Can Nature be Supersymmetric?

Mikhailfest

Benasque, September 2019

Jorge Zanelli (z@cecs.cl)
CECs –Valdivia (Chile)

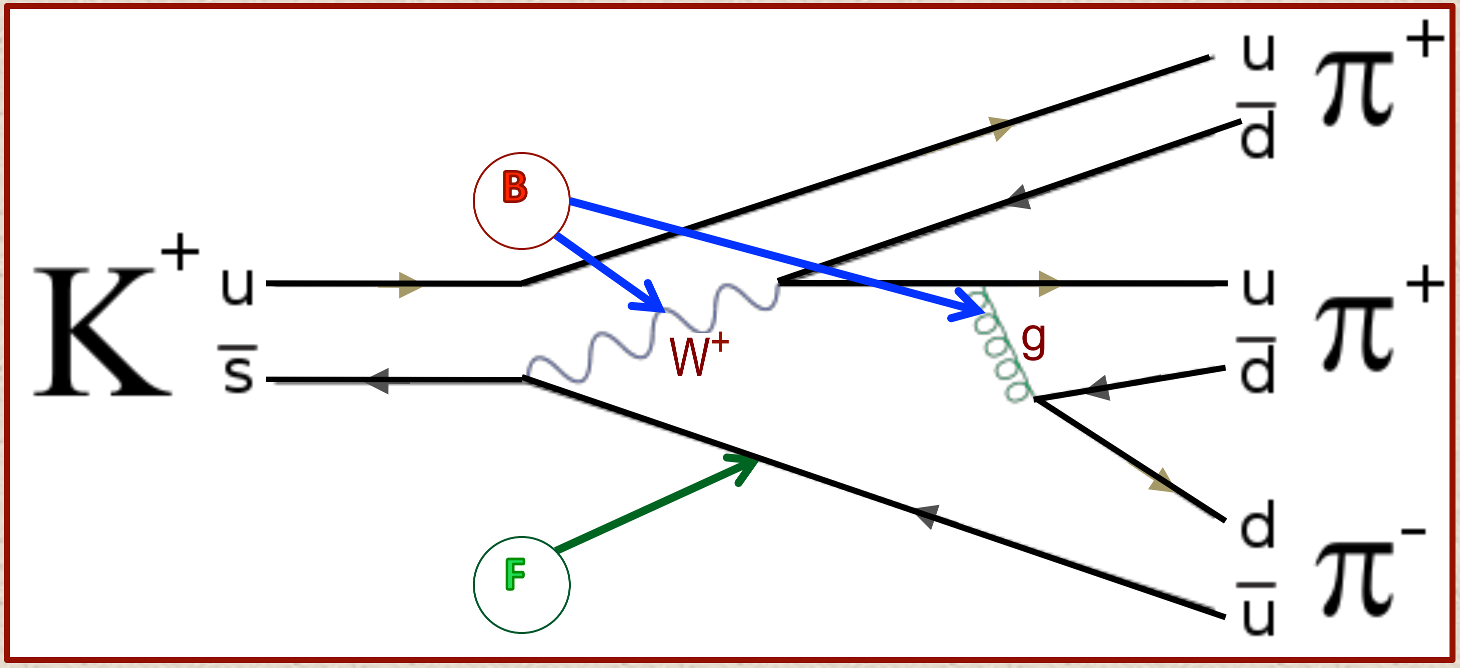
with *P. D. Alvarez*, *A. Guevara, P. Pais, E. Rodríguez, P. Salgado and M. Valenzuela* arXiv:1109.3944, 1306.1247, 1505.03834, 1606.05239
+ L. Andrianopoli, B. Cerchiai, R. D'Auria, and M. Trigiante (Torino)

It was inevitable to fall in love with SUSY in the 70s:

- Unifies *internal* and *spacetime* symmetries
- Makes fermions and bosons necessary
- Restricts particle multiplets
- Relates masses and coupling constants
- Provides $E \geq 0$ theorems, stability
- Improves renormalizability
- Protects hierarchy (respects energy scales)
- Can accommodate gravity

However …

Fermions and Bosons play very different roles in nature:

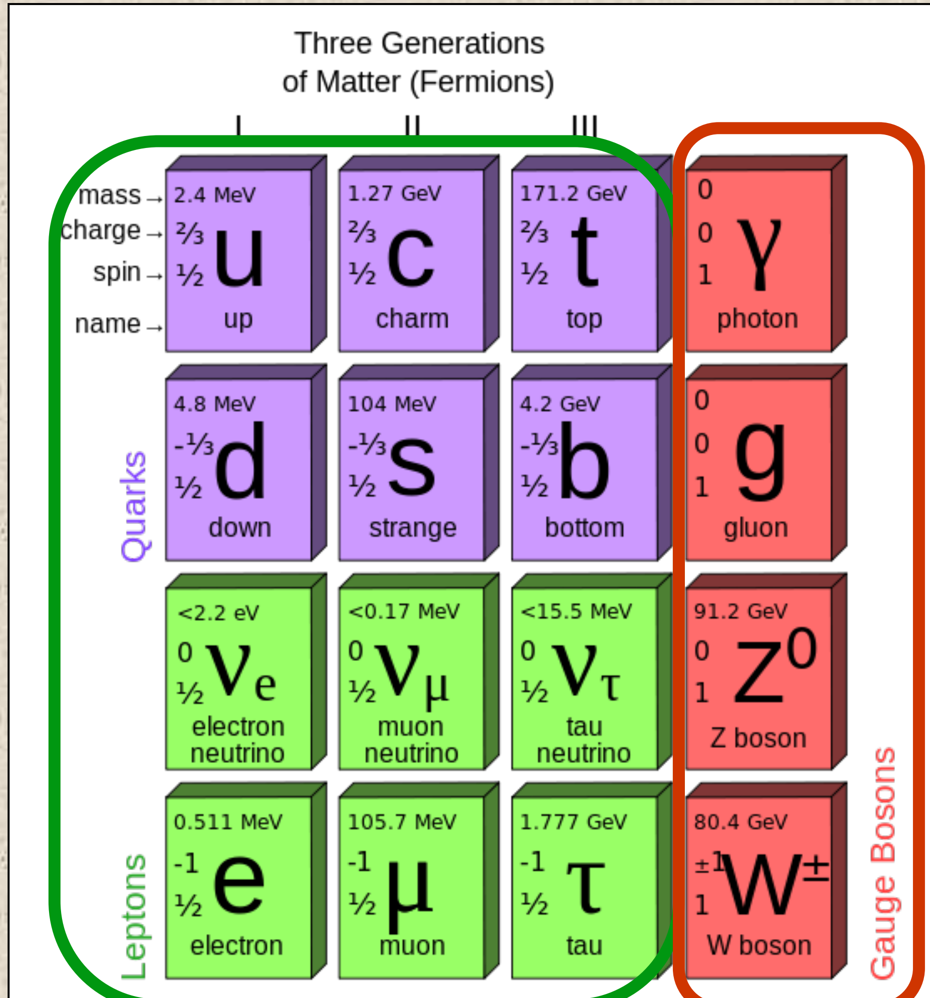


Standard Model

F s=1/2

Building blocks of matter

- Conserved currents
- Gauge vectors (sections)
- Spacetime scalars (zero forms)
- Lorentz spinors
- 1st order field eqs.



B s=1

Carriers of interactions

- Not conserved
- Gauge potentials (connections)
- Spacetime vectors (one-forms)
- Lorentz scalars
- 2nd order field eqs.

(Higgs boson ?)

Optimism in the 90's

Although SUSY was not seen in the SM, it should be found in the next generation of accelerators:

"The assumption that field theories have a Fermi-Bose symmetry leads to predictions which will be tested in the next decade, certainly at the LHC at CERN, and possibly earlier at the Tevatron at Fermilab.

...

For some of the predicted supersymmetric partners, upper and lower limits on their masses can be given, so that not finding these supersymmetric particles will be a serious problem for SUSY. On the other hand, *discovery of supersymmetric particles will rank, with quantum mechanics, special and general relativity, and gauge theories, among the most important physical discoveries of our century.*"

F. Ruiz Ruiz & P. van Nieuwenhuizen, 1996

Optimism in the 90's turned into pesimism 20 years later...

Selected for a Viewpoint in *Physics*
PRL **107,** 221804 (2011) PHYSICAL REVIEW LETTERS week ending 25 NOVEMBER 2011

Search for Supersymmetry at the LHC in Events with Jets and Missing Transverse Energy

S. Chatrchyan *et al.*[*]
(CMS Collaboration)

(Received 12 September 2011; published 21 November 2011)

A search for events with jets and missing transverse energy is performed in a data sample of *pp* collisions collected at $\sqrt{s} = 7$ TeV by the CMS experiment at the LHC. The analyzed data sample ...osity of 1.14 fb^{-1}. In this search, a kinematic variable α_T is used as the ...s with genuine and misreconstructed missing transverse energy. No ...model expectation is found. Exclusion limits in the parameter space of ...metric extension of the standard model are set. In this model, squark ...ed at 95% C.L. Gluino masses below 1.1 TeV are also ruled out at ...al scalar mass parameter below 500 GeV.

CIENTIFIC MERICAN™ 2011 NATIONAL MAGAZINE AWARD WINNER

Search ScientificAmerican.com

e | News & Features | Blogs | Multimedia | Education | Citizen Science | Topics

cientific American Magazine » May 2012

| More Science Tweet 26

Is Supersymmetry Dead?

The grand scheme, a stepping-stone to string theory, ... high on physicists' wish lists. But if no solid evidence ... soon, it could begin to have a serious PR problem

By Davide Castelvecchi | April 25, 2012 | 29

THE SCIENCES

Supersymmetry Fails Test, Forcing Physics to Seek New Ideas

With the Large Hadron Collider unable to find the particles that the theory says must exist, the field of particle physics is back to its "nightmare scenario"

By Natalie Wolchover, Quanta Magazine on November 29, 2012

The Unification Dream

1960s:

Internal gauge symmetries $[T, T] \sim T$

$[J, T] = 0$

External space-time symmetries $[J, J] \sim J$

Can these two algebras be part of a larger Lie algebra?

Coleman–Mandula Theorem (1967): Space-time and internal symmetries can only be combined in a trivial way (direct product of Lie groups)

Haag–Łopuszański–Sohnius theorem (1975)

Internal gauge symmetries

$[T, T] \sim T$

$[J, T] = 0$

External space-time symmetries

$[J, J] \sim J$

SUSY

$[T, Q] \sim Q$

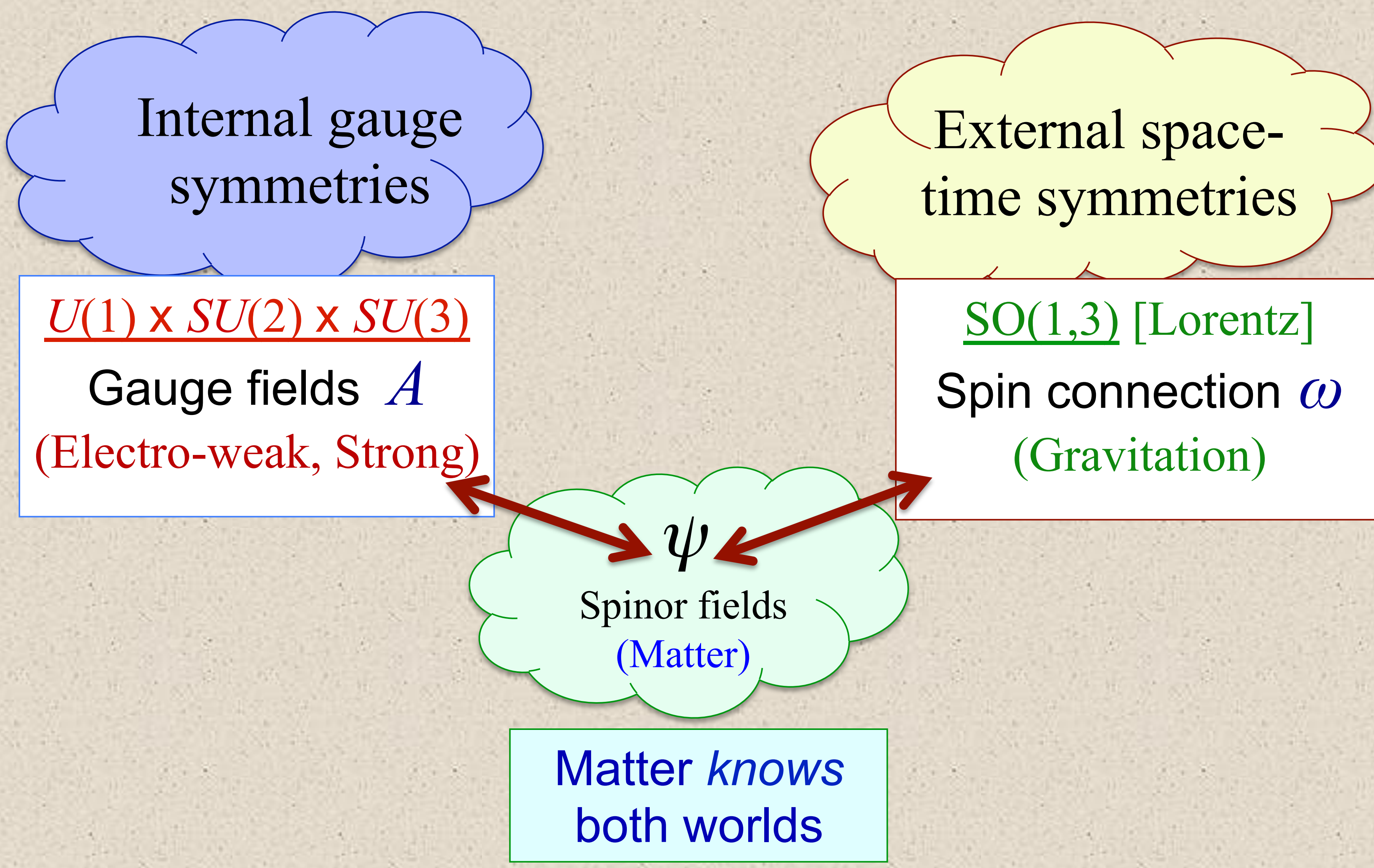
$[J, Q] \sim Q$

Supersymmetry

$\{Q, Q\} \sim T + J$

Spacetime and gauge symmetries can be part of a larger ***graded*** *Lie algebra*

Local symmetries and interactions



Standard (rigid/global) supersymmetry

$$\begin{bmatrix} B' \\ F' \end{bmatrix} = Q \begin{bmatrix} B \\ F \end{bmatrix} = \begin{bmatrix} s_{BB} & s_{BF} \\ s_{FB} & s_{FF} \end{bmatrix} \begin{bmatrix} B \\ F \end{bmatrix}$$

"Vector" representation

Global (rigid) supersymmetry:

$$\{Q^{\alpha}, \overline{Q}_{\beta}\} = H(\Gamma_0)^{\alpha}_{\beta}, \quad [H, Q] = 0$$

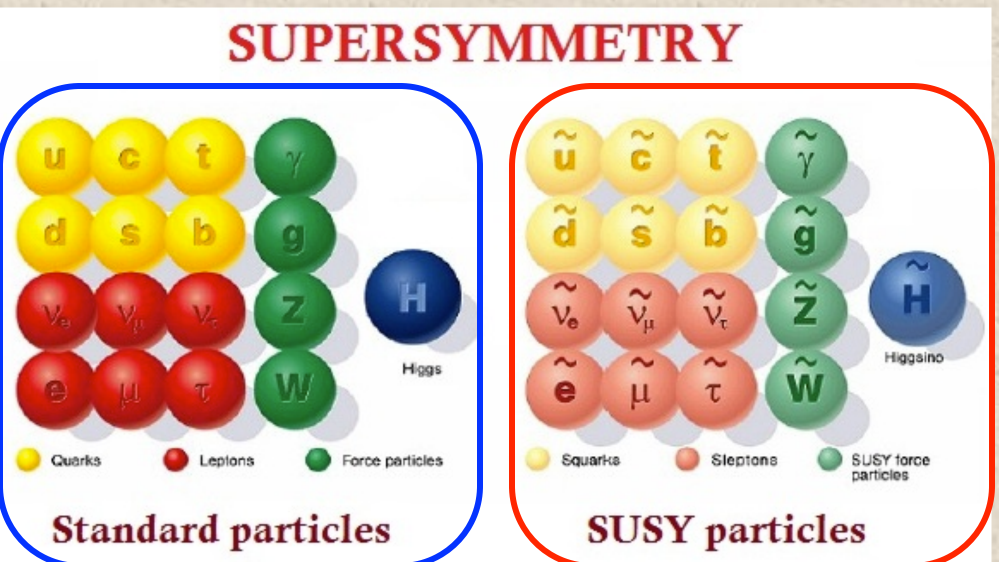
- ➔ Energy states are degenerate: $m_B = m_F$
- ➔ Equal numbers of *B*- and *F*-states

Not even approximately true:

SUSY must be strongly broken

at presently attainable energies

“SUSY-SM”: for each observed particle, include a partner with equal mass and other q-numbers, but different spin:

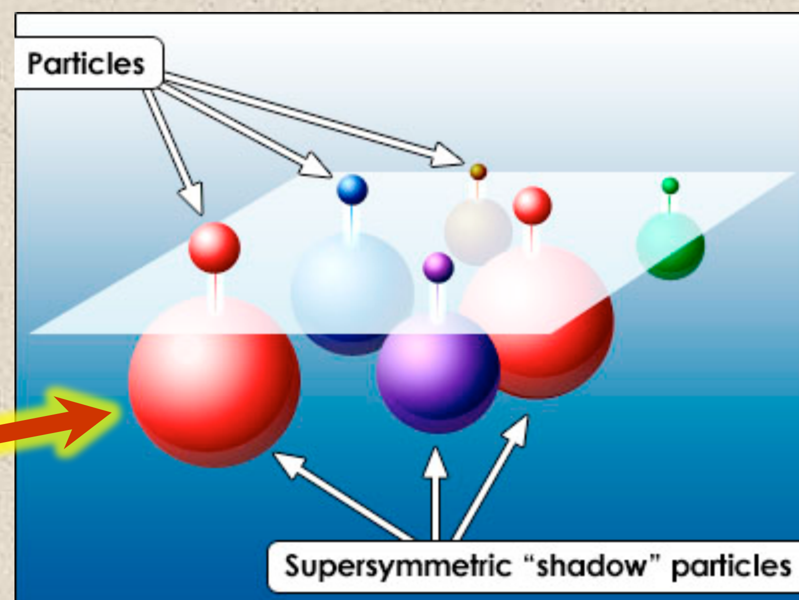


Particles	Spin	Superpartners	Spin
Graviton	2	*Gravitino*	3/2
Photon	1	*Photino*	1/2
Gluon	1	*Gluino*	1/2
$W^{\pm}$	1	*Wino*$^{\pm}$	1/2
Z^{0}	1	*Zino*	1/2
Higgs	0	*Higgsino*	1/2
Electron	1/2	*Selectron*	0
Muon	1/2	*Smuon*	0
Tau	1/2	*Stau*	0
Neutrino	1/2	*Sneutrino*	0
Quark	1/2	*Squark*	0

Currently observed

Not observed

- SUSY breaking mechanism?
- Why are these so heavy?



If the Lord had consulted my opinion, I would have suggested something considerably simpler...

Alfonso X, the Wise, commenting on Ptolemy's epicycles (~ 1280)

Could we do better?

Wish list

Combine B and F under a local (super)symmetry that:

- Respects their different roles as connection (B) and sections (F) in a fiber bundle, respectively
- Does not introduce duplicate fields (no superpartners)
- Gives the right kinetic terms and couplings
- Contains only spins 1 and ½ (the rest can be composites)
- Allows for different masses
- Allows for curved, dynamic spacetime

Unconventional SUSY

Basic idea:

Combine an internal gauge connection A^r_μ,
a spinor ψ^α and the Lorentz connection ω^{ab}
into a single connection field:

$$\mathcal{A} \sim A^r \mathrm{K}_r + \bar{Q}_\alpha \psi^\alpha + \bar{\psi}_\alpha Q^\alpha + \tfrac{1}{2}\omega^{ab}\mathrm{J}_{ab} + \cdots$$

Internal gauge generators | *Complex SUSY generators* | *Lorentz generators* | *Needed to close the algebra*

Technical problem:

$\mathcal{A}$ is a connection 1-form ➔ the spinor ψ^{α} must also be a 1-form, $\psi^{\alpha}=\psi^{\alpha}{}_{\mu}dx^{\mu}$. ➔ s=3/2 gravitino (standard sugra)

… There is an alternative:

$$\mathcal{A}_{\mu} \sim A_{\mu}\mathrm{K} + \overline{Q}_{\alpha}(\Gamma_{\mu})^{\alpha}_{\beta}\psi^{\beta} + \overline{\psi}_{\alpha}(\Gamma_{\mu})^{\alpha}_{\beta}Q^{\beta} + \cdots$$

where

$$\Gamma_{\mu} = \gamma_{a} e^{a}_{\mu}$$

Standard s=1/2 spinor

Dirac matrices (tangent space) $\{\gamma_{a},\gamma_{b}\} = \eta_{ab}\mathrm{I}$

Vielbein (soldering form): Needed to project Clifford algebra from tangent space onto the spacetime manifold.

Example in 3 dimensions

Consider a connection for an algebra that includes internal, spacetime, and supersymmetry generators:

$$\mathcal{A} = A^A \mathrm{T}_A + \tfrac{1}{2}\omega^{ab}\mathrm{J}_{ab} + \overline{\psi}^r_\alpha (\Gamma)^\alpha{}_\beta Q^\beta_r + \overline{Q}^r_\alpha (\Gamma)^\alpha{}_\beta \psi^\beta_r$$

1-form bosons

0-form fermions

$\mathrm{T}_A \rightarrow SU(2)$

$\mathrm{J}_{ab} \rightarrow SO(1,2)$

$Q^\beta_r,\ \overline{Q}^r_\alpha \rightarrow SUSY$

Superalgebra $su(1,2|2)$

The one-form $\mathcal{A} = A^A \mathrm{T}_A + \frac{1}{2}\omega^{ab}\mathrm{J}_{ab} + \bar{\psi}^r_\alpha (\Gamma)^\alpha_\beta Q^\beta_r + \bar{Q}^r_\alpha (\Gamma)^\alpha_\beta \psi^\beta_r$ transforms as a *connection* under the $su(1,2|2)$ superalgebra:

SO(1,2): $\left[\mathrm{J}^{ab}, \mathrm{J}^{cd}\right] = \eta^{bc}\mathrm{J}^{ad} - \eta^{ac}\mathrm{J}^{bd} + \eta^{ad}\mathrm{J}^{bc} - \eta^{bd}\mathrm{J}^{ac}$,

SU(2): $\left[\mathrm{T}_A, \mathrm{T}_B\right] = i\varepsilon_{ABC}\mathrm{T}_C$, $\quad \left[\mathrm{T}_A, \mathrm{J}^{ab}\right] = 0$

SUSY: $\left\{Q^\alpha_r, \bar{Q}^s_\beta\right\} = i\delta^\alpha_\beta \mathrm{T}_A \left(\sigma^A\right)^s_r + \frac{1}{2}\delta^s_r \mathrm{J}^{ab}(\Gamma_{ab})^\alpha{}_\beta$

$\left[\mathrm{J}^{ab}, Q\right] = \frac{1}{2}\Gamma^{ab}Q; \quad \left[\mathrm{J}^{ab}, \bar{Q}\right] = -\frac{1}{2}\Gamma^{ab}\bar{Q}$,

$\left[\mathrm{T}, Q\right] \sim Q; \quad \left[\mathrm{T}, \bar{Q}\right] \sim -\bar{Q}$,

All the fields are scalars under general coordinate transformations.

➡ *General covariance is automatically built in.*

This is kinematics. What is the action?

Action principles

The action is the integral of a gauge-invariant *D*-form.

$$I = \int L(\mathcal{A}) = \int L(A, \psi, \ldots)$$

There are two basic options:

- Chern-Simons (odd *D* only)

$$L_{2n+1} = \left\langle \mathcal{A}(d\mathcal{A})^{n} + c_1 \mathcal{A}^3 (d\mathcal{A})^{n-1} + \cdots + c_n \mathcal{A}^{2n+1} \right\rangle$$

and

- Yang-Mills (any *D*)

$$L_D = \tfrac{1}{2}\left\langle \mathcal{F} \wedge {}^*\mathcal{F} \right\rangle$$

$$= \tfrac{1}{4} g^{\alpha\mu} g^{\beta\nu} \left\langle \mathcal{F}_{\alpha\beta} \mathcal{F}_{\mu\nu} \right\rangle \sqrt{g}\, dx^D$$

where $\mathcal{F} \equiv d\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$, and $\langle \quad \rangle$ = Trace.

Simplest Examples

The simplest SUSY algebra contains a single $U(1)$ generator (K), one (complex) spinor charge (Q^α, $\overline{Q}_\alpha$), and the Lorentz generators J_{ab}.

This is sufficient in $3D$ and almost sufficient in $4D$:

$3D$ ➔ $su(1,2|2)$, $osp(2|2)$, in $4D$ ➔ $usp(4|2)$, $usp(2,2|2)$

The resulting theory is bound to contain gravity:

Fermions ➔ Vielbein ➔ metric

Susy ➔ Local Lorentz symmetry

Metric theory with local Lorentz invariance.

Gravity

Isn't this supergravity?

a. Three dimensions

JHEP **04** (2012) 058, arXiv:1109.3944

Given a connection $\mathcal{A}$ for some gauge group, the Chern-Simons form defines a gauge (quasi-) invariant action,

$$L = \tfrac{1}{2}\left\langle \mathcal{A} d\mathcal{A} + \tfrac{2}{3}\mathcal{A}\mathcal{A}\mathcal{A} \right\rangle$$

where the bracket is the invariant trace in the algebra.

$SU(2)$-CS $\qquad$ $SO(1,2)$-CS (gravity)

$$L = \overbrace{\tfrac{1}{2}\mathrm{Tr}[AdA + \tfrac{2}{3}A^3]}^{SU(2)\text{-}CS} + \overbrace{\tfrac{1}{2}[\omega^a{}_b d\omega^b{}_a + \tfrac{2}{3}\omega^a{}_b\omega^b{}_c\omega^c{}_a]}^{SO(1,2)\text{-}CS\ (\text{gravity})}$$

$$-2\bar{\psi}(\partial\!\!\!/ + iA\!\!\!/ - \tfrac{1}{4}\Gamma_a \omega\!\!\!/^{ab}\Gamma_b)\psi|e|d^3x \leftarrow \text{Dirac}$$

$$-\bar{\psi}\psi\, e^a T_a \leftarrow \text{Torsional coupling}$$

A rather ordinary-looking system

In the case of $U(1)$, the action is

$$I = \int AdA + [\tfrac{1}{2}\omega^a{}_b d\omega^b{}_a + \tfrac{1}{3}\omega^a{}_b\omega^b{}_c\omega^c{}_a]$$

$$- \int \overline{\psi}(i\Gamma^a E_a^\mu \nabla_\mu + \mu)\psi\sqrt{-g}d^3x$$

where $\nabla_\mu \equiv \partial_\mu - iA_\mu + \tfrac{1}{4}\omega^{ab}_\mu \Gamma_{ab}$,
and $\mu\sqrt{-g}d^3x = e^a T_a$, fermion mass (constant).

Lagrangian for long wavelength limit of *Graphene*, allowing for curvature and torsion in the 2+1 spacetime.

It's supersymmetric and includes gravity, but it's not SUGRA

The action that describes graphene in this regime is invariant (up to surface terms) under local $U(1)\times SO(2,1)$, and under SUSY:

$$\delta A_\mu = -\tfrac{i}{2}(\bar{\varepsilon}\Gamma_\mu\psi + \bar{\psi}\Gamma_\mu\varepsilon)$$

$$\delta\omega^a_\mu = \bar{\varepsilon}\Gamma^a\Gamma_\mu\psi + \bar{\psi}\Gamma^a\Gamma_\mu\varepsilon$$

$$\delta\psi = \tfrac{1}{3}\nabla\!\!\!/\,\varepsilon$$

$$\delta e^a_\mu = 0$$

➔ The metric $g_{\mu\nu} = \eta_{ab}e^a_\mu e^b_\nu$ is SUSY invariant

Field equations:

$$\delta A: \qquad F^A = \tfrac{i}{2}\varepsilon_{abc}\overline{\psi}\mathrm{T}^A\Gamma^a\psi e^b e^c \qquad (1)$$

$$(F^A_{\mu\nu} = \varepsilon_{\mu\nu\lambda} j^{\lambda A})$$

$$\delta\omega^{ab}: \qquad R^{ab} = -2\overline{\psi}\psi e^a e^b \qquad (2)$$

Fermion mass

$$\delta\overline{\psi}: \qquad [\not{\partial} + i\not{A} - \tfrac{1}{4}\gamma^a \not{\omega}_{ab}\gamma^b + \mu]\psi = 0 \qquad (3)$$

$$\delta e^a: \qquad \overline{\psi}\varepsilon_{abc}\gamma^c[\overleftarrow{d}e^b - e^b\overrightarrow{d} + 2iAe^b]\psi = 2\overline{\psi}\psi T_a \qquad (4)$$

- Standard equations for CS electrodynamics, gravity & spin ½ in 2+1 dimensions.
- ψ gets "mass" from torsion: $\mu = \eta_{ab} e^a_\mu T^b_{\nu\lambda}\varepsilon^{\mu\nu\lambda}$

$$DT^a = 0 \Rightarrow T_a = \tfrac{1}{6}\mu\varepsilon_{abc}e^b e^c, \quad \mu = \text{const} \quad ✔$$

b. Four dimensions

In 4D:

- Minimal susy extension of $U(1)$ leads to $osp(4|2)$:

$$\mathcal{A} = A\mathrm{K} + \bar{Q}\Gamma\psi + \bar{\psi}\Gamma Q + J_a f^a + \tfrac{1}{2} J_{ab}\omega^{ab}$$

$U(1)$, $SU(\mathrm{N})$ Gauge

Complex Lorentz spinor

$SO(3,2)$ anti- de Sitter

- Curvature: $$\mathcal{F} = d\mathcal{A} + \mathcal{A}\mathcal{A}$$ $$= F_0 K + \bar{Q}_i \mathcal{F}^i + J_a \mathcal{F}^a + \tfrac{1}{2} J_{ab}\mathcal{F}^{ab}$$

- Lagrangian

$$L = -\tfrac{1}{4}\langle \mathcal{F} \wedge \tilde{\mathcal{F}} \rangle = -\tfrac{1}{4}[\mathcal{F}_0 * \mathcal{F}_0 + \bar{\mathcal{F}}^i \Gamma_5 \mathcal{F}^i + \tfrac{1}{2}\varepsilon_{abcd}\mathcal{F}^{ab}\mathcal{F}^{cd}]$$

SUSY is broken: No AdS-invariant trace in 4D

4D Lagrangian (identifying $f^a_\mu = \mu e^a_\mu$):

$$L = \Big\{ -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ \tfrac{i}{2}[\bar{\psi}\overleftarrow{\nabla}\psi - \bar{\psi}\overrightarrow{\nabla}\psi] + \bar{\psi}\Gamma_5\Gamma_a T^a\psi$$
$$+ \mu^{-2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\Gamma_5\psi)^2]\Big\}\sqrt{-g}\, d^4x$$
$$-\tfrac{1}{16}\varepsilon_{abcd}[R^{ab} - \mu^2 e^a e^b][R^{cd} - \mu^2 e^c e^d]$$

Maxwell / YM

Dirac

Nambu-Jona Lasinio

Einstein +cc

- Standard couplings: $\nabla_\nu = \partial_\nu - iA_\nu + \frac{1}{4}\Gamma_{ab}\omega^{ab}_\nu - \frac{i\mu}{2}\Gamma_\nu$
- No $\partial_\mu \partial_\nu \psi$ - terms: fermions behave as standard matter
- Cosmological constant $\Lambda \sim \mu^2$ ➔ *de Sitter*
- Newton's constant $G \sim \mu^{-2}$

Phenomenological, low energy, 4D theory.

SUSY breaking

Under SUSY transformations (in 3D),

$$\delta A_\mu = -\tfrac{i}{2}(\bar{\varepsilon}\Gamma_\mu\psi + \bar{\psi}\Gamma_\mu\varepsilon)$$

$$\delta\omega^a_\mu = \bar{\varepsilon}\Gamma^a\Gamma_\mu\psi + \bar{\psi}\Gamma^a\Gamma_\mu\varepsilon$$

$$\delta\psi = \tfrac{1}{3}\not{\nabla}\varepsilon \ , \qquad \delta e^a_\mu = 0$$

the superconnection transforms as expected: $\delta\mathcal{A} = \not{\nabla}\varepsilon$.

A bosonic vacuum ($\psi = 0$) is invariant provided $\varepsilon(x)$ satisfies subsidiary condition

$$\boxed{\not{\nabla}\varepsilon = 0}$$

If spacetime admits $\mathcal{N}$ globally defined Killing spinors there are $\mathcal{N}$ unbroken global (rigid) SUSYs; the rest are spontaneously broken: the action is invariant, not the vacuum.

For $D=2n$, the only invariant $2n$-forms are characteristic classes (Chern - Weil theorem).

Consequently, there can be no locally SUSY-invariant actions in even D.

There might exist SUSY-invariant configurations, but that symmetry would be spoiled in a dynamical background.

SUSY, like Poincaré invariance, would be an approximate symmetry in a small enough region of spacetime –except for the very rare cases of maximally symmetric spaces–.

Overview and summary

Unconventional SUSY:

Internal gauge symmetry with all fields as parts of the same connection.

F (matter) = sections

B (interactions) = connections

Packaged together in a single gauge connection

- Only s = ½,1 fundamental fields (s =0, 3/2, 2 composite)

- Gravity is automatically included:
  - Spinors in tangent space → Vielbein
  - Local SUSY → local Lorentz algebra

Gravity is not only allowed, it is necessary!

Composite fields

- The metric is not a fundamental field

$$g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu \quad \sim \text{"graviton"} (s=2)$$

- The $s = 3/2$ field is not a fundamental either

$$\chi_\mu = e^a_\mu \gamma_a \psi = \gamma_\mu \psi \quad \sim \text{"gravitino"} (s=3/2)$$

N.B.: In SUGRA the gravitino satisfies $\gamma^\mu \chi_\mu = 0$, which projects out the spin ½ part. Here we keep the s=½ sector and project out the gravitino:

$$\chi_\mu = \gamma_\mu \psi \Rightarrow (\delta^\mu_\nu - \tfrac{1}{D} \gamma_\nu \gamma^\mu) \chi_\mu \equiv 0$$

Ours is the discarded sector of supergravity

Links to other theories

- 3D u-SUSY ≈ Electrons in a in a graphene sheet with curvature and torsion;

$$R^{ab} = \bar{\psi}\psi e^a e^b \Rightarrow R^{ab} e_b = 0 \Rightarrow DT^a = 0$$

$$\Rightarrow T^a = \mu \varepsilon^{abc} e_b e_c$$ Constant torsion (μ ~ electron mass) ✔

$$\Rightarrow d(\bar{\psi}\psi) = 0$$ Constant electron density ✔

- In arXiv:1801.08081[hep-th], Andrianopoli et al have shown that our 3D model emerges as a bounday theory of 4D $\mathcal{N}$=2 AdS SUGRA.

- In arXiv:1711.03220 [hep-th], Gomes et al have found that our 4D model emerges as a dimensional reduction of 5D Chern - Simons SUGRA.

U-SUSY summary

- ✓ Standard kinetic terms for all fields
- ✓ Standard couplings between fermions and bosons
- ✓ No SUSY pairs, no matching d.o.f.
- ✓ Bosons remain massless, fermions may acquire mass
  m_F = integration constant ~ torsion ⟷ cosmological const.
- ✓ SUSY implies Einstein-Hilbert gravity (& no gravitini)
- ★ In *3D* SUSY can be spontaneously broken by the background
- ★ In *4D* SUSY is explicitly broken down to $SO(3,1) \times SU(\ldots)$ due to inexistence of $SO(4,1)$-invariant action [Chern-Weil thm].

“If [supersymmetry] was true, it would have been discovered long ago”

P.A.M.Dirac (1976)

Maybe we have been living with SUSY all along but perhaps we have been looking for the wrong signals.

The reports of my death have been greatly exaggerated...

(Mark Twain)

Happy birthday, Mikhail!