Quantum emitter(s) ultrastrongly coupled to a cavity

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Fundación BBVA

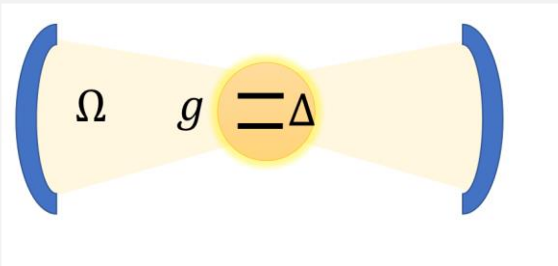
Outline

1. Light-matter beyond the Rotating Wave Approximation

2. Ultrastrongly dissipative quantum Rabi model

3. Spin squeezing generation

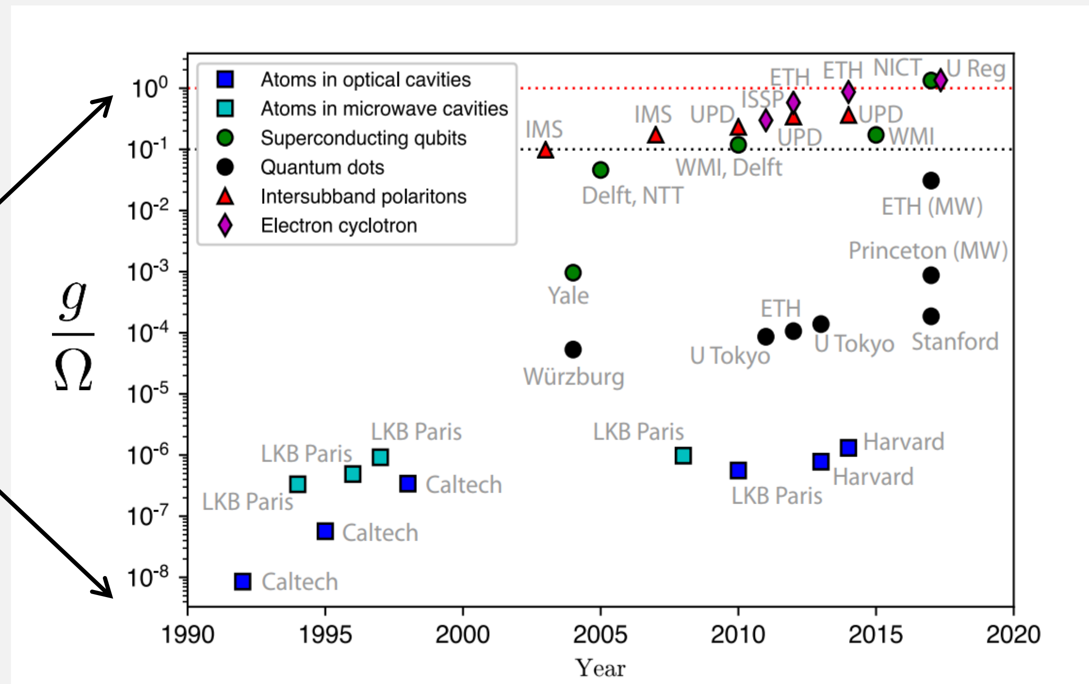
Cavity QED



- Quantum Rabi model

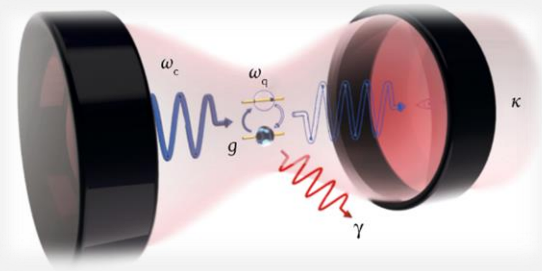
$$H = \frac{\Delta}{2}\sigma^z + \Omega b^\dagger b + g\sigma^x(b + b^\dagger)$$

8 orders of magnitude for exploring light-matter interaction



Reviews por Kockum *et al*, Nat Rev Phys (2019) y Forn-Díaz et al, arxiv:2018

The experimental state of the art (cavity QED)



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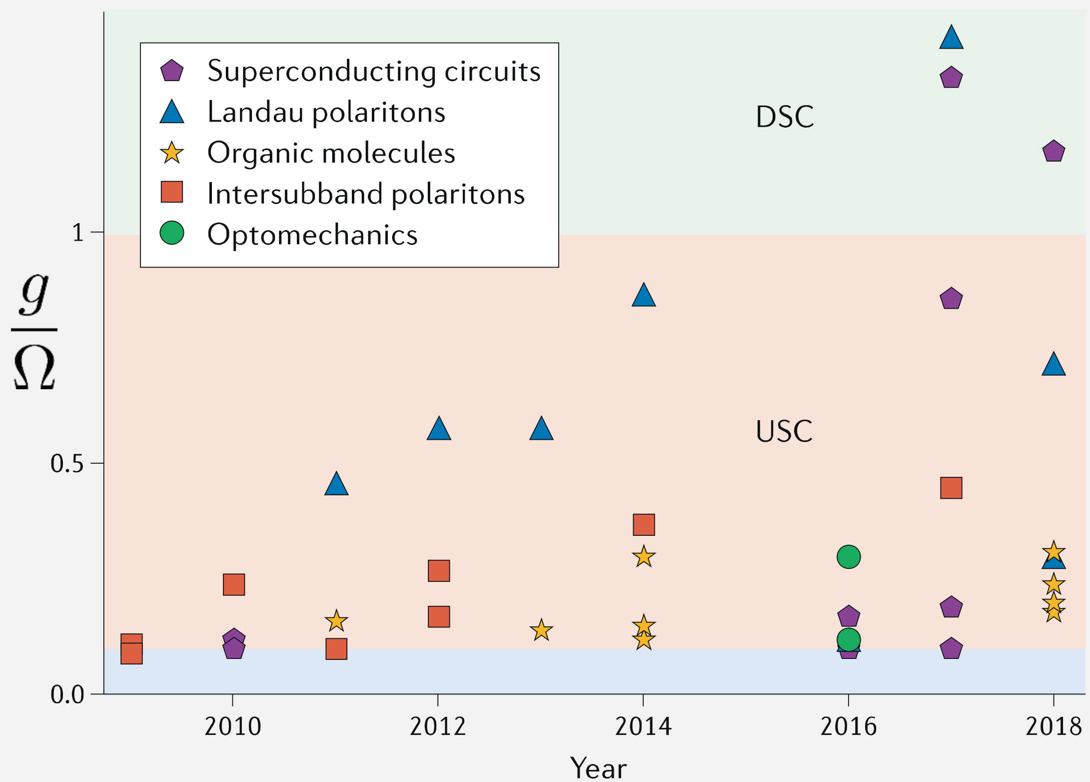


$\frac{\Delta}{g} \to 0$

RWA fails:
$\sigma^+ a^\dagger + h.c.$
play a role.
FULL MODEL

$H_I \cong g\sigma^+ a + \mathrm{h.c.}$
RWA (JC model)

Reviews por Kockum *et al*, Nat Rev Phys (2019) y Forn-Díaz et al, arxiv:2018

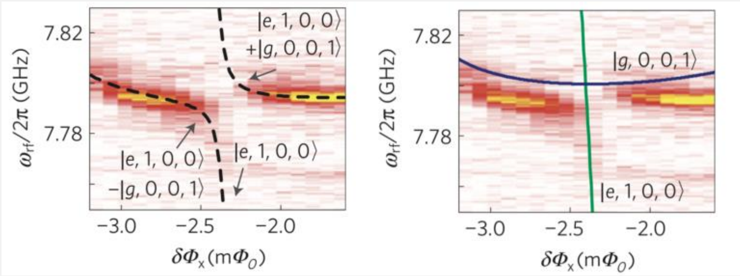
RWA fails →USC (Ultrastrong Coupling Regime)

$$H = \frac{\Delta}{2}\sigma^z + \Omega b^\dagger b + g\sigma^x(b + b^\dagger)$$

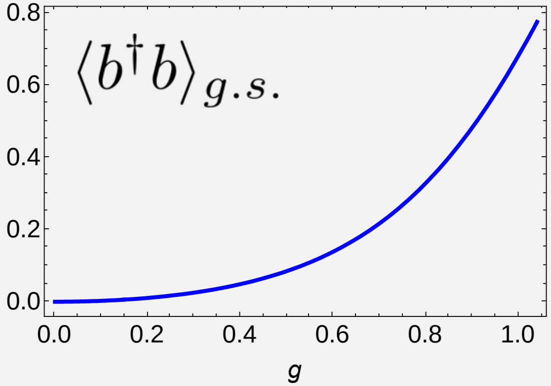
- The terms $\sigma^+ b^\dagger + h.c.$:

  - Transitions $|n\downarrow\rangle \leftrightarrow |n+1\uparrow\rangle$ allowed

T Niemczyk



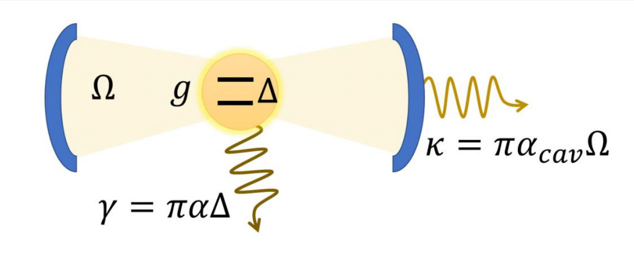
Niemczyk *et al* Nat. Phys. 2010



  - The g.s. is "non trivial"
    $\left(\sigma^+ b^\dagger + h.c.\right)|0\downarrow\rangle \neq 0$

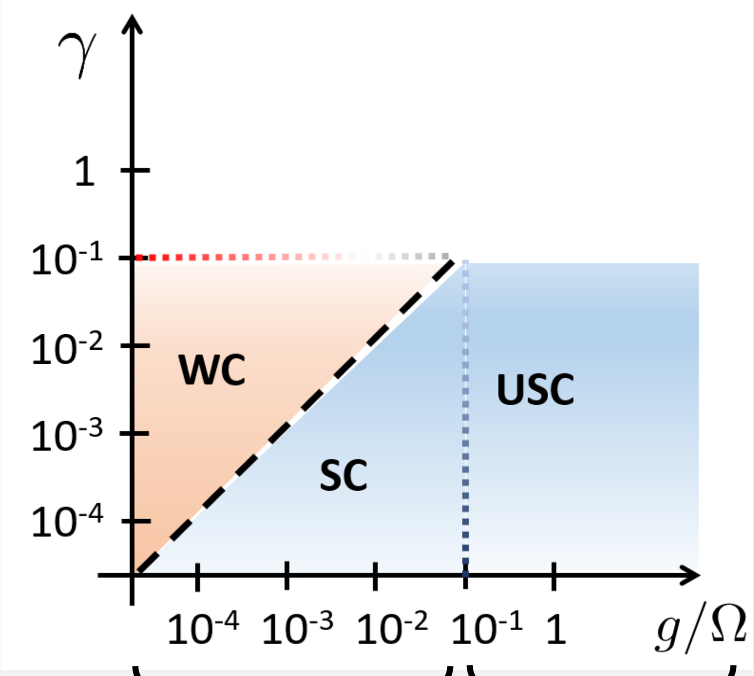
⇒dressed atom $\langle\sigma_z\rangle \succ -1$ and field

light-matter regimes



dissipation

$$H = \frac{\Delta}{2}\sigma^z + \Omega b^\dagger b + g\sigma^x(b + b^\dagger) + H_{\gamma,\kappa}$$



dissipation via QME
(pert. theory)

$$\partial_t \varrho = -i[H, \varrho] + D[\varrho]$$

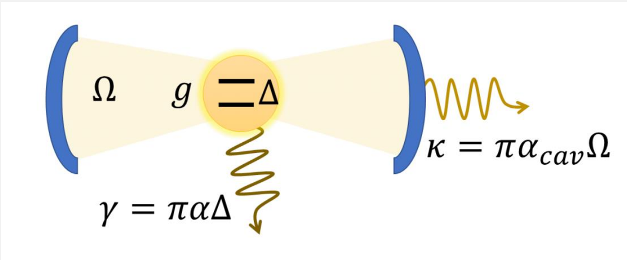
RWA

$$H_I \cong g\sigma^+ a + \text{h.c.}$$

non RWA

$$H_I = g\sigma^x(b + b^\dagger)$$

light-matter regimes



dissipation

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QME approach fails

dissipation via QME
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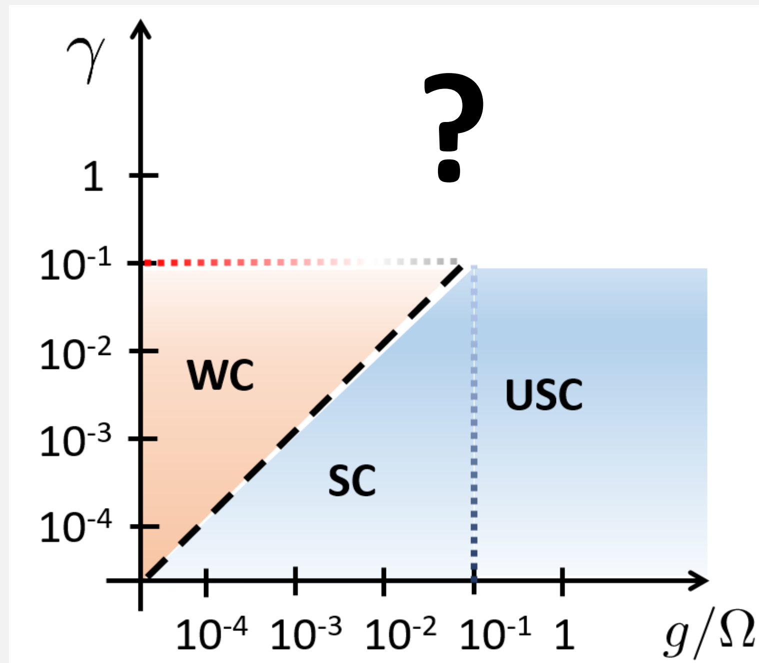

RWA

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non RWA

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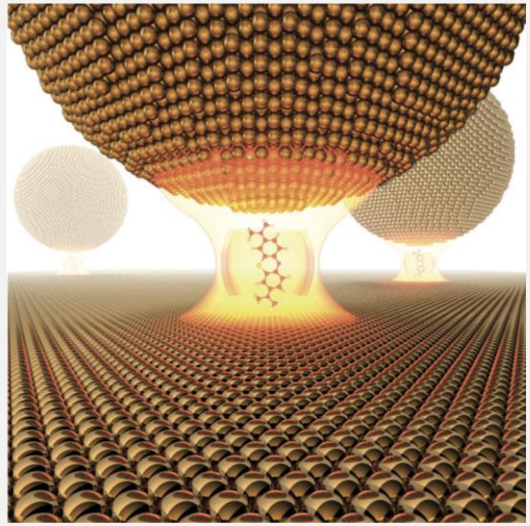
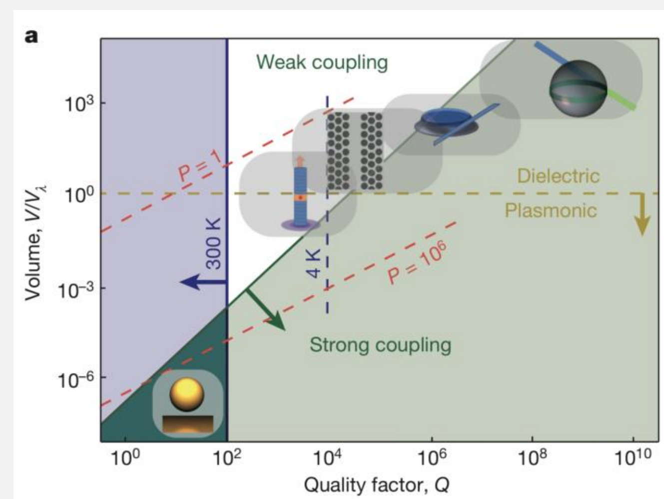
Benasque: Quantum nanophotonics



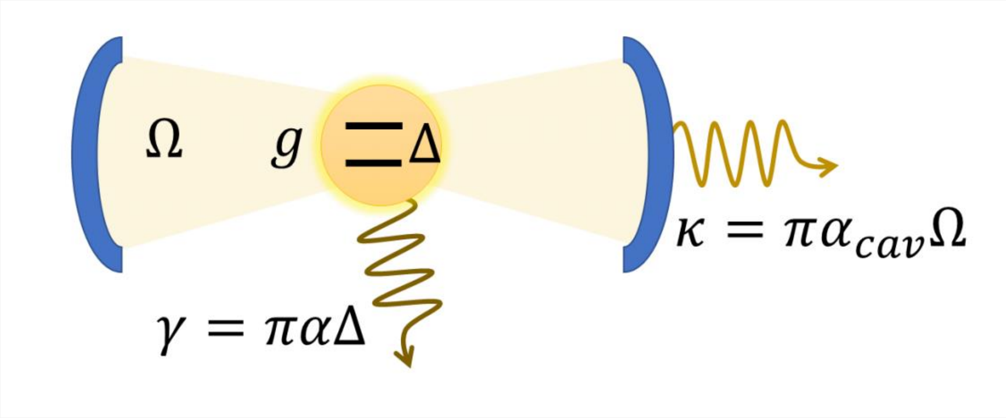
Increase the coupling → reduce the field volume → reduce the Quality factor

$$g \sim \frac{1}{\sqrt{V}}$$

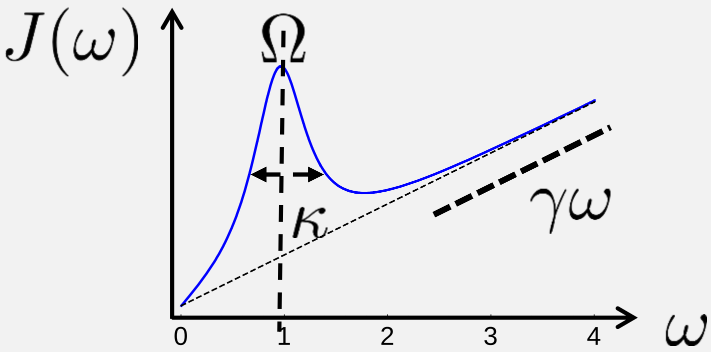


Chikkaraddy *et al* Nature (2016)

Cavity QED = spin-boson



$$H = \frac{\Delta}{2}\sigma_z + \sigma_x \sum_{k'}^{2N+1} c_{k'}(b_{k'}^\dagger + b_{k'}) + \sum_{k'}^{2N+1} \omega_{k'} b_{k'}^\dagger b_{k'} \ .$$



$$J(\omega) = \pi\gamma\omega + \frac{4g^2\pi\kappa\Omega^2\omega}{(\Omega^2-\omega^2)^2 + (\pi\kappa\Omega\omega)^2}$$

DZ, García-Ripoll PRA (2019)

Polaron in a nutshell

- Unitary transform that disentangles emitter(s) and environment (bosons)

$$|GS\rangle = U_p|0;\mathbf{0}\rangle$$

- Displaced resonators (interpolate between $\Delta = 0$ and $g_k = 0$)

$$U_p = \exp\left[-\sigma_x \sum (f_k b_k^\dagger - f_k^* b_k)\right] \ ,$$

- f_k varational parameters:

$$f_k = \frac{g_k}{\Delta_r + \omega_k}, \text{ and } \Delta_r = \Delta e^{-2\sum_k |f_k|^2}.$$

Silbey Harris J Chem Phys (1984)

- It turns out that in the polaron picture ($H_p = U_p^\dagger \, H \, U_p$)

$$H_p|0;\mathbf{0}\rangle = E_{gs}|0;\mathbf{0}\rangle$$

Low-Energy dynamics in the Polaron frame

- In the Polaron picture, $H_p = U_p^\dagger \, H \, U_p$:

$$H_p = \Delta_r \sigma^+ \sigma^- + \sum_{k=1}^{N} \omega_k b_k^\dagger b_k - 2\Delta_r \left(\sigma^+ \sum_{k=1}^{N} f_k b_k + \mathrm{H.c.} \right)$$

$$- 2\Delta_r \sigma_z \sum_{k,p=1}^{N} f_k^* f_p b_k^\dagger b_p$$

$$+ \frac{\Delta}{2} + \sum_{k=1}^{N} (\omega_k |f_k|^2 - g_k^* f_k - f_k^* g_k) + \mathrm{h.o.t.}$$

- RWA physics (Wigner-Weisskopf, Rabi Oscillations)

- TLS renormalization

Low-Energy dynamics in the Polaron frame

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$$- 2\Delta_r \sigma_z \sum_{k,p=1}^{N} f_k^* f_p b_k^\dagger b_p$$

$$+ \frac{\Delta}{2} + \sum_{k=1}^{N} (\omega_k |f_k|^2 - g_k^* f_k - f_k^* g_k) + \text{h.o.t.}$$

- RWA physics (Wigner-Weisskopf, Rabi Oscillations)

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Spin-boson model

- A TLS coupled to a bosonic environment

$$H = \frac{\Delta}{2}\sigma^z + \sum_k \omega_k\, b_k^\dagger b_k + \sigma^x \sum_k \left(g_k b_k^\dagger + \text{h.c.}\right)$$

- Spectral function

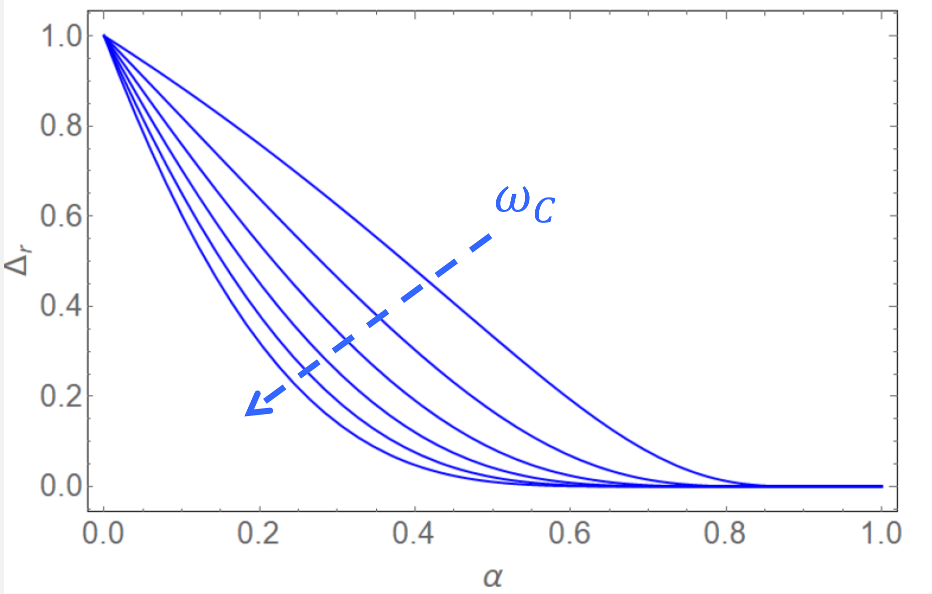
$$J(\omega) = 2\pi \sum_k g_k^2(\omega_k - \omega)$$

- Ohmic: $J(\omega) \sim \omega$, with cutoff ω_C

- Qubit freq. renormalization:

$$\frac{\Delta_r}{\Delta} = \left(\frac{\Delta}{\omega_c}\right)^{\frac{\alpha}{1-\alpha}}$$

Leggett *et al*, Rev Mod Phys (1987)



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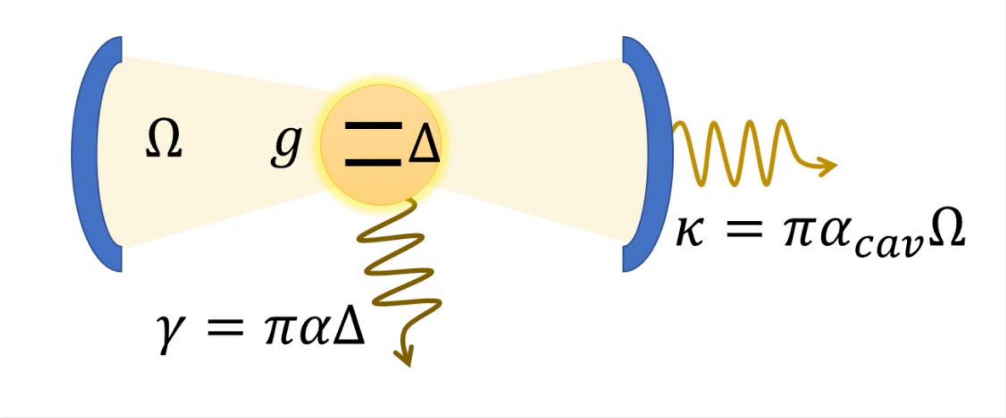
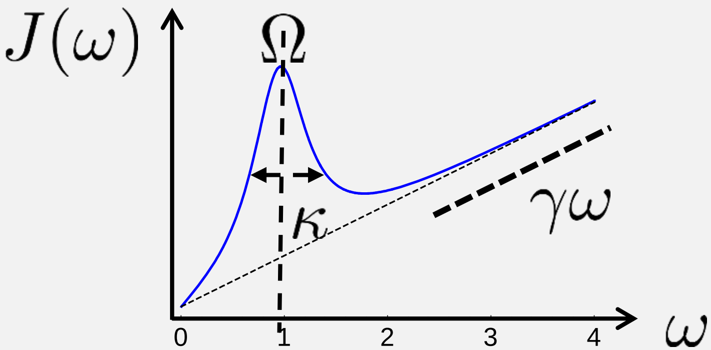


$$H = \frac{\Delta}{2}\sigma_z + \sigma_x \sum_{k'}^{2N+1} c_{k'}(b_{k'}^\dagger + b_{k'}) + \sum_{k'}^{2N+1} \omega_{k'} b_{k'}^\dagger b_{k'} \ .$$



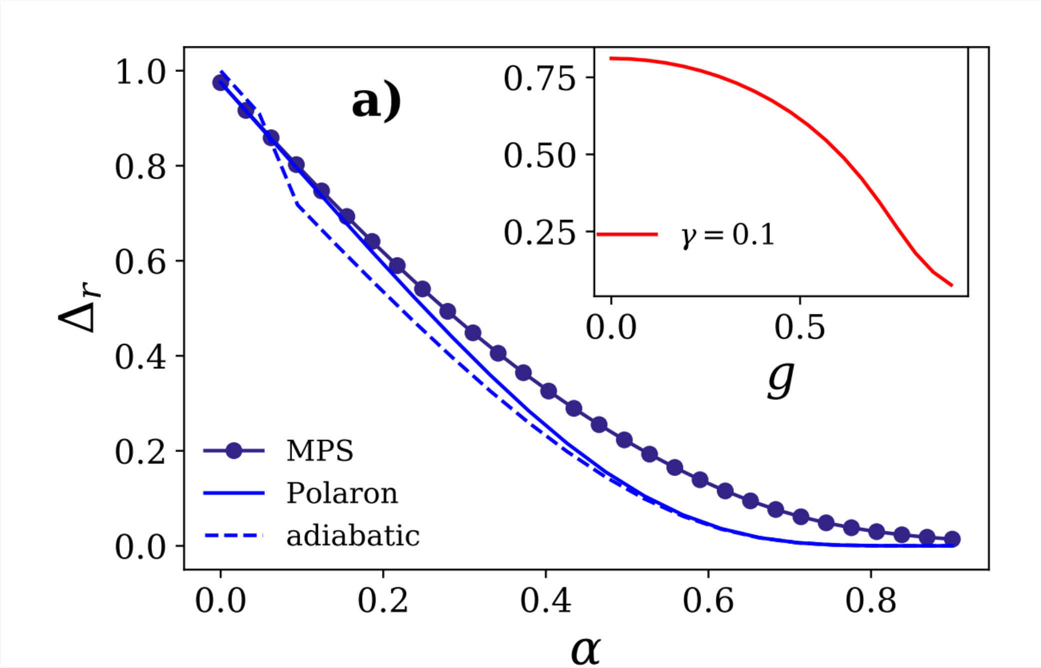
$$J(\omega) = \pi\gamma\omega + \frac{4g^2\pi\kappa\Omega^2\omega}{(\Omega^2-\omega^2)^2 + (\pi\kappa\Omega\omega)^2}$$

DZ, García-Ripoll PRA (2019)

Qubit renormalization freq.

$$\Delta_r = \Delta \mathrm{e}^{-1/2 \int_0^{\omega_c} J(\omega)/(\omega+\Delta_r)^2}$$
$$g_r \cong g$$
$$\Omega_r = \Omega$$

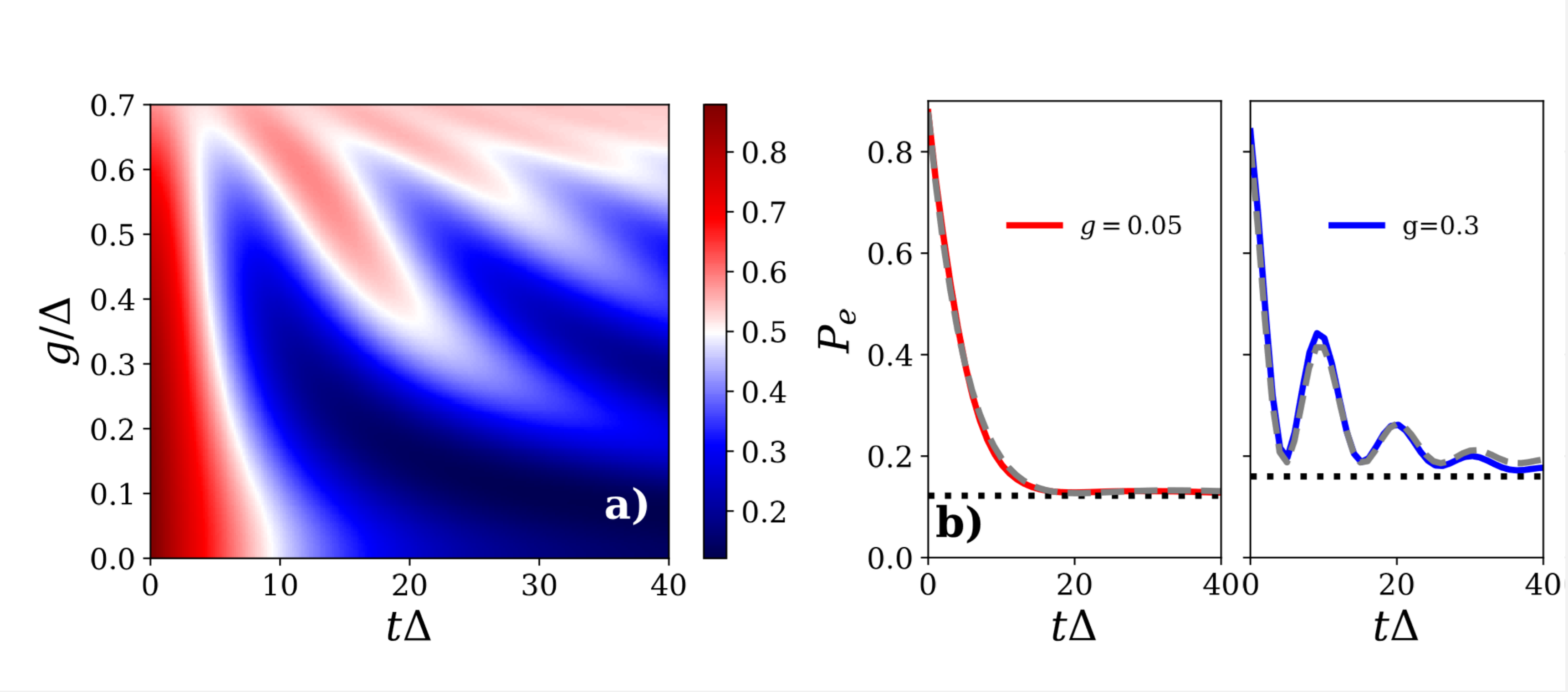
- Only the qubit (nonlinear element) is renormalized.



→ New resonance condition:
$\Omega = \Delta_r$

DZ, García-Ripoll PRA (2019)

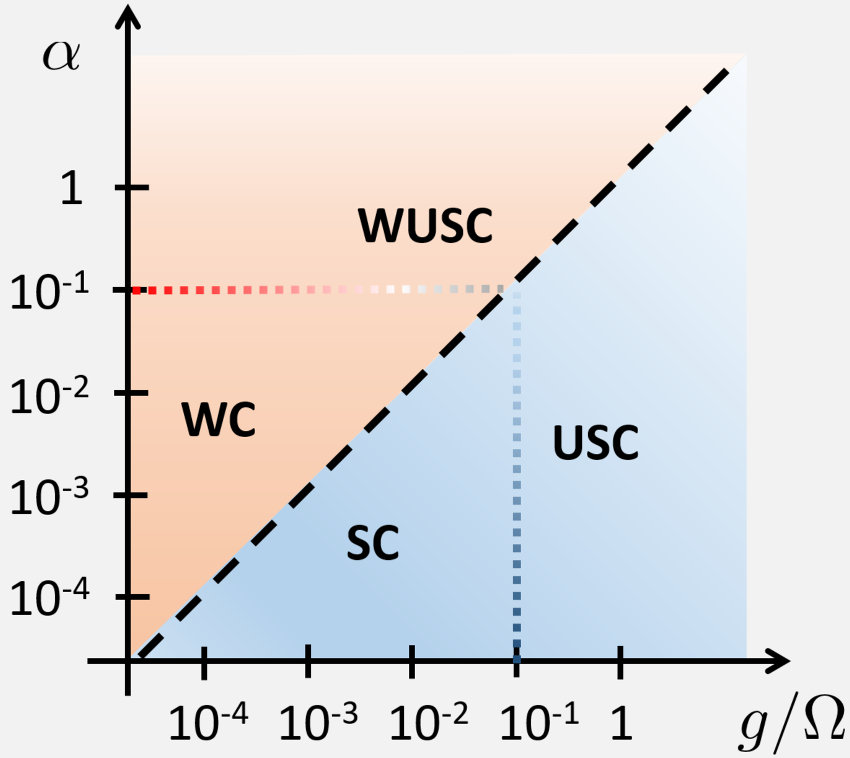
Rabi oscillations



- Renormalized resonance $\Delta_r = \Omega$
- Dynamics understood via polaron $P_e \cong \left(1 - P_e^{eq}\right) P_e^{RWA}(\Delta_r) + P_e^{eq}$
- Onset of Rabi oscillations $g > |\gamma_r - \kappa|/4$

DZ, García-Ripoll PRA (2019)

light-matter beyond QME regime



DZ, García-Ripoll PRA (2019)

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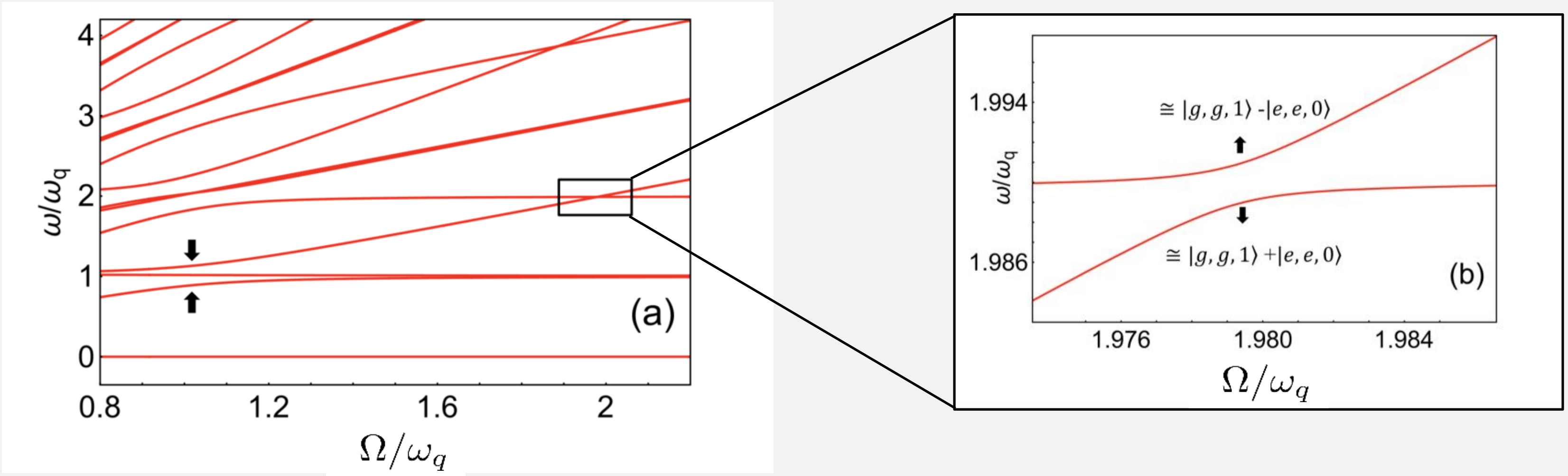
1 photon can simultaneously excite 2 atoms

$$H = \frac{\Delta}{2}(\sigma_1^z + \sigma_2^z) + \frac{\epsilon}{2}(\sigma_1^x + \sigma_2^x) + \Omega b^\dagger b + g(\sigma_1^x + \sigma_2^x)(b + b^\dagger)$$

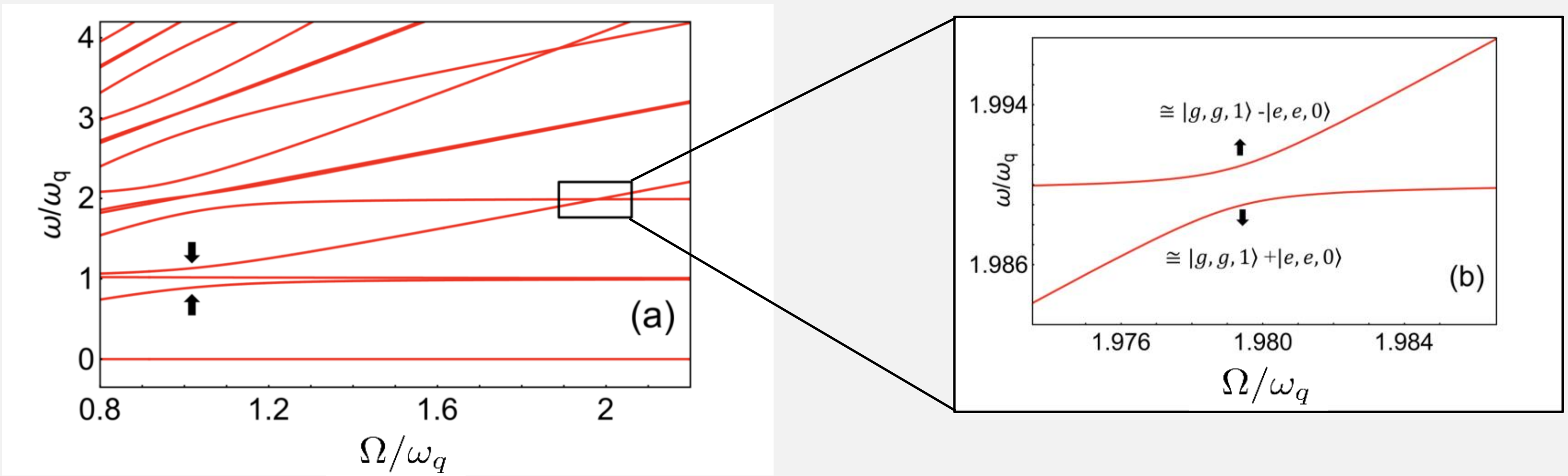


- $|g, g, 1\rangle \leftrightarrow |e, e, 0\rangle$
- No number conserving terms
- Parity Breaking
- 3rd order $\sim g^3/\omega_q^2$

Garziano *et al* PRL (2016)

1 photon can simultaneously excite 2 atoms

$$H = \frac{\omega_q}{2}(\sigma_1^z + \sigma_2^z) + \Omega b^\dagger b + g\Big(\cos\theta(\sigma_1^x + \sigma_2^x) + \sin\theta(\sigma_1^z + \sigma_2^z)\Big)(b + b^\dagger)$$



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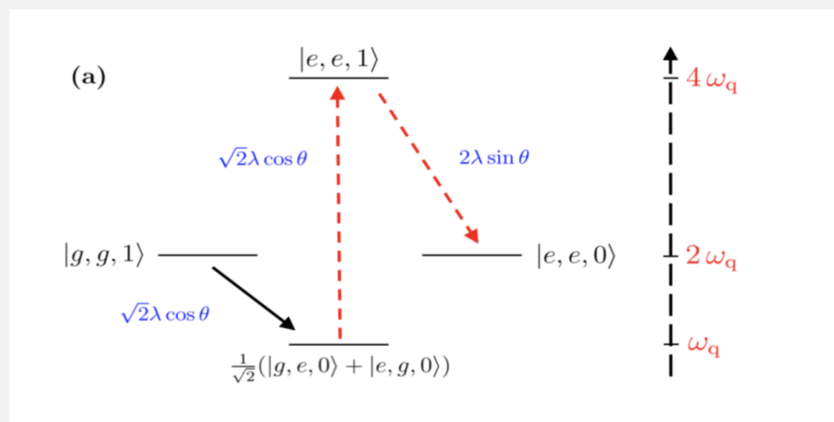
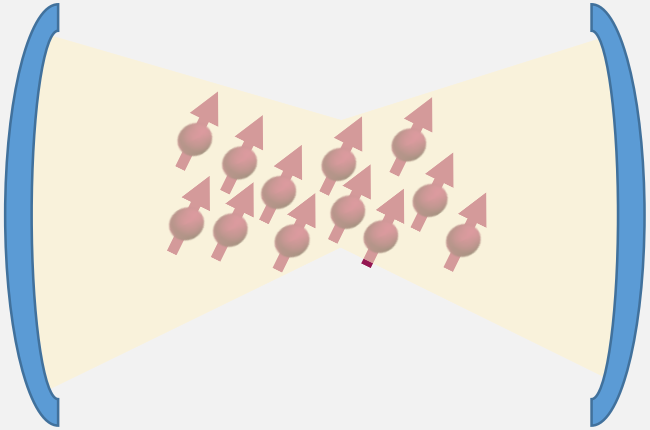


Garziano *et al* PRL (2016)

Generalization to ***N*** spins: Dicke model



parity breaking

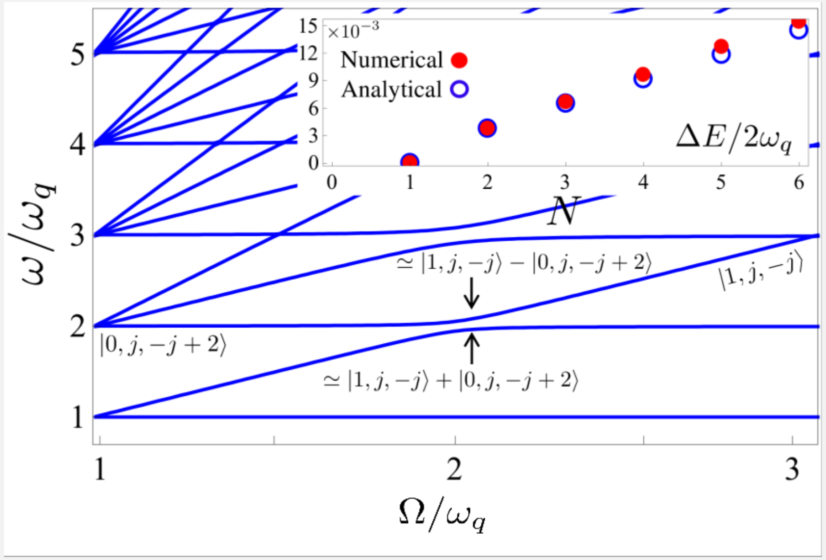
$$\hat{H} = \frac{\Delta}{2}\hat{J}_z + \frac{\epsilon}{2}\hat{J}_x + \Omega\,\hat{a}^\dagger\hat{a} + g(\hat{a}+\hat{a}^\dagger)\hat{J}_x$$

$$\hat{J}_\alpha = \sum_i^N \hat{\sigma}_\alpha^i \quad (\alpha = x, y, z)$$

- At the resonance $\Omega = 2\,\omega_q = 2\sqrt{\Delta^2+\epsilon^2}$ there is a gap. The transition:

$$\hat{a}^\dagger|0,j,-j\rangle \leftrightarrow \hat{J}_+^2|0,j,-j\rangle$$

- The gap grows with *N* (~superradiance)



Macri, Nori, Savasta, DZ, arxiv:1902.10377

Effective model

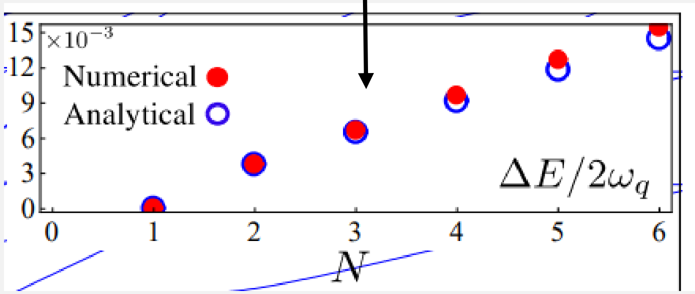
- 3rd order pert. theory (James´ method)

$$\hat{H}_{\rm eff} = g_{\rm eff}\left(\hat{a}\hat{J}_+^2 + \hat{a}^\dagger \hat{J}_-^2\right) \, ,$$

with

$$g_{\rm eff} = -\frac{4g^3\cos^2\theta\sin\theta}{3\omega_q^2}$$

gap @ anticrossing as *N*



- Spin-spin interaction (entanglement)

$$|\psi(0)\rangle = \cos\varphi|0,j,-j\rangle + \sin\varphi|1,j,-j\rangle$$

$$|\psi(t)\rangle = \cos\varphi|0,j,-j\rangle + \sin\varphi\cos(g_{\rm eff}t)|1,j,-j\rangle - i\sin\varphi\sin(g_{\rm eff}t)|0,j,-j+2\rangle$$

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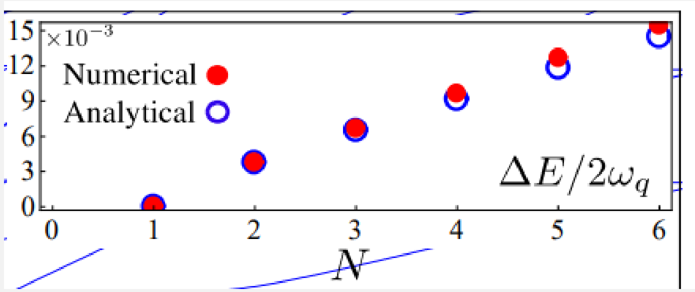

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$$\downarrow$$

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Macri, Nori, Savasta, DZ, arxiv:1902.10377

Spin squeezing

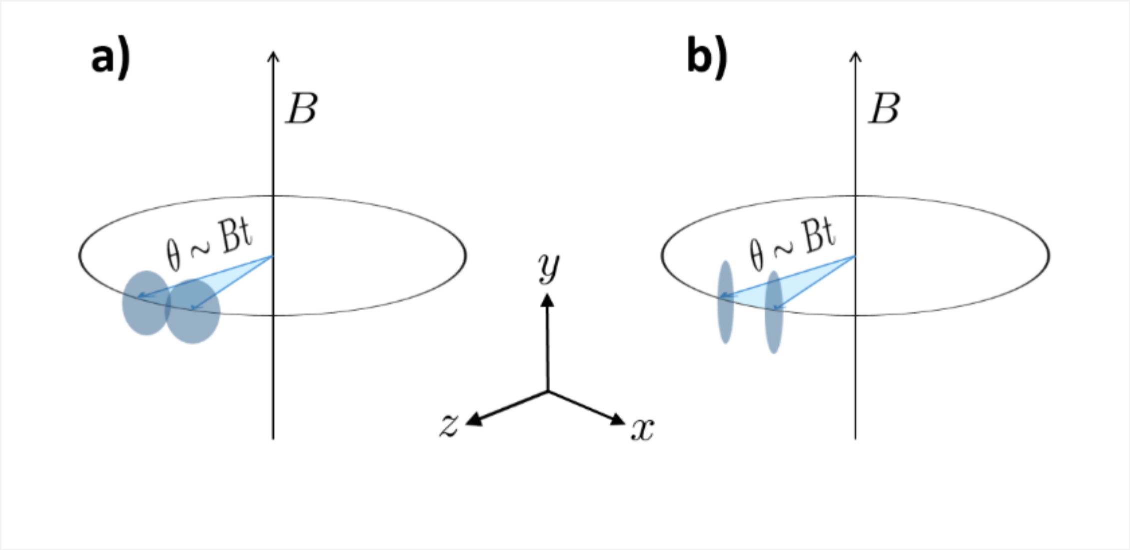
- Heisenberg:

$$(\Delta J_x)^2\,(\Delta J_y)^2 \geq \frac{1}{4}|\langle J_z\rangle|^2$$

- Squeezing def.

$$(\Delta J_y)^2 < \frac{1}{2}|\langle J_z\rangle| \rightarrow \xi^2 = \frac{2(\Delta J_y)^2}{j}$$

- A resource to metrology (magnetometer):



Macroscopic spin squeezing

- Including dissipation (cavity leakage + TLS dissipation)

$$\dot{\hat{\varrho}} = -i[\hat{H}, \hat{\varrho}] + \kappa\mathcal{D}[\hat{a}] + \frac{\gamma}{N}\mathcal{D}[\hat{J}_-]$$

- Squeezing evolves as two-axis twisting Hamiltonian

$$\frac{d\xi^2}{dt} = -\big(i4g_{\rm eff}N\langle\hat{a}\rangle + \gamma\big)\xi^2 + \gamma\,,$$

- It is optimal $\xi^2 \sim \frac{1}{N}$ and it squeezes exponential in time

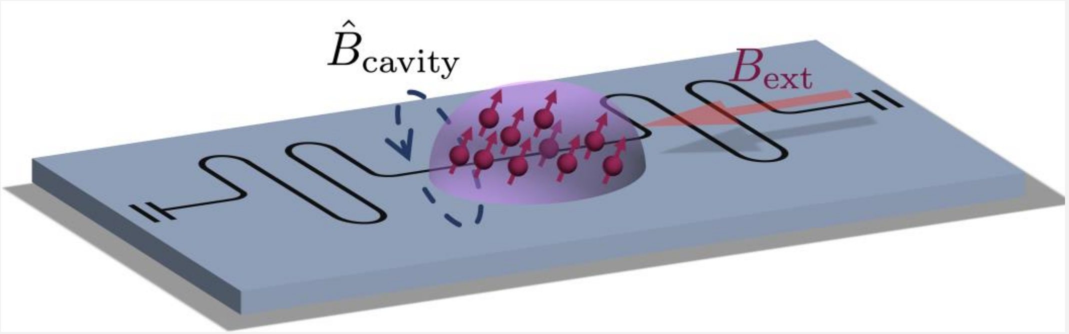
- Compare with bad-cavity limit schemes: global dissipation $\xi^2 \sim \frac{1}{\sqrt{N}}$

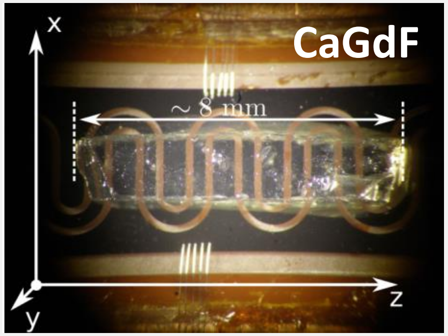
Sorensen and Molmer PRA 2002 , Borreegaard *et al* NJP 2017, Norcia *et al* Science 2018, etc ...

Macri, Nori, Savasta, DZ, arxiv:1902.10377

Dicke model with hybrid systems

- Magnetic molecules coupled to superconducting circuits

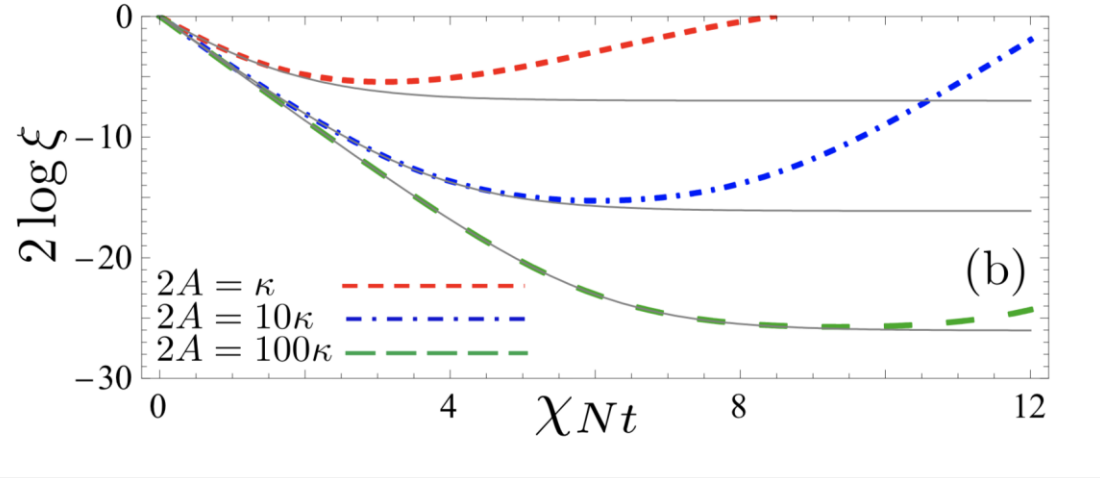




$$H \sim -\vec{\mu}\,\vec{B} \rightarrow g\,J_x(a^+ + a)$$

- Driving the cavity
  $\sqrt{n_{ph}} = 2\frac{A}{\kappa}$

- Reaches a max. squeezing:
  $\xi^2 \sim \frac{\gamma}{g_{eff}N\sqrt{n_{ph}}}$



Macri, Nori, Savasta, DZ, arxiv:1902.10377

USC Collaborators & funding

- Luis Martín-Moreno (ICMA-Zaragoza)
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- Salvatore Savasta (U Messina)
- Franco Nori (Riken-Tokio)
- Vincezo Macri (Riken-Tokio)
- Eduardo Sánchez-Burillo (MPQ-Garching)

GOBIERNO DE ESPAÑA — MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

QUANTERA

Quantum Materials and Devices

Fundación BBVA

araid

Conclussions

1. Polaron picture to describe noise (dissipation) beyond the QME approach

DZ, García-Ripoll PRA (2019)

2. USC application: spin-spin interactions for e.g. optimal squeezing generation

Macri, Nori, Savasta, DZ, arxiv:1902.10377

Thank you very much!