Flat Bands Systems

Matteo Rizzi
Universität zu Köln & Forschungszentrum Jülich

Benasque, Atomtronics, 10.05.2019

Synthetic Quantum Matter

Bottom-up engineering

trapped ions, superconducting qubits, quantum dots, NV centers, etc.

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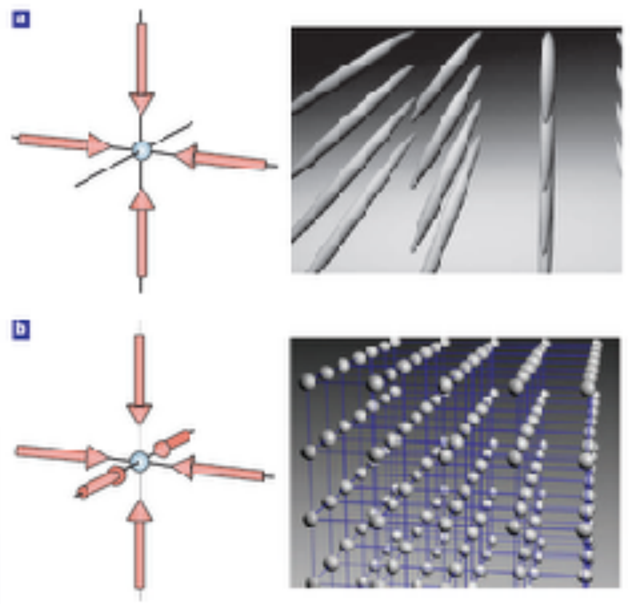

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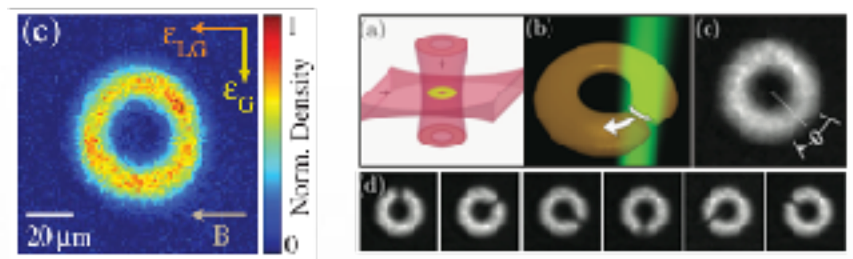
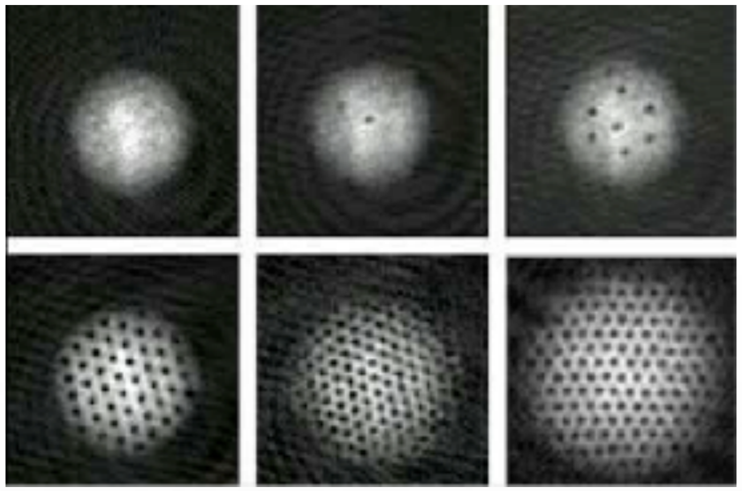


Ramanathan et al., PRL 106, 130401 (2011);



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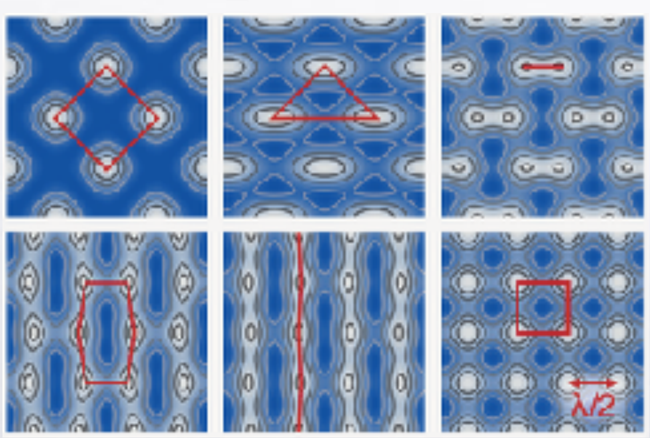
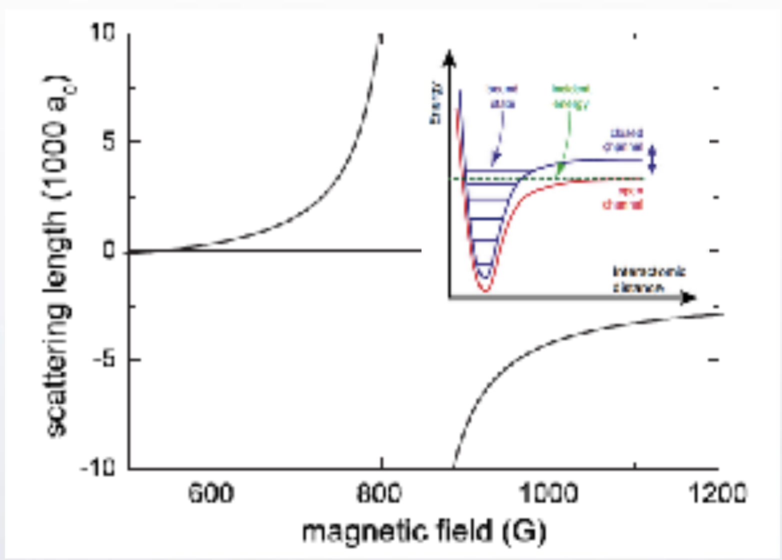
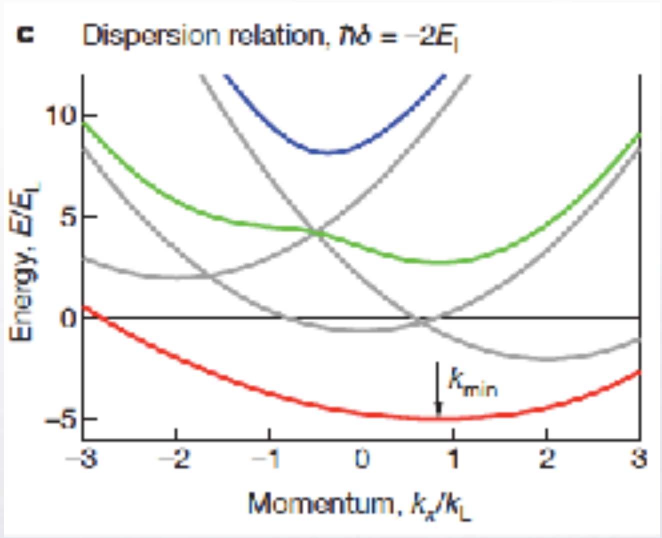


L. Tarruell, et al., Nature **483**, 302 (2012)



C.Chin,et al.,RMP **82**,1225 (2010)



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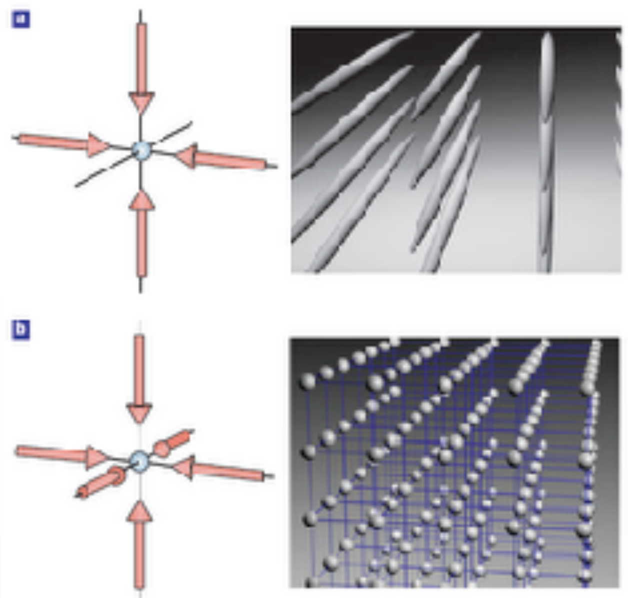

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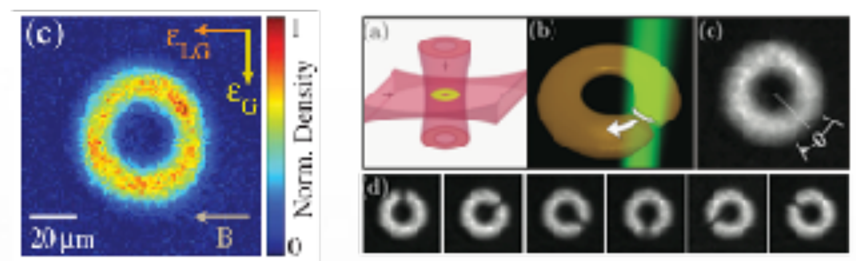

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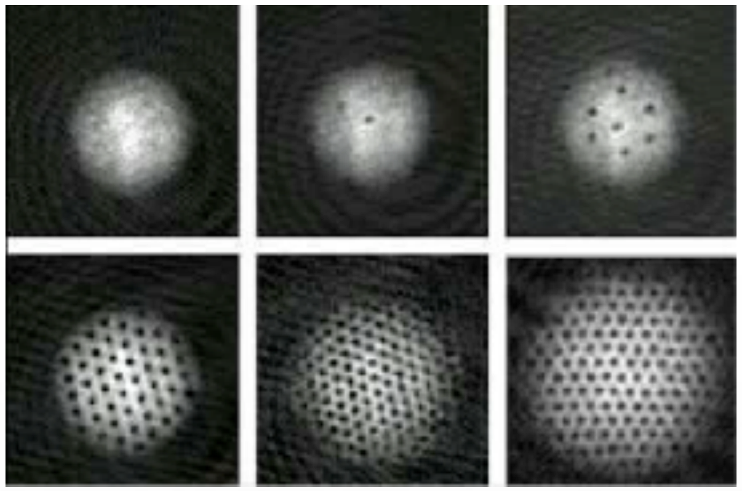

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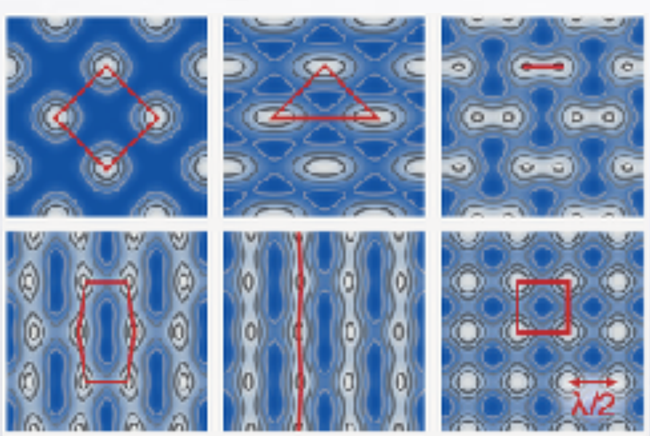

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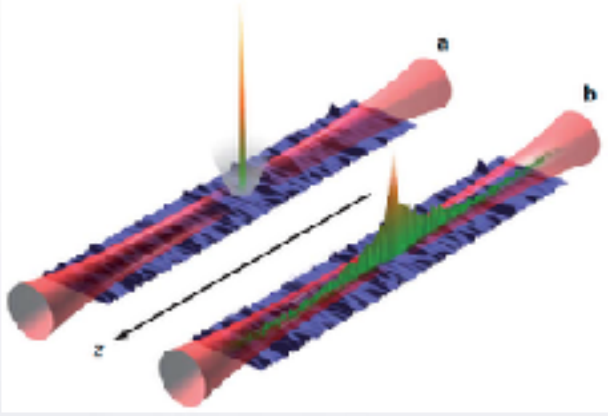
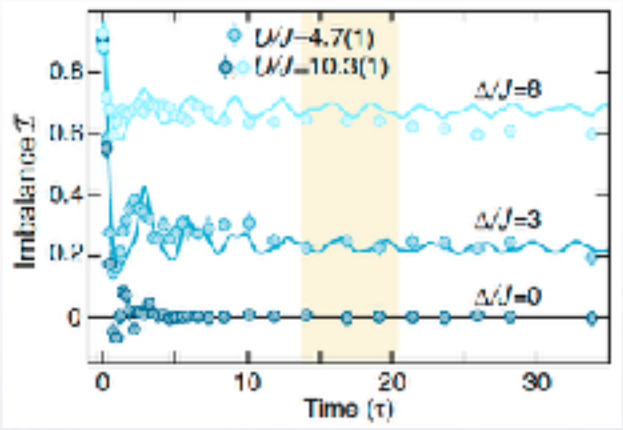
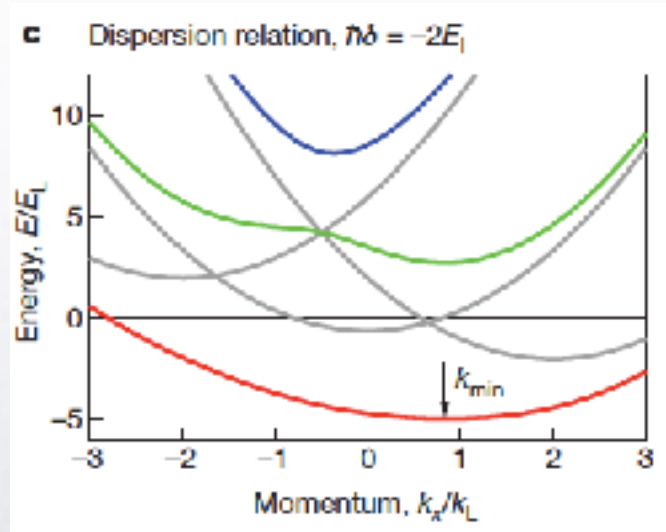


J. Billy, et al.,
Nature 453, 891 (2008)



M. Schreiber, et al.,
Science 349, 842 (2015)



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General Picture

Geometrical constraints + Background gauge fields

↓

Frustration, large degeneracy,
single-particle topological character,
flat-band dispersion

+

Interactions btw. constituents

↙ ↘

Fractional topological phases
here fermions
range q => fraction 1/(q+1)

Collective transport properties:
pair-transport in bosons, etc.

see works by Tovmasyan, Peotta, Huber, Törma (& others) for enhanced fermionic pairing, too

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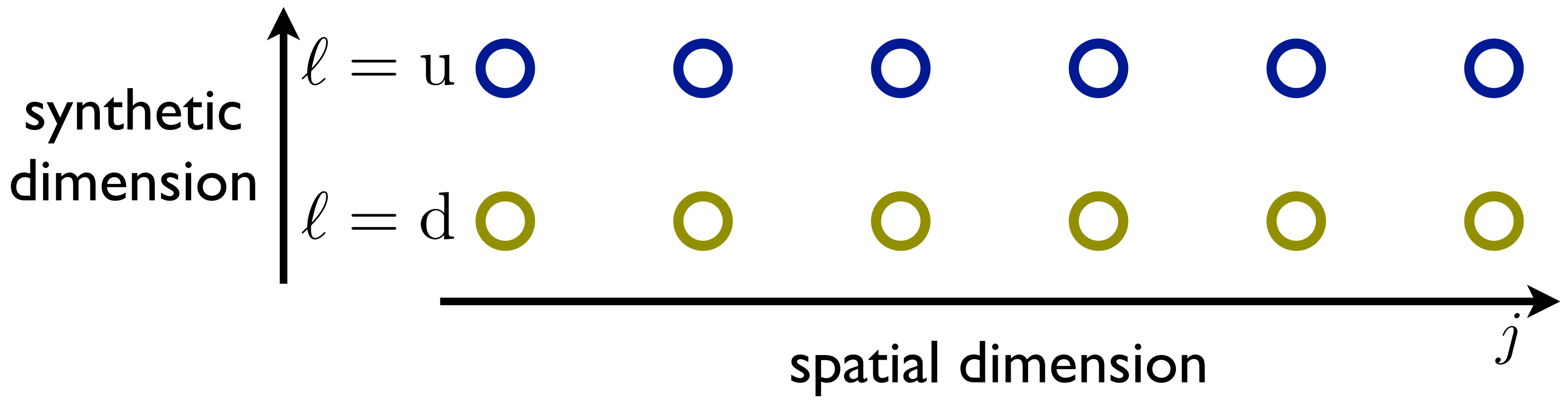
From standard to flat bands



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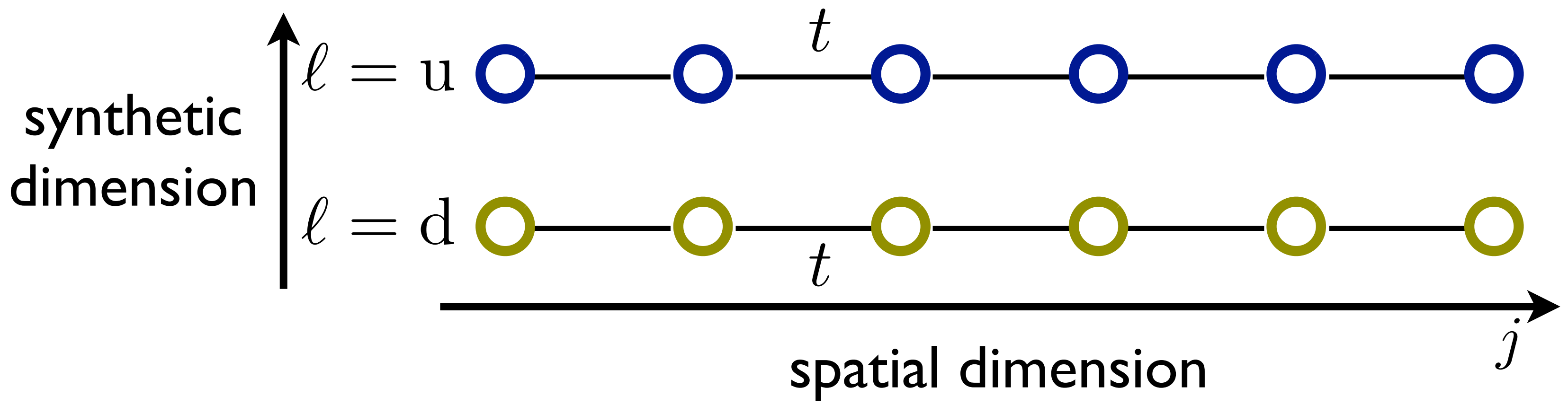

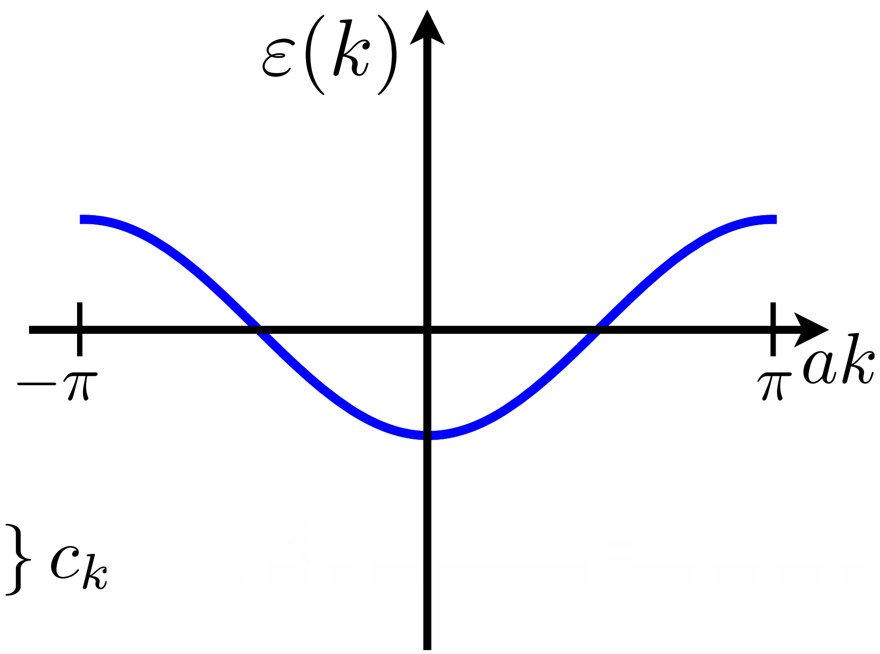

$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ \quad -2t\cos(ak)\ \sigma_0 + \quad \} c_k$$

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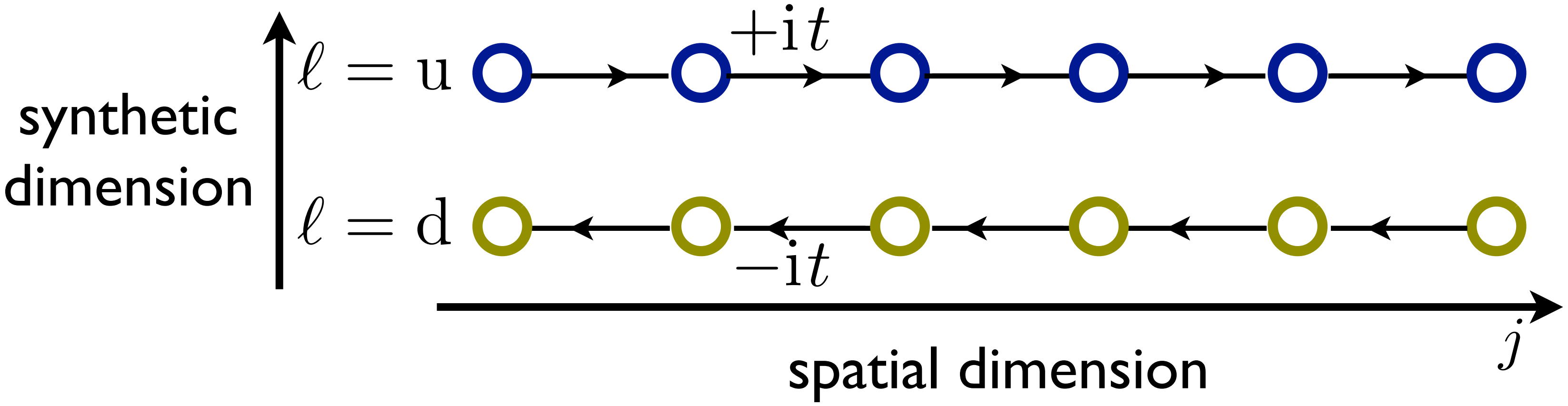

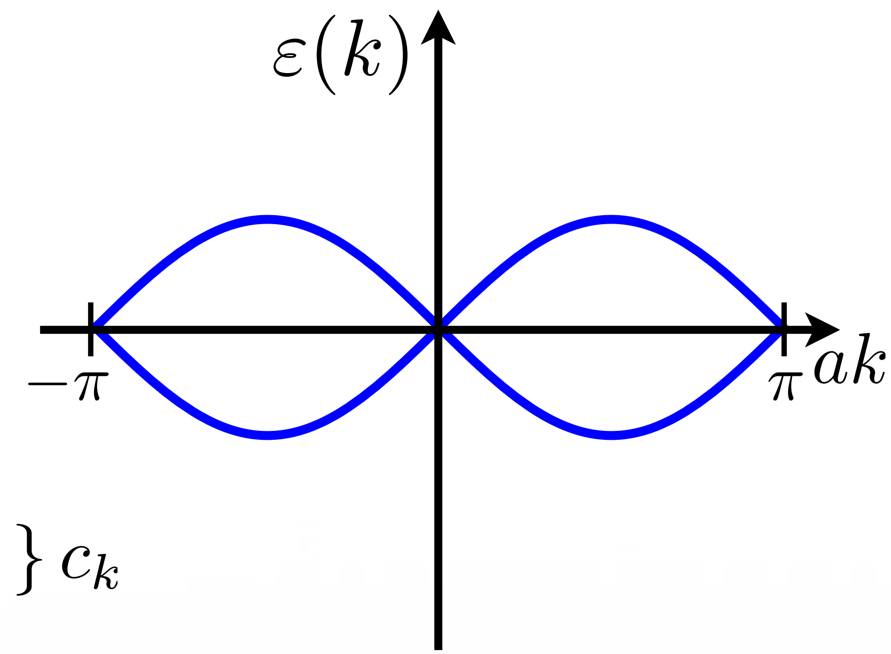

$$\mathcal{H}_0 = \sum_k c_k^\dagger \{[\qquad + 2t\sin(ak)]\,\sigma_3 + \qquad \}\, c_k$$

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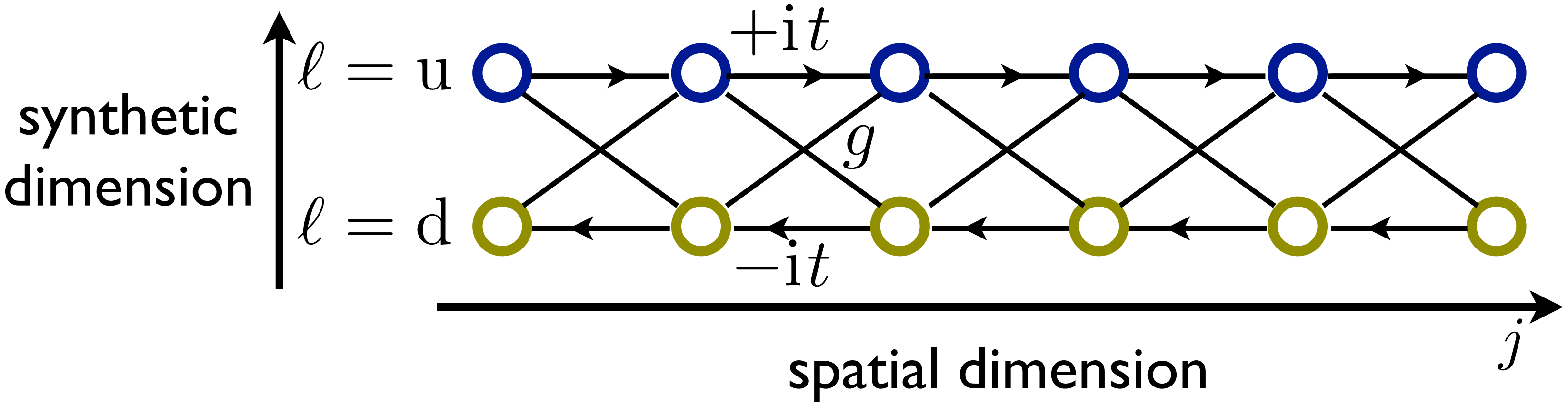
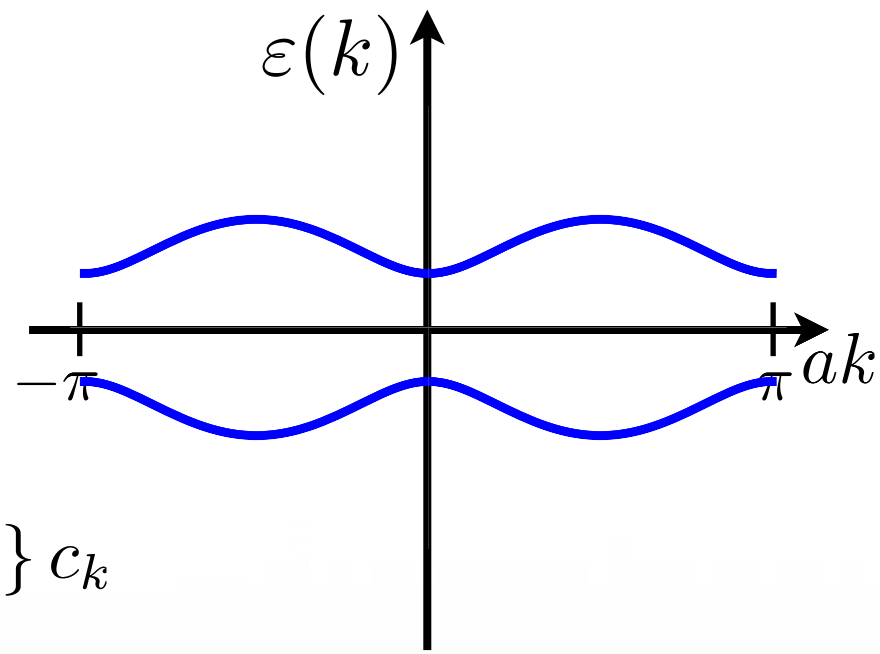
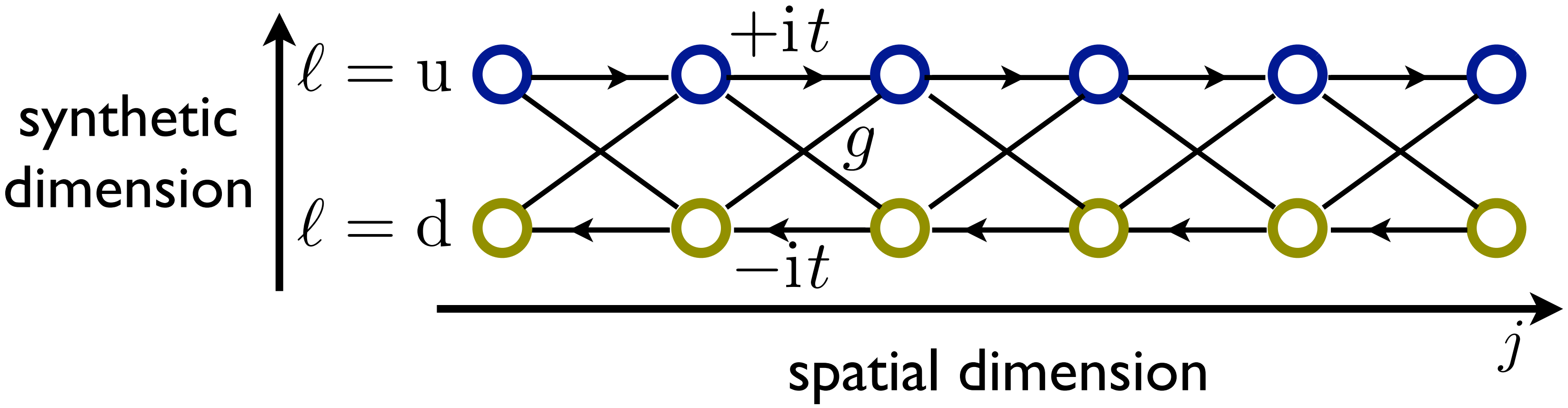
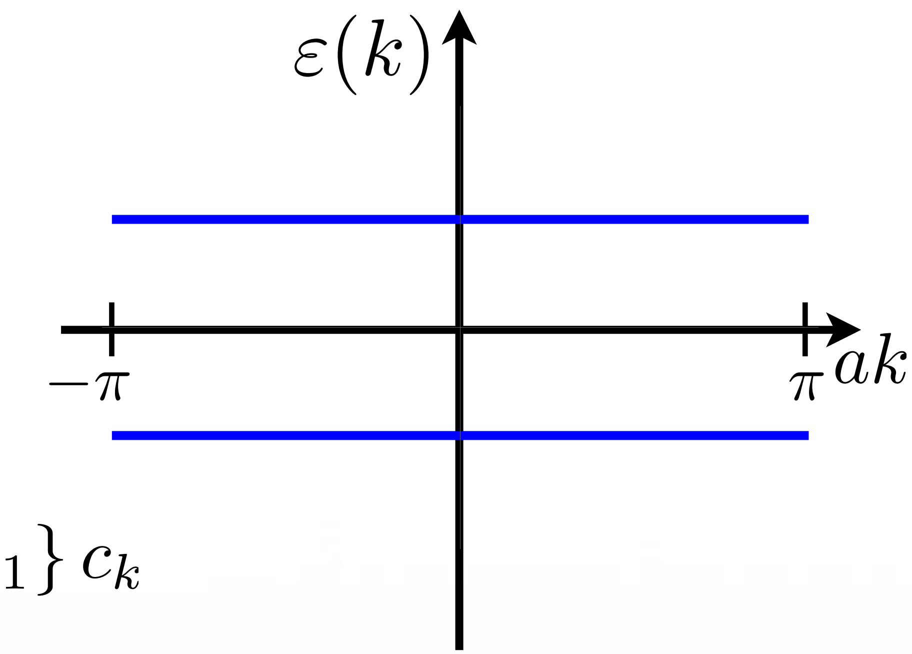




$$\mathcal{H}_0 = \sum_k c_k^\dagger \{[\qquad + 2t\sin(ak)]\,\sigma_3 + [\qquad - 2g\cos(ak)]\,\sigma_1\}\, c_k$$

Flat topological bands





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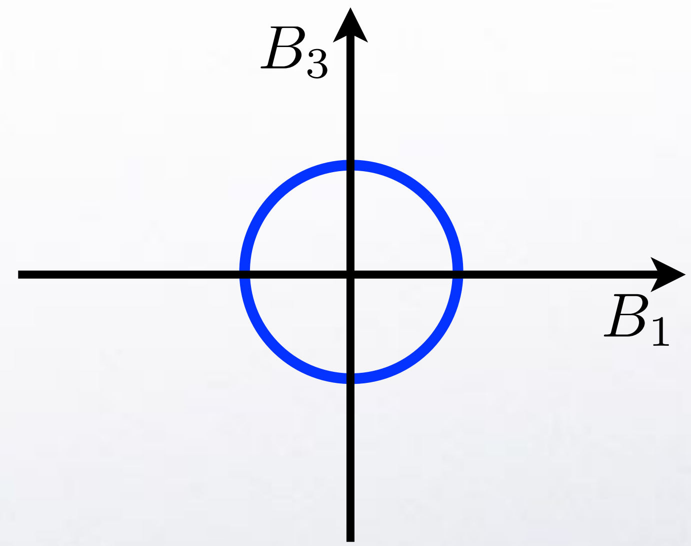
$g = t$ flat bands, with topological character!

$$\mathcal{H}_0 = \sum_{k\in \mathrm{BZ}} c_k^\dagger \{\mathbf{B}(k)\cdot\boldsymbol{\sigma}\}\, c_k$$

constrained in (B_1, B_3) plane by chiral symmetry:

$$\sigma_2\mathcal{H}_0(k)\sigma_2 = -\mathcal{H}_0(k)$$



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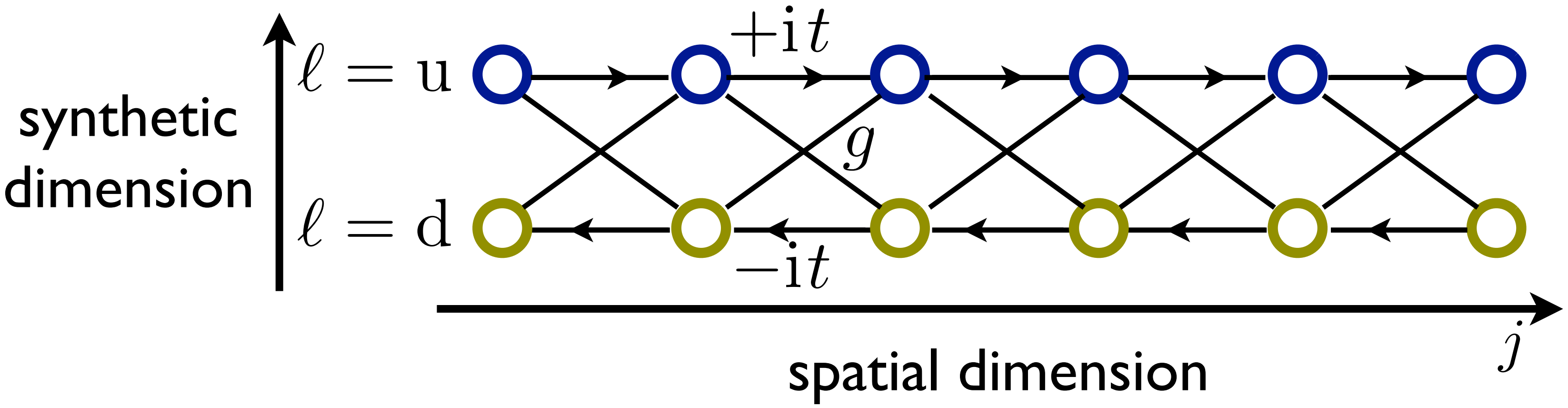

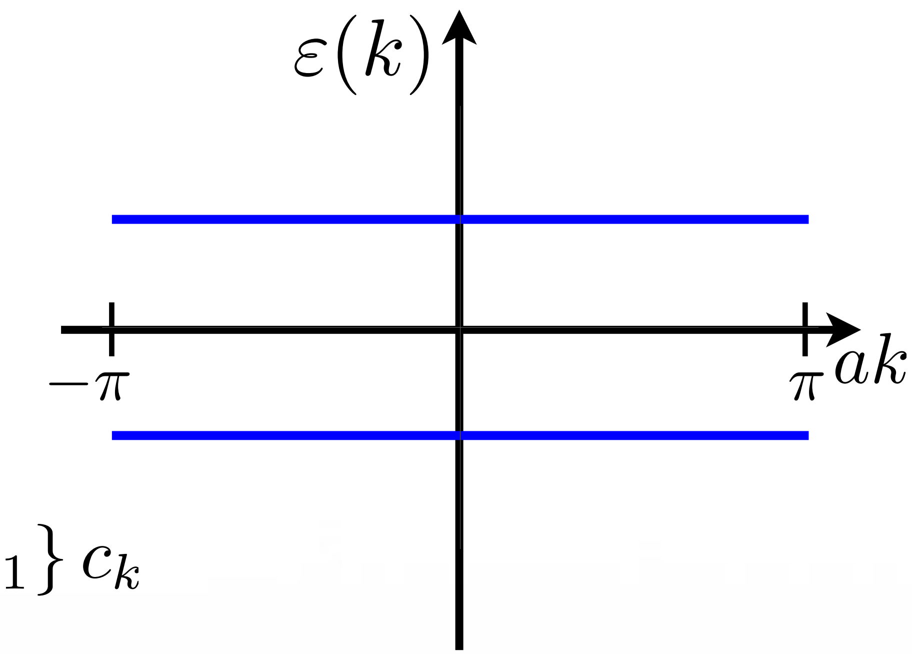

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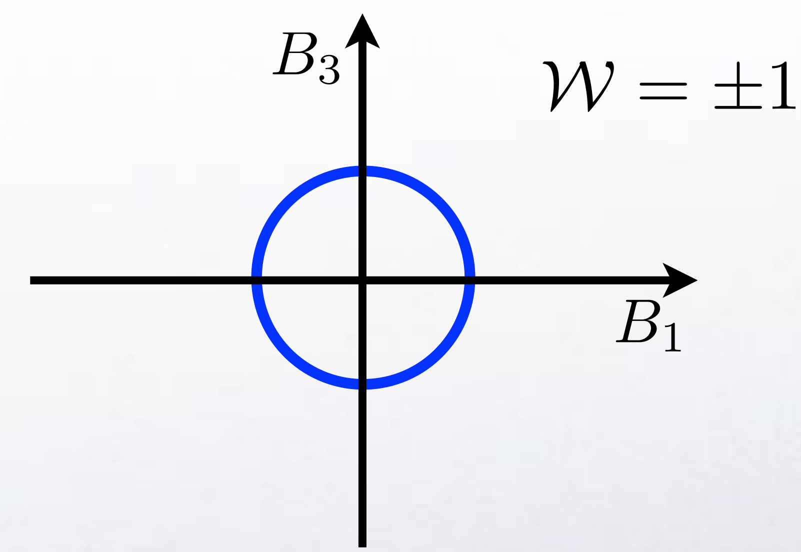

Symmetry Protected Topological (SPT) Order

$$\mathcal{A}_\pm(k) = \langle \varepsilon_\pm(k)|\mathrm{i}\partial_k|\varepsilon_\pm(k)\rangle \qquad \varphi_{\mathrm{Zak},\pm} = \int_{\mathrm{BZ}} \mathrm{d}k\, \mathcal{A}_\pm(k) = \pi$$

e.g., S. Ryu, et al., NJP **12**, 065010 (2010), D. Xiao, et al., RMP **82**, 1959 (2010)

φ_{Zak} measured in cold gases via Ramsey interferometry

M. Atala, et al., Nat. Phys. 9, 795 (2013)

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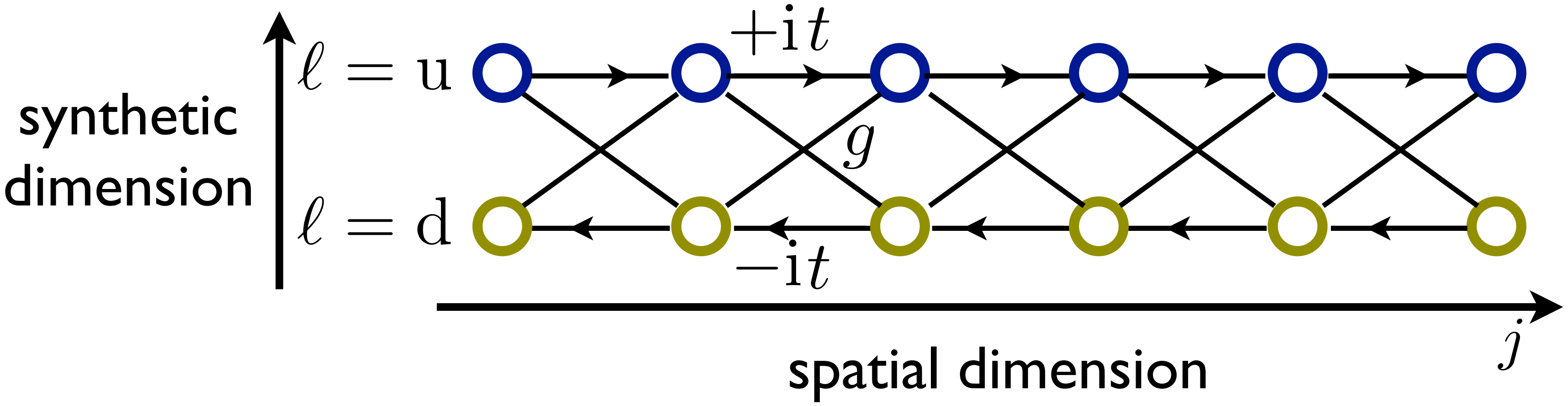
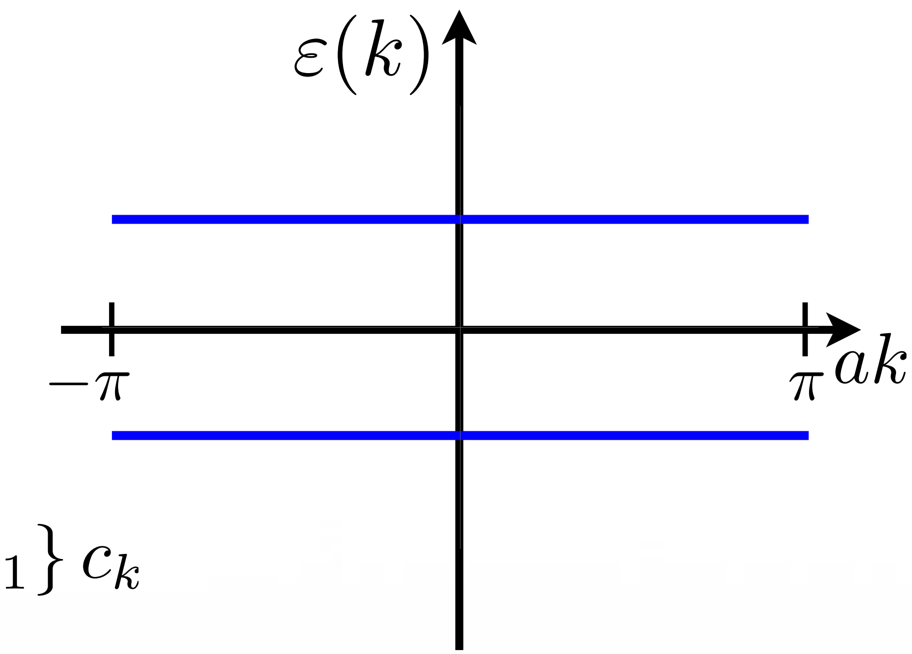
SPT - Trivial transition





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$g = t$ flat bands, with topological character! *(SPT)*

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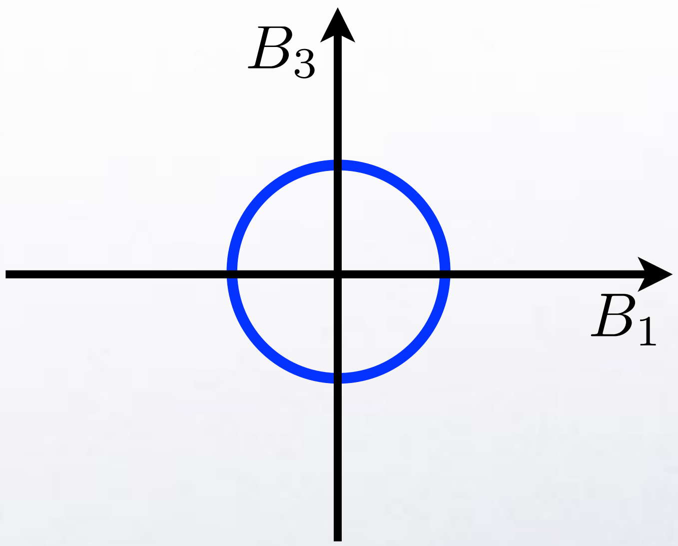

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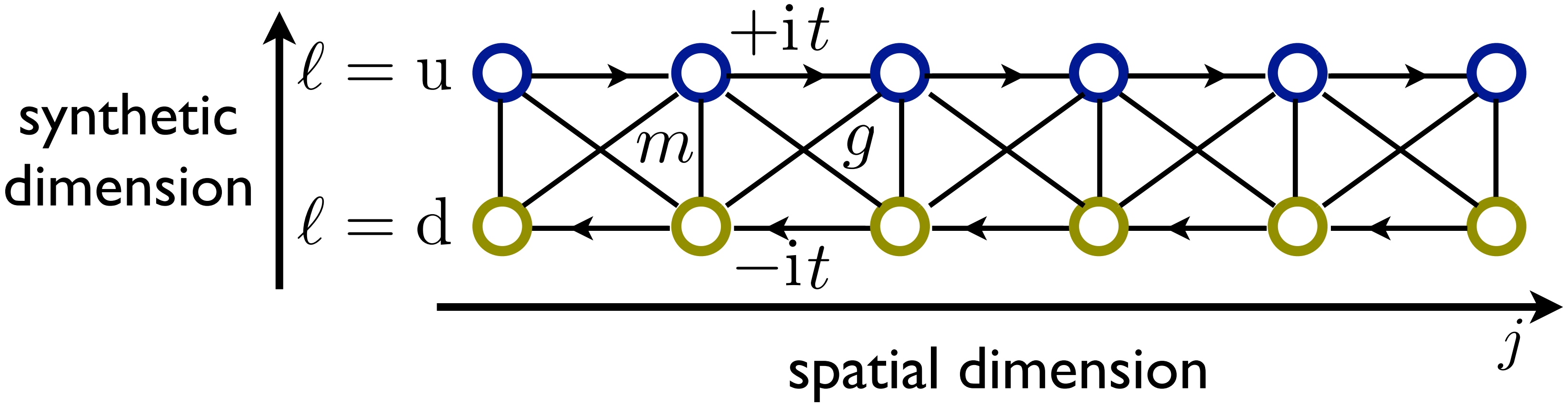

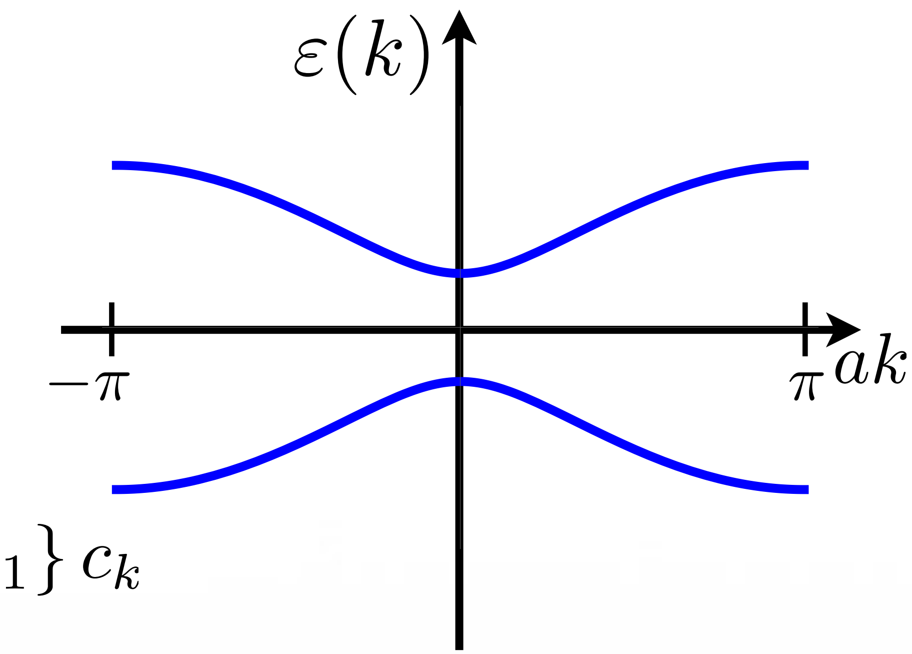

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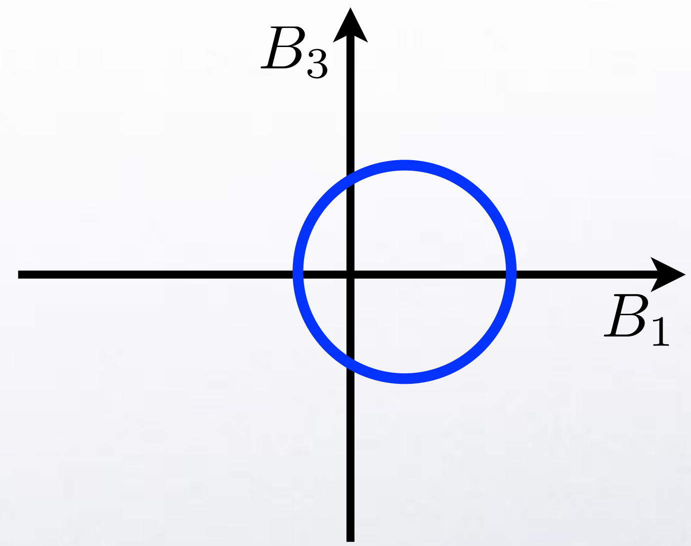

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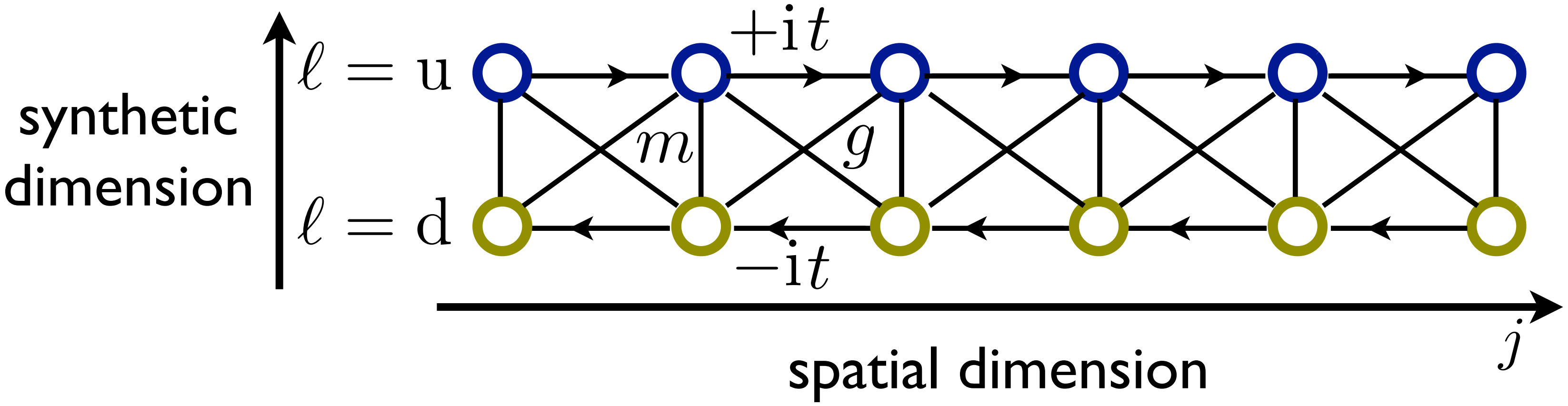

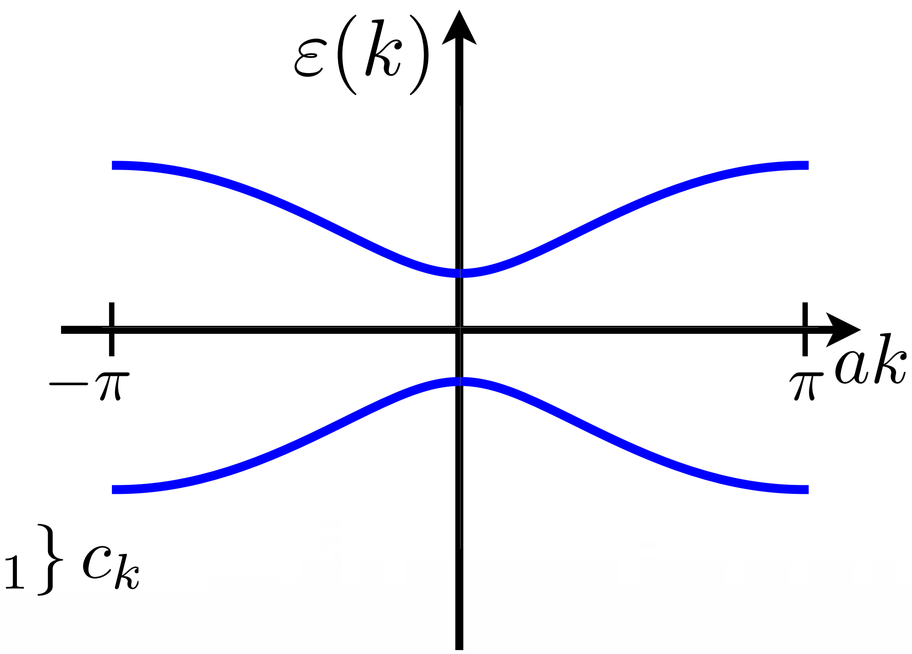

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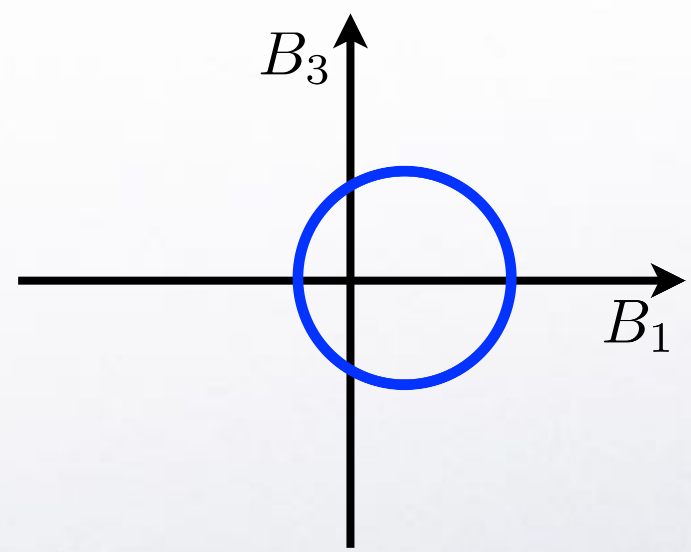

$$T: \ \sigma_1\mathcal{H}_0^*(-k)\sigma_1 = +\mathcal{H}_0(k)$$
$$C: \ \sigma_3\mathcal{H}_0^*(-k)\sigma_3 = -\mathcal{H}_0(k)$$

⇨ Class **BDI** of AZ table

Altland, Zirnbauer, PRB **55**, 1142 (1997)

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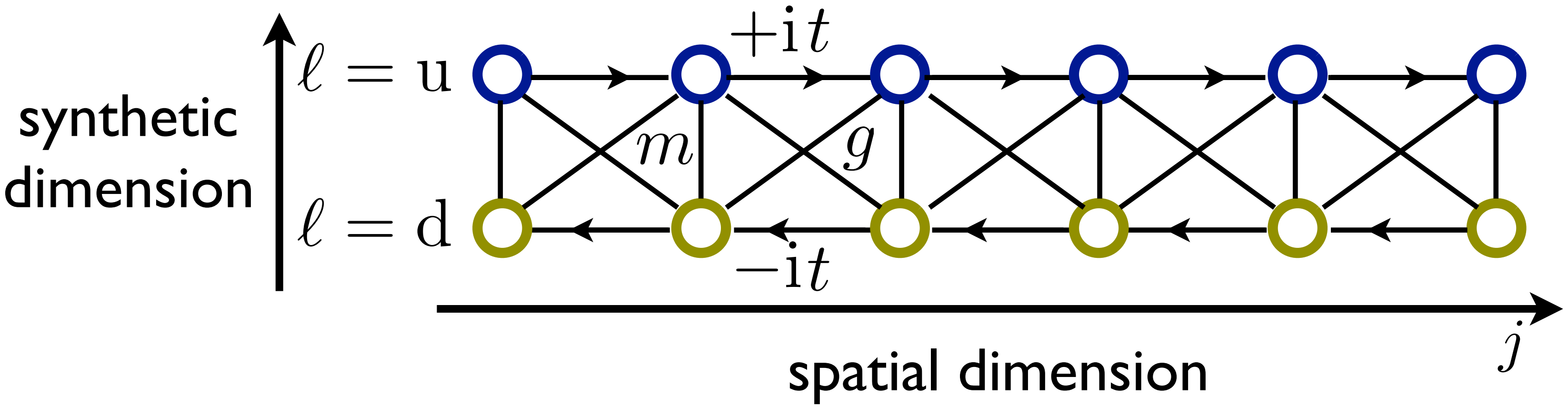

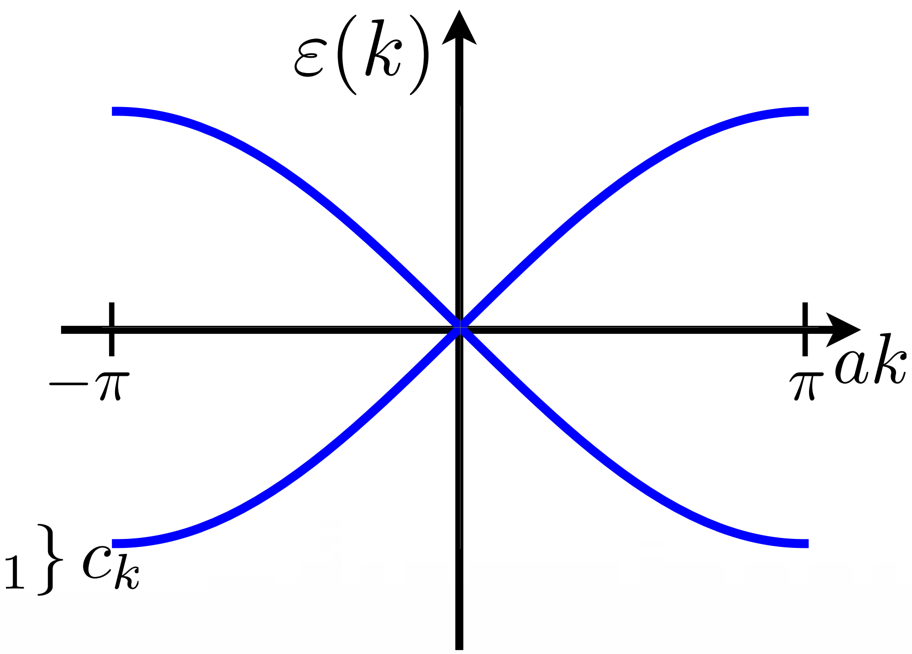

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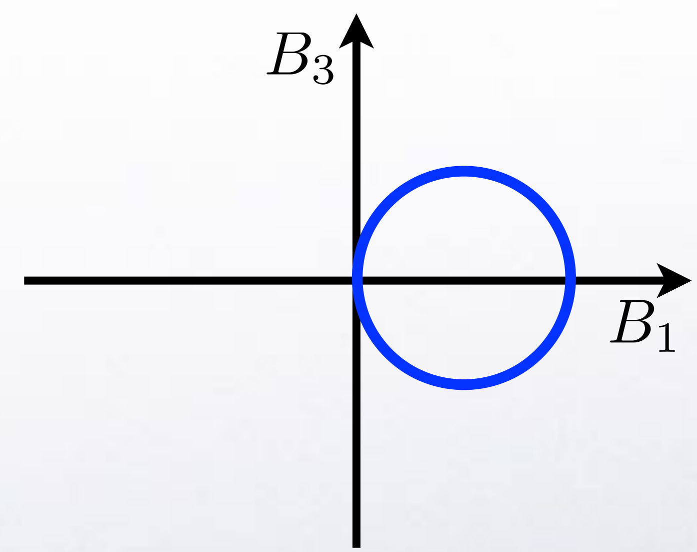

$m = g = t$ <u>isolated Dirac cone</u>

[no Fermion doubling]

Creutz, Horváth, PRD **50**, 2297 (1994)
Creutz, PRL **83**, 2636 (1999)

matteo.rizzi@fz-juelich.de

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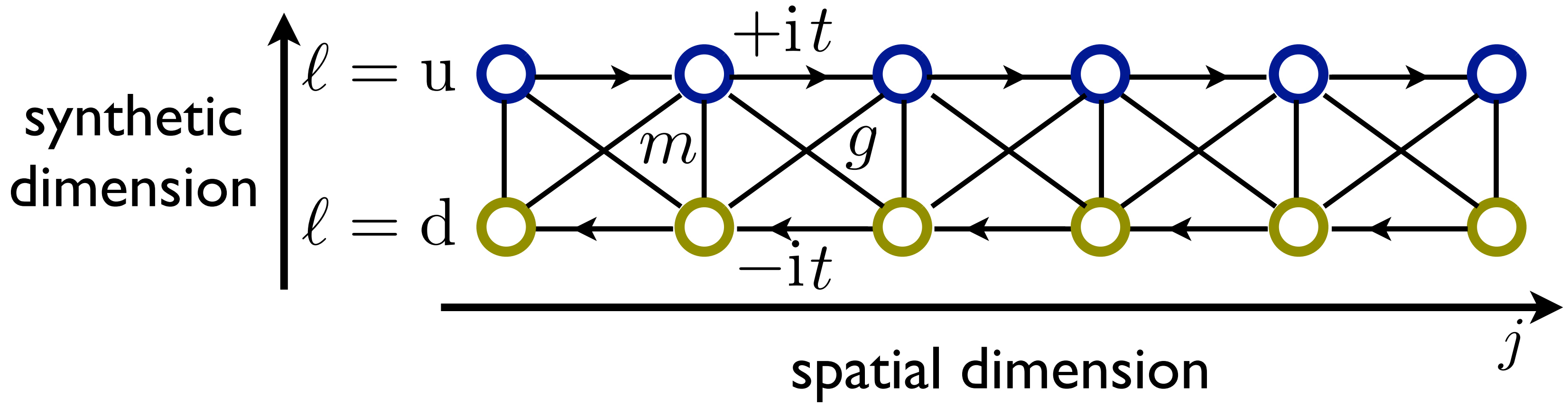

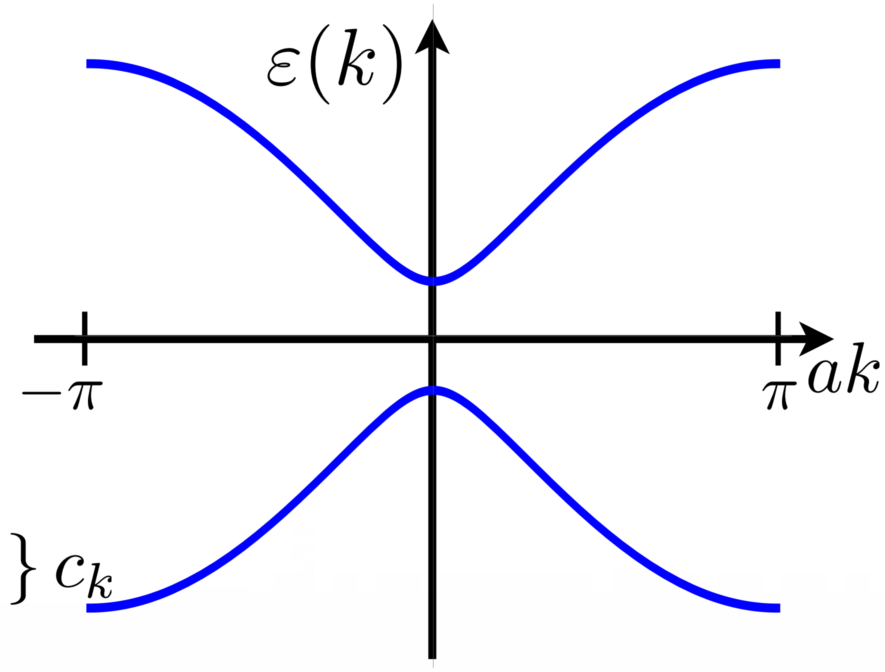

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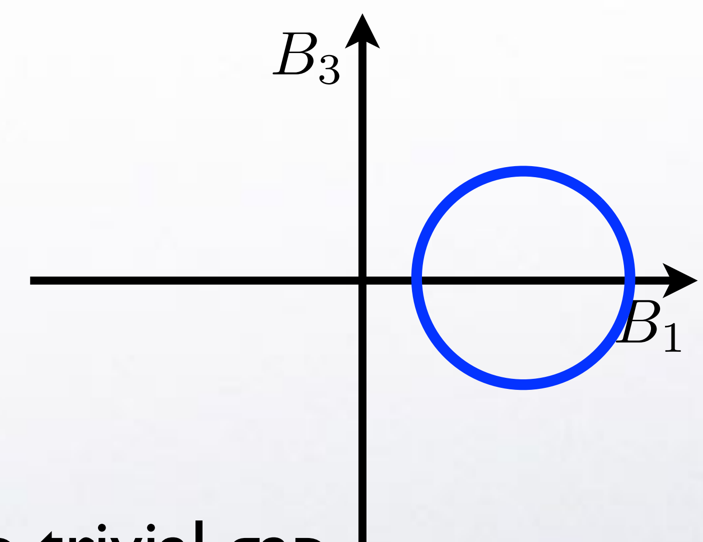

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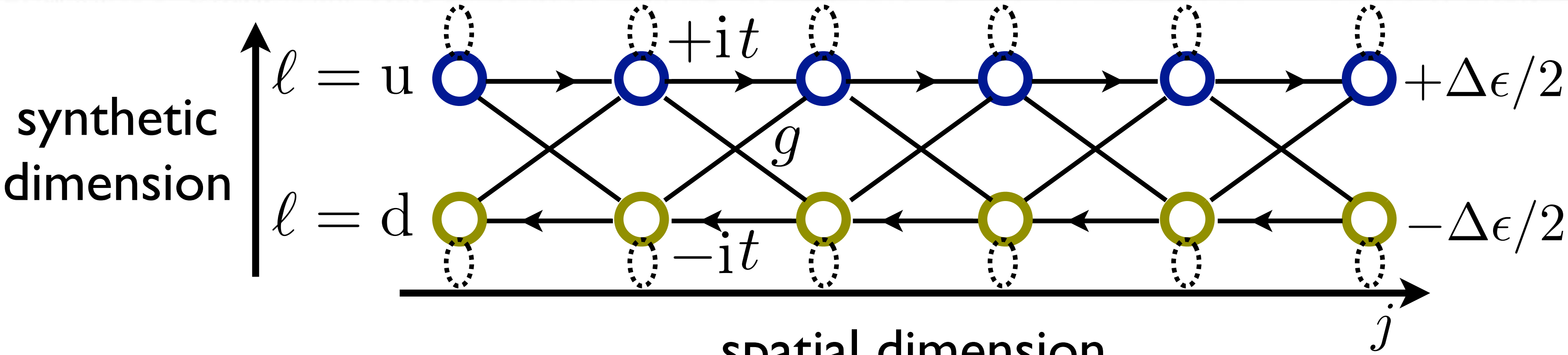
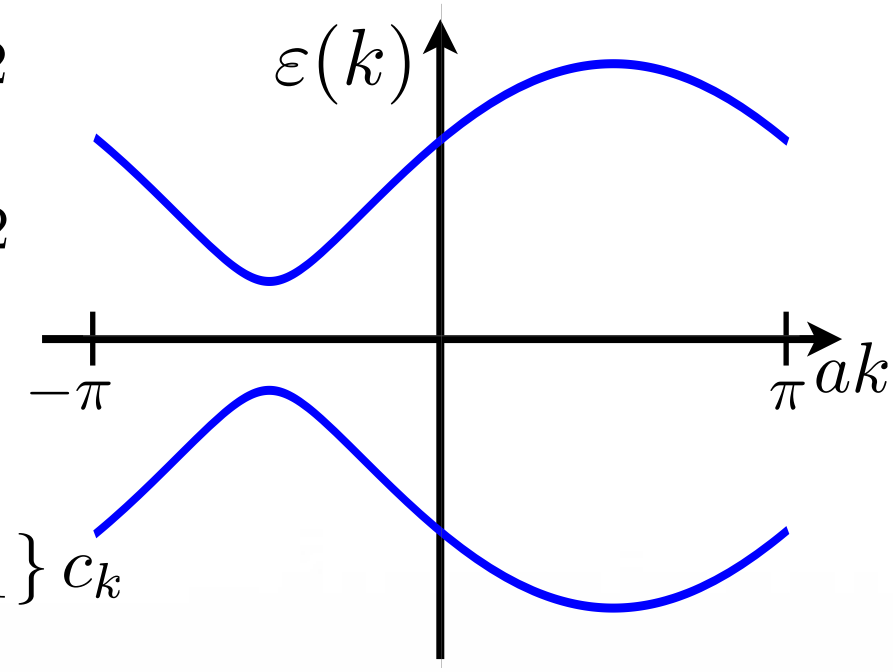
$m > g = t$ transition to trivial gap
$\mathcal{W} = 0 \quad \varphi_{\mathrm{Zak}} = 0$
no more edge states

AIII SPT phase



J. Jünemann, et al. (MR), PRX **7**, 031057 (2017)



$$\mathcal{H}_0 = \sum_k c_k^\dagger \left\{ \left[\Delta\epsilon/2 + 2t\sin(ak) \right] \sigma_3 + \left[\quad - 2g\cos(ak) \right] \sigma_1 \right\} c_k$$

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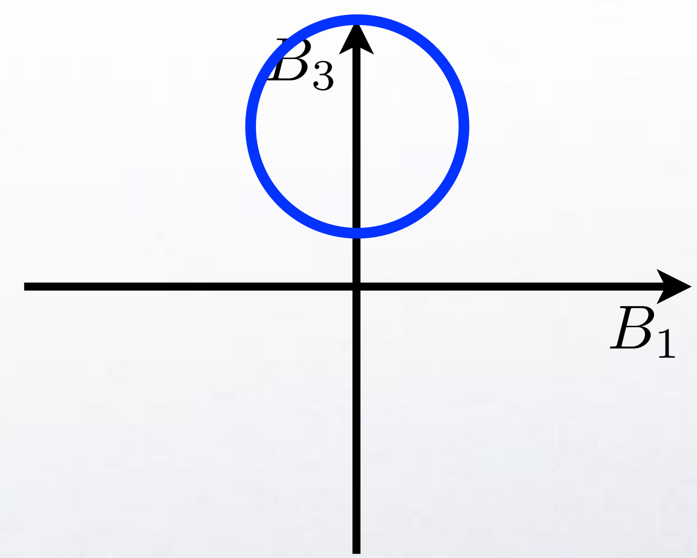


Zeeman $\Delta\epsilon$ breaks T & C symmetry!

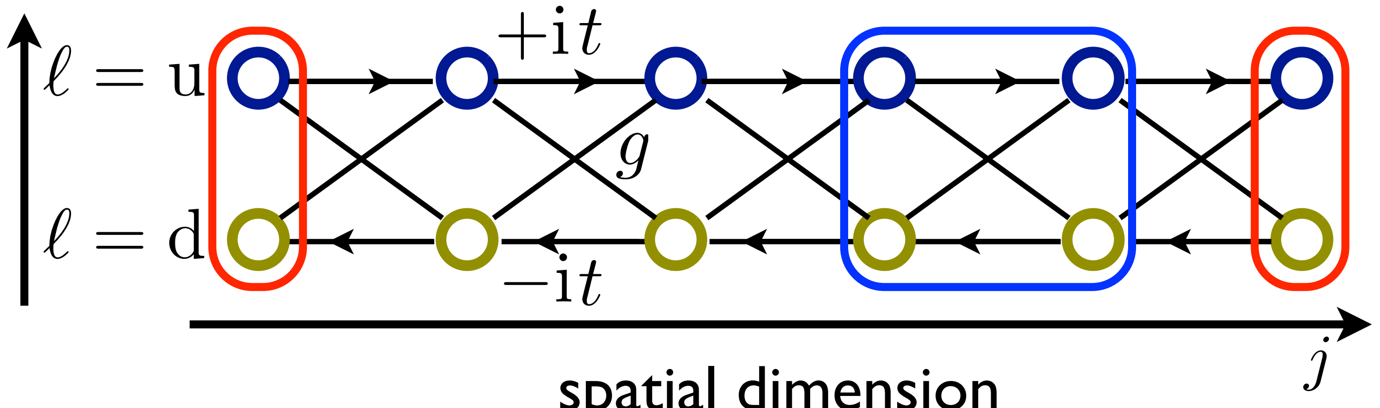
$\nexists U_{T/C}$ s.t. $U_\alpha \mathcal{H}_0^*(-k) U_\alpha^\dagger = \pm \mathcal{H}_0(k)$ ⇨

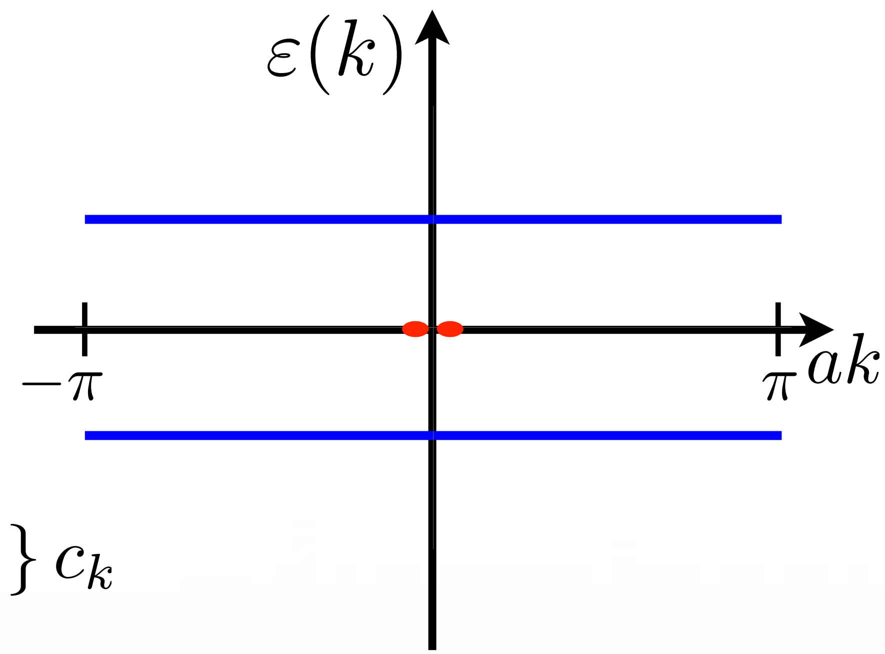
Class **AIII** of AZ table

Altland, Zirnbauer, PRB **55**, 1142 (1997)

[another scheme Velasco, Paredes, PRL **119**, 115301 (2017)]

Aharanov-Bohm cages





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Basis of localized states $w^\dagger_{j,\pm}$, a.k.a. Aharanov-Bohm cages

Vidal, Mosseri, & Doucot, PRL **81**, 5888 (1998)

OBC: Mid-gap zero-energy edge states ($l^\dagger$,$r^\dagger$), *[bulk-edge correspondence]*

S. Ryu and Y. Hatsugai, PRL **89**, 077002 (2002)
P. Delplace, et al., PRB **84**, 195452 (2011)

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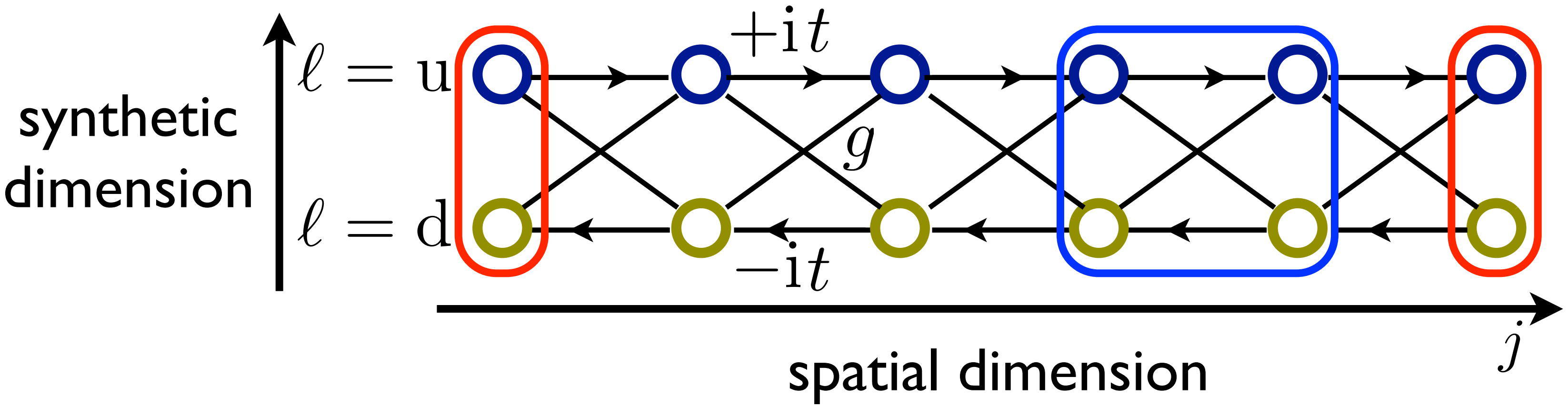

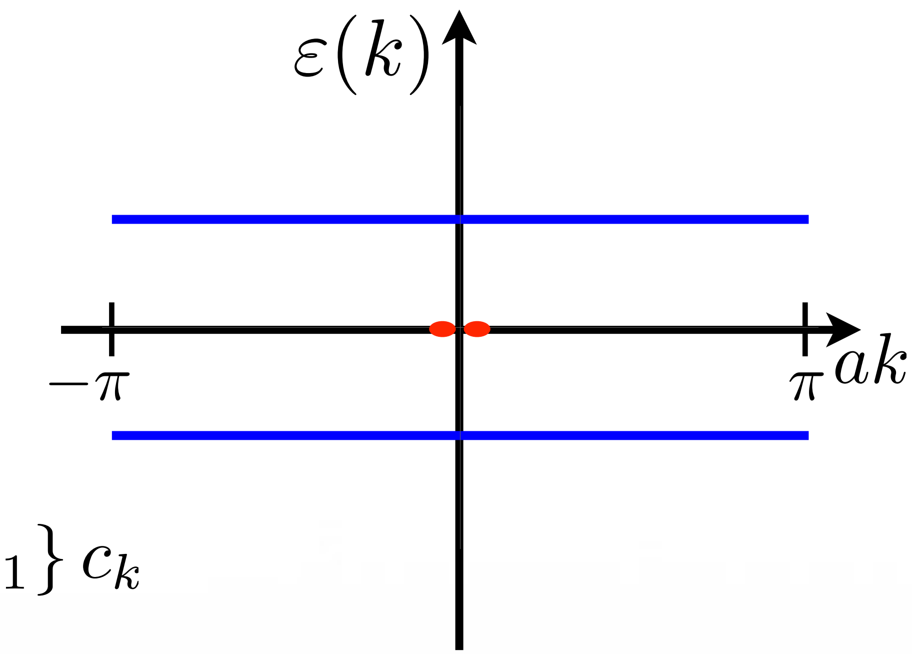

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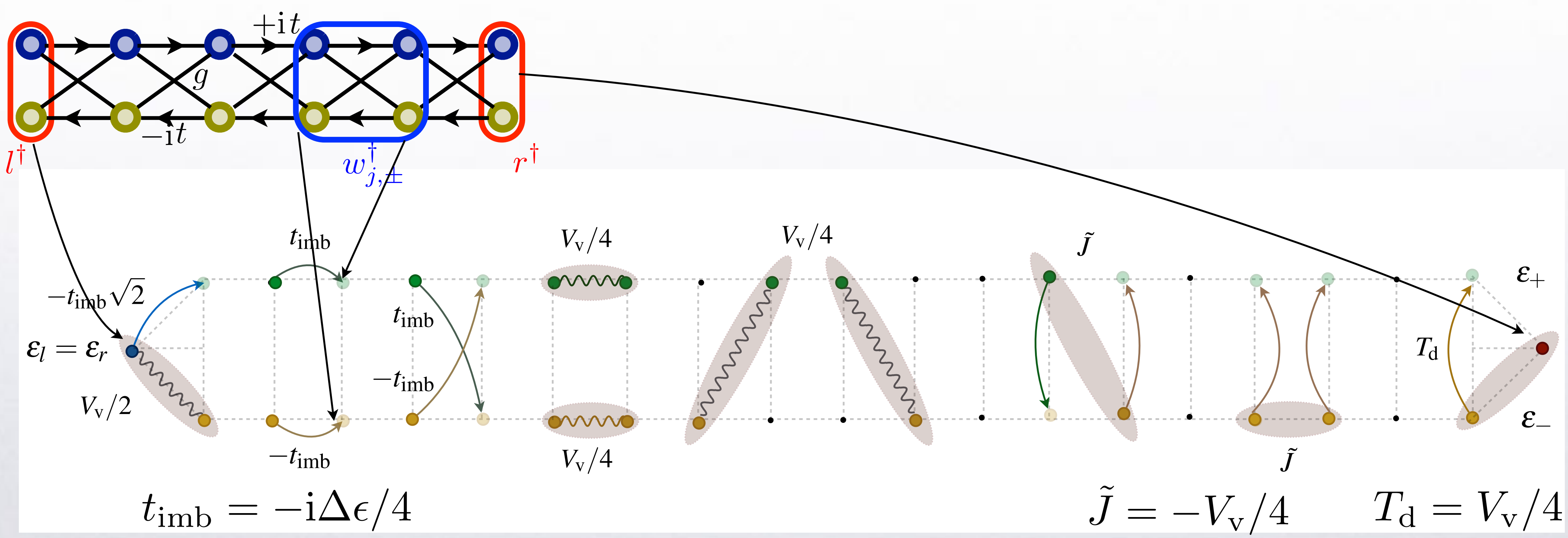
OBC: Mid-gap zero-energy edge states ($l^\dagger$,$r^\dagger$),
[bulk-edge correspondence]
S. Ryu and Y. Hatsugai, PRL **89**, 077002 (2002)
P. Delplace, et al., PRB **84**, 195452 (2011)

a *workhorse* for flat-band & SPT physics:
Tovmasyan, van Nieuwenburg, & Huber, PRB **88**, 220510(R) (2013)
Takayoshi, Katsura, Watanabe, & Aoki, PRA **88**, 063613 (2013)
Huber & Altman, PRB **82**, 184502 (2010)
Tovmasyan, Peotta, Törmä, & Huber, PRB **94**, 245149 (2016)
Sticlet, Seabra, Pollmann, & Cayssol, PRB **89**, 115430 (2014)

Bragg techniques
to measure edge states
in ultracold cold atoms
Goldman, Beugnon, Gerbier,
PRL 108, 255303 (2012)

Effective model

on the basis of Aharanov-Bohm cages



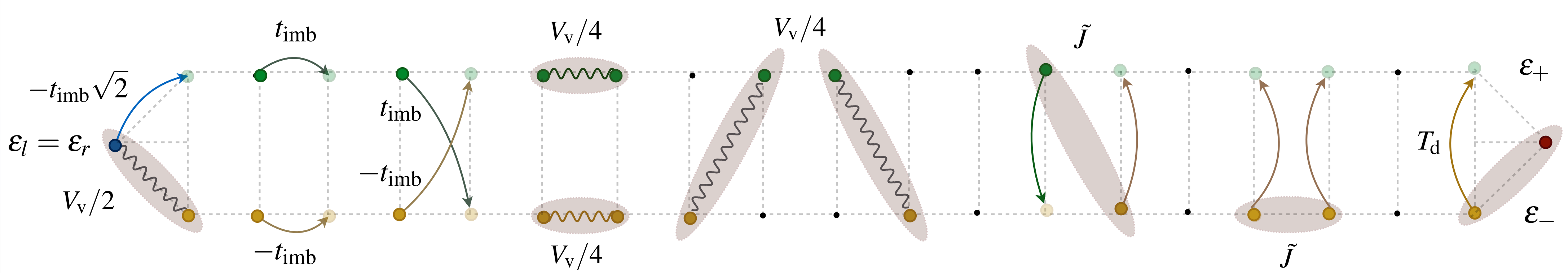
Effective model

on the basis of Aharanov-Bohm cages

exotic Hubbard model

[without dipolar atoms or other "strange" schemes]

- imbalance induced hopping
- n.n. interactions
- pair tunnelling
- density-assisted tunnelling



$t_{\rm imb} = -\mathrm{i}\Delta\epsilon/4$ $\tilde{J} = -V_{\rm v}/4$ $T_{\rm d} = V_{\rm v}/4$

Effective model

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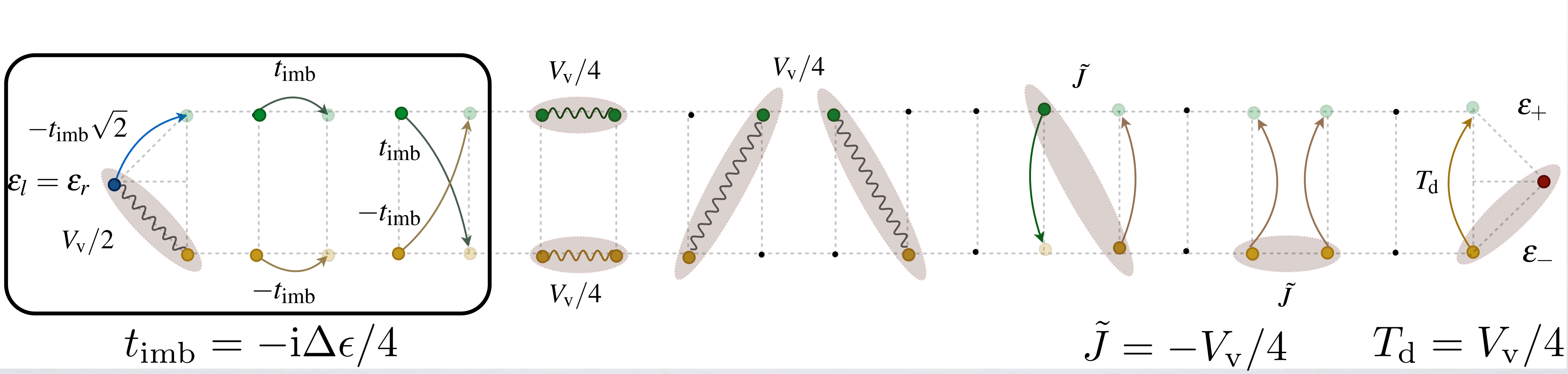

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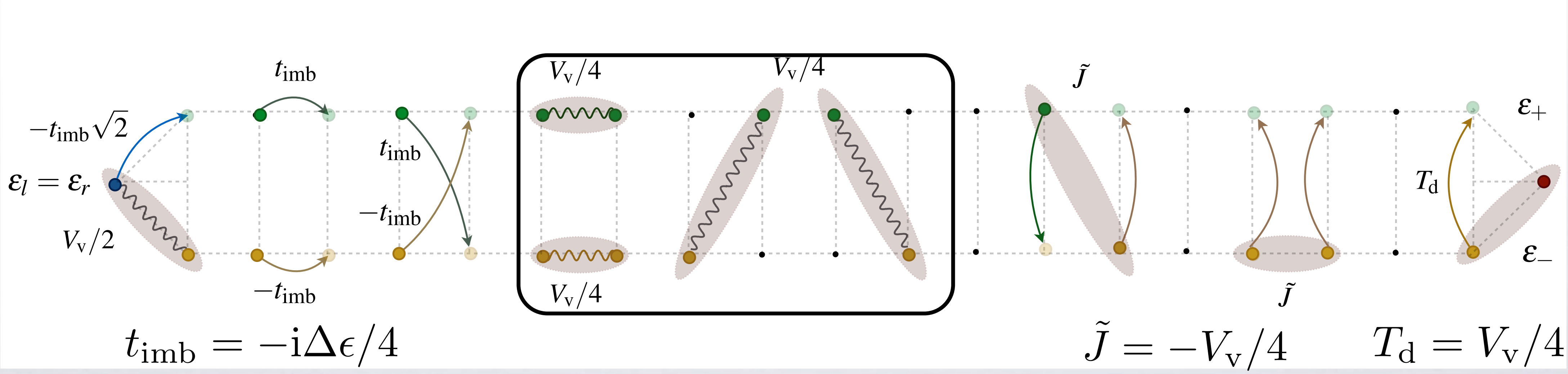

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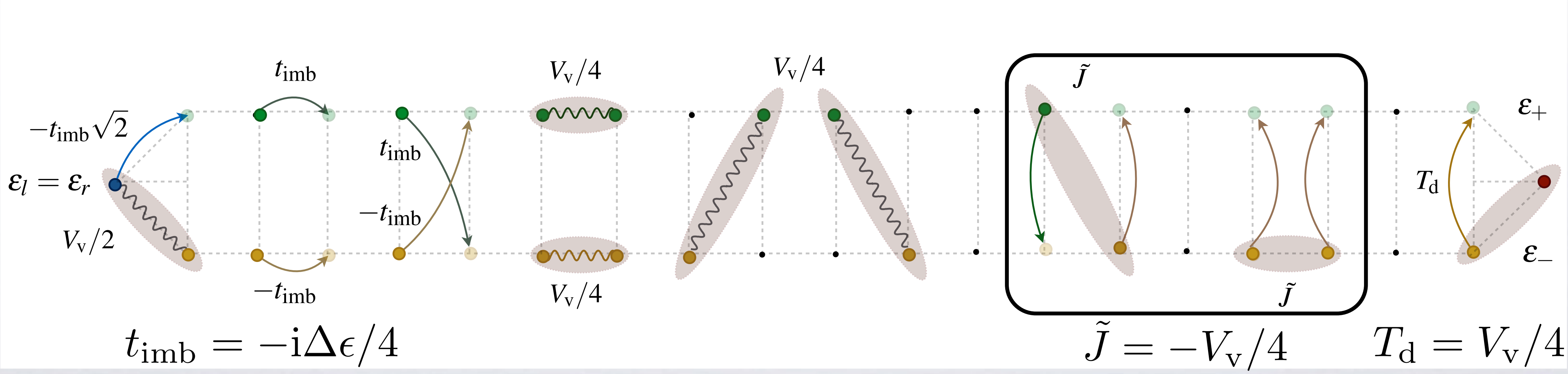

Effective model

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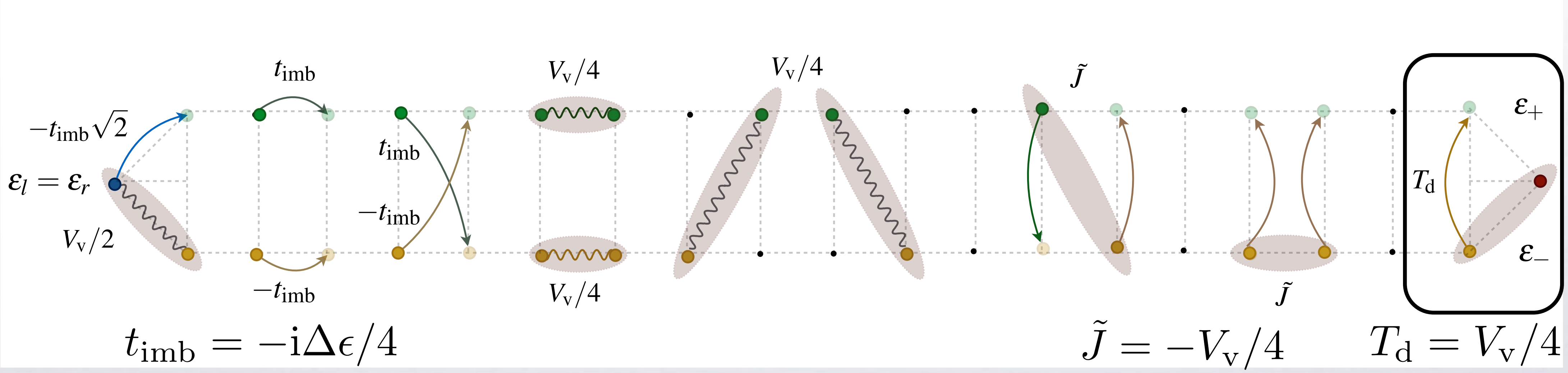


matteo.rizzi@fz-juelich.de

New J. Phys. ***21**, 043048 (2019)*

Effective model

on the basis of Aharanov-Bohm cages

exotic Hubbard model

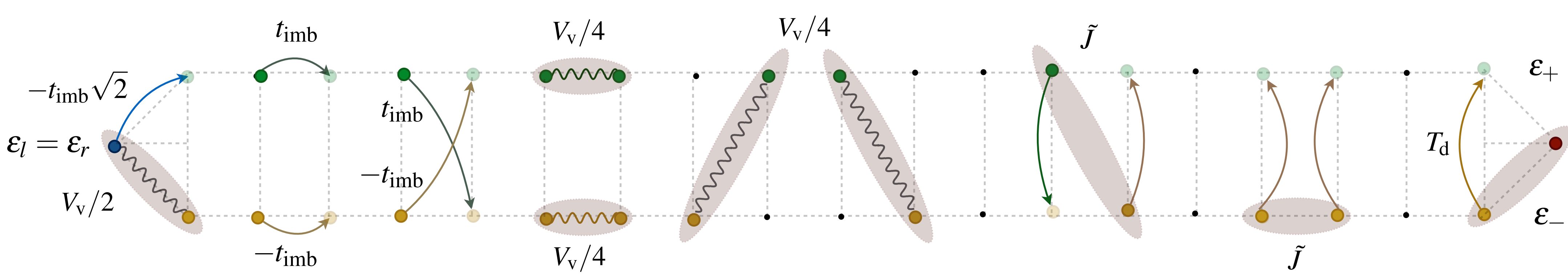
[without dipolar atoms or other "strange" schemes]

- imbalance induced hopping
- n.n. interactions
- pair tunnelling
- density-assisted tunnelling

bulk-mediated *[à la Fano-Anderson]*

edge-edge interactions

J. Jünemann, et al., PRX **7**, 031057 (2017)

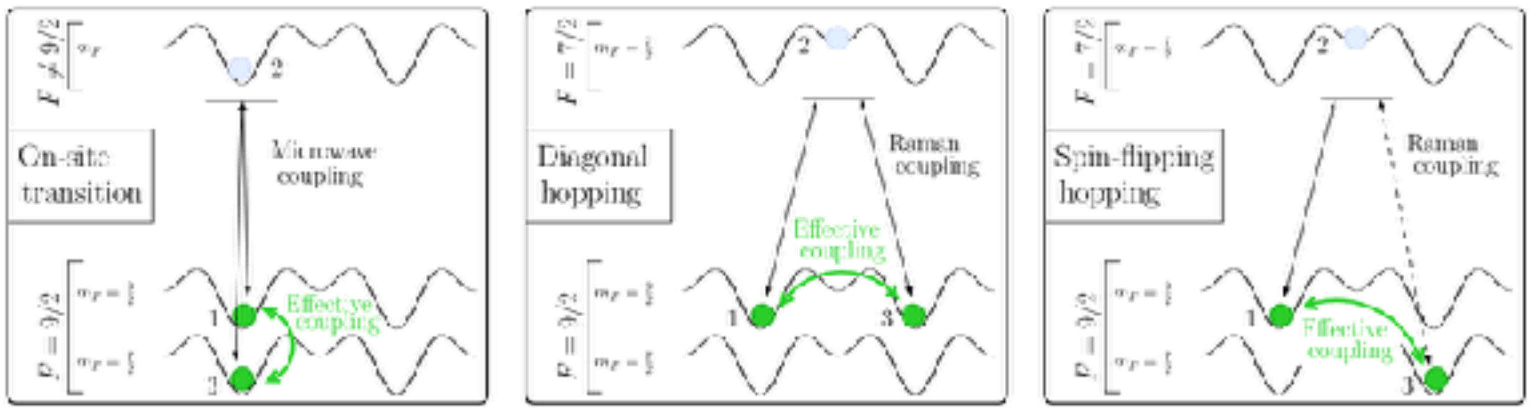


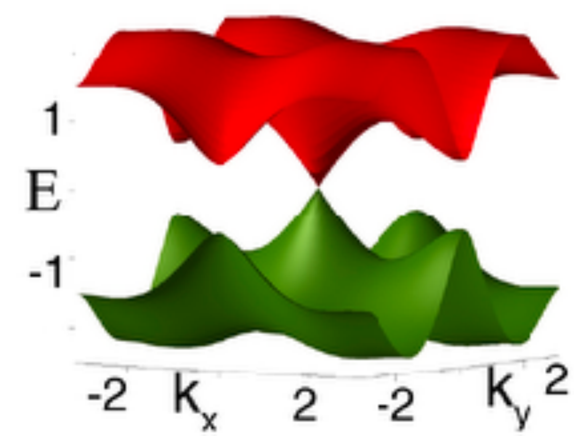
$t_{imb} = -i\Delta\epsilon/4$ $\qquad$ $\tilde{J} = -V_v/4$ $\qquad$ $T_d = V_v/4$

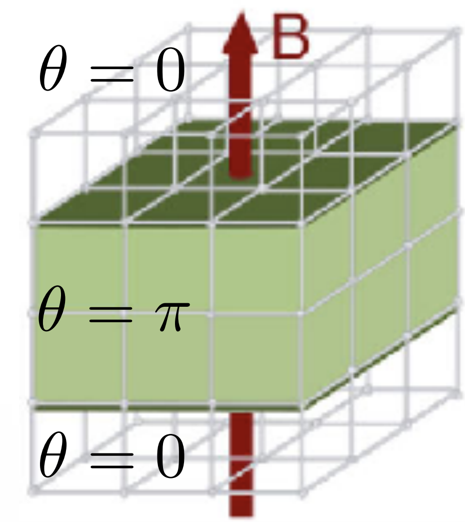
Experimental schemes

An optical-lattice-based quantum simulator for relativistic field theories and topological insulators

A. Bermudez, et al., PRL **105**, 190404 (2010);
L. Mazza, et al., NJP **14** 015007 (2012);
MR, PoS - SISSA **193**, 036 (2014)

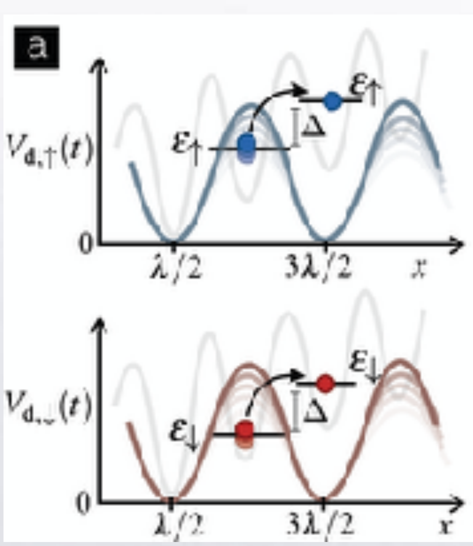


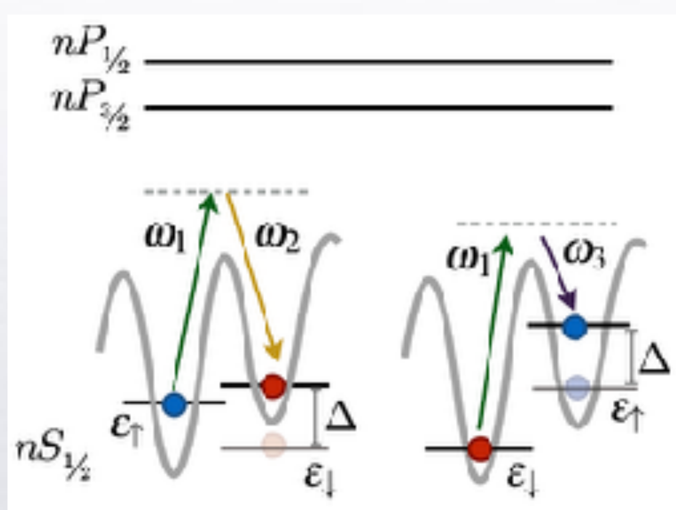




A shaken lattice proposal for the Creutz ladder

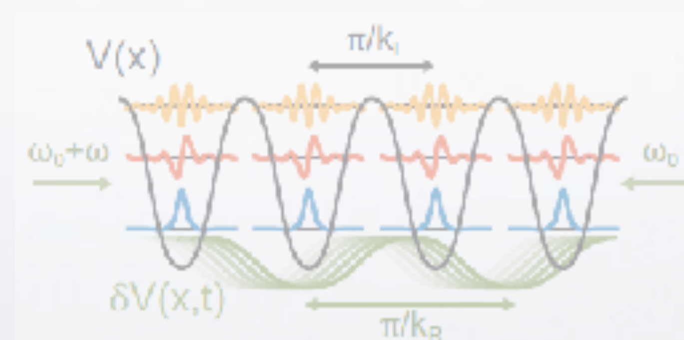
J. Jünemann, et al., PRX **7**, 031057 (2017)

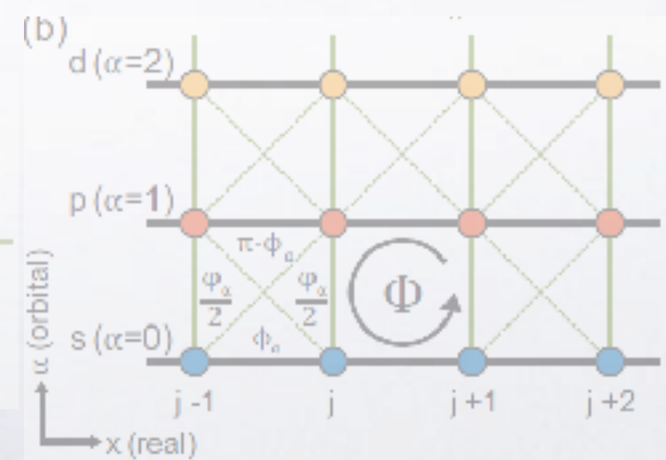




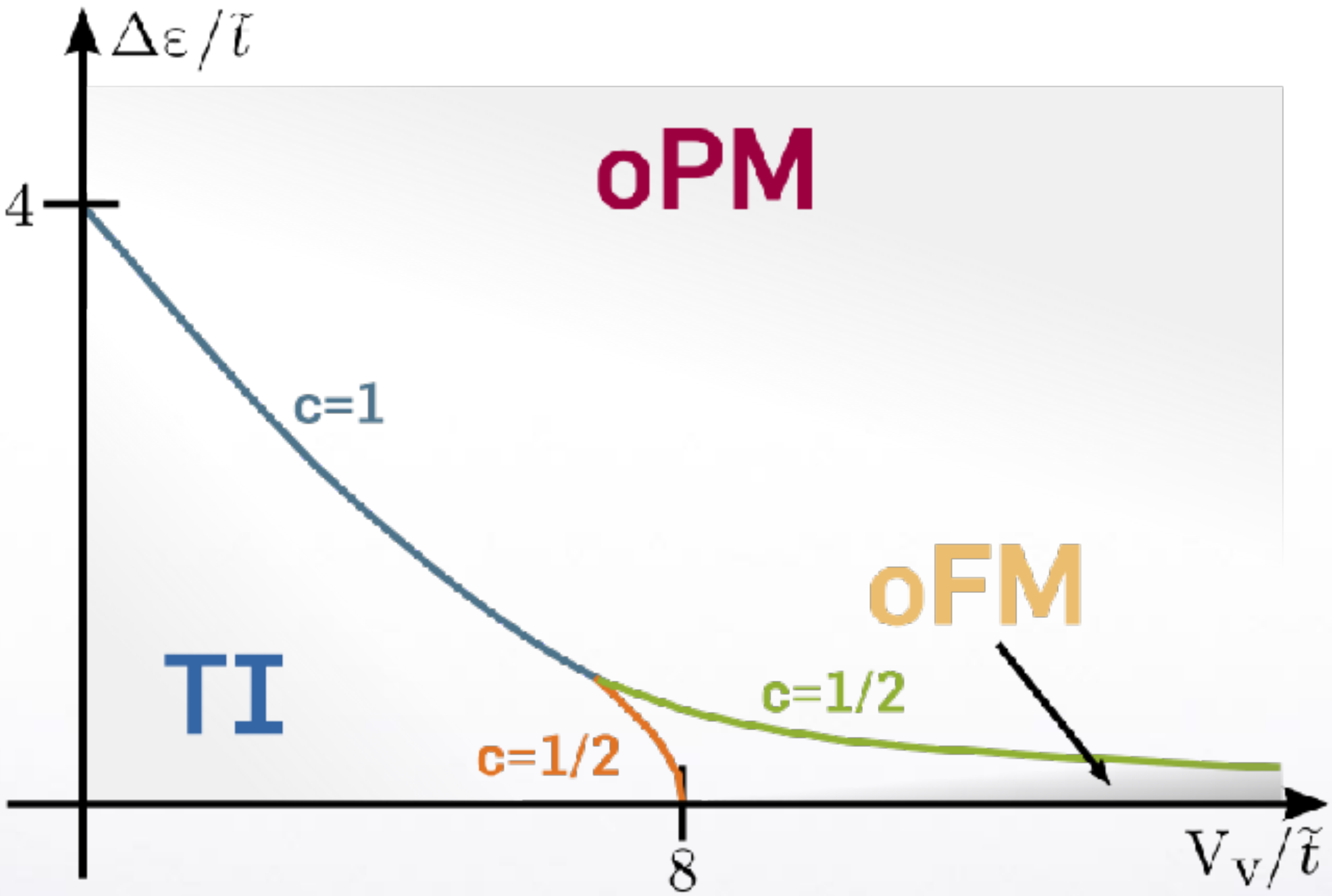
Realization of a cross-linked chiral ladder by orbital-momentum coupling

J.H. Kang, J.H. Han, & Y. Shin, arXiv:1807.01444 & 1902.10304





Half-filling $\nu = 1$ results



J. Jünemann, et al. (MR), PRX **7**, 031057 (2017)

A. Bermudez, et al. (MR), Ann. Phys. **339**, 149 (2018)

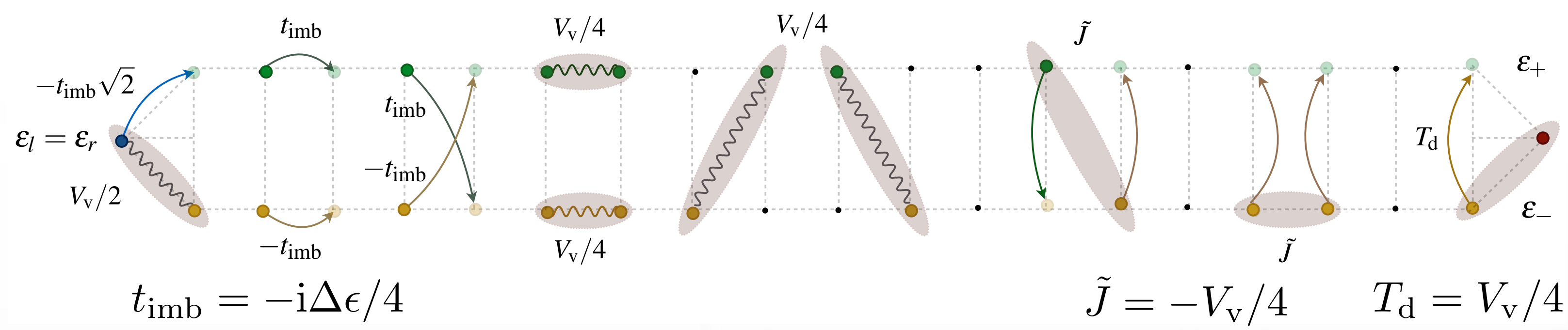
E. Tirrito, et al. (MR), PRB **99**, 125106 (2019)

Topological CDW

Can intrinsically interacting, gapped, SPT phases arise at $\nu = N/L \neq 1$ for which non-interacting system is gapless?

Creutz-Hubbard ladder

Projection on lowest band & approximate cage basis: … n.n. interactions …



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Creutz-Hubbard ladder

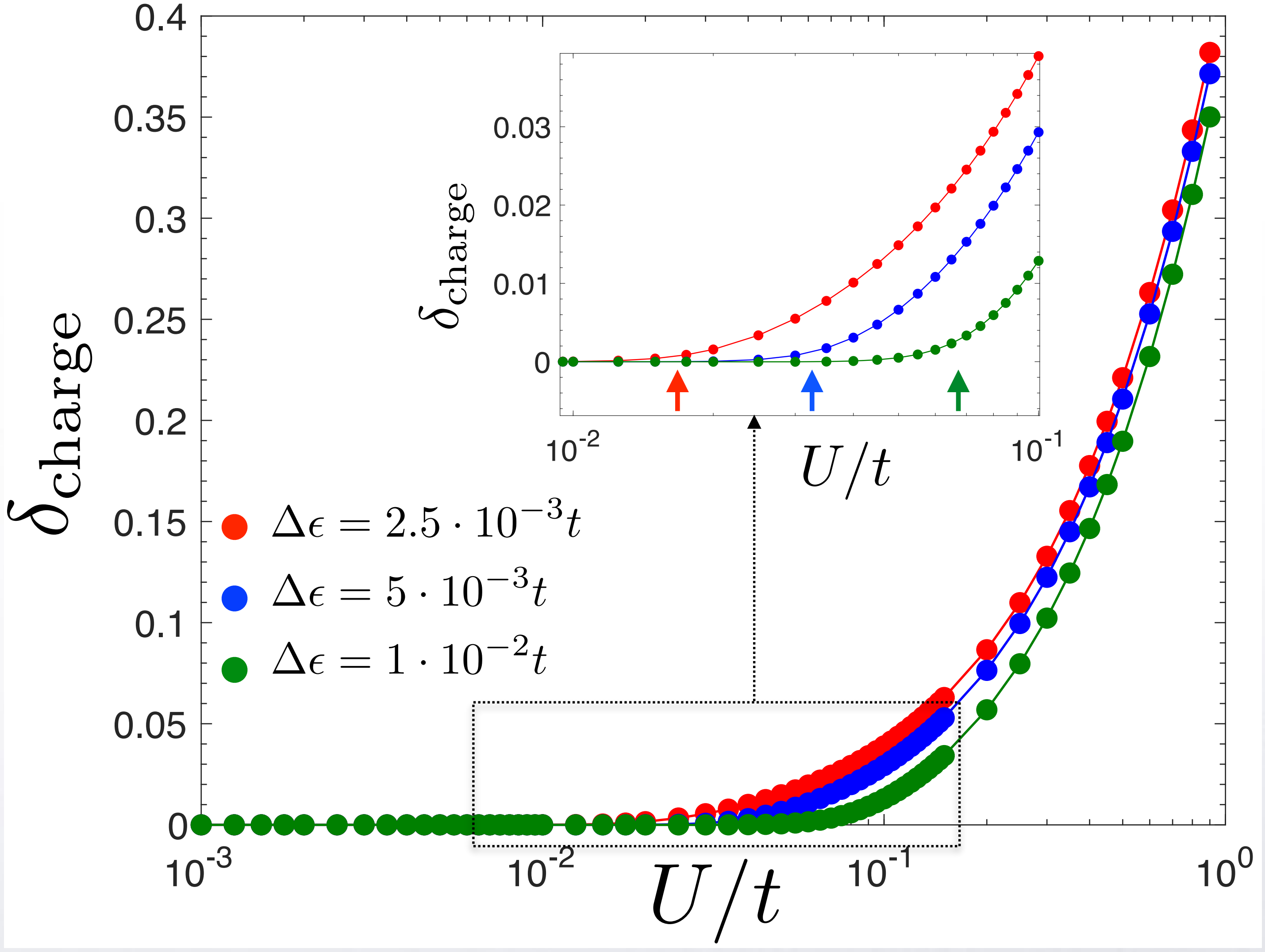
↓

Projection on lowest band
& approximate cage basis:
… n.n. interactions …

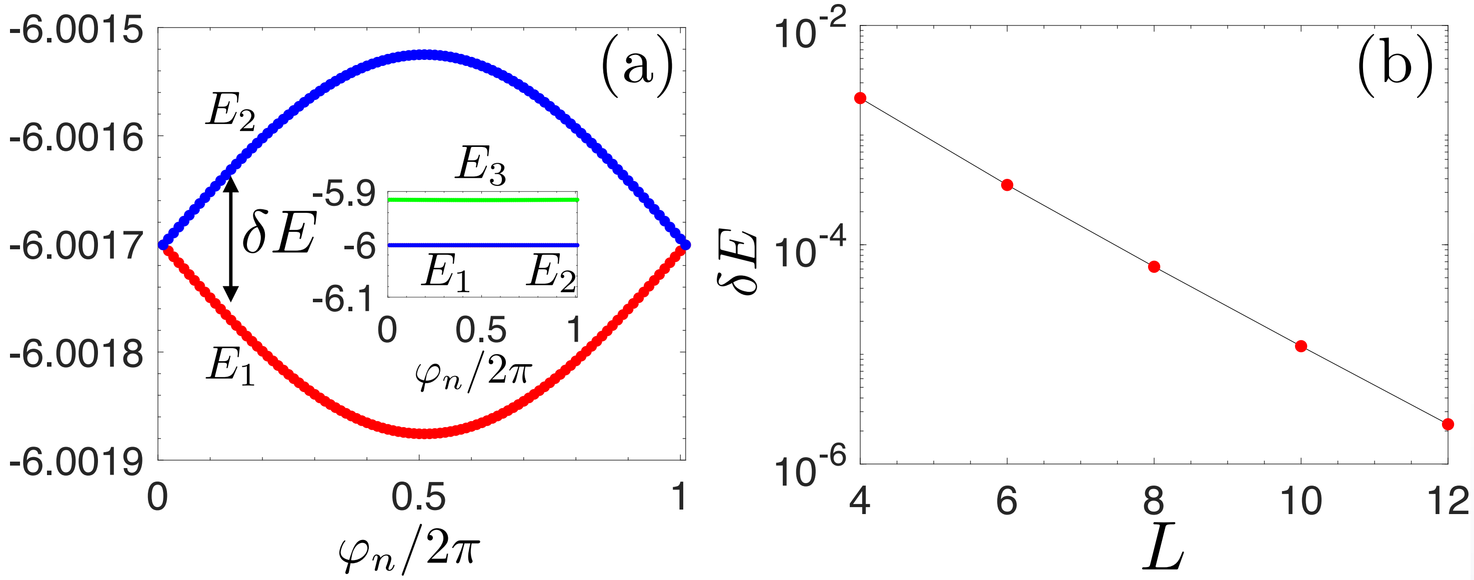
↓

gapped phase at $\nu = \frac{1}{2}$ & critical U/t

$$\delta_{\rm charge} = E_1(N) - \tfrac{1}{2}\left[E_1(N+1) + E_1(N-1)\right]$$



Topological CDW



gapped phase at $\nu = \frac{1}{2}$ & critical U/t

$$\delta_{\text{charge}} = E_1(N) - \frac{1}{2}\left[E_1(N+1) + E_1(N-1)\right]$$

&

double degeneracy with PBC

$$\delta_{\text{spin},\alpha} = E_\alpha(N) - E_1(N)$$

Twisted PBC $t \to t\exp(i\varphi/L)$

Topological CDW

generalized SPT invariant

destroyed by

$$\hat{H}_{\rm SB} = iM \sum_j \left(\hat{c}^\dagger_{j,\uparrow} \hat{c}_{j,\downarrow} - {\rm H.c.} \right) \propto \sigma_y$$

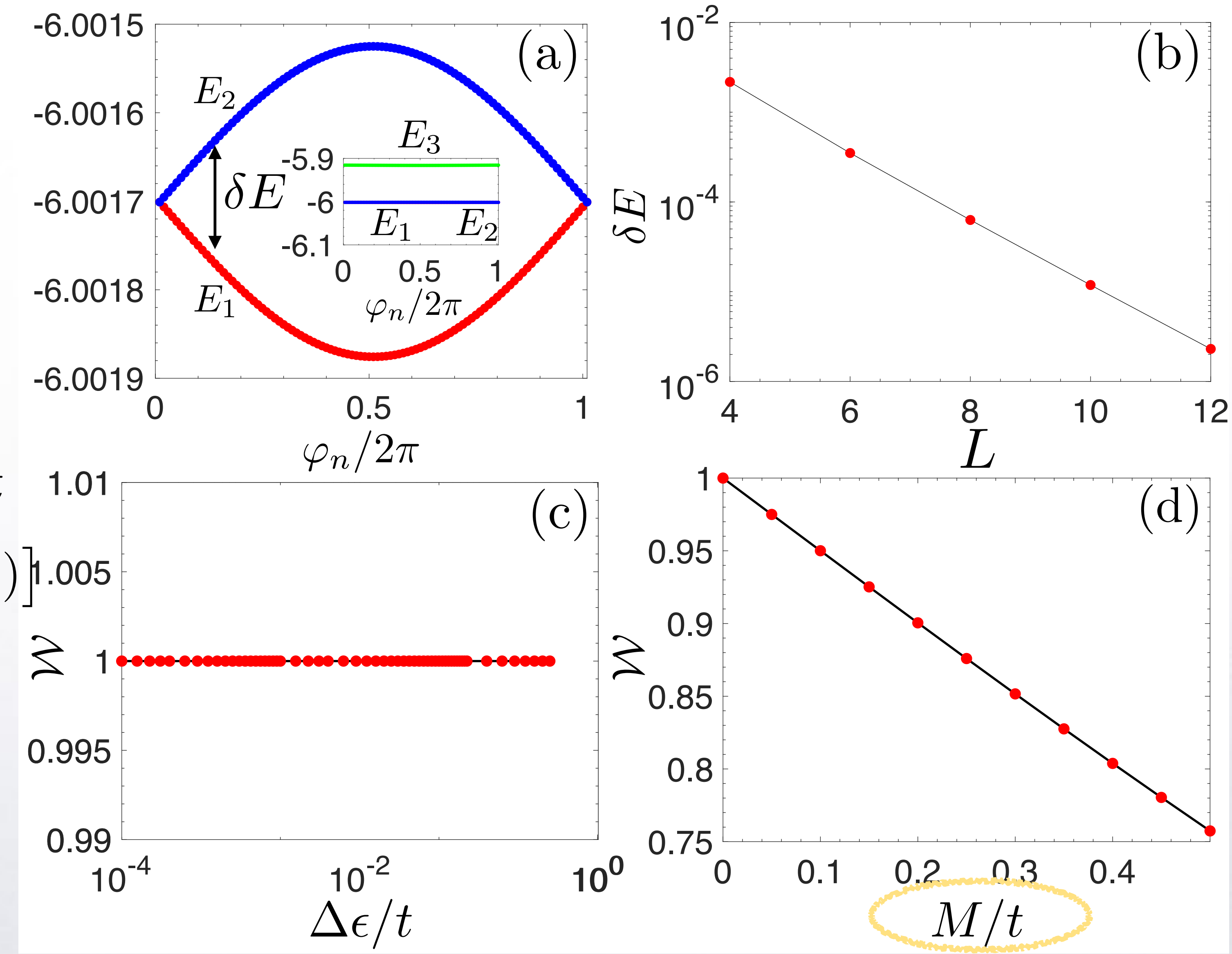
explicitly breaking chiral symm.

$$U_S = \sigma_y \qquad U_S H_0(k) U_S^\dagger = -H_0(k)$$

$$U_S H_{\rm SB} U_S^\dagger = +H_{\rm SB}$$

$$\mathcal{W} = \frac{i}{\pi} \int_0^{2\pi} d\varphi \, {\rm Tr} \left[\langle \Psi_\alpha(\varphi) | \partial_\varphi | \Psi_\beta(\varphi) \rangle \right]$$

F. Wilczek & A. Zee., PRL **52**, 2111 (1984)



gapped phase at $\nu = \frac{1}{2}$ & critical U/t

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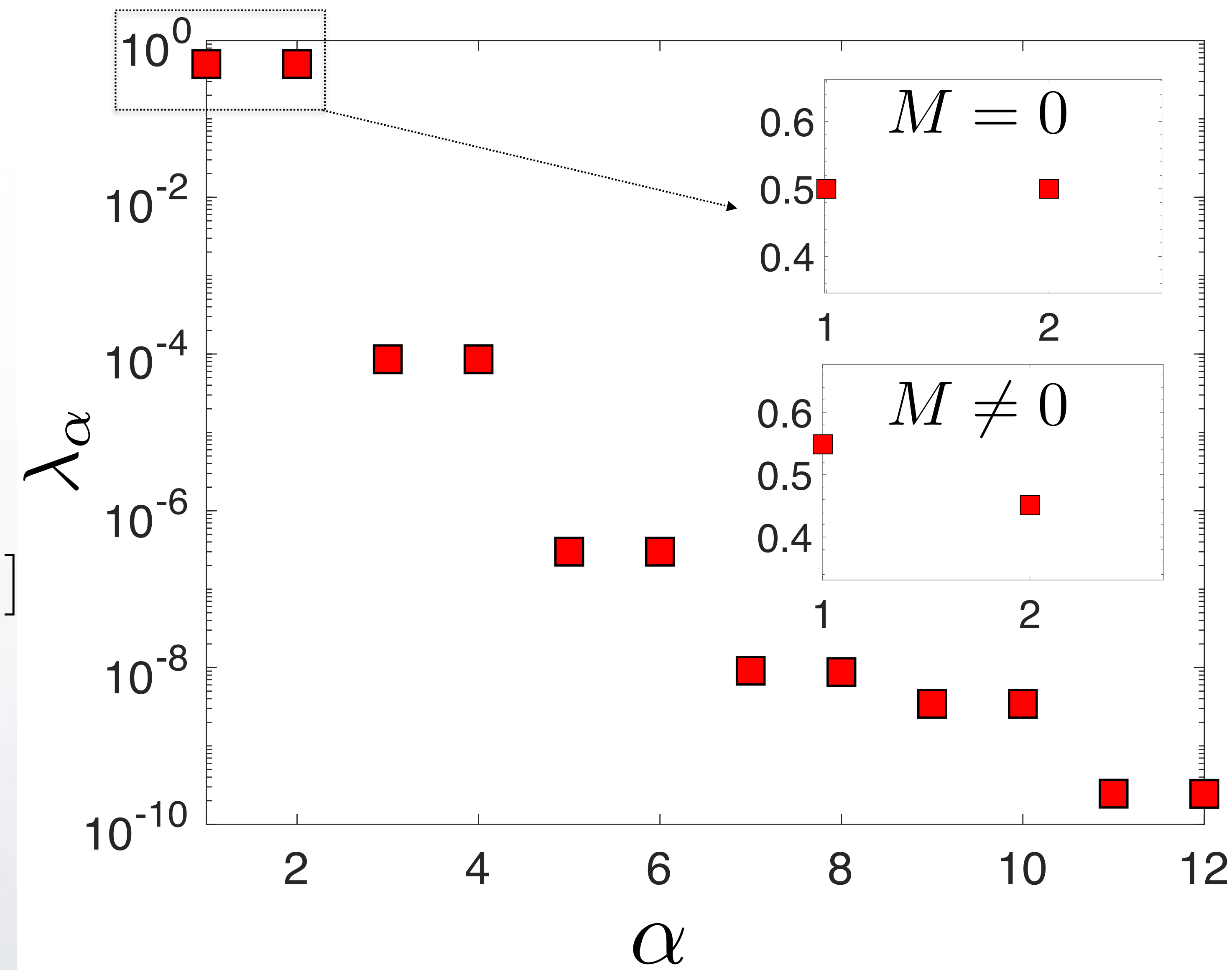
double degeneracy with PBC

$$\delta_{\mathrm{spin},\alpha} = E_\alpha(N) - E_1(N)$$

Same happens for
for entanglement spectrum!



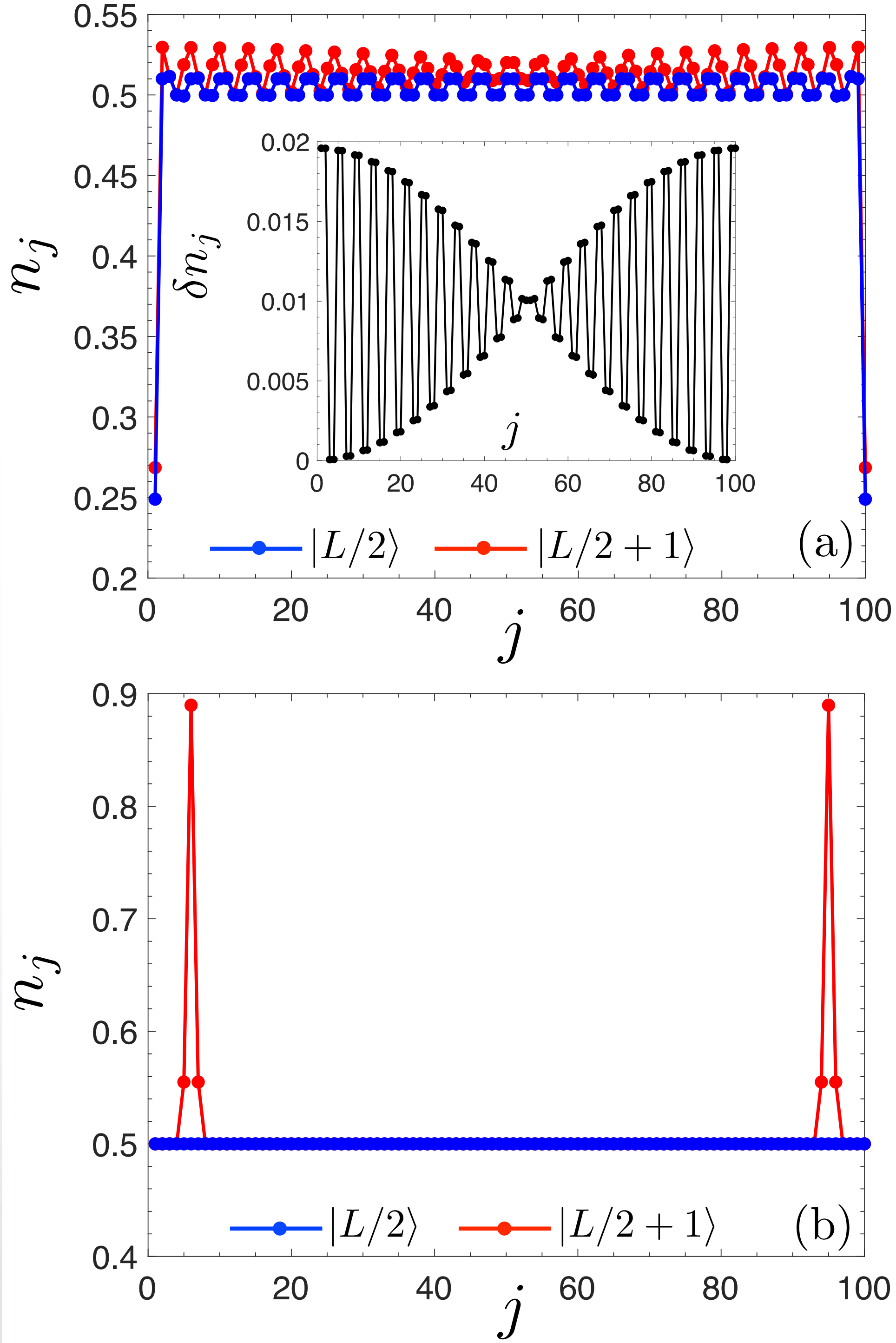
Topological CDW

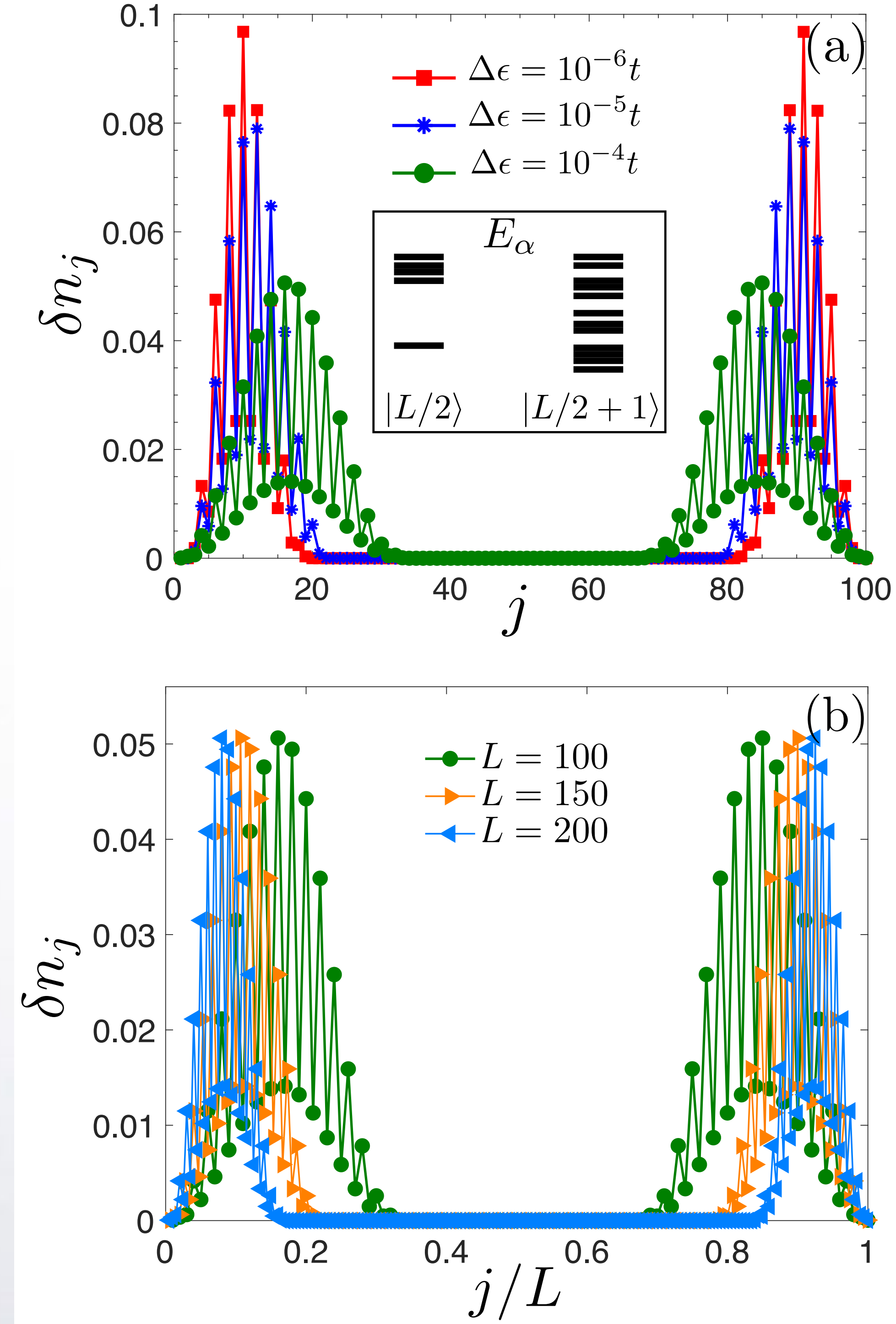
Reminiscent edge state physics, however not at zero energy…

$U = 0$

$\Delta\epsilon = 0$

$U = t$





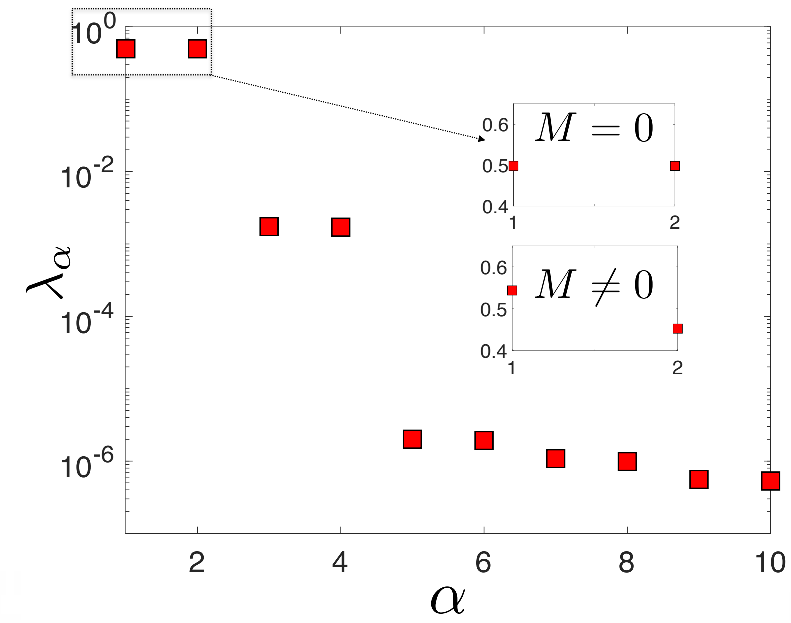
$U = t$

$\Delta\epsilon = 10^{-4}$

Open directions

- Bosonization *(not shown)* predicts a
  devil's staircase *at filling 1/(q+1) for int. range q*

  e.g., $\nu = 1/3$ for $U = 2V = t$



- Up to now, BDI/AIII/spatial inversion SPT's: other classes? Extension to higher-dim. flat-band topological systems?

- apparent absence of bulk-edge correspondence at zero energy: why?

- experimental detection via mean chiral displacement or similar?

General Picture

Geometrical constraints + Background gauge fields

↓

Frustration, large degeneracy,
single-particle topological character,
flat-band dispersion

+

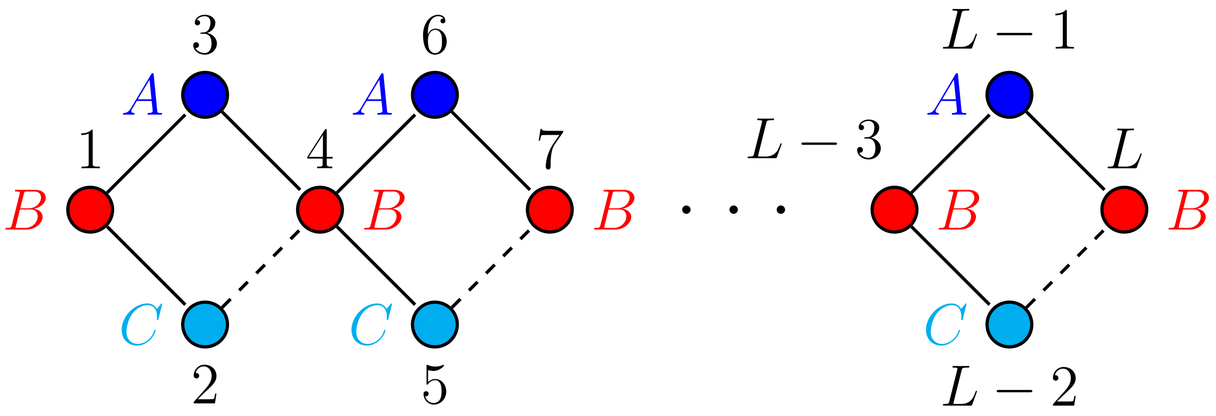
Interactions btw. constituents

Fractional topological phases
fermions
range q => fraction 1/(q+1)

Collective transport properties:
here pair-transport in bosons, etc.

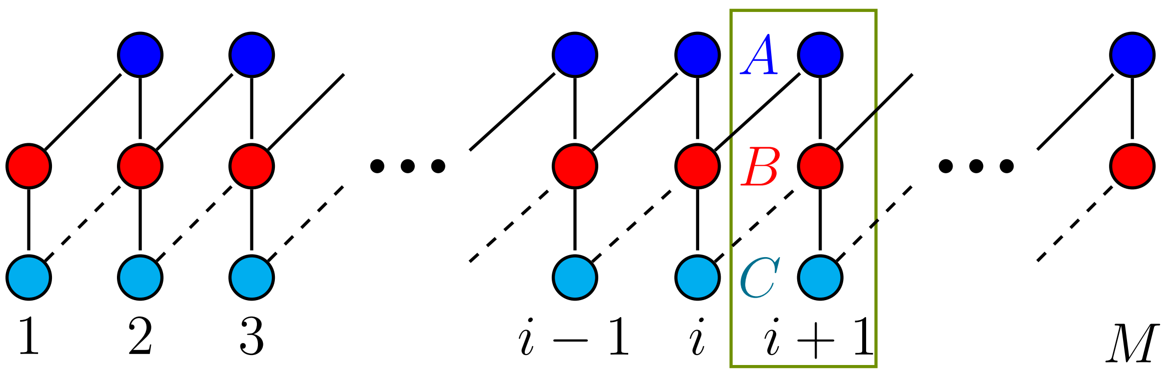
see works by Tovmasyan, Peotta, Huber, Törma (& others) for enhanced fermionic pairing, too

The Model: AB cages



$$\hat{H}_0 = -J \sum_{j,\ell} \sum_{\alpha,\beta} T^{(\ell)}_{\alpha,\beta} \; \hat{b}^\dagger_{j+\ell,\alpha} \hat{b}_{j,\beta}$$

The Model: AB cages

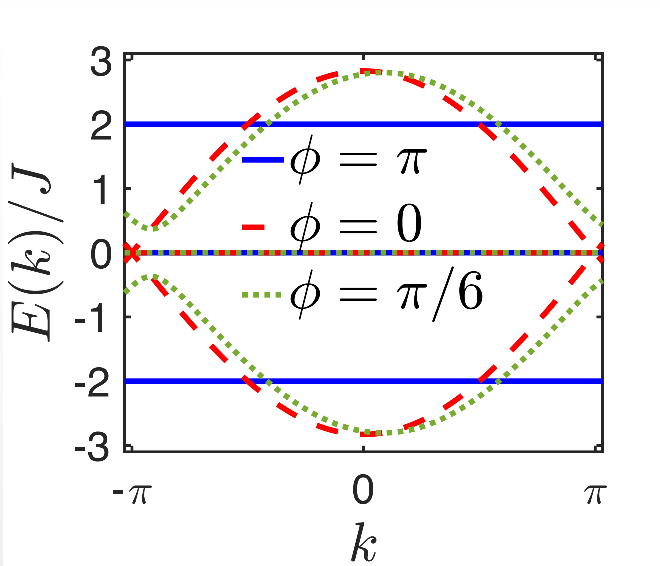


$$\hat{H}_0 = -J \sum_{j,\ell} \sum_{\alpha,\beta} T^{(\ell)}_{\alpha,\beta} \, \hat{b}^\dagger_{j+\ell,\alpha} \hat{b}_{j,\beta}$$

$$T^{(0)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T^{(+1)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & e^{i\phi} \\ 0 & 0 & 0 \end{pmatrix}, \quad T^{(-1)} = \left(T^{(+1)}\right)^\dagger$$

$$E_\tau(k) = 2J\tau\sqrt{1+\cos\left(k-\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)}$$

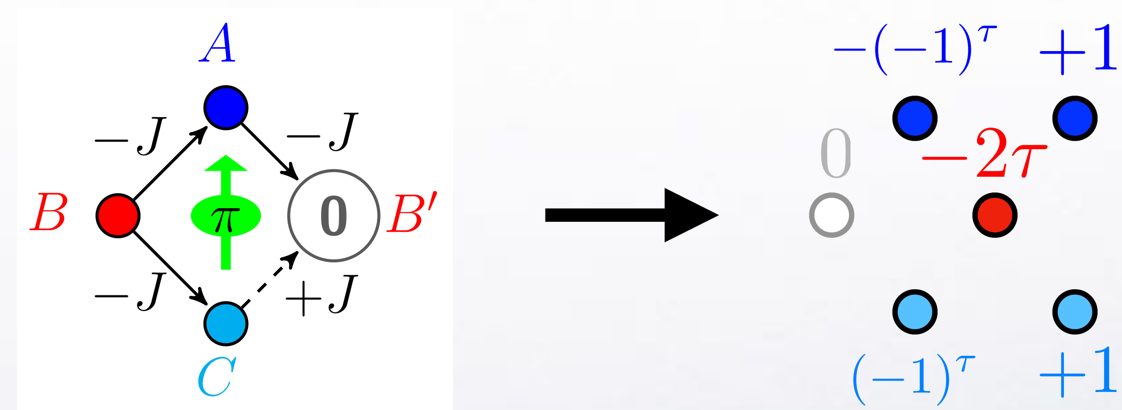
$\tau = 0, \pm 1$



Fully flat-bands (all!) at half-flux!

Aharanov-Bohm interference

Doucot, Vidal, PRL **88**, 227005 (2002)



Sub-lattice symmetry

$$\Gamma H_0(k) \Gamma = -H_0(k)$$

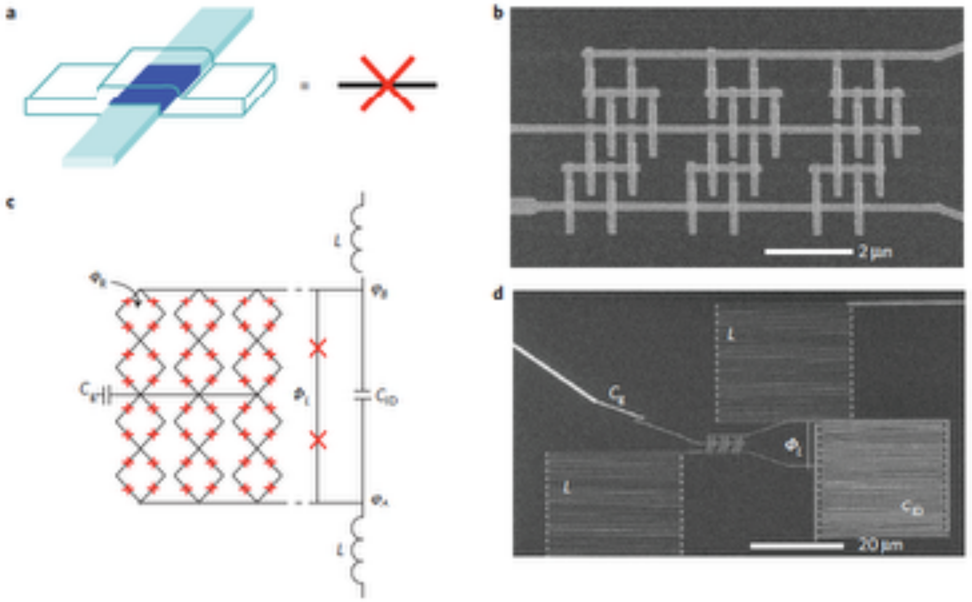
$$\Gamma = \mathrm{diag}\{-1, +1, -1\}$$

No quantized topological invariant, though it appears in squared *H* …

Kremer, et al., arXiv:1805.05209

Experimental realisations

Josephson Junction Arrays

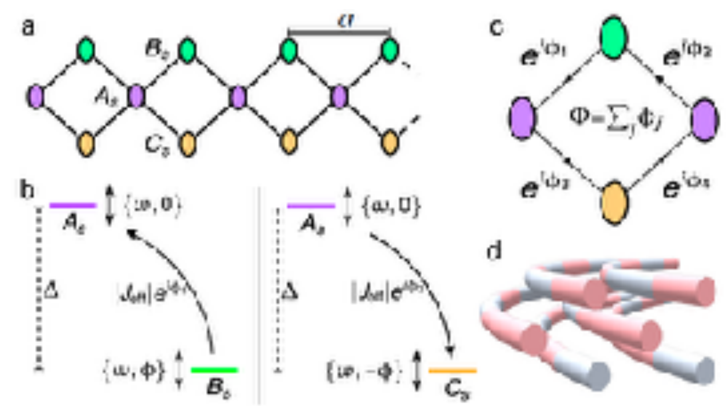


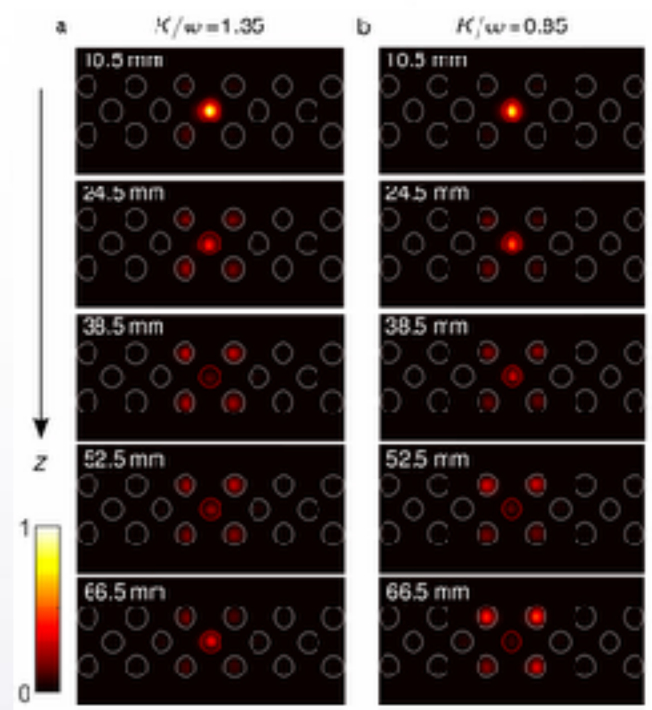
Gladchenko, et al., Nat. Ph. **5**, 48 (2009)

Cold atoms !?

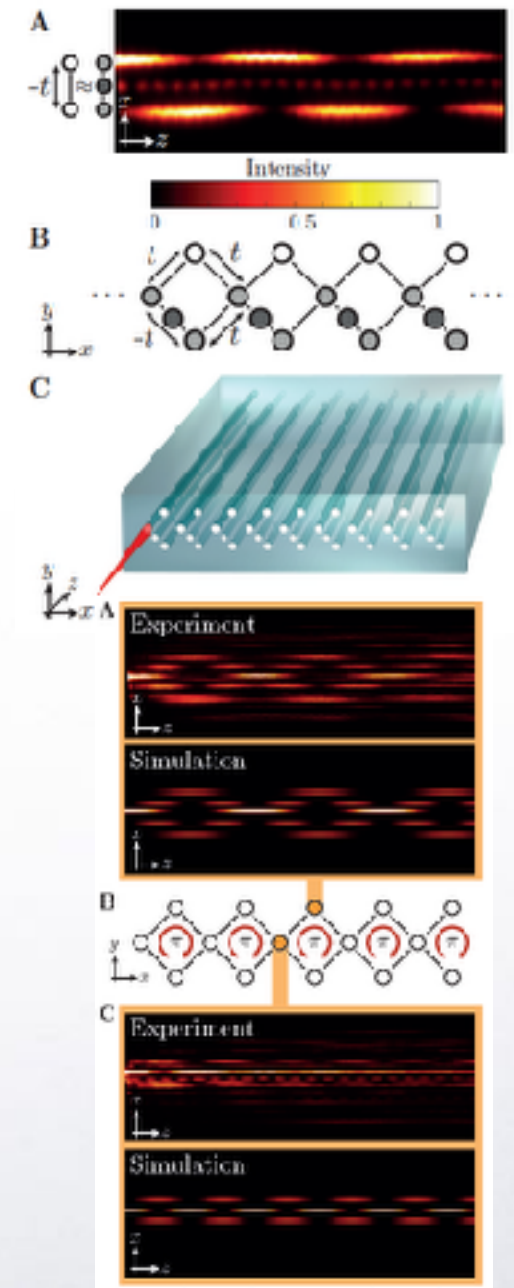
- real space: superlattices or DMD
  or synthetic dimensions
  (e.g., angular momentum!)
- shaking of single links
  or laser-assisted tunneling
- tunable interactions
- … bosons / fermions …
  Pelegrí, et al., arXiv:1807.08096

Photonic Waveguides





Mukherjee, et al.,
arXiv:1805.03564



Kremer, et al.,
arXiv:1805.05209

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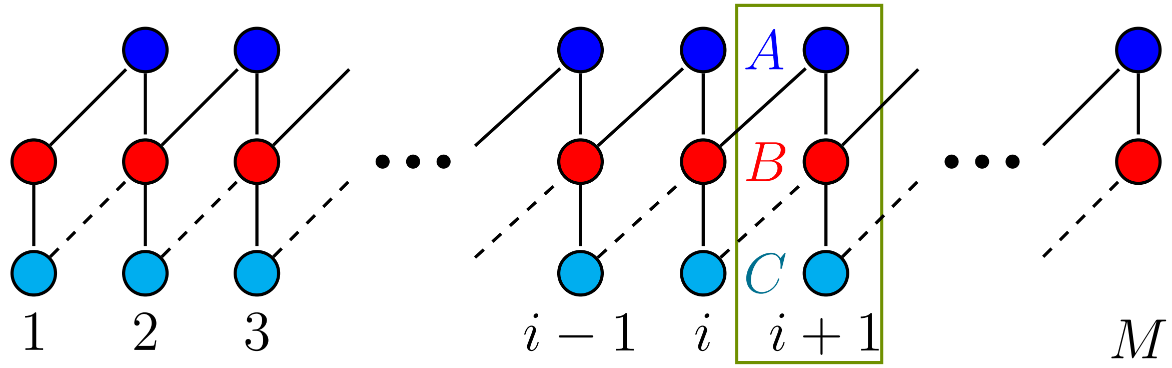
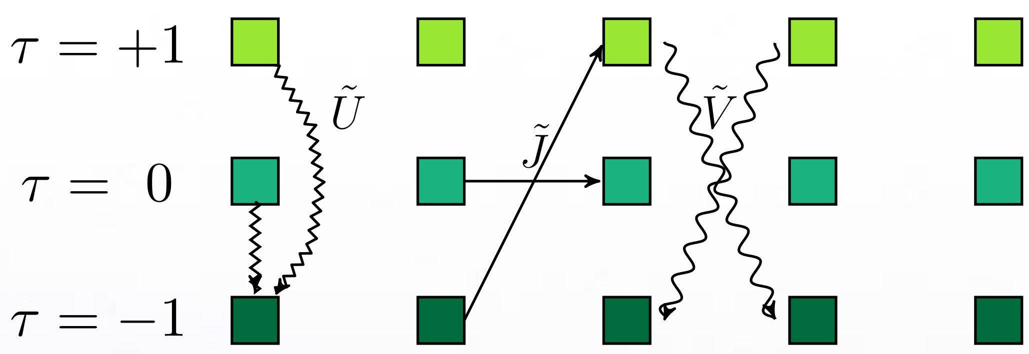
The Model: Z_2 symmetry



$$\hat{H}_0 = -J \sum_{j,\ell} \sum_{\alpha,\beta} T^{(\ell)}_{\alpha,\beta} \; \hat{b}^\dagger_{j+\ell,\alpha} \hat{b}_{j,\beta}$$

$$\hat{H}_I = \frac{U}{2} \sum_j \sum_\alpha \hat{n}_{j,\alpha}(\hat{n}_{j,\alpha} - 1)$$

$$\hat{H}_0^{(\phi=\pi)} = \sum_j \sum_{\tau \in \{0,\pm\}} 2J\tau \, \hat{w}^\dagger_{j,\tau} \hat{w}_{j,\tau}$$



$$\widetilde{U},\ \widetilde{J},\ \widetilde{V} \propto U$$

$$\hat{H}_I^{(\mathrm{bulk})} = \sum_j \Big(\widetilde{U}_{\tau_1,\tau_2,\tau_3,\tau_4} \hat{w}^\dagger_{j,\tau_1} \hat{w}^\dagger_{j,\tau_2} \hat{w}_{j,\tau_3} \hat{w}_{j,\tau_4} + \sum_{\ell \in \{\pm 1\}} \widetilde{V}^{(\ell)}_{\tau_1,\tau_2,\tau_3,\tau_4} \hat{w}^\dagger_{j+\ell,\tau_1} \hat{w}^\dagger_{j,\tau_2} \hat{w}_{j,\tau_3} \hat{w}_{j+\ell,\tau_4} + \sum_{\ell \in \{\pm 1\}} \widetilde{J}^{(\ell)}_{\tau_1,\tau_2,\tau_3,\tau_4} \hat{w}^\dagger_{j+\ell,\tau_1} \hat{w}^\dagger_{j+\ell,\tau_2} \hat{w}_{j,\tau_3} \hat{w}_{j,\tau_4} \Big)$$

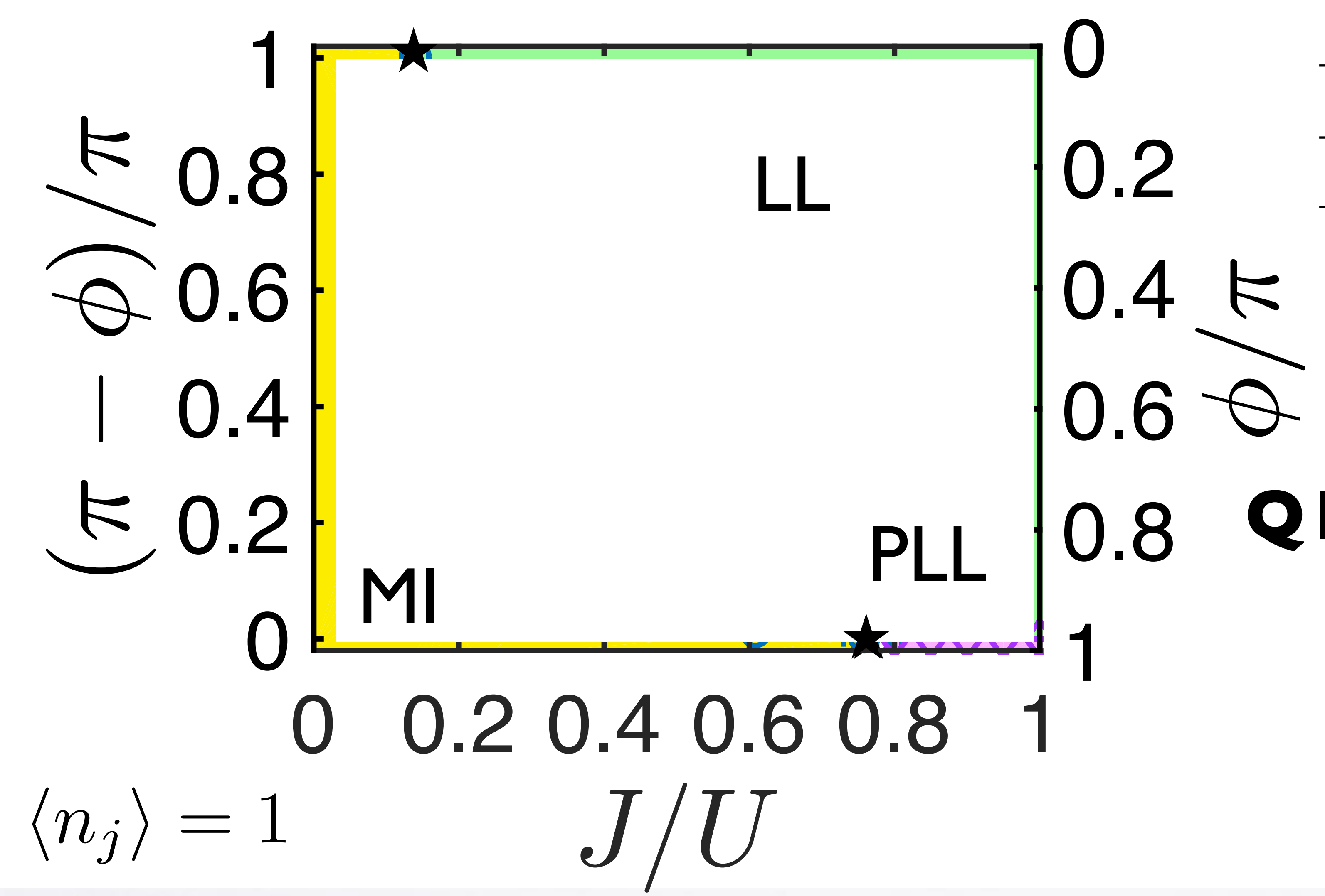
$$\left[\hat{H}_{\mathrm{BH}}, \hat{P}_j\right] = 0 \quad \forall j \quad \text{with} \quad \hat{P}_j \equiv \exp\left[i\pi \sum_\tau \hat{w}^\dagger_{j,\tau} \hat{w}_{j,\tau} \right]$$

Exotic Hubbard model with emergent **local parity conserved**!

If a gapless liquid exists, it should be made of pairs!

Doucot, Vidal, PRL **88**, 227005 (2002) & Tovmasyan, et al., arXiv:rXiv:1805.04529

The phase diagram



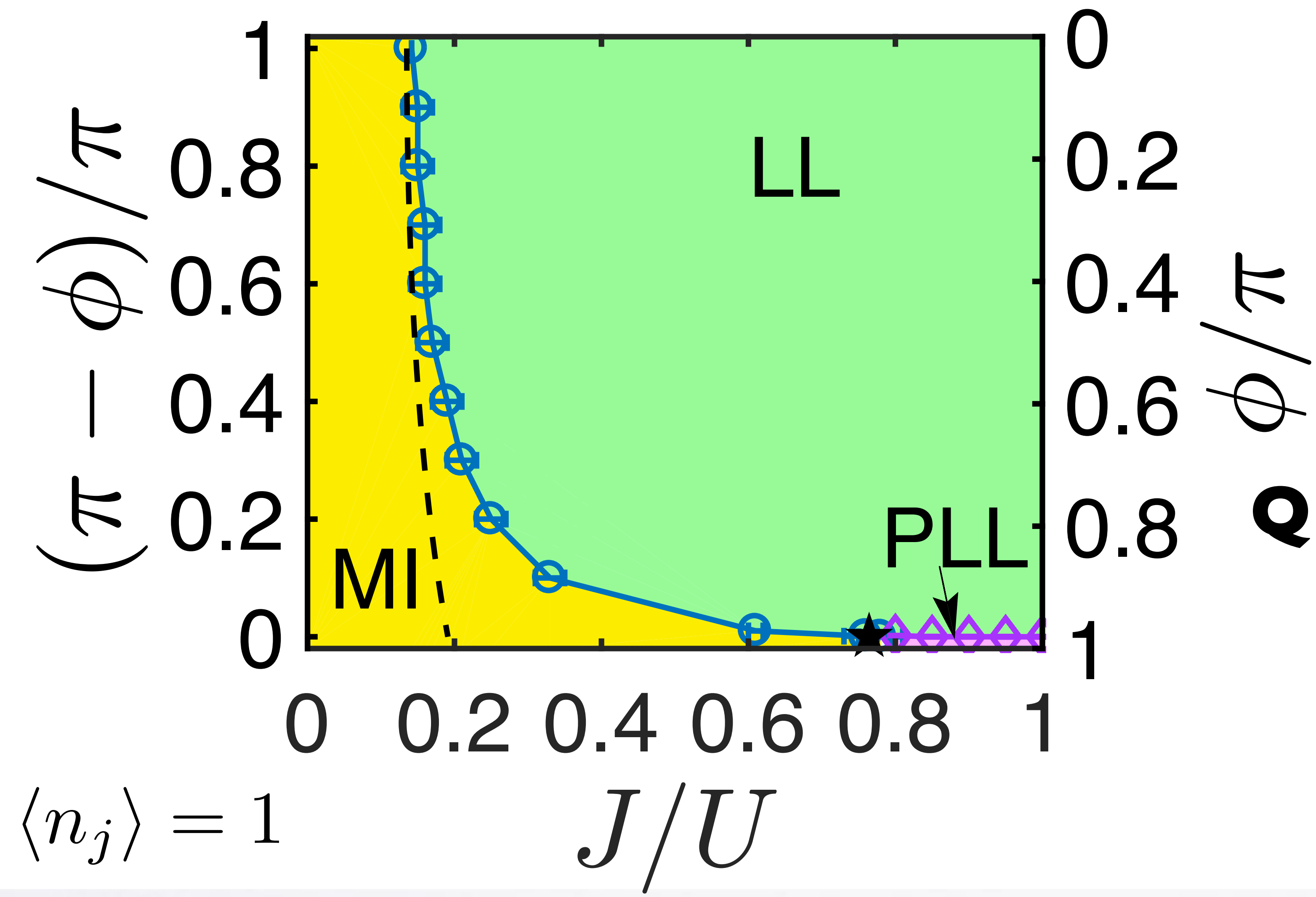
	$\langle \mathbf{b}_j^\dagger \mathbf{b}_{j+r} \rangle$	$\langle (\mathbf{b}_j^\dagger)^2 (\mathbf{b}_{j+r})^2 \rangle$
MI	$e^{-r/\xi}$	$e^{-2r/\xi+\dots}$
LL	$r^{-\kappa_s}$	$r^{-2\kappa_s+\dots}$
PLL	$e^{-r/\xi'}$	$r^{-\kappa_p}$

Q1: how robust is the PLL phase?

Q2: some Ising transition between PLL & LL ?

$$\mathbf{b}_j \simeq \widetilde{\mathbf{b}}_j \sigma_j^z$$

The phase diagram



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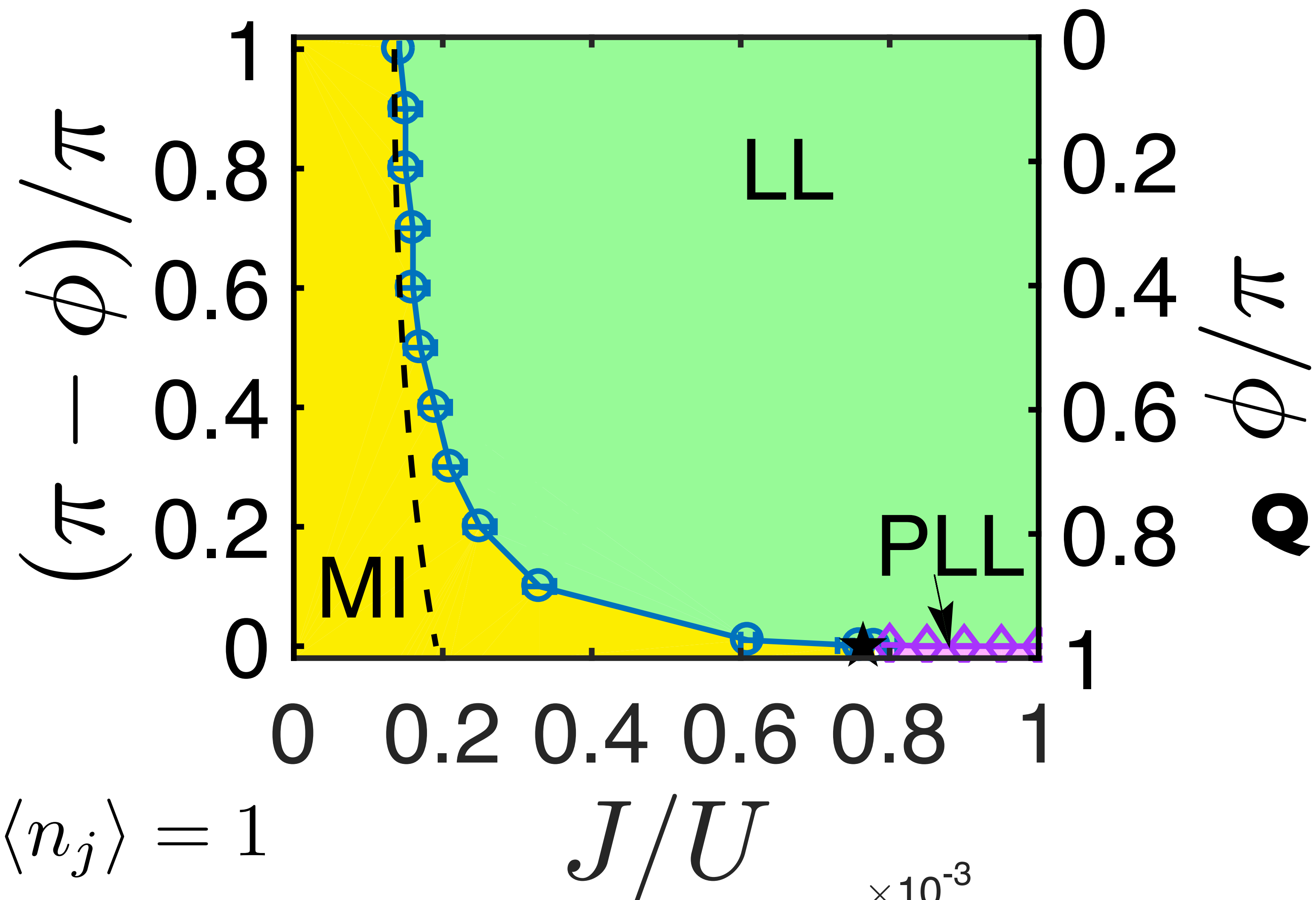
*PRB **98**, 184508 (2018)*

The phase diagram



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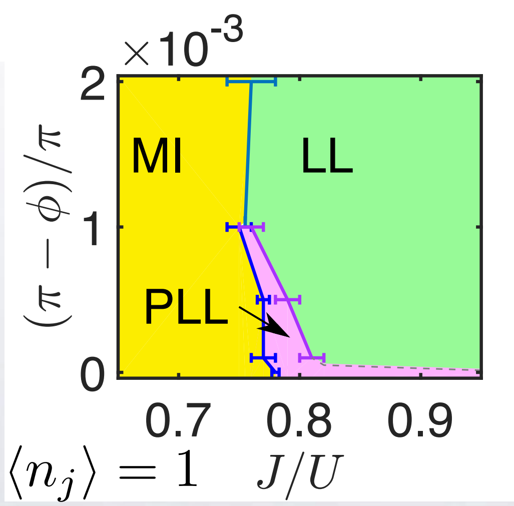
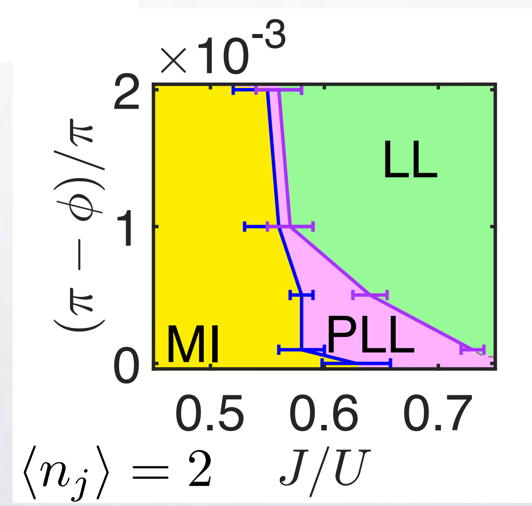
Q1: how robust is the PLL phase?

Q2: some Ising transition between PLL & LL ?

$$\mathbf{b}_j \simeq \widetilde{\mathbf{b}}_j \sigma_j^z$$

unfortunately
compatible with some
$|\pi - \phi_c| \simeq \mathrm{e}^{-J/U}$
[or other fast decays…]



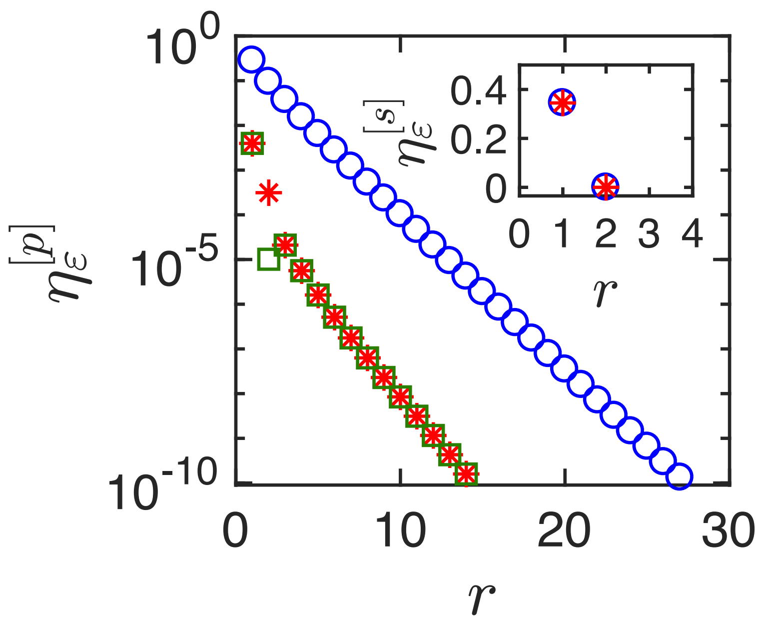


higher filling
helps a bit …

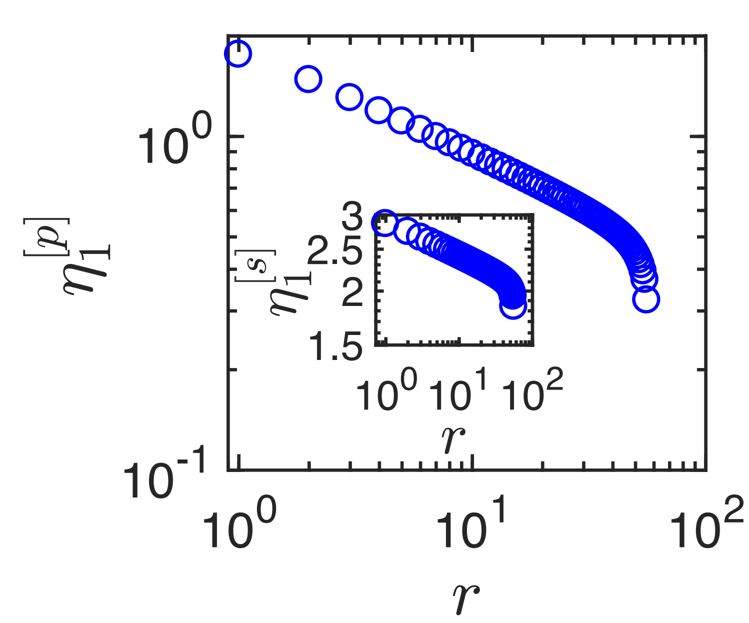
Traditional observables

correlation matrices $\eta_\epsilon^{[s](i,i+r)} = \mathrm{spec}\left(\langle \hat{b}^\dagger_{i,\alpha} \hat{b}_{i+r,\beta} \rangle\right)$ $\eta_\epsilon^{[p](i,i+r)} = \mathrm{spec}\left(\langle (\hat{b}^\dagger_{i,\alpha})^2 (\hat{b}_{i+r,\beta})^2 \rangle\right)$

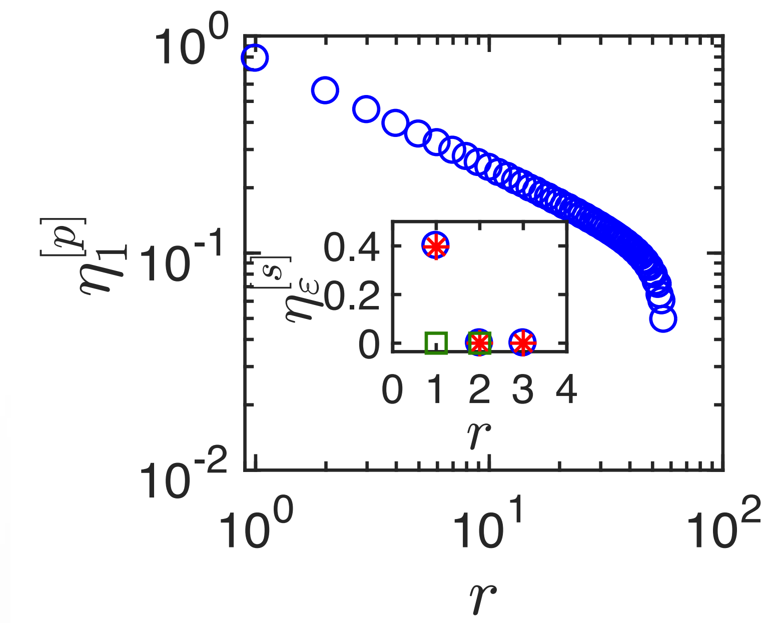
MI



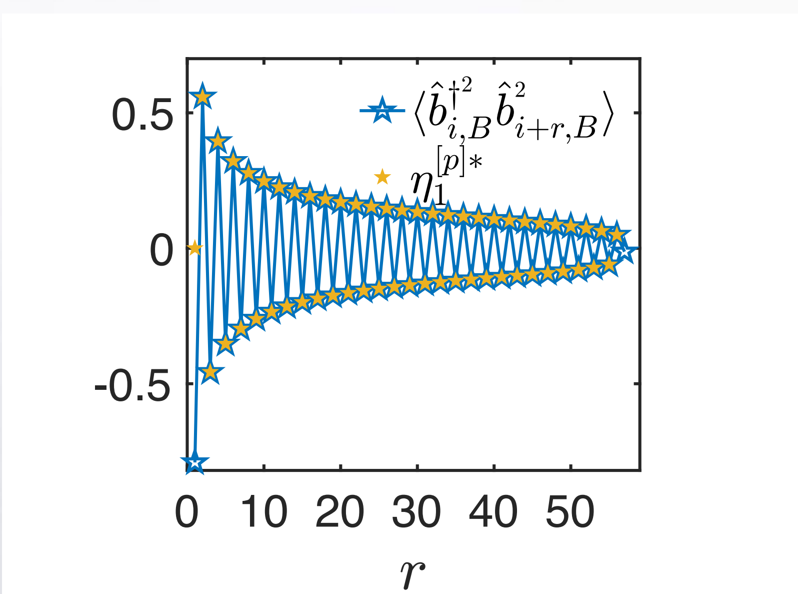
LL

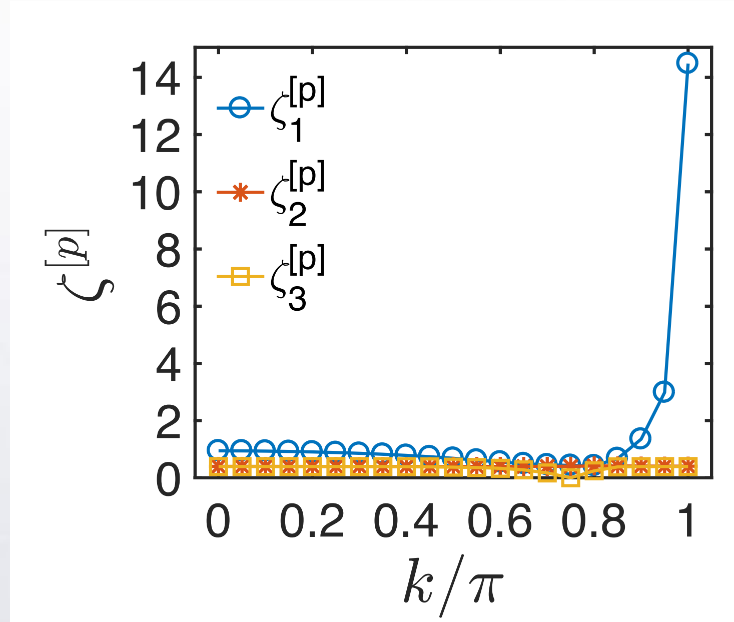


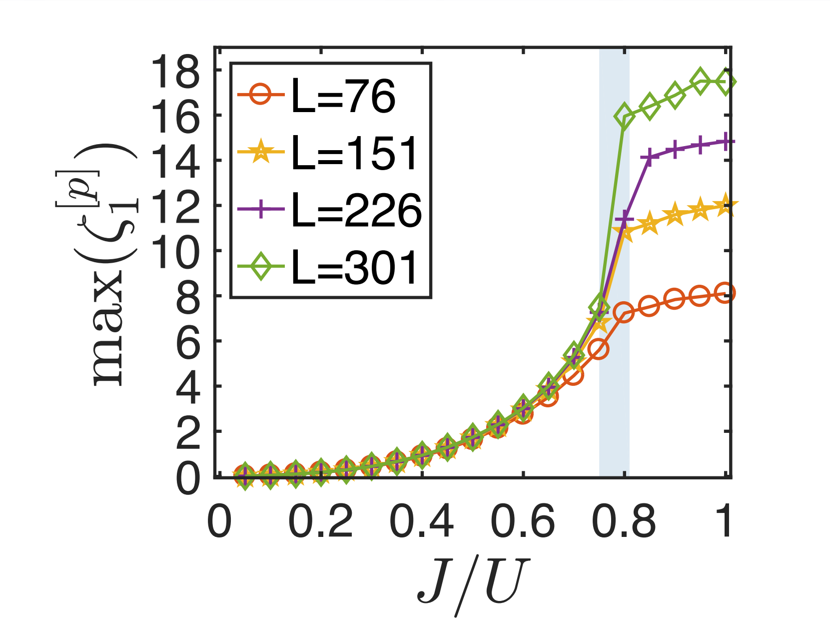
PLL



peaked structure factor at $k = \pi$ $\zeta_\epsilon^{[s/p]}(k) = \mathrm{spec}\left(\mathrm{FT}(\langle \ldots [s/p] \ldots \rangle)\right)$





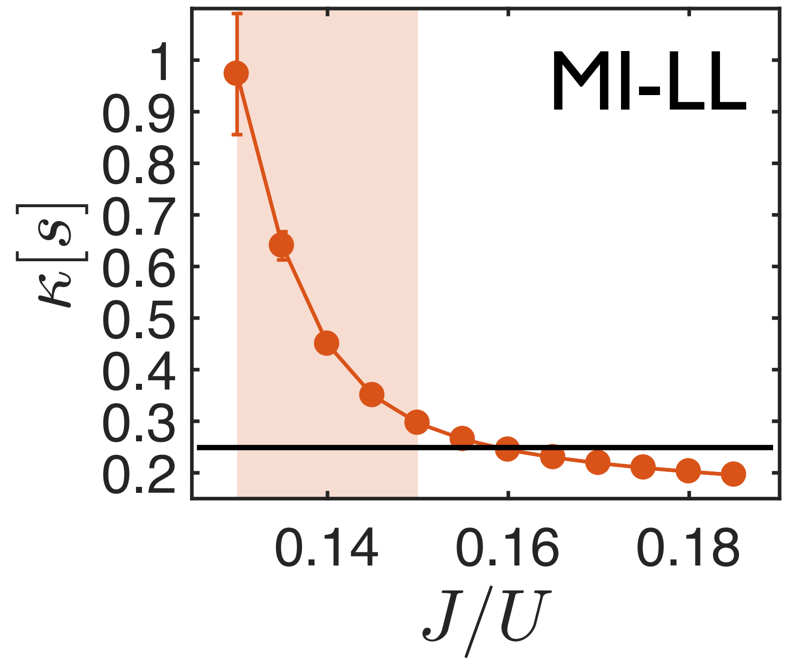


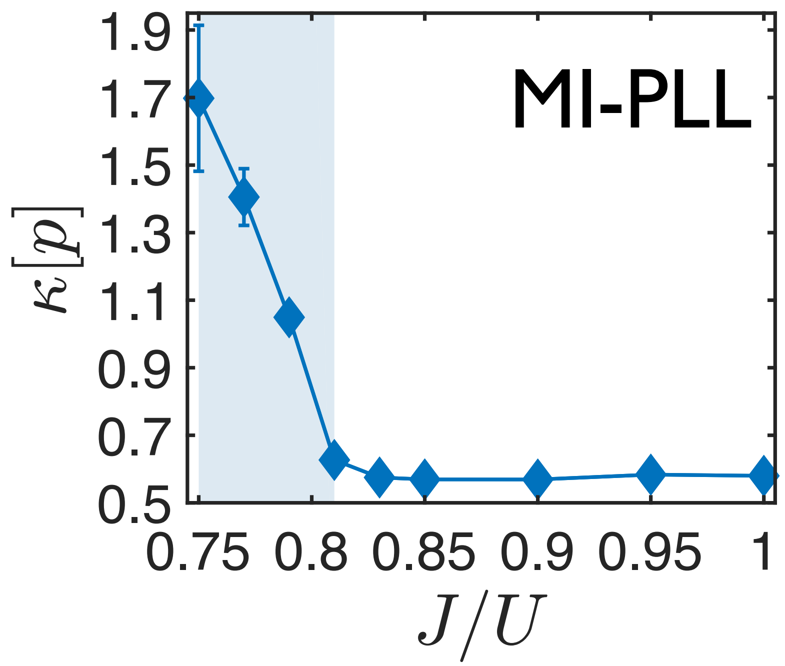
Traditional observables

correlation matrices $\eta_\epsilon^{[s](i,i+r)} = \mathrm{spec}\left(\langle \hat{b}_{i,\alpha}^\dagger \hat{b}_{i+r,\beta} \rangle\right)$ $\eta_\epsilon^{[p](i,i+r)} = \mathrm{spec}\left(\langle (\hat{b}_{i,\alpha}^\dagger)^2 (\hat{b}_{i+r,\beta})^2 \rangle\right)$

$$\eta_{\max}(r) \simeq r^{-\kappa}$$

$$2\kappa = K$$





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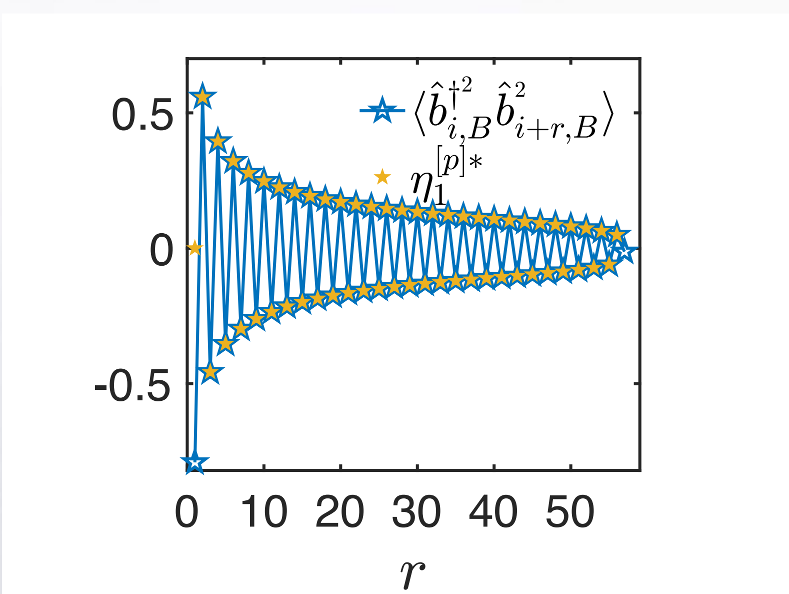

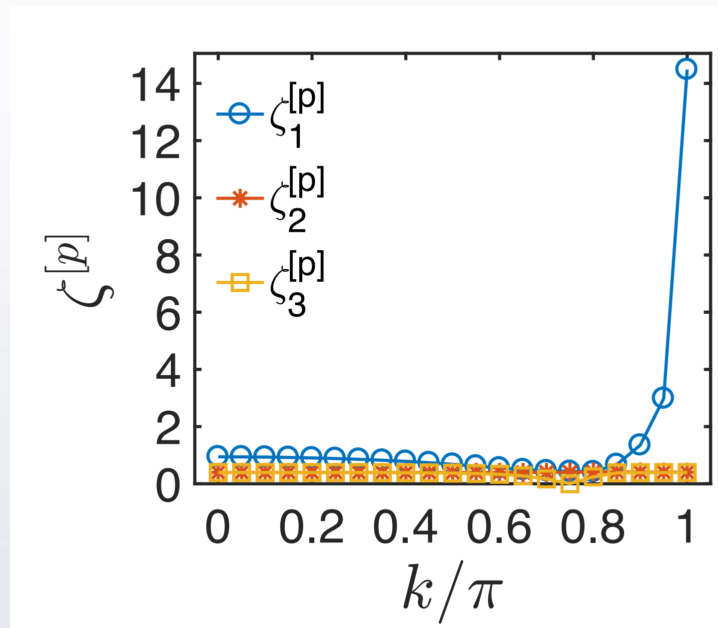

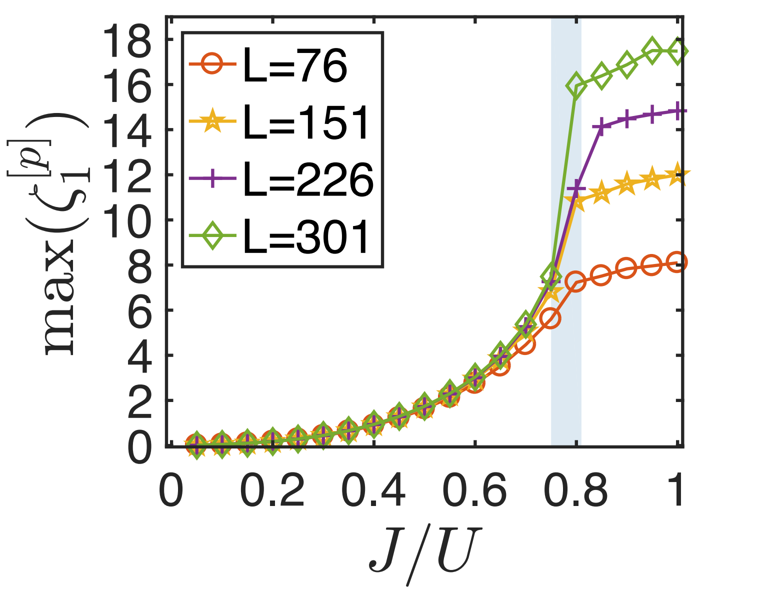

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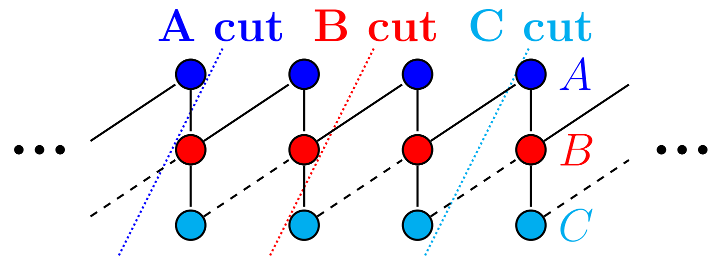
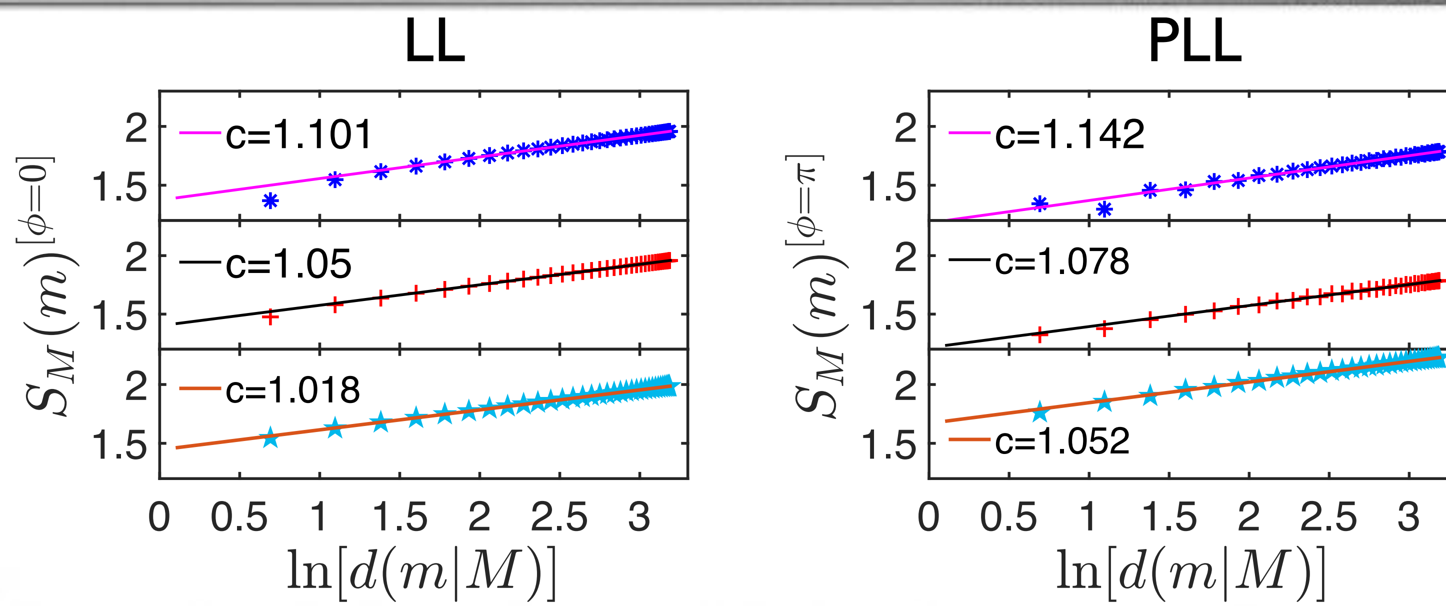
PRB ***98****, 184508 (2018)*

Entanglement properties



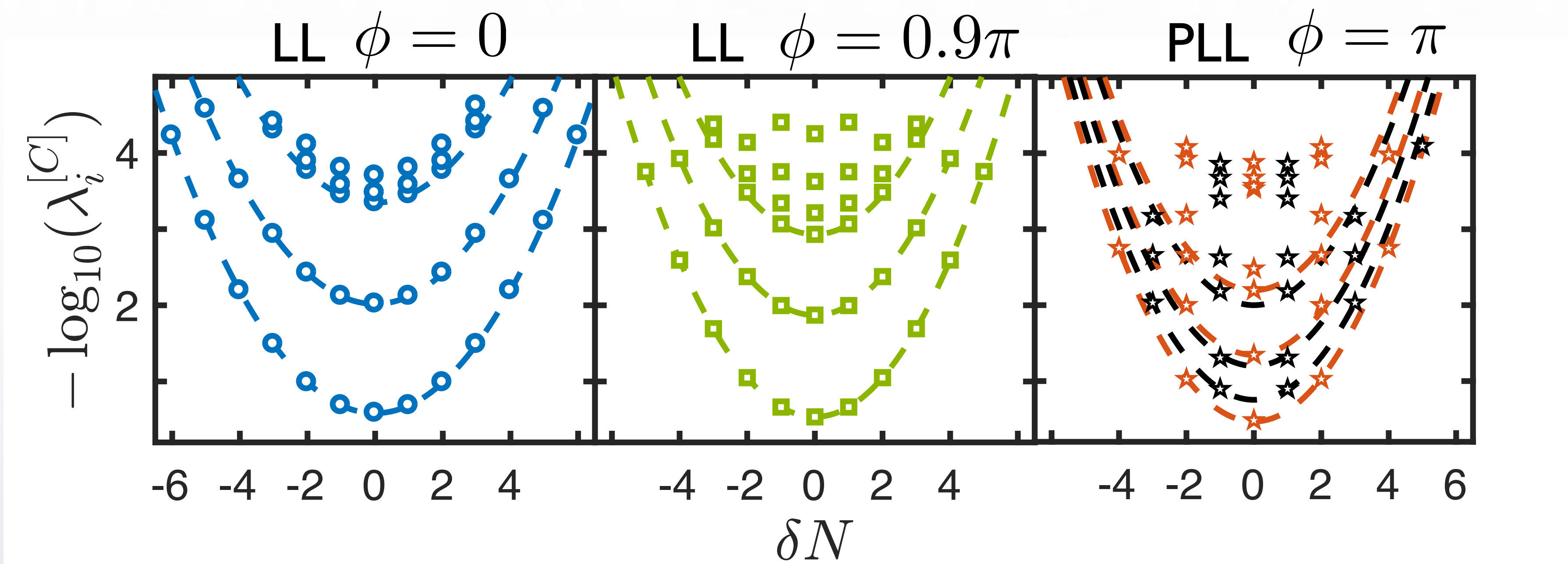
$$S_M(m) = \frac{c}{6}\ln\left[\frac{M}{\pi}\sin\left(\frac{\pi m}{M}\right)\right] + A + \mathcal{O}\left(\frac{1}{m}\right)$$

Vidal, et al., PRL **90**, 227902 (2003)
Calabrese & Cardy, JSTAT P06002 (2004)



same CFT central charge <u>BUT</u> *different* entanglement spectrum

working tool:
Tensor Networks



entanglement quantities are gaining experimental relevance …

e.g., Islam, et al., Nature **528**, 77 (2015) — Dalmonte, et al., Nat. Ph., **14**, 827 (2018)

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*PRB **98**, 184508 (2018)*

Open directions

- comparison of PLL robustness to Creutz-ladder flat bands:
  any relation to topological invariants?
  Ising model !? (we find no c=3/2 line)

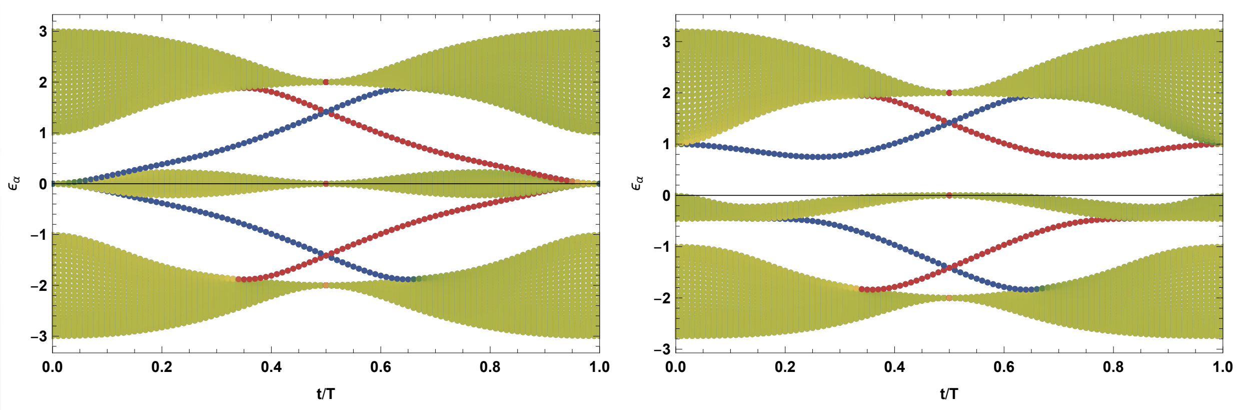
Tovmasyan, et al., PRB **88**, 220510 (2013)
Takayoshi, et al., PRA **88**, 063613 (2013)
Jünemann, et al. (MR), PRX **7**, 031057 (2017)

- exploration of "square root TI" character:
  quantized pumping schemes?

$$\mathcal{H}_{\diamond\pi} \longrightarrow \gamma_{\mathrm{Zak}} \notin \{0, \pi\}$$

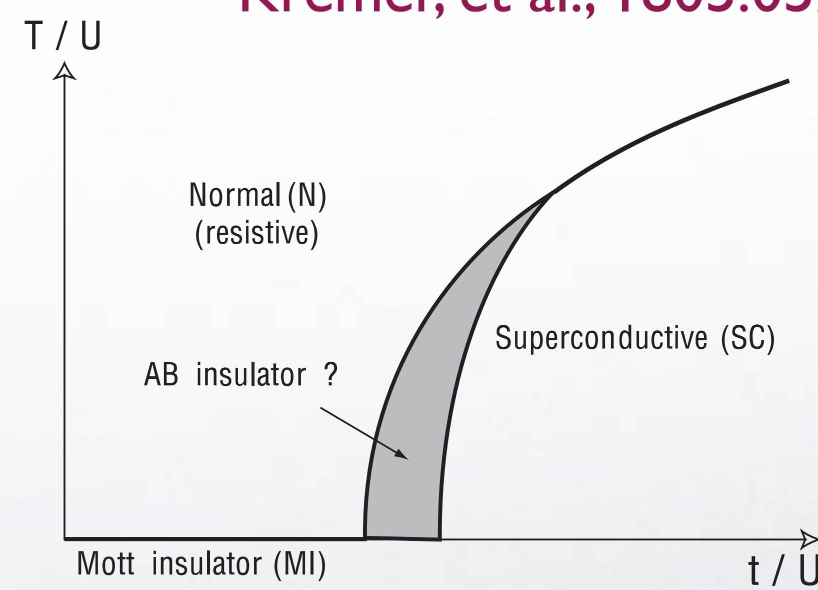
$$\mathcal{H}^2_{\diamond\pi} = \mathcal{H}_{\mathrm{trivial}} \oplus \mathcal{H}_{\mathrm{SSH}}$$

Kremer, et al., 1805.05209



Tausendpfund, MR, unpublished

- revival of 2D AB-cages: glassy phase at hand!?

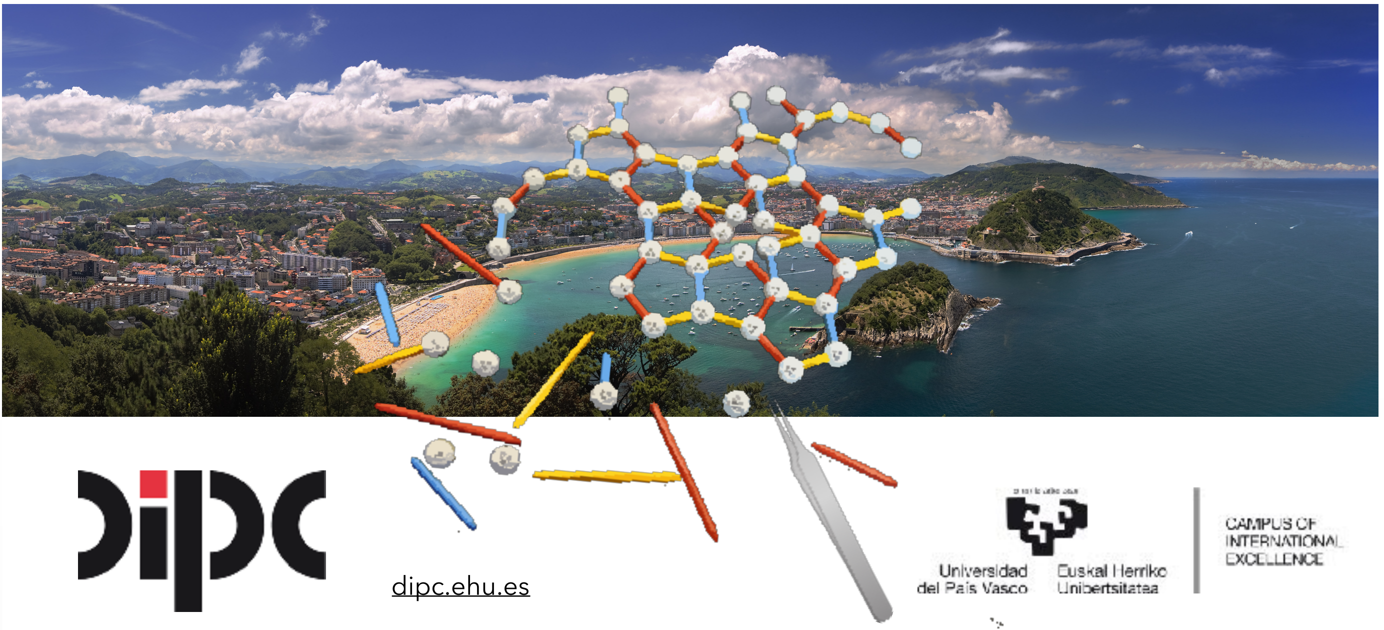


MR, et al., PRB **73**, 144511 (2006)

- many-body dynamics in presence of extensive local symmetries: MBL-like?

Brenes, et al., PRL **120**, 030601 (2018)

Some Advertisement



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Tilman Esslinger (Zurich)
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A. Haller

+ N. Tausendpfund & J. Nothhelfer

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M. Filippone (Uni Geneva)

L. Mazza (Paris-Sud)

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all of you for your attention!