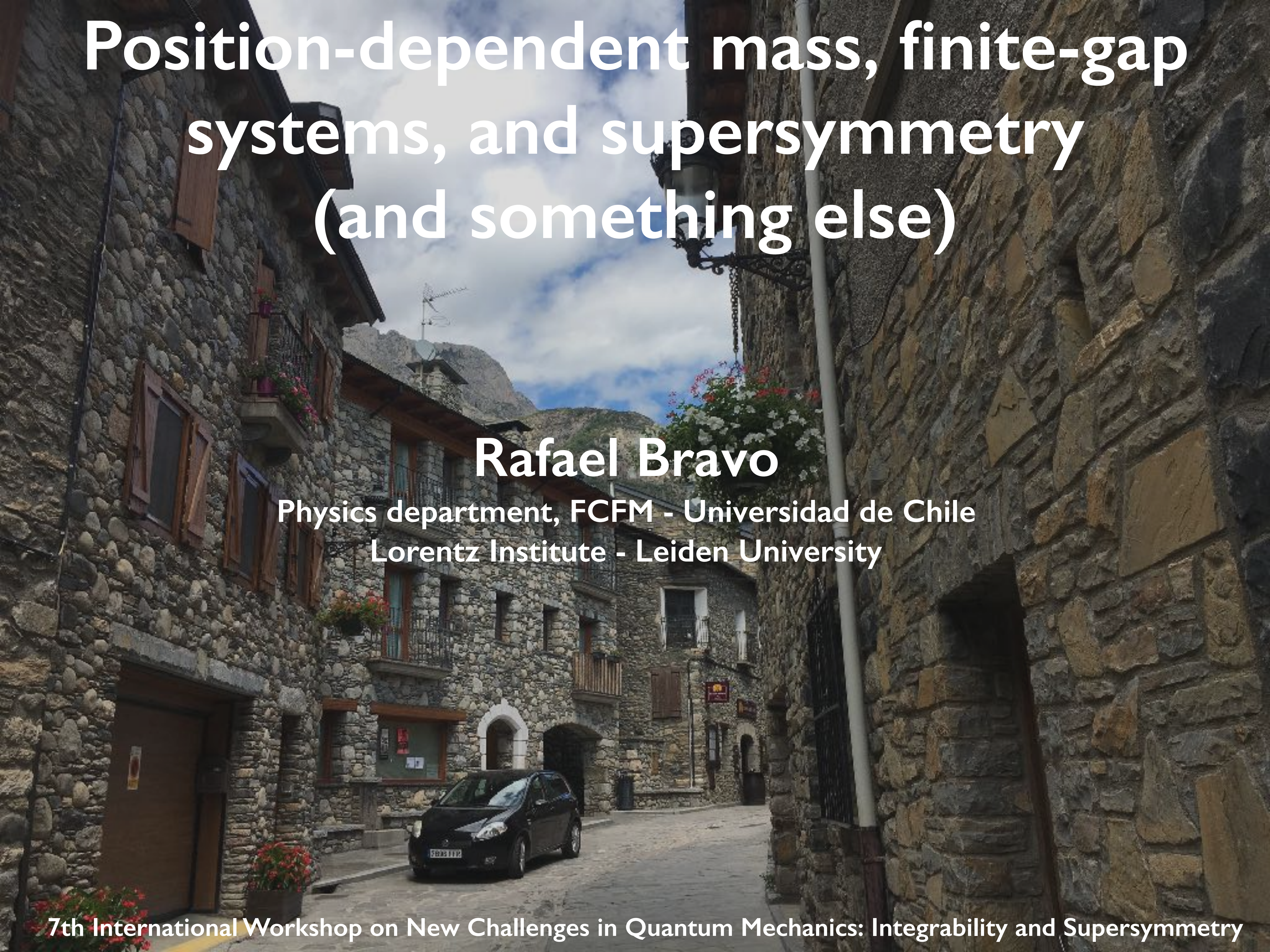

Position-dependent mass, finite-gap systems, and supersymmetry (and something else)

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SUSY-QM

Based on: Phys. Rev. D93 (2016) 105023
In collaboration with Mikhail A. Plyushchay

SUSY from a fictitious similarity transformation

$$h_1 = p^2$$

$$h_\zeta = \zeta(x) p \frac{1}{\zeta^2(x)} p \zeta(x) = \left(-i \zeta(x) p \frac{1}{\zeta(x)} \right) \left(i \frac{1}{\zeta(x)} p \zeta(x) \right)$$

Classically

$$h_\zeta = h_1$$

Let us consider the quantum analog of the factorization as

$$A_\zeta = \frac{1}{\zeta(x)} \frac{d}{dx} \zeta(x)$$

It is straightforward to find the following relations

$$H_\zeta = A_\zeta^\dagger A_\zeta = A_{1/\zeta} A_{1/\zeta}^\dagger$$

$$H_{1/\zeta} = A_{1/\zeta}^\dagger A_{1/\zeta} = A_\zeta A_\zeta^\dagger$$

SUSY from a fictitious similarity transformation

$$H_\zeta = -\frac{d}{dx^2} + W^2 - W', \quad W = \frac{d}{dx}\ln\zeta$$

At the end of the day we have

$$\mathcal{H}_\zeta = \mathrm{diag}(H_\zeta, H_{1/\zeta}) \qquad \mathcal{Q}_{\zeta+} = A_\zeta^\dagger \sigma_+$$

The N = 2 SUSY is generated

$$[\mathcal{H}_\zeta, \mathcal{Q}_{\zeta\pm}] = 0, \quad \mathcal{Q}_{\zeta\pm}^2 = 0, \quad \{\mathcal{Q}_{\zeta+}, \mathcal{Q}_{\zeta-}\} = \mathcal{H}_\zeta$$

Kinetic term with a PDM and SUSY

Let us consider a particle of mass $M(x) = \frac{1}{2} m(x)$

$$L(x) = \frac{1}{4} m(x) \dot{x}^2 - u(x)$$

The corresponding EOM

$$\ddot{x} = -2 \frac{u'(x)}{m(x)} - \frac{1}{2} \frac{m'(x)}{m(x)} \dot{x}^2$$

It is possible to make the point transformation

$$f(x) = \frac{1}{\sqrt{m(x)}} \qquad x \to \chi \qquad d\chi = \frac{dx}{f(x)}$$

$$\chi(x) = \int^x \frac{d\eta}{f(\eta)} \qquad x(\chi) = \int^\chi d\eta \, \varphi(\eta)$$

Kinetic term with a PDM and SUSY

$$L(\chi, x) = \frac{1}{4}\dot{\chi}^2 - U(\chi)$$

Now the EOM reads

$$x = x(\chi)$$
$$\ddot{\chi} = -2U'(\chi)$$

The canonical transformation $(x, p) \to (\chi, P), \quad P = f(x)p$

Corresponds to the previous point transformation

$$h_{m(x)} = \frac{1}{m(x)}p^2 + u(x) \quad \to \quad h_1 = P^2 + U(\chi)$$

Kinetic term with a PDM and SUSY

If we consider the PDM kinetic term $h_{m(x)} = \frac{1}{m(x)} p^2$

$$h_{f,\zeta} = \left(-if(x)\zeta(x)p\frac{1}{\zeta(x)} \right) \left(i\frac{1}{\zeta(x)}p\zeta(x)f(x) \right)$$

$$H_{f,\zeta} = f\zeta \frac{d}{dx}\frac{1}{\zeta^2}\frac{d}{dx}\zeta f = A^{\dagger}_{f,\zeta} A_{f,\zeta}$$

Implementing a similitude transformation

$$\mathcal{A} = f^{1/2} A_{f,\sigma} f^{-1/2}$$
$$\mathcal{A}^{\dagger} = f^{1/2} A^{\dagger}_{f,\sigma} f^{-1/2}$$

Kinetic term with a PDM and SUSY

In terms of χ

$$\mathcal{A} = \Phi^{-1}(\chi)\frac{d}{d\chi}\Phi(\chi) = \frac{d}{d\chi} + \mathcal{W}$$

$$\mathcal{A}^\dagger = \Phi(\chi)\frac{d}{d\chi}\Phi^{-1}(\chi) = -\frac{d}{d\chi} + \mathcal{W}$$

Where

$$\Phi(\chi) = \varphi(\chi)^{1/2}\Sigma(\chi)$$

$$\varphi(\chi) = f(x(\chi))$$

$$\Sigma(\chi) = \zeta(x(\chi))$$

$$\mathcal{W} = \frac{d}{d\chi}\ln\Phi(\chi)$$

Then

$$H_+ = \mathcal{A}\mathcal{A}^\dagger, \quad H_- = \mathcal{A}^\dagger\mathcal{A}$$

$$H_+ = -\frac{d^2}{d\chi^2} + \mathcal{W}^2 + \mathcal{W}' = -\frac{d^2}{d\chi^2} + V_+$$

$$H_- = -\frac{d^2}{d\chi^2} + \mathcal{W}^2 - \mathcal{W}' = -\frac{d^2}{d\chi^2} + V_-$$

$$U_\pm = V_\pm + u(x(\chi))$$

A reflectionless system

Just to illustrate one of the system that we studied

$$h_{m(x)} = p^2(1-\alpha^2 x^2)^2 + \frac{c^2}{2}x^2$$

Applying the recipe we found

$$H_+ = -\frac{d^2}{d\chi^2} + \gamma^2$$

$$H_- = -\frac{d^2}{d\chi^2} + \gamma^2 - \frac{2\gamma^2}{\cosh^2(\alpha\chi)}$$

$$\gamma = \gamma(\alpha, c)$$

The mass term or from now on, the non-canonical mass was

$$m(x) = \frac{1}{(1-\alpha^2 x^2)^2}$$

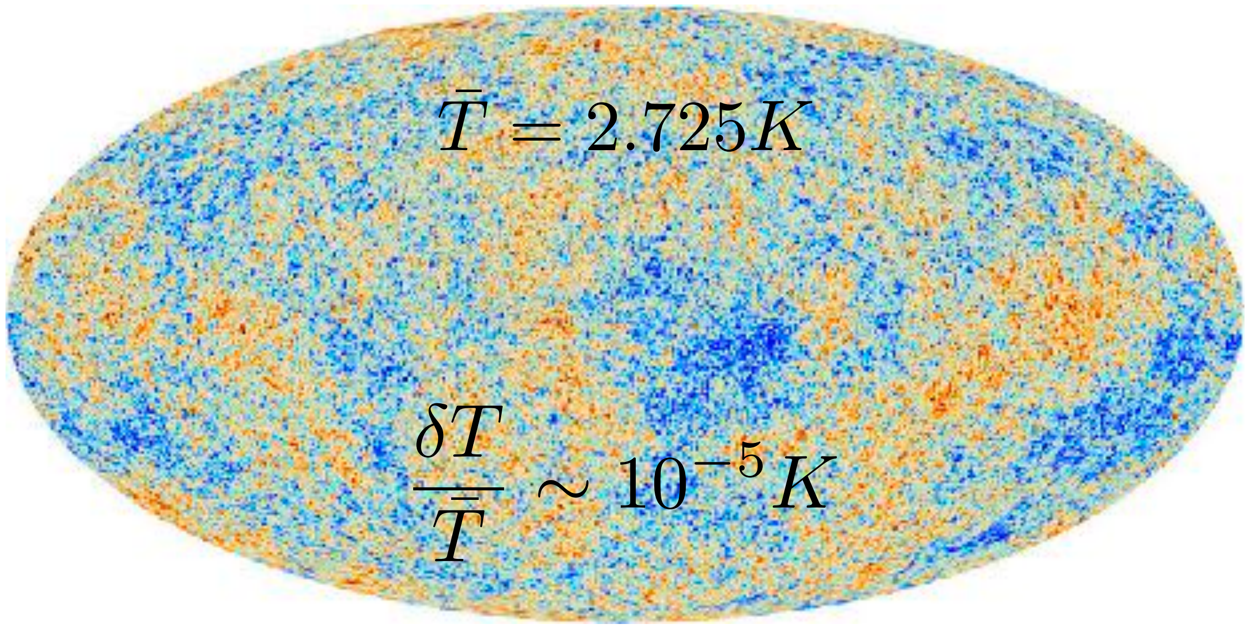
$$ds^2 = \frac{4d\mathbf{x}^2}{(1-\mathbf{x}^2)^2}$$

COSMOLOGY

Based on: 1906.05772 (accepted in JCAP)
In collaboration with Gonzalo A. Palma and Simón Riquelme

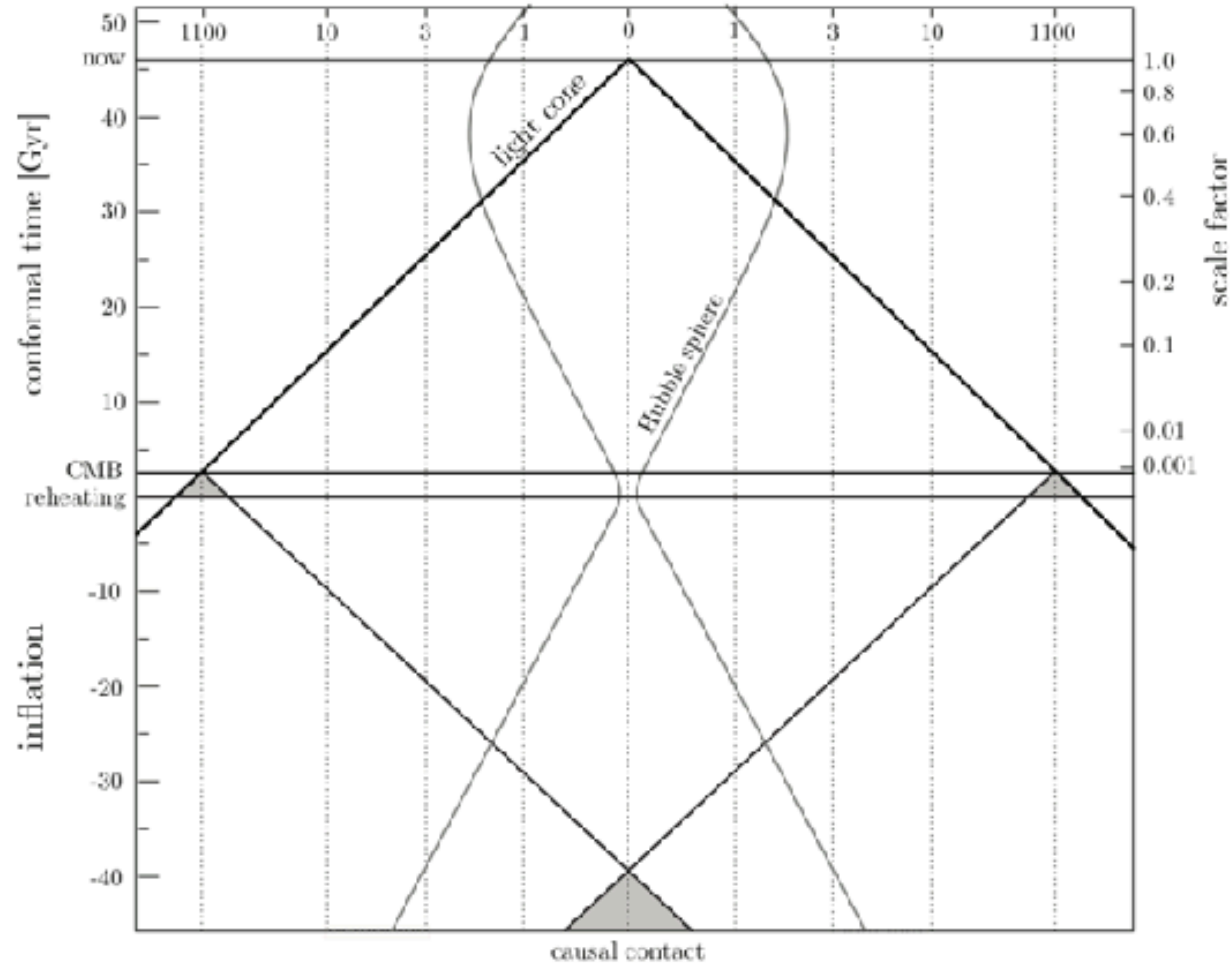
Cosmic Inflation

Why is the CMB so uniform ?



$$\bar{T} = 2.725K$$

$$\frac{\delta T}{\bar{T}} \sim 10^{-5} K$$



$$\frac{d}{dt}(aH)^{-1} < 0$$

$$\Updownarrow$$

$$\ddot{a} > 0 \quad \Longleftrightarrow \quad \epsilon \equiv -\frac{\dot{H}}{H^2} < 1$$

Single-field Inflation

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

$$\phi(t,\mathbf{x}) = \phi_0(t) + \delta\phi(t,\mathbf{x})$$

Background

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \partial_{\phi_0}V = 0$$

Comoving gauge $\delta\phi(t,\mathbf{x}) \equiv 0$

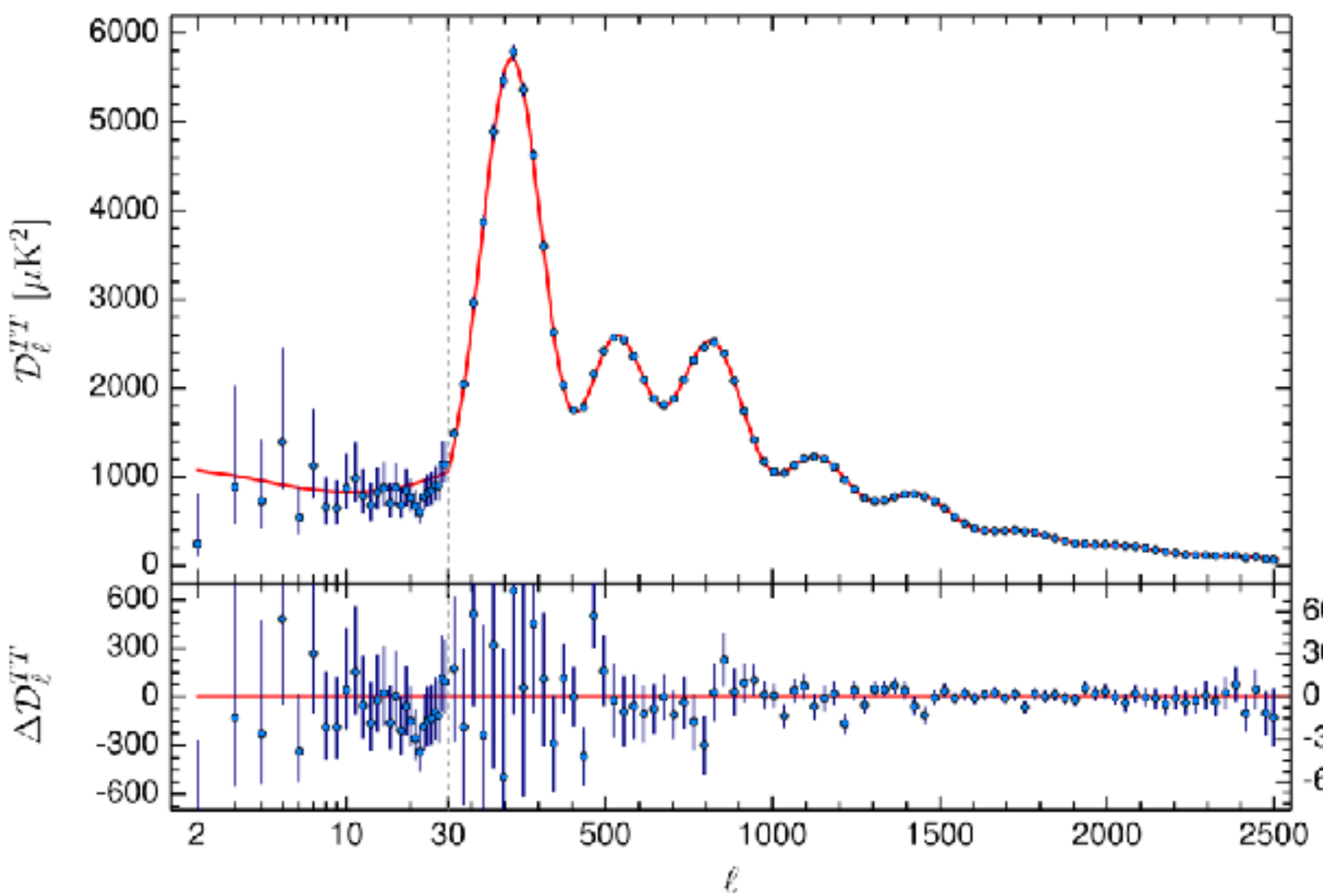
$$ds^2 = -N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$\gamma_{ij} = a^2(t)[(1+2\zeta(t,\mathbf{x}))\delta_{ij} + h_{ij}]$$

Single-field Inflation: Scalar perturbations

$$\langle 0|\hat{\zeta}^\dagger_{\mathbf{k}}\hat{\zeta}_{\mathbf{k}'}|0\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')\frac{2\pi^2}{k^3}P_\zeta(k) \qquad P_\zeta(k) = \frac{H^2}{8\pi^2\epsilon M_{Pl}^2}$$

$$\mathcal{D}_\ell^{TT} = \frac{\ell(\ell+1)}{4\pi^3}\int dk\, k^2 P_\zeta(k) T_\ell^2$$



PLANCK collaboration (2018)

Single-field Inflation: Tensor perturbations

$$P_h(k) = \frac{2H^2}{\pi^2 M_{\rm Pl}^2}$$ Has not been observed

It is possible to constraint through observation, the ratio between the spectra

$$r = \frac{P_h}{P_\zeta} \qquad r < 0.062 \quad (95\% CL)$$

BICEP2/Keck collaboration (2018)

Near future surveys, could achieve $r \sim 0.01$

CLASS
CMB S4
Simons Observatory

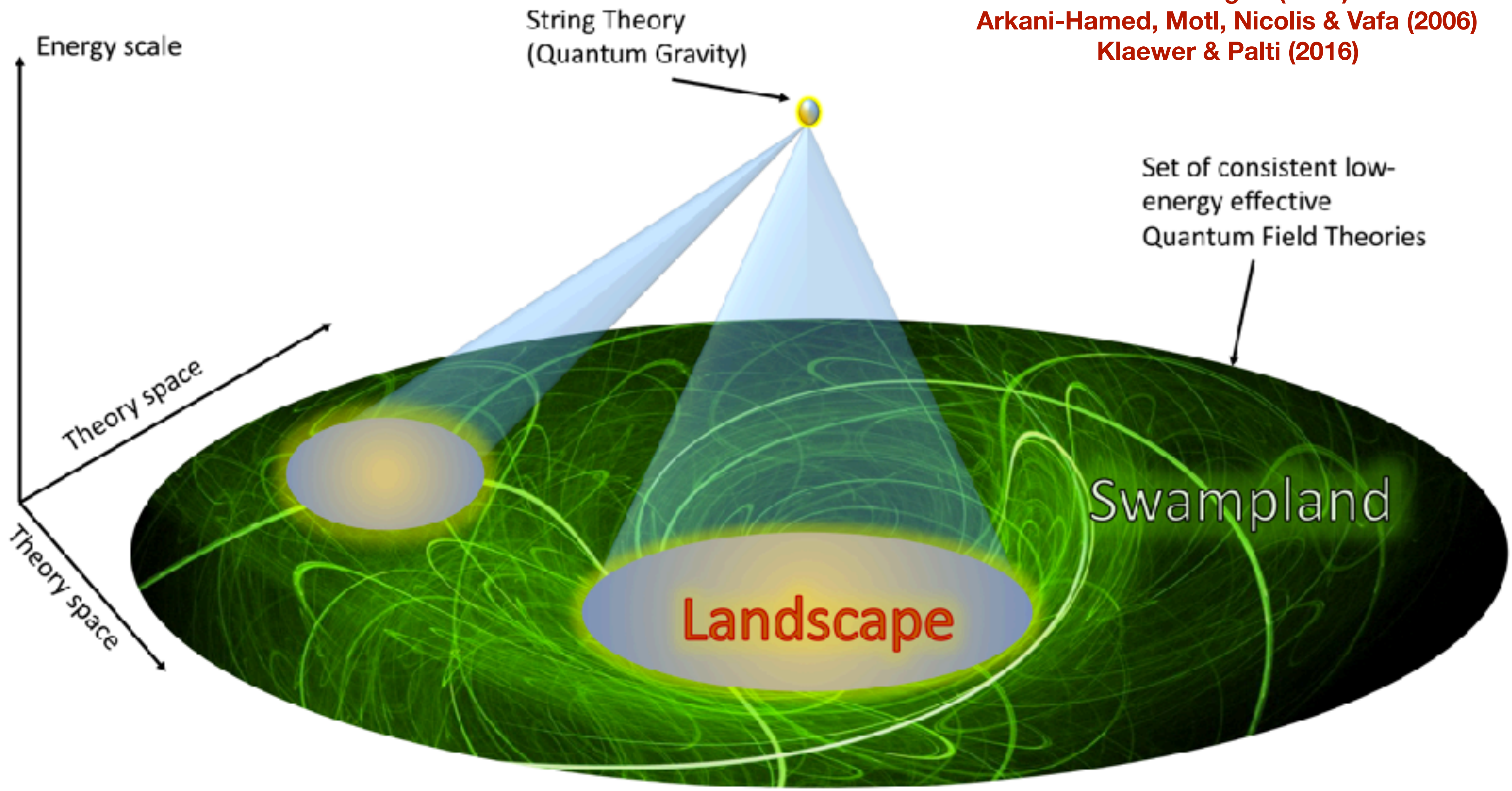
Additionally, exists a relation between the field displacement and the tensor-to-scalar ratio

$$\frac{\Delta\phi}{M_{Pl}} \gtrsim \mathcal{O}(1)\sqrt{\frac{r}{0.01}}$$

Lyth (1997)

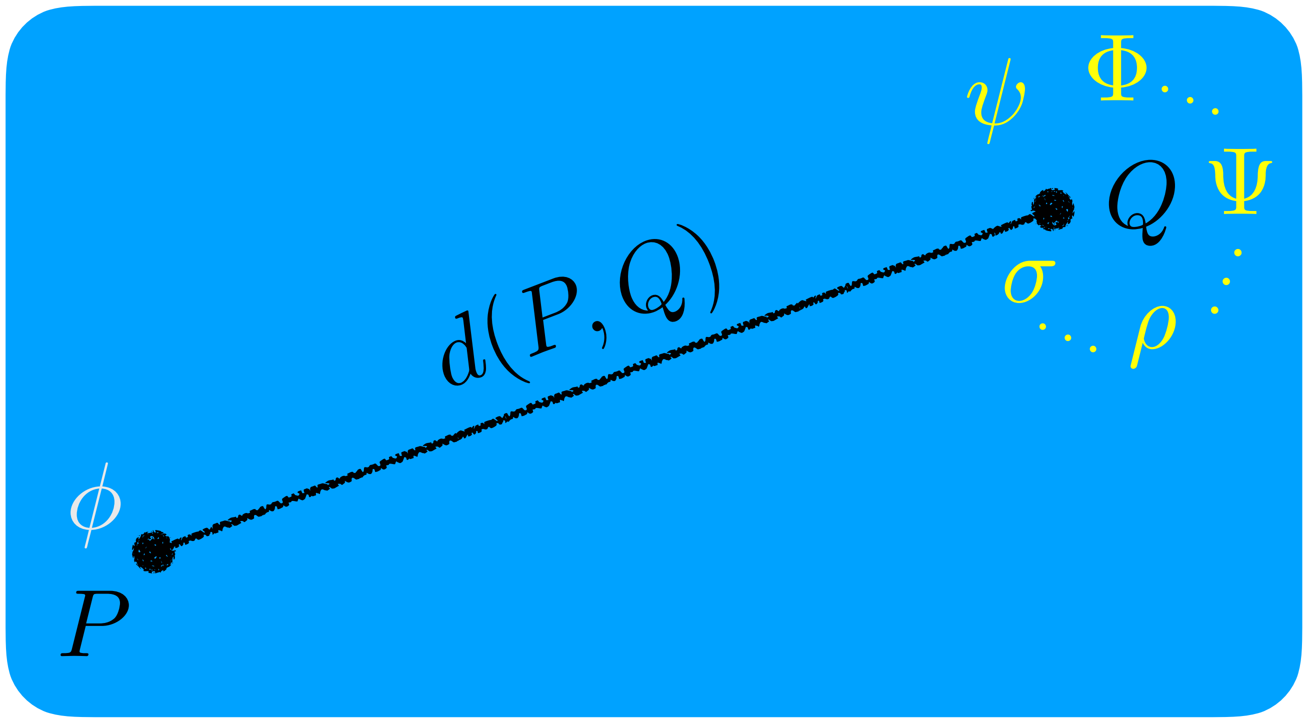
The Swampland program

Vafa (2005)
Vafa & Ooguri (2007)
Arkani-Hamed, Motl, Nicolis & Vafa (2006)
Klaewer & Palti (2016)



The distance conjecture

From string compactifications it is possible to find a tower of massless states at finite distances



$$M(Q) = M(P)e^{-\nu d(Q,P)}$$

K-K or Winding modes

$$\Updownarrow$$

Weak gravity Conjecture

The requirement $d(Q,P) < M_{\rm Pl}$ applies when the distance is a geodesic

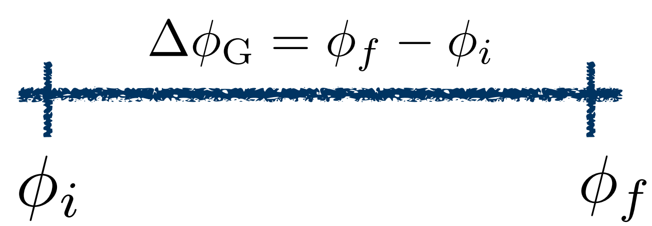
$$\Delta\phi_{\rm G} < \mathcal{O}(1) M_{\rm Pl}$$

The problem

If primordial gravitational waves are detected in the near future, then the inflaton necessarily had super-Planckian displacements

Agrawal, Obied, Steinhardt & Vafa (2018)
Obied, Ooguri, Spodyneiko & Vafa (2018)

$$\Delta\phi \gtrsim \mathcal{O}(1) M_{\rm Pl} \qquad\qquad \Delta\phi_{\rm G} < \mathcal{O}(1) M_{\rm Pl}$$



If one is interested in completing inflation in string theory, how can we overcome this issue?

Multi-Scalar Field Theories

Multi-scalar field theories

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \gamma_{ab}(\phi) \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right] + \Delta S_\Lambda$$

$\mathcal{O}(1)$ Wilson coefficients

$$\Delta S_\Lambda \supset -\frac{1}{4} \int d^4x \sqrt{-g} g^{\mu\nu} \frac{f_{abcd}}{\Lambda^2} \Delta\phi^c \Delta\phi^d \partial_\mu \phi^a \partial_\nu \phi^b$$

EFT cut-off

Effective field-space metric

$$\gamma^\Lambda_{ab}(\phi) \equiv \gamma_{ab} + \frac{f_{abcd}}{2\Lambda^2} \Delta\phi^c \Delta\phi^d + \ldots$$

Riemann Normal Coordinates

$$\gamma^\Lambda_{ab}(\phi) = \delta_{ab} - \frac{1}{3} \mathbb{R}^\Lambda_{acbd}(\phi_\star) \phi^c \phi^d + \ldots$$

Multi-scalar field theories

The Riemann tensor

$$\mathbb{R}^{\Lambda}_{abcd} = \mathbb{R}_{abcd} + \frac{1}{\Lambda^2} g_{abcd}(f) + \dots$$

Riemann-symmetrized linear combination of f *

Characteristic mass scale curvature $R_0 \longrightarrow \mathbb{R} \sim R_0^{-2}$

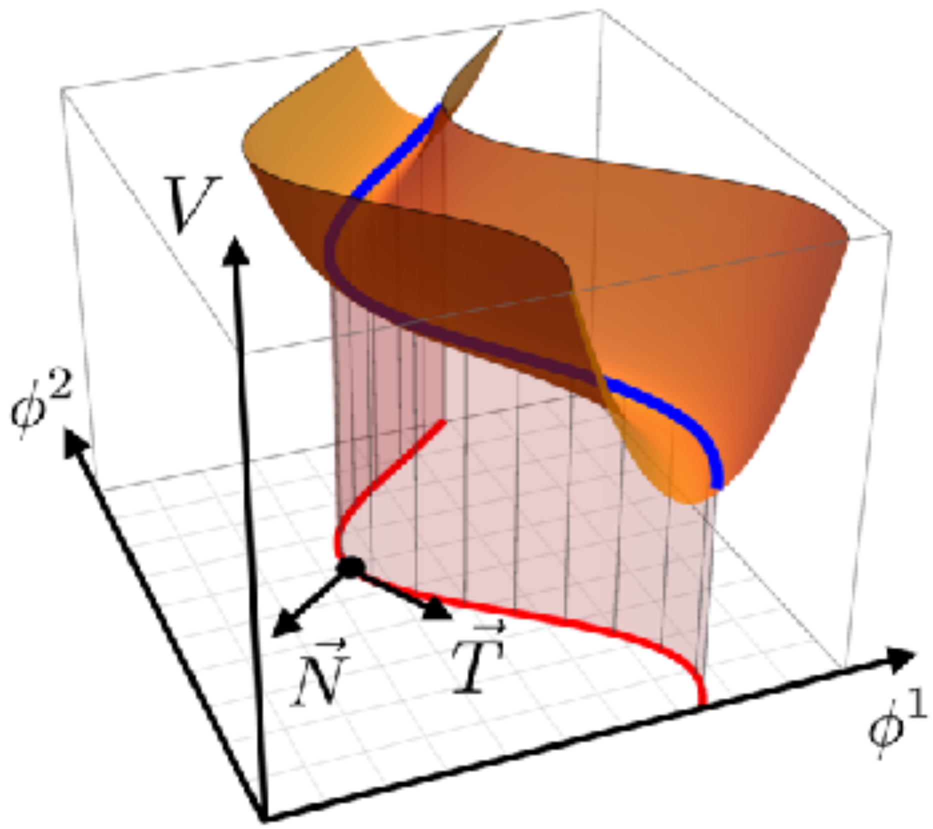
- ☐ $R_0 > \Lambda$: The theory is indistinguishable from a theory with flat geometry $\gamma_{ab} = \delta_{ab}$
- ☑ $R_0 < \Lambda$: It is possible study genuine non-trivial effects from γ_{ab}

For our purposes

$$\Lambda = M_{\mathrm{Pl}}$$

* $g_{abcd}(f) = \frac{1}{2}(f_{adbc} - f_{dbac} - f_{acbd} + f_{cbad})$

Multi-scalar field theories



$$D_t\dot{\phi}^a + 3H\dot{\phi}^a + \gamma^{ab}V_b = 0$$

$$D_t X^a = \dot{X}^a + \Gamma^a_{bc} X^b \dot{\phi}^c$$

The Hamilton-Jacobi (or fake SUGRA) potential

$$V(\phi) = 3W^2(\phi) - 2\gamma^{ab}W_a(\phi)W_b(\phi)$$

The system admits exact non-geodesic solutions

$$H = W$$
$$\dot{\phi}^a = -2\gamma^{ab}W_b(\phi)$$

Inflation in a Hyperbolic field-space

M.C. Escher - *Heaven and Hell*

$$\mathcal{L} = -\frac{1}{2}\frac{(\partial\phi)^2 + \phi^2(\partial\theta)^2}{\left(1-\frac{\phi^2}{6\alpha}\right)^2} - V(\phi)$$

Kallosh & Linde (2015)
Achúcarro, Kallosh, Linde, Wang & Welling (2017)

The scalar curvature of the field-space

$$\mathbb{R} = -\frac{8}{3\alpha} \qquad \alpha \equiv \frac{R_0^2}{3}$$

The geodesics are given by $\ddot{\phi}^a + \Gamma^a_{bc}\dot{\phi}^a\dot{\phi}^b = 0$

Since we have a metric in field-space, it is possible to compute the proper field distance

$$\Delta\phi = \int dt\sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}$$

Overcoming the distance conjecture

Solving the EOM and matching the initial conditions it is possible to find

$$\Delta\phi_{\mathrm{G}} = 2\sqrt{\frac{2}{|\mathbb{R}|}}\,\mathrm{arcsinh}\left(\frac{1}{2}\sqrt{\frac{|\mathbb{R}|}{2}}\Delta\phi_{\mathrm{NG}}\right)$$

Which yields the nice relation

$$\Delta\phi_{\mathrm{G}} < M_{\mathrm{Pl}} < \Delta\phi_{\mathrm{NG}}$$

We can produce observable primordial gravitational waves without the need of geodesic super-Planckian field displacements

Summary

- One dimensional Systems with PDM can be treated with the inclusion of SUSY in particular systems with a "Poincaré disk" mass

- We have used the very same geometry in the context of Multi-field inflation in order to produce sizable primordial gravitational waves without the need of super-Planckian geodesic field displacements, overcoming the Swampland distance conjecture

Happy birthday, Mikhail!