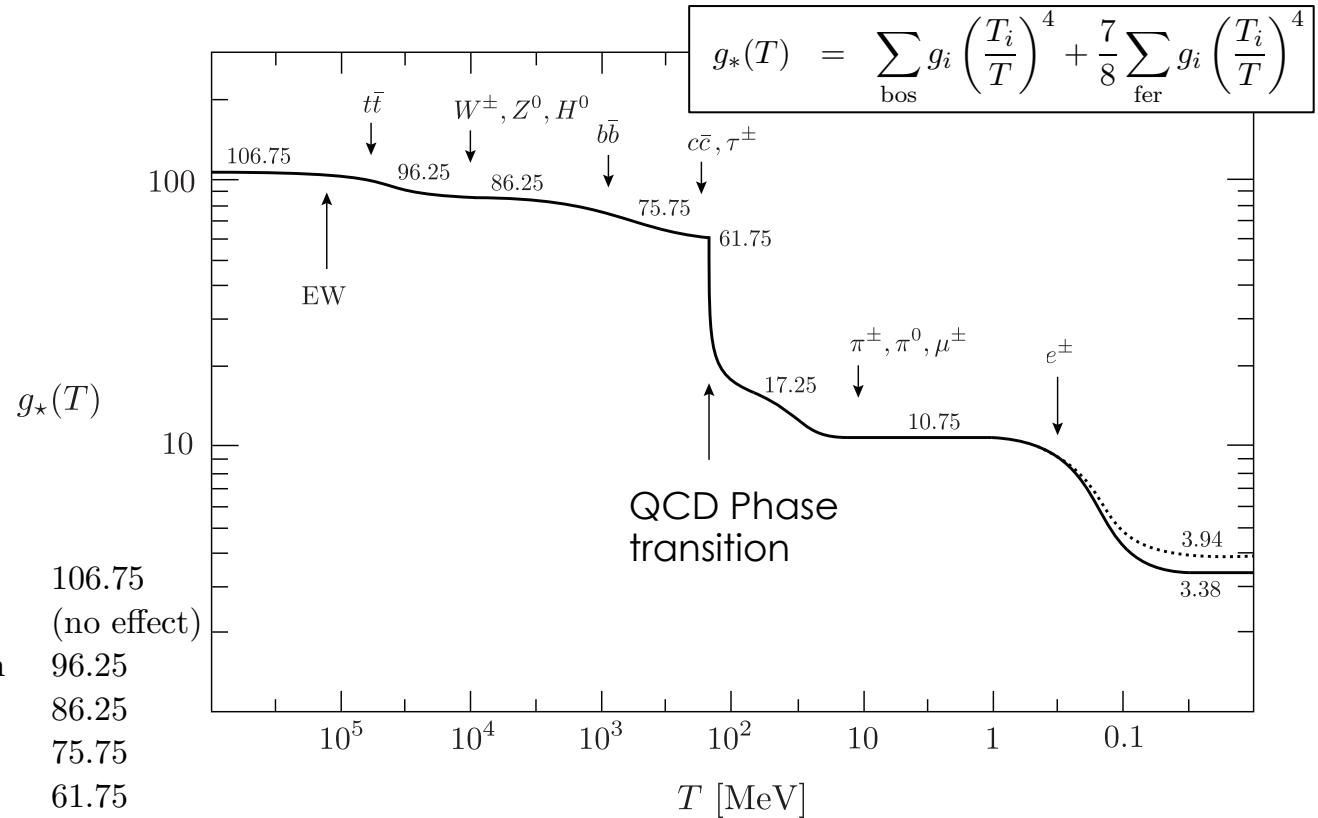
For non-relativistic particles, we have

$$n = g_{eff} \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T} \longrightarrow Y_{eq} = \frac{45}{2\pi^4} \left( \frac{\pi}{8} \right)^{1/2} \frac{g_{eff}}{g_{*s}} \left( \frac{m}{T} \right)^{3/2} e^{-m/T}$$

# Number of relativistic degrees of freedom in the Standard Model

|                             |                   |  |                  |                                 |                              |                          |
|-----------------------------|-------------------|--|------------------|---------------------------------|------------------------------|--------------------------|
| Quarks                      | $t$               | $174.2 \pm 3.3\text{GeV}$                | $\bar{t}$        | spin= $\frac{1}{2}$<br>3 colors | $g = 2 \cdot 2 \cdot 3 = 12$ |                          |
|                             | $b$               | $4.20 \pm 0.07\text{GeV}$                | $\bar{b}$        |                                 |                              |                          |
|                             | $c$               | $1.25 \pm 0.09\text{GeV}$                | $\bar{c}$        |                                 |                              |                          |
|                             | $s$               | $95 \pm 25\text{MeV}$                    | $\bar{s}$        |                                 |                              |                          |
|                             | $d$               | $3\text{--}7\text{MeV}$                  | $\bar{d}$        |                                 |                              |                          |
|                             | $u$               | $1.5\text{--}3.0\text{MeV}$              | $\bar{u}$        |                                 |                              | <hr/> 72                 |
| Gluons                      | 8 massless bosons |  |                  | spin=1                          | $g = 2$                      | 16                       |
| Leptons                     | $\tau^-$          | $1777.0 \pm 0.3\text{MeV}$               | $\tau^+$         | spin= $\frac{1}{2}$             | $g = 2 \cdot 2 = 4$          |                          |
|                             | $\mu^-$           | $105.658\text{MeV}$                      | $\mu^+$          |                                 |                              |                          |
|                             | $e^-$             | $510.999\text{keV}$                      | $e^+$            |                                 |                              | <hr/> 12                 |
|                             | $\nu_\tau$        | $< 18.2\text{MeV}$                       | $\bar{\nu}_\tau$ | spin= $\frac{1}{2}$             | $g = 2$                      |                          |
|                             | $\nu_\mu$         | $< 190\text{keV}$                        | $\bar{\nu}_\mu$  |                                 |                              |                          |
|                             | $\nu_e$           | $< 2\text{ eV}$                          | $\bar{\nu}_e$    |                                 |                              | <hr/> 6                  |
| Electroweak<br>gauge bosons | $W^+$             | $80.403 \pm 0.029\text{GeV}$             |                  | spin=1                          | $g = 3$                      |                          |
|                             | $W^-$             | $80.403 \pm 0.029\text{GeV}$             |                  |                                 |                              |                          |
|                             | $Z^0$             | $91.1876 \pm 0.0021\text{GeV}$           |                  |                                 |                              |                          |
|                             | $\gamma$          | $0 \quad (< 6 \times 10^{-17}\text{eV})$ |                  |                                 | $g = 2$                      | <hr/> 11                 |
| Higgs boson (SM)            | $H^0$             | <b>125.5 GeV</b>                         |                  | spin=0                          | $g = 1$                      | <hr/> 1                  |
|                             |                   |  |                  |                                 |                              | <hr/>                    |
|                             |                   |  |                  |                                 |                              | $g_f = 72 + 12 + 6 = 90$ |
|                             |                   |  |                  |                                 |                              | $g_b = 16 + 11 + 1 = 28$ |

# Number of relativistic degrees of freedom in the Standard Model



|                  |                         |             |   |
|------------------|-------------------------|-------------|---|
| $T \sim 200$ GeV | all present             | 106.75      |   |
| $T \sim 100$ GeV | EW transition           | (no effect) |   |
| $T < 170$ GeV    | top annihilation        | 96.25       |   |
| $T < 80$ GeV     | $W^\pm, Z^0, H^0$       | 86.25       |   |
| $T < 4$ GeV      | bottom                  | 75.75       |   |
| $T < 1$ GeV      | charm, $\tau^-$         | 61.75       |   |
| $T \sim 150$ MeV | QCD transition          | 17.25       | $(u,d,g \rightarrow \pi^{\pm,0}, \quad 37 \rightarrow 3)$ |
| $T < 100$ MeV    | $\pi^\pm, \pi^0, \mu^-$ | 10.75       | $e^\pm, \nu, \bar{\nu}, \gamma$ left                      |
| $T < 500$ keV    | $e^-$ annihilation      | (7.25)      | $2 + 5.25(4/11)^{4/3} = 3.36$                             |



QCD Phase transition  $T \sim 150 \text{ MeV}, t \sim 20 \mu\text{s}.$

The temperature and thus the quark energies have fallen so that the quarks lose their asymptotic freedom

There are no more free quarks and gluons; the quark-gluon plasma has become a hadron gas

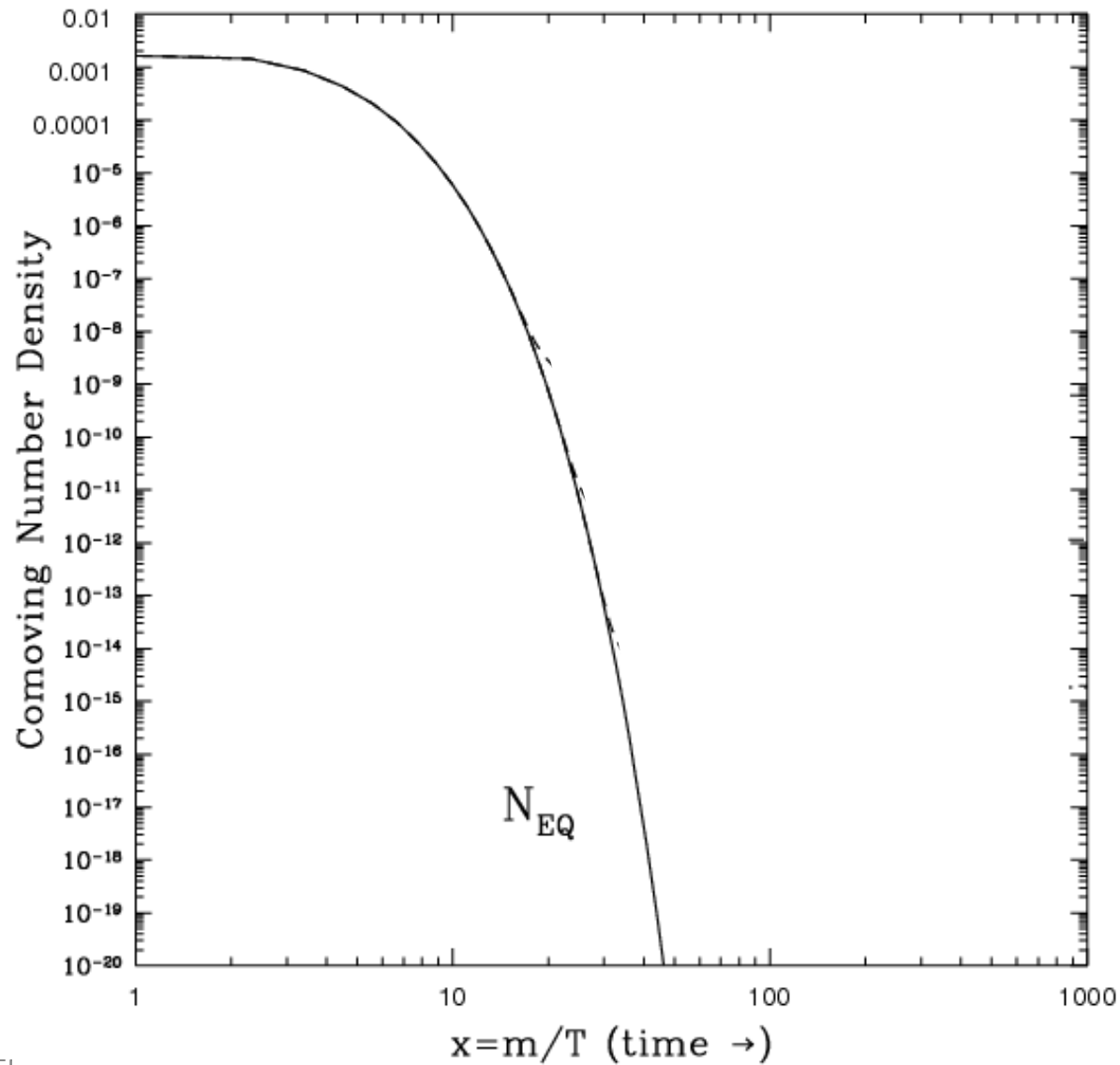
The lightest baryons are the nucleons: the proton and the neutron. The lightest mesons are the pions

all except pions are nonrelativistic below the QCD phase transition temperature.

Thus the only particle species left in large numbers are the pions ( $g=3$ ), muons (4), electrons (4), neutrinos ( $2 \times 3$ ), and the photons (2).

$$g_*=17.25$$

$$Y_{eq} = \frac{45}{2\pi^4} \left(\frac{\pi}{8}\right)^{1/2} \frac{g_{eff}}{g_{*s}} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$



## EXAMPLE 1.1

It is easy to estimate the value of the Yield that we need in order to reproduce the correct DM relic abundance,  $\Omega h^2 \approx 0.1$ , since

$$\Omega h^2 = \frac{\rho_\chi}{\rho_c} h^2 = \frac{m_\chi n_\chi h^2}{\rho_c} = \frac{m_\chi Y_0 s_0 h^2}{\rho_c}, \quad (1.9)$$

where  $Y_0$  corresponds to the DM Yield today and  $s_0$  is today's entropy density. We can assume that the Yield did not change since DM freeze-out and therefore

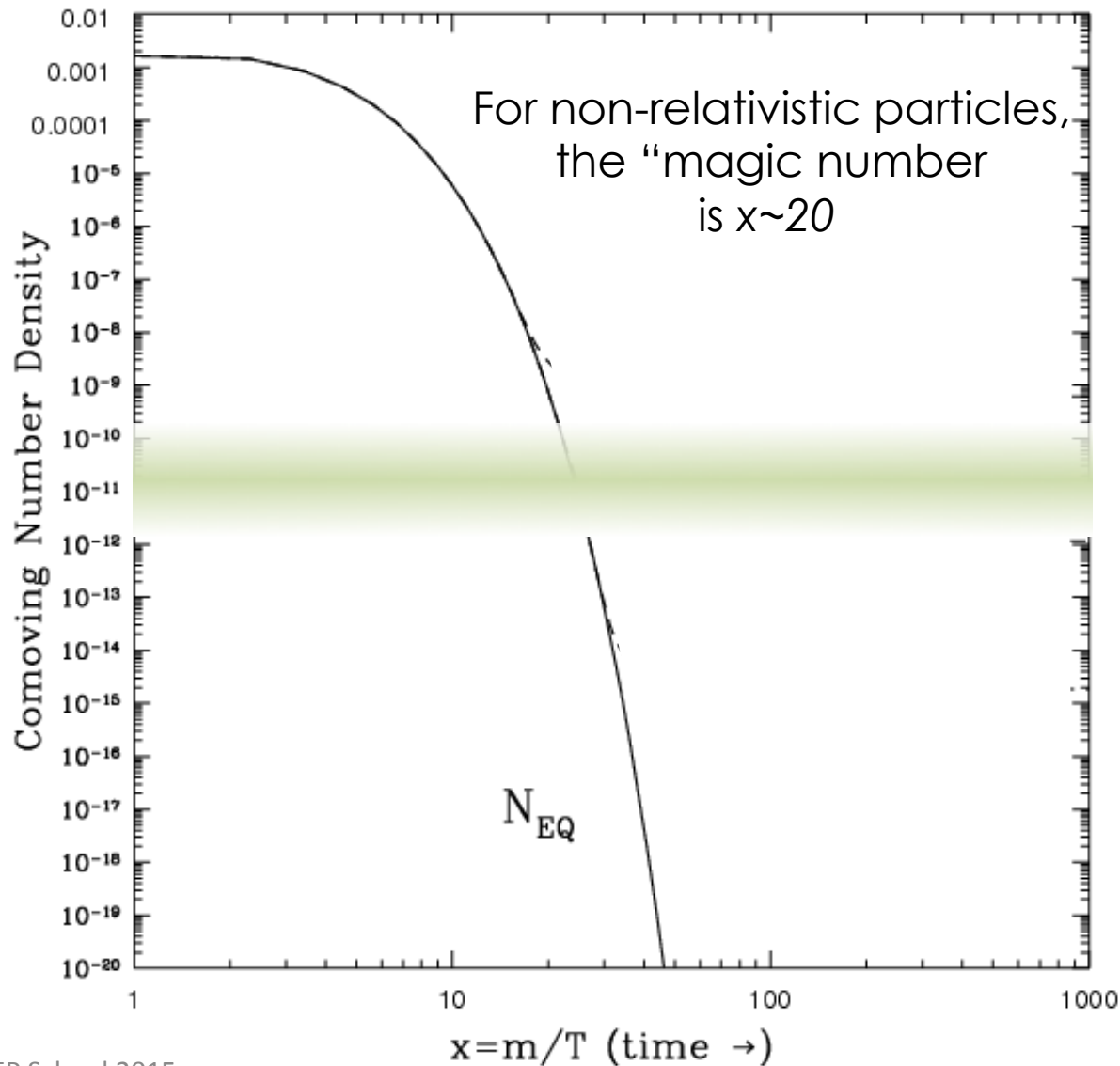
$$\Omega h^2 = \frac{m_\chi Y_f s_0 h^2}{\rho_c}. \quad (1.10)$$

Using the measured value  $s_0 = 2970 \text{ cm}^{-3}$  and the value of the critical density  $\rho_c = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3}$ , as well as Planck's result on the DM relic abundance we arrive at

$$Y_f \approx 3.55 \times 10^{-10} \left( \frac{1 \text{ GeV}}{m_\chi} \right). \quad (1.11)$$



$$Y_{eq} = \frac{45}{2\pi^4} \left(\frac{\pi}{8}\right)^{1/2} \frac{g_{eff}}{g_{*s}} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$



For DM masses in the  
range 1 GeV – 1 TeV

The time evolution of the phase space distribution function is dictated by Liouville's operator (which ensures conservation of density in the phase space) and the Collisional operator, which encodes number changing processes

$$\hat{L}[f] = C[f]$$

The Liouville operator can be written in a covariant way

$$\hat{L} = \frac{d}{d\tau} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma_{\sigma\rho}^\mu p^\sigma p^\rho \frac{\partial}{\partial p^\mu}$$

Where the affine connection is related to derivatives of the metric as follows

$$\Gamma_{\nu\lambda}^\mu = \frac{1}{2} g^{\mu\sigma} (g_{\sigma\nu,\lambda} + g_{\sigma\lambda,\nu} - g_{\nu\lambda,\sigma})$$

Notice that this terms incorporates gravity and the actual geometry of space-time.

If we apply this to the FRW metric, which only depends on  $t$  and  $E$

$$f(x^\mu, p^\mu) = f(t, E)$$

We find that Liouville operator can be greatly simplified

Exercise 1

$$\begin{aligned}\hat{L} &= E \frac{\partial}{\partial t} - \Gamma_{\sigma\rho}^0 p^\sigma p^\rho \frac{\partial}{\partial E} \\ &= E \frac{\partial}{\partial t} - H |\mathbf{p}|^2 \frac{\partial}{\partial E}\end{aligned}$$

Ultimately, we are interested in the time evolution of the number density

$$n = \frac{g}{2\pi^3} \int f(\mathbf{p}) d^3p$$



Thus, we integrate Liouville's operator in the momentum space

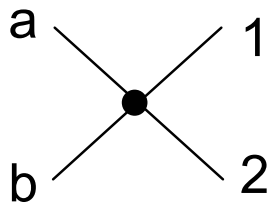
$$\frac{g}{2\pi^3} \int \hat{L}[f] d^3\mathbf{p} = \frac{g}{2\pi^3} \int C[f] d^3\mathbf{p}$$

### Exercise 2

Prove the following relation

$$\frac{g}{(2\pi)^3} \int \frac{d^3\vec{p}}{E} \left[ E \frac{\partial f}{\partial t} - H |\vec{p}|^2 \frac{\partial f}{\partial E} \right] = \frac{dn}{dt} + 3Hn$$

Where we have divided by E for convenience



$$d\Pi_i = \frac{g_i}{2\Pi^3} \frac{d^3\mathbf{p}_i}{2E_i}$$

a,b=WIMP

1,2=SM (light) particles

No CP violation in DM sector

$$|\mathcal{M}_{12 \rightarrow AB}|^2 = |\mathcal{M}_{AB \rightarrow 12}|^2$$

Energy Conservation

$$f_A f_B = f_A^{eq} f_B^{eq} = e^{-\frac{E_A + E_B}{T}} = e^{-\frac{E_1 + E_2}{T}} = f_1^{eq} f_2^{eq}$$

$$\begin{aligned} \frac{g}{2\pi^3} \int \frac{C[f]}{E} d^3\mathbf{p} &= - \int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta(p_A + p_B - p_1 - p_2) \\ &\quad \left[ |\mathcal{M}_{12 \rightarrow AB}|^2 f_1 f_2 - |\mathcal{M}_{AB \rightarrow 12}|^2 f_A f_B \right] \\ &= -\langle \sigma v \rangle (n^2 - n_{eq}^2) \end{aligned}$$

We have defined the thermally averaged annihilation cross section

$$\langle \sigma v \rangle \equiv \frac{1}{n_{eq}^2} \int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta(p_A + p_B - p_1 - p_2) |\mathcal{M}|^2 f_1^{eq} f_2^{eq}$$

## Non-relativistic species

$$\frac{dn}{dt} + 3Hn - \langle \sigma v \rangle (n^2 - n_{eq}^2)$$

- $\frac{dY}{dt} = \frac{d}{dt} \left( \frac{n}{s} \right) = \frac{d}{dt} \left( \frac{a^3 n}{a^3 s} \right) = \frac{1}{a^3 s} \left( 3a^2 \dot{a} n + a^3 \frac{dn}{dt} \right) = \frac{1}{s} \left( 3Hn + \frac{dn}{dt} \right)$

- $x = \frac{m}{T}$

$$\frac{d}{dt}(a^3 s) = 0 \rightarrow \frac{d}{dt}(aT) = 0 \rightarrow \frac{d}{dt} \left( \frac{a}{x} \right) = 0 \quad \longrightarrow \quad \frac{dx}{dt} = Hx$$

$$\frac{dY}{dt} = \frac{dY}{dx} \frac{dx}{dt} = \frac{dY}{dx} Hx$$

$$\frac{dY}{dx} = -\frac{\lambda \langle \sigma v \rangle}{x^2} (Y^2 - Y_{eq}^2)$$

$$\lambda \equiv \frac{2\pi^2}{45} \frac{M_P g_{*s}}{1.66 g_*^{1/2}} m$$

### Exercise 3

$$\lambda \equiv \frac{2\pi^2}{45} \frac{M_P g_{*s}}{1.66 g_*^{1/2}} m$$

$$\frac{dY}{dx} = -\frac{\lambda \langle \sigma v \rangle}{x^2} (Y^2 - Y_{eq}^2)$$

$$\Delta_Y \equiv Y - Y_{eq}$$



$$\Delta_Y = -\frac{\frac{dY_{eq}}{dx}}{Y_{eq}} \frac{x^2}{2\lambda \langle \sigma v \rangle}, \quad 1 < x \ll x_f$$

$$\Delta_{Y_\infty} = Y_\infty = \frac{x_f}{\lambda \left( a + \frac{b}{3x_f} \right)}, \quad x \gg x_f$$

This leads to :

$$\begin{aligned} \Omega h^2 &= \frac{m_\chi Y_\infty s_0 h^2}{\rho_c} \\ &\approx \frac{10^{-10} \text{ GeV}^{-2}}{\left( a + \frac{b}{60} \right)} \\ &\approx \frac{10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\left( a + \frac{b}{60} \right)} \end{aligned}$$

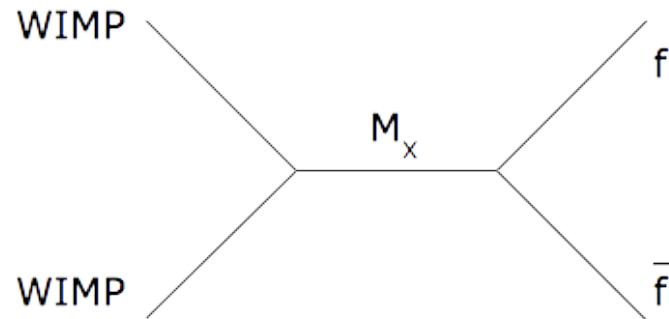


- Very different scales conjure up to lead to the electroweak scale

A typical electroweak scale cross section for a non-relativistic particle

$$\sigma v \approx \alpha^2 \frac{m^2}{M_W^2} = G_F^2 m^2$$

$$G_F \approx 10^{-5} \text{ GeV}^{-2}$$



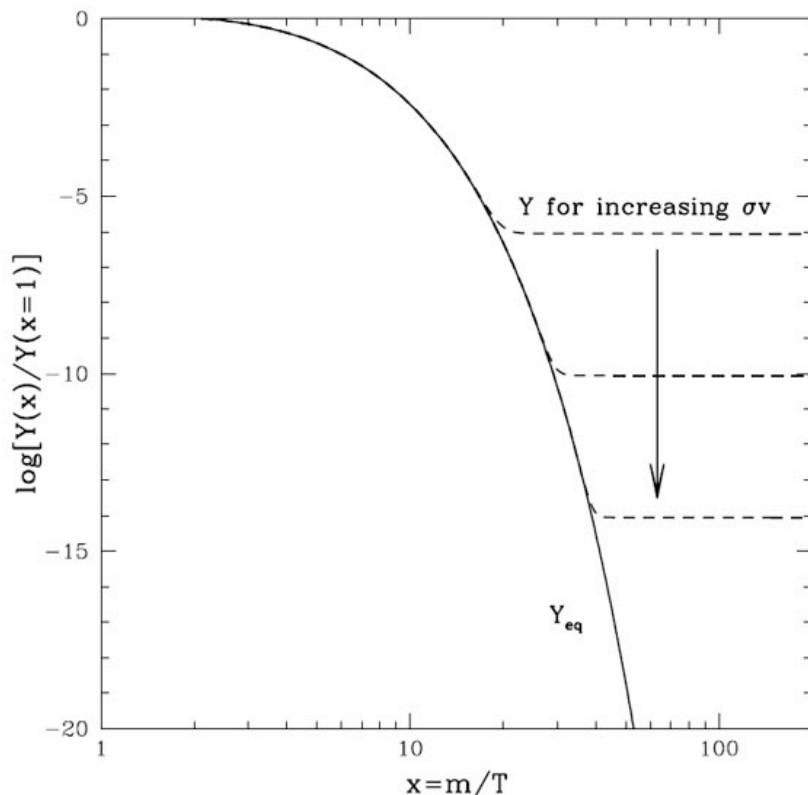
Notice that this implies  $\Omega h^2 \sim \frac{1}{\langle \sigma_A v \rangle} \sim \frac{1}{m^2}$  (non-relativistic particle)

Imposing  $\Omega \leq 1 \rightarrow m \leq 340 \text{ TeV}$  (Griest, Kamionkowski '90)

WIMPs can be thermally produced in the early universe in just the right amount

The freeze-out temperature (and hence the relic abundance) depends on the DM annihilation cross-section

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle (n^2 - n_{eq}^2)$$



$$\Omega_{\chi} h^2 \simeq const. \cdot \frac{T_0^3}{M_{Pl}^3 \langle\sigma_A v\rangle} \simeq \frac{0.1 \text{ pb} \cdot c}{\langle\sigma_A v\rangle}$$

$$T_o \approx 10^{-13} \text{ GeV}$$

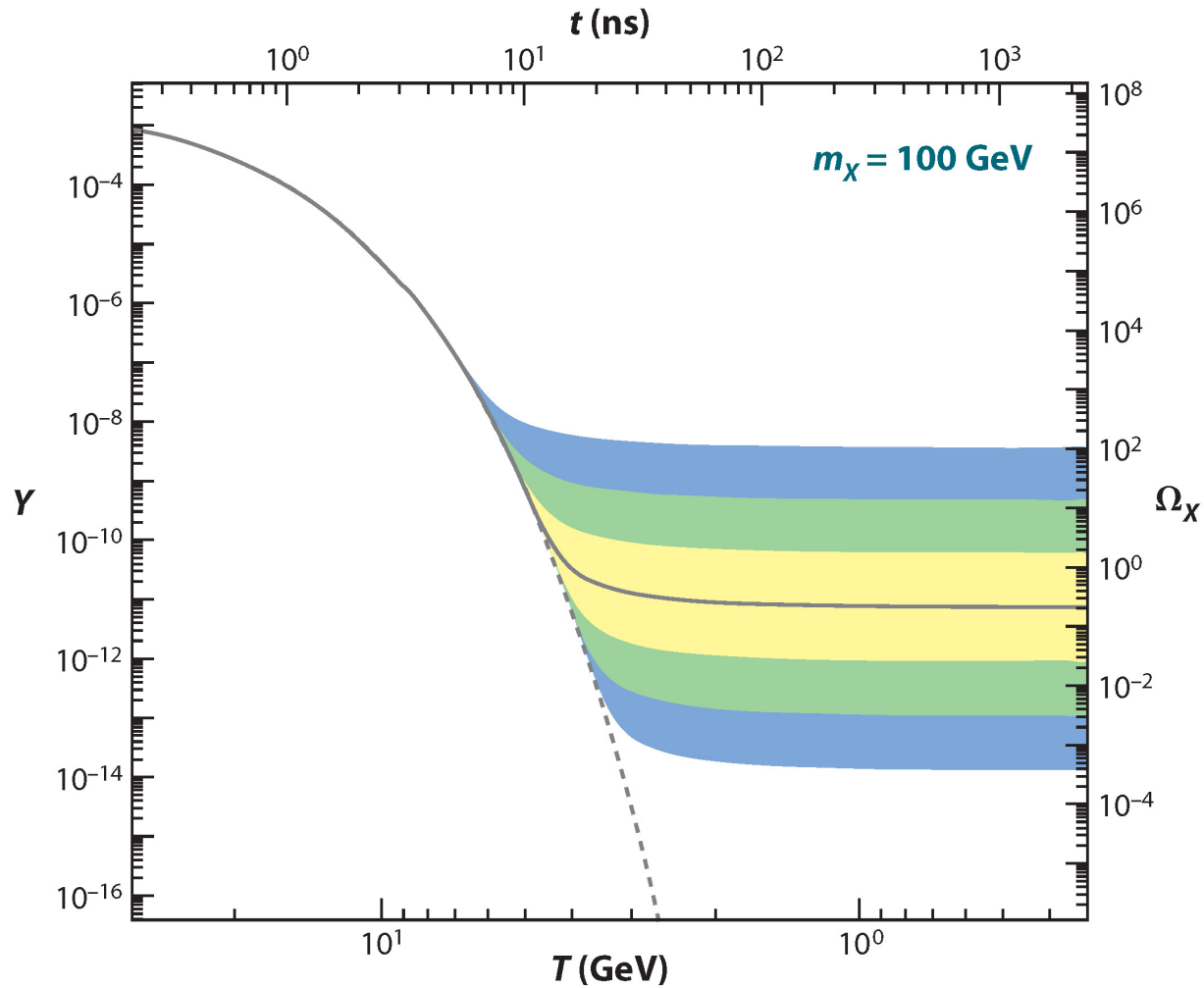
$$H_{100} = 100 \text{ km sec}^{-1} \text{ Mpc} \approx 10^{-42} \text{ GeV}$$

$$M_{Planck} = 1/G_N^{1/2} = 10^{19} \text{ GeV}$$

A generic (electro)Weakly-Interacting Massive Particle can reproduce the observed relic density.

This slide was left  
intentionally dark

$$Y_{eq} = \frac{45}{2\pi^4} \left(\frac{\pi}{8}\right)^{1/2} \frac{g_{eff}}{g_{*s}} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$





# Special cases

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- The low-temperature expansion for the annihilation cross section

$$\sigma_A v = a + \frac{b}{x}$$

is not valid in some cases:

Resonant annihilation

Thresholds

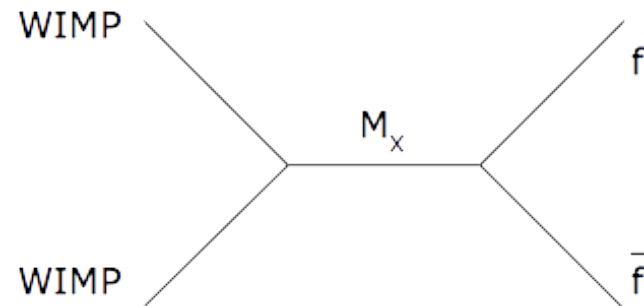
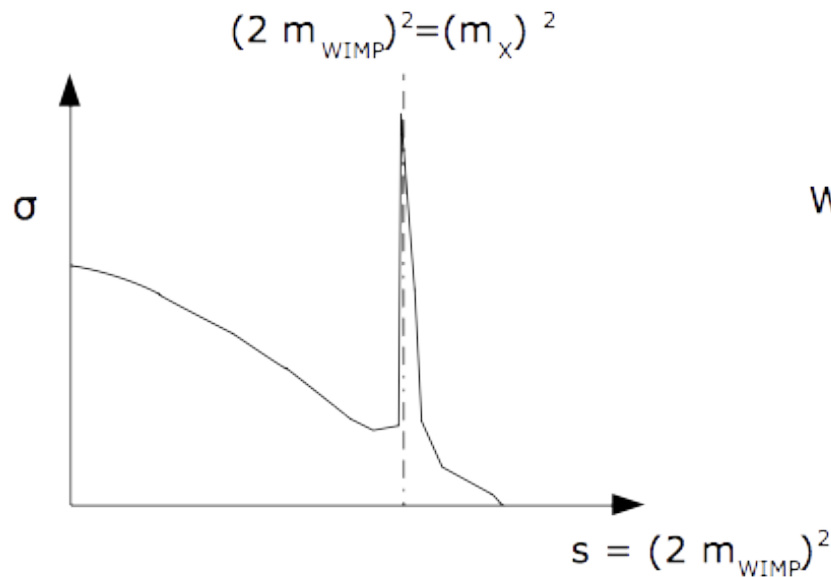
(Gondolo, Gelmini '91)

Coannihilations with other particles close in mass

(Griest, Seckel '91)

# Special cases

- Resonant annihilation:



The resonant increase in the cross section implies a sharp decrease in the relic abundance.

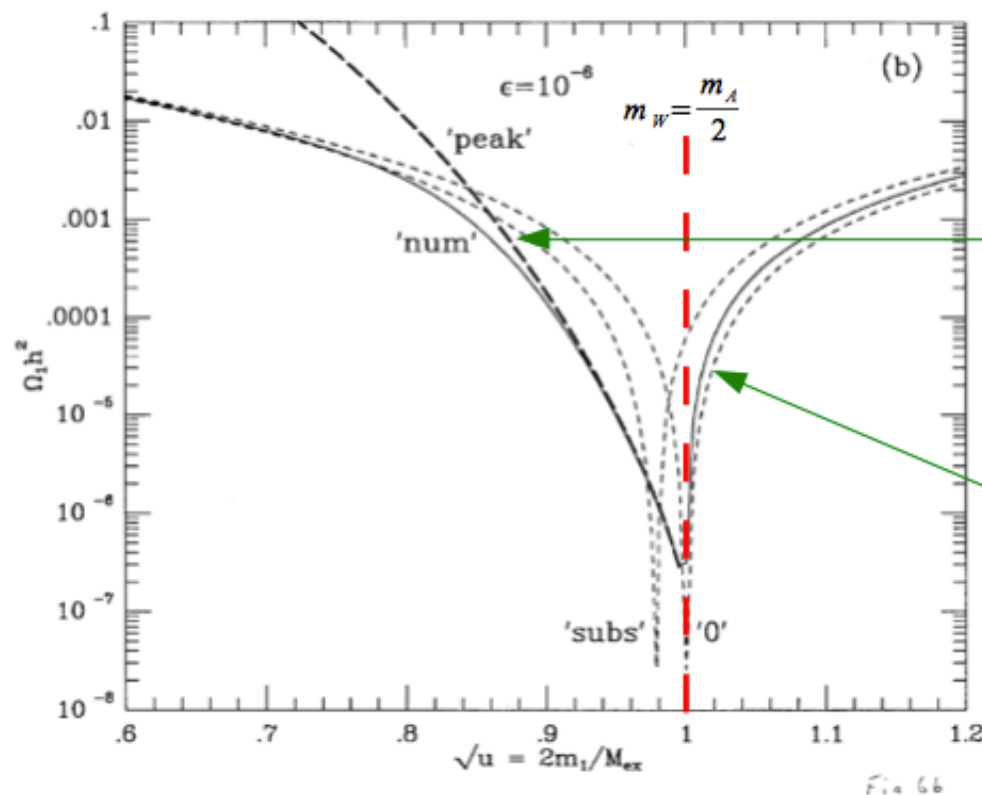
General expression for thermal average of annihilation cross section

(Gondolo, Gelmini '91)

# Special cases

- Resonant annihilation

The annihilation cross section is significantly increased in the pole of the propagator.



As a consequence, the relic density decreases rapidly.

Thermal motion allows resonant annihilation when

$$m_W < \frac{m_A}{2}$$

This is not possible for

$$m_W > \frac{m_A}{2}$$

(Griest, Seckel '91)

However....

- Not applicable to non-thermal candidates (for which the relic abundance is calculated differently, such as decays of heavier particles) (e.g., gravitinos and axinos)
- Non-standard Cosmology, e.g.,

The presence of scalar fields in the Early Universes may induce a period of a much higher expansion rate. WIMPs decouple earlier and have a much larger relic abundance (by several orders of magnitude).

(Catena, Fornengo, Masiero, Pietroni, Rosati, '04)

Changes in the cosmology also affect the calculation of relic abundance for non-thermal dark matter. E.g., quintessence-motivated kination models.

(Gómez, Lola, Pallis, Rodríguez-Quintero '08)

Late entropy production which dilutes the DM density

See, e.g., (Giudice, Kolb, Riotto '01)

E.g., decay of heavy sterile neutrinos

(Abazajian, Koushiappas '06)

(Asaka, Shaposhnikov, Kusenko '06)

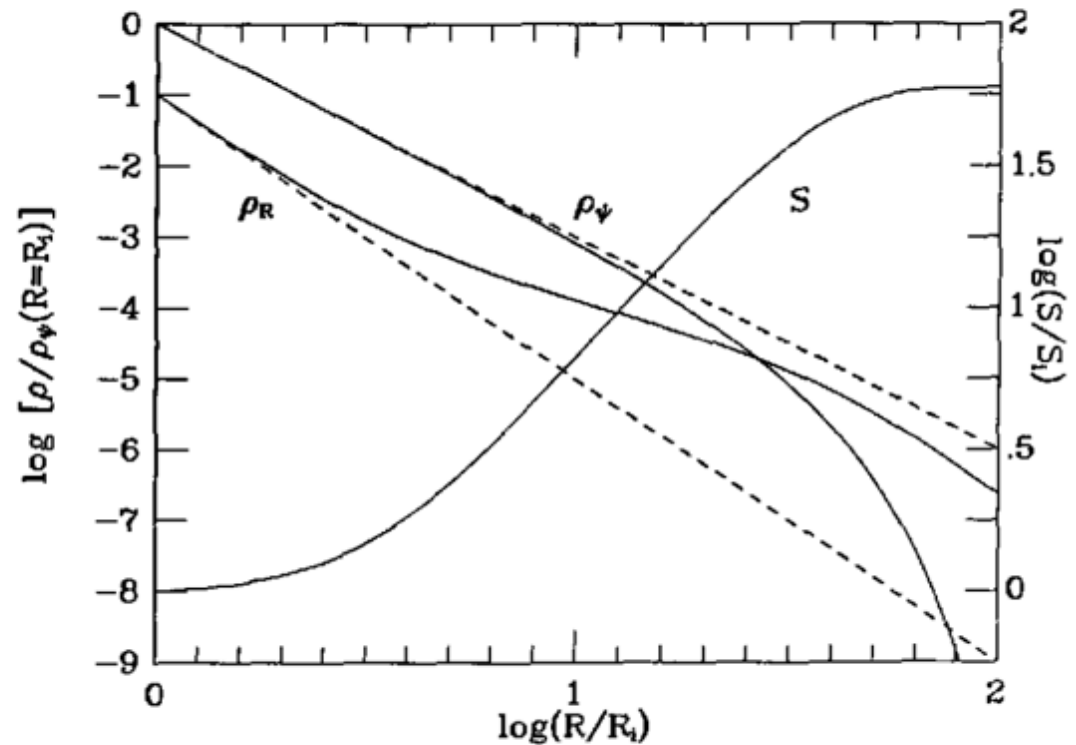


Late (out of equilibrium) decay of semistable particles induce an injection of entropy.

The only constraint to be considered is not to spoil BBN predictions

$$T_R > 5 \text{ MeV}$$

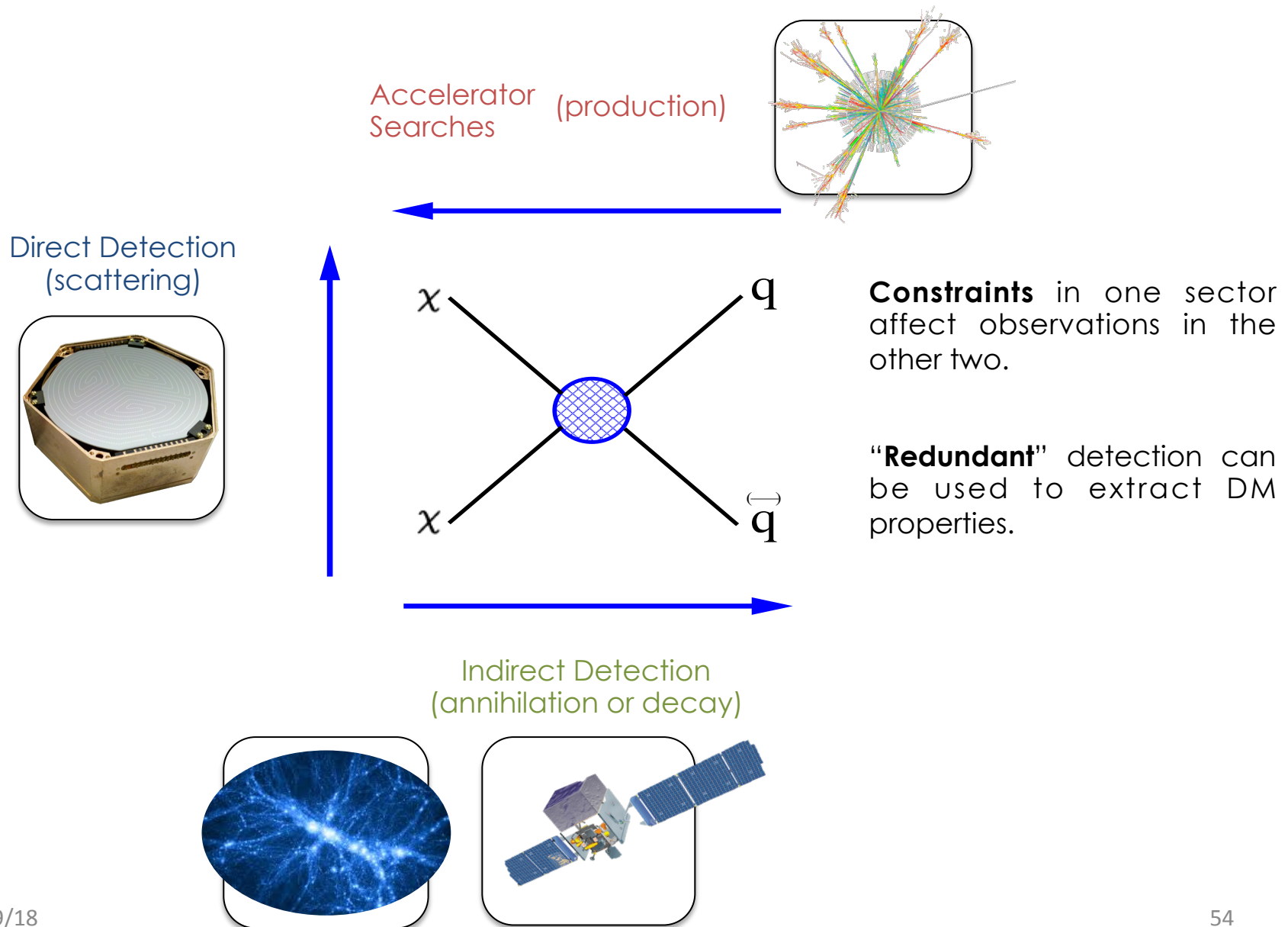
There are various possibilities depending on whether  $T_R$  is smaller or larger than the DM freeze out temperature and on whether the decay produces more DM particles.



(Kolb, Turner Chapt.5)

$$T_R > T_f$$

... probing **DIFFERENT** aspects of their interactions with ordinary matter



In the past ~20 yrs we have had numerous potential signatures for DM. Some remain unexplained while many have been attributed to backgrounds or statistical fluctuations.

**These are shaping our theoretical approach to the DM problem making us look in (often conflicting) directions**

### Astro/Cosmo Probes

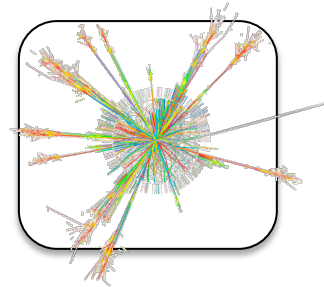


Warm DM (Simulations)

Self-interacting DM

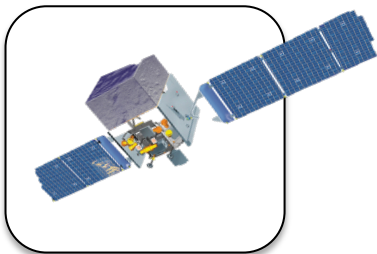
3.5 keV line

LHC



Diphoton at 750 GeV

### Indirect Detection

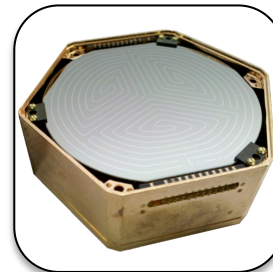


PAMELA-AMS

Fermi-LAT:  
- Galactic Centre  
- 135 gamma line

511 eV emission

### Direct Detection

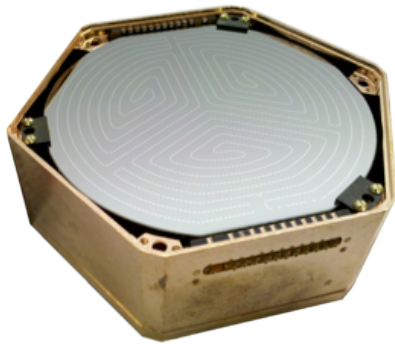


DAMA annual modulation

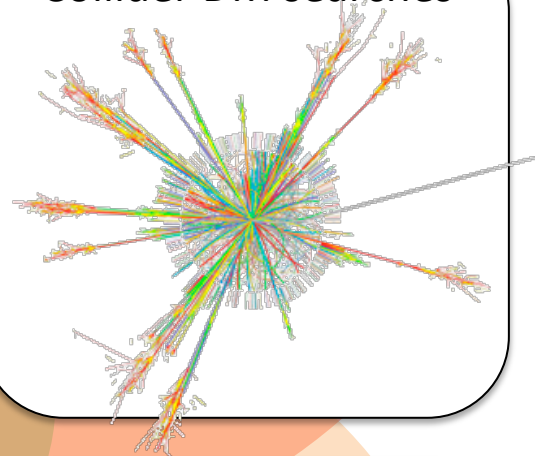
Low-mass craze  
(CDMS, CoGeNT, CRESST)

Dark matter **MUST BE** searched for in different ways...

Direct DM detection



Collider DM searches



Astro/Cosmo probes



Indirect DM detection

