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Lagrangian Quantum Mechanics for indistinguishable fermions: a self-consistent universe.

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Abstract

This work corresponds to a paradigmatic classical mechanic approach to quantum mechanics and, as consequence, the paradigm of expanding universe is replaced for a universe of contracting particles which allows explaining the cosmological redshift because as the time progresses the hydrogen atoms absorb smaller wavelengths.

Quantum particles are defined as linearly independent indistinguishable normalized classical bi-spinor fields with quartic interactions, this matter allows defining positive energy spectra and to evade the problems with infinities associated to quantization procedure. To have a consistent particle interpretation in each inertial system, a large N approach for the number of fermions must be imposed.

The following model, based in dynamical mass generation methods, explains the quark confinement and the hadronic mass behavior in a trivial form and allows oscillation of low massive neutrinos inside of massive matter.

1 Introduction

In Quantum Mechanics a system is described by its wave function ψ, the equations that rule this wave function come from either Heisenberg's Matrix Mechanics [1] or Schrödinger's Equation [2], and from the initial condition of the wave function [3]. The wave function has a probabilistic interpretation, in which $\int_{\mathrm{V}} \psi_D^\dagger \psi_D d\mathrm{V} = P(\mathrm{V}, t)$ corresponds to the probability that when collapsing the particle does so inside of the volume V, though it can also be interpreted as the fraction of the particle that is in the volume V, thinking that the process of observation collapse the particle by through to imposes non-linear interactions.

This paper tries to break the paradigm of quantum fields theories by **evading the quantization scheme** and studying the **fermionic quanta** [4] that conform the matter as **indistinguishable classic spinorial material waves** that have distributions of charges and mass. The charge currents correspond to couplings between fermionic particles and the broken gauge symmetry fields, understanding the later **"a la" Maxwell's** [5].

The characteristic **discrete spectrum** of quantum mechanics is obtained demanding for the **normalization** of the classical fermions, and **Pauli's exclusion principle** comes from demand **linear independence** among the set of occupied classical fermions. In a Lagrangian formalism this condition looks like $\Delta L = \lambda_{ij}(\int_V \psi_i^\dagger \psi_j dV - \delta_{ij})$, whit λ_{ij} Lagrangian multipliers, but this term is not Lorentz invariant and finally it is necessary choose $\lambda_{ij} = 0$ to not break such symmetry. Thus, the normalization condition does not change the equation of motion, but discriminate among the algebraical solutions. The **large N approach**, allows have **a particle interpretation for each inertial reference frame**, because wave function density $\psi_i^\dagger \psi_i = j_i^0$ is a part of a four-vector j_i^μ, $\mu = 0, 1, 2, 3$, and then the normalization condition it is not Lorentz invariant, then if you work with a finite number of particles in an inertial reference frame, in some else reference system, the observers can not observe the same number of normalized particles, but if it is introduced an infinite number of particles (where a lot of them are in a **massless zero energy state**) the no normalized states in the target reference frame, can be understood as a lot of particles that condensate into the no normalized state. Obviously, for lower velocities, the Lorentz boost practically does not change the normalizations, thus, all observer in a laboratory see the same particles.

The **illusion of probabilistic collapse of particles** could be understood classically in analogy to the election of one numbered ball among a lot of different numbered balls in an opaque box, which is classically full defined by the initial conditions of the full system, but look like a random process if the observers do not know the initial conditions and can not calculate evolution for the election process. These are not occult variables, only **the observer is not aware of all** of them.

The classical analogous of QED and QCD, allow to describe **oscillation of asymptotically massless neutrinos inside of other matter** because the indistinguishability between fermions gives mass to neutrinos when these share space whit massive fermions, and the confinement of quarks as the condition that the color currents and masses be zero. The interaction of QCD sector with other interactions, change the later equilibrium condition, by a new equilibrium between interaction forces, which allows hadronic masses greater than the sum of isolated quark masses, due to existence of a color mass as consequence of the color currents that are necessaries inside of the hadrons to maintain the interaction forces in equilibrium.

Rewrite the laws of **quantum physics in classical field theory formalism** allows **the unification of particle physics and gravitation**, because the modern theories of gravitation connect the energy of a system with the curvature of space in Lagrangian form [6–11]. In this area, two phenomena that are still not well understood are: Dark Matter [12] and Dark Energy [13], which correspond to behaviours of the velocities of the objects that make up the galaxies very different from Newtonians that have led to think the existence of additional massive particles, which are not possible to observe through other non-gravitational interactions, and an energy that expands the universe in accelerated form, which has been associated with an energy of which we can not know the origin as a cosmological constant, between other models. **Gravity with fermions demands torsion** due to the spin density of fermions, as consequence, torsion deform the movement of particles in the space and it has already been shown how this can reproduce the movement for object in galaxies **explaining Dark matter** [14].

The theorem of **singularity of Penrose-Hawking** demand as condition for existence of singularities type black hole that **the physical system has a positive definite density of energy** [15]. What invites the search of a method of dynamic mass generation that provides only positive definite terms to the energy-momentum tensor of the system, squares of real numbers.

From the analysis of a Friedman-Lemaître-Robertson-Walker for a perfect fluid of fermions is shown an accelerated contraction of the space inside of matter, which invites to think that the universe is not expanding but the matter is contracted.

Some of most interesting contents in this work are:

- to construct a quantum theory that evades quantization procedures,
- to study a analogous for the Dirac equation with positive definite energies that describes both particles and anti-particles,
- to construct a system of interacting indistinguishable fermions, which describes pair creation-annihilation in classical form,
- construct a method of dynamical mass generation for each quantum classical analogue for Standard Model's fields, replacing the roll of Higgs particle [16] by fermionic condensates.
- to explain the oscillation of asymptotically massless fields (neutrinos) as result of the indistinguishability between the mass of such particles and the mass of the particles that they go through,
- to understand the hadronic masses and the confinement of quark as a equilibrium condition between all interaction forces inside of hadrons,
- to study a inverse inflationary point of view in which the matter contracts, allowing, over time, the universe to look larger with respect to the size of the matter.

In the Sec. 2, the duality between Hartree-Fock energies for contact interaction and the Lagrangian energies for classical fermionic quanta it is shown. Also it is studied a method of dynamical charge and mass generation that defines positive energies for fermions. In Sec. 3, the model is generalized for interacting indistinguishable fermions. In Sec. 4, is presented a quantum classical model analogous to Standard model, in which the roll of Higg's boson is replaced for fermionic condensates, observing neutrinos's oscillation in matter and quark confinement.

In Sec. 5, it is introduced the gravitational formalism for fermions in curved space-time, and as consequence of the variable masses in the model it is shown, in Sec. 6, how a contraction of matter arises.

2 Contact interaction and QM-CM duality

For a system of quantum particles the Hartree-Fock approach allows to define the quantum mechanical wave function for N particles in the form

$$\psi = \psi_1(\vec{x}_1, t)\psi_2(\vec{x}_2, t)\cdots\psi_N(\vec{x}_N, t) \tag{1}$$

with the normalization condition

$$\langle\psi_i|\psi_j\rangle = \int_V \psi_i^\dagger(\vec{x}, t)\psi_j(\vec{x}, t)dV = \delta_{ij}, \tag{2}$$

for $i, j = 1, \ldots, N$, where $\vec{x}_i$ are the coordinates of the particle i and $\vec{x}$ are and auxiliary coordinate system. Via canonical quantization approach it is possible to define a Hamiltonian for N particles in the form

$$H_0 = \sum_{i=1}^{N} \hat{H}_i(\vec{x}_i, t) + \frac{1}{2}\sum_{i<j}^{N} V_{ij}(\vec{x}_i, \vec{x}_j, t) \tag{3}$$

and also, if are introduced pure quantum auto-interacting potentials is possible ensures the indistinguishability of the fermions

$$H = \sum_{i=1}^{N} \left(\hat{H}_i(\vec{x}_i, t) + \frac{1}{2}\psi_i^\dagger(\vec{x}_i, t) V_{ii}(\vec{x}_i, t)\psi_i(\vec{x}_i, t)\gamma^0 \right) + \frac{1}{2}\sum_{i<j}^{N} V_{ij}(\vec{x}_i, \vec{x}_j, t). \tag{4}$$

For fermions $H_i = -i\hbar c\gamma^0\gamma^a\partial_{x_i^a}$, where γ^μ are the Dirac gamma matrices $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{I}_4$, $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, $\mu = 0, 1, 2, 3$, $a = 1, 2, 3$. If it is demanded that the interaction of fermion is by contact, then Lorentz invariant fermion-fermion interaction take the form $V_{ij} = g\gamma^0 \otimes \gamma^0\delta(\vec{x}_i - \vec{x}_j)$ and $V_{ii} = g\gamma^0$, g a positive coupling constant, then the Hartree-Fock energy for the system of particles, $E_{\mathrm{HF}} = \langle\psi H\psi\rangle$, takes the form

$$E_{\mathrm{HF}} = \sum_{i=1}^{N} \int_V \psi_i^\dagger(\vec{x}, t)\hat{\vec{v}} \cdot \hat{\vec{P}}\psi_i(\vec{x}, t)\mathrm{d}V + \frac{g}{2}\int_V \left(\sum_{i=1}^{N} \bar{\psi}_j(\vec{x}, t)\psi_i^\dagger(\vec{x}, t) \right)^2 \mathrm{d}V, \tag{5}$$

where $\bar{\psi} = \psi^\dagger\gamma^0$ and $\hat{v}^a = \dot{\hat{x}}_i^a = c\gamma^0\gamma^a$.

To optimize the Hartree-Fock energy, each isolated particle must be defined by the variational principle

$$\frac{\delta\Delta E[\psi, \lambda_{ij}]}{\delta\psi_i^\dagger(\vec{x}_i)} = 0, \quad \Delta E[\psi] = i\hbar\langle\psi\partial_t\psi\rangle - \langle\psi H\psi\rangle + \sum_{i,j=1}^{N} \lambda_{ij}(\langle\psi_i|\psi_j\rangle - \delta_{ij}), \tag{6}$$

that gives the equations of motion

$$i\hbar\partial_t\psi_i = \hat{\vec{v}} \cdot \hat{\vec{P}}\psi_i + g\left(\sum_{j=1}^{N} \bar{\psi}_j\psi_j \right)\gamma^0\psi_i, \tag{7}$$

where to ensures the Pauli's exclusion principle the linear independence of the fermions has been added by through the variational principle

$$\frac{\delta E[\psi, \lambda_{ij}]}{\delta\lambda_{ij}} = (\langle\psi_i|\psi_j\rangle - \delta_{ij}) = 0, \tag{8}$$

here λ_{ij} are lagrangian multipliers and have being chosen $\lambda_{ij} = 0$ to ensures Lorentz covariance of equation of motion, behind of this election is the necessity of add a set of Lagrangian multipliers that demand normalization conditions for each inertial reference frame because the normalization condition is not Lorentz invariant. Thus, to be able to construct a particle picture in each reference frame, it is necessary introduce a large N number of fermions so that normalized states that transform to no normalized ones could be interpreted in the transformed inertial reference frame as condensates of a set of normalized particles. The energy of each isolated particle is

$$\epsilon_i(t) = \int_V \left(\psi_i^\dagger\hat{\vec{v}} \cdot \hat{\vec{P}}\psi_i + \bar{\psi}_i\psi_i\sum_{j=1}^{N}\bar{\psi}_j\psi_j \right)\mathrm{d}V, \tag{9}$$

but the total energy is not the addition of the isolated energies because the energy of the interaction between particles must be added one time, then the Hartree Fock energy of the system of particles is

$$E_{\mathrm{HF}} = \langle H\rangle = \sum_i \epsilon_i(t) - \Delta E[\psi]. \tag{10}$$

In the next is exposed a model of dynamical mass generation for fermions that reproduces in lagrangian form the later Hartree-Fock formalism.

The presented model allows reject the canonical quantization procedure and study fermions as classical spinorial fields where ΔE takes the role of Lagrangian. Thus, Yang Mills interactions and Einstein-Cartan theory can be added in to the large N Lagrangian approach for fermions, allowing to define a kind of Grand Unification Theory.

Dirac sea: a static solution ψ_0 defined by $\psi_0^\dagger\psi_0 = \rho_0 = \text{cte}$ and $\bar{\psi}_0\psi_0 = 0$ is a no energetic solution to Hartree-Fock selff-consitence equations [7] and corresponds to an analogous to the Dirac sea that defines a density of particles without mass (nor charge) as the vacuum.

2.1 Dirac equation

From Dirac's work [4] it is possible to formulate the following factorization of the Klein-Gordon equation:

$$\left(-\hbar^2c^2\partial_\mu\partial^\mu + m^2c^4\right)\mathbb{I}_\gamma\Psi = \left(i\hbar c\gamma^\mu\partial_\mu + mc^2\right)\left(i\hbar c\gamma^\mu\partial_\mu - mc^2\right)\Psi = 0 \tag{11}$$

where, $x^\mu = (ct, \vec{x})$ are the four coordinates, Ψ is a 2-spinor and $\gamma^0 = \sigma_3 \otimes \mathbb{I}_2$, $\gamma^j = i\sigma_2 \otimes \sigma_j$ and $\gamma^5 = \sigma_1 \otimes \mathbb{I}_2$ are the Dirac's matrices, $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{I}_4$, It can be differentiated between the states annihilated only by either $(i\hbar c\gamma^\mu\partial_\mu + mc^2)$ or $(i\hbar c\gamma^\mu\partial_\mu - mc^2)$, or only after the application of both of them.

Fermions will be called the states ψ_D^+ satisfying

$$\left(i\hbar c\gamma^\mu\partial_\mu - mc^2\right)\psi_D^+ = 0, \tag{12}$$

and anti-fermions the ψ_D^- states satisfying

$$\left(i\hbar c\gamma^\mu\partial_\mu + mc^2\right)\psi_D^- = 0, \tag{13}$$

these equations describe free particles of spin $\frac{\hbar}{2}$ and mass m.

For each fermion, the continuity equation

$$\partial_\mu\left(\bar{\psi}_D^\pm\gamma^\mu\psi_D^\pm\right) = 0 \tag{14}$$

is satisfied, which allows to define $|\mathcal{N}|^2 = \int_V (\psi_D^\pm)^\dagger\psi_D^\pm dV$ as a conserved quantity, $\partial_0|\mathcal{N}|^2 = 0$, where V is the volume in which the particle exists. This conserved quantity is finite under physical boundary conditions, for instance: if the current $\vec{J}$ and its derivatives tend to zero at infinity, or if the particle there exist, e.g., in a sphere or a box. It is evaded the study of Klein-Gordon bosons because they can not be probabilistically described.

The later continuity equation is also behind the conservation and quantization of the total charge in the case of electromagnetism. To maintain the charge conserved, it is necessary to demand

$$Q = Q\int_{V(\psi)}\psi^\dagger\psi dV \tag{15}$$

for any physical fermion ψ, where $V(\psi_D)$ is some volume in which the free particle it is distributed such that the total charge of the plane wave is Q.

But when analyzing the Dirac equation some problems are found. It has problem to describes the phenomenon known as pair's creation-anihilation, defines negative energy densities for anti particles and present an Dirac sea of negative infinite energy and charge [21]. The later does not allow to define gravitational matter in order to fill the Penrose-Hawking singularity theorem.

2.2 Dirac equation from a principle of least action and dynamical mass generation.

From the principle of stationary action it is possible to obtain the dynamic of a Dirac equation through the following classical Lagrangian systems

$$\mathcal{L}_D^{\pm} = \frac{i\hbar c}{2}(\bar{\psi}_D^{\pm}\gamma^{\mu}\partial_{\mu}\psi_D^{\pm} - \partial_{\mu}\bar{\psi}_D^{\pm}\gamma^{\mu}\psi_D^{\pm}) \mp mc^2\bar{\psi}_D^{\pm}\psi_D^{\pm} \tag{16}$$

evading quantization procedure.

The equation of motion describe positive and negative densities of energy

$$\frac{1}{2}\left(\psi_D^{\pm\dagger}\hat{E}\psi_D^{\pm} - (\hat{E}\psi_D^{\pm\dagger})\psi_D^{\pm}\right) = \pm mc^2\psi_D^{\pm\dagger}\gamma^0\psi_D^{\pm} + \frac{1}{2}\left(\psi_D^{\pm\dagger}\hat{\vec{v}}\cdot\hat{\vec{P}}\psi_D^{\pm} - (\hat{\vec{P}}\psi_D^{\pm\dagger})\cdot\hat{\vec{v}}\psi_D^{\pm}\right), \tag{17}$$

as γ^0 has eigenvalues ± 1 then both particles and antiparticles have solutions with positive and negative energy densities, where $\psi_D^{\pm\dagger}\psi_D^{\pm}$ is a distribution of a conserved quantity in space. Note that

$$\frac{1}{2}\left(\psi_D^{\pm\dagger}\hat{E}^2\psi_D^{\pm} + (\hat{E}^2\psi_D^{\pm\dagger})\psi_D^{\pm}\right) = m^2c^4\psi_D^{\pm\dagger}\psi_D^{\pm} + c^2\frac{1}{2}\left(\psi_D^{\pm\dagger}\hat{\vec{P}}^2\psi_D^{\pm} + (\hat{\vec{P}}^2\psi_D^{\pm\dagger})\psi_D^{\pm}\right). \tag{18}$$

To explain the mass gap and define a positive energy density a dynamical method of mass generation [22–24] it is proposed. The method generates a positive variable Lagranian masses for free particles and antiparticles, but positive and negative Dirac masses. The later will be behind of the possibility of describes in Lagrangian form the pair creation-annihilation.

The lagrangian for Dirac's particles with dynamical mass generation takes the form

$$\mathcal{L} = \frac{i\hbar c}{2}(\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \partial_{\mu}\bar{\psi}\gamma^{\mu}\psi) - \frac{g}{2}(\bar{\psi}\psi)^2, \tag{19}$$

which describes a dynamical mass for fermions

$$m_D(x)c^2 = g\bar{\psi}\psi \tag{20}$$

and then, the equation of motion for the fermion is

$$\left(i\hbar c\partial_{\mu}\gamma^{\mu} - g\bar{\psi}\psi\right)\psi = 0, \tag{21}$$

which only describes positive densities of energy

$$\frac{1}{2}\left(\psi^{\dagger}\hat{E}\psi - (\hat{E}\psi^{\dagger})\psi\right) = g(\psi^{\dagger}\gamma^0\psi)^2 + \frac{1}{2}\left(\psi^{\dagger}\hat{\vec{v}}\cdot\hat{\vec{P}}\psi - (\hat{\vec{P}}\psi^{\dagger})\cdot\hat{\vec{v}}\psi\right), \tag{22}$$

because velocity in special relativity is related with the momentum and energy in the form $v = \frac{pc^2}{E}$ and then $\psi^{\dagger}\hat{\vec{v}}\cdot\hat{\vec{P}}\psi$ has the same sign that $\psi^{\dagger}\hat{E}\psi$, $\hat{P}^i = -i\hbar\partial_i$, $\hat{v}^i = c\gamma^0\gamma^i$, also, the modulus of the energy grow with the momentum

$$\frac{1}{2}\left(\psi^{\dagger}\hat{E}^2\psi + (\hat{E}^2\psi^{\dagger})\psi\right) = g^2(\psi^{\dagger}\gamma^0\psi)^2\psi^{\dagger}\psi + \frac{1}{2}\left(\psi^{\dagger}\hat{\vec{P}}^2\psi + (\hat{\vec{P}}^2\psi^{\dagger})\psi\right), \tag{23}$$

and then, due to the positive definite mass in Eq. [22] the energies are positive definite.

For a single fermion, the mass must be identical to the fermion mass, thus, for at rest fermion the constant coupling is fixed in the form

$$g = mc^2V_0 \tag{24}$$

where V_0 is the volume of a rest fermion and m is its Dirac mass.

The plane wave solution are the same that the set of positive energy plane waves of Dirac's particles (12) and antiparticles (13), because for Dirac's plane waves

$$\bar{\psi}^{+}_{D\pm}\psi^{+}_{D\pm} = \pm\frac{1}{V_0}, \quad \bar{\psi}^{-}_{D\mp}\psi^{-}_{D\mp} = \pm\frac{1}{V_0}, \tag{25}$$

where sub-index $\pm$ corresponds to the sign of the energy of the plane wave, see Appendix [7]. For a single plane waves the consistence equation of fermion corresponds to reproduce the Dirac mass from the mass condensate $m_D(x) = m$ and for antifermions $m_D(x) = -m$, as consequence of (25), only the solutions with sub-index $+$ are solutions of non linear Dirac's equation, and then no negative energy stationary solution exist.

Note that particles and antiparticles in the presented model are solutions of the same equation, and then those are different phases of the same fermionic matter.

Most general solutions corresponds to variable mass density condensate $\rho_M(x) = mV_0(\bar{\psi}\psi)^2$. For variable Dirac mass $m_D(x) = mV_0\bar{\psi}\psi$ to be a physical solution is necessary demand that the particle that solves the system respect the normalization condition.

In this non-linear formalism, it is necessary to define the following electromagnetic current

$$J^{\mu}_{Q} = QV_0\bar{\psi}\psi \cdot \bar{\psi}\gamma^{\mu}\psi, \tag{26}$$

which for plane waves takes the form $J^{\mu}_{Q} = Q\bar{\psi}\gamma^{\mu}\psi$ for particles and $J^{\mu}_{Q} = -Q\bar{\psi}\gamma^{\mu}\psi$ for antiparticles.

The 00 component of the energy-momentum tensor for this auto-interacting single particle reproduces the Hartree-Fock energy, $\int_V T^{00}dV = \epsilon - \frac{g}{2}\int_V(\bar{\psi}\psi)^2dV$, $\epsilon = \int_V \frac{i\hbar}{2}(\psi^{\dagger}\partial_t\psi - (\partial_t\psi^{\dagger})\psi)dV$. To ensures the Pauli's exclusion principle we can add via lagrangian multipliers the conditions of normalization and linear independence of the fermions states. But to maintain the Lorentz invariance of the equation of motion it is necessary choose all lagrangian multipliers zero, and to allow the particle interpretation in al the frame it is necessary a system with an infinite number of particles, $N \to \infty$, to can in each inertial coordinate system define a set of particles supporting a particle interpretation.

3 System of indistinguishable interacting fermions

Fermions are indistinguishable particles, then a theory of N fermions must be global $U(N)$ invariant, due to all possible changes of the base that define the occupied states, {occ}. Taking this in count, it is possible to define the following lagrangian

$$\mathcal{L} = \sum_{\text{occ}} \frac{i\hbar c}{2}(\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \partial_{\mu}\bar{\psi}\gamma^{\mu}\psi) - \frac{g}{2}\left(\sum_{\text{occ}}\bar{\psi}\psi\right)^2 \tag{27}$$

which together to a Yang Mills field allows a consistent channel for pair creation-annihilation, that conserve $\int_V T^{0\mu}dV$ in the process. Adding a gauge field via Yang-Mills theory [25–27] to (27) it can be defined the lagrangian:

$$\mathcal{L}_T = \mathcal{L} + \sum_a A^a_{\mu}J^{a\mu} + \mathcal{L}_{YM}, \tag{28}$$

where $\mathcal{L}_{YM} = -\frac{1}{4\mu_0}\text{Tr}[F_{\mu\nu}F^{\mu\nu}]$, $F_{\mu\nu} = \frac{i}{q}\left[\partial_{\mu} - iq\tau^a A^a_{\mu}, \partial_{\nu} - iq\tau^a A^a_{\nu}\right]$, is the Yang-Mills lagrangian, τ^a are the generators of the broken symmetry group, $a = 1,\ldots,l$ with l the number of generators of gauge group, and $J^{a\mu}$ are the 4-current that generates the gauge fields, which take the form:

$$J^{a\mu} = QV_0\left(\sum_{\text{occ}}\bar{\psi}\psi\right)\left(\sum_{\text{occ}}\bar{\psi}\tau^a\otimes\gamma^{\mu}\psi\right) \tag{29}$$

and a chiral version

$$J^{a\mu}_{\mathrm{Ch}} = Q_{\mathrm{Ch}} V_0 \left(\sum_{\mathrm{occ}} \bar{\psi}\gamma^5\psi \right) \left(\sum_{\mathrm{occ}} \bar{\psi}\tau^a \otimes \gamma^\mu\gamma^5\psi \right). \tag{30}$$

For electromagnetism the current is conservative, $\tau = 1$, i.e., $\partial_\mu J^\mu = 0$, and implies the conservation of the electric charge $Q_{\mathrm{T}} = \int_V J^0 d\mathrm{V} = Q(N^+ - N^-) = zQ$, where $z \in \mathbb{Z}$, that corresponds to the charge quantization, and is given by the charge of the particles plus the antiparticles.

For a vacuum solution ψ_0, $\bar{\psi}_0\psi_0 = 0$, $\psi_0^\dagger\psi_0 = \rho_0$, immediately the total charge is zero, evading the problems of infinite energy and charge associated to the Dirac sea.

The Lagrangian (28) gives the following equations of motion: for fermions

$$\left(i\hbar c\gamma^\mu\partial_\mu + QV_0 A^a_\mu \left(\sum_{\mathrm{occ}} \bar{\psi}\psi \right) \tau^a \otimes \gamma^\mu - mc^2 V_0 \sum_{\mathrm{occ}} \bar{\psi}\psi + QV_0 A^a_\mu \sum_{\mathrm{occ}} \bar{\psi}\tau^a \otimes \gamma^\mu\psi \right) \psi = 0, \tag{31}$$

and for gauge fields

$$\partial^\mu F^a_{\mu\nu} + q f^{abc} A^{b\mu} F^c_{\mu\nu} = \mu_0 J^a_\nu. \tag{32}$$

where $F^a_{\mu\nu}\tau^a = F_{\mu\nu}$.

The energy of the fermion it is described by the identity

$$\psi^\dagger \hat{E}\psi = g(\psi^\dagger\gamma^0\psi)^2 + \psi^\dagger\hat{\vec{v}} \cdot \hat{\vec{P}}\psi - 2QV_0 \left(\sum_{\mathrm{occ}} \bar{\psi}\psi \right) \left(\sum_{\mathrm{occ}} \bar{\psi}\tau^a \otimes \gamma^\mu\psi \right) A^a_\mu, \tag{33}$$

thus, the broken gauge symmetry fields A^a_μ affect the energy of particles adding a no necessary positive term. To maintain the mass condensate independent of the broken gauge symmetry fields, it is possible choose a condition for gauge symmetry fields inside of fermionic matter:

$$A^a_\mu \sum_{\mathrm{occ}} \bar{\psi}\tau^a \otimes \gamma^\mu\psi = 0 \tag{34}$$

and a chiral version

$$A^a_\mu \left(Q \sum_{\mathrm{occ}} \bar{\psi}\psi \sum_{\mathrm{occ}} \bar{\psi}\tau^a \otimes \gamma^\mu\psi + Q_{\mathrm{Ch}} \sum_{\mathrm{occ}} \bar{\psi}\gamma^5\psi \sum_{\mathrm{occ}} \bar{\psi}\tau^a \otimes \gamma^\mu\gamma^5\psi \right) = 0 \tag{35}$$

the gauge condition (35) fixes the potential vector perpendicular to the particle's current density. This condition arises in natural form if they are understood the charges as Lagrangian multipliers.

Those gauge condition fix positive the energy density [33], which takes the form:

$$\psi^\dagger \hat{E}\psi = g\left(\psi^\dagger\gamma^0\psi\right)^2 + \psi^\dagger\hat{\vec{v}} \cdot \hat{\vec{P}}\psi \tag{36}$$

and reduces the equation of motion to

$$\left(i\hbar c\gamma^\mu\partial_\mu + QV_0 A^a_\mu \left(\sum_{\mathrm{occ}} \bar{\psi}\psi \right) \tau^a \otimes \gamma^\mu - mc^2 V_0 \sum_{\mathrm{occ}} \bar{\psi}\psi \right) \psi = 0, \tag{37}$$

which look as a usual Charged Dirac equation but with the charge and mass changed by non homogeneous distribution $Q(x) = QV_0 \sum_{\mathrm{occ}} \bar{\psi}\psi$ and $m(x) = mV_0 \sum_{\mathrm{occ}} \bar{\psi}\psi$ respectively.

When particles are orbited by interacting pairs $\psi^+\psi^-$, this last are confined by the interaction generates by the orbited particles, as in a quantum potential well.

In order to study the capability of this system to generate interaction between particles and antiparticles the conservation laws must be checked, which fix the coupling constant μ_0 between gauge fields and fermions.

For this theory the total Belinfante-Rosenfeld energy-momentum tensor [28] is:

$$T^{\mu\nu} = T^{\mu\nu}_{m=0} + T^{\mu\nu}_{\psi} + T^{\mu\nu}_{\text{em}} \tag{38}$$

$$T^{\mu\nu}_{m} = \frac{i\hbar c}{4}\sum_{\text{occ}}(\bar{\psi}\gamma^{\mu}\partial^{\nu}\psi - (\partial^{\nu}\bar{\psi})\gamma^{\mu}\psi) + (\mu \leftrightarrow \nu) \tag{39}$$

$$T^{\mu\nu}_{\psi} = -\eta^{\mu\nu}\frac{g}{2}\left(\sum_{occ}\bar{\psi}\psi\right)^2 \tag{40}$$

$$T^{\mu\nu}_{\text{Gauge}} = -\frac{1}{\mu_0}\text{Tr}\left[F^{\mu\alpha}F^{\nu}{}_{\alpha}\right] + \frac{1}{4\mu_0}\eta^{\mu\nu}\text{Tr}\left[F^{\alpha\beta}F_{\alpha\beta}\right]. \tag{41}$$

The set of states occ it is fixed by a self-consistent set of states of positive energy and an infinite set of pairs of particles that annihilate between them accomplish the role of Dirac sea. The energy momentum tensor reproduces the Hartree-Fock energy and due that the dynamic of wave function is definite by the non-linear Dirac equation and the normalization condition, it is possible to ensures the quantum spectrum of stationary particles, because normalization conditions discriminates among all algebraical solutions.

With the electromagnetic formalism of the present theory, it is possible find a consistent annihilation process because for separated at rest fermion and antifermion the total energy is $\int_V(T^{0\mu} - T^{0\mu}_{\text{em}})dV = mc^2$ (not $2mc^2$ because the non linear formalism) meanwhile for a superposed fermion and antifermion pair $\int_V(T^{0\mu} - T^{0\mu}_{\text{em}})dV = 0$ and then annihilation process it is possible by conservation law if $\Delta T^{0\nu}_{\text{em}} = mc^2$ with boundary condition over fields that they and their derivatives tend to zero in the spacial infinity.

A chiral mass term can be constructed, [22], adding the next term to the Lagrangian

$$\mathcal{L}_{m\text{ch}} = -\frac{g_{\text{Ch}}}{2}\left(\sum_{\text{occ}}\bar{\psi}\gamma^5\psi\right)^2 \tag{42}$$

which defines a Dirac's mass term in the form

$$m_{\text{Ch}}c^2 = -g_{\text{Ch}}\left(\sum_{\text{occ}}\bar{\psi}\gamma^5\psi\right)\cdot\gamma^5. \tag{43}$$

Also, for systems with different families of flavours, it is possible construct a positive like Yukawa coupling in the form

$$\mathcal{L}_{\text{Yu}} = -\frac{1}{2}\left(\sum_{\text{occ}}\bar{\psi}\hat{Y}\psi\right)^2 = -\frac{1}{2}\left(\sum_{\text{occ}}Y_{ij}\bar{\psi}_i\psi_j\right)^2 \tag{44}$$

where (Y_{ij}) is a constant $n_f \times n_f$ matrix, $\bar{\psi} = (\bar{\psi}_1, \bar{\psi}_2, \ldots, \bar{\psi}_{n_f})$, n_f is the number of fundamental fermions, which it is easily generalized for chiral mass terms in the form $\mathcal{L}_{\text{YuCh}} - \frac{1}{2}\left(\sum_{\text{occ}}Y_{\text{Ch}ij}\bar{\psi}_i\gamma^5\psi_j\right)^2$. The mas term produced by Yukawa term is

$$m_{\text{Yu}}c^2 = \left(\sum_{\text{occ}}Y_{ij}\bar{\psi}_i\psi_j\right)\cdot(Y_{kl}). \tag{45}$$

Neutrinos's oscillation: all the particles in the present system solve the same Dirac equation, the fermion states that travel together to the mass condensate are massive particles, thus, an asymptotically massless particle when impacting with a non-zero mass condensate sector must satisfy a massive Dirac equation which allows the oscillation to other generations of particles.

About the spectrum: the interaction $-\frac{g}{2}\left(\sum_{\text{occ}}\bar{\psi}\psi\right)^2$ breaks the positive spectrum because only demand that the mass of the vacuum particle-antiparticle background must be zero, but do not impose conditions over the energy and momenta. In absence of mass condensate, the only requirement over occ is be composed for constant states annihilated by the kinetik operator $\hat{\vec{v}}\hat{\vec{P}}$ which ensures the finite density of the background because other solutions are polynomial with poles in the infinities.

For massive solutions it is necessary at least a solution that give mass to system being possible in each point separate occ in two set of fermions one that give mass to the system occ$'$ and a set that corresponds to the background Bg,

$$\sum_{\text{Bg}}\bar{\psi}\psi = 0 \tag{46}$$

that fix the background mass and charge of the fermions in zero. For example, if we have a particle plane wave ψ_1^+ that generates mass m this must has positive energy, but their background for such interaction can be composed for other particle plane wave ψ_2^+ and a negative energy plane wave ψ_3^-, but do not impose conditions over the energies, momenta, nor spin of the background. Thus, the interaction $-\frac{g}{2}\left(\sum_{\text{occ}}\bar{\psi}\psi\right)^2$ for homogeneous condensates allows backgrounds of positive and negative densities of energies, spin and momenta, but for inhomogeneous condensates is necessary that the background be composed for pairs of particles and exactly their antiparticles because in the inhomogeneous case $\bar{\psi}\psi$ takes different shapes for each solutions, unlike the homogeneous case where $\bar{\psi}\psi$ takes the same form for all plane waves, until a change of sign for solutions with negative or positive energy.

A form of break explicitly the problems with the backgrounds and recover the positive spectra defining no observable backgrounds is aggregate an additional mass term associated to the mass of electromagnetic field inside of matter $-\frac{1}{2}\left(\sum_{\text{occ}}\bar{\psi}\gamma^\mu\gamma^5 rA_\mu\psi\right)^2$, where r is a real coupling constant that defines the mass of gauge fields and the contribution of these to the charge density of fermions. This interaction term imposes conditions over the spin, energy and momenta of the backgrounds, if the background do not contribute to the mass of the gauge fields then it fixes the spin density of the background in zero

$$\frac{\hbar}{2}\sum_{\text{Bg}}\bar{\psi}\gamma^i\gamma^5\psi = \sum_{\text{Bg}}\psi^\dagger S_i\psi = \frac{1}{c^2}\sum_{\text{Bg}}\psi^\dagger\hat{v}_i S_3\hat{v}_3\psi = 0, \tag{47}$$

and the z component of the background momenta in zero

$$\frac{\hbar c}{2}\sum_{\text{Bg}}\bar{\psi}\gamma^0\gamma^5\psi = \sum_{\text{Bg}}\psi^\dagger S_3\hat{v}_3\psi = 0, \tag{48}$$

as a consequence of the later identities the total momenta in zero, thus, the energy of the positive energy states of the baground is opposite to the energy of the negative energy estates, fixing the total energy of the background in zero, which allows recovering the positive spectra. For fermions with different flavours is necessary generalize r by $\hat{r}$ a constant hermitian matrix and for different broken gauge symmetries the later condition must be deformed to ensures that the observables of the vacuum vanish, as total color and weak currents in addition to total angular momentum, spin energy and momentums of the background.

The set occ$'$ can not generate negative energy densities because for relativistic classical mechanics $\frac{\langle\vec{v}\rangle}{\psi^\dagger\psi} = \frac{\langle\hat{\vec{P}}\rangle c^2}{\langle\hat{E}\rangle}$, because the modulus of energy grow with the momenta, thus, $\langle\hat{\vec{v}}\hat{\vec{P}}\rangle$ has the sign of $\langle\hat{E}\rangle$ and due that the sum of the energies of such estates corresponds to

$$\sum_{\text{occ}'}\langle\hat{E}\rangle = \sum_{\text{occ}'}\langle\hat{\vec{v}}\hat{\vec{P}}\rangle + \sum_{\text{occ}',j}(r_j(x))^2, \tag{49}$$

where $\{r_j(x)\}_{j=1}^{\ell}$ are ℓ real valued condensates of the states occ′, then $|\sum_{\text{occ}'}\langle\hat{E}\rangle| > |\sum_{\text{occ}'}\langle\hat{\vec{v}}\hat{\vec{P}}\rangle|$, thus, from $\sum_{\text{occ}'_j}(r_j(x))^2 > 0$ necessarily $\sum_{\text{occ}'}\langle\hat{E}\rangle = |\sum_{\text{occ}'}\langle\hat{E}\rangle| > 0$. And as consequence, the energy density of the fermions $E_\psi(x) = \sum_{\text{occ}'}\langle\hat{E}\rangle - \frac{1}{2}\sum_{\text{occ}',j}(r_j(x))^2$ too is greater than zero, $\langle\hat{A}\rangle = \frac{1}{2}\left(\psi^\dagger\hat{A}\psi + (\hat{A}\psi)^\dagger\psi\right)$ with † the transposed conjugate.

The interacting vacuum is not exactly a vacuum but a fluctuation of this, because the full vacuum state is not interacting, but the fluctuations interact with the external fields allowing separation of particle-antiparticle pairs, Fluctuations generate charge and mass, then, corresponds to positive energy states.

4 Consequences on the standard model.

By using this non-linear formalism for fermions also it is possible breaks the gauge symmetries evading the Higgs's mechanism [16], replacing it for a fermionic condensate,

For standard model's particles, it is necessary change the electromagnetic gauge field in previous analysis by the gauge fields of electromagnetism: A^μ, $SU(2)_L$ weak theory: W^1, W^2 and W^3, and $SU(3)_c$ chromodynamics: G^a_μ, $a = 1, \ldots, 8$. the Yang Mills lagrangian takes the form

$$\mathcal{L}_{SMG} = -\frac{1}{4\mu^A}A^{\mu\nu}A_{\mu\nu} - \frac{1}{4\mu^W}\text{Tr}\left[W^{\mu\nu}W_{\mu\nu}\right] - \frac{1}{4\mu^G}\text{Tr}\left[G^{\mu\nu}G_{\mu\nu}\right], \tag{50}$$

where $X^{\mu\nu} = \frac{i}{c_X}\left[\partial^\mu - ic_X\tau^a_X X^\mu, \partial^\nu - ic_x\tau^a_X X^\nu\right]$, $X = A, W, G$.

The mass of the broken gauge symmetry bosons can be generated in a lot of form, by example through a Proca term,

$$\mathcal{L}_{\text{P}} = -\sum_{X,a_X}\frac{1}{2}\left(\sum_{r,\text{occ}}\bar{\psi}C^X_r\tau^{a_X}_r\psi\right)^2 X^{a_X\mu}X^{a_X}_\mu, \tag{51}$$

where C_r is a coupling constant for each representation τ^a_r of the broken gauge symmetry generator τ^a. The sign of $X^{a\mu}X^a_\mu$ fix a condition over $X^{a\mu}$ to maintain the energy positive. A positive definite option to Proca mass term, comes from Higgs's mechanism [16], in the form

$$\mathcal{L}_{mG} = -\sum_{X,a_X}\frac{1}{2}\left(\sum_{r,\text{occ}}\bar{\psi}\gamma^\mu\gamma^5C^X_r\tau^{a_X}_r\psi X^{a_X\mu}\right)^2 \tag{52}$$

which ensures the positive energies of the model, and Yang-Mills charges of the backgrounds in zero, ever is possible separate the broken gauge fields in two or mare interaction terms like this, separating, by example, color fields with electroweak fields.

In this form it is defined a mass for broken gauge fields

$$M^{a_X\mu\nu} = \sum_{r,\text{occ}}\bar{\psi}\gamma^\mu\gamma^5C^X_r\tau^{a_X}_r\psi \cdot \sum_{r,\text{occ}}\bar{\psi}\gamma^\nu\gamma^5C^X_r\tau^{a_X}_r\psi \tag{53}$$

and defines a additional fermion's mixing mass term

$$m_G(x)c^2 = \sum_{X,a_X}\sum_{r,\text{occ}}X^{a_X\mu}\bar{\psi}\gamma^\mu\gamma^5C^X_r\tau^{a_X}_r\psi \cdot \sum_{r,\text{occ}}X^{a_X\nu}\bar{\psi}\gamma^\nu\gamma^5C^X_r\tau^{a_X}_r. \tag{54}$$

which allows explain the hadronic mass behaviour, due that additional mass term appears from the broken gauge symmetry fields inside of the quarks. The dynamical mass generation method allows flavour oscillation of massless neutrinos over the massive matter, due in collisions the indistinguishability of the neutrinos with other massive matter allows neutrino identify the mass of the massive particle as its.

The current generating broken gauge symmetry fields can be defined in the following form

$$J^{Xa\mu} = \sum_{\text{occ}} \bar{\psi}\psi \sum_{\text{occ}} \bar{\psi}\tau_r^{Xa} Q V_r^X \Gamma^\mu \psi \tag{55}$$

and chiral current as

$$J_{\text{Ch}}^{Xa\mu} = \sum_{\text{occ}} \bar{\psi}\Gamma^5\psi \sum_{r,\text{occ}} \bar{\psi}\tau_r^{Xa}\Gamma^\mu\Gamma^5 Q V_{r,\text{Ch}}^X \psi, \tag{56}$$

where $\Gamma^\alpha \equiv \mathbb{I}_{24} \times \gamma^\alpha$ and $\alpha = 0, 1, 2, 3, 5,$, for the standard model fermionic particles. For a like plane wave weak theory $Q_r^W = -Q_{r\text{Ch}}^W = \frac{g'}{2}$, and for a strong theory $Q_r^G = g_s$. For electromagnetism the coupling charges are contained in $\hat{Q}V$, a diagonal matrix which element are the charges times the volume of at rest isolated particles for leptons, and the notation is extended for fix the coupling constants for quarks

$$\hat{QV} = \text{diag}\left(Q_e V_e \mathbb{I}_4, Q_{\nu_e} V_{\nu_e} \mathbb{I}_4, Q_\mu V_\mu \mathbb{I}_4, Q_{\nu_\mu} V_{\nu_\mu} \mathbb{I}_4, Q_\tau V_\tau \mathbb{I}_4, Q_{\nu_\tau} V_{\nu_\tau} \mathbb{I}_4,\right. \tag{57}$$

$$\left. Q_u V_u \mathbb{I}_{12}, Q_d V_d \mathbb{I}_{12}, Q_c V_c \mathbb{I}_{12}, Q_s V_s \mathbb{I}_{12}, Q_t V_t \mathbb{I}_{12}, Q_b V_b \mathbb{I}_{12}\right) \tag{58}$$

In this formalism, weak interaction happens through W_3 boson, because the formalism do not need defines a weak angle to construct a massless field, it is only necessary vanish the mass of Yang-Mills fields in vacuum. It is accomplish because far from the particles densities

$$\sum_{r,\text{occ}} \bar{\psi} C_r^X \tau_r^{ax} \Gamma^\mu \Gamma^5 \psi \sim 0. \tag{59}$$

Confinement: to no generate gluon fields that oscillate particles from vacuum states and to be able to have systems of quarks that travel free in a stable form, it is necessary that the color currents being zero, ot the same

$$\sum_{r,\text{occ}} \bar{\psi}\tau_r^{Xa} Q_r^X \Gamma^\mu \psi \sim 0, \tag{60}$$

for all a. To solve it inside of quark matter it is necessary to choose packages of quarks that can move free: i)Just one quark can not solve it, explaining the confinement, ii) but it can be done by a quark-antiquark vacuum state (color-anticolor pair), $\sum_{i=1,2} \bar{\psi}_i \psi_i = 0$, $\psi_1 = C(\psi_2)$ and $\psi_2 = C(\psi_1)$, which is possible if ψ_i are vacuum states of energy zero. iii) Three consistent quarks of different colours, for instance, if three vectors are taken $\psi_1 = (\psi, 0, 0)^T$, $\psi_2 = (0, \psi, 0)^T$ and $\psi_3 = (0, 0, \psi)^T$ and replaced on (59) and (60), with τ the Gell-Mann matrices, the consistency of the confined triplet is checked, iv) more complex cases are possibles corresponding to composition of ii) and iii).

A pure classical QCD theory, fix stable (confined) object if no color current are generated for the system, thus the classical solutions of Yang Mills theory coupled to fermionic condensates has free solutions for particle antiparticle pairs (Mesons) and triplets of identical states but different colors (Baryons) and other more complex cases. But it is practically impossible find a stable isolated quark because it generates automatically high color currents that generate dynamic over the quark and their background (analogue to Dirac sea), at difference of color-anticolor pairs or triplet of quarks, where the color currents are only generated from the wave functions differences reducing the intensity of the currents and generating more stable objets as mesons and baryons. Due the different coupling constant it is not possible expect for identical wave functions for different flavours of quarks, but these differences allow construct the zoology of Hadrons, in which by example uuu is less stable than uud because the triplet uuu is pushed away by the positive electric charge of the u quarks generating color currents higher than the generates by *uud* which it is pulled away for the negative electric charge of *d* quark which reduces the differences among wave functions.

If one quark in a confined doublet or triplet starts to excite by their interactions (Electric charge), immediately the particles and/or antiparticles generates mass and color current, and with this, massive an attractive gluon fields that pull the particles to maintain together. Which could model the confinement and the hadronic mass behaviour.

When quarks are pushed away the energy of bosons gauge grows at the point that can exit a orbiting particle-antiparticle pair that, when confining with the initial particles, will pull up gluons to equilibrium.

5 Dirac equation in curved space time and gravitation.

The fermionic matter in curved space-times [10, 11], is described by following Dirac equations:

$$\left(i\hbar c\gamma^{\mu}(\partial_{\mu}+\Gamma_{\mu}) - mc^2 V_0\bar{\psi}\psi\right)\psi = 0, \tag{61}$$

where Γ_{μ} is redefined as the spin connection, $\bar{\psi} = \psi^{\dagger}h$ with h the spin metric $h = h^{\dagger}$, $h\gamma^{\mu}h^{-1} = (\gamma^{\mu})^{\dagger}$, $h\gamma^5 h^{-1} = -(\gamma^5)^{\dagger}$, $\{\gamma^{\mu},\gamma^{\nu}\} = 2g_{\mu\nu}\mathbb{I}_4$. The lagrangian for free fermionic quanta can be written in the following form

$$\mathcal{L}_{\psi} = i\hbar c\sum_{\text{occ}}\left(\frac{1}{2}\psi^{\dagger}h\gamma^{\mu}\nabla_{\mu}\psi - \frac{1}{2}(\gamma^{\mu}\nabla_{\mu}\psi)^{\dagger}h\psi\right) - \frac{C}{2}\left(\sum_{\text{occ}}\bar{\psi}\psi\right)^2 \tag{62}$$

which defines a positive energy for isolated particles, $\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}$.

It is necessary to note that the conserved current for each quantum in this formalism corresponds to

$$J^{\mu}_{\psi} = \sqrt{-g}\bar{\psi}\gamma^{\mu}\psi, \quad \partial_{\mu}J^{\mu}_{\psi} = 0 \tag{63}$$

which allows a normalization condition for the fermions $\int_V \sqrt{-g}\bar{\psi}\gamma^0\psi d^3x = 1$, and with this, a quantization of the total electric charge.

Modern gravitation theory couple the spin connection and metric to matter through spin density condensate and stress-energy density tensor condensate.

For this system the total spin density condensate is

$$\Sigma^{abc} = \frac{i\hbar c}{2}\sum_{\text{occ}}\bar{\psi}\gamma^{[a}\gamma^{b}\gamma^{c]}\psi, \tag{64}$$

which in an Einstein-Cartan theory is responsible for the torsion of the space time, also, torsion has showed be an realistic method to explain Dark matter [14].

The metric of the space time is defined by the Belifante-Rosenfeld stress-energy tensor condensate, which On Shell takes the form

$$T^{\mu\nu}_{B} = \frac{i\hbar c}{4}\sum_{\text{occ}}\left(\bar{\psi}\gamma^{\mu}\nabla^{\nu}\psi + \bar{\psi}\gamma^{\nu}\nabla^{\mu}\psi - (\nabla^{\nu}\bar{\psi})\gamma^{\mu}\psi - (\nabla^{\mu}\bar{\psi})\gamma^{\nu}\psi\right) - g^{\mu\nu}\frac{mV_0c^2}{2}\left(\sum_{\text{occ}}\bar{\psi}\psi\right)^2, \tag{65}$$

$\nabla_{\mu}\psi = (\partial_{\mu} + \Gamma_{\mu})\psi$ through the usual Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{B\mu\nu} \tag{66}$$

and the spin connection it is completely defined by the equations

$$[\Gamma_\mu, \gamma^\nu] = \partial_\mu \gamma^\nu + \Gamma^\nu_{\mu\rho}\gamma^\rho \tag{67}$$

$$\Gamma_\mu = \frac{1}{8}\Gamma^\nu_{\mu\rho}[\gamma_\nu, \gamma^\rho] + M_\mu, \tag{68}$$

where $\Gamma^\nu_{\mu\rho}$ is the Riemann-Christoffel event connection

$$\Gamma^\nu_{\mu\rho} = \frac{1}{2}g^{\nu\alpha}\left(\partial_\mu g_{\alpha\lambda} + \partial_\lambda g_{\alpha\mu} - \partial_\alpha g_{\mu\lambda}\right). \tag{69}$$

The light should to satisfy the perpendicularity condition of the broken gauge symmetry fields to the current density condensates. As physical particles are no localized, its distribution constrain to the broken gauge symmetry fields along the space, which can explain the gravitational lensing. Thus, light moves attached to the cosmic halos of the quantum particles. If this is not sufficient condition for explain the effect of fermions to light geodesic, mechanism for coupling spin connection and gauge fields have been studied in the literature [29].

6 Contraction of a static fermion

To obtain a consistent gravitational theory of classical fermion matter, it is not necessary that $\psi^\dagger h\gamma^0\hat{E}\psi > 0$, but it is that the 00 component of energy momentum tensor be greater then zero. For Dirac particles the lagrangian energy density looks like $T^{D0}{}_0 = \psi_D^\dagger h\gamma^0\hat{E}\psi_D$, but in presence of dynamical mass generation it takes the form

$$T_{B00} = g_{00}\left(\frac{C}{2}(\bar{\psi}\psi)^2 + \frac{c}{2}\left(\bar{\psi}\gamma^i\hat{P}_i\psi - (\hat{P}_i\bar{\psi})\gamma^i\psi\right)\right), \tag{70}$$

and if it is chosen $g^{00} \geq 0$ as consequence of $\psi^\dagger h\gamma^0\hat{E}\psi > 0$ is described a positive definite energy. At this point it is necessary to note the difference between the Dirac's mass of the particles, which appears in Dirac's operator (a matrix, linear in $\bar{\psi}_i$ and ψ_i), and the inertial/gravitational mass, a scalar quadratic in $\bar{\psi}_i$ and ψ_i, differentiating between quantum particle physic masses and the gravitational masses of such particles, being the later which defines the critical density of the universe.

A first approximation to the observable universe is a set of $N \sim 10^{80}$ Hydrogen atoms this matter can be a 2-fermion state without total charge nor total spin, their mas condensate must be the proton mass density inside of the volume of protons V_p, the mass distribution of the electron will be obviated.

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric is a metric ansatz for the Einstein Equations that describes a perfect fluid

$$ds^2 = c^2dt^2 - a(t)^2\left(dx^2 + dy^2 + dz^2\right), \tag{71}$$

for the present lagrangian formalism, this metric gives a positive definite energy, $T^{00} > 0$, $h = \gamma^0$, $\gamma^0(x) = \gamma^0$, $\gamma^i(x) = \frac{1}{a(t)}\gamma^i$, $\nabla_0 = \frac{1}{c}\left(\partial_t + \frac{3}{2}\frac{\dot{a}(t)}{a(t)}\right)$, $\nabla_i = \partial_i$.

The present quantum approach for a stationary hydrogen atom describes the following Friedmann equations

$$\left(\frac{a'(t)}{a(t)}\right)^2 = \frac{4\pi G}{3}\rho_{\mathrm{M}} \geq 0, \quad \rho_{\mathrm{M}} = \frac{g}{c^2}\left(\sum_{\mathrm{occ}}\bar{\psi}\psi\right)^2 \tag{72}$$

and

$$\frac{a''(t)}{a(t)} = -\frac{8\pi G}{3}\rho_{\mathrm{M}} \leq 0, \tag{73}$$

which defines an accelerated contraction of the space inside of the matter in the universe, generating an optional interpretation to the usual idea of expansion of universe. Here the redshift of photons of distant galaxies is explained no from a expansion of the vacuum but from a reduction of the size of particles, thus, a photon generated at t_0 in a galaxy of particles of radius $a(t_0)r_p(0)$ is absorbed by particles of lower radius, $r_p(t) = a(t)r_p(0)$ at time $t > t_0$, it is assumed that all particles contract in the same form in all space starting from the approach of a initial homogeneous universe in which al particles lives in the same space the same but the inhomogeneities in initial the mass condensate necessarily generates inhomogeneities in the contraction of atoms in different places of the universe.

For a single particle the mass density condensate is $\rho_{\rm M} = \frac{g}{c^2}\left(\bar{\psi}\psi\right)^2$ and the normalization condition looks as

$$\int_{V_0} dV\sqrt{-g(x)}\bar{\psi}\gamma^0(x)\psi = 1 \tag{74}$$

which defines $\bar{\psi}\psi = \frac{1}{V_0 a(t)^3}$, it is due o the volume in which one particle lies decay in the form $V_p(t) = a(t)^3 V_p(0)$ to maintain the normalization condition. This is in concordance with the Friedman equations which define the mass density condensate in the form

$$\rho_M = \frac{\rho_0}{a(t)^6} = \frac{1}{12\pi G(t_f - t)^2}, \quad a(t) = \sqrt[3]{\sqrt{12\pi G\rho_0}(t_f - t)} \tag{75}$$

where t_f is an integration constant and $0 \le t \le t_f$. In this case the wave functions of the pair of fermions that describes a simplified spin zero hydrogen atom look as

$$\psi_\uparrow = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{e^{-i\frac{m_p c^2}{\hbar}\int_0^t a^{-3}(u)du}}{\sqrt{V_0 a^3(t)}}, \quad \psi_\downarrow = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \frac{e^{-i\frac{m_p c^2}{\hbar}\int_0^t a^{-3}(u)du}}{\sqrt{V_0 a^3(t)}}, \tag{76}$$

inside of the volume $V_p(t)$ in which particle lies, and zero elsewhere, obviously a better approximation must demand the continuity of the mass condensate, in which step function could be changed by a continuous distribution approach. Now it is possible to define a time parameter τ that transform these states in plane waves

$$\tau = \int_0^t a^{-3}(u)du = -\frac{\ln\left(1 - \frac{t}{t_f}\right)}{\sqrt{12\pi G\rho_0}} \tag{77}$$

and the scale factor takes an exponential form

$$\tilde{a}(\tau) = a(t(\tau)) = \left(12\pi G\rho_0 {t_f}^2\right)^{\frac{1}{6}} e^{-\sqrt{12\pi G\rho_0}\tau} \tag{78}$$

where the role of the integration constant ρ_0 is to define a time scale $\tau_{\rho_0} = \frac{1}{\sqrt{12\pi G\rho_0}}$. For time τ coordinate system $h' = h$, $\gamma'^0(x') = \frac{1}{\tilde{a}(\tau)^3}\gamma^0$, $\gamma'^i(x') = \frac{1}{\tilde{a}(\tau)}\gamma^i$, $\nabla'_0 = \frac{1}{c}\left(\partial_\tau - \frac{3}{2}H_0\right)$, $\nabla'_i = \partial_i$, $x' = (c\tau, \vec{x})$.

Red-shift: light inside of the matter move such that $d^2s = c^2dt^2 - a_M^2(t)d^2\vec{x} = 0$ or the same $\frac{c}{f} = a(t)\lambda(t)$, thus, a photon travelling in the space shows a change in their wavelength depending of the media in which travel $\lambda(t) = \frac{\lambda_v}{a_M(t_e)}$. In the process of emission-absorption of photons happens three stages emission, travel in vacuum and absorption: for $t = t_i$ the wavelength inside of the source is $\lambda(t_e) = \frac{\lambda_v}{a_{p_e}(t_e)}$, where $a_{p_e}(t_e)$ is the scale factor at time t_i of the atom source, for $t_i < t < t_f$, the photon travels in the vacuum and $\lambda(t) = \frac{c}{f} = \lambda_v$, where the scale factor of the vacuum is $a_v(t) = 1$ and finally for $t_f > t_i$, in the target atom $\lambda(t_o) = \frac{\lambda_v}{a_{p_o}(t_o)}$, where $a_{p_f}(t_f)$ is the scale factor of the target hydrogen atom at time t_o.

Under these conditions the redshift parameter looks as

$$z = \frac{\lambda(t_o) - \lambda(t_e)}{\lambda_v} = \frac{1}{a_{p0}(t_o)} - \frac{1}{a_{p_e}(t_e)}, \tag{79}$$

if it is possible defines a similar evolution for all hydrogen atoms, or the same, $a_{p_o}(t) \sim a_{p_e}(t) \sim a_p(t)$ then Eq. (79) takes the form

$$z = \frac{1}{a_p(t_o)} - \frac{1}{a_p(t_e)}. \tag{80}$$

The time t corresponds to the cosmological time, while τ is the particles times, the transformation from t to τ time map the zero of $a(t)$ to the infinity of τ, identifying τ with the time that particles measure.

Thus, in later eras the hydrogen atoms absorbed higher wavelengths explaining the redshift in the cosmological observations.

Hubble's Law: using a telescoping series and the definition of limit it is possible to obtain the Hubble's law in the form:

$$\frac{1}{\tilde{a}_p(\tau_o)} - \frac{1}{\tilde{a}_p(\tau_e)} = \lim_{N\to\infty} \sum_{j=0}^{N-1} \left(\frac{1}{\tilde{a}_p(\tau_j + \Delta\tau)} - \frac{1}{\tilde{a}_p(\tau_j)} \right) = -\int_{\tau_e}^{\tau_o} \frac{\dot{\tilde{a}}_p(\tau)}{\tilde{a}_p^2(\tau)} d\tau, \tag{81}$$

where $\tau_0 = \tau_e$ and $\tau_N = \tau_o$, then

$$z = \int_{\tau_e}^{\tau_o} H(\tau) \frac{d\tau}{\tilde{a}(\tau)} = H_0 \int_{\tau_e}^{\tau_o} \frac{d\tau}{\tilde{a}(\tau)} = \frac{H_0}{c} D, \tag{82}$$

where the Hubble function it is defined as $H(\tau) = -\frac{\dot{\tilde{a}}_p(\tau)}{\tilde{a}_p(\tau)} = H_0$ and $D = c\int_{\tau_e}^{\tau_o} \frac{d\tau}{\tilde{a}(\tau)}$.

From definition of ρ, $H(t) = \frac{H_0}{a(t)^3}$ where $\rho_0 = \frac{3H_0{}^2}{4\pi G} \sim 1.8355 \times 10^{-26} \frac{kg}{m^3}$. Fixing $a_p(0) = 1$ it is possible defines a relation between the scale factor of matter and the observed redshift of firsts radiations in the form

$$a_p(t) = \tilde{a}_p(\tau) = \frac{1}{1+z} \tag{83}$$

and then the differential of Eq (82) is $dz = (1+z)^4 H_0 dt$ and then the total time "t" of the universe

$$t_f = \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)^4} = \frac{1}{3H_0} = \frac{1}{\sqrt{12\pi G \rho_0}}, \tag{84}$$

time t_f takes a estimated value $1.47 \times 10^{17} s = 4.67 \times 10^9 years$ lower then the range of times of the visible universe $13 \times 10^9 years$, but for particles $dz = (1+z)H_0 d\tau$, the total possible redshift correspons with a infinite time τ,

$$\int_0^{\tau_f} d\tau = \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)} = \infty, \tag{85}$$

being t_f in this coordinate system a scale parameter for the contraction of the particles,

$$a(t) = \left(1 - \frac{t}{t_f}\right)^{1/3} = e^{-\frac{\tau}{t_f}} = \tilde{a}(\tau), \quad \tau = -t_f \ln\left(1 - \frac{t}{t_f}\right). \tag{86}$$

While particles contract, their normalization conditions allows a mass conservation, because the volume of integration contract as the particle volume, thus, the mass density of the atom is

$$\frac{\rho_0}{\tilde{a}(\tau)^3} = \sqrt{-g(t)_{ij}} \rho(t) = \frac{m_p}{V_p(t)}, \tag{87}$$

which allows us estimate the age τ_{Now} of the isolated particles from the Hubble parameter and the mass and volume of proton now because the current mass density of proton, as well as the simplifyed atom in present model gives the identity

$$\frac{\rho_0}{\tilde{a}(\tau_{\text{Now}})^3} \sim \frac{1.67 \times 10^{-27} kg}{10^{-45} m^3}, \tag{88}$$

which defines age for isolated particles $\tau_{\text{Now}} = 4.9646 \times 10^{18} s$ or $157 \times 10^9 years$.

For $\tau = 0$, the scale factor of the atoms is the same that of the vacuum $a_p(0) = a_v = 1$, and are the anisotropies in the mass condensate generated by Yang-Mills interactions which will be the responsible for the generation of the future vacuum, the volume of the simplified atom is $0.0911 m^3$ and if the vacuum at this moment is the same that the actual volume of the universe, $V = 4 \times 10^{80}$, then the total "atoms" of the observed universe $N = 10^{80}$ would correspond to the 2% of the volume of the space, but a real hydrogen atom it is $\sim 6 \times 10^{14}$ times the volume of the proton, thus, the orbitals of different electrons would occupy the same space which would generate non linear couplings among the atoms due to the contact interaction and the indistinguishability of all the fermions. In a universe like this the sky is completely full and only receive radiation from the closer atoms, as atoms inside of a material.

The cosmic microwave background corresponds to see a bigger hydrogen gas of diluted particles that full the sky and due to a shielding effect does not let us see the radiation of lower density eras. Thus, the estimated age of biggers globular clusters and expansive models, $\tau' \sim 4.1 \times 10^{17} s$ or 13 billions of years [31, 32], corresponds to the observable time inside of a big curtain of particles that contract generating shining stars and, also, letting to see beyond.

For τ before $\tau = 0$ particles travel together and with strong couple conditions because the masses of each atom depends on the wave functions of all the atoms, it primordial material, could occupy all the universe or be a particle in a bigger universe. In such era the estimate acceleration ratio of the contraction of universe takes the form

$$\frac{a''(t)}{a(t)} \sim -\frac{8\pi G}{3} C \left(\frac{N(t)}{V_p(t)}\right)^2 = -\frac{8\pi G}{3} C \left(\frac{N_T}{V_u(t')}\right)^2, \tag{89}$$

where $N(t) = N_T \frac{V_p(t)}{V_u(t)}$ is the ratio of protons in the volume of a proton, N_T is the number of particle in the current visible universe and $V_u(t)$ show the evolution of the volume of the current observable universe.

6.1 Contraction of particles as a cosmological model

The later analysis of gravitating fermions allows to describe the following eras of the universe.

- i) the first era, which starts in $\tau = -\infty$, $V_u(-\infty) = \infty$, is characterized by a universe of volume $V_u(\tau)$ in which all particles are together and strongly coupled. In this era the mass condensate generated is low dense but high nonlinear coupled, the particles are high dependent of the others because all occupy the same space, thus, to define the equation of motion of one particle is necessary know the wave function of all of them. In general this self-concistence condition for wave-functions is ever a condition but when atoms are isolated is possible to obviate the contribution of other atoms to their masses because their mass distribution are localized far away from the isolated atom.

  In a continuous approach, if it is cut the continuous distributions in little step function, it is possible understand that the space practically not contracts far from the focus of mass condensate, where distribution of the particle is practically zero (vacuum), while the space inside of the matter do it in dependence of the local density.

This era end when the inhomogeneities in the primordial mass condensate allows generate vacuum and then isolated set of coupled atoms, this era continue to the moment in which is possible differentiate among the majority of atoms or when the size of vacuum is of the order of the volume occupy for the particles.

- ii) the second era has not a clear beginning, but it is the era of defined atoms, which contract in function of their own mass distribution, because are rounded by a lot of vacuum. In this era the volume of atoms descend exponentially. From the point of view of expanding universe, define a initial plateau in the expansion, follows by a accelerated era, it is because particles that decrease exponentially in a "fix" volume universe (what happens when the volume of vacuum is similar to the volume of the universe) could be understand as particles of fix volume in an exponentially expanding universe.

Other responsible for acceleration and deceleration of contraction of the universe are the Particle-antiparticles separation because the energy of Yang-Mills fields can transform in mass condensate.

This model describes the eras of relative accelerating expansion from the point of view of accelerate contraction of the matter, a first era of accelerated contraction of a higher number of particles coupled because occupy the same space, a transition era in which the particles separate among them and a final accelerated era in which the high density of each isolated particle accelerates their contraction.

The chain of eras described here, allows to think in a fractal universe in which particles are little "universes" composed by little fermions and also our visible universe could be part of a bigger fermion inside of a bigger "universe". In each fractal copy of "universe" the velocity of light in the respective "vacuum" must be different, such that the beginning of respective era of contraction of "isolated" fermions correspond with the correspondence between the scale factor of the "particles" and the respective "vacuum".

7 Outlook

In this work, a lagrangian method for the study of identical fermions has been built, which preserves the electric charge, quantizes it and describes the creation-annihilation of pairs. But in these systems the number of particles is conserved, and what is traditionally known as "creation of pairs", here looks as a oscillation of a pair of particle-antiparticle from a low energy composed state to states in scattering zones. In this positive energy formalism the Dirac sea is replaced by a set of fermion solutions which generates a finite density but zero contributions to energy-momenta, mass and charge. By using this model with broken gauge symmetry, it is possible to avoid the annoying infinities that appear in other quantum theories, because conservation laws demand that the observed conserved quantities remains invariant while the system evolves, then if initial condition defines finite conserved quantities then these remain finite.

To maintain positive the energy density along the space a method of dynamical mass generation has been utilized. This method defines a variable mass and charge for fermion that are defined by fermionic condensates, which imposes self consistence conditions between the Dirac potentials and the occupied states of the system and broken gauge symmetry bosons.

The method can be generalized to higher order interaction of order $2(2n)$ in fermions and $2l$ in broken gauge fields $n \geq l$, n, l natural numbers, interaction terms of order $2(2n)$ in fermions and even order in gauge fields demands gauge conditions. It is only possible to avoid these gauge conditions if the constant coupling matrix vanish, being, in this case, the mass term of the broken gauge fields the only responsible for the broken gauge symmetry.

The mass behaviour of this model allows the empowerment of oscillation of low massive neutrinos inside of massive matter, and describes an expanding universe by through a point of view of accelerated contraction of particles.

The probabilistic observation in the collapse of wave function here is described by the nolinearities in the interaction between particles in the wave and the particles in the target. Each colapse depends on the initial condition of all particles and the inability of obtain all this information is which do that the collapse looks probabilistic.

To describe a realistic model it is necessary understand the fermions as solar systems of particle-antiparticle pairs with the effective fermion in the centre. And in this direction it is necessary map the typical collectivity surrounding each particle. Also it is necessary fix the coupling constant, being the coupling constants of fermionic condensates to gauge fields related with the resistance of materials, due that it appears as potency terms in action, similar to $P = \frac{V^2}{R}$.

Higg's boson is replaced by a set of dynamical fermionic condensates, which give mass to fermions and broken gauge symmetry fields and allow explain hadronic mass behaviour as an equilibrium condition between color and electromagnetic forces.

A cosmological model it is presented for a non charged universe composes of spin zero hydrogen atoms and an analysis for no continuous media is do it. It is observed as inside of hydrogen atoms a scale factor that contract the particles generates a relative expansion of the universe but from the point of view of contracting particles. In this form it is evaded the necessity of a Dark Energy or cosmological constant.

Appendix

For Dirac equation equation (12) the physical solutions with momentum $\vec{p}$ correspond to two solutions of positive energy, $E = \sqrt{m^2c^4 + c^2\vec{p}^2}$,: $\psi^+_{D\uparrow}(\vec{p}, x^\mu) = \mathcal{N} u_\uparrow(\vec{p}) e^{-i\frac{\sqrt{m^2c^4+\vec{p}^2c^2}t}{\hbar} + i\frac{\vec{p}\cdot\vec{x}}{\hbar}}$ and $\psi^+_{D\downarrow}(\vec{p}, x^\mu) = \mathcal{N} u_\downarrow(\vec{p}) e^{-i\frac{\sqrt{m^2c^4+\vec{p}^2c^2}t}{\hbar} + i\frac{\vec{p}\cdot\vec{x}}{\hbar}}$

$$u_\uparrow(\vec{p}) = \begin{pmatrix} 1 \\ 0 \\ \frac{cp_z}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \\ \frac{cp_x+icp_y}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \end{pmatrix}, \quad u_\downarrow(\vec{p}) = \begin{pmatrix} 0 \\ 1 \\ \frac{cp_x-icp_y}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \\ \frac{-cp_z}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \end{pmatrix}, \tag{A.1}$$

and two solutions negative energy, $E = -\sqrt{m^2c^4 + c^2\vec{p}^2}$,: $\psi^-_{D\uparrow}(\vec{p}, x^\mu) = \mathcal{N} v_\uparrow(\vec{p}) e^{i\frac{\sqrt{m^2c^4+\vec{p}^2c^2}t}{\hbar} + i\frac{\vec{p}\cdot\vec{x}}{\hbar}}$ and $\psi^-_{D\downarrow}(\vec{p}, x^\mu) = \mathcal{N} v_\downarrow(\vec{p}) e^{i\frac{\sqrt{m^2c^4+\vec{p}^2c^2}t}{\hbar} + i\frac{\vec{p}\cdot\vec{x}}{\hbar}}$

$$v_\uparrow(\vec{p}) = \begin{pmatrix} \frac{-cp_z}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \\ \frac{-cp_x-icp_y}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \\ 1 \\ 0 \end{pmatrix}, \quad v_\downarrow(\vec{p}) = \begin{pmatrix} \frac{-cp_x+icp_y}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \\ \frac{cp_z}{\sqrt{m^2c^4+c^2\vec{p}^2}+mc^2} \\ 0 \\ 1 \end{pmatrix}, \tag{A.2}$$

where $\mathcal{N} = \sqrt{\frac{mc^2}{2V_0}}\sqrt{\sqrt{m^2c^4 + c^2\vec{p}^2} + mc^2}$ with V_0 is the volume in which a fermion in the ground state is contained. by estimating the volume of a electron using classical radius of the electron we can find a magnitude for V_0, $V_0 \sim 10^{-44}[m]^3$. If the volume is finite then the spectrum is discrete, e.g., the momentum quantization comes from demanding periodic boundary conditions on the borders of the volume V. But if this volume tends to infinity, the spectrum becomes continuous. On the other hand, if potential wells are added to the free particle potential, as example: the coulomb potential, discrete spectrum it is added to the continuous scattering spectrum.

For the plane wave states of Dirac equations $\psi^\dagger_{D\pm}\psi_{D\pm} = \frac{\sqrt{m^2c^4+c^2\vec{p}^2}}{mc^2V_0}$ inside of the volume $V(\psi_D)$ and zero out of this; and the total volume in $V(\psi_D)$ is $\mathrm{V}(|E|) = V_0\frac{mc^2}{\sqrt{m^2c^4+c^2\vec{p}^2}}$.

The free particle solutions generates a complete set for Dirac equation, hence it is possible to expand localized solutions in a series of these states. The sub-indices $\uparrow$ and $\downarrow$ correspond to the z component of the intrinsic angular momentum in the at rest particle's ($\vec{p} = 0$). To obtain the antiparticle's states enough to compute $\psi^-_{Ds}(E,p) = C(\psi^+_{D-s}(-E,-p)) = \gamma^c(\psi^+_{D-s}(-E,-p))^*$ from the states $\psi^+_{D-s}(-E,-p)$ of the particle spectrum, $(\gamma^\mu)^*\gamma^c = \gamma^c\gamma^\mu$, $(\gamma^c)^\dagger\gamma^c = 1$ and, in the Dirac representation, $\gamma^c = i\sigma_3 \otimes \sigma_2$.
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Additional Information

Competing interests

The author declares no competing interests.