A Simple Model

Three types of entropy evolution  $_{\rm OOOOO}$ 

Conclusion

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# Eternal life of entropy in non-Hermitian quantum systems

Thomas Frith

#### City, University of London

#### 7th International Workshop on New challenges in Quantum Mechanics: Integrability and Supersymmetry

Benasque

2nd September 2019

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A. Fring and T. Frith, Phys. Rev. A ,100, 010,102 (2019).

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# Outline

#### Introduction

- Non-Hermitian QM
- Von Neumann entropy
- A Simple Model
  - PT Symmetry
  - Calculation of the metric
- O Three types of entropy evolution
  - Eternal life of entropy
- Conclusions

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# Why Study Non-Hermitian Systems?

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## Why Study Non-Hermitian Systems?

What happens in quantum mechanics when our Hamiltonian is non-Hermitian,  $H^{\dagger} \neq H$ ?

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# Why Study Non-Hermitian Systems?

What happens in quantum mechanics when our Hamiltonian is non-Hermitian,  $H^{\dagger} \neq H$ ?

Real eigenvalues are still possible!

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## Why Study Non-Hermitian Systems?

What happens in quantum mechanics when our Hamiltonian is non-Hermitian,  $H^{\dagger} \neq H$ ?

Real eigenvalues are still possible!

Hamiltonians with PT-symmetry give real eigenvalues.

 $PT: p \rightarrow p, \quad x \rightarrow -x, \quad i \rightarrow -i$ 

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## Why Study Non-Hermitian Systems?

What happens in quantum mechanics when our Hamiltonian is non-Hermitian,  $H^{\dagger} \neq H$ ?

Real eigenvalues are still possible!

Hamiltonians with PT-symmetry give real eigenvalues.



Figure: Bender and Boettcher, 1998

$$H=p^2+(ix)^\epsilon$$

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$$h(t) = h^{\dagger}(t), \quad H(t) \neq H^{\dagger}(t)$$



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$$h(t) = h^{\dagger}(t), \quad H(t) \neq H^{\dagger}(t)$$

$$h(t)\phi(t) = i\hbar\partial_t\phi(t), \quad H(t)\Phi(t) = i\hbar\partial_t\Phi(t).$$

Time dependent Dyson operator

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$$\phi(t) = \eta(t)\Phi(t).$$

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Theory			

$$h(t) = h^{\dagger}(t), \quad H(t) \neq H^{\dagger}(t)$$

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Time dependent Dyson operator

$$\phi(t) = \eta(t) \Phi(t).$$

 $\implies$  Time dependent Dyson equation

$$h(t) = \eta(t) H(t) \eta(t)^{-1} + i\hbar \partial_t \eta(t) \eta(t)^{-1}$$

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Theory			

$$h(t) = h^{\dagger}(t), \quad H(t) \neq H^{\dagger}(t)$$

$$h(t)\phi(t) = i\hbar\partial_t\phi(t), \quad H(t)\Phi(t) = i\hbar\partial_t\Phi(t).$$

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 $\implies$  Time dependent quasi-Hermiticity relation

$$H^{\dagger}(t)\rho(t)-\rho(t)H(t)=i\hbar\partial_{t}\rho(t).$$

where  $\rho(t) = \eta^{\dagger}(t) \eta(t)$ 

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Observables			

Observables o(t) in the Hermitian system must be self-adjoint for real eigenvalues.



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Observables			

Observables o(t) in the Hermitian system must be self-adjoint for real eigenvalues. Observables O(t) in the non-Hermitian O(t) are quasi Hermitian

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Observables			

Observables o(t) in the Hermitian system must be self-adjoint for real eigenvalues.

Observables  $\mathcal{O}(t)$  in the non-Hermitian  $\mathcal{O}(t)$  are quasi Hermitian

$$o(t) = \eta(t)\mathcal{O}(t)\eta^{-1}(t).$$

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Observables			

Observables o(t) in the Hermitian system must be self-adjoint for real eigenvalues.

Observables  $\mathcal{O}(t)$  in the non-Hermitian  $\mathcal{O}(t)$  are quasi Hermitian

$$o(t) = \eta(t)\mathcal{O}(t)\eta^{-1}(t).$$

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Then we have

 $egin{aligned} &\langle \phi(t) \mid \! o(t) \phi(t) 
angle = \langle \Psi(t) \mid \! 
ho(t) \mathcal{O}(t) \Psi(t) 
angle \; . \ &
ho\left(t
ight) = \eta^{\dagger}\left(t
ight) \eta\left(t
ight) ext{ is vital!} \end{aligned}$ 

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# Von Neumann entropy

#### Hermitian

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# Von Neumann entropy

#### Hermitian

$$\varrho_{h} = \sum_{i} p_{i} \left| \phi_{i} \right\rangle \left\langle \phi_{i} \right|,$$

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# Von Neumann entropy

#### Hermitian

$$\varrho_h = \sum_i p_i \ket{\phi_i} \langle \phi_i |,$$

$$i\partial_t\varrho_h = \left[h, \varrho_h\right],$$

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# Von Neumann entropy

#### Hermitian

$$\varrho_{h} = \sum_{i} p_{i} \left| \phi_{i} \right\rangle \left\langle \phi_{i} \right|,$$

$$i\partial_t \varrho_h = [h, \varrho_h],$$

$$S_h = -tr \left[ \varrho_h \ln \varrho_h \right].$$

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# Von Neumann entropy

#### Hermitian

$$\varrho_{h} = \sum_{i} p_{i} |\phi_{i}\rangle \langle\phi_{i}|,$$

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Substitute time-dependent Dyson equation into von Neumann equation,

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# Von Neumann entropy

#### Hermitian

$$\varrho_{h} = \sum_{i} p_{i} |\phi_{i}\rangle \langle\phi_{i}|,$$
$$i\partial_{t}\varrho_{h} = [h, \varrho_{h}],$$

$$S_h = -tr \left[ \varrho_h \ln \varrho_h \right].$$

Substitute time-dependent Dyson equation into von Neumann equation,

$$h = \eta H \eta^{-1} + i\hbar \partial_t \eta \eta^{-1} \to i \partial_t \varrho_h = [h, \varrho_h].$$

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Von Neum	ann entropy		

$$\varrho_h = \eta \varrho_H \eta^{-1}$$



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Von Neur	nann entropy		

$$\varrho_h = \eta \varrho_H \eta^{-1}$$

#### Non-Hermitian

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Von Neum	nann entropy		

$$\varrho_h = \eta \varrho_H \eta^{-1}$$

### Non-Hermitian

$$\varrho_{H} = \sum_{i} p_{i} \left| \psi_{i} \right\rangle \left\langle \psi_{i} \right| \rho,$$

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$$\varrho_h = \eta \varrho_H \eta^{-1}$$

## Non-Hermitian

$$\varrho_{H} = \sum_{i} p_{i} |\psi_{i}\rangle \langle\psi_{i}|\rho,$$
$$i\partial_{t}\varrho_{H} = [H, \varrho_{H}],$$

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$$\varrho_h = \eta \varrho_H \eta^{-1}$$

## Non-Hermitian

$$\varrho_{H} = \sum_{i} p_{i} |\psi_{i}\rangle \langle\psi_{i}|\rho,$$
$$i\partial_{t}\varrho_{H} = [H, \varrho_{H}],$$
$$S_{H} = -tr [\varrho_{H} \ln \varrho_{H}].$$

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$$\varrho_h = \eta \varrho_H \eta^{-1}$$

## Non-Hermitian

$$\begin{split} \varrho_{H} &= \sum_{i} p_{i} |\psi_{i}\rangle \langle\psi_{i}|\,\rho, \\ i\partial_{t}\varrho_{H} &= \left[H, \varrho_{H}\right], \\ S_{H} &= -tr\left[\varrho_{H}\ln\varrho_{H}\right]. \end{split}$$

$$S_{H} = -\sum_{i} \lambda_{i} \ln \lambda_{i} = S_{h}.$$

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# A Simple Model

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# System/bath coupled harmonic oscillators

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## System/bath coupled harmonic oscillators

$$H = \nu a^{\dagger} a + \nu \sum_{n=1}^{N} q_n^{\dagger} q_n + (g + \kappa) a^{\dagger} \sum_{n=1}^{N} q_n + (g - \kappa) a \sum_{n=1}^{N} q_n^{\dagger},$$

u, g,  $\kappa \in \mathcal{R}$  are time-independent parameters.

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$$H = \nu a^{\dagger} a + \nu \sum_{n=1}^{N} q_n^{\dagger} q_n + (g + \kappa) a^{\dagger} \sum_{n=1}^{N} q_n + (g - \kappa) a \sum_{n=1}^{N} q_n^{\dagger},$$

 $\nu,\, {\it g},\, \kappa \in {\cal R}$  are time-independent parameters.

$\mathcal{PT}$ symmetry		

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$$H = \nu a^{\dagger} a + \nu \sum_{n=1}^{N} q_n^{\dagger} q_n + (g + \kappa) a^{\dagger} \sum_{n=1}^{N} q_n + (g - \kappa) a \sum_{n=1}^{N} q_n^{\dagger},$$

u, g,  $\kappa \in \mathcal{R}$  are time-independent parameters.

# $\mathcal{PT}$ symmetry $PT: i \rightarrow -i, a \rightarrow -a, a^{\dagger} \rightarrow -a^{\dagger}, q_n \rightarrow -q_n, q_n^{\dagger} \rightarrow -q_n^{\dagger}$

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$$H = \nu a^{\dagger} a + \nu \sum_{n=1}^{N} q_n^{\dagger} q_n + (g + \kappa) a^{\dagger} \sum_{n=1}^{N} q_n + (g - \kappa) a \sum_{n=1}^{N} q_n^{\dagger},$$

u, g,  $\kappa \in \mathcal{R}$  are time-independent parameters.

# $\mathcal{PT} \text{ symmetry}$ $PT: i \to -i, a \to -a, a^{\dagger} \to -a^{\dagger}, q_n \to -q_n, q_n^{\dagger} \to -q_n^{\dagger}$ $E_{m,N}^{\pm} = m \left( \nu \pm \sqrt{N} \sqrt{g^2 - \kappa^2} \right).$

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$$H = \nu a^{\dagger} a + \nu \sum_{n=1}^{N} q_n^{\dagger} q_n + (g + \kappa) a^{\dagger} \sum_{n=1}^{N} q_n + (g - \kappa) a \sum_{n=1}^{N} q_n^{\dagger},$$

 $\nu,\, {\it g},\, \kappa \in {\cal R}$  are time-independent parameters.

#### $\mathcal{PT}$ symmetry

$$PT: i \to -i, a \to -a, a^{\dagger} \to -a^{\dagger}, q_n \to -q_n, q_n^{\dagger} \to -q_n^{\dagger}$$
$$E_{m,N}^{\pm} = m \left( \nu \pm \sqrt{N} \sqrt{g^2 - \kappa^2} \right).$$

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 $\mathcal{PT}$  symmetry spontaneously broken when  $g<\kappa$ 

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# Calculating the metric

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# Calculating the metric

#### Generators

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# Calculating the metric

#### Generators

$$\begin{split} N_A &= a^{\dagger} a, \quad N_Q = \sum_{n=1}^N q_n^{\dagger} q_n, \quad N_{AQ} = N_A - \frac{1}{N} N_Q - \frac{1}{N} \sum_{n \neq m} q_n^{\dagger} q_m, \\ A_x &= \frac{1}{\sqrt{N}} \left( a^{\dagger} \sum_{n=1}^N q_n + a \sum_{n=1}^N q_n^{\dagger} \right), \quad A_y = \frac{i}{\sqrt{N}} \left( a^{\dagger} \sum_{n=1}^N q_n - a \sum_{n=1}^N q_n^{\dagger} \right). \end{split}$$

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# Calculating the metric

#### Generators

$$egin{aligned} &\mathcal{N}_A = a^\dagger a, \quad \mathcal{N}_Q = \sum_{n=1}^N q_n^\dagger q_n, \quad \mathcal{N}_{AQ} = \mathcal{N}_A - rac{1}{N} \mathcal{N}_Q - rac{1}{N} \sum_{n 
eq m} q_n^\dagger q_m, \ &\mathcal{A}_X = rac{1}{\sqrt{N}} \left( a^\dagger \sum_{n=1}^N q_n + a \sum_{n=1}^N q_n^\dagger 
ight), \quad \mathcal{A}_Y = rac{i}{\sqrt{N}} \left( a^\dagger \sum_{n=1}^N q_n - a \sum_{n=1}^N q_n^\dagger 
ight). \end{aligned}$$

#### Algebra

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# Calculating the metric

#### Generators

$$\begin{split} N_A &= a^{\dagger} a, \quad N_Q = \sum_{n=1}^N q_n^{\dagger} q_n, \quad N_{AQ} = N_A - \frac{1}{N} N_Q - \frac{1}{N} \sum_{n \neq m} q_n^{\dagger} q_m, \\ A_x &= \frac{1}{\sqrt{N}} \left( a^{\dagger} \sum_{n=1}^N q_n + a \sum_{n=1}^N q_n^{\dagger} \right), \quad A_y = \frac{i}{\sqrt{N}} \left( a^{\dagger} \sum_{n=1}^N q_n - a \sum_{n=1}^N q_n^{\dagger} \right). \end{split}$$

#### Algebra

$$[N_A, N_Q] = 0, \ [N_A, N_{AQ}] = 0, \ [N_A, A_x] = -iA_y, \ [N_A, A_y] = iA_y, [N_Q, A_x] = iA_y, \ [N_Q, A_y] = -iA_x, \ [N_{AQ}, A_x] = -2iA_y, \ [N_{AQ}, A_y] = 2iA_x.$$

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Calculating	g the metric		

$$H = \nu N_A + \nu N_Q + \sqrt{N}gA_x - i\sqrt{N}\kappa A_y.$$



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Calculatin	g the metric		

$$H = \nu N_A + \nu N_Q + \sqrt{N}gA_x - i\sqrt{N}\kappa A_y.$$

$$\rho\left(t\right) = \eta^{\dagger}\left(t\right)\eta\left(t\right)$$



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Calculating	the metric		

$$H = \nu N_A + \nu N_Q + \sqrt{N}gA_x - i\sqrt{N}\kappa A_y.$$

$$\rho\left(t\right) = \eta^{\dagger}\left(t\right)\eta\left(t\right)$$

$$h(t) = \eta(t) H \eta^{-1}(t) + i\hbar \partial_t \eta(t) \eta^{-1}(t)$$

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Calculatir	ig the metric		

$$H = \nu N_A + \nu N_Q + \sqrt{N}gA_x - i\sqrt{N}\kappa A_y.$$

$$\rho\left(t\right) = \eta^{\dagger}\left(t\right)\eta\left(t\right)$$

$$h(t) = \eta(t) H \eta^{-1}(t) + i\hbar \partial_t \eta(t) \eta^{-1}(t)$$

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Ansatz

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Calculating	g the metric		

$$H = \nu N_A + \nu N_Q + \sqrt{N}gA_x - i\sqrt{N}\kappa A_y.$$

$$\rho(t) = \eta^{\dagger}(t) \eta(t)$$

$$h(t) = \eta(t) H \eta^{-1}(t) + i\hbar \partial_t \eta(t) \eta^{-1}(t)$$

Ansatz

$$\eta(t) = e^{\beta(t)A_y} e^{\alpha(t)N_{AQ}},$$

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$$H = \nu N_A + \nu N_Q + \sqrt{N}gA_x - i\sqrt{N}\kappa A_y.$$

$$\rho(t) = \eta^{\dagger}(t) \eta(t)$$

$$h(t) = \eta(t) H \eta^{-1}(t) + i\hbar \partial_t \eta(t) \eta^{-1}(t)$$

Ansatz

С

$$\eta(t) = e^{\beta(t)A_y} e^{\alpha(t)N_{AQ}},$$

#### Coupled differential equations

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$$H = \nu N_A + \nu N_Q + \sqrt{N}gA_x - i\sqrt{N}\kappa A_y.$$

$$\rho\left(t\right) = \eta^{\dagger}\left(t\right)\eta\left(t\right)$$

$$h(t) = \eta(t) H \eta^{-1}(t) + i\hbar \partial_t \eta(t) \eta^{-1}(t)$$

Ansatz

С

$$\eta(t) = e^{\beta(t)A_y} e^{\alpha(t)N_{AQ}},$$

#### Coupled differential equations

$$\begin{split} \dot{\alpha} &= -\tanh\left(2\beta\right)\left[\sqrt{N}g\cosh\left(2\alpha\right) + \sqrt{N}\kappa\sinh\left(2\alpha\right)\right],\\ \dot{\beta} &= \sqrt{N}\kappa\cosh\left(2\alpha\right) + \sqrt{N}g\sinh\left(2\alpha\right). \end{split}$$

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# Calculating the metric

#### Solution for $\alpha$

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#### Solution for $\alpha$

$$\tanh\left(2\alpha\right) = \frac{-Ng\kappa + \dot{\beta}\sqrt{\dot{\beta}^2 + N\left(g^2 - \kappa^2\right)}}{Ng^2 + \dot{\beta}^2}$$

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#### Solution for $\alpha$

$$\tanh\left(2\alpha\right) = \frac{-Ng\kappa + \dot{\beta}\sqrt{\dot{\beta}^2 + N\left(g^2 - \kappa^2\right)}}{Ng^2 + \dot{\beta}^2}$$

#### Solution for $\beta$

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#### Solution for $\alpha$

$$anh(2lpha) = rac{-Ng\kappa + \doteta\sqrt{\doteta^2 + N\left(g^2 - \kappa^2
ight)}}{Ng^2 + \doteta^2}.$$

#### Solution for $\beta$

$$\ddot{eta}+2 anh(2eta)\left[Ng^2-N\kappa^2+\dot{eta}^2
ight]=0.$$

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#### Solution for $\alpha$

$$anh(2lpha) = rac{-Ng\kappa + \dot{eta}\sqrt{\dot{eta}^2 + N\left(g^2 - \kappa^2
ight)}}{Ng^2 + \dot{eta}^2}.$$

#### Solution for $\beta$

$$\ddot{\beta} + 2 \tanh(2\beta) \left[ Ng^2 - N\kappa^2 + \dot{\beta}^2 \right] = 0.$$
  
 $\sinh(2\beta) = \sigma$ 

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#### Solution for $\alpha$

$$\tanh(2\alpha) = \frac{-Ng\kappa + \dot{\beta}\sqrt{\dot{\beta}^2 + N(g^2 - \kappa^2)}}{Ng^2 + \dot{\beta}^2}$$

#### Solution for $\beta$

$$\ddot{\beta} + 2 \tanh (2\beta) \left[ Ng^2 - N\kappa^2 + \dot{\beta}^2 \right] = 0.$$
  
sinh  $(2\beta) = \sigma$   
 $\ddot{\sigma} + 4N \left( g^2 - \kappa^2 \right) \sigma = 0,$ 

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#### Solution for $\alpha$

$$\tanh(2\alpha) = \frac{-Ng\kappa + \dot{\beta}\sqrt{\dot{\beta}^2 + N(g^2 - \kappa^2)}}{Ng^2 + \dot{\beta}^2}.$$

#### Solution for $\beta$

$$\ddot{\beta} + 2 \tanh (2\beta) \left[ Ng^2 - N\kappa^2 + \dot{\beta}^2 \right] = 0.$$
  

$$\sinh (2\beta) = \sigma$$
  

$$\ddot{\sigma} + 4N \left( g^2 - \kappa^2 \right) \sigma = 0,$$
  

$$\sigma = \frac{c_1}{\sqrt{g^2 - \kappa^2}} \sin \left( 2\sqrt{N}\sqrt{g^2 - \kappa^2} \left( t + c_2 \right) \right),$$

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# Calculating the metric

#### Hermitian Hamiltonian

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#### Hermitian Hamiltonian

$$h(t) = \nu N_{A} + \nu N_{Q} + \mu(t) A_{x},$$

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#### Hermitian Hamiltonian

$$h(t) = \nu N_{A} + \nu N_{Q} + \mu(t) A_{x},$$

$$\mu(t) = \frac{(g^2 - \kappa^2)\sqrt{N}\sqrt{c_1^2 + g^2 - \kappa^2}}{c_1^2 + 2(g^2 - \kappa^2) - c_1^2\cos\left(4\sqrt{N}\sqrt{g^2 - \kappa^2}(t + c_2)\right)}.$$

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# Three types of entropy evolution

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# Von Neumann entropy

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# Von Neumann entropy

#### Initial State

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# Von Neumann entropy

#### Initial State

$$\left|\phi\left(0
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angle=\sin\gamma\left|1_{a}0_{q}
ight
angle+rac{\cos\gamma}{\sqrt{N}}\sum_{i=1}^{N}\left|0_{a}1_{i}
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angle,$$

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# Von Neumann entropy

#### Initial State

$$\left|\phi\left(0
ight)
ight
angle=\sin\gamma\left|1_{a}0_{q}
ight
angle+rac{\cos\gamma}{\sqrt{N}}\sum_{i=1}^{N}\left|0_{a}1_{i}
ight
angle,$$

#### State at time t

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# Von Neumann entropy

#### Initial State

$$\left|\phi\left(\mathbf{0}
ight)
ight
angle=\sin\gamma\left|\mathbf{1}_{a}\mathbf{0}_{q}
ight
angle+rac{\cos\gamma}{\sqrt{N}}\sum_{i=1}^{N}\left|\mathbf{0}_{a}\mathbf{1}_{i}
ight
angle,$$

#### State at time t

$$\begin{aligned} |\phi(t)\rangle &= e^{-i\nu t} \left( \sin\gamma \sin\mu_{I}(t) + \cos\gamma \cos\mu_{I}(t) \right) |1_{a}0_{q}\rangle \\ &+ \frac{e^{-i\nu t}}{\sqrt{N}} \left( \sin\gamma \cos\mu_{I}(t) - \cos\gamma \sin\mu_{I}(t) \right) \sum_{i=1}^{N} |0_{a}1_{i}\rangle \,. \end{aligned}$$

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# Von Neumann entropy

#### Initial State

$$\left|\phi\left(0
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ight
angle=\sin\gamma\left|1_{a}0_{q}
ight
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ight
angle,$$

#### State at time t

$$\begin{aligned} |\phi(t)\rangle &= e^{-i\nu t} \left( \sin\gamma \sin\mu_{I}(t) + \cos\gamma \cos\mu_{I}(t) \right) |1_{a}0_{q}\rangle \\ &+ \frac{e^{-i\nu t}}{\sqrt{N}} \left( \sin\gamma \cos\mu_{I}(t) - \cos\gamma \sin\mu_{I}(t) \right) \sum_{i=1}^{N} |0_{a}1_{i}\rangle \,. \end{aligned}$$

$$\mu_{I}(t) = \int^{t} \mu(s) ds = \frac{1}{2} \arctan\left(\frac{\sqrt{c_{1}^{2} + g^{2} - \kappa^{2}} \tan\left(2\sqrt{N}\sqrt{g^{2} - \kappa^{2}}(t + c_{2})\right)}{\sqrt{g^{2} - \kappa^{2}}}\right).$$
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## Von Neumann entropy

#### Reduced density matrix

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### Reduced density matrix

$$\varrho_{a}(t) = Tr_{q}[\varrho_{h}(t)] = \begin{pmatrix} (\sin\gamma\sin\mu_{I}(t) + \cos\gamma\cos\mu_{I}(t))^{2} & 0\\ 0 & (\sin\gamma\cos\mu_{I}(t) - \cos\gamma\sin\mu_{I}(t))^{2} \end{pmatrix}.$$

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#### Reduced density matrix

$$\varrho_{a}(t) = Tr_{q}[\varrho_{h}(t)] = \begin{pmatrix} (\sin\gamma\sin\mu_{I}(t) + \cos\gamma\cos\mu_{I}(t))^{2} & 0\\ 0 & (\sin\gamma\cos\mu_{I}(t) - \cos\gamma\sin\mu_{I}(t))^{2} \end{pmatrix}.$$

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### Eigenvalues

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#### Reduced density matrix

$$\varrho_{a}(t) = Tr_{q}[\varrho_{h}(t)] = \begin{pmatrix} (\sin\gamma\sin\mu_{I}(t) + \cos\gamma\cos\mu_{I}(t))^{2} & 0\\ 0 & (\sin\gamma\cos\mu_{I}(t) - \cos\gamma\sin\mu_{I}(t))^{2} \end{pmatrix}.$$

### Eigenvalues

$$\lambda_1(t) = (\sin \gamma \sin \mu_I(t) + \cos \gamma \cos \mu_I(t))^2,$$
  
$$\lambda_2(t) = (\sin \gamma \cos \mu_I(t) - \cos \gamma \sin \mu_I(t))^2,$$

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#### Reduced density matrix

$$\varrho_{a}(t) = Tr_{q}\left[\varrho_{h}(t)\right] = \begin{pmatrix} (\sin\gamma\sin\mu_{I}(t) + \cos\gamma\cos\mu_{I}(t))^{2} & 0\\ 0 & (\sin\gamma\cos\mu_{I}(t) - \cos\gamma\sin\mu_{I}(t))^{2} \end{pmatrix}.$$

#### Eigenvalues

$$\lambda_1(t) = (\sin \gamma \sin \mu_I(t) + \cos \gamma \cos \mu_I(t))^2,$$
  
$$\lambda_2(t) = (\sin \gamma \cos \mu_I(t) - \cos \gamma \sin \mu_I(t))^2,$$

 $S_{h,a}(t) = -\lambda_1(t) \log [\lambda_1(t)] - \lambda_2(t) \log [\lambda_2(t)] = S_{H,a}(t).$ 

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### Unbroken regime

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### Unbroken regime



Figure: Von Neumann entropy as a function of time and varied bath size, with  $c_1 = 1, g = 0.7, \kappa = 0.3$ 

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### Exceptional point



Figure: Von Neumann entropy as a function of time and varied bath size, with  $c_1=1,~g=\kappa$ 

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### Broken regime



Figure: Von Neumann entropy as a function of time and varied bath size, with  $c_1 = 1$ , g = 0.3,  $\kappa = 0.7$ . The asymptote is at  $S_{t\to\infty} \approx 0.3521$ 

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# Conclusion

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#### • Framework for Von Neumann entropy

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- Framework for Von Neumann entropy
- $\bullet\,$  Three different types of entropy evolution  $\to \mathcal{PT}$  regimes

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- Framework for Von Neumann entropy
- $\bullet\,$  Three different types of entropy evolution  $\to \mathcal{PT}$  regimes

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• In the broken regime, entropy decays to finite minimum

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- Framework for Von Neumann entropy
- $\bullet\,$  Three different types of entropy evolution  $\to \mathcal{PT}$  regimes

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- In the broken regime, entropy decays to finite minimum
- Quantum computing- maintaining entanglement

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- Framework for Von Neumann entropy
- $\bullet\,$  Three different types of entropy evolution  $\to \mathcal{PT}$  regimes

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- In the broken regime, entropy decays to finite minimum
- Quantum computing- maintaining entanglement
- Further models

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### Questions?

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