Comb tensor networks.
Criticality on a comb lattice

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8 March 2019

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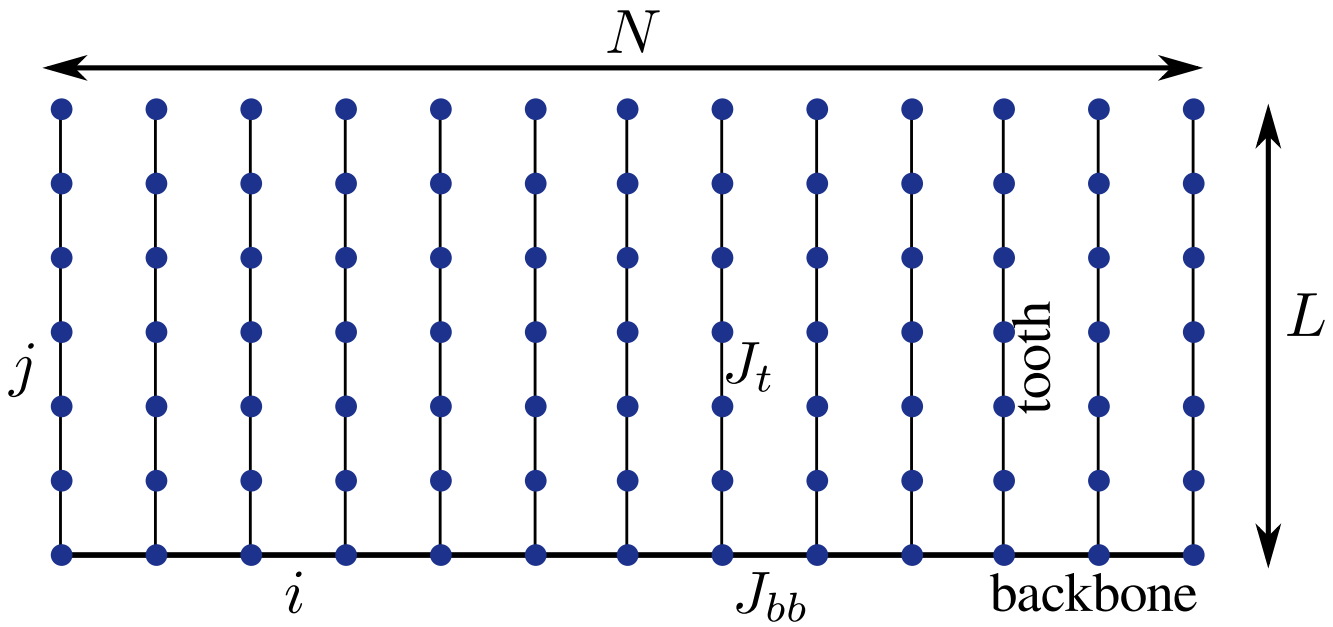
arXiv:1903.00432

Scope

- Comb tensor networks
- Spin-1 comb
  - Emergent spin-1/2 chain - critical backbone
  - "Higher-order" edge states
- Spin-1/2 comb - critical teeth
  - Effective length of critical chains: $L \to 2L \to L-1$
  - Finite-size scaling of the energy gap
- Transverse field Ising comb - critical teeth
  - Induced longitudinal field
  - Transition or crossover?
- Outlook

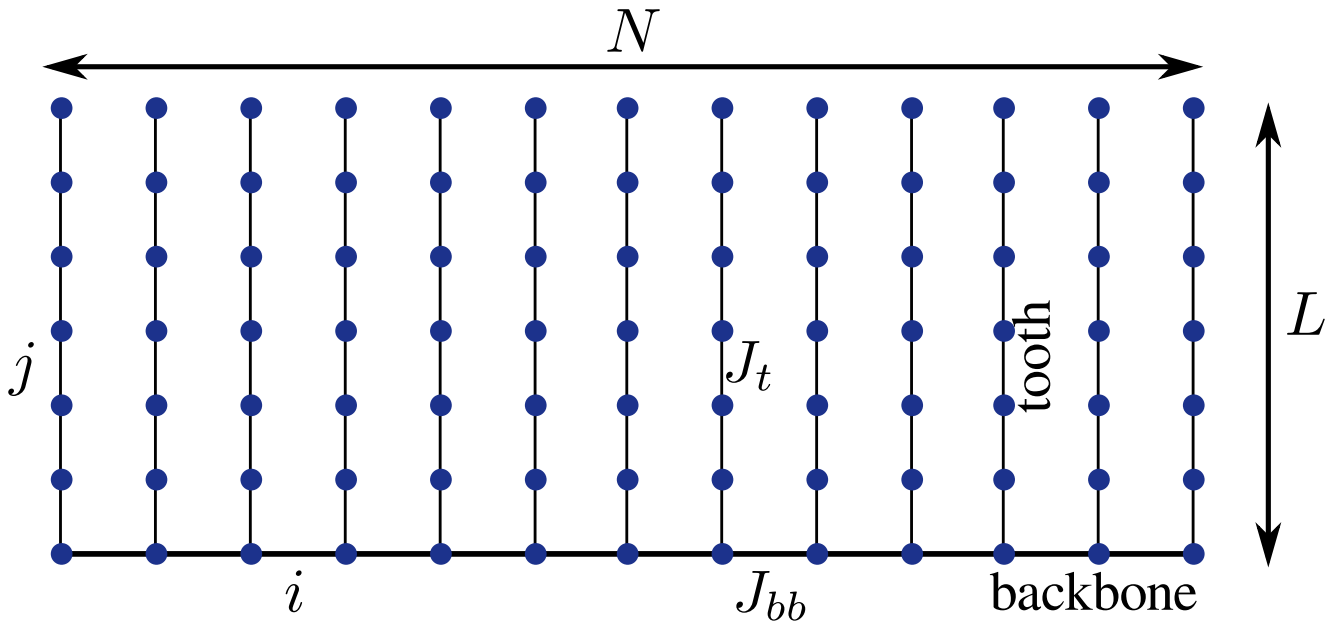
Comb geometry



- Spin chains (teeth) coupled through one edge
- Highly decorated spin chain (backbone)

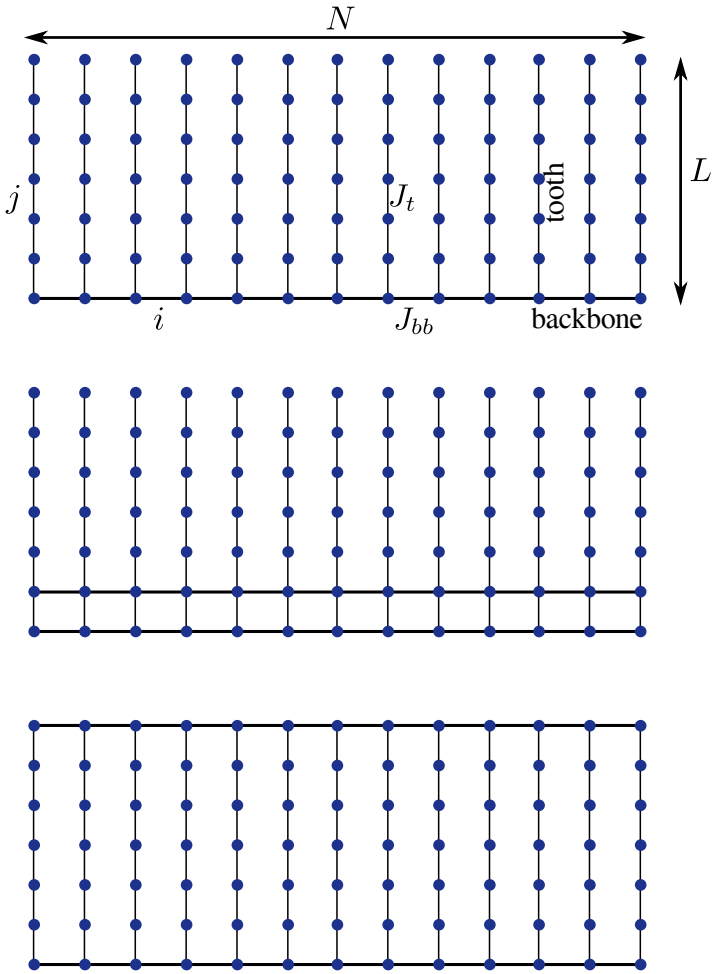
One dimensional... in which direction?

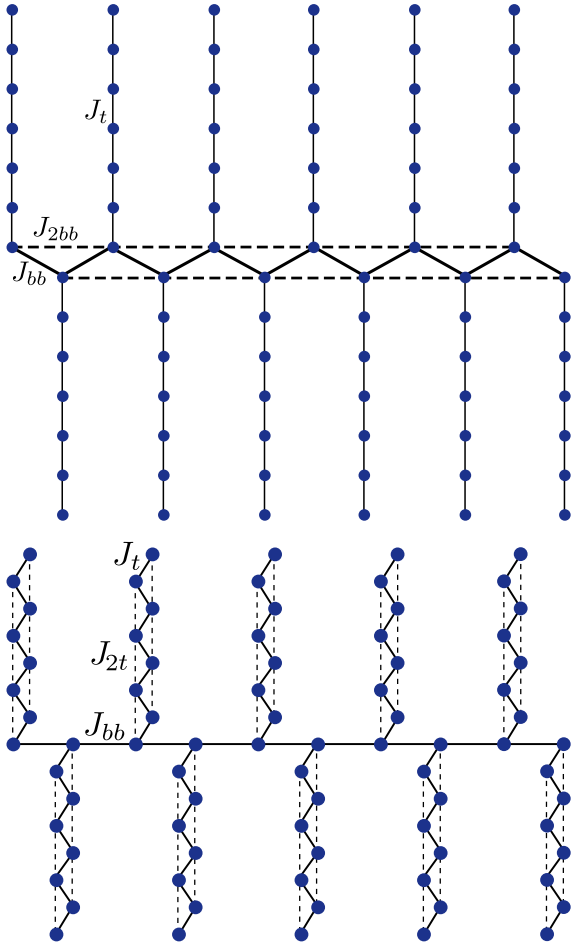
Comb geometry



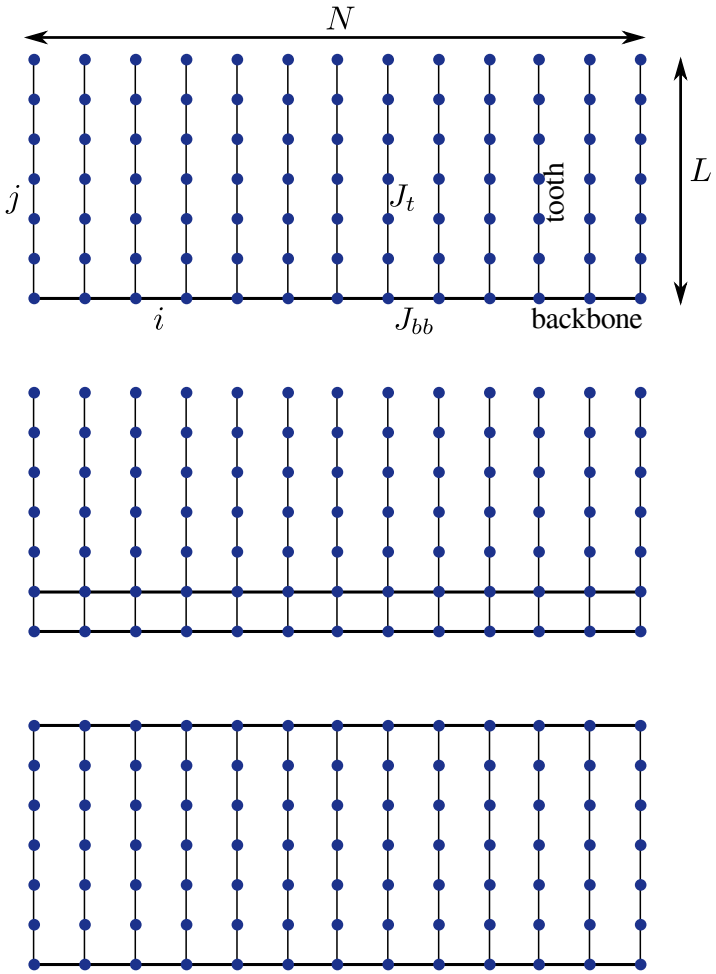
- **Y-DMRG:** Guo, White, Phys. Rev. B **74**, 060401 (2006)
- **Fork tensor networks:**
  Holzner, Weichselbaum, von Delft, Phys. Rev. B **81**, 125126 (2010);
  Bauernfeind, Zingl, Triebl, Aichhorn, Evertz, Phys. Rev. X **7**, 031013 (2017)

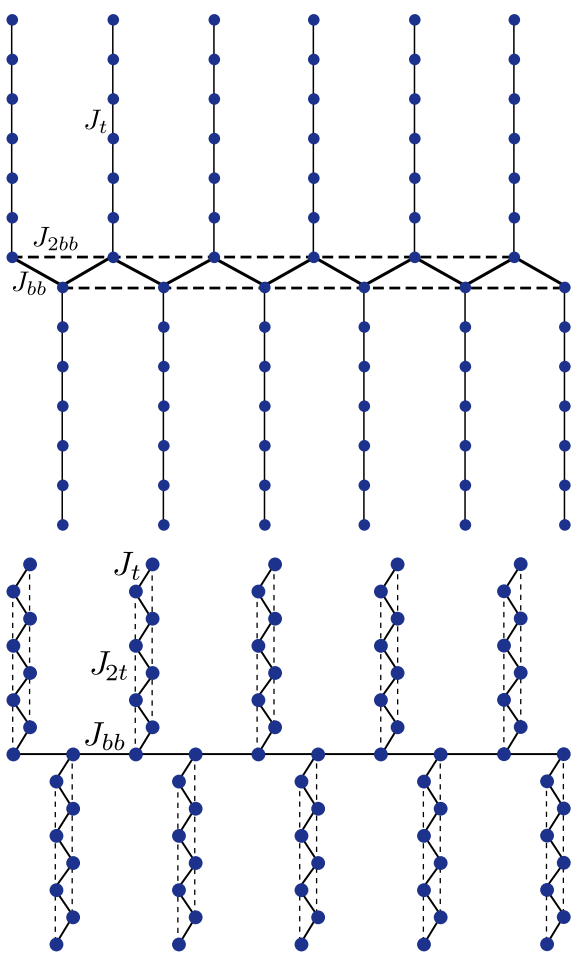
Comb geometry

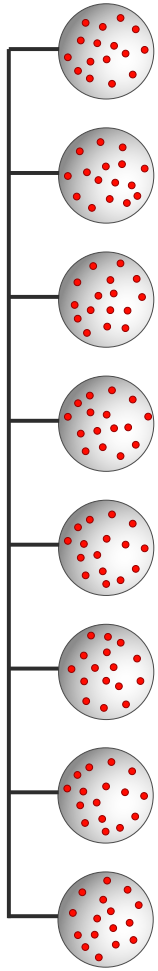




Comb geometry

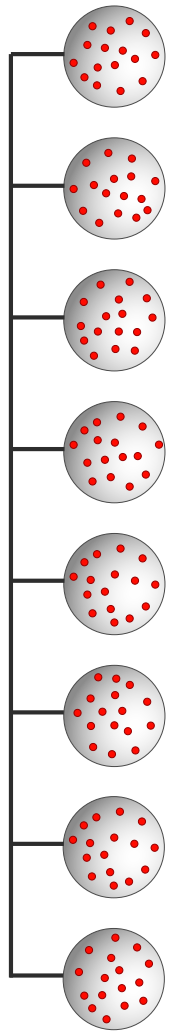




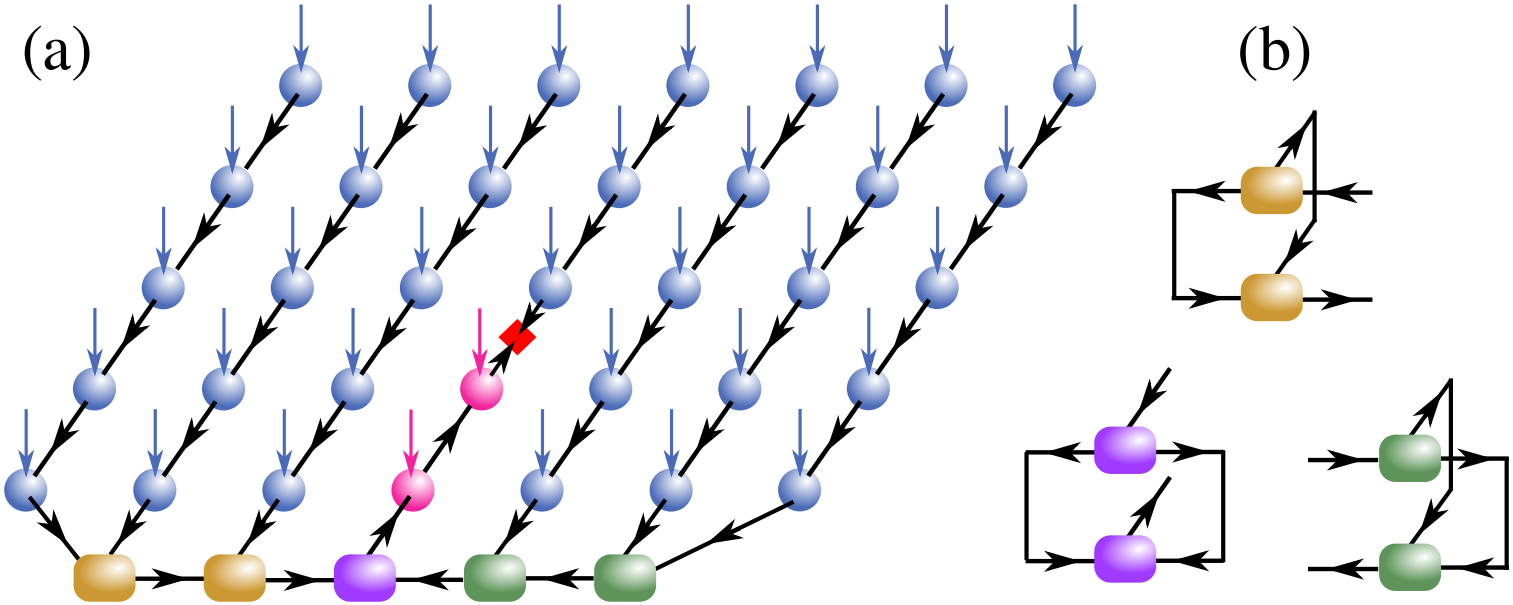


Generic comb

- **The goal is to split two channels of entanglement: along the backbone and within the tooth**
- Finite-size clusters form local degrees of freedom
- Ad-lib complicated interactions within the clusters (DMRG-limited)
- The wave-function is expected to obey the area law



A comb network. Mixed-canonical form

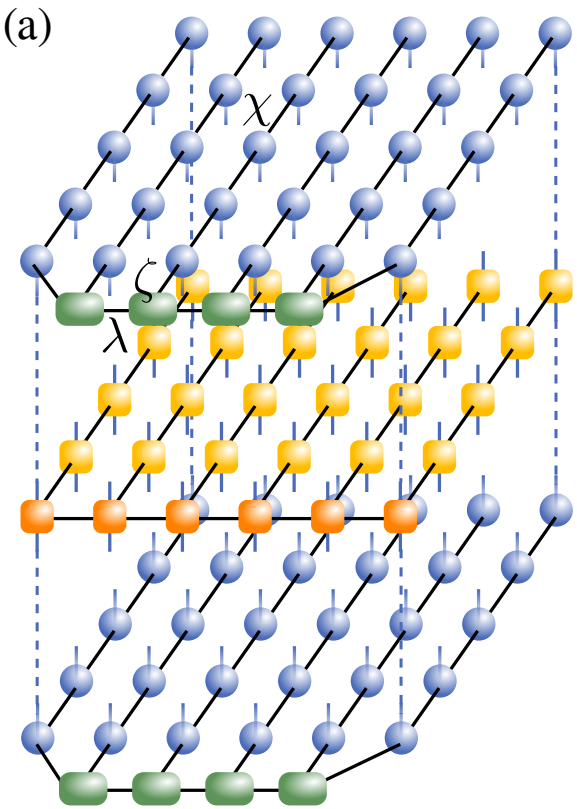


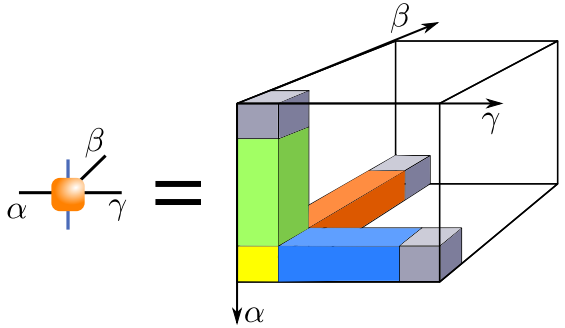
Auxiliary backbone tensors:

- Each tensor is at most of rank 3
- Split degrees of freedom on a backbone

NC, White, arXiv:1903.00432

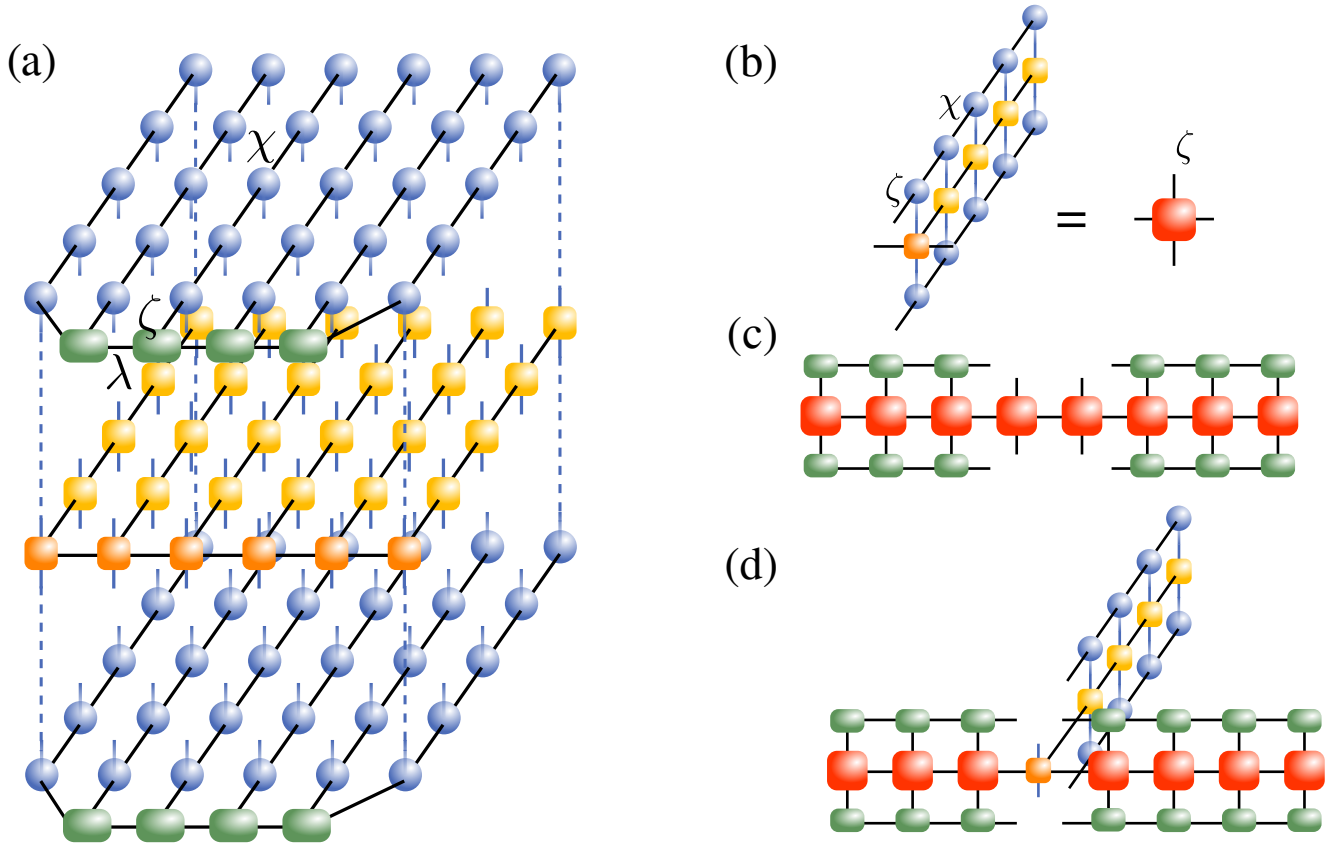
Variational optimization





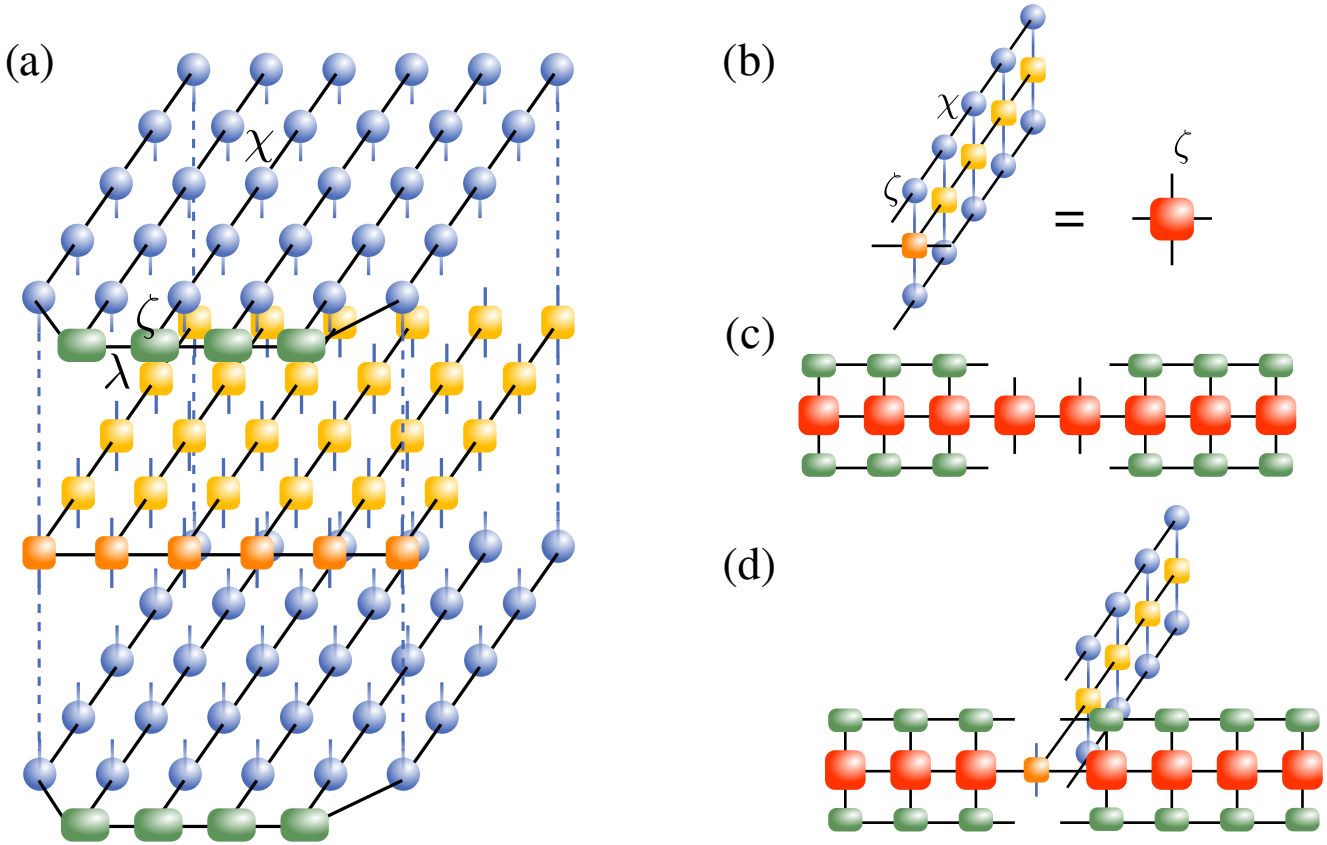
- Hamiltonian in terms of local tensors - PEPO

Variational optimization



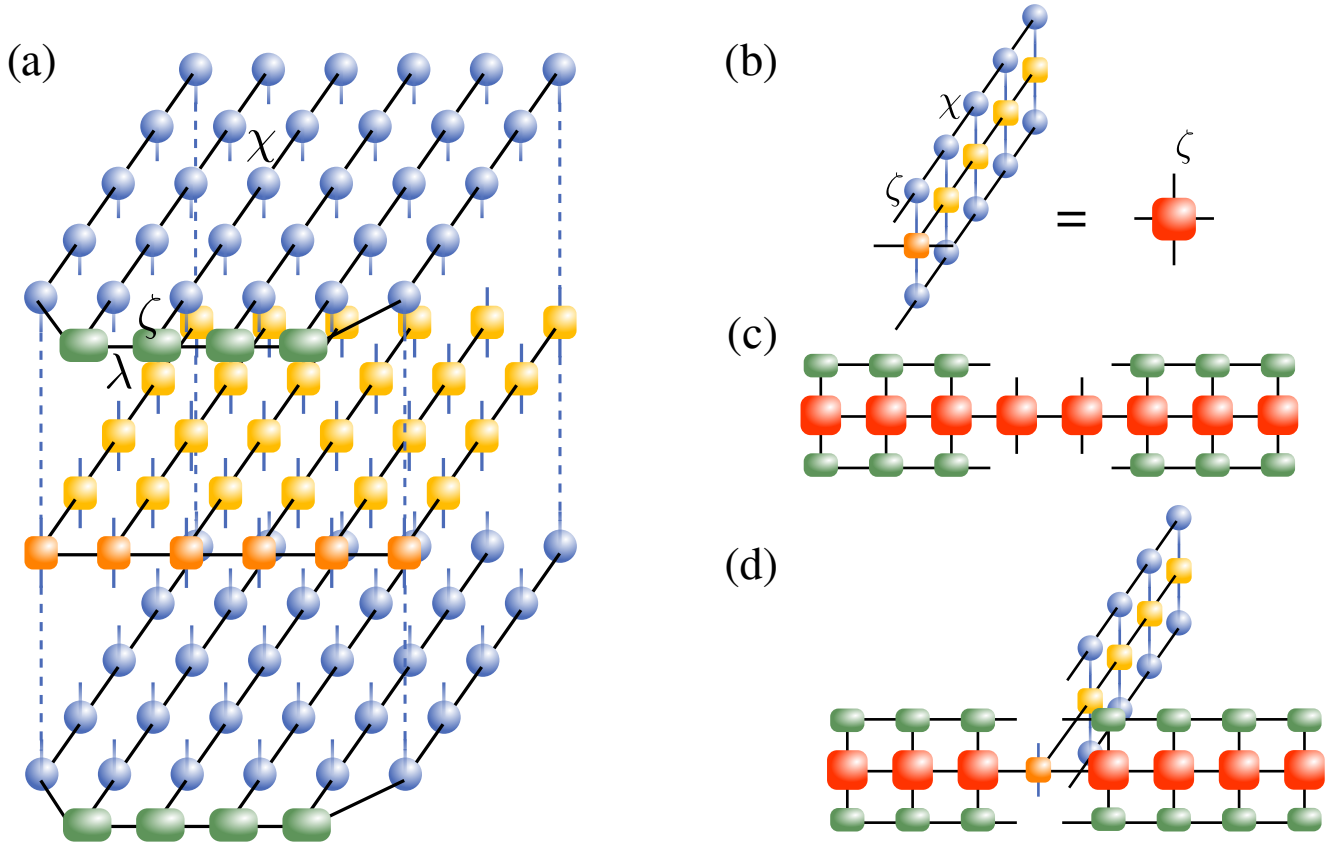
- Hamiltonian in terms of local tensors - PEPO
- Optimization within the tooth = DMRG

Variational optimization



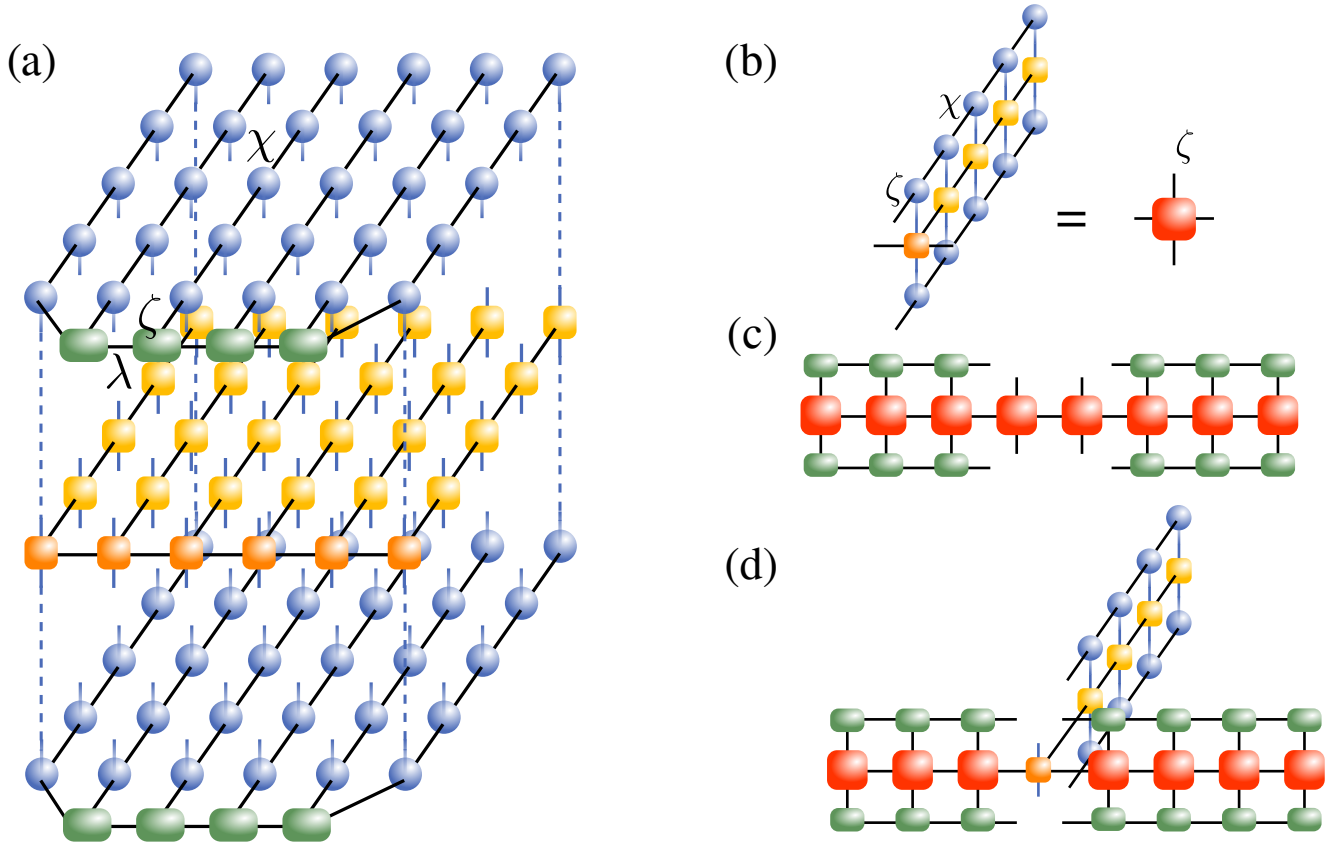
- Hamiltonian in terms of local tensors - PEPO
- Optimization within the tooth = DMRG
- Fully contracted tooth can be viewed as an MPO with fat physical bonds

Variational optimization



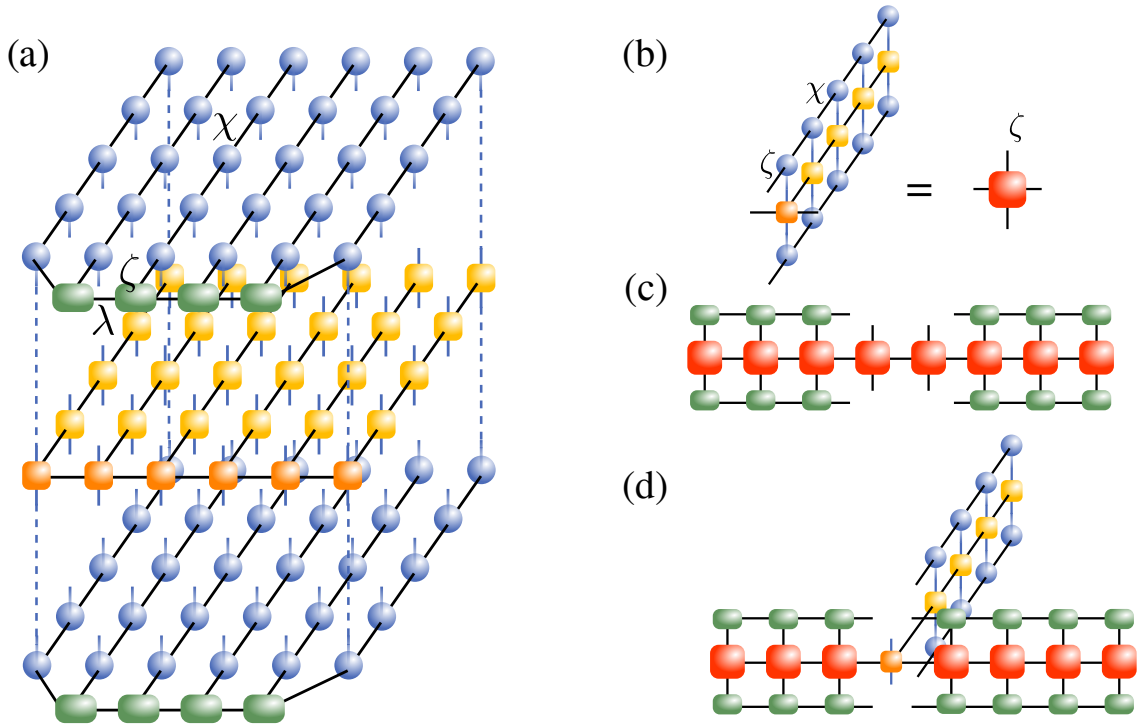
- Optimization within the tooth = DMRG
- Fully contracted tooth can be viewed as an MPO with fat physical bonds
- Optimization of two backbone tensors = DMRG

Variational optimization



- Optimization within the tooth = DMRG
- Optimization of two backbone tensors = DMRG
- Connect update ≠ DMRG and involves three environments

Complexity

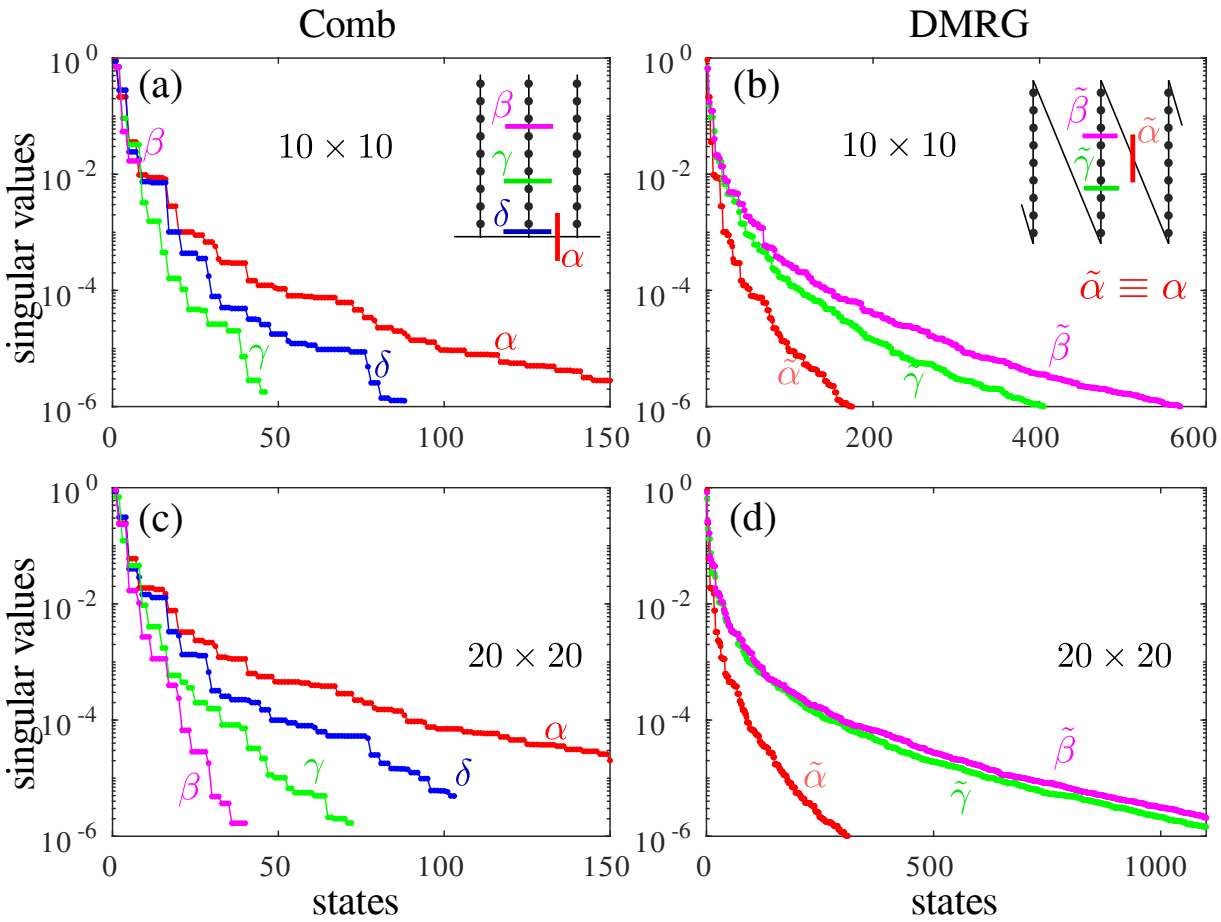


Complexity ($\chi \approx \zeta \approx \lambda \approx D$)

- Backbone update: D^5
- Connect update: D^4
- Tooth update: D^3

For AKLT-like states (finite ξ, ζ) the complexity is λ^3

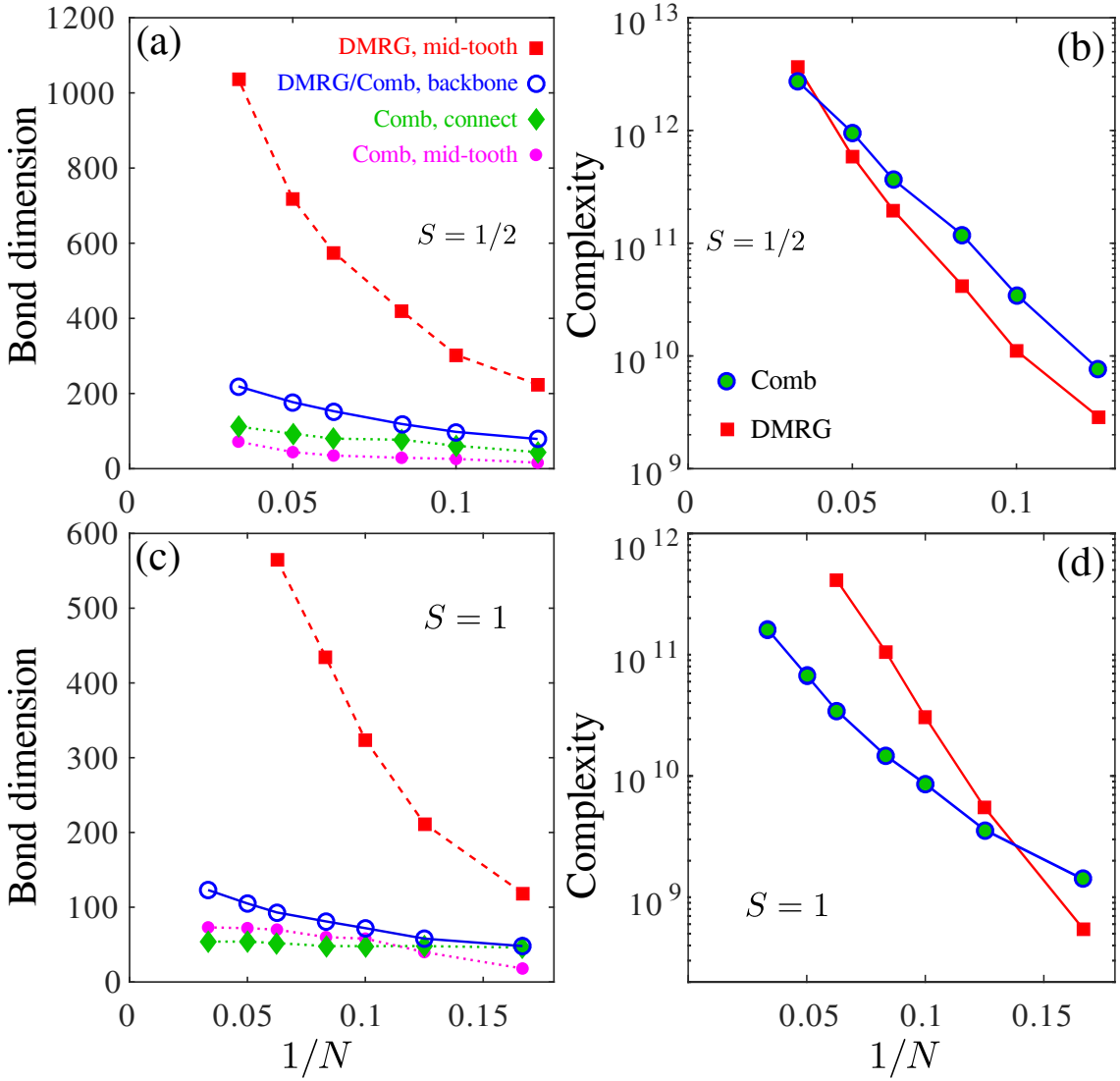
DMRG versus comb. Schmidt values



- Heisenberg spin-1/2
- Backbone cut is the same for the comb and for the DMRG
- DMRG: the largest bond dimension is inside the tooth
- Comb: the bond dimension decreases upon approaching the tip of the tooth

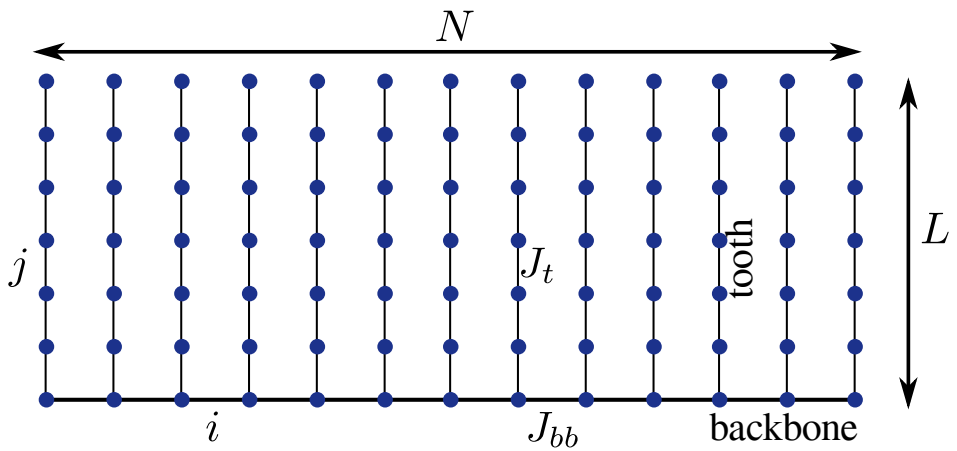
NC, White, arXiv:1903.00432

DMRG versus comb. Complexity



Spin-1 Heisenberg comb

$$H = J_{bb} \sum_{i=1}^{N-1} \mathbf{S}_{i,1} \cdot \mathbf{S}_{i+1,1} + J_t \sum_{i=1}^{N} \sum_{j=1}^{L-1} \mathbf{S}_{i,j} \cdot \mathbf{S}_{i,j+1},$$



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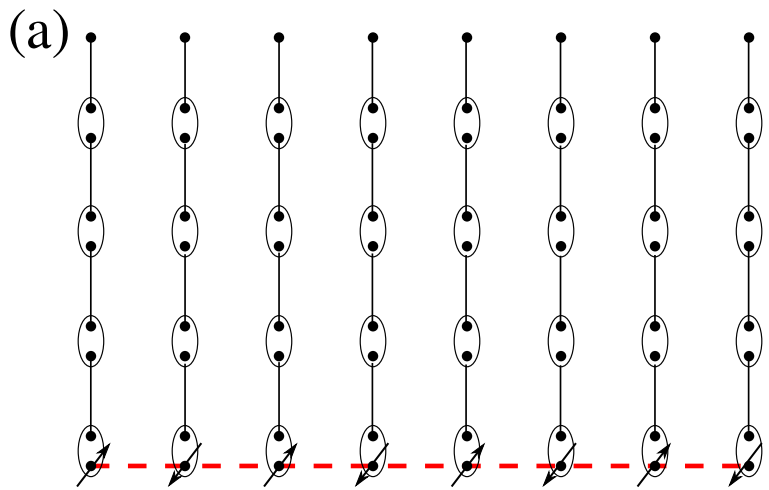
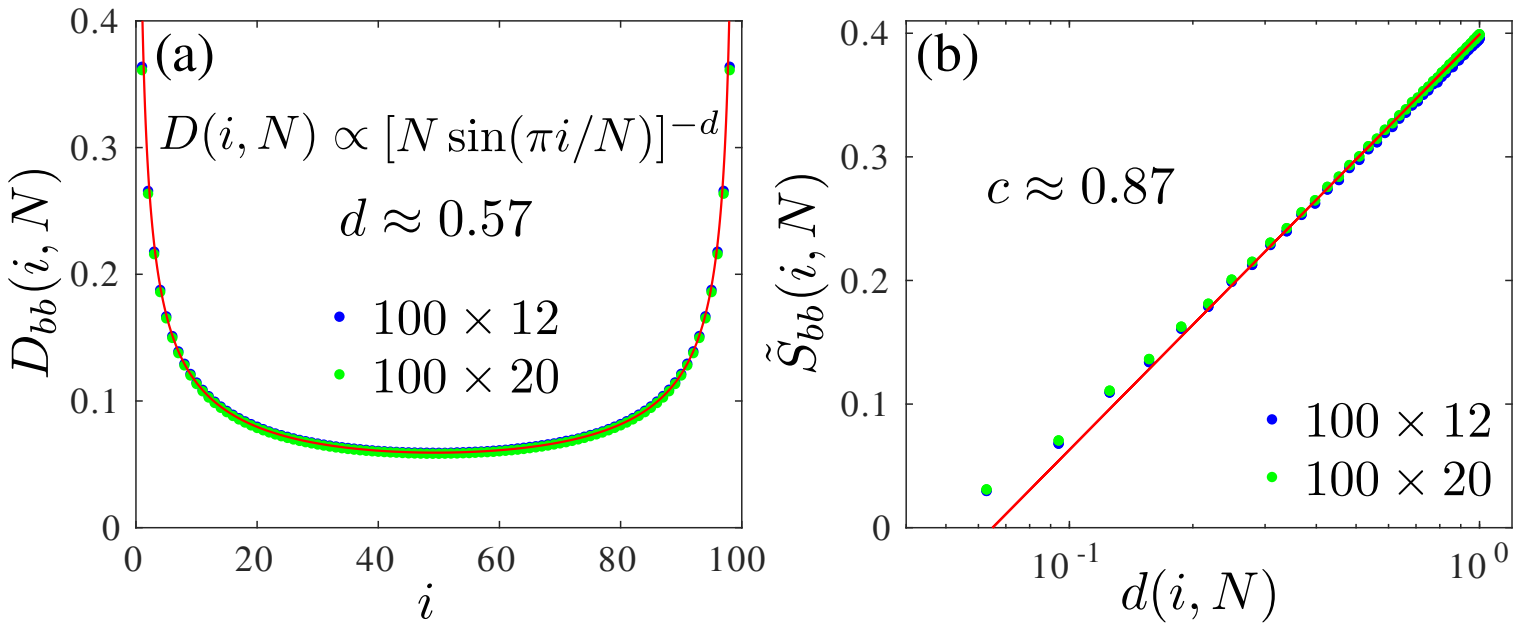
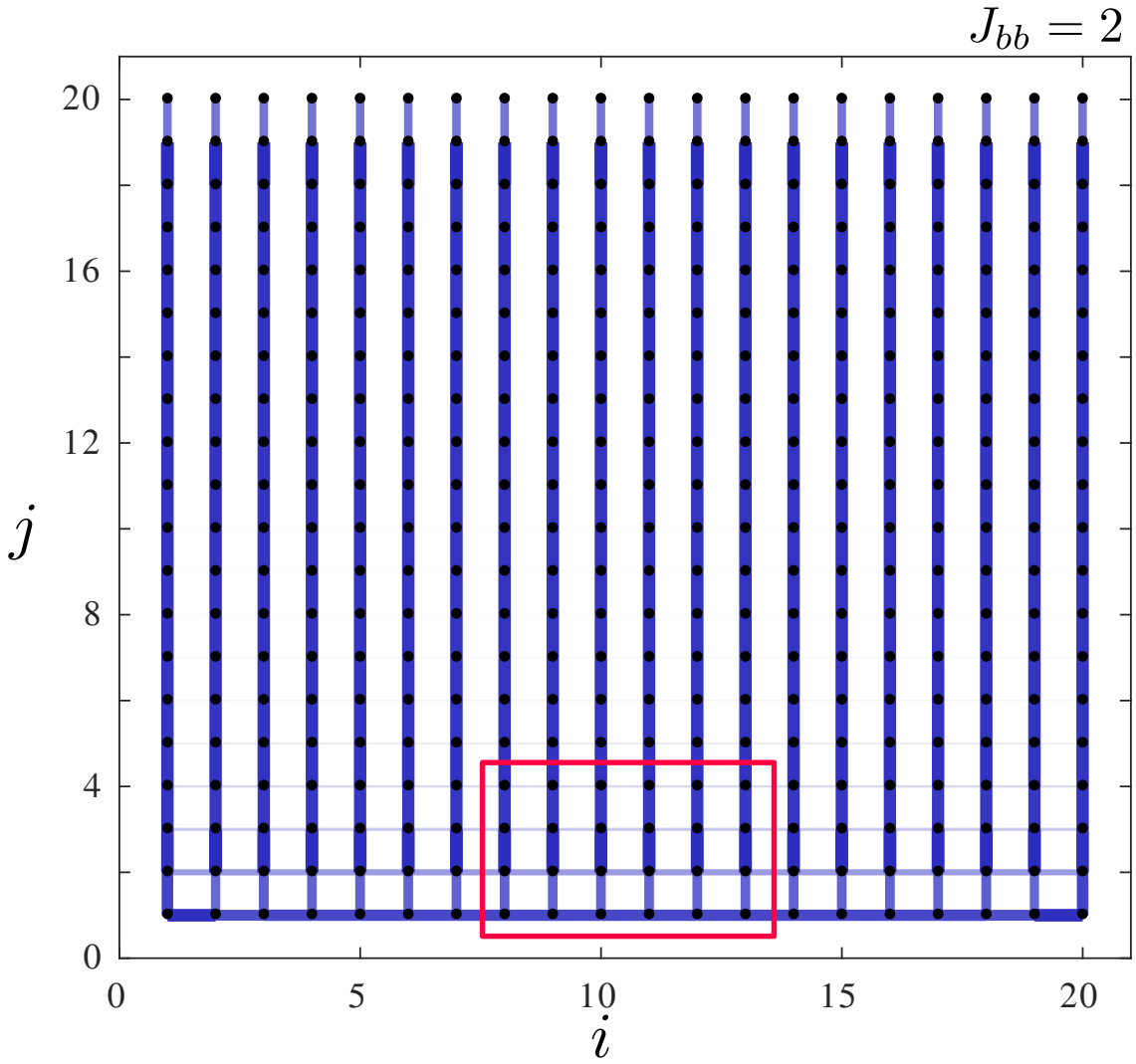


Emergent spin-1/2 chain

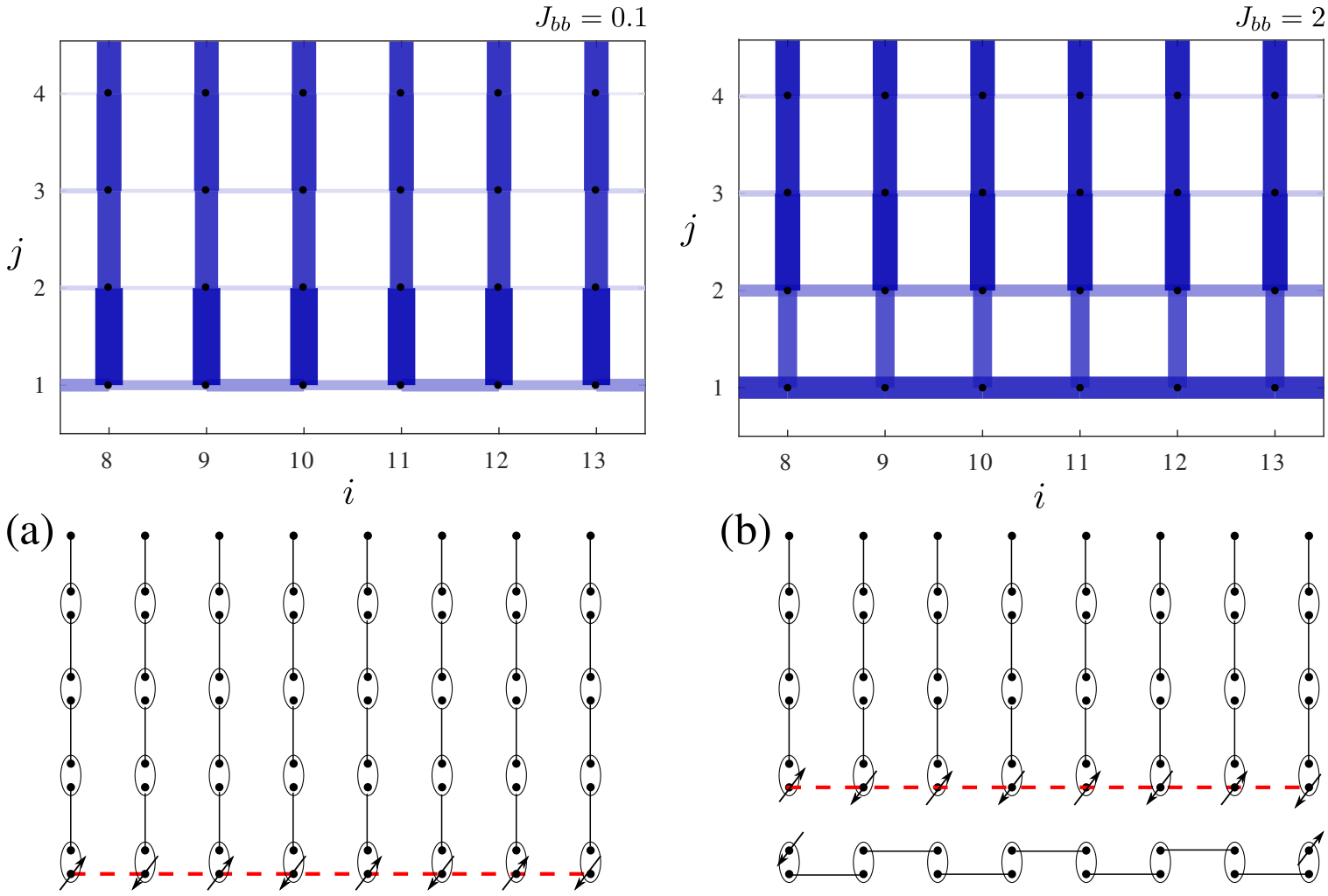


CFT prediction for WZW $SU(2)_1$: $d = 1/2$ and $c = 1$

Spin-1 comb. Correlations

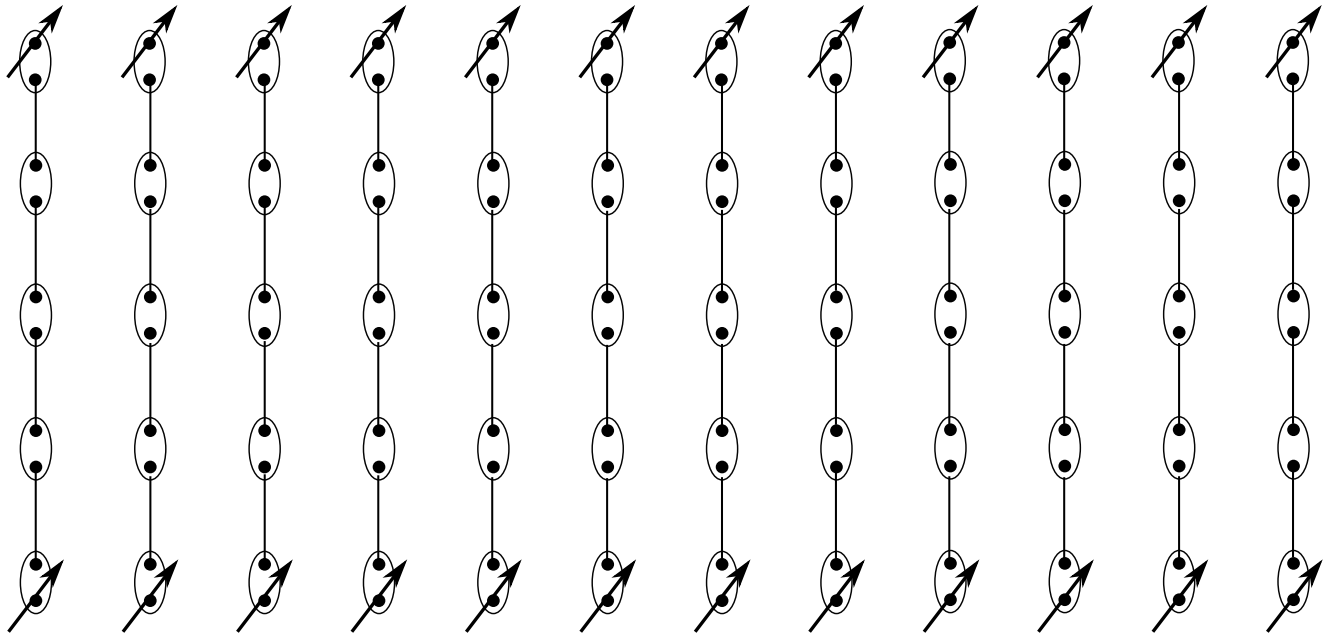


Spin-1 comb



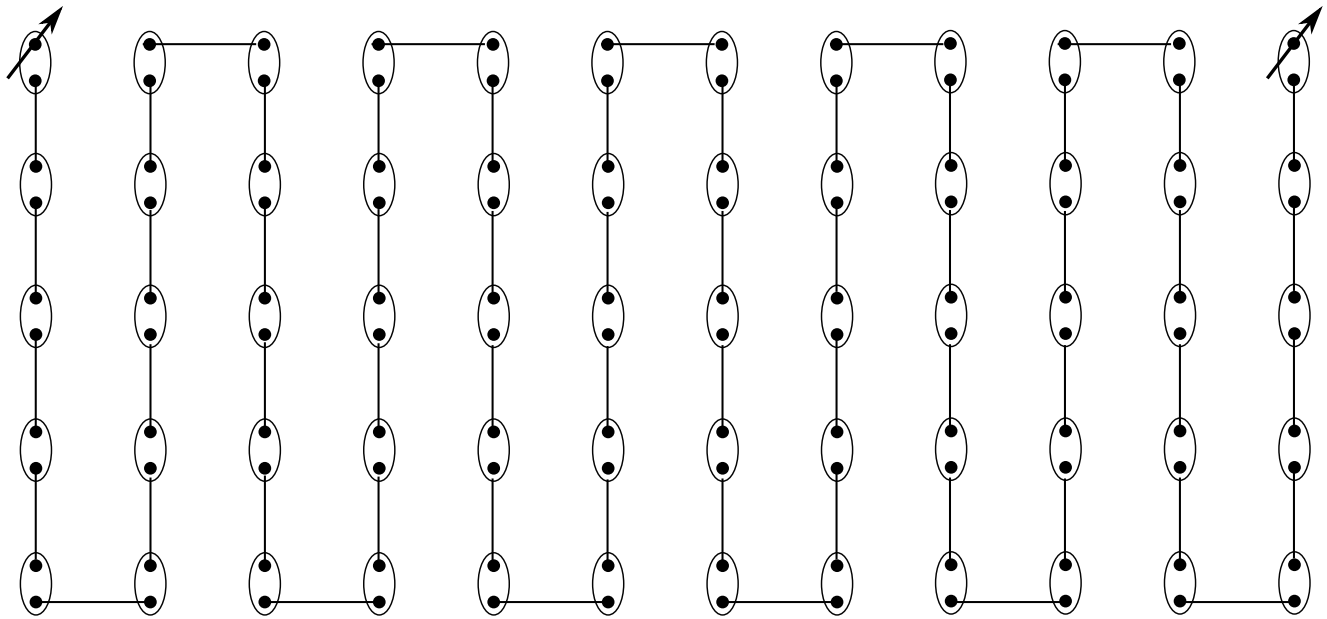
Higher-order edge states

- Tooth with odd number of sites
- Edge states of each tooth couple to a triplet



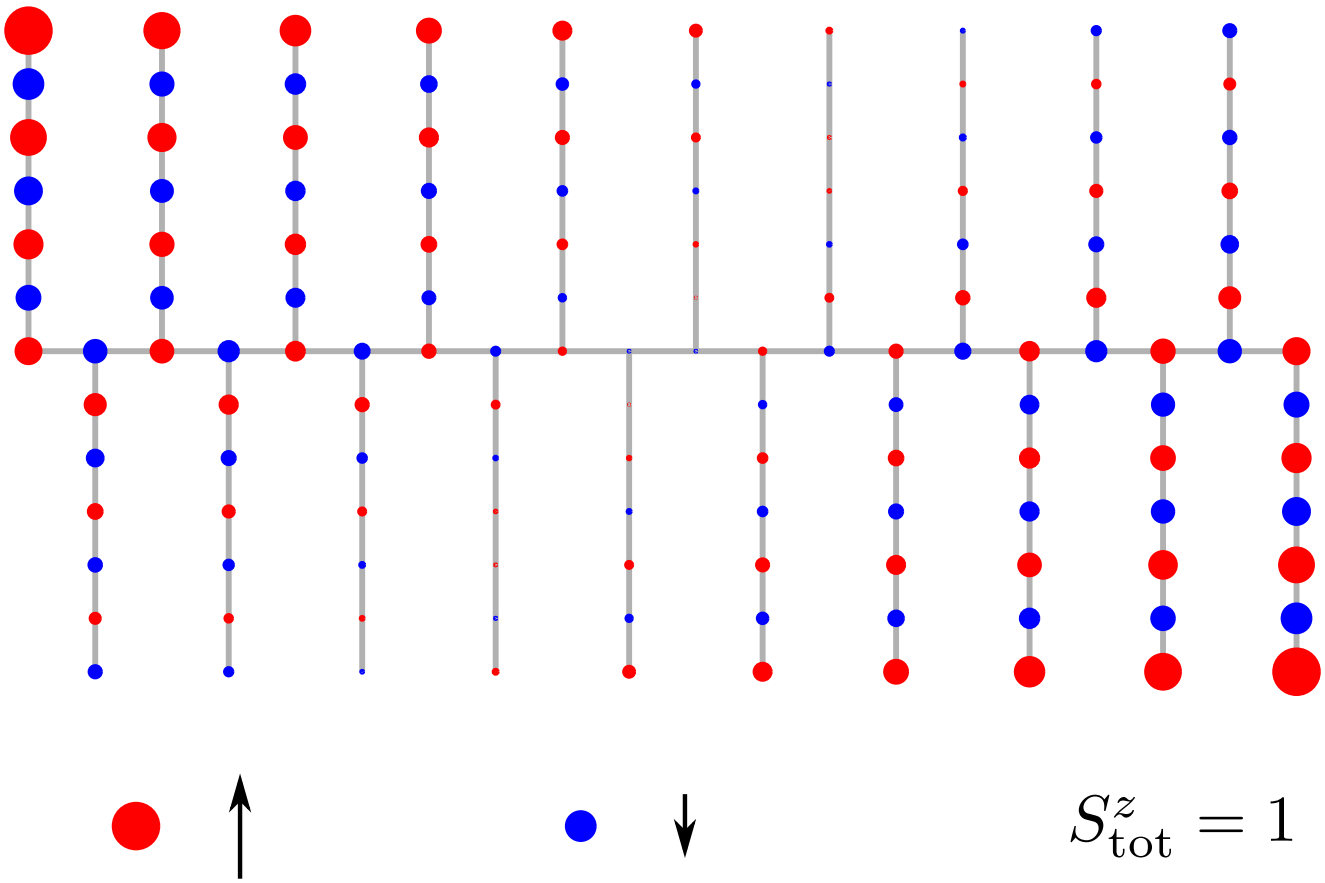
Higher-order edge states

- Tooth with odd number of sites
- Edge states of each tooth couple to a triplet
- Effective spin-1 chain - Haldane state → **Edge states**



Higher-order edge states

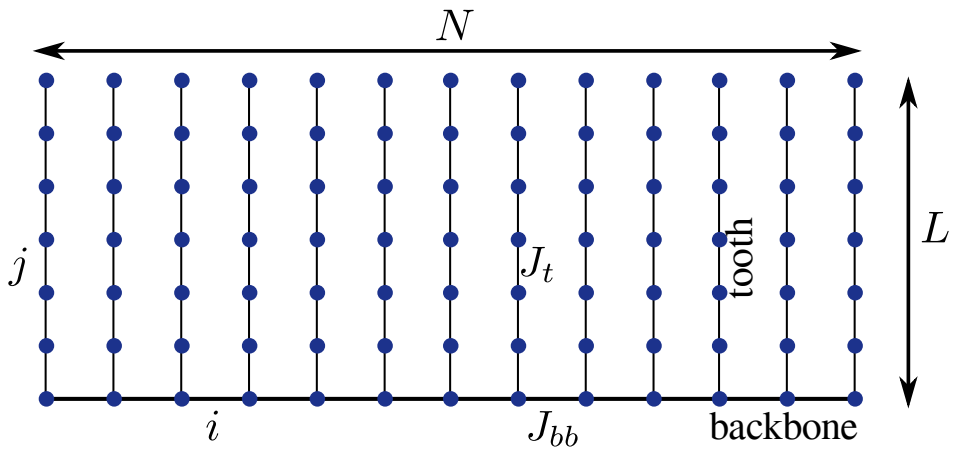
$$J_{bb} = J_t$$



NC, White, arXiv:1903.00432

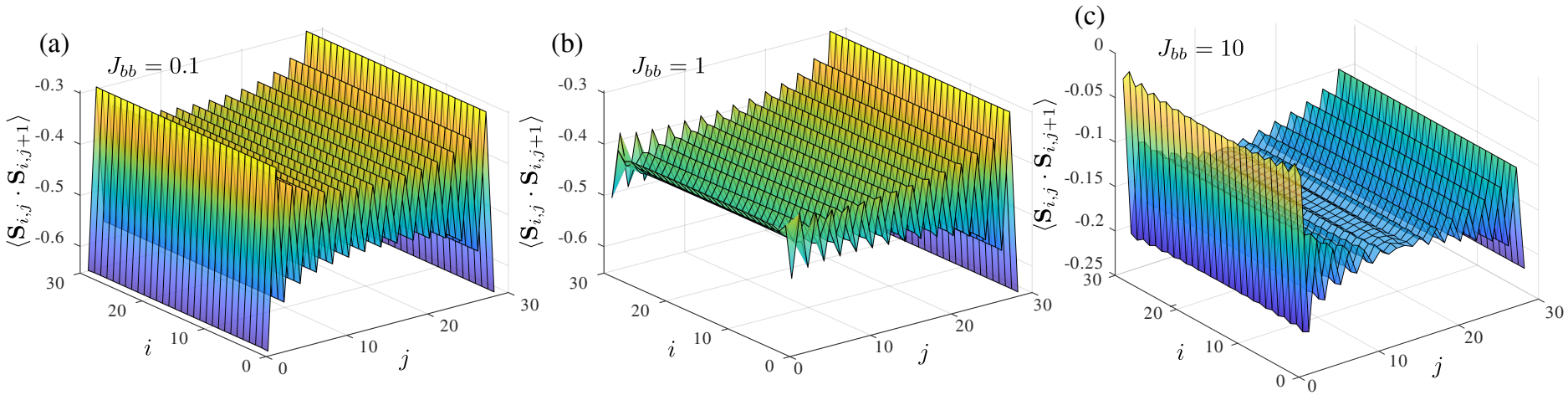
Spin-1/2 Heisenberg comb

$$H = J_{bb} \sum_{i=1}^{N-1} \mathbf{S}_{i,1} \cdot \mathbf{S}_{i+1,1} + J_t \sum_{i=1}^{N} \sum_{j=1}^{L-1} \mathbf{S}_{i,j} \cdot \mathbf{S}_{i,j+1},$$

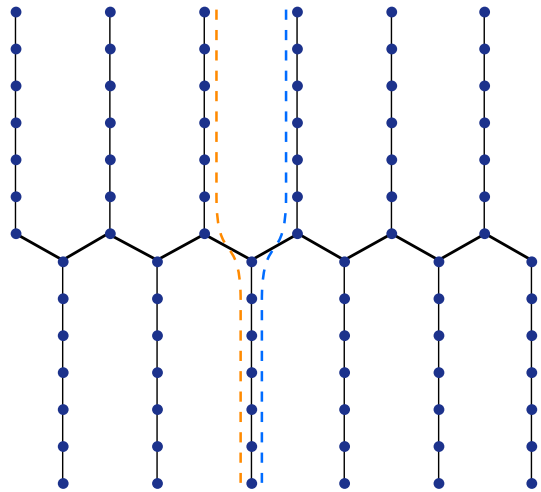


Critical teeth!

Spin-1/2 Heisenberg comb. Tooth correlations

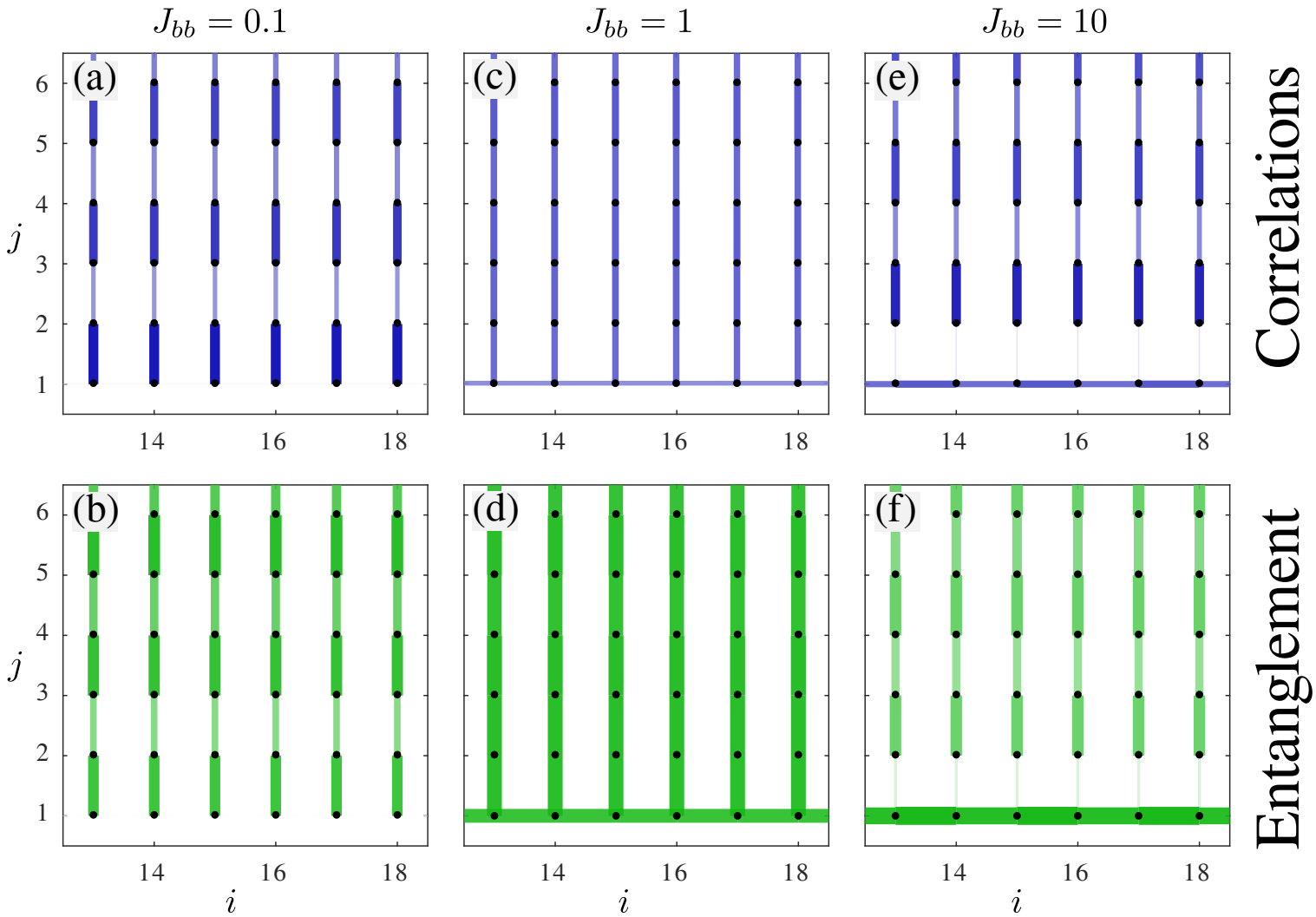


- $J_{bb} \ll J_t$: Decoupled chains with L sites
- $J_{bb} \approx J_t$: Chains with $2L$ sites
- $J_{bb} \gg J_t$: Decoupled chains with $L-1$ sites

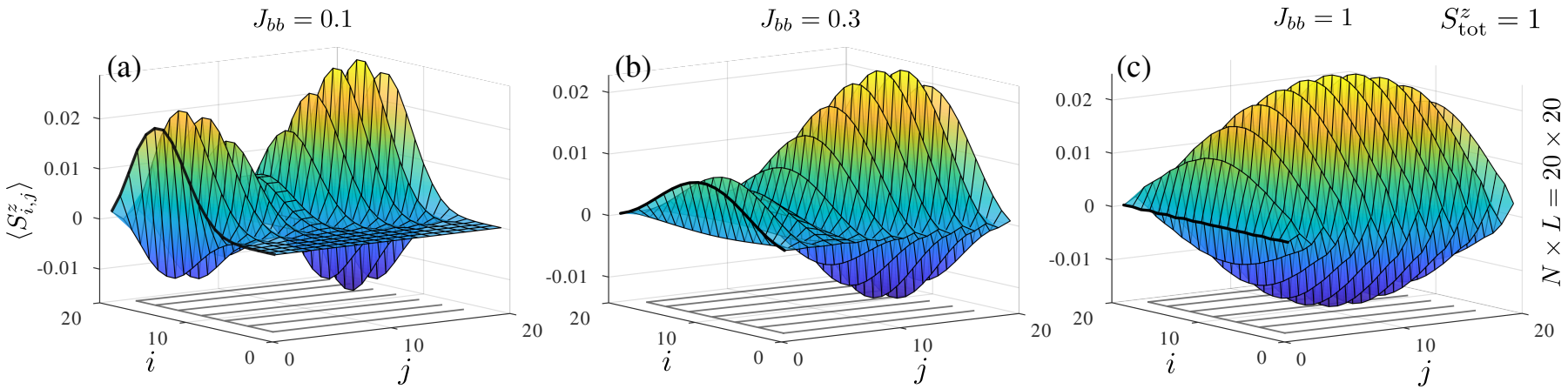


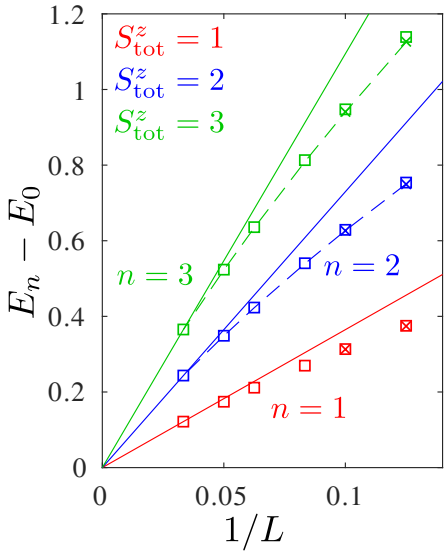
Two chains: Eggert and Affleck, Phys. Rev. B **46**, 10866 (1992)

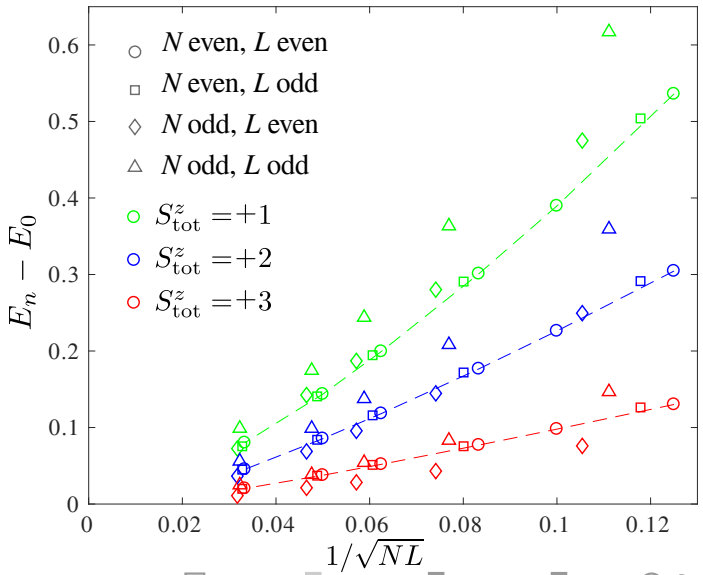
Nearest-neighbor correlations and entanglement



Local magnetization







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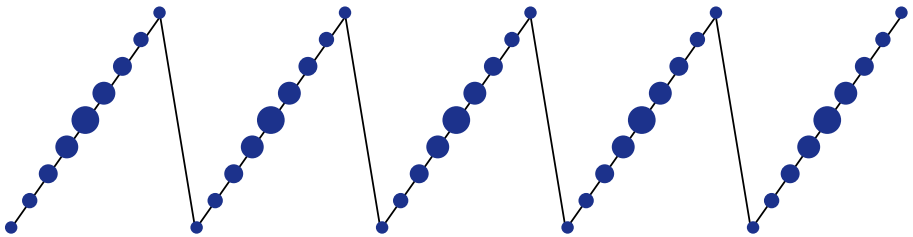
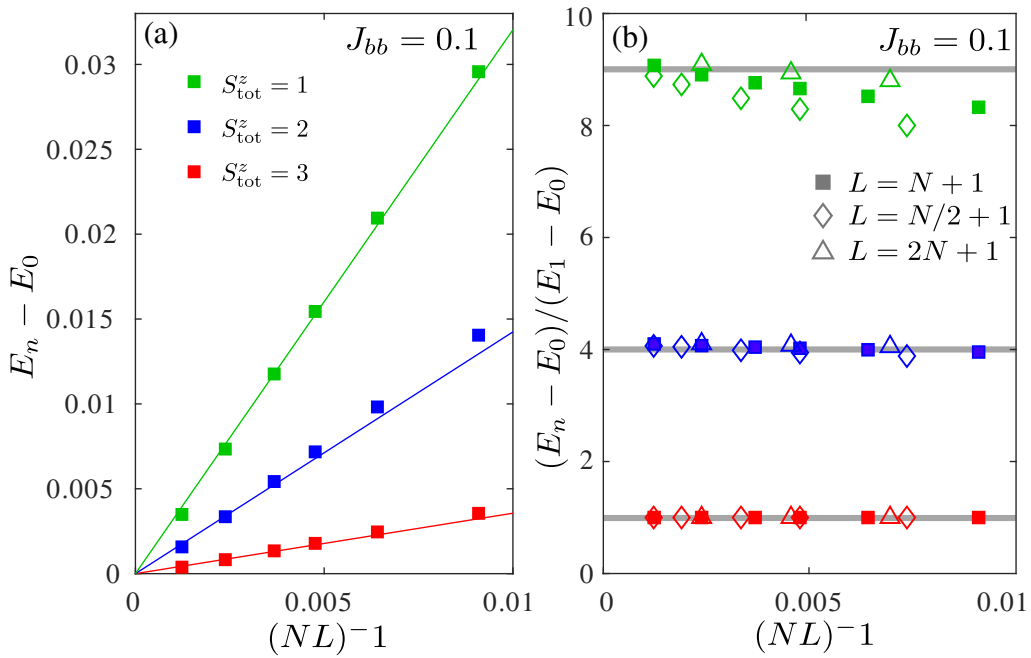
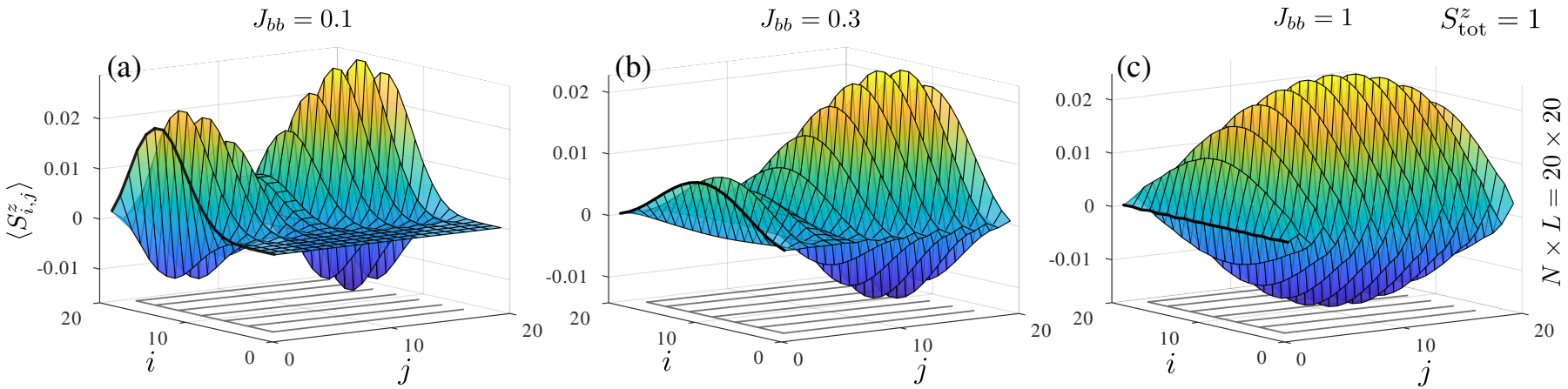
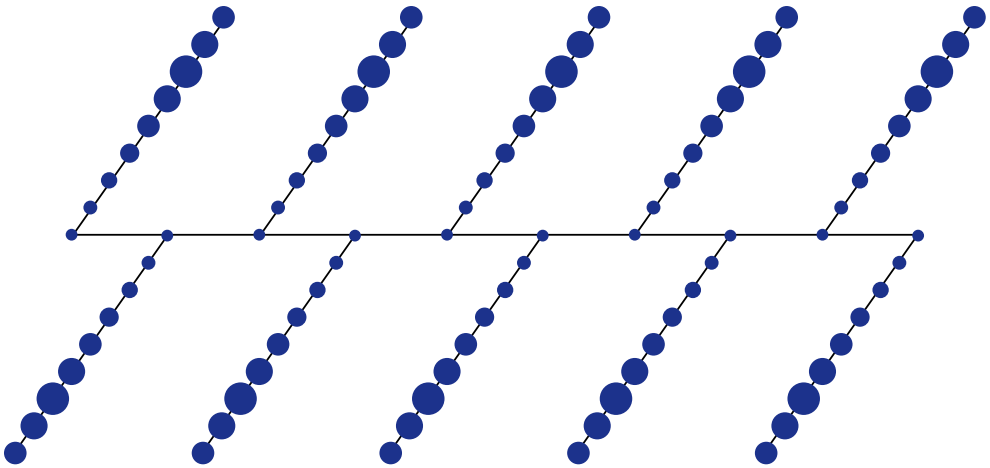


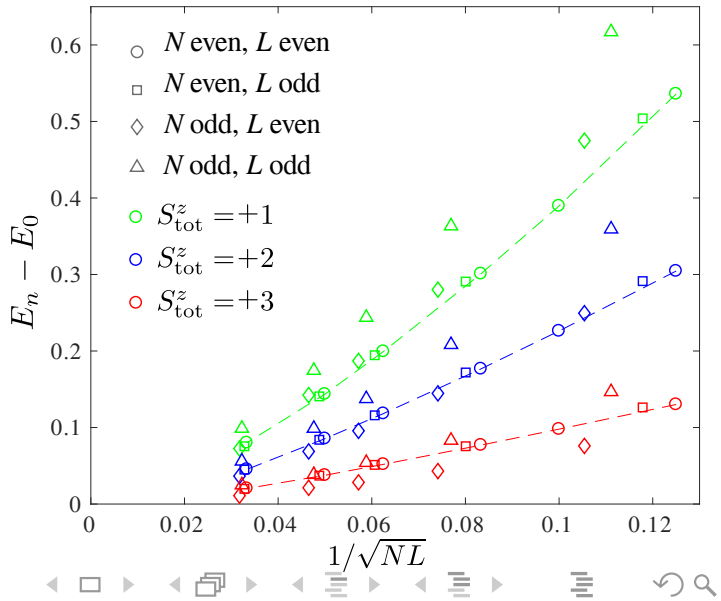


- Teeth with odd number of sites
- Spin-1/2 state on each tooth
- Ground-state - critical spin-1/2 chain of length (NL)

Local magnetization



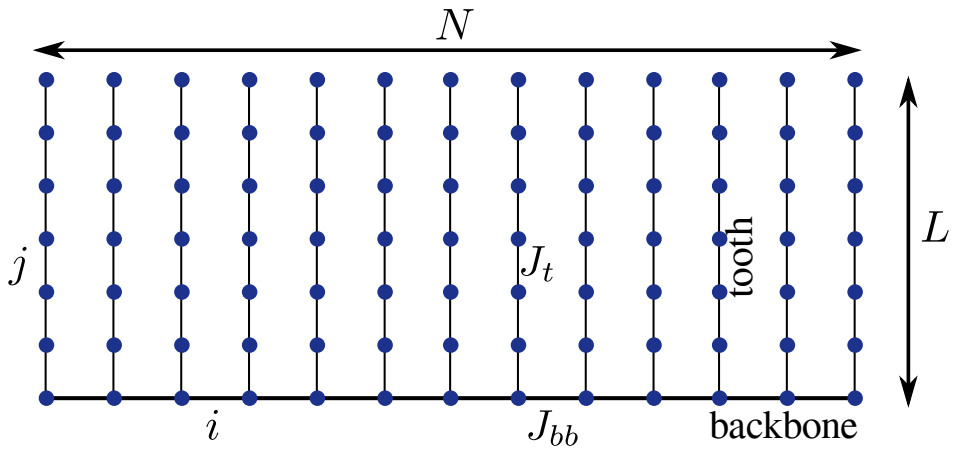




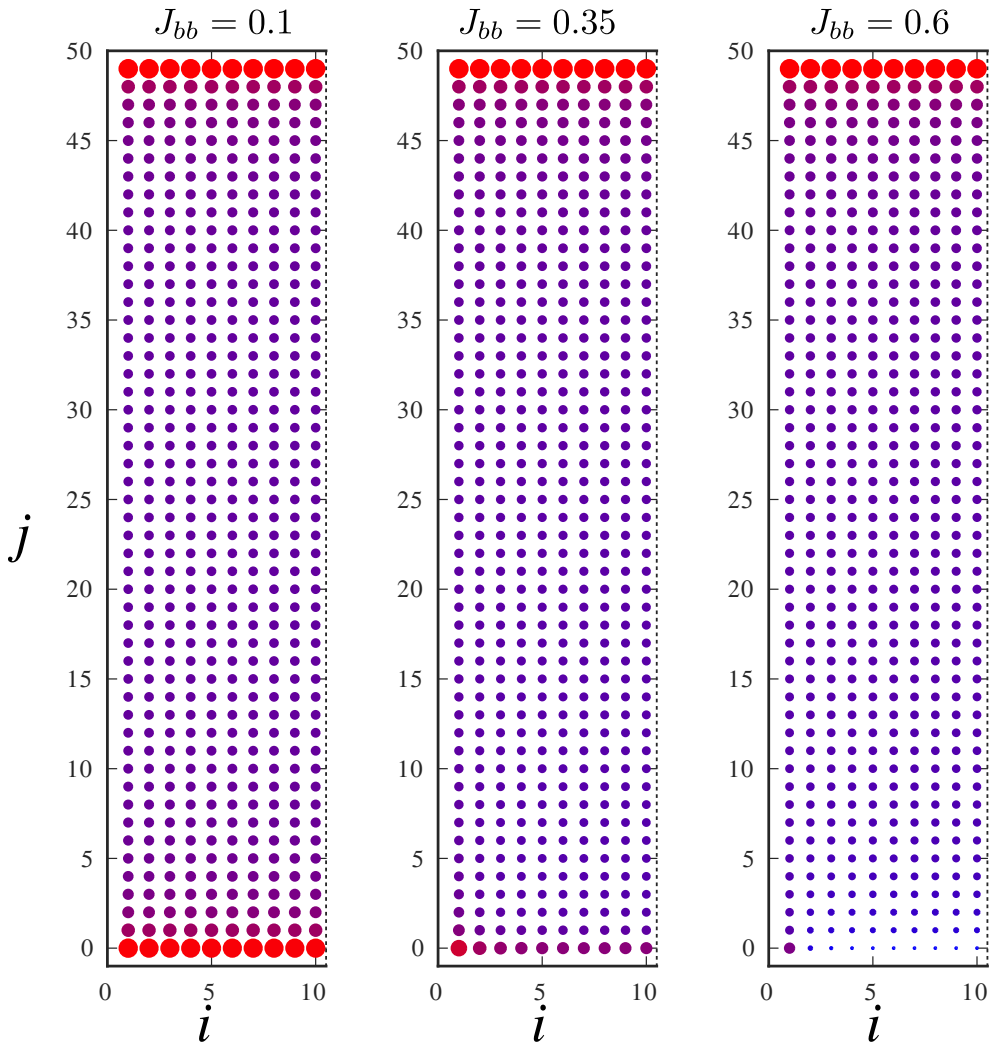
Transverse-field Ising comb

$$H = J_{bb} \sum_{i=1}^{N-1} {S^x}_{i,1} {S^x}_{i+1,1} + J_t \sum_{i=1}^{N} \sum_{j=1}^{L-1} {S^x}_{i,j} {S^x}_{i,j+1} + h \sum_{i=1}^{N} \sum_{j=1}^{L} S^z_{i,j},$$

Critical teeth: $h = J_t/2$

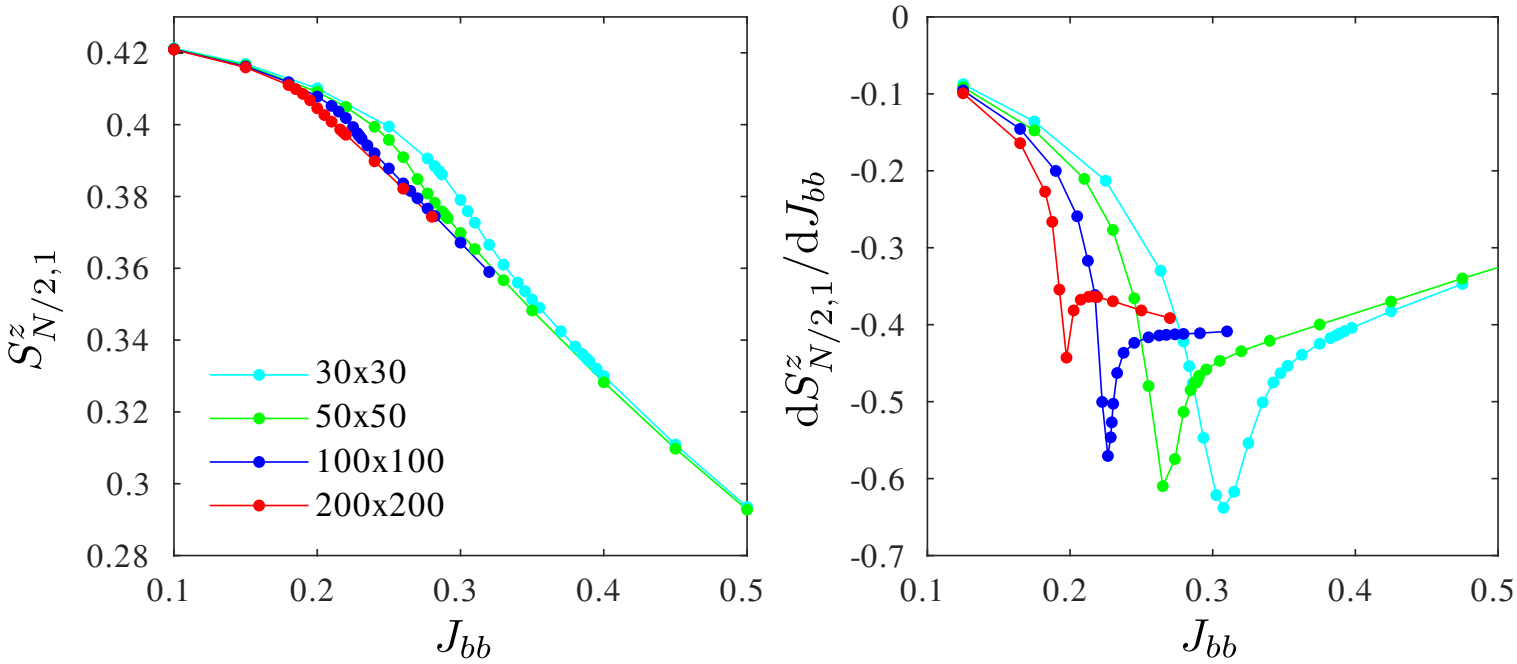


Local magnetization $S^z_{i,j}$



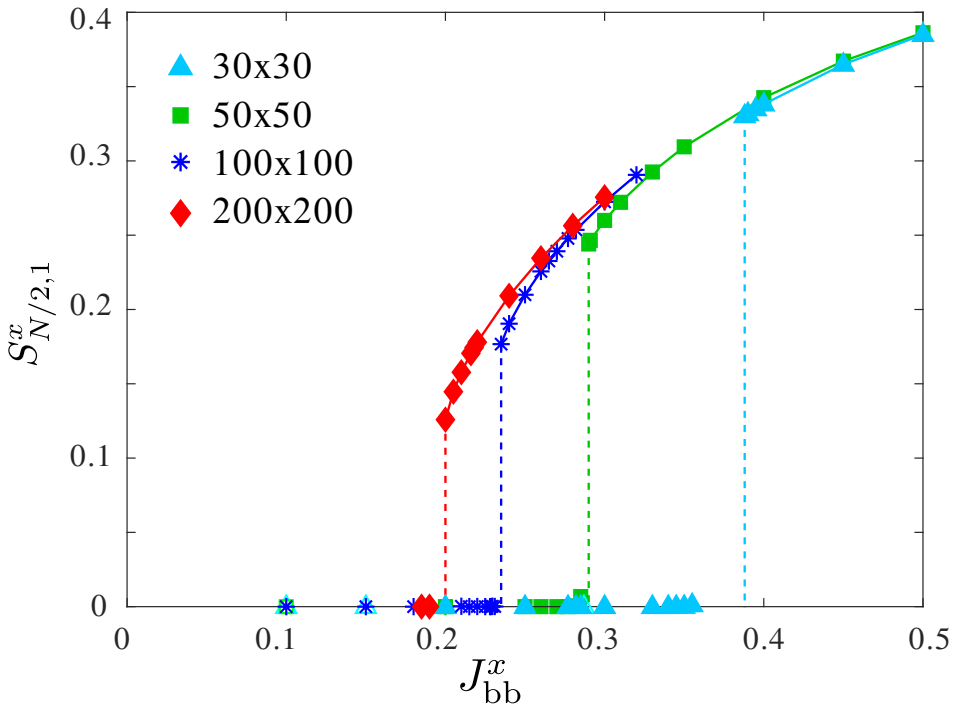
- Tips of the teeth are polarized along the field
- Polarization on the backbone decreases with increasing J_{bb}
- Special type of edge states appear
- Teeth induce non-uniform longitudinal field
  $h^x_i = J_t S^x_{i,2} \neq \text{const}$

Total magnetization



- Phase transition or a crossover?
- Enormous finite-size effect

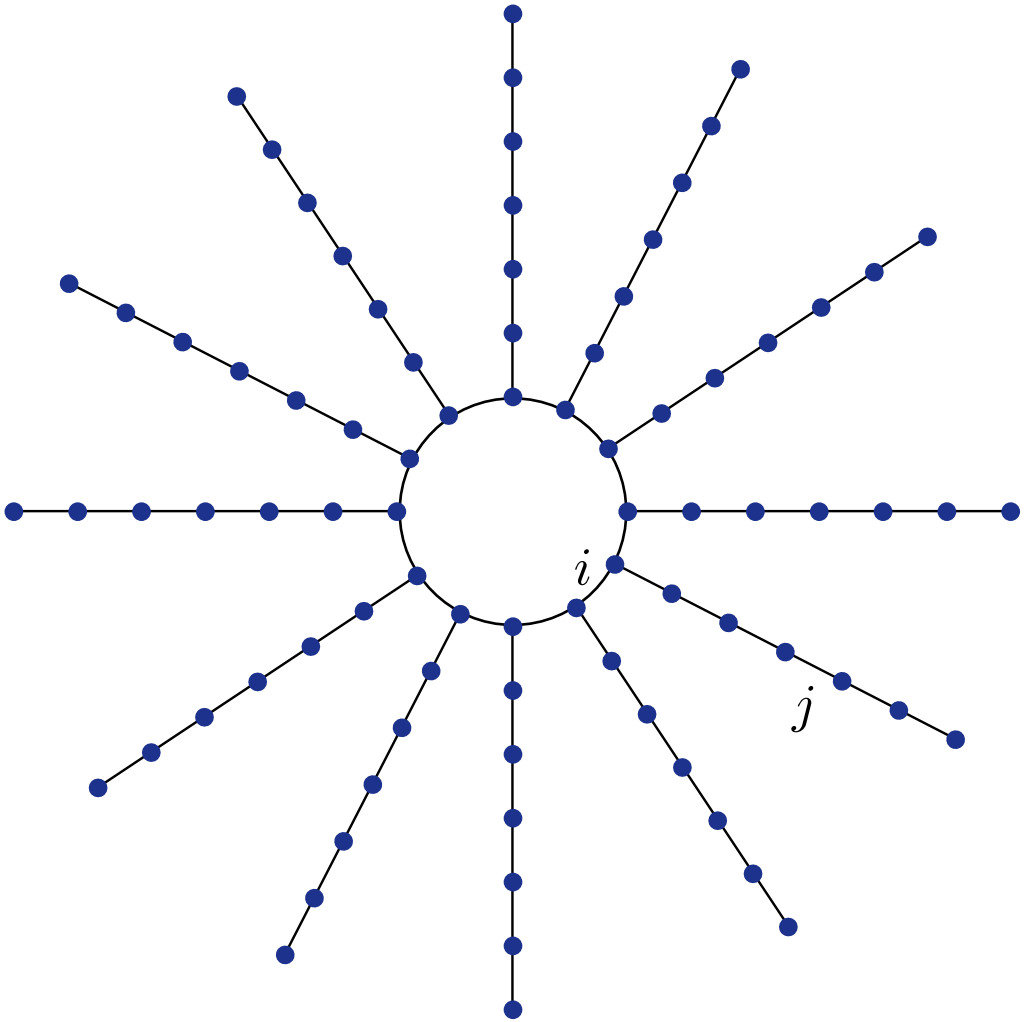
Local polarization along x-axis



First-order transition?

Continuous transition in the thermodynamic limit?

Transverse field Ising comb



Periodic boundary conditions $\rightarrow$ **Uniform longitudinal field**

Outlook

- Comb lattice - quasi-one-dimensional system
- Exotic critical behavior induced by the backbone interaction
- Competing dimensions: *gapless* teeth + *gapped* backbone, etc.
- Direction-dependent correlation length, central charge, critical exponent, etc.
- Flexible and powerful algorithm
  ...and many geometries to play with