Disorder effects on SPT edge mode locality

Based on arXiv:1901.02891

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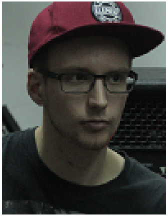
Acknowledgements



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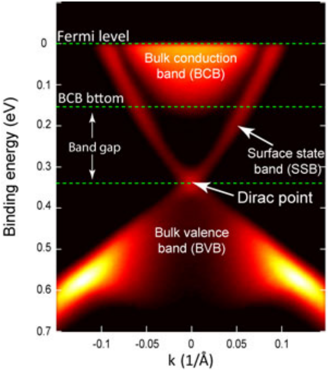
1 Introduction
- Symmetry protected topological phases (SPT)
- XZX cluster Hamiltonian
- SPTs and perturbations

2 Constructing edges modes in the presence of disorder
- with an XX interaction
- with a Heisenberg interaction

3 Conclusion and outlook

Symmetry protected topological phases (SPTs)



Bi_2Se_3 edge state visible in ARPES data[1]

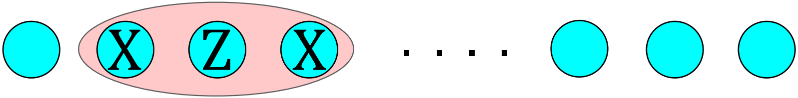
The simplest of the topological phases. Examples include:

- Kitaev chains
- AKLT model
- Topological insulators and more!

Feature edge modes that are stable to perturbations that don't break the protecting symmetry.

[1] Y. Chen et al., Science **325**, 178 (2009)

The XZX cluster Hamiltonian

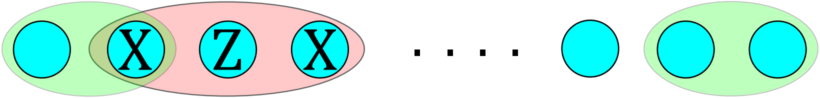


Warm-up exercise (and today's featured model): The XZX cluster Hamiltonian

$$H_{XZX} = -\sum_{j=2}^{N-1} X_{j-1} Z_j X_{j+1}$$

Symmetric under $\mathcal{T} = \left(\prod_{j=1}^{N} Z_j\right) K$, same phase as AKLT.

The XZX cluster Hamiltonian



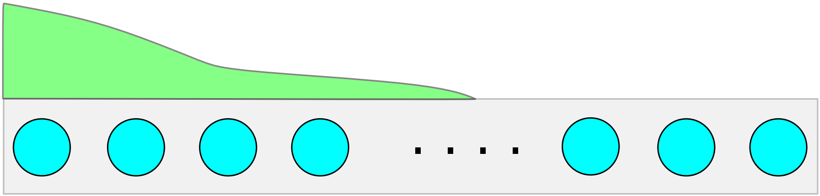
Besides the symmetry, there are 6 operators which commute with H_{XZX}, 3 at each edge $(\mathcal{E}_L, \mathcal{E}_R)$

- $\mathcal{X}_L = X_1$
- $\mathcal{Y}_L = Y_1 X_2$
- $\mathcal{Z}_L = Z_1 X_2$
- $\mathcal{X}_R = X_N$
- $\mathcal{Y}_R = X_{N-1} Y_N$
- $\mathcal{Z}_R = X_{N-1} Z_N$

Each edge has its own Pauli algebra corresponding to an emergent spin-$\frac{1}{2}$, enforcing a 4-fold degeneracy.

Moving away from the solvable point

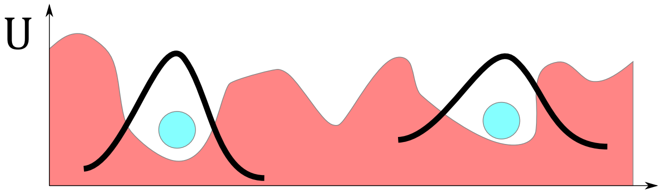
The universe dislikes solvable models. What happens when other interactions are included?



The edge modes spread. This causes decoherence since we never have access to the full edge state.

Disorder to localise states

Disorder localises bulk eigenstates in 1D systems.[2]



How might this help edge modes?

[2] D. Basko, I. Aleiner, and B. Altshuler, Annals of Physics **321**, 1126 (2006)
[3] D. A. Huse et al., Phys. Rev. B **88**, 014206 (2013).

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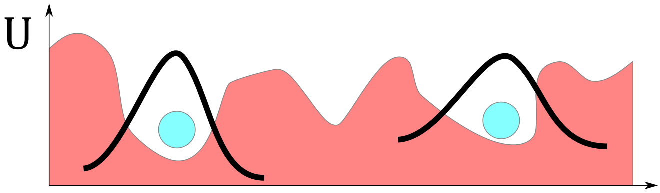

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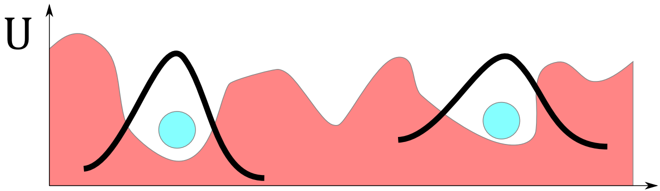

How might this help edge modes?

- Pessimistic: Edge modes are insensitive to bulk features.
- Optimistic: Edge modes broaden by becoming dressed with bulk modes. Localised bulk $\Longrightarrow$ More localised edge[3]

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[3] D. A. Huse et al., Phys. Rev. B **88**, 014206 (2013).

Disordered and perturbed XZX Hamiltonian

We'll test this by constructing the edge modes of

$$H = -\sum_{j=2}^{N-1}(1+h_j)X_{j-1}Z_jX_{j+1} - JV,\ \ h_i \in \left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$$

directly, for 2 different perturbations.

$$V_{XX} = \sum_{j=1}^{N-1} X_jX_{j+1} + Y_jY_{j+1}$$

$$V_{Heis} = \sum_{j=1}^{N-1} X_jX_{j+1} + Y_jY_{j+1} + Z_jZ_{j+1}$$

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- Why this model of disorder?

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- Why this model of disorder?
  - We want to see if and how much disorder *improves* localisation, so $\Delta = 0$ should still be an SPT.
- Why these two perturbations?
  - We are very limited in system size for V_{Heis}, so we want to benchmark with a free fermion problem.

Edge modes with V_{XX}

In the presence of V_{XX}, we can solve for the edge modes of

$$H = -\sum_{j=2}^{N-1}(1+h_j)X_{j-1}Z_jX_{j+1} - J\sum_{j=1}^{N-1}X_jX_{j+1} + Y_jY_{j+1}$$

using a Jordan-Wigner transformation to Majorana operators

$$\gamma_j = Z_1\dots Z_{j-1}X_j \quad \text{and} \quad \bar{\gamma}_j = Z_1\dots Z_{j-1}Y_j,$$

yielding a quadratic fermion Hamiltonian

$$H = -i\sum_{j=2}^{N-1}(1+h_j)\bar{\gamma}_{j-1}\gamma_{j+1} - J\sum_{j=1}^{N-1} i\bar{\gamma}_j\gamma_{j+1} + i\gamma_j\bar{\gamma}_{j+1}$$

Edge modes with V_{XX}

We can diagonalise H with a quick real SVD

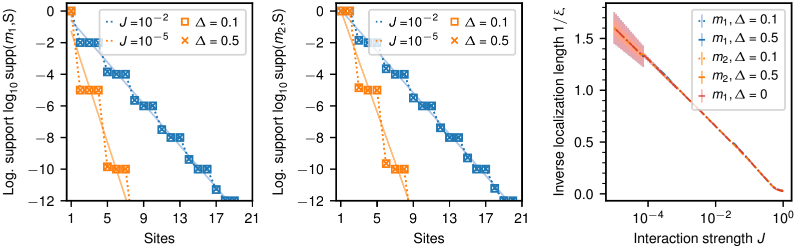
$$m_j = \sum_{k=1}^{N} Q_{j,k}\gamma_k, \qquad \bar{m}_j = \sum_{k=1}^{N} \bar{Q}_{j,k}\bar{\gamma}_k \tag{1}$$

The smallest singular values corerspond to the edge modes

$$H = \underbrace{i\sigma_1 m_1 \bar{m}_1 + i\sigma_2 m_2 \bar{m}_2}_{\text{edge mode spectrum}} + \dots$$

We can plot the supports of m_1 and m_2

Edge modes with V_{XX}



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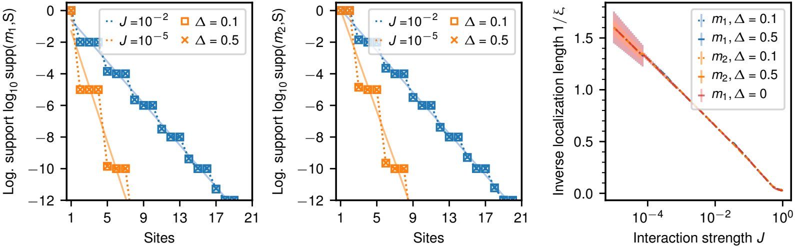
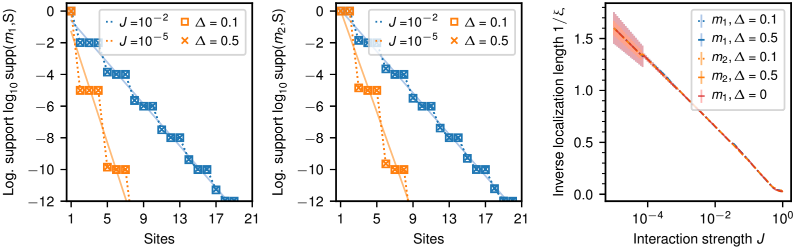


- Exponential localisation which worsens as J increases

Edge modes with V_{XX}



- Exponential localisation which worsens as J increases
- *No disorder dependence whatsover*

Edge modes with V_{Heis}

Switching to our other perturbation, we have

$$H = -\sum_{j=2}^{N-1}(1+h_j)X_{j-1}Z_jX_{j+1} - J\sum_{j=1}^{N-1} X_jX_{j+1} + Y_jY_{j+1} + Z_jZ_{j+1}$$

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However, adding a ZZ coupling complicates our lives severely, since

- we can't use JW to solve directly, as

$$Z_jZ_{j+1} \Longrightarrow \gamma_j\bar{\gamma}_j\gamma_{j+1}\bar{\gamma}_{j+1}$$

Edge modes with V_{Heis}

Our solution is to attempt to construct them directly, using a method developed to find locally conserved quatities in MBL[4]. It is based on the following logic

- Start with H_0 which has a conserved quantity $\mathcal{O}$
- Perturb to $H = H_0 + V$
- After a long time, $\mathcal{O}$ relaxes to its infinite time average $\bar{\mathcal{O}}$
- $\bar{\mathcal{O}}$ is close to the new conserved quantity $\mathcal{O}_{pert.}$, so we reconstruct it.

[4] M. Goihl et al., Phys. Rev. B **97**, 134202 (2018)

Edge modes with V_{Heis}

What do we use as our ansatz $\mathcal{O}$?

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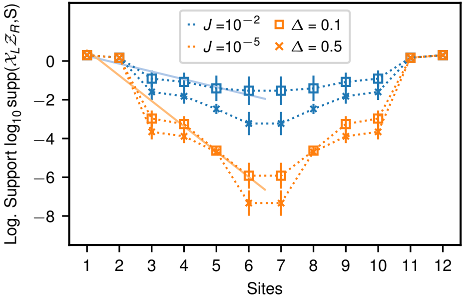
- We need to use $\mathcal{O} = \mathcal{E}_L \mathcal{E}_R$, which can relax to an operator which commutes with H
  - 9 choices of initial pairs, we'll focus on $\mathcal{X}_L \mathcal{Z}_R = X_1 Z_N X_{N-1}$ and $\mathcal{Y}_L \mathcal{Y}_R = X_2 Y_1 Y_N X_{N-1}$

Initialising with $\mathcal{X}_L \mathcal{Z}_R$



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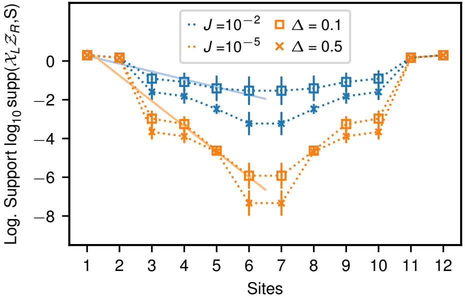
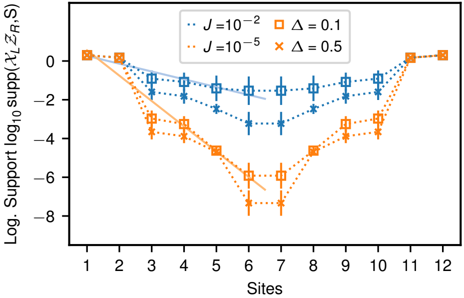


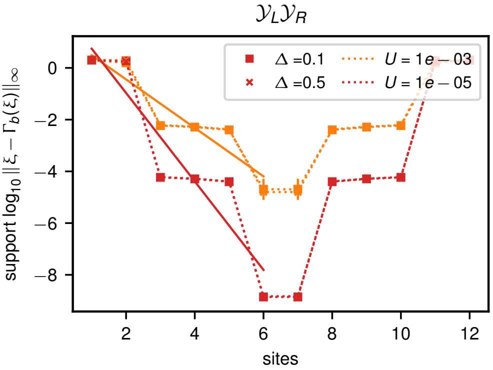
- Appearence of disorder dependence (it helps!)

Initialising with $\mathcal{X}_L\mathcal{Z}_R$

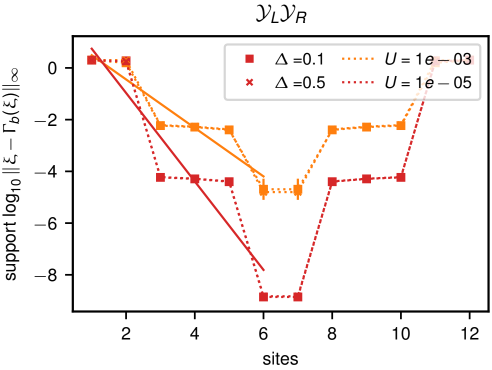


- Appearence of disorder dependence (it helps!)
- Localisation of edge modes is slightly worse than non-interacting case

Initialising with $\mathcal{Y}_L\mathcal{Y}_R$

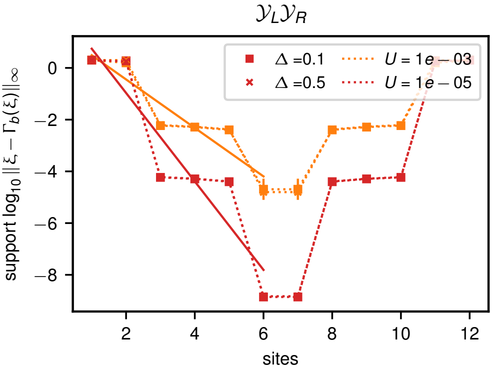


Initialising with $\mathcal{Y}_L\mathcal{Y}_R$



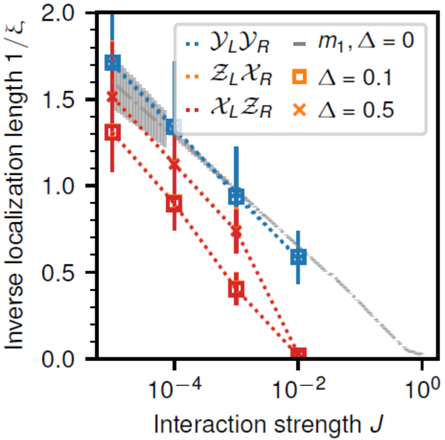
- Disappearence of disorder dependence!

Initialising with $\mathcal{Y}_L\mathcal{Y}_R$



- Disappearence of disorder dependence!
- Better localised than $\mathcal{X}_L\mathcal{Z}_R$

Behaviour of the localisation length



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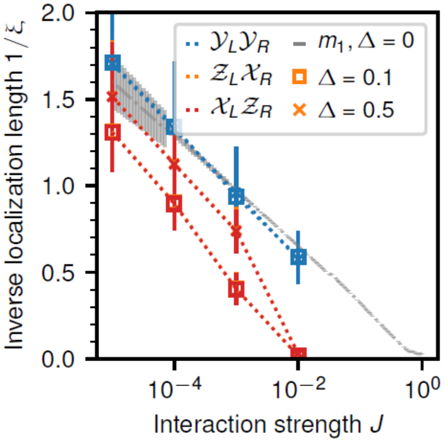

- $\mathcal{Y}_L\mathcal{Y}_R$ behaves as if the system did not have a ZZ term, in a system 3x larger.

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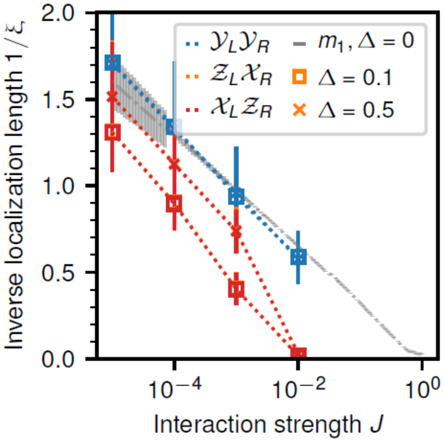

- $\mathcal{Y}_L\mathcal{Y}_R$ behaves as if the system did not have a ZZ term, in a system 3x larger.
- Disorder moves $\mathcal{X}_L\mathcal{Z}_R$ localisation towards non-interacting behaviour

Conclusions and outlook

- SPT edge mode behaviour depends strongly on the perturbations introduced.
    - Free-fermion-type perturbations produce edge modes with no disorder dependence
    - Introduction of many-body interactions leads to the appearance of disorder effects

- Edge modes are not affected uniformly by disorder.
    - $\mathcal{Y}$-type edge modes show both enhanced localisation and disorder insensitivity.

With these results in mind, one should check to see what effects this has on edge qubit coherence times. (work in progress)

Bonus Slides

Edge modes with V_{Heis}

Operationally, we can compute $\mathcal{O}_{pert.}$ as follows

- Diagonalise H, get energy basis $|k\rangle$ and diagonalisation unitary U_D
- Compute infinite time average $\bar{\mathcal{O}} = \sum_k \langle k|\mathcal{O}|k\rangle |k\rangle\langle k|$
- Put $\langle k|\mathcal{O}|k\rangle$ in decreasing order, reorder columns of U_D, get $\tilde{U}_D$
- $\mathcal{O}_{pert.} = \tilde{U}_D Z \tilde{U}_D^\dagger$

[2] M. Goihl et al., Phys. Rev. B **97**, 134202 (2018)

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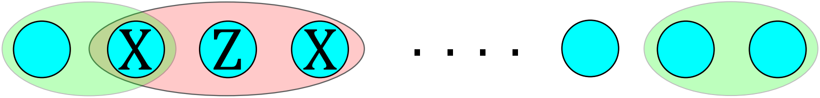
- $\mathcal{O}_{pert.} = \tilde{U}_D Z \tilde{U}_D^{\dagger}$

The exact diagonalisation and sorting make this expensive, so we are limited to smaller system sizes.

[2] M. Goihl et al., Phys. Rev. B **97**, 134202 (2018)

The XZX cluster Hamiltonian



None of the edge operators commute with $\mathcal{T}$, thus the symmetries are not accidental. Breaking the degeneracy is difficult requiring either

- explicit symmetry breaking
- a phase transition
- a non-local term which couples both edges together

Edge modes with V_{XX}

We can write this more compactly

$$H = i \sum_{j,k=1}^{N} \gamma_j C_{j,k} \bar{\gamma}_k$$

with the coupling matrix

$$C_{j,k} = \begin{cases} J & \text{if} \quad j = k+1 \\ -J & \text{if} \quad j = k-1 \\ -(1+h_{j+1}) & \text{if} \quad j = k-2. \end{cases}$$