Characterising Optical Lattice Depths *with* Simple Atomic Dynamics

Ben Beswick
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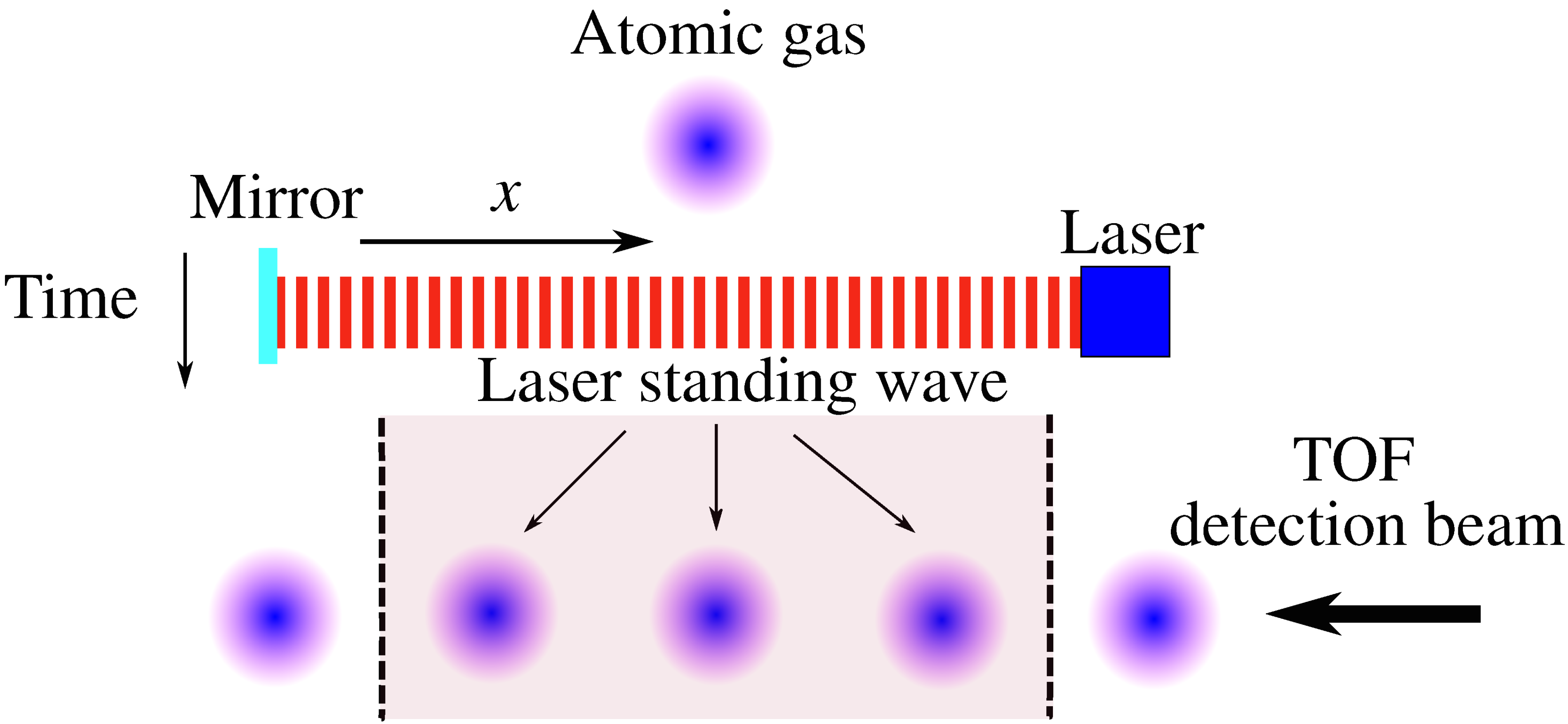
Simon Gardiner

Conceptual Setup

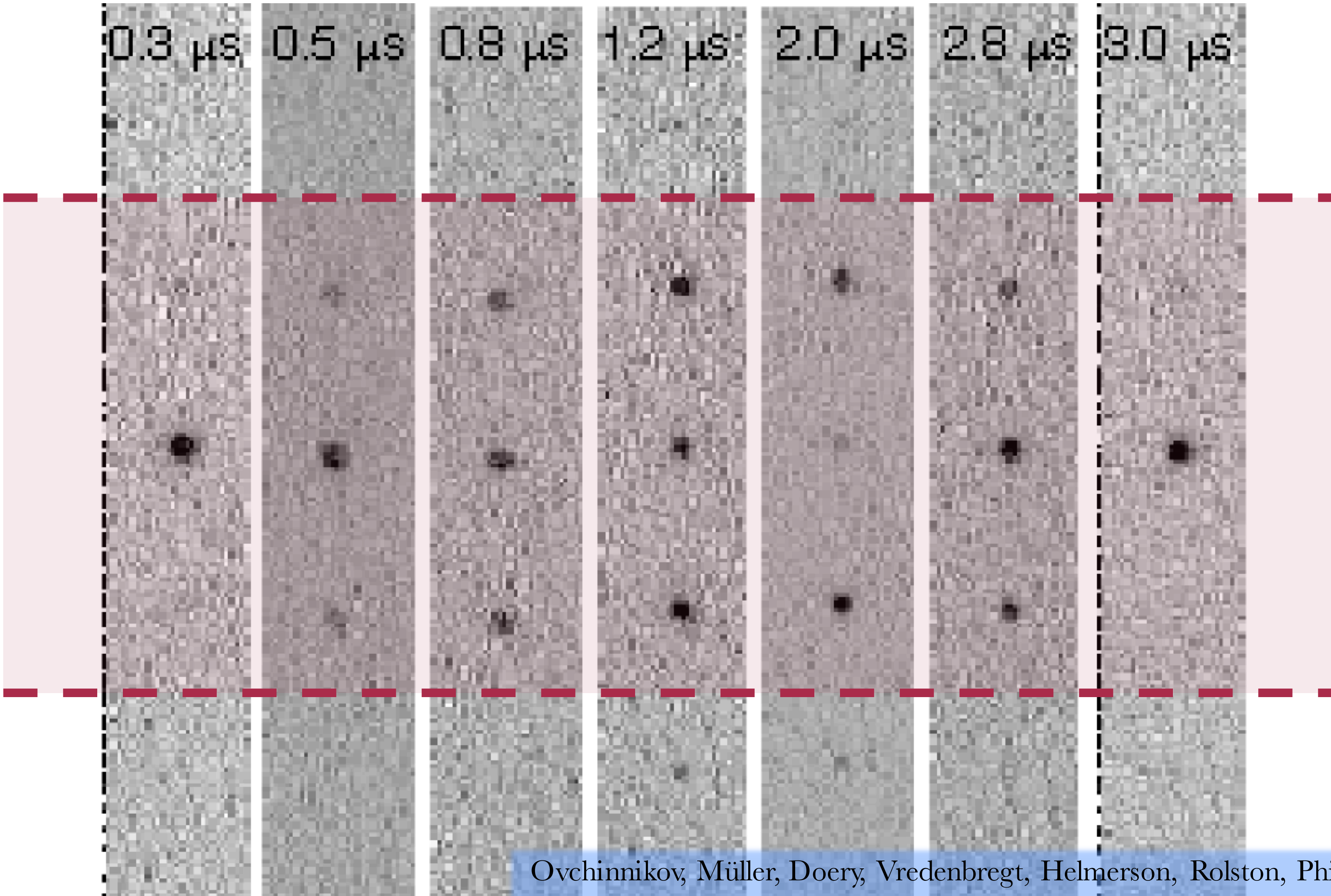


Prehistory (NIST, 1999)

- Form weakly interacting sodium BEC (approximately **zero momentum** initial motional state)
- Turn on **off-resonant laser standing wave** (optical lattice)
- Consider dynamics after various times (**duration** of **single** standing wave laser pulse)
- Time-of-flight measurement (maps **momentum distribution** onto **spatial distribution**)

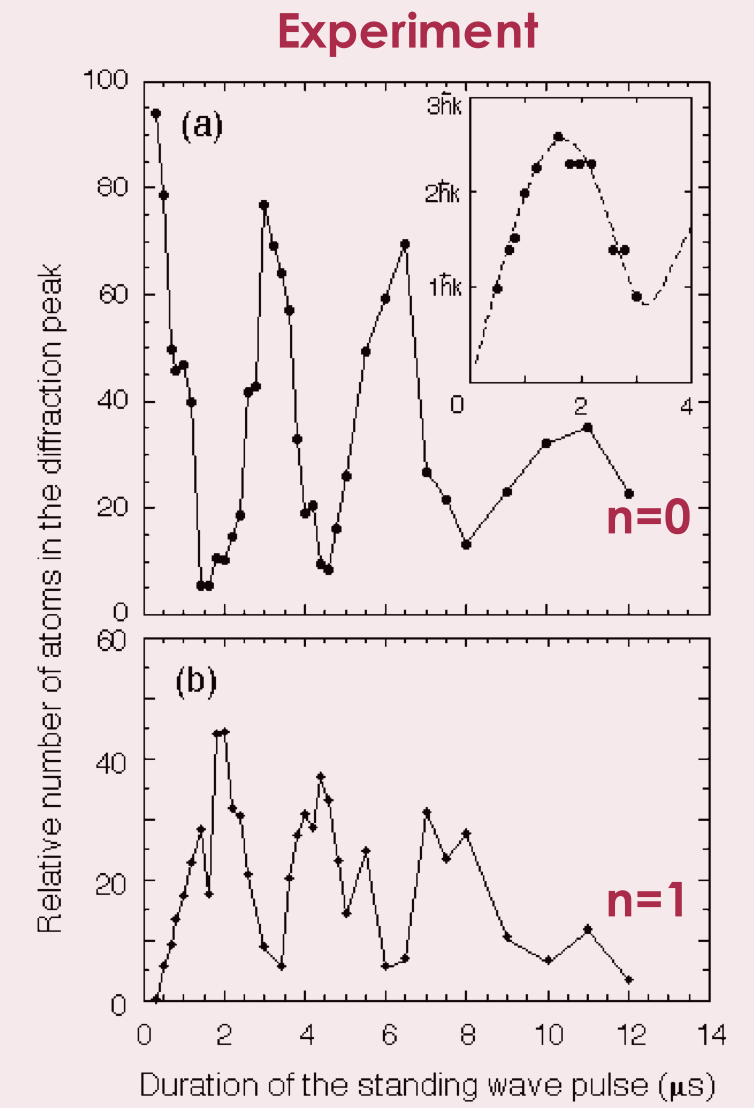
Ovchinnikov, Müller, Doery, Vredenbregt, Helmerson, Rolston, Phillips
Physical Review Letters **83**, 284 (1999)

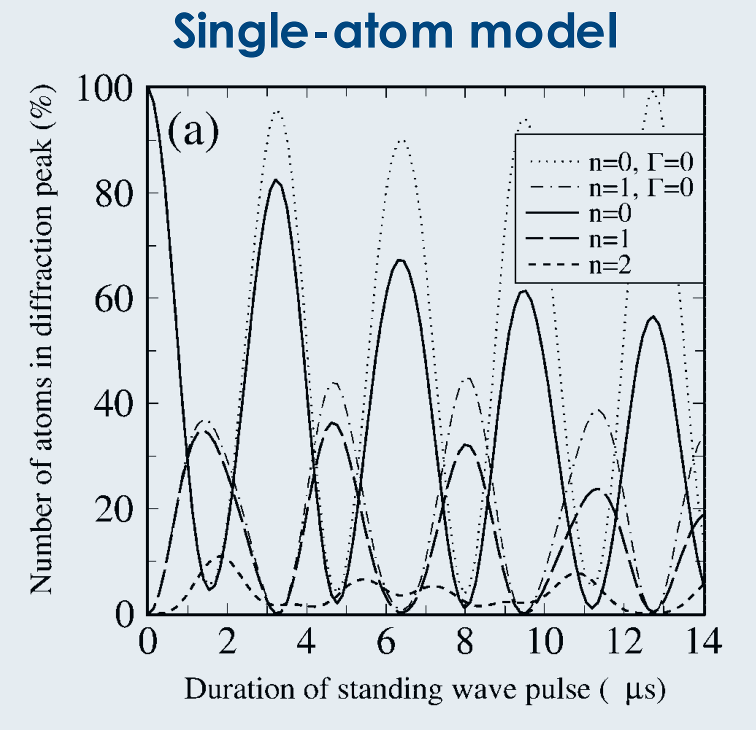
NIST Time of Flight Data



Ovchinnikov, Müller, Doery, Vredenbregt, Helmerson, Rolston, Phillips
Physical Review Letters **83**, 284 (1999)

Rabi Oscillations





- Describe **potential** as $\propto \cos(Kx)$
- **Oscillations** between $|0\rangle$ and

$$\frac{|\hbar K\rangle + |-\hbar K\rangle}{\sqrt{2}}$$

(n=0, n=1 diffraction orders)

May Wish to Know **Weak** Optical Lattice Depths

PRL **109,** 243003 (2012) PHYSICAL REVIEW LETTERS week ending 14 DECEMBER 2012

Precision Measurement of Transition Matrix Elements via Light Shift Cancellation

C. D. Herold,* V. D. Vaidya, X. Li, S. L. Rolston, and J. V. Porto

Joint Quantum Institute, University of Maryland and NIST, College Park, Maryland 20742, USA

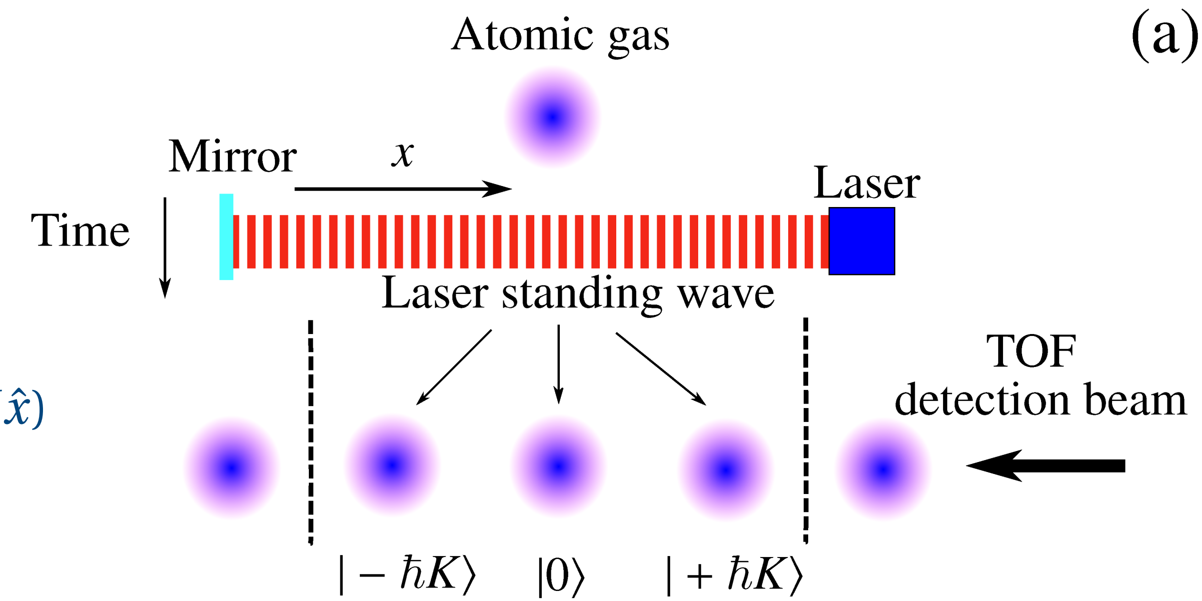
M. S. Safronova

Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA

(Received 20 August 2012; published 14 December 2012)

- "**Precise knowledge of atomic transition strengths is important in …**
  - development of ultraprecise atomic clocks,
  - studies of fundamental symmetries,
  - [studies of] degenerate quantum gases,
  - quantum information,
  - plasma physics, and
  - astrophysics."

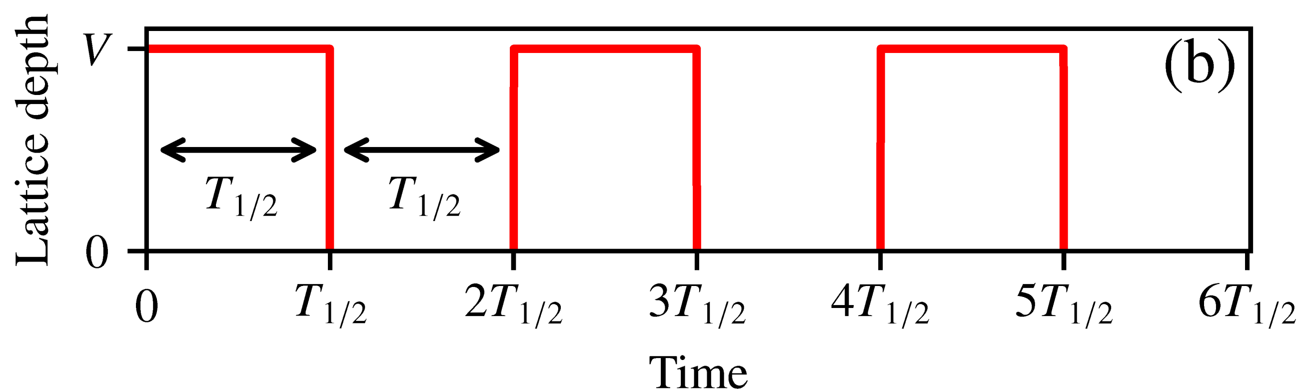
Resonantly Timed Multiple Short Pulses



- Alternating Hamiltonians

$$\hat{H}_{\text{Latt}} = \frac{\hat{p}^2}{2M} - V\cos(K\hat{x})$$

$$\hat{H}_{\text{Free}} = \frac{\hat{p}^2}{2M}$$



- Applied for duration

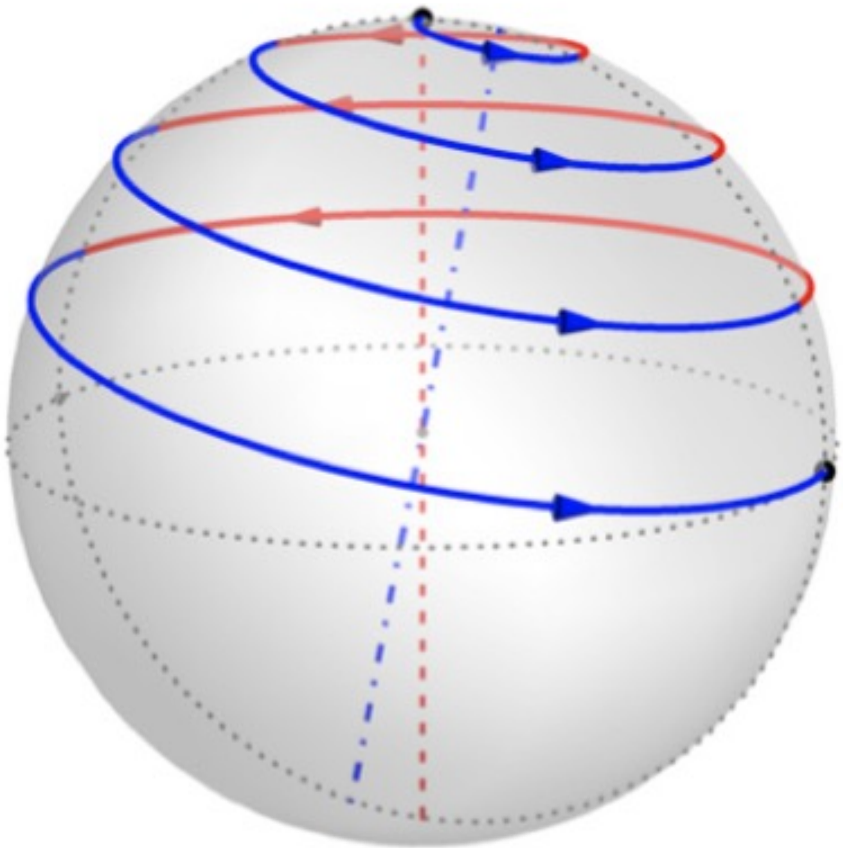
$$T_{1/2} = \frac{2\pi M}{\hbar K^2} = \frac{h}{8E_R}$$

(**half-Talbot** time)

Bloch Sphere Picture

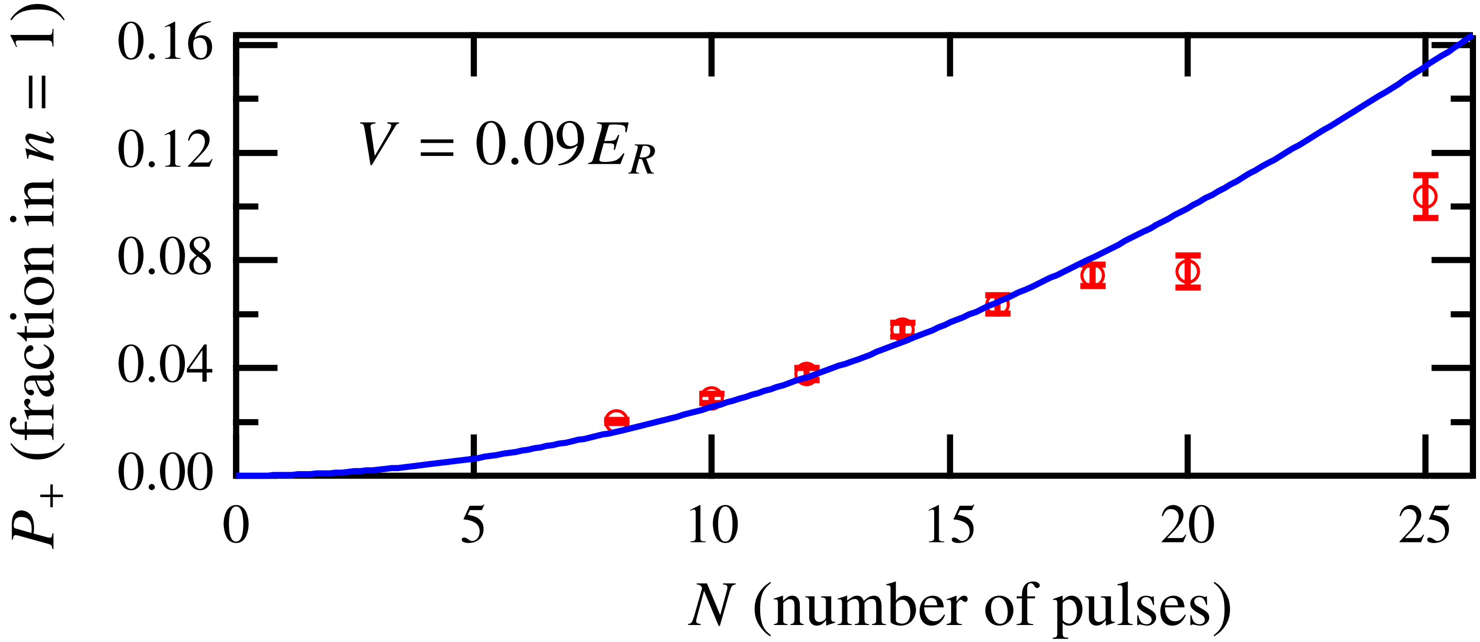
Herold, Vaidya, Li, Rolston, Porto, Safronova
Physical Review Letters **109**, 243003 (2012)

- Multipulse sequence **coherently adds** effect of each pulse
- **Blue** shows evolution with lattice on
- **Red** shows evolution with lattice off



Quadratic Population of First Diffraction Order

Herold, Vaidya, Li, Rolston, Porto, Safronova
Physical Review Letters **109**, 243003 (2012)



- Assuming “small” VN (and zero initial momentum), then P_+ grows **approximately quadratically**

- V may be deduced by **numerical fit**

A Closer Analytic Treatment

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Beswick, Hughes, Gardiner
Physical Review A **99**, 013614 (2019)

- System **spatially periodic**, hence (Bloch's theorem) partition momentum $(\hbar K)^{-1} p = k + \beta$, where $k \in \mathbb{Z}, \quad \beta \in [-1/2, 1/2)$
- Quasimomentum β conserved; momentum states with different quasimomentum **do not couple**
- Time evolution within particular **quasimomentum subspace** governed by Floquet operator

$$\hat{F}(\beta) = \hat{F}(\beta)_{\mathrm{Free}} \hat{F}(\beta)_{\mathrm{Latt}}$$
$$= \exp\left(-i\left[\frac{\hat{k}^2 + 2\hat{k}\beta}{2}\right]2\pi\right)\exp\left(-i\left[\frac{\hat{k}^2 + 2\hat{k}\beta}{2} - V_{\mathrm{eff}}\cos(\hat{\theta})\right]2\pi\right)$$

$$V_{\mathrm{eff}} = VM/\hbar^2K^2, \quad \hat{\theta} = K\hat{x}$$

Zero Temperature (Zero Quasimomentum Space)

Beswick, Hughes, Gardiner
Physical Review A **99**, 013614 (2019)

- By symmetry, evolution remains in **symmetric subspace**; minimally $|0\rangle$ and $\dfrac{|\hbar K\rangle + |-\hbar K\rangle}{\sqrt{2}}$
- Essentially a Rabi system with **periodic phase changes** to excited state; populations evolve as

$$P_0(N, V_{\text{eff}}) = 1 - A\sin^2(N\phi/2)$$

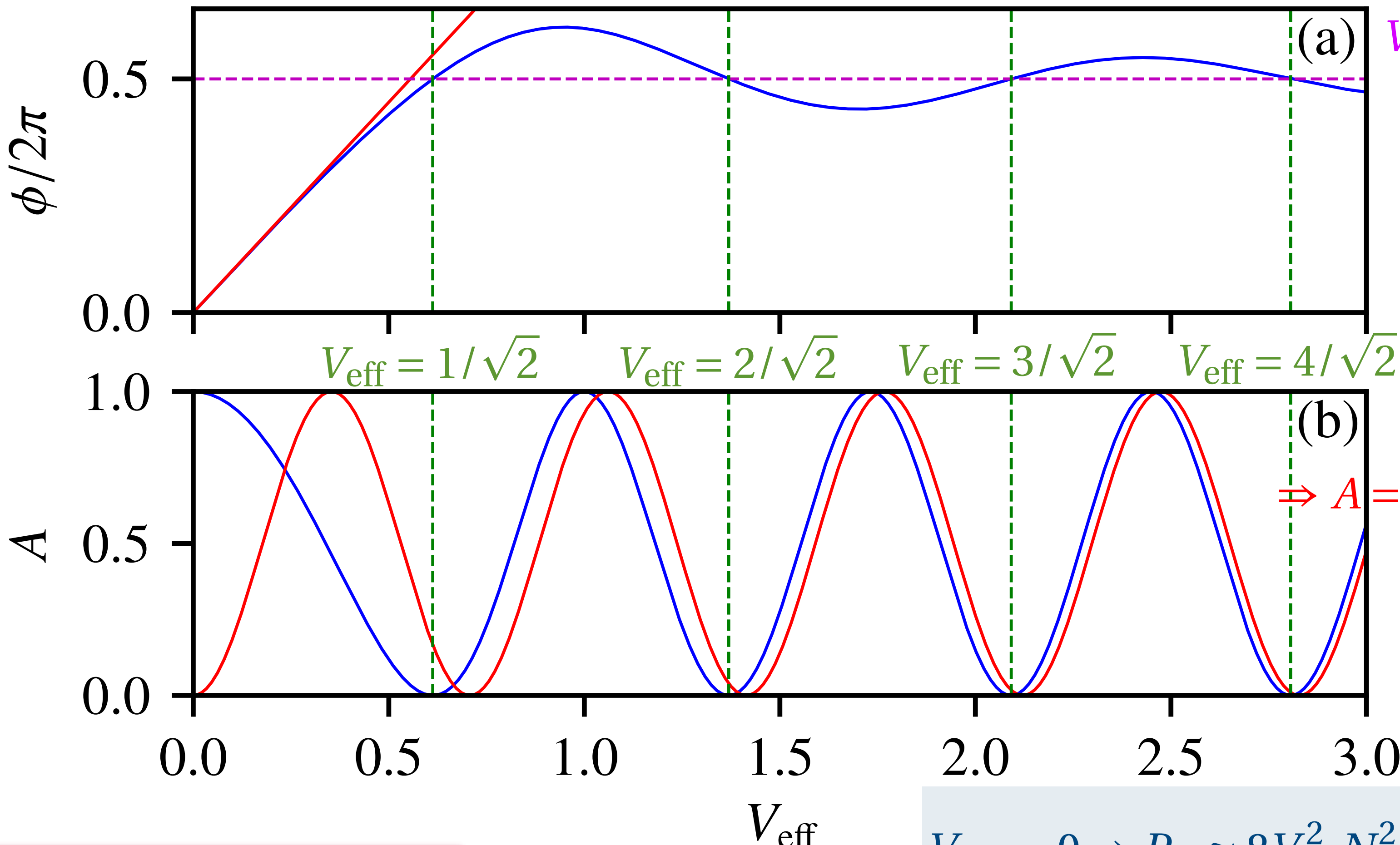
$$P_+(N, V_{\text{eff}}) = A\sin^2(N\phi/2)$$

$$A = \frac{8V_{\text{eff}}^2 \sin^2\left(\pi\sqrt{1+8V_{\text{eff}}^2}/2\right)}{8V_{\text{eff}}^2 + \cos^2\left(\pi\sqrt{1+8V_{\text{eff}}^2}/2\right)}$$

$$\phi = 2\arctan\left(\frac{\sqrt{8V_{\text{eff}}^2 + \cos^2\left(\pi\sqrt{1+8V_{\text{eff}}^2}/2\right)}}{\sin\left(\pi\sqrt{1+8V_{\text{eff}}^2}/2\right)}\right)$$

Limiting Behaviours

$V_{\text{eff}} \to 0 \Rightarrow \phi = 4\sqrt{2}V_{\text{eff}}$



$V_{\text{eff}} \to \infty$

$\Rightarrow \phi = \pi$

$V_{\text{eff}} \to \infty$

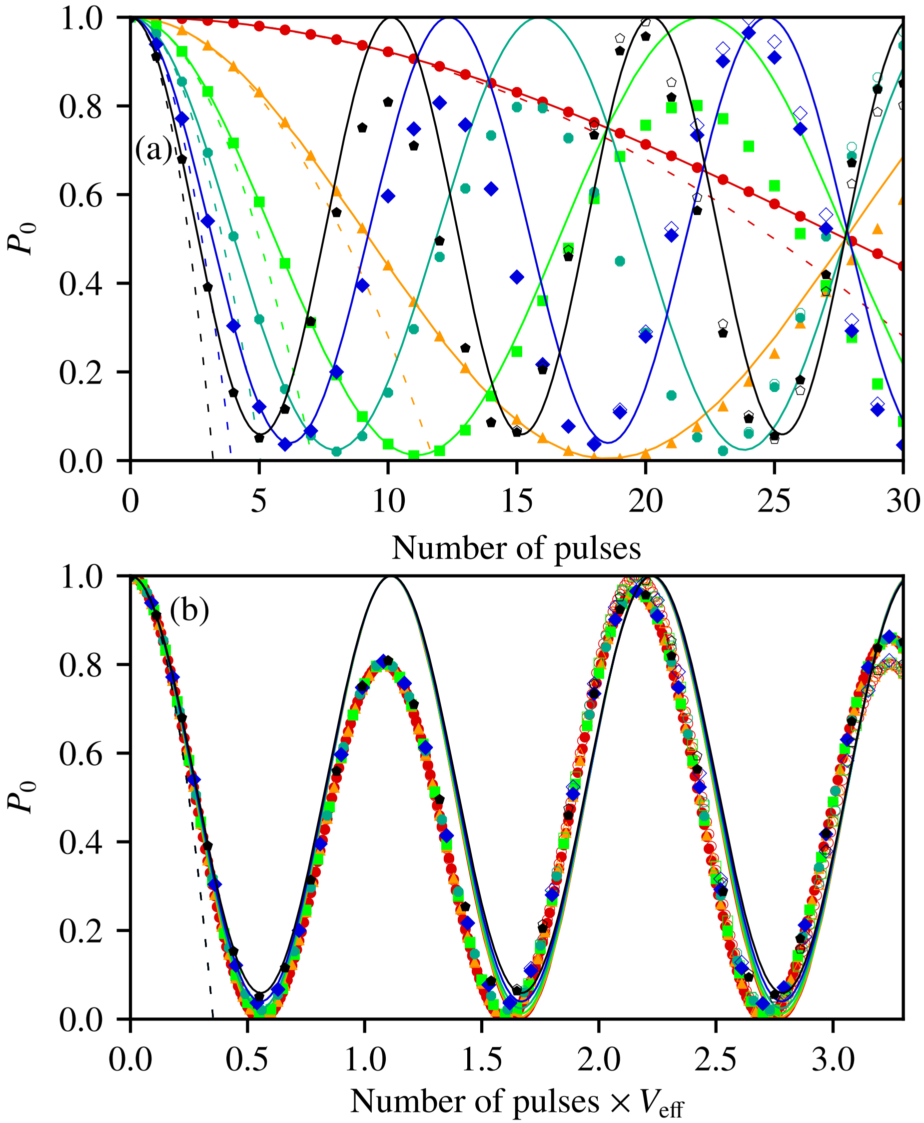
$\Rightarrow A = \sin^2(\sqrt{2}\pi V_{\text{eff}})$

Beswick, Hughes, Gardiner
Physical Review A **99**, 013614 (2019)

$$V_{\text{eff}} \to 0 \Rightarrow P_+ \approx 8V_{\text{eff}}^2 N^2 = \frac{1}{8}\left(\frac{V}{E_R}\right)^2 N^2$$

$$\Rightarrow P_0 \approx 1 - 8V_{\text{eff}}^2 N^2$$

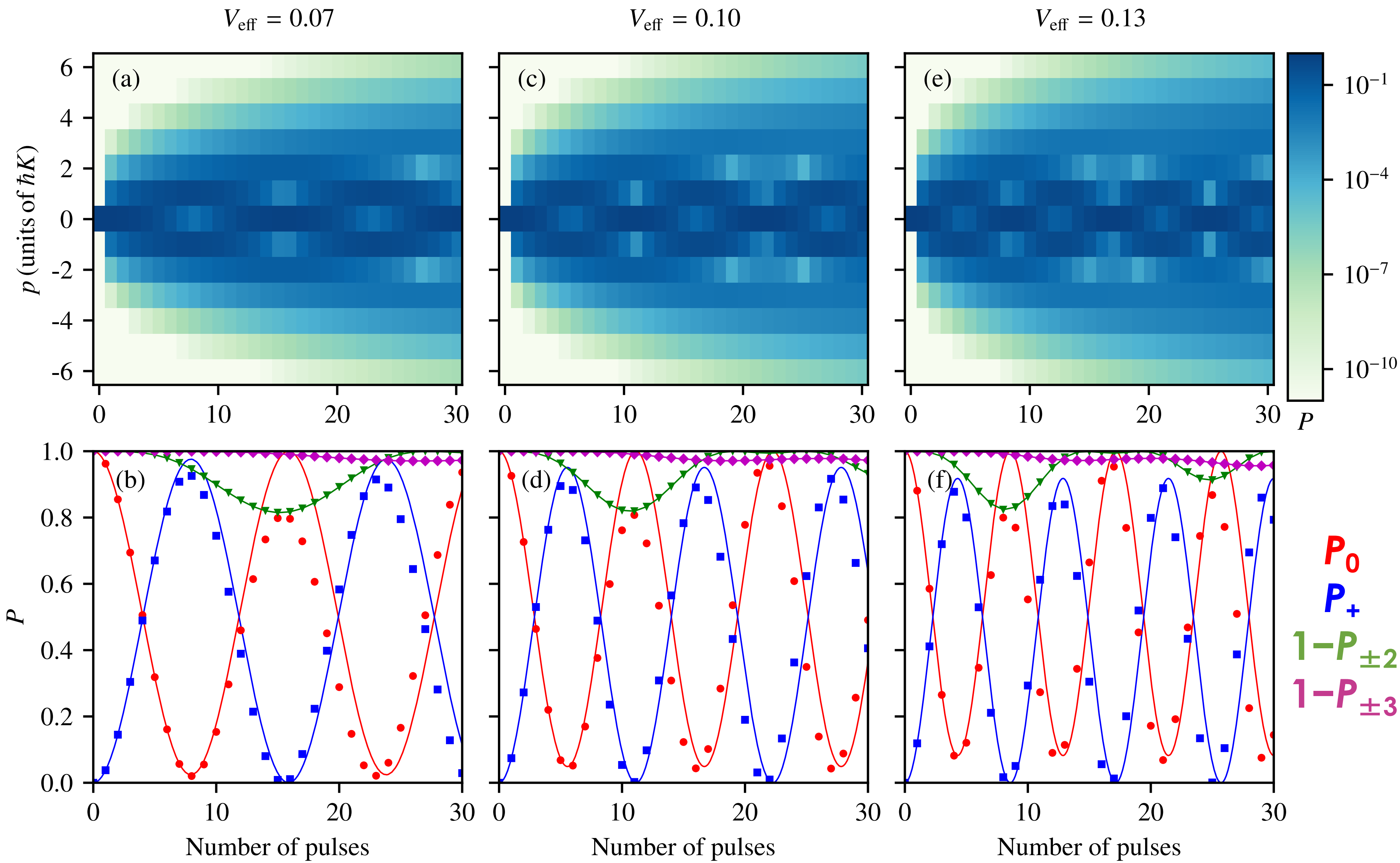
Universal Behaviour



Beswick, Hughes, Gardiner
Physical Review A **99**, 013614 (2019)

- Solid lines: **analytical estimates**
- Dashed lines: **quadratic approximation**
- Solid markers: **full numerics**
- Hollow markers: **zeroth, first, second diffraction orders** only

Population Beyond First Diffraction Order



Atomtronics 3
Benasque, Spain, 5–18 May 2019

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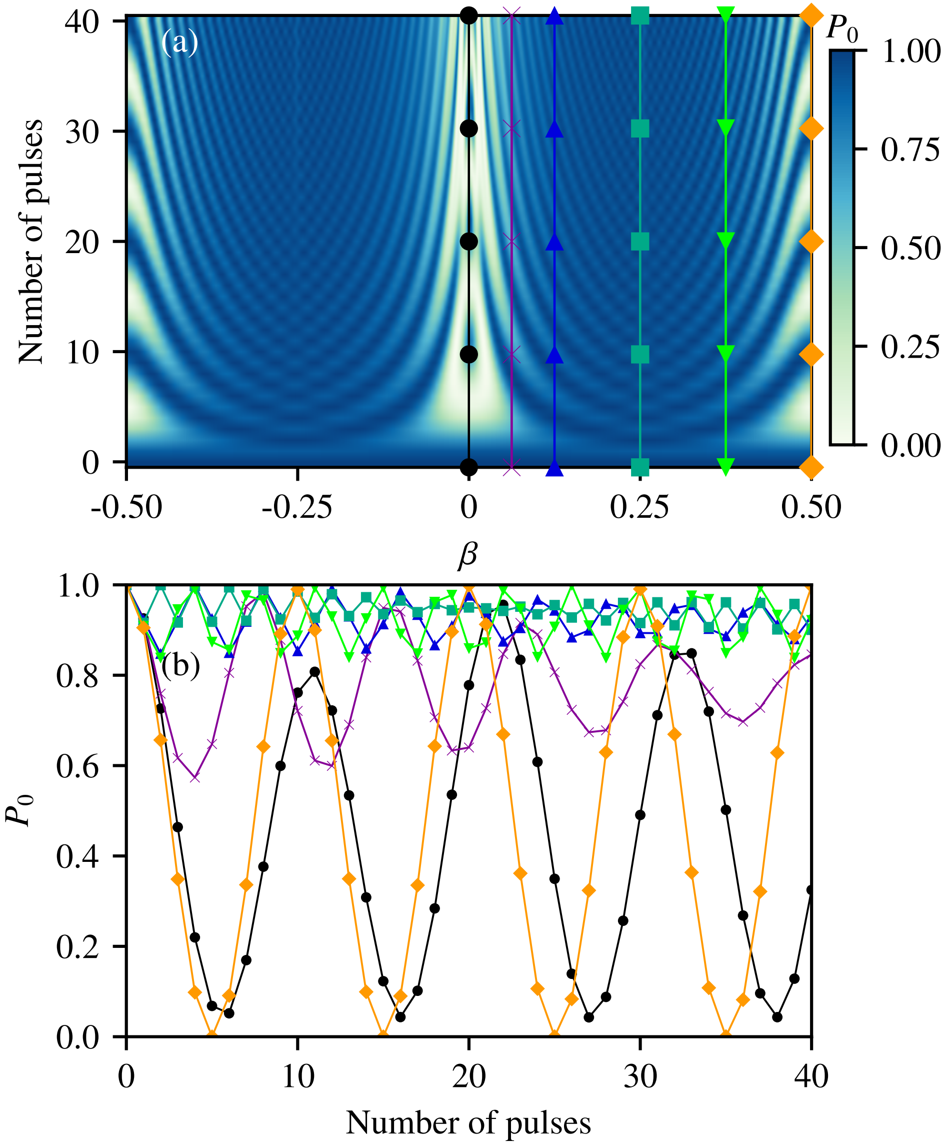
Dynamics in Other Quasimomentum Spaces



$V_{\rm eff} = 0.1$

Beswick, Hughes, Gardiner
Physical Review A **99**, 013614 (2019)

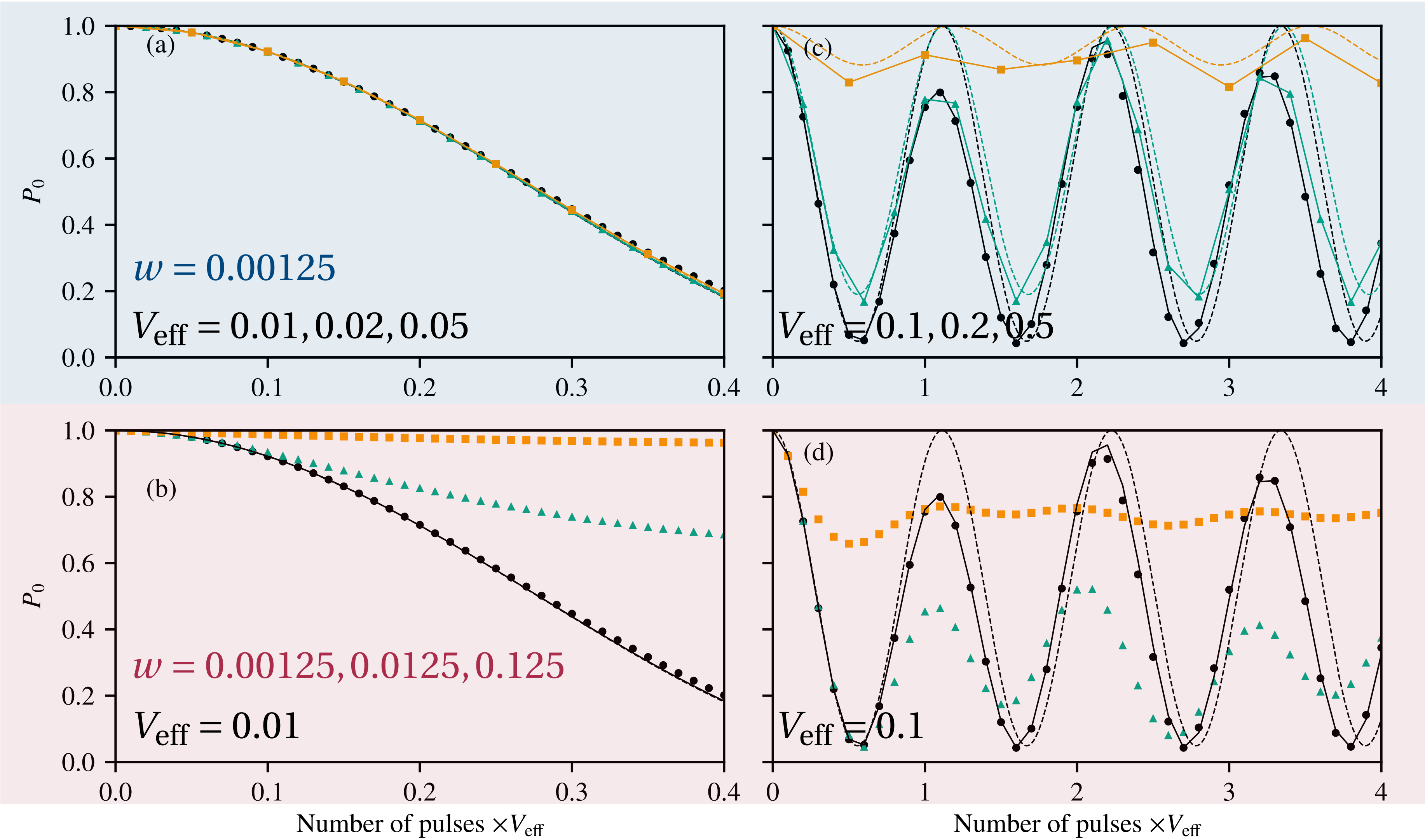
- Relevant for **finite temperature response**
- Consider rescaled **Maxwell-Boltzmann distribution**

$$D_{k=0}(\beta) = \frac{1}{w\sqrt{2\pi}} \exp\left(\frac{-\beta^2}{2w^2}\right)$$

- Corresponds to temperature

$$\mathcal{T}_w = \hbar^2 K^2 w^2 / M k_{\rm B}$$

Finite Temperature Response



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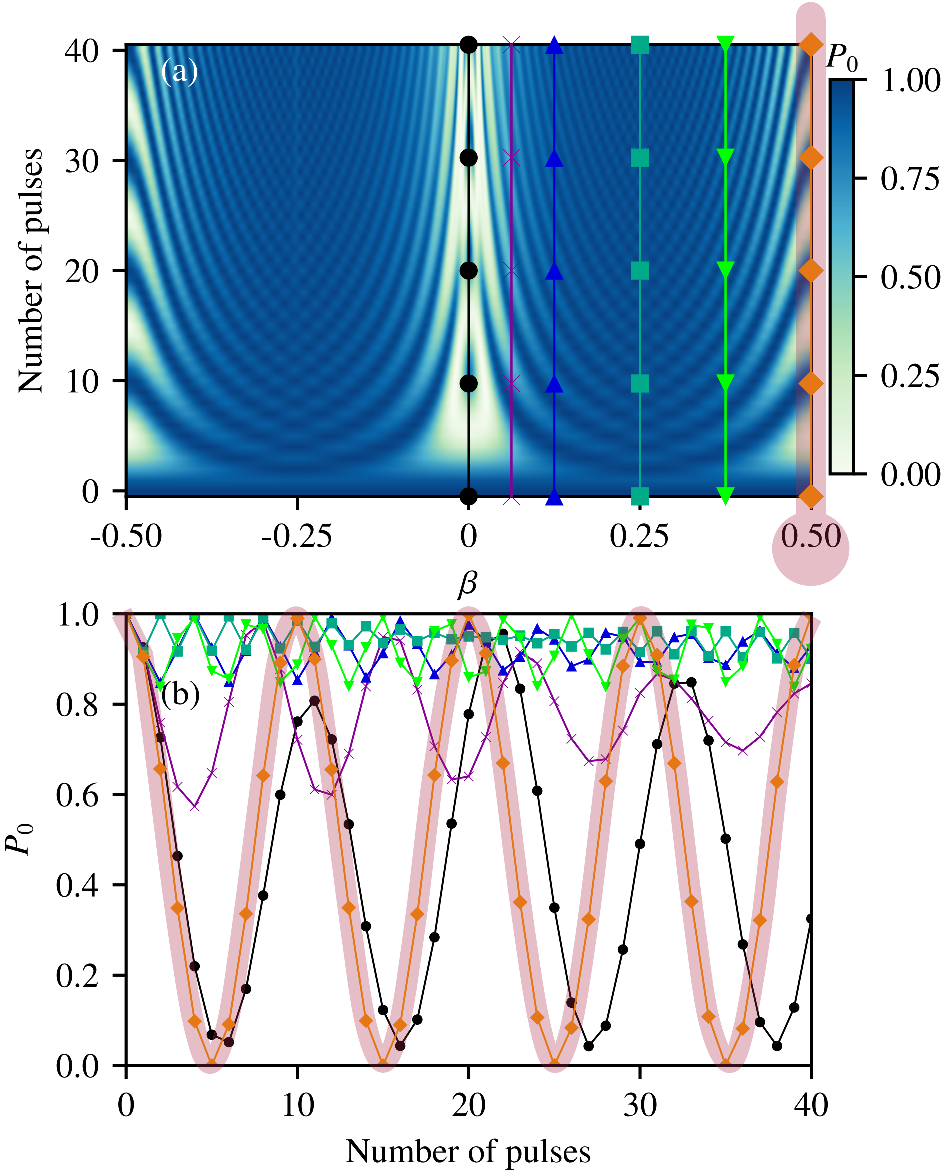
Dynamics when Quasimomentum = 1/2



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- Floquet operator

$$\hat{F}(\beta = 1/2) = \hat{F}(1/2)_{\mathrm{Free}}\hat{F}(1/2)_{\mathrm{Latt}}$$

$$= \exp\left(-i\left[\hat{k}(\hat{k}+1)\right]\pi\right)$$

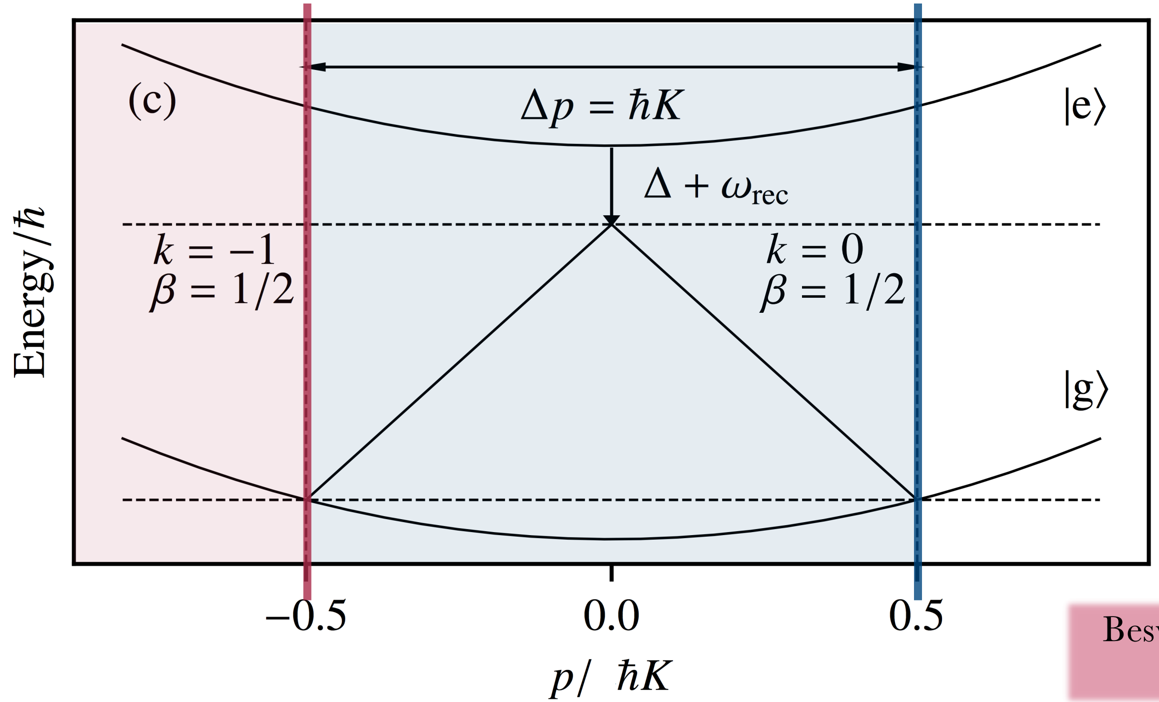
$$\times \exp\left(-i\left[\hat{k}(\hat{k}+1)/2 - V_{\mathrm{eff}}\cos(\hat{\theta})\right]2\pi\right)$$

simplifies to

$$\hat{F}(\beta = 1/2)$$

$$= \exp\left(-i\left[\hat{k}(\hat{k}+1)/2 - V_{\mathrm{eff}}\cos(\hat{\theta})\right]2\pi\right)$$

Continuous Diffraction with Quasimomentum = 1/2



Beswick, Hughes, Gardiner
arXiv:1903.04011

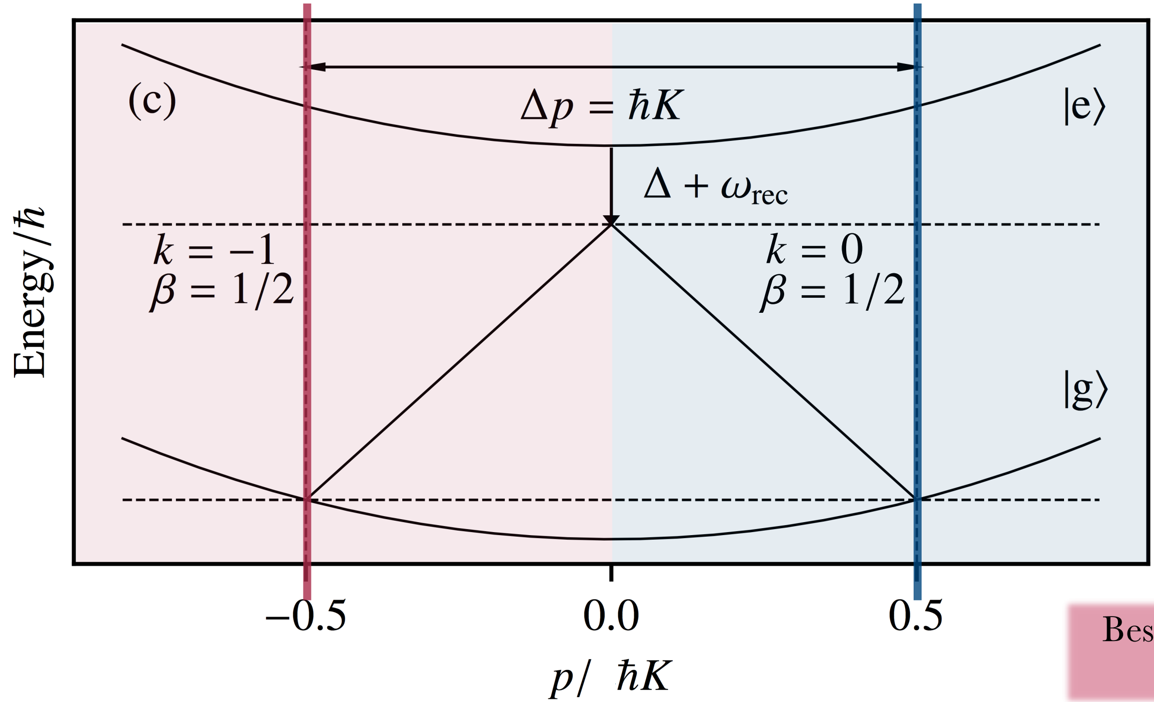
- Simultaneous energy and quasimomentum conservation

- Resonant **Raman Rabi coupling** of two **discrete** states

$$H^{2\times 2}_{\mathrm{Latt}} = \begin{pmatrix} 1/4 & -V_{\mathrm{eff}}/2 \\ -V_{\mathrm{eff}}/2 & 1/4 \end{pmatrix}$$

$$P_0 = \cos^2(V_{\mathrm{eff}}\tau/2),$$
$$P_{-1} = \sin^2(V_{\mathrm{eff}}\tau/2),$$

Continuous Diffraction with Quasimomentum = 1/2



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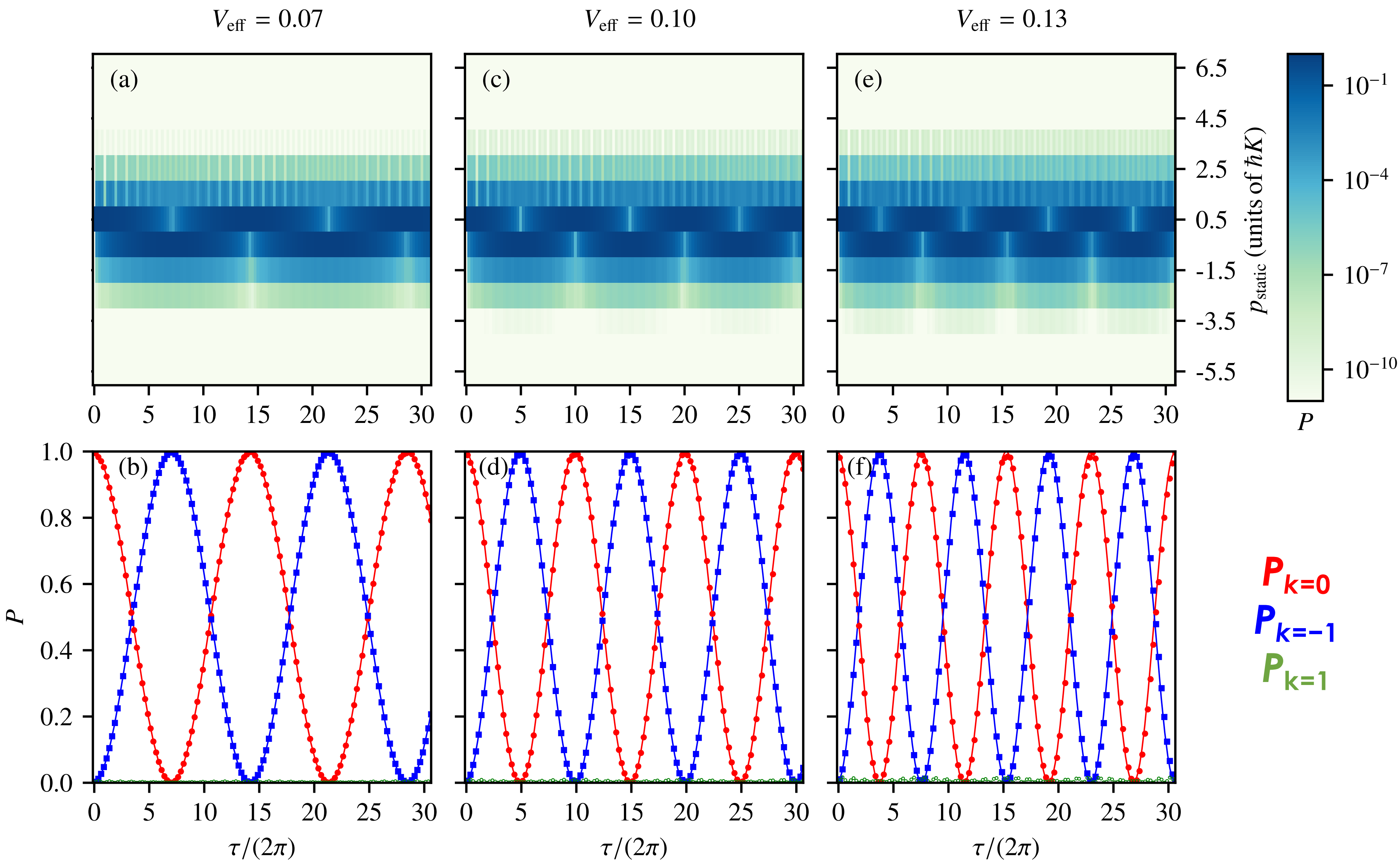
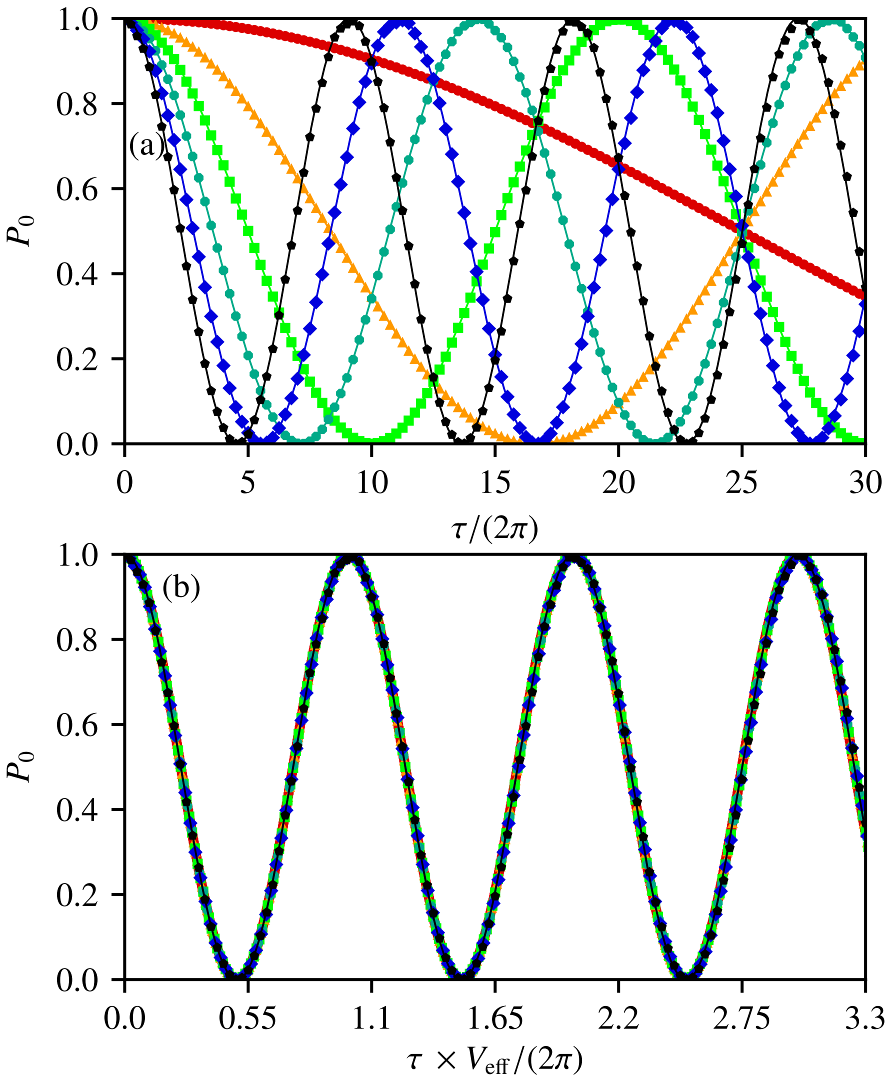
Population Beyond First Diffraction Order



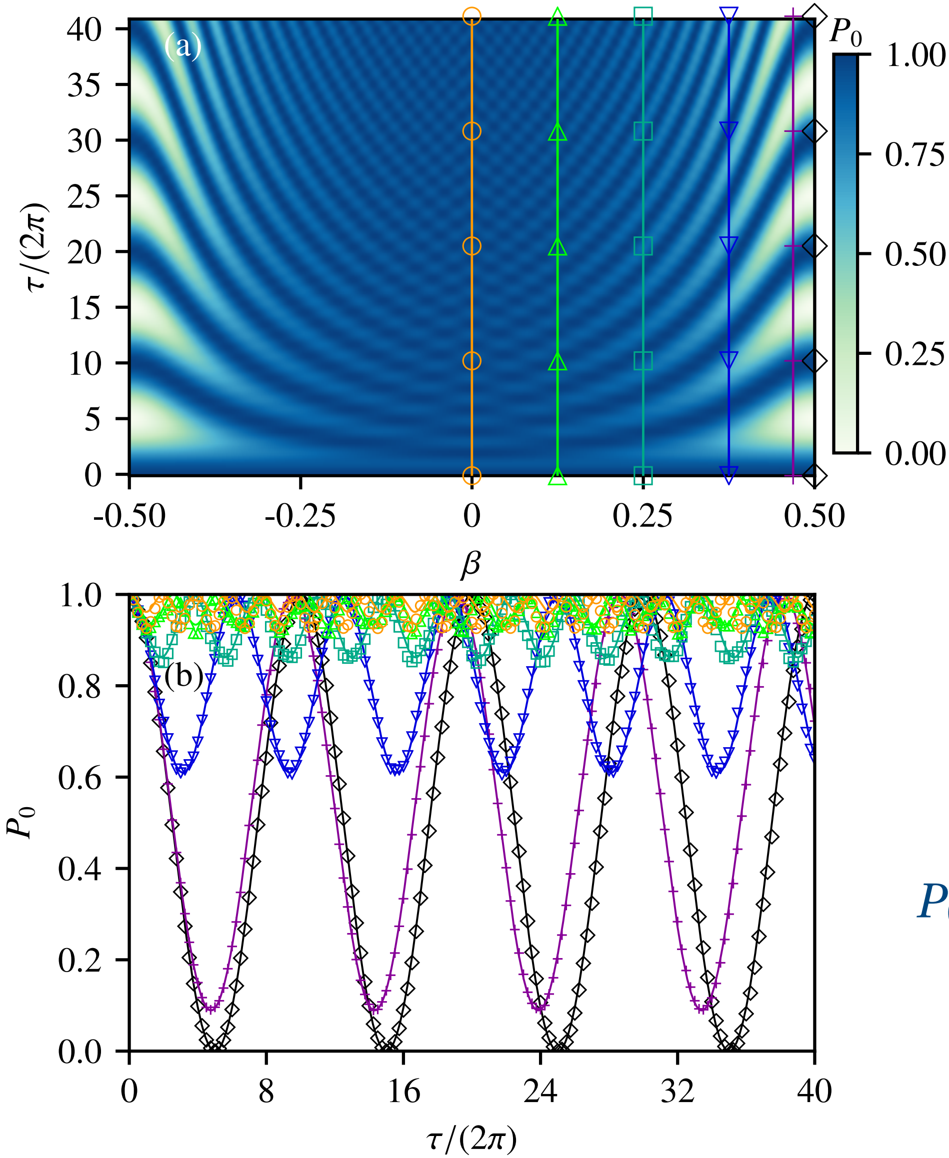
Universal Behaviour



Beswick, Hughes, Gardiner
arXiv:1903.04011

- **Continuous time evolution** in terms of $\tau = (\hbar K^2 / M)\, t$ (solid line analytics)
- **Exact numerics** for $V_{\mathrm{eff}} = 0.01$ to $V_{\mathrm{eff}} = 0.11$ (markers)

Other Quasimomentum Spaces



$$V_{\text{eff}} = 0.1$$

Beswick, Hughes, Gardiner
arXiv:1903.04011

- **Off-resonant** Rabi model

$$H^{2\times2}_{\text{Latt}}(\beta) = \begin{pmatrix} \beta^2/2 & -V_{\text{eff}}/2 \\ -V_{\text{eff}}/2 & (1-2\beta+\beta^2)/2 \end{pmatrix}$$

- Yields population dynamics

$$P_0(\beta) = 1 - \frac{V_{\text{eff}}^2 \sin^2\left(\sqrt{(\beta-1/2)^2 + V_{\text{eff}}^2}\,\frac{\tau}{2}\right)}{(\beta-1/2)^2 + V_{\text{eff}}^2}$$

Zoom in on Zero Quasimomentum Space (Orange)

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$V_{\text{eff}} = 0.1$

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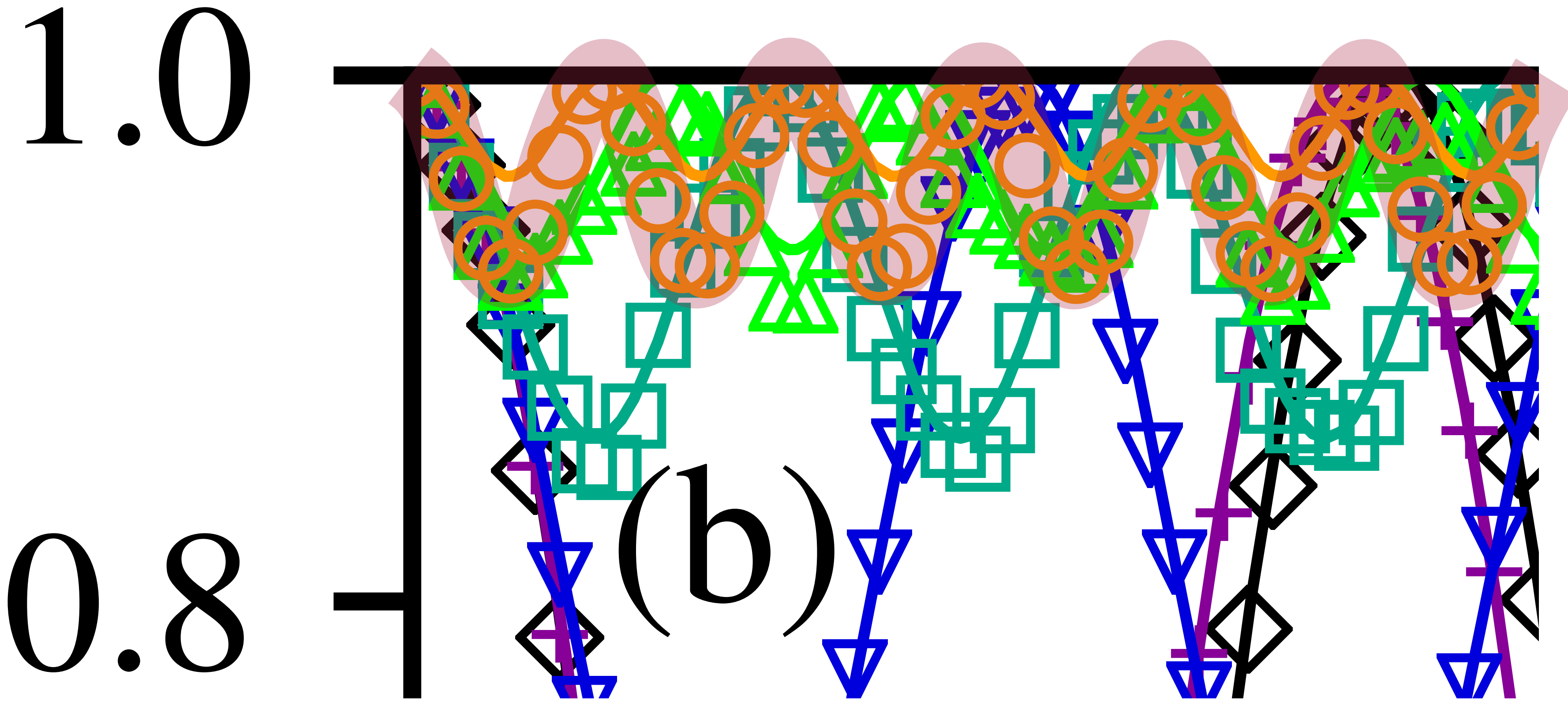
arXiv:1903.04011



Finite Temperatures

Beswick, Hughes, Gardiner
arXiv:1903.04011

- Consider (displaced) **Maxwell-Boltzmann distribution**

$$D_{k=0}(\beta, w) = \frac{1}{w\sqrt{2\pi}} \exp\left(\frac{-(\beta - 1/2)^2}{2w^2}\right)$$

- Population **remaining** in $k = 0$ band given by

$$P_0(w) = \int_0^1 D_{k=0}(\beta, w) P_0(\beta)\, \mathrm{d}\beta$$

- Can be determined from (for **sufficiently narrow** distribution)

$$P_0(\rho) = 1 - \frac{1}{\sqrt{2\pi}\rho} \int_{-\infty}^{\infty} \exp\left(\frac{-\gamma^2}{2\rho^2}\right) \frac{1}{\gamma^2 + 1} \sin^2\left(\frac{\sqrt{\gamma^2 + 1}}{2}\phi\right) \mathrm{d}\gamma$$

$$\gamma = (\beta - 1/2)/V_{\mathrm{eff}}, \quad \phi = V_{\mathrm{eff}}\tau, \quad \rho = w/V_{\mathrm{eff}}$$

Finite Temperature Analytics

Beswick, Hughes, Gardiner
arXiv:1903.04011

- **Steady state** given by

$$P_{0,\phi\to\infty}(\rho) = \frac{1}{2\rho}\sqrt{\frac{\pi}{2}}\exp\left(\frac{1}{2\rho^2}\right)\mathrm{Erfc}\left(\frac{1}{\sqrt{2}\rho}\right)$$

- Possible to determine

$$P_0(\rho) = 1 - \sum_{s=0}^{\infty}\sum_{q=0}^{s}\left\{\frac{(-\phi^2)^{s+1}s!}{[2(s+1)]!}\right\}\left\{\frac{-(2q)!}{[2(q!)^2(s-q)!]}\right\}\left\{\left(\frac{\rho^2}{2}\right)^q\right\}$$

$$= 1 - \sum_{q=0}^{\infty}\left(\frac{\rho}{2}\right)^{2q}\frac{(2q)!}{q!^2}\left\{\left(\frac{\phi}{2}\right)^{2(q+1)}\left[\left(\frac{2}{\phi}\right)\frac{\mathrm{d}}{\mathrm{d}(\phi/2)}\right]^q\left[\frac{\sin^2(\phi/2)}{(\phi/2)^2}\right]\right\}$$

- Collapses to **zero-temperature case** for $q=0$. Can in principle add $q\neq 0$ **low-temperature corrections**, although each such term **individually diverges** as $\phi\to\infty$ (long-time limit)

Finite Temperature and Decay to Steady State

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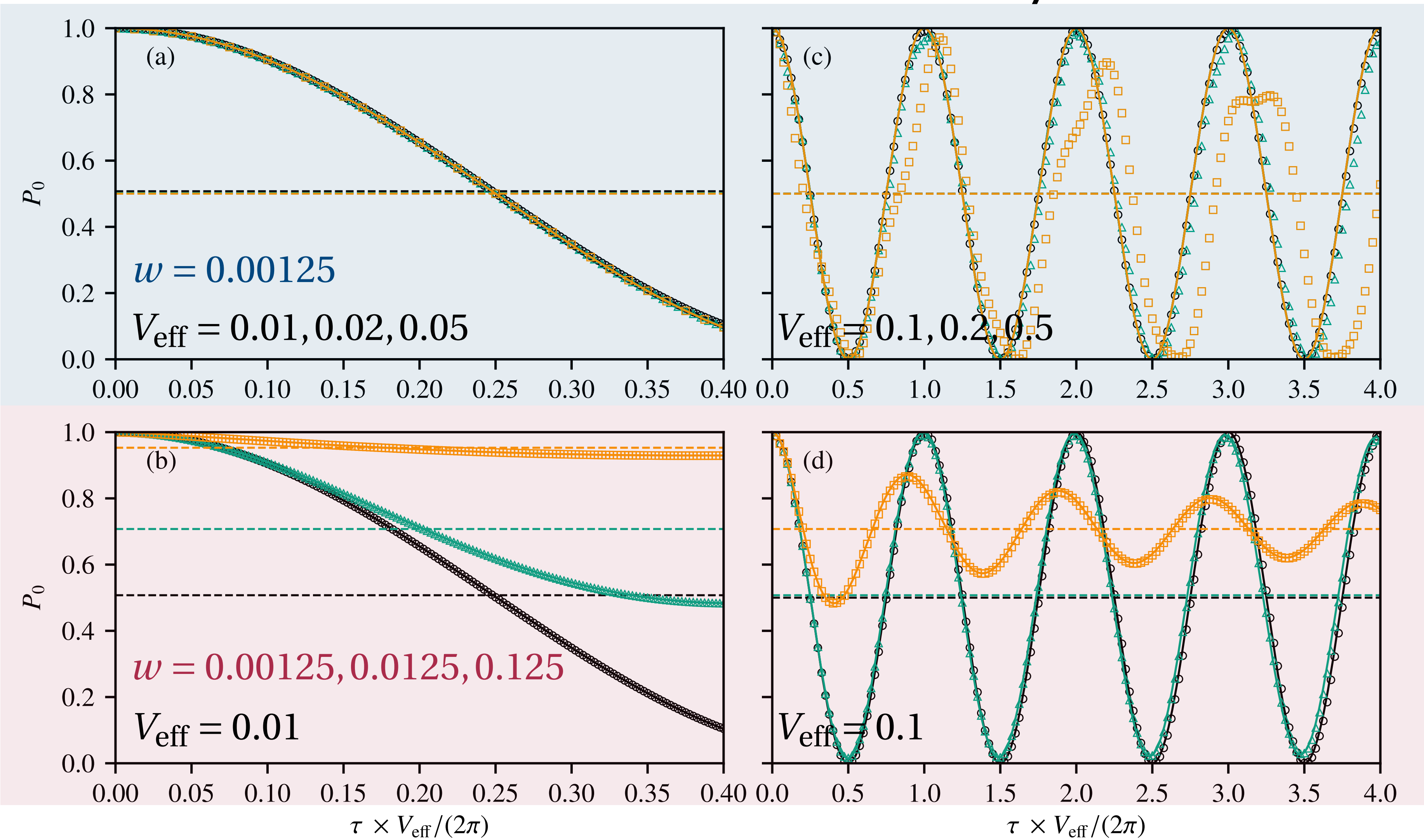
Dashed horizontal lines indicate steady state



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Finite Temperature (Pulsed, Momentum Zeroed)

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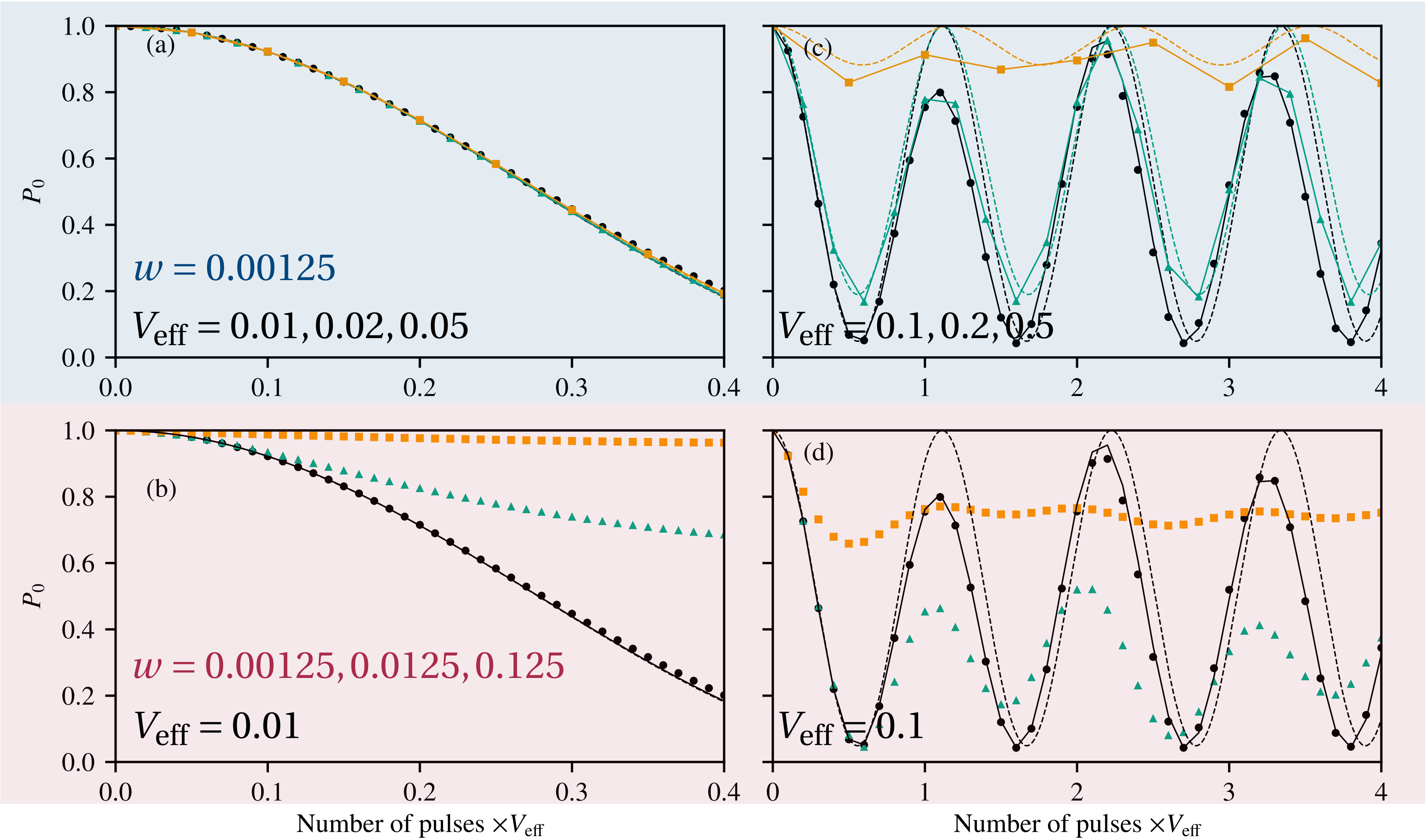

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Finite Temperature (Continuous, Momentum Offset)

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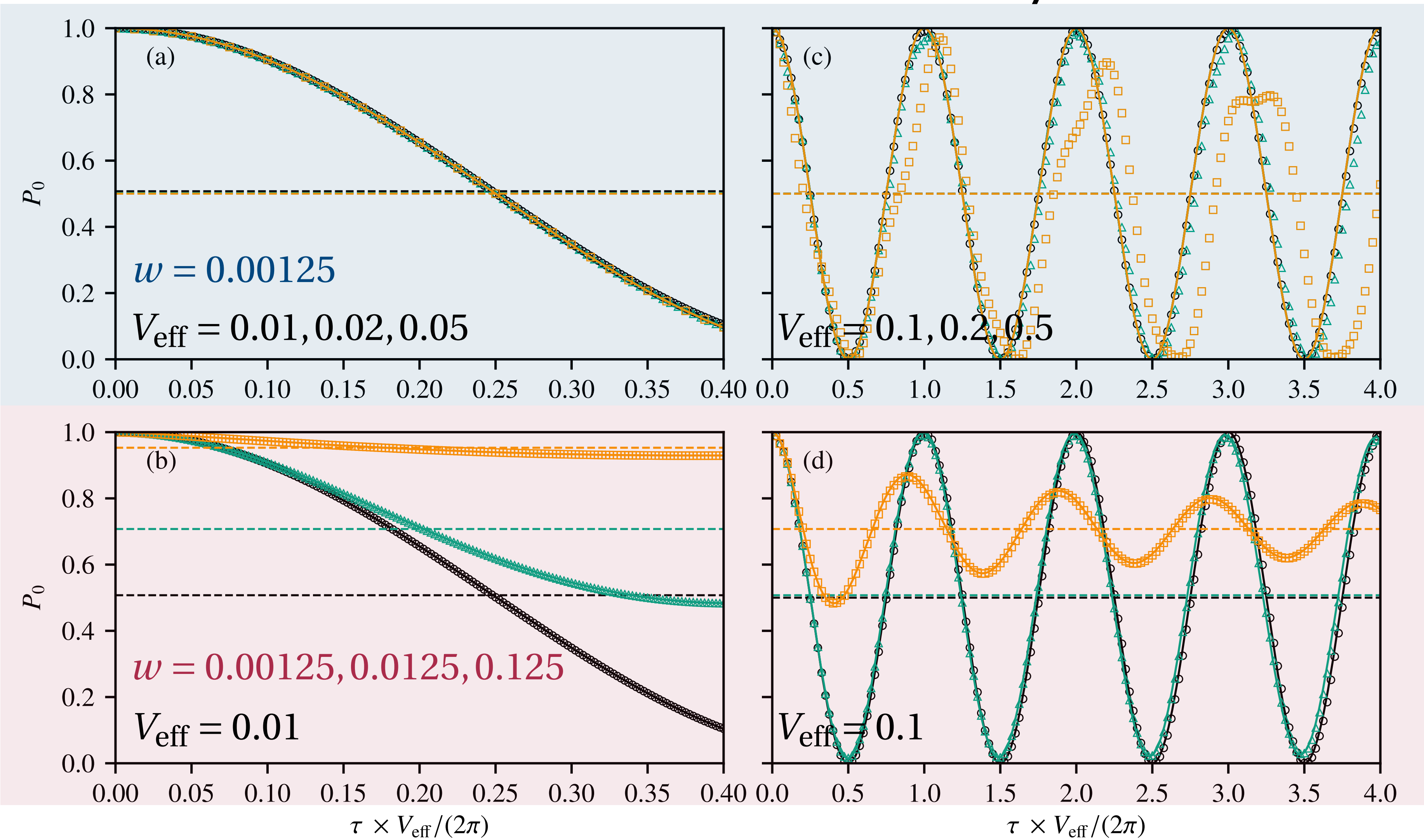
Dashed horizontal lines indicate steady state



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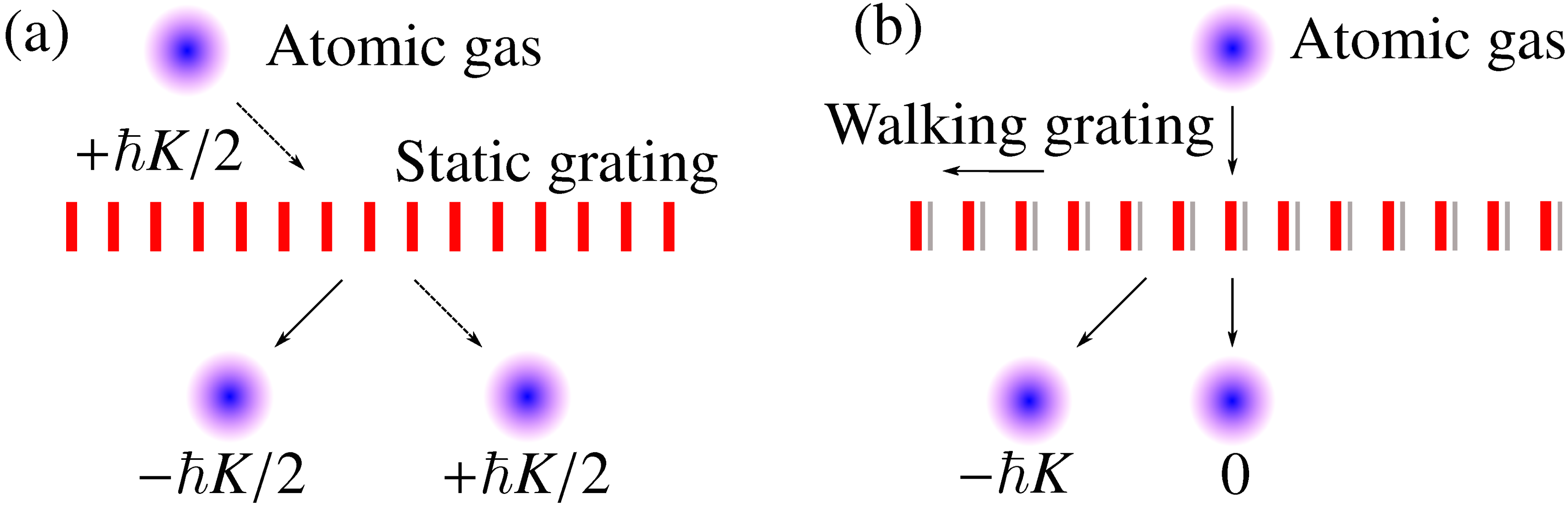
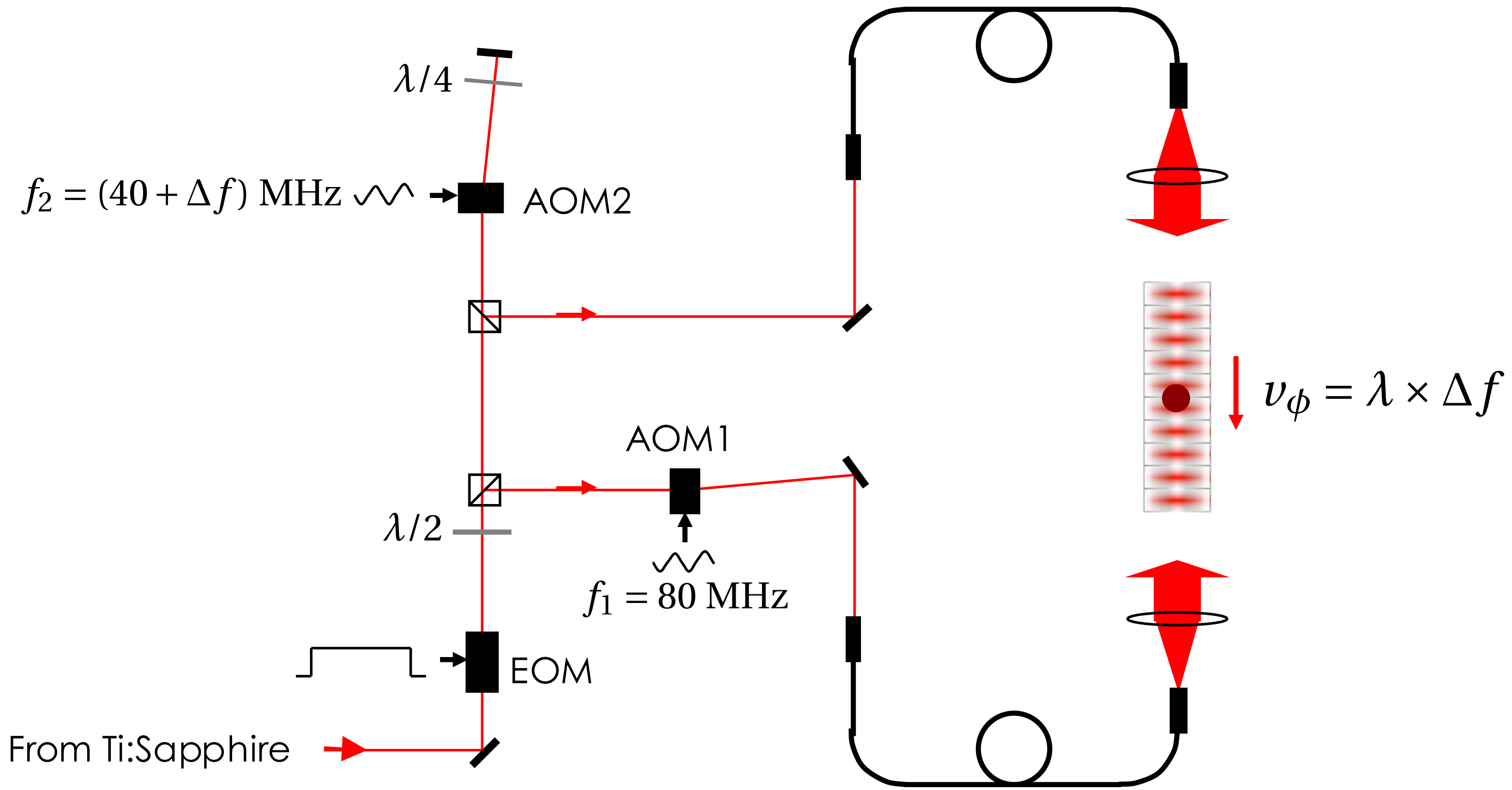
Equivalence to Static Gas and Walking Lattice



- Consider, in lab frame $\tilde{H}_{\mathrm{Latt}} = \dfrac{\hat{p}^2}{2M} - V\cos\left(K\left[\hat{x} + v_\phi t\right]\right)$
- If $v_\phi = \hbar K/2M$, then zero initial momentum in **lab frame** equivalent to $\hbar K/2$ initial momentum in **comoving frame** in which lattice **appears to be static**

Example: Switchable Walking Wave (Caesium)



- Walking wave controllable to near **arbitrary precision** via frequency difference between two acousto-optic modulators
- Such elements **frequently in place** for optical lattice experiments

Conclusion

- This does not seem too difficult!

Acknowledgements

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