Optical nonlinear processes in semiconductors: an ab-initio description

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European Theoretical Spectroscopy Facility

Outline

1.Introduction: definition and general features 2.SHG in condensed matter

- Screening, Excitons and Local fields
- GaAs
- 3.Nonlinear processes with static fields
 - Theoretical description
 - Electronic and ionic part
 - EFISH

Second harmonic generation



(Long wavelength limit)

Interest for Second Harmonic Generation: in condensed matter

• Probe for materials :

Sensitivity to local symmetries and selection rules for electronic transitions in $\chi^{(2)}$ gives access to states with different symmetries, compared to linear optics

Surfaces Thin films Interfaces Nanowires

•Development and characterisation of new materials

New optical devices



Interest for Second Harmonic Generation: in ultrafast processes

Ultrafast probe for materials : time-resolved SHG

Ultrafast Demagnetization in Cr₂O₃



V. G. Sala et al, Phys. Rev. B 94 014430 (2016)

Ultrafast Reversal of the Ferroelectric Polarization in LiNbO₃



R. Mankowsky *et al*, Phys. Rev. Lett. **118** 197601 (2017)

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Band theory



Band theory



Independent particle approximation: *All the electrons make independent transitions* (IPA) Fermi golden rule

Independent Particle Approximation



Second-order response : in "real life"

Independent Particle Approximation

$$\chi_{abc}^{(2)}(-2\omega,\omega,\omega) = \frac{-ie3}{\hbar^2 m^3 \omega^3 V} \sum_{nml} \int d\vec{k} \frac{1}{E_m - E_n - 2\omega - 2i\eta} \\ \times \left[f_{nl}(\vec{k}) \frac{p_{nm}^a(\vec{k}) \left\{ p_{ml}^b(\vec{k}) p_{\ln}^c(\vec{k}) \right\}}{E_l - E_n - \omega - i\eta} + f_{ml}(\vec{k}) \frac{p_{nm}^a(\vec{k}) \left\{ p_{ml}^b(\vec{k}) p_{\ln}^c(\vec{k}) \right\}}{E_m - E_l - \omega - i\eta} \right]$$

Dipole approximation (optical processes)

Screening



Creation of a hole



Hole + (N-1) electrons







Hole + (N-1) electrons

reaction: polarisation, screening

Back to band structure



GW or Scissor operator

Excitonic effects



Electron-hole interaction



Bethe Salpeter Equation (2-particles equation)

or

Time-Dependent Density-Functional Theory (TDDFT)

1-particle picture ----> 2-particle picture

Excitonic effects



1-particle picture ----> 2-particle picture

Towards the macroscopic polarization

External and total fields

$$\mathbf{E}_{\text{ext}}(\omega) \longleftrightarrow \mathbf{E}_{\text{tot}}(\omega)$$

induced fields — large and irregular fluctuations over the atomic scale

Macroscopic average

Average over distances :

- ✓ large compared to the cell diameter
- small compared to the wavelength of the external perturbation



 $\mathbf{P}_{M} = \chi^{(2)} \mathbf{E}_{tot} \mathbf{E}_{tot}$

The difference between the microscopic fields and the averaged (macroscopic) fields is called local fields.

Macroscopic susceptibility

Zinc-blende symmetry

$$\chi_{xyz}^{(2)}(2\mathbf{q}, 2\omega) = \frac{-i}{12q_x q_y q_z} \left[\epsilon_M(2\mathbf{q}, 2\omega)\right] \left[\epsilon_M(\mathbf{q}, \omega)\right]^2 \chi^{(2)}(2\mathbf{q}, \mathbf{q}, \mathbf{q}, \omega, \omega)$$

- ϵ_{M} : macroscopic dielectric function
- $\chi^{(2)}$: second order response function

Response functions

Dyson equation

$$\chi^{(1)} = \chi_0^{(1)} + \chi_0^{(1)} (v + f_{xc}) \chi^{(1)}$$

Coulomb potential
Non-interacting
response function
(Kohn-Sham)
$$f_{xc}(\mathbf{r}, t, \mathbf{r}', t') = \frac{\delta V_{xc}(\mathbf{r}, t)}{\delta \rho(\mathbf{r}', t')}$$

1st order

Response functions

Dyson equation in momentum space

 $\chi^{(1)}(\mathbf{q} + \mathbf{G}, \mathbf{q} + \mathbf{G}', \omega) = \chi_0^{(1)}(\mathbf{q} + \mathbf{G}, \mathbf{q} + \mathbf{G}', \omega) \\ + \sum_{\mathbf{G}''\mathbf{G}'''} \chi_0^{(1)}(\mathbf{q} + \mathbf{G}, \mathbf{q} + \mathbf{G}'', \omega) \left[v(\mathbf{q} + \mathbf{G}'', \mathbf{q} + \mathbf{G}''') + f_{xc}(\mathbf{q} + \mathbf{G}'', \mathbf{q} + \mathbf{G}''', \omega) \right] \chi^{(1)}(\mathbf{q} + \mathbf{G}''', \mathbf{q} + \mathbf{G}', \omega)$



$\chi^{(2)}$ for GaAs



Screening (scissor)

Exp: S. Bergfeld and W. Daum, PRL (2003)

$\chi^{(2)}$ for GaAs



Exp: S. Bergfeld and W. Daum, PRL (2003)

Screening (scissor)

Screening and local fields

$\chi^{(2)}$ for GaAs



Exp: S. Bergfeld and W. Daum, PRL (2003)

Full calculation

Screening (scissor)

Screening and local fields

Exciton (Long range static kernel)

$$f_{xc} = \frac{\alpha}{q^2}$$

ϵ_{M} for GaAs

$$\chi_{xyz}^{(2)}(2\mathbf{q}, 2\omega) = \frac{-i}{12q_x q_y q_z} \left[\epsilon_M(2\mathbf{q}, 2\omega)\right] \left[\epsilon_M(\mathbf{q}, \omega)\right]^2 \chi^{(2)}(2\mathbf{q}, \mathbf{q}, \mathbf{q}, \omega, \omega)$$

ϵ_{M} for GaAs

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Linear dielectric function

- TDDFT (Long rang kernel)
- Similar results with BSE

ϵ_{M} for GaAs

$$\chi_{xyz}^{(2)}(2\mathbf{q}, 2\omega) = \frac{-i}{12q_x q_y q_z} \left[\epsilon_M(2\mathbf{q}, 2\omega)\right] \left[\epsilon_M(\mathbf{q}, \omega)\right]^2 \chi^{(2)}(2\mathbf{q}, \mathbf{q}, \mathbf{q}, \omega, \omega)$$



Linear dielectric function

- TDDFT (Long rang kernel)
- Similar results with BSE

In common : static approximation

$\chi^{(2)}$ and ϵ_{M} for GaAs





 $\chi^{(2)}$ evaluated with the experimental dielectric functions

Good agreement with the experiment The kernel should be improved in this region (adiabatic approximation)

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Why static fields?

Investigated charge carrier effects in silicon membranes using a femtosecond laser WI Ndebeka, PH Neethling, CM Steenkamp, H Stafast, EG Rohwer Frontiers in Optics, Optical Society of America, 2015



Second harmonic generation at the interface

EFISH : Electric Field Induced Second Harmonic The time dependence of the resulting EFISH signal is a measure of the rate of trap site generation and population, by both electrons and holes being pumped across the interface

$$P_i(2\omega) = \sum_{jkl} \left[\chi_{ijk}^{(2)}(-2\omega;\omega,\omega) + 3\chi_{ijkl}^{(3)}(-2\omega;\omega,\omega,0) \mathcal{E}_l \right] E_j(\omega) E_k(\omega)$$

Why static fields?

Non centro-symmetric materials, without charge accumulation

$$\chi_{ijk}^{(2)}(-2\omega;\omega,\omega) + 3\chi_{ijkl}^{(3)}(-2\omega;\omega,\omega,0) \mathcal{E}_l$$

Ex : Charge accumulation

$$\chi^{(3)}_{ijkl}(-2\omega;\omega,\omega,0) \ \mathcal{E}_l$$

Centro-symmetric materials, with an applied static field

Electro-optic effect : definition

Optical properties of a material can be modified by an applied electric field.

The refractive index changes as a function of the applied field





Electro-optic effect : definition

Linear electro-optic effect (Pockels effect, 1893)



Discovered well before the laser

Can be described as a nonlinear process inside the material

Linear electro-optic effect (LEO) requires non-centrosymmetric materials

Quadratic electro-optic effect (DC Kerr effect, 1875)



Electro-optic effect : definition

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Linear electro-optic effect (LEO) requires non-centrosymmetric materials

- Towards high speed optical modulators : Giant electro-optic effect in Si/SiGe coupled quantum wells
- Strained Si photonic crystal waveguide: data processing in all-silicon components
- Quasiperfect phase matching conditions in homogeneous crystals based on controlled birefrengence
- nondestructive and noninvasive probe of surfaces and interfaces in semiconductors

Linear electro-optic effect

$$\tilde{\epsilon}_{ij} = \epsilon_{ij} + \sum_{k} 8\pi \, \chi^{(2)}_{ijk}(\omega) \, \mathcal{E}_k$$

Second order susceptibility

$$\chi_{ijk}^{(2)}(\omega) = \lim_{\omega_2 \to 0} \chi_{ijk}^{(2)}(-\omega - \omega_2; \omega, \omega_2)$$

Perturbation theory

« Dressed » Hamiltonian

$$\tilde{H} = H_0 + \mathcal{E}.\mathbf{r}$$

$$\widetilde{\epsilon}_{ij}$$
 obtained from the eigenstates of \widetilde{H}

Non perturbative in \mathcal{E} DC-Kerr effect included

$${\mathcal E}$$
 : Static field

Linear electro-optic effect

$$\tilde{\epsilon}_{ij} = \epsilon_{ij} + \sum_{k} 8\pi \, \chi^{(2)}_{ijk}(\omega) \, \mathcal{E}_k$$

Second order susceptibility

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$${\mathcal E}$$
 : Static field

Previous work:

- Mainly performed for SHG
- ω-dependent susceptibility: Independent Particle Approximation [1]
- Static susceptibility : based on the Berry phase (ferroelectric oxides) [2,3]

[1] J. L. P. Hughes and J. E. Sipe, Phys. Rev. B 53, 10751 (1996)
[2] M. Veithen, X. Gonze and P. Ghosez, Phys. Rev. B 71, 125107 (2005)
[3] M. Veithen, PhD thesis, Université de Liège, Belgium (2005)

$$\mathbf{P}^{(2)}(\omega) = 2 \, \chi^{(2)}(-\omega;\omega,0) \, \mathbf{E}(\omega) \, \mathcal{E}$$
LEO susceptibility tensor

Modified dielectric tensor

Impermeability tensor

η=ε-1

$$\tilde{\varepsilon}_{ij}(\omega) = \varepsilon_{ij}(\omega) + \sum_{k} 8\pi \chi_{ijk}^{(2)}(-\omega;\omega,0)\mathcal{E}_{k}$$
$$\tilde{\eta}_{ij}(\omega) = \eta_{ij}(\omega) + \sum_{k} r_{ijk}(\omega) \mathcal{E}_{k}$$
Electro-optic coefficient

$$\chi_{ijk}^{(2)}(-\omega;\omega,0) = -\frac{1}{8\pi} n_i^2(\omega) n_j^2(\omega) r_{ijk}(\omega)$$

General expression for $\chi^{(2)}$ (\mathbf{q}_1, ω_1) (\mathbf{q}_2, ω_2)

$$\chi_{0}^{(2)}(\mathbf{q},\mathbf{q}_{1},\mathbf{q}_{2},\omega_{1},\omega_{2}) = \frac{2}{V} \sum_{n,m,p} \sum_{\mathbf{k}} \frac{\langle \phi_{n,\mathbf{k}} | e^{-i\mathbf{q}\mathbf{r}} | \phi_{m,\mathbf{k}+\mathbf{q}} \rangle \langle \phi_{m,\mathbf{k}+\mathbf{q}} | e^{i\mathbf{q}_{1}\mathbf{r}_{1}} | \phi_{p,\mathbf{k}+\mathbf{q}_{2}} \rangle \langle \phi_{p,\mathbf{k}+\mathbf{q}_{2}} | e^{i\mathbf{q}_{2}\mathbf{r}_{2}} | \phi_{n,\mathbf{k}} \rangle}{E_{n,\mathbf{k}} - E_{m,\mathbf{k}+\mathbf{q}} + \omega_{1} + \omega_{2} + 2i\eta} \\ \times \left(\frac{f_{n,\mathbf{k}} - f_{p,\mathbf{k}+\mathbf{q}_{2}}}{E_{n,\mathbf{k}} - E_{p,\mathbf{k}+\mathbf{q}_{2}} + \omega_{2} + i\eta} + \frac{f_{m,\mathbf{k}+\mathbf{q}} - f_{p,\mathbf{k}+\mathbf{q}_{2}}}{E_{p,\mathbf{k}+\mathbf{q}_{2}} - E_{m,\mathbf{k}+\mathbf{q}} + \omega_{1} + i\eta} \right) + \left[(\mathbf{q}_{1},\omega_{1}) \leftrightarrow (\mathbf{q}_{2},\omega_{2}) \right]$$

 $\omega_2
ightarrow 0 \qquad \mathbf{q}_2
ightarrow 0 \qquad$ Special care for the limit: correct ordering

(1)
$$q_2 \to 0$$

(2) $\omega_2 \to 0$

Brut force numerical evaluation can be problematic

k.p perturbation theory

$$\begin{split} \chi_{0}^{(2)}(\hat{\mathbf{q}}, \hat{\mathbf{q}}_{1}, \hat{\mathbf{q}}_{2}, \omega, 0) &= \frac{1}{V} \sum_{\mathbf{k}} \sum_{n,m,p} \sigma_{n,m,p} \hat{\mathbf{r}}_{nm,\mathbf{k}}(\hat{\mathbf{q}}) \Big[\hat{\mathbf{r}}_{mp,\mathbf{k}}(\hat{\mathbf{q}}_{1}) \, \hat{\mathbf{r}}_{pn,\mathbf{k}}(\hat{\mathbf{q}}_{2}) \left(-\frac{f_{np}}{(E_{nm,\mathbf{k}} + \tilde{\omega})E_{np,\mathbf{k}}} \right. \\ &- \frac{f_{mp}}{(E_{nm,\mathbf{k}} + \tilde{\omega})(E_{pm,\mathbf{k}} + \tilde{\omega})} + \frac{1}{2} \frac{f_{nm} E_{np,\mathbf{k}}}{(E_{nm,\mathbf{k}} + \tilde{\omega})(E_{nm,\mathbf{k}})^{2}} - \frac{1}{2} \frac{f_{nm} E_{pm,\mathbf{k}}}{(E_{nm,\mathbf{k}} + \tilde{\omega})^{2}E_{nm,\mathbf{k}}} \\ &+ \frac{1}{2} \frac{f_{mp} E_{np,\mathbf{k}}}{(E_{pm,\mathbf{k}} + \tilde{\omega})(E_{pm,\mathbf{k}})^{2}} + \frac{1}{2} \frac{f_{mp} E_{nm,\mathbf{k}}}{(E_{pm,\mathbf{k}} + \tilde{\omega})^{2}E_{pm,\mathbf{k}}} + \frac{1}{2} \frac{f_{np} (E_{pm,\mathbf{k}} + E_{nm,\mathbf{k}})}{(E_{np,\mathbf{k}} + \tilde{\omega})(E_{np,\mathbf{k}})^{2}} \right) \\ &+ \hat{\mathbf{r}}_{pn,\mathbf{k}}(\hat{\mathbf{q}}_{1}) \, \hat{\mathbf{r}}_{mp,\mathbf{k}}(\hat{\mathbf{q}}_{2}) \left(-\frac{f_{np}}{(E_{nm,\mathbf{k}} + \tilde{\omega})(E_{np,\mathbf{k}} + \tilde{\omega})} - \frac{f_{mp}}{(E_{nm,\mathbf{k}} + \tilde{\omega})(E_{np,\mathbf{k}})^{2}} \right) \\ &+ \frac{1}{2} \frac{f_{np} E_{pm,\mathbf{k}}}{(E_{np,\mathbf{k}} + \tilde{\omega})(E_{np,\mathbf{k}})^{2}} - \frac{1}{2} \frac{f_{nm} E_{pm,\mathbf{k}}}{(E_{nm,\mathbf{k}} + \tilde{\omega})(E_{nm,\mathbf{k}})^{2}} + \frac{1}{2} \frac{f_{nm} E_{np,\mathbf{k}}}{(E_{nm,\mathbf{k}} + \tilde{\omega})^{2}E_{nm,\mathbf{k}}} \\ &+ \frac{1}{2} \frac{f_{np} E_{pm,\mathbf{k}}}{(E_{np,\mathbf{k}} + \tilde{\omega})(E_{np,\mathbf{k}})^{2}} - \frac{1}{2} \frac{f_{np} E_{nm,\mathbf{k}}}{(E_{nm,\mathbf{k}} + \tilde{\omega})(E_{nm,\mathbf{k}})^{2}} + \frac{1}{2} \frac{f_{np} (E_{np,\mathbf{k}} + \tilde{\omega})^{2}E_{nm,\mathbf{k}}}{(E_{nm,\mathbf{k}} + \tilde{\omega})(E_{pm,\mathbf{k}})^{2}} \right] \end{split}$$

Interband transitions Intraband transitions

Present status of the calculation:

- No local field included (restricted to weakly inhomogeneous materials)
- Screening: Scissor operator (\rightarrow additional terms)
- Inclusion of the excitonic effect: long-range kernel

Scalar Dyson-equation

$$\chi^{(2)}(\mathbf{q}, \mathbf{q}_1, \mathbf{q}_2, \omega_1, \omega_2) = \chi_0^{(2)}(\mathbf{q}, \mathbf{q}_1, \mathbf{q}_2, \omega_1, \omega_2) \left[1 + \frac{\alpha}{q_1^2} \chi^{(1)}(\mathbf{q}_1, \omega_1) \right] \left[1 + \frac{\alpha}{q_2^2} \chi^{(1)}(\mathbf{q}_2, \omega_2) \right] + \chi_0^{(1)}(\mathbf{q}, \omega) \frac{\alpha}{q^2} \chi^{(2)}(\mathbf{q}, \mathbf{q}_1, \mathbf{q}_2, \omega_1, \omega_2)$$

Preliminary results

Electro-optic effect vs Second harmonic generation



$$\lim_{\omega \to 0} \chi_{\text{SHG}}^{(2)}(-2\omega; \omega, \omega) = \lim_{\omega \to 0} \chi_{\text{LEO}}^{(2)}(-\omega; \omega, 0)$$

A surprising experimental result !

GaAs In the transparent region

Shg
$$|\chi^{(2)}_{xyz}(-2\omega;\omega,\omega)|=200 \text{ pm/V}$$

LEO
$$|\chi_{xyz}^{(2)}(-\omega;\omega,0)|=100 \text{ pm/V}$$

$$\lim_{\omega \to 0} \chi_{\text{SHG}}^{(2)}(-2\omega; \omega, \omega) = \lim_{\omega \to 0} \chi_{\text{LEO}}^{(2)}(-\omega; \omega, 0) \qquad \text{Expected !!!}$$



3 contributions:

- electronic
- ionic
- piezzoelectric

Electronic contribution: ω >1eV direct transition from valence electrons

Ionic contribution: ω < 10⁻³ eV ionic lattice displacements

Piezzo contribution: modification of the shape of the unit cell due to electric forces

Electronic + ionic \rightarrow clamped coefficient

Electronic + ionic + piezzo→ unclamped coefficient

The relative amplitude of these contributions depends on the frequency range and on the material

A surprising experimental result !

GaAs In the transparent region

Electronic

Shg
$$|\chi^{(2)}_{xyz}(-2\omega;\omega,\omega)|=200 \text{ pm/V}$$

Leo
$$|\chi_{xyz}^{(2)}(-\omega;\omega,0)| = 100 \text{ pm/V}$$

Electronic+ionic Contributions @1eV

Contribution @1eV

Faust-Henry coefficient:
$$C^{FH} = \frac{\chi^{(2)ionic}}{\chi^{(2)electronic}}$$

Extracted from Raman scattering efficiency of TO and LO phonon modes

$$C^{FH} = -0,51$$

Measured at ω =1eV



S. Adachi, GaAs and Related Materials: Bulk Semiconducting and Superlattice Properties (World Scientific, Teaneck, NJ, 1994).

Cubic SiC: zinc-blende symmetry

Electro-optic coefficient

$$r_{ijk}(\omega) = 8\pi \frac{\chi_{ijk}^{(2)}(-\omega;\omega,0)}{n_i^2(\omega)n_j^2(\omega)}$$

Electronic contribution

Clamped coefficient (e+i)

Clamped coefficient (α -kernel)





10

12

 r_{41}^{S} (pm/V)

 2.7 ± 0.5 1.37^{1}

2.15¹

2.40¹

Cubic SiC: zinc-blende symmetry $r_{xyz}^{S,e}$: QPA $---r_{xyz}^{S}$: QPA $---r_{xyz}^{S}$: QPA+ α $\left|r_{xyz}(\omega)\right| \, \left[\mathrm{pm/V}
ight]$ Electro-optic coefficient 20 $r_{ijk}(\omega) = 8\pi \frac{\chi_{ijk}^{(2)}(-\omega;\omega,0)}{n_i^2(\omega)n_j^2(\omega)}$ 10 0 2 6 8 0 4 Electronic contribution ω (eV) Clamped coefficient (e+i) Clamped coefficient (α -kernel) $\chi_{xyz}^{(2)}$ (pm/V) α (a.u.) $C^{FH} = +0.35$ Expt. OPA 0 24.8 $OPA + \alpha_{static}$ 0.3 39.1 G. L. Harris, Properties of Silicon Carbide (IET, Stevenage, $OPA + \alpha$ 0.5 55.2

1995).

EFISH

Electronic part of the $\chi^{(3)}$

Cubic SiC: zinc-blende symmetry



Importance of the ionic contribution?

Nothing like the Faust-Henry coefficient is available

What's next?

- * Electronic contributions for LEO and EFISH: done!
- * Precise evaluation of the Faust-Henry coefficient for LEO : $C^{FH} = \frac{\chi^{(2)ionic}}{\chi^{(2)electronic}}$

Experimental values only for simple materials available

Theoretical values for $\omega = 0$ (poor agreement)

Frequency-dependent coefficient required

Material design **TeraHertz Generation** for optical modulators (a) (\mathbf{a})

$$\chi_{ijk}^{(2)}(-\omega;\omega,0) = \chi_{kij}^{(2)}(0;-\omega,\omega)$$

* Importance of the ionic contribution for EFISH



Formalism and GaAs : E. Luppi and H. Hübener (PhD) LSI, Ecole Polytechnique

LEO and EFISH : L. Prussel (PhD) LSI, Ecole Polytechnique

Thank you for your attention