Entanglement in Strongly Correlated Systems

Centro de Ciencias de Benasque
Pedro Pascual

Fractionalized &
Correlated matter

Part I
"Smoking gun" probes for U(1) spin liquids with fermi surfaces

Democracy

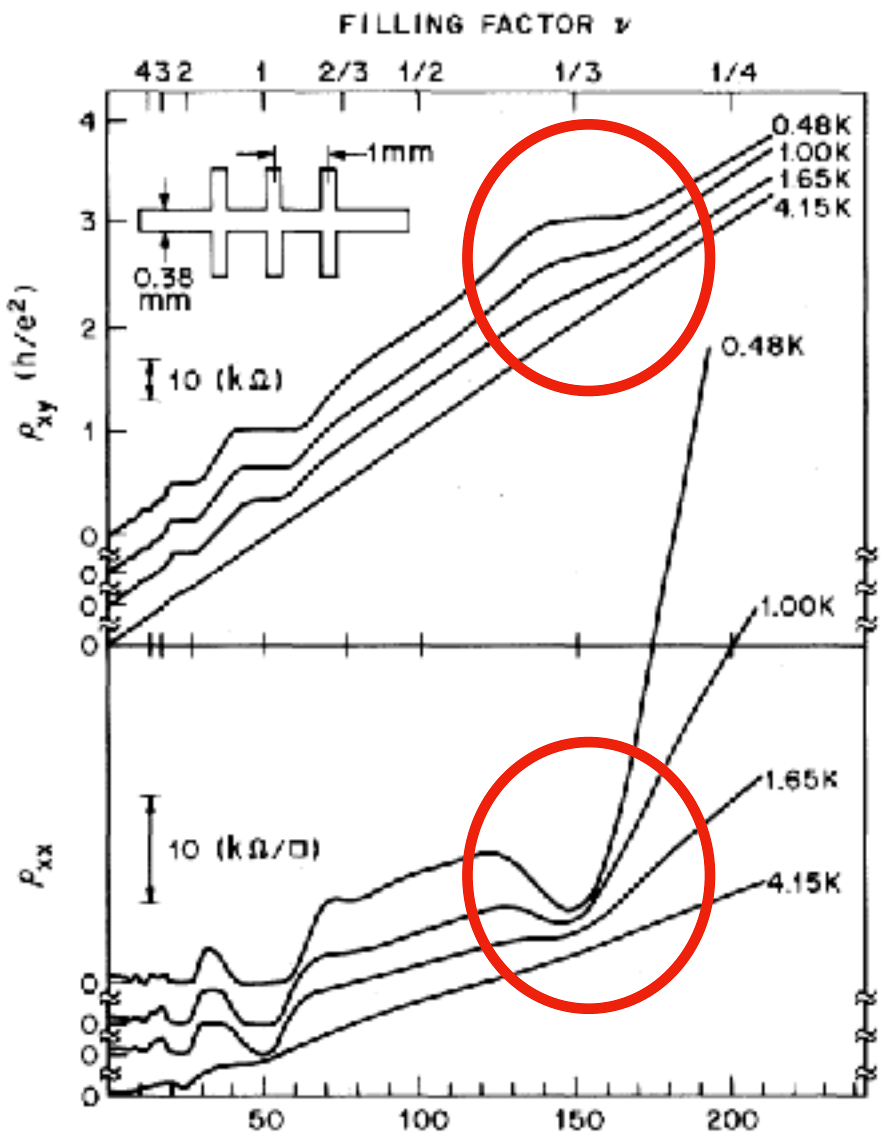
Part II - A
The Berry Curvature Dipole of metals

Part II - B
Quantum electric field lines in QDM and 6 vertex models

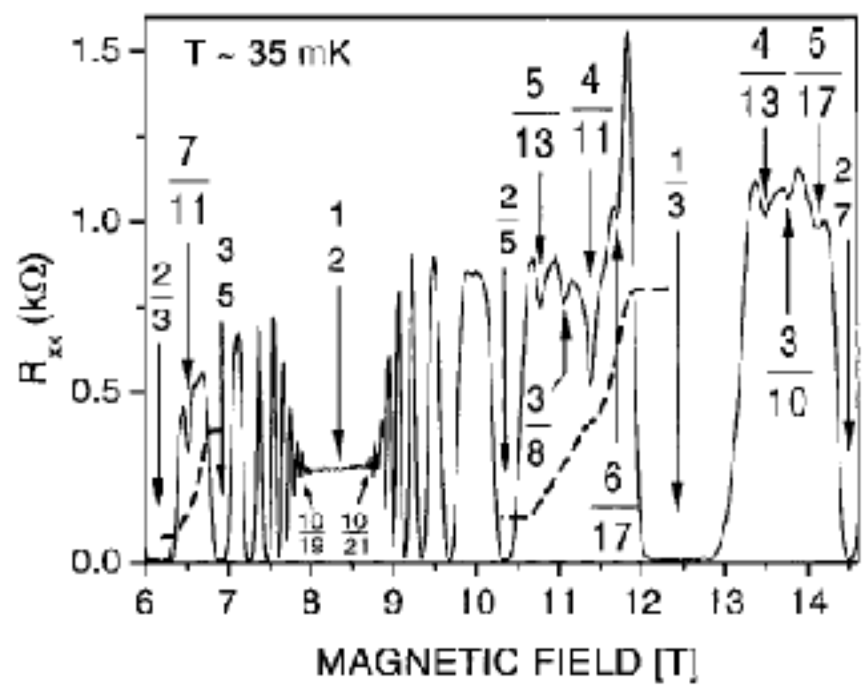
Inti Sodemann
Max-Planck Institute for the Physics of Complex Systems
Dresden, Germany

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,[(a),(b)] H. L. Stormer,[(a)] and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)



After 37 years no reasonable explanation for the fractional plateaus which is not a fractionalized state of matter has appeared

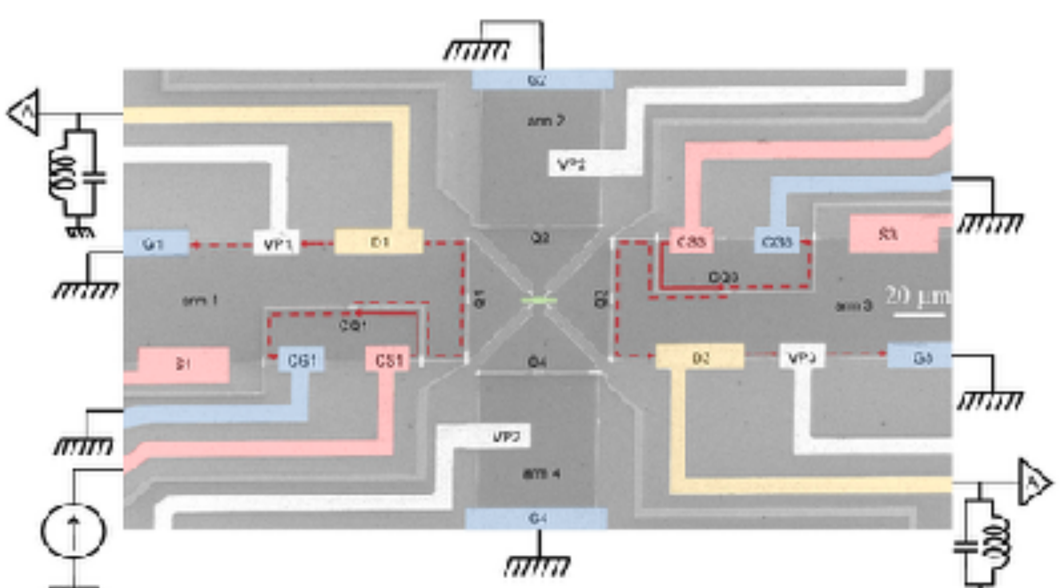


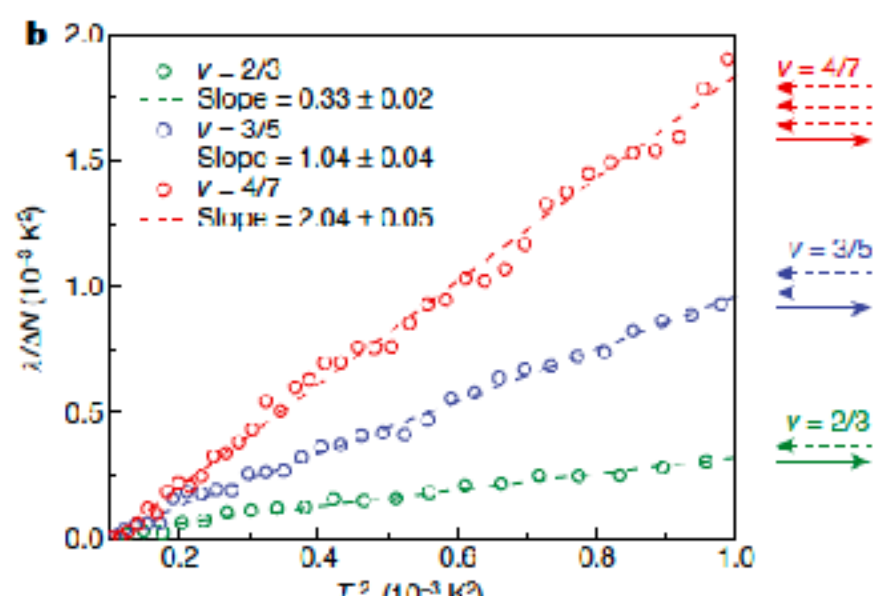
W. Pan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer,
K. W. Baldwin, and K. W. West
Phys. Rev. Lett. **90**, 016801, 2003

Smoking gun evidence beyond transport?

Smoking gun evidence beyond transport?

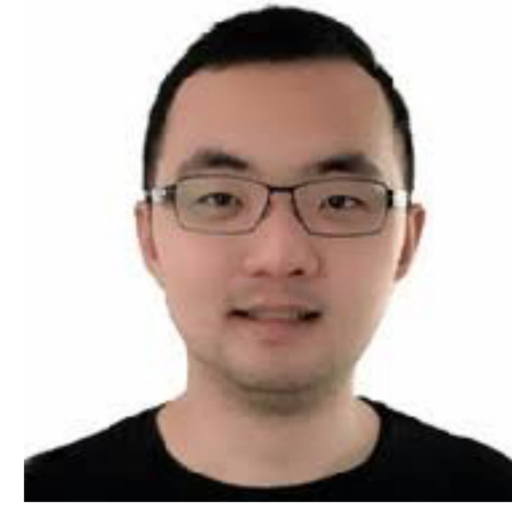
Quantized heat transport:





Mitali Banerjee, Moty Heiblum, Amir Rosenblatt, Yuval Oreg, Dima E. Feldman, Ady Stern & Vladimir Umansky, Nature 545, 75 (**2017**)

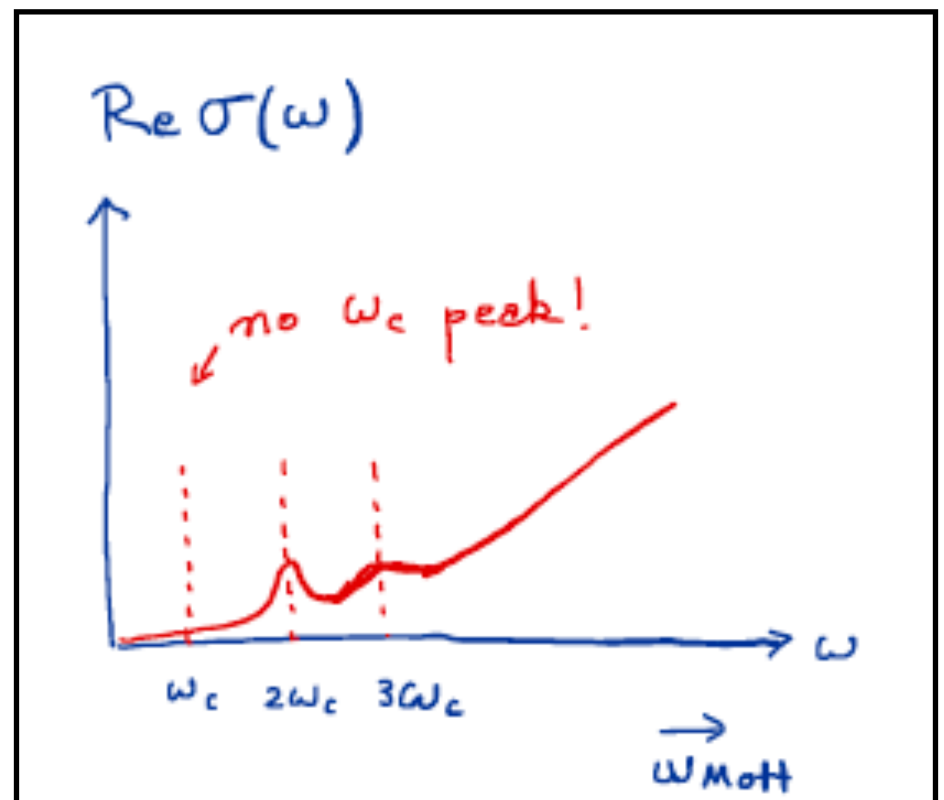
Outline Part I
"Smoking gun" probes for U(1) spin liquids with gapless fermions



Peng Rao

1) Spinons in U(1) spin liquids are electrically neutral particles but they develop Landau levels under physical magnetic fields.

2) These liquids display cyclotron resonance and magnetisation oscillations (de-Haas van alphen) and are an insulators.

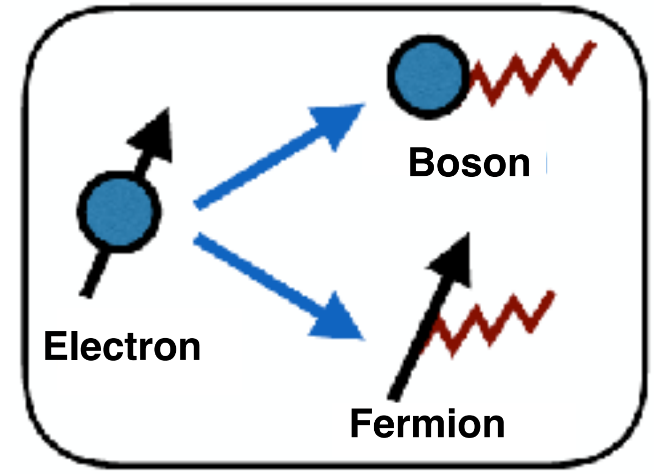


Spinon Fermi surface and Composite Fermi liquid

Slave-boson

Spinfull neutral fermion ↓

Electron → $c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$ ← Spinless charged boson

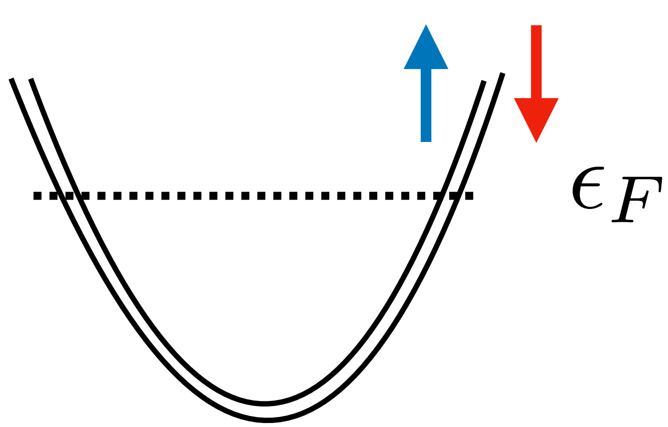


Composite fermi surface:

Bosons form Laughlin state:

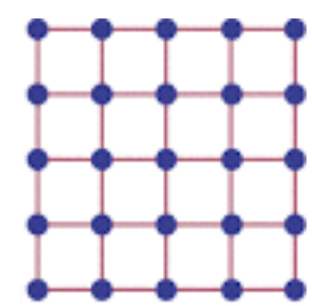
$$\Psi_b = \prod_{i<j} (z_i - z_j)^2 e^{-\frac{|z_i|^2}{4l^2}}$$

Fermion forms fermi sea:

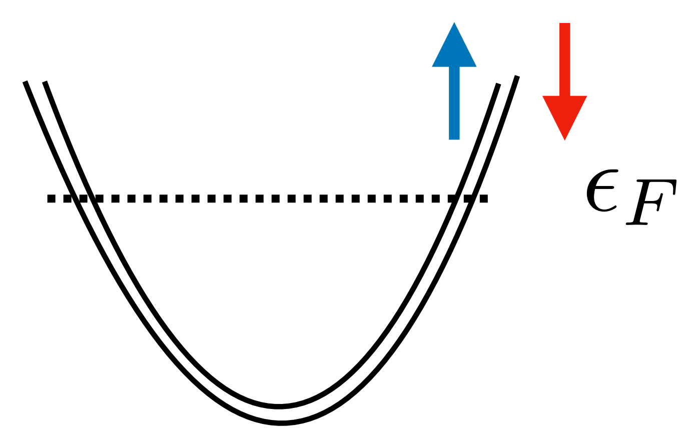


Spinon fermi surface:

Bosons form trivial Mott insulator:



Fermion forms fermi sea:



Spinon Fermi surface

Slave-boson

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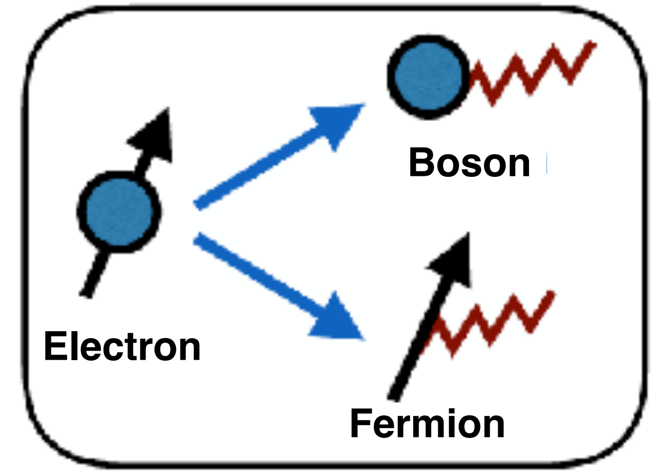

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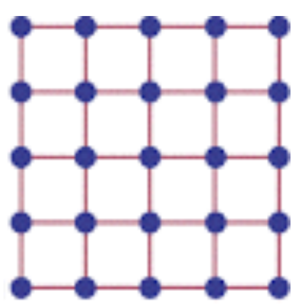
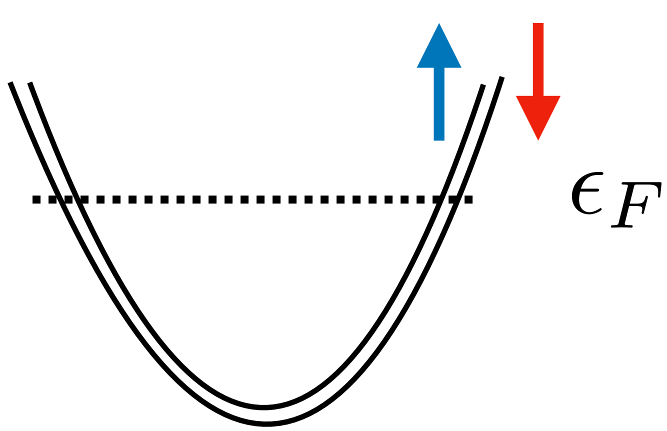


$$\sum_\sigma \langle c^\dagger_{r\sigma} c_{r\sigma} \rangle = \langle b^\dagger_r b_r \rangle = n \in \mathbb{Z}$$

Fermion forms fermi sea:



$$\sum_\sigma \langle c^\dagger_{r\sigma} c_{r\sigma} \rangle = \sum_\sigma \langle f^\dagger_{r\sigma} f_{r\sigma} \rangle = n \ \mathbf{odd}$$

Simplest case: electrons in half-filled lattice

Spinon Fermi surface

Slave-boson

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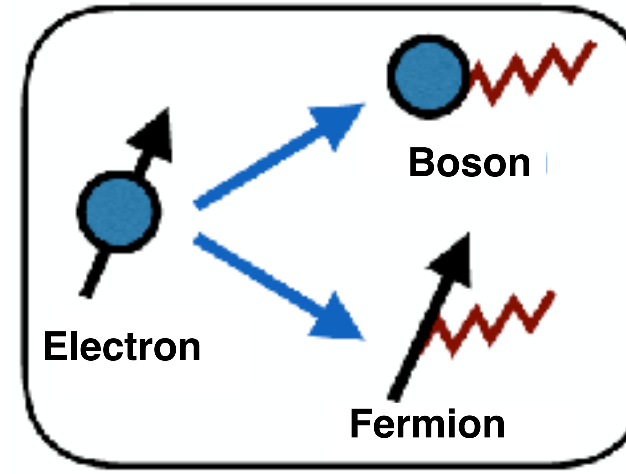
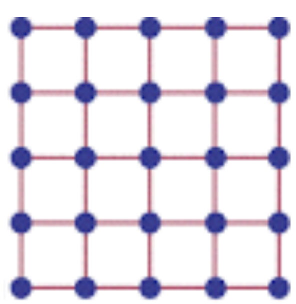
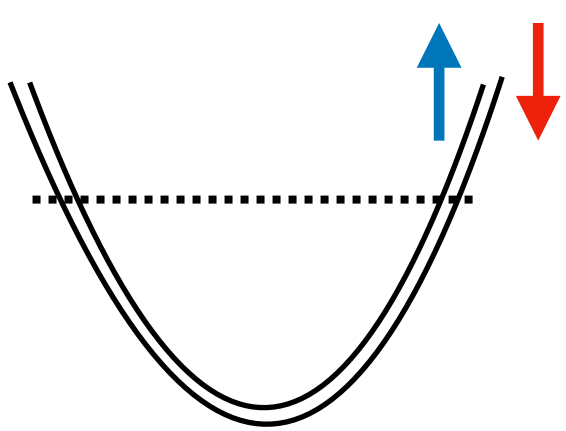


Spinon fermi surface:

Bosons form trivial Mott insulator:



Fermion forms fermi sea:



ϵ_F

Simplest case: electrons in half-filled lattice

Compact U(1) gauge field $q^{\text{spinon}}_{U(1)} = 1$

$$q^{\text{boson}}_{U(1)} = -1$$

Is compact QED coupled to a fermi surface stable to confinement?

Probably yes in 2+1D

Sung-Sik Lee, Phys. Rev. B **78**, 085129 (2008)
Instanton has infinite scaling dimension

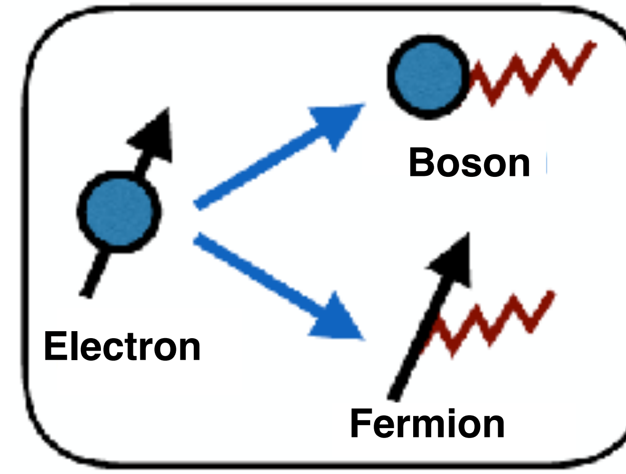
Yes in 3+1D and above

Spinon Fermi surface

Slave-boson

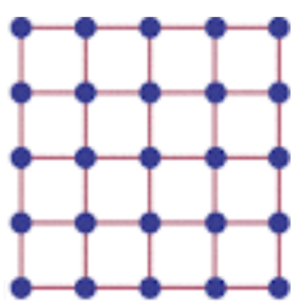
Spinfull neutral fermion ↓

Electron ⟶ $c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$ ⟵ Spinless charged boson

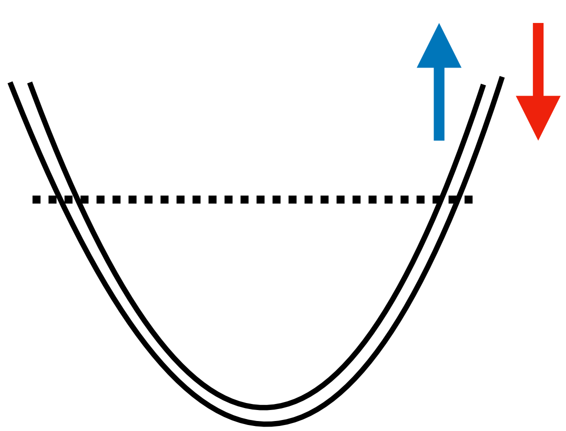


Spinon fermi surface:

Bosons form trivial Mott insulator:



Fermion forms fermi sea:



ϵ_F

Simplest case: electrons in half-filled lattice

Compact U(1) gauge field $q^{\text{spinon}}_{U(1)} = 1$

$$q^{\text{boson}}_{U(1)} = -1$$

Perhaps no exactly solvable model (a la Kitaev) exists for this phase of matter

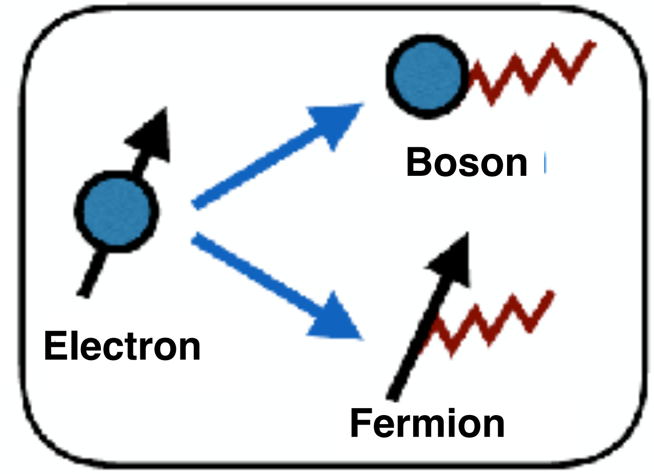
“Non-Fermi liquid”:

Spinon fermi surface $C \sim T^{2/3}$

Landau Fermi liquid $C \sim T$

Motrunich, Phys. Rev. B **72**, 045105 (2005)

Where could it be?



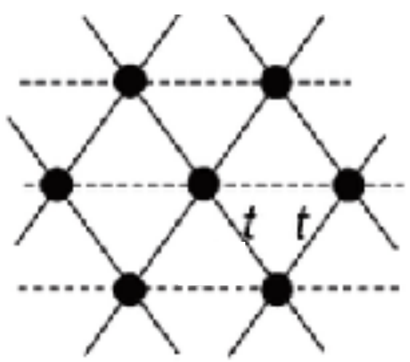
Fermion ↘

Electron ⟶ $c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$ ⟵ **boson**

Triangular lattice Hubbard model at half-filling

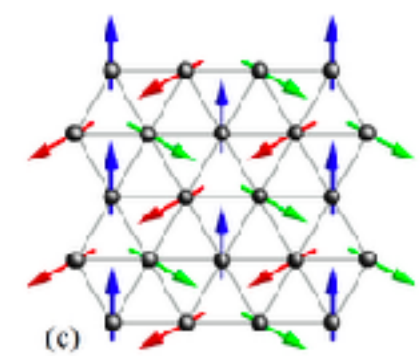
$t \gg U$ | $t \sim U$ | $t \ll U$

Metal



Spinon fermi surface?

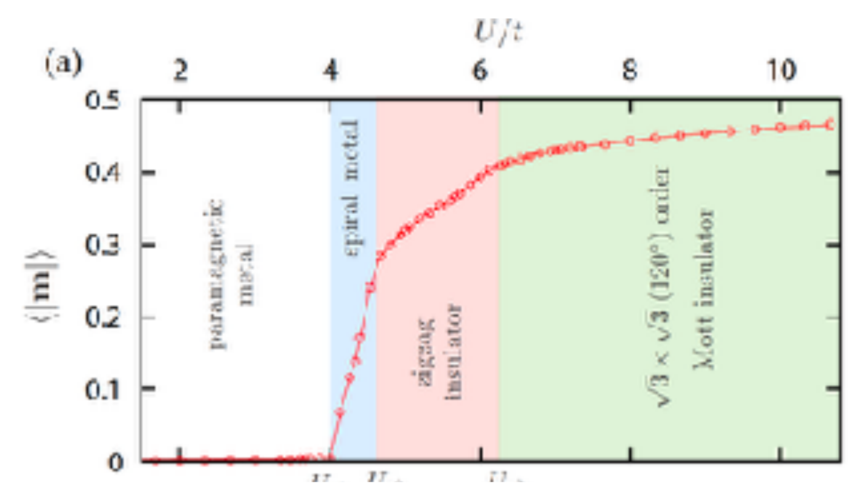
120 AFM



Motrunich, PRB 72, 045105 (2005)

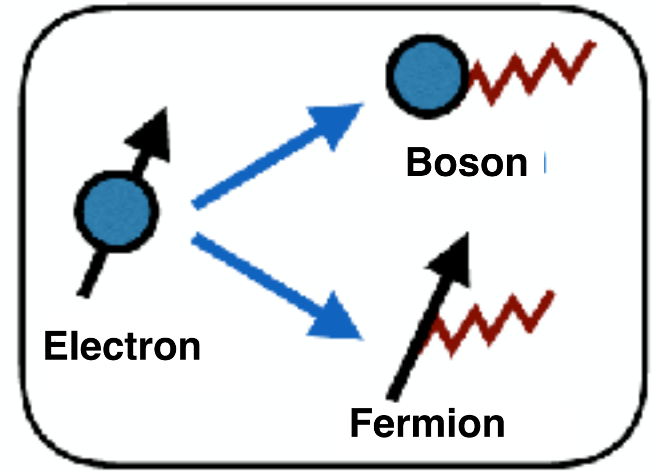
SS. Lee and PA. Lee, PRL 95, 036403 (2005)

Spin wave theory



Gia-Wei Chern et al. PRB **97**, 035120 (2018)

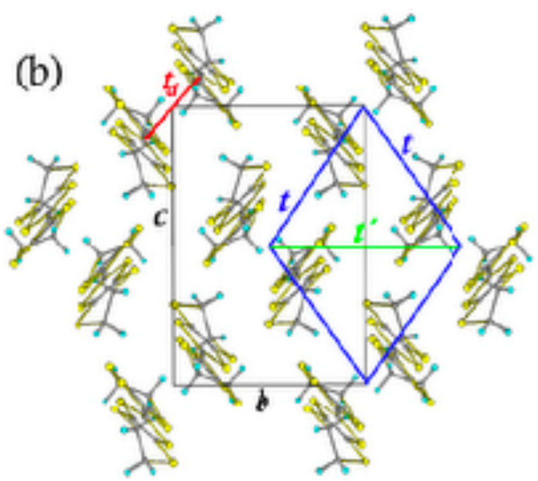
Where could it be?

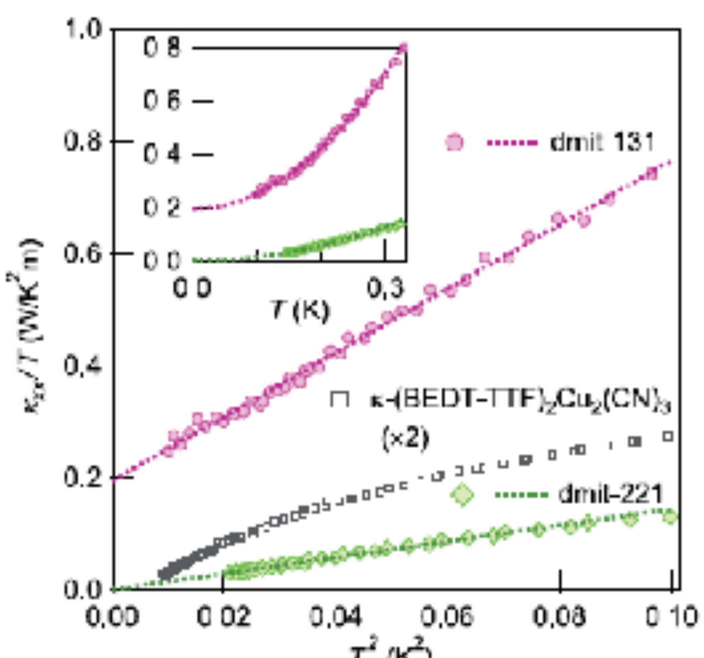


Fermion ↘

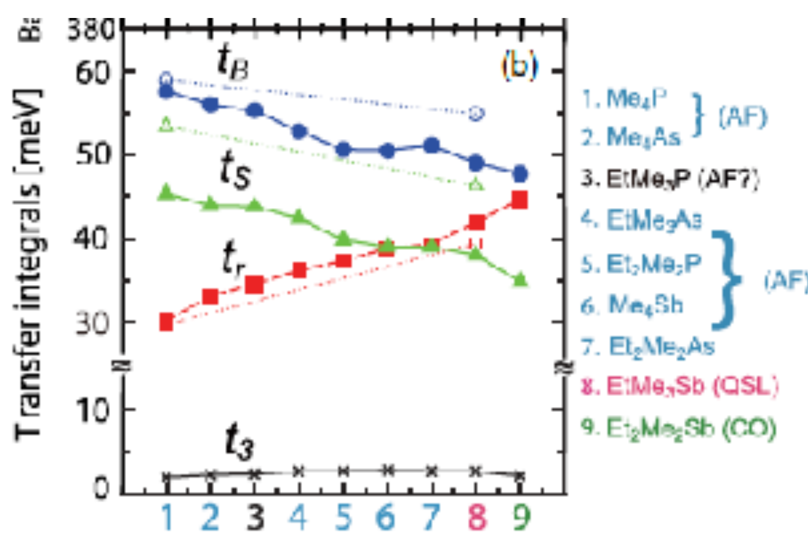
Electron ⟶ $c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$ ⟵ **boson**

Triangular lattice organic materials

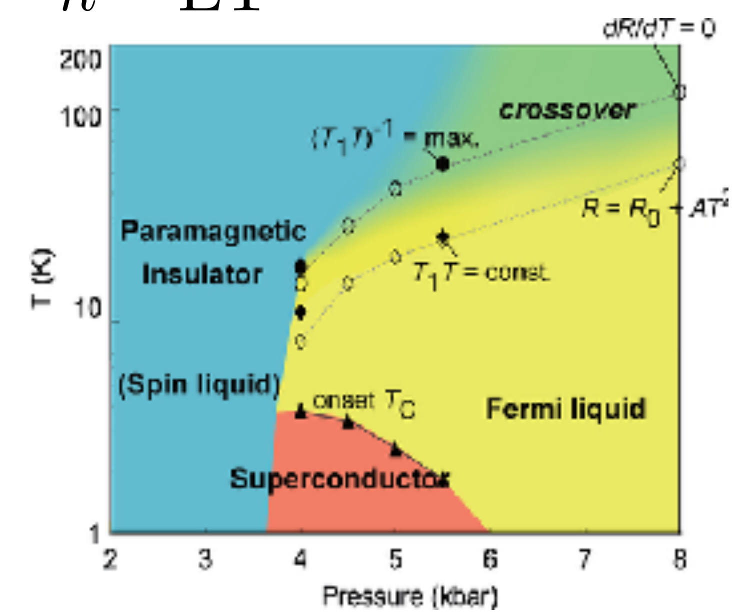




d-mit

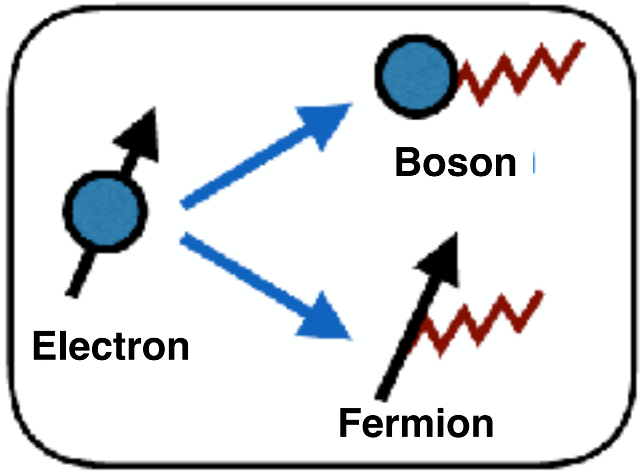


$\kappa - \mathrm{ET}$



d-mit conducts heat like a metal in spite of being an electrical insulator

Low energy effective theory



Fermion

Electron → $c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$ ← **boson**

carries physical charge

U(1) gauge theory:

$$\mathcal{L} = \mathcal{L}_f(p-a) + \mathcal{L}_b(p+a-A) + \cdots$$

anything gauge invariant and allowed by symmetry

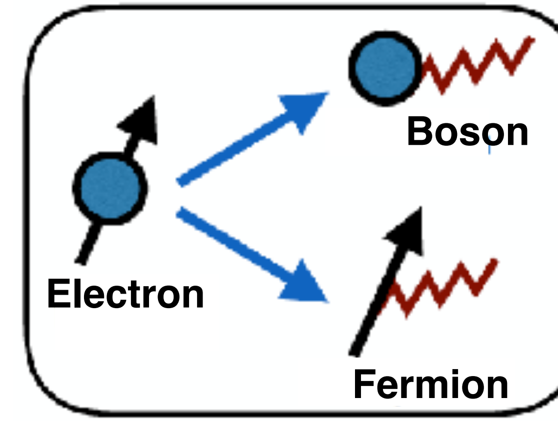
↓ **Integrate out bosons**

$$\mathcal{L} = \mathcal{L}_f(p-a) + \frac{\epsilon}{2}(e-E)^2 - \frac{1}{2\mu}(b-B)^2 + \cdots$$

Internal fields want to "track" probe physical fields

Spinons in magnetic fields

$$c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$$



U(1) gauge theory:

$$\mathcal{H} = \mathcal{H}_f(p-a) + \frac{\epsilon}{2}(e-E)^2 + \frac{1}{2\mu}(b-B)^2 + \cdots$$

Magnetic field present:

$$\chi_f = \frac{g}{24\pi m_f}$$

$$E \approx E_f(b) + \frac{(b-B)^2}{2\mu}$$

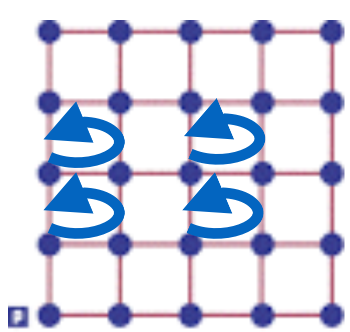
$$T \gg \omega_f = \frac{b}{m_f} \qquad E_f(b) \approx \frac{\chi_f}{2} b^2$$

$$\boxed{b \approx \frac{1}{1+\mu\chi_f} B}$$

Physically:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

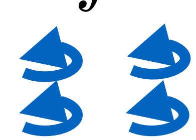
$$j_b \neq 0$$



$$\mathbf{b} = \nabla \times \mathbf{a}$$

$$j_f = j_b$$

$$f^\dagger$$



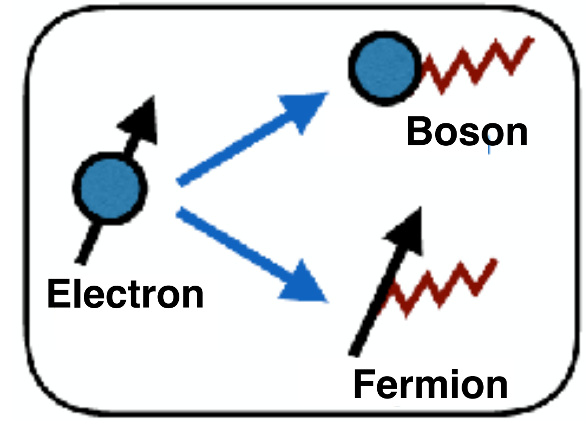
$$M_f = M_b$$

Diamagnet:

$$\boxed{4\pi M \approx -\frac{\chi_f}{1+\mu\chi_f} B}$$

Quantum oscillations of spinons

$$c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$$

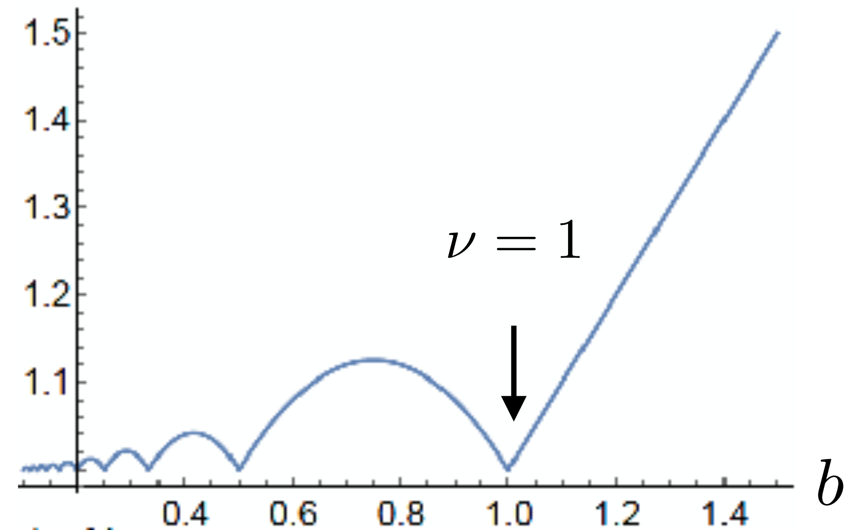


Magnetic field present:

$$E \approx E_f(b) + \frac{(b-B)^2}{2\mu}$$

$$T \ll \omega_f = \frac{b}{m_f}$$

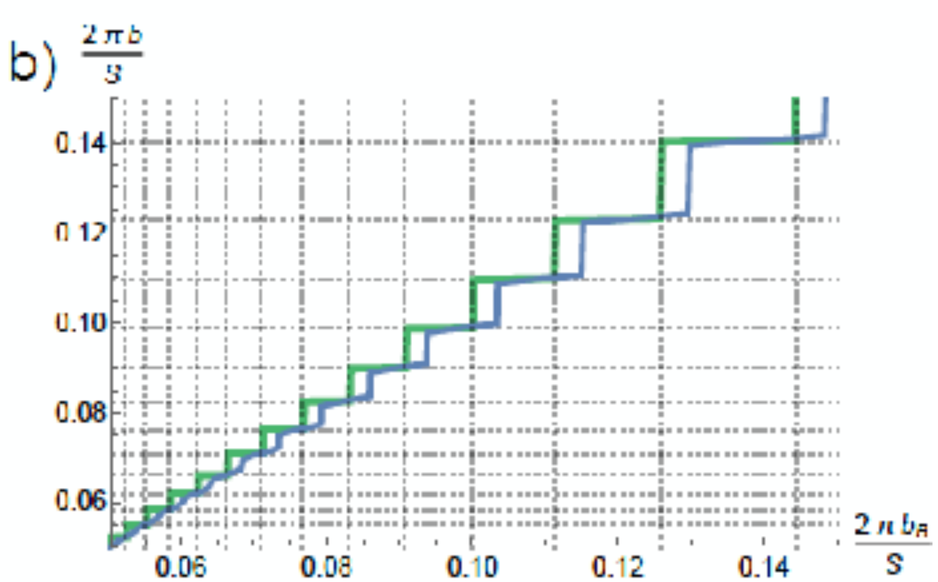
$E_f(b)/E_f(0)$

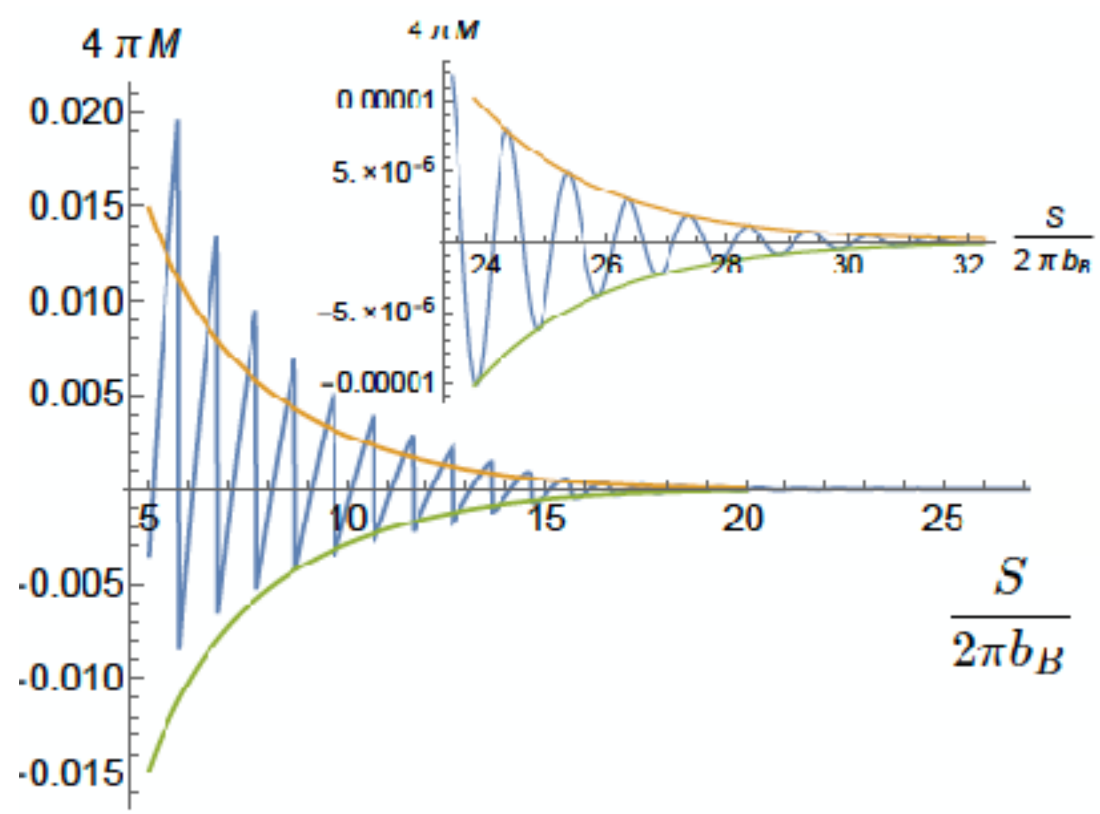


Motrunich, PRB (2006)

Katsura, Nagaosa & Lee, PRL (2010)

Quantum oscillations:



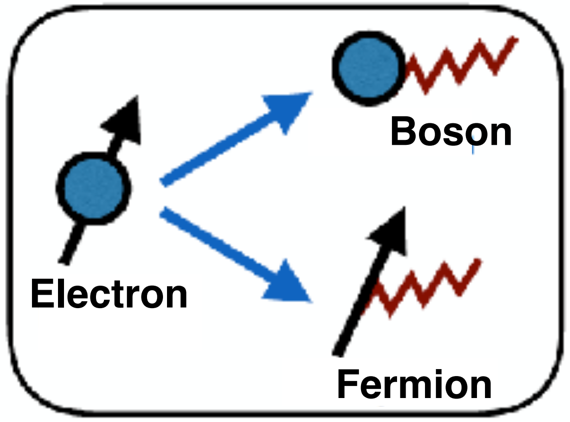


Chowdhury, Sodemann, and Senthil, Nature Comms 9, 1766 (2018).

Sodemann, Chowdhury, and Senthil, PRB 97, 045152 (2018).

Spinons with electric fields

$$c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$$



Electric field present:

$$\mathcal{L} = \sum_i \frac{m_f v_i^2}{2} + v_i a + \frac{\epsilon L^2}{2}(\dot{a} - \dot{A})^2$$

Maxwell-Ampere equation ⟷ **“Gauge freezing” of the Kohn mode**

$$\sum_i v_i + \epsilon L^2(\ddot{a} - \ddot{A}) = 0$$

No response to DC electric fields

Ioffe-Larkin composition rule:

$$\sigma^{-1}(\omega) = \sigma_f^{-1}(\omega) + \sigma_b^{-1}(\omega)$$

$$\rho(\omega) = \rho_f(\omega) + \rho_b(\omega)$$

$$Re\sigma(\omega) \approx \omega^2 \frac{\epsilon^2}{D\tau}$$

$$\sigma_f(\omega) \approx \frac{iD}{\omega + i/\tau}$$

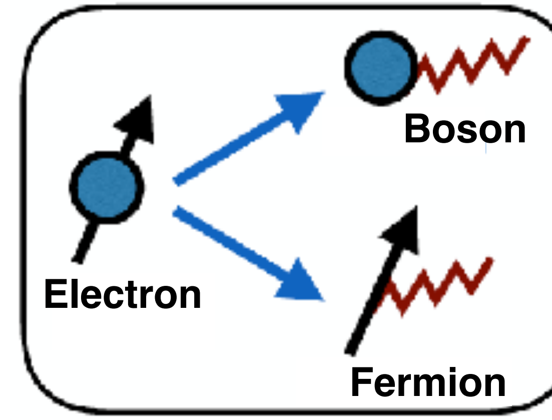
Metal

$$\sigma_b(\omega) \approx -i\omega\epsilon$$

Insulator

L. B. Ioffe and A. I. Larkin, Phys. Rev. B 39, 8988 (1989).

Power law optical absorption



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$$\sigma^{-1}(\omega) = \sigma_f^{-1}(\omega) + \sigma_b^{-1}(\omega)$$

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Metal

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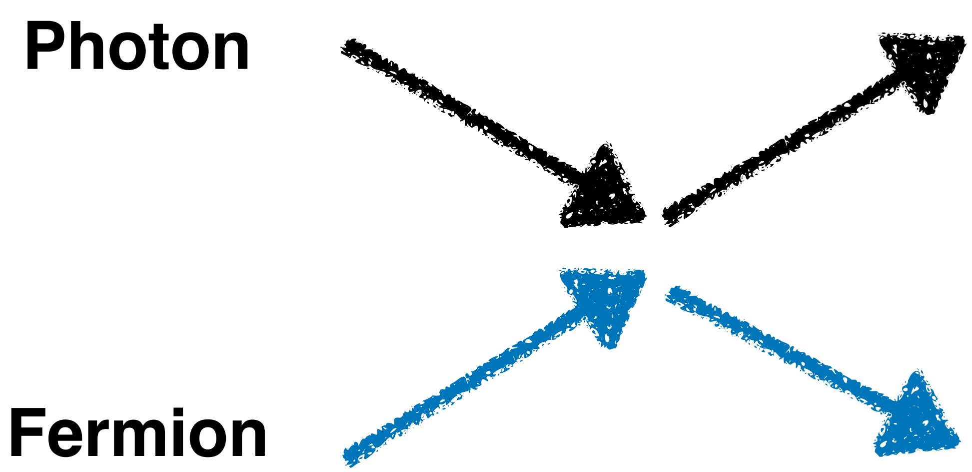
Insulator

$$\rho(\omega) = \rho_f(\omega) + \rho_b(\omega)$$



$$\frac{1}{\tau} \approx \omega \left(\frac{\omega}{\Lambda}\right)^{1/3}$$

P. A. Lee and N. Nagaosa, Phys. Rev. B 46, 5621 (1992)

Ng and P. A. Lee, Phys. Rev. Lett. 99, 156402 (2007)

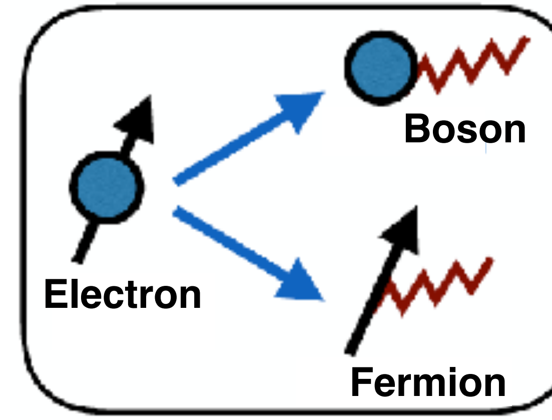
Clean

$$Re\sigma(\omega) \approx \omega^{\frac{10}{3}} \frac{\epsilon^2 \Lambda^{1/3}}{D}$$

With impurities

$$Re\sigma(\omega) \approx \omega^2 \frac{\epsilon^2}{D\tau_0}$$

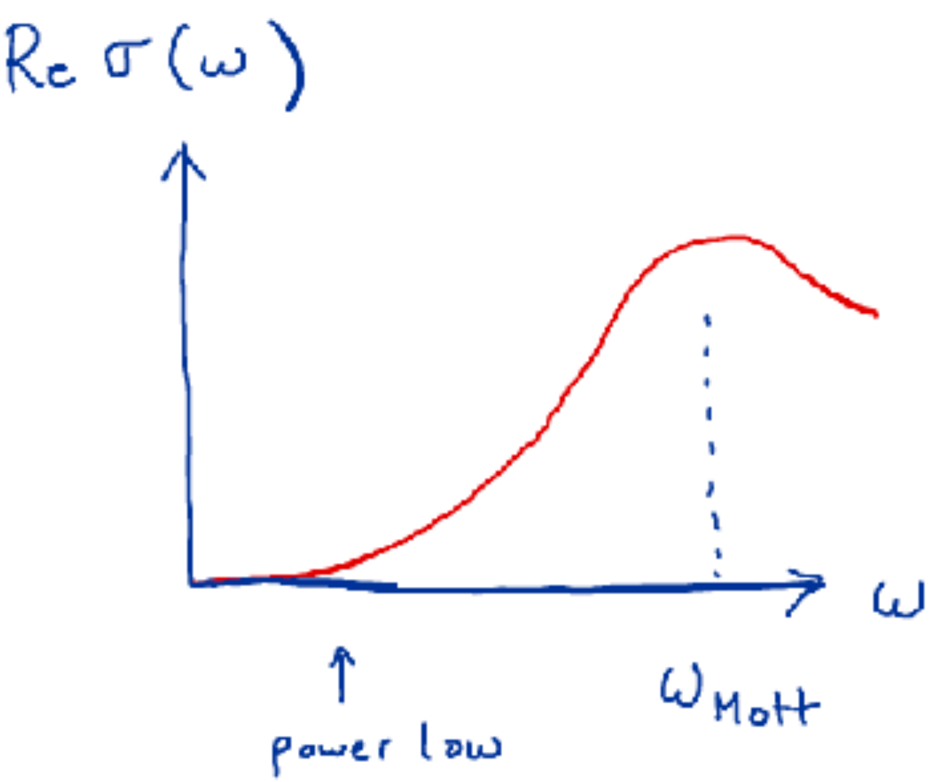
How to distinguish it from “dirt”?



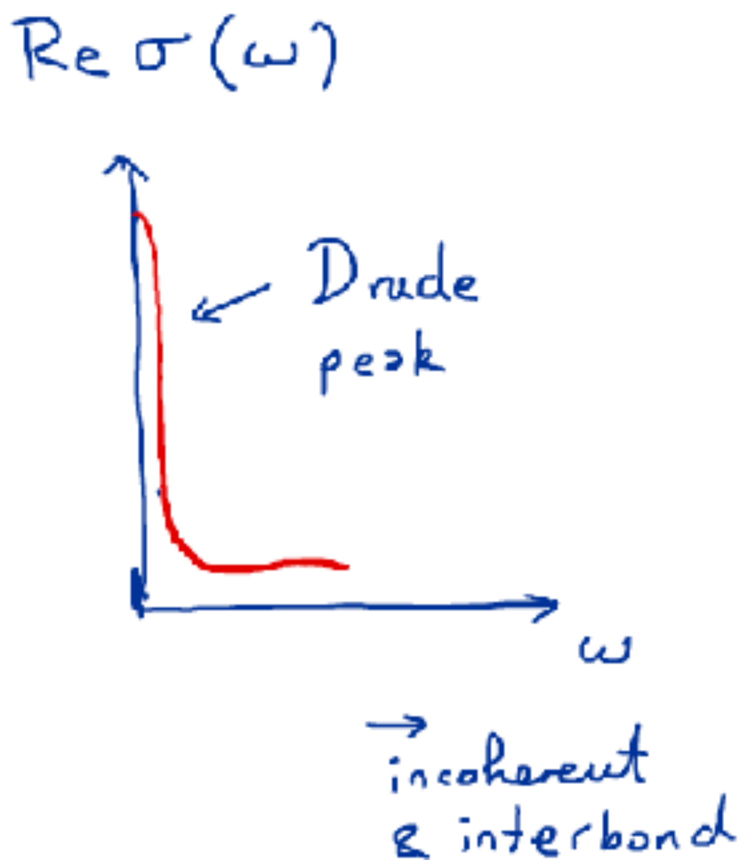
Optical conductivity

$$Re\sigma(\omega) \approx \omega^2 \frac{\epsilon^2}{D\tau_0}$$

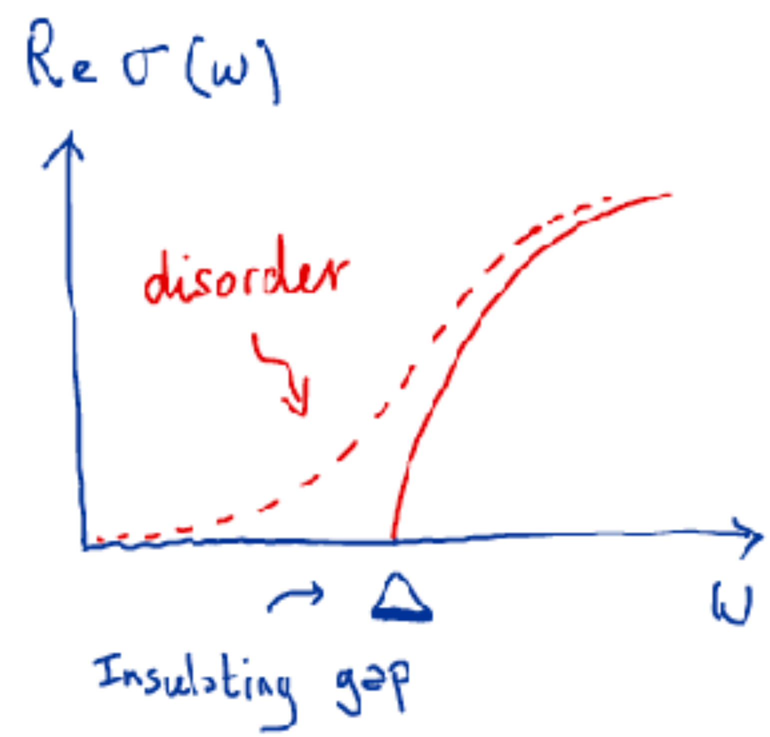
Gauged fermi surface



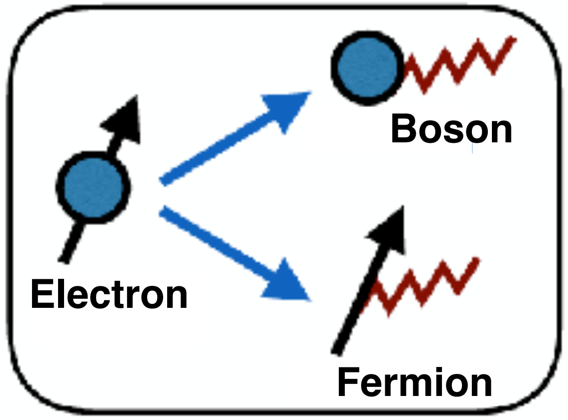
Ordinary metal



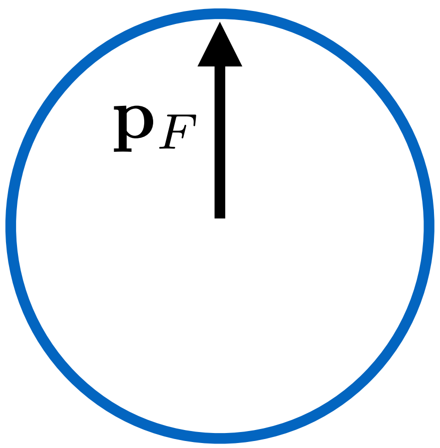
Ordinary insulator



Cyclotron resonance

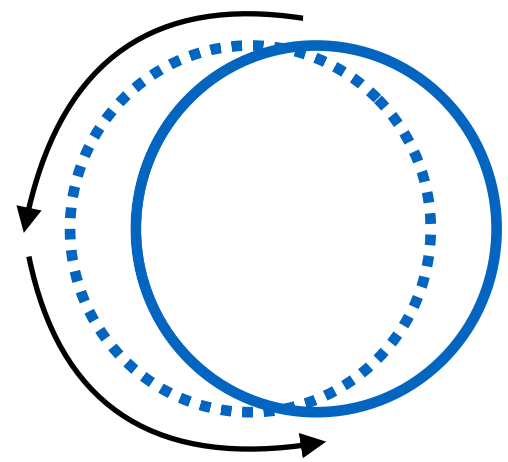


Cyclotron resonance metals:

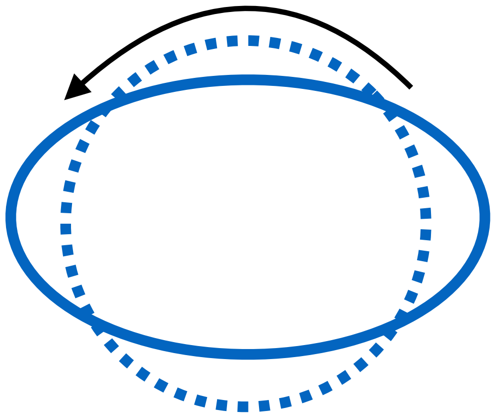


Kohn mode: ω_c

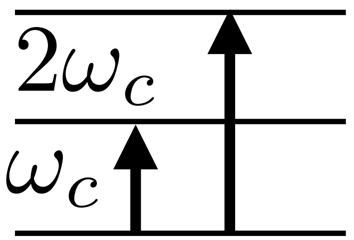
$T_1 = \frac{2\pi}{\omega_c}$

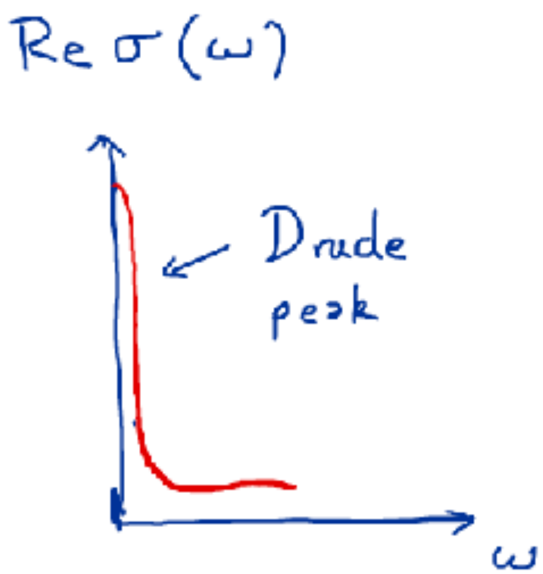


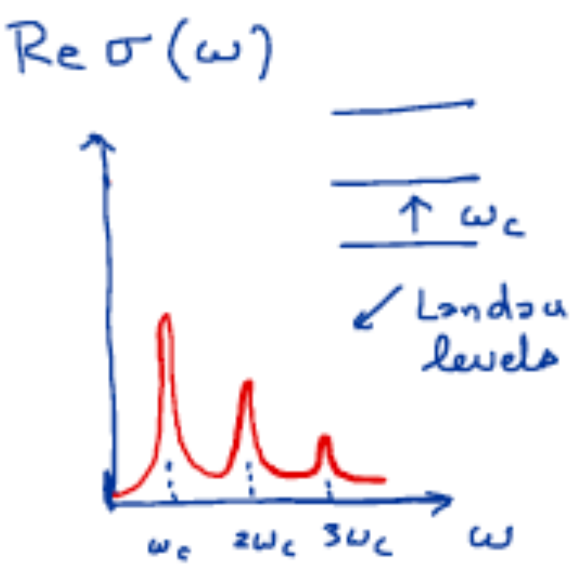
Second mode: $2\omega_c$



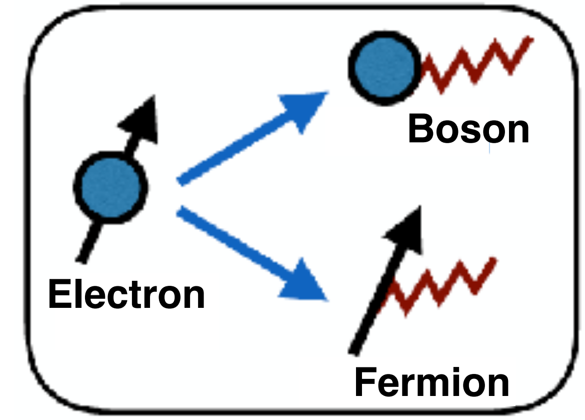
$T_2 = \frac{T_1}{2}$





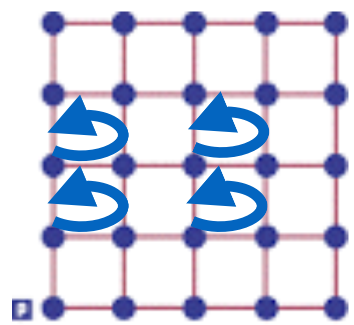


Cyclotron resonance spinons



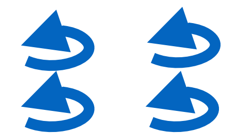
$\mathbf{B} = \nabla \times \mathbf{A}$

$j_b \neq 0$



$\mathbf{b} = \nabla \times \mathbf{a}$

$f^\dagger$

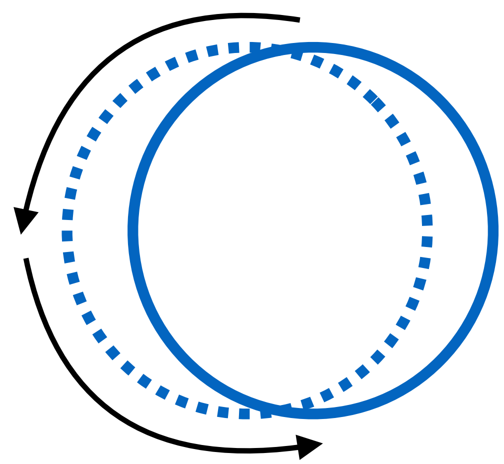


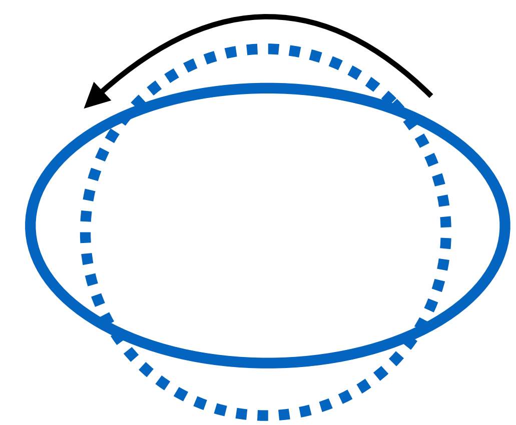
Kohn mode: ω_c

Second mode: $2\omega_c$

“Gauge freezing” of the Kohn mode

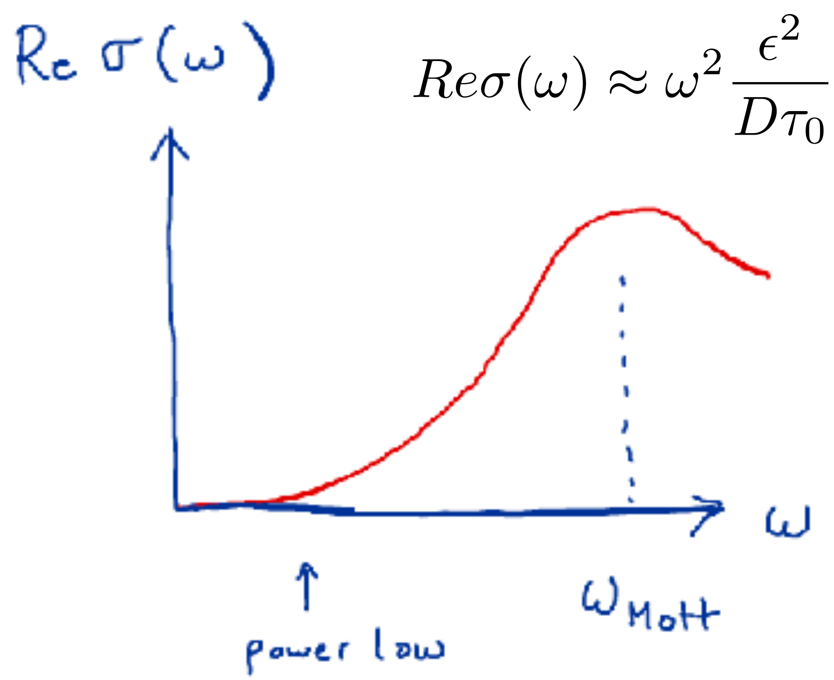
$$T_1 = \frac{2\pi}{\omega_c}$$

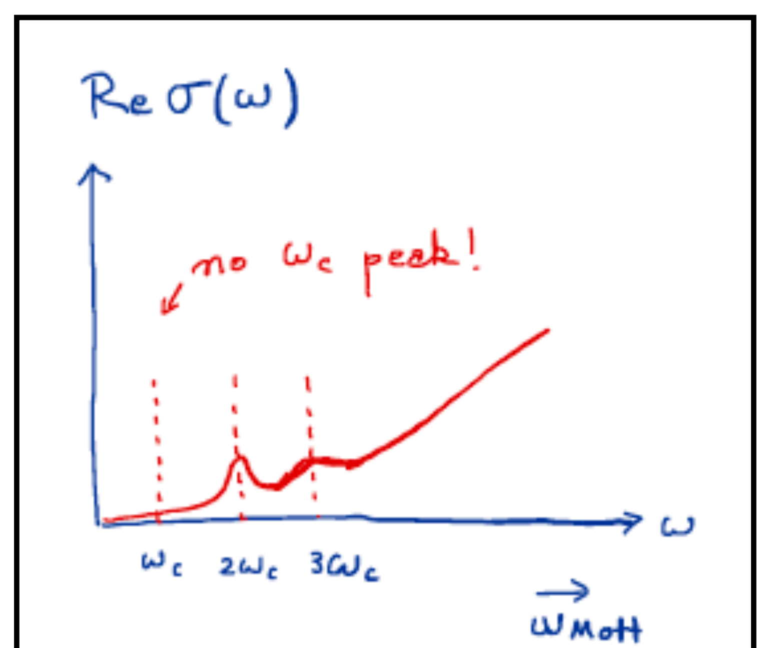




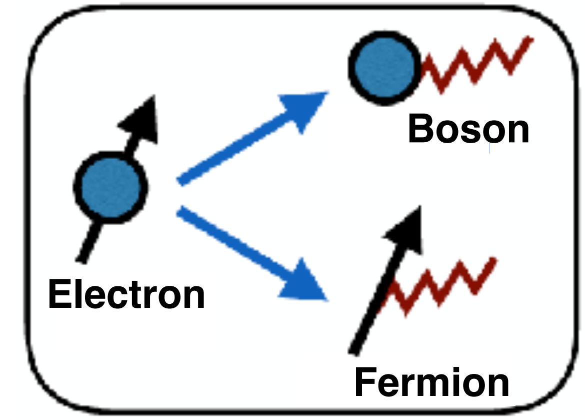
$$T_2 = \frac{T_1}{2}$$

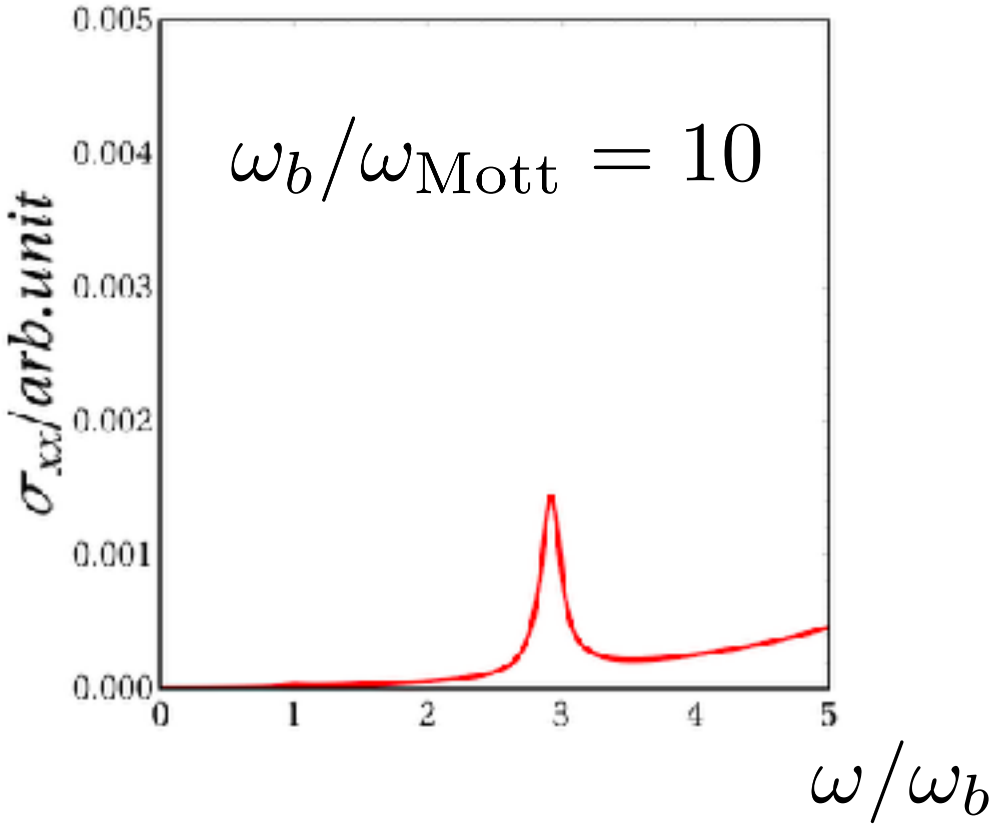
$$Re\sigma(\omega) \approx \omega^2 \frac{\epsilon^2}{D\tau_0}$$





Cyclotron resonance spinons

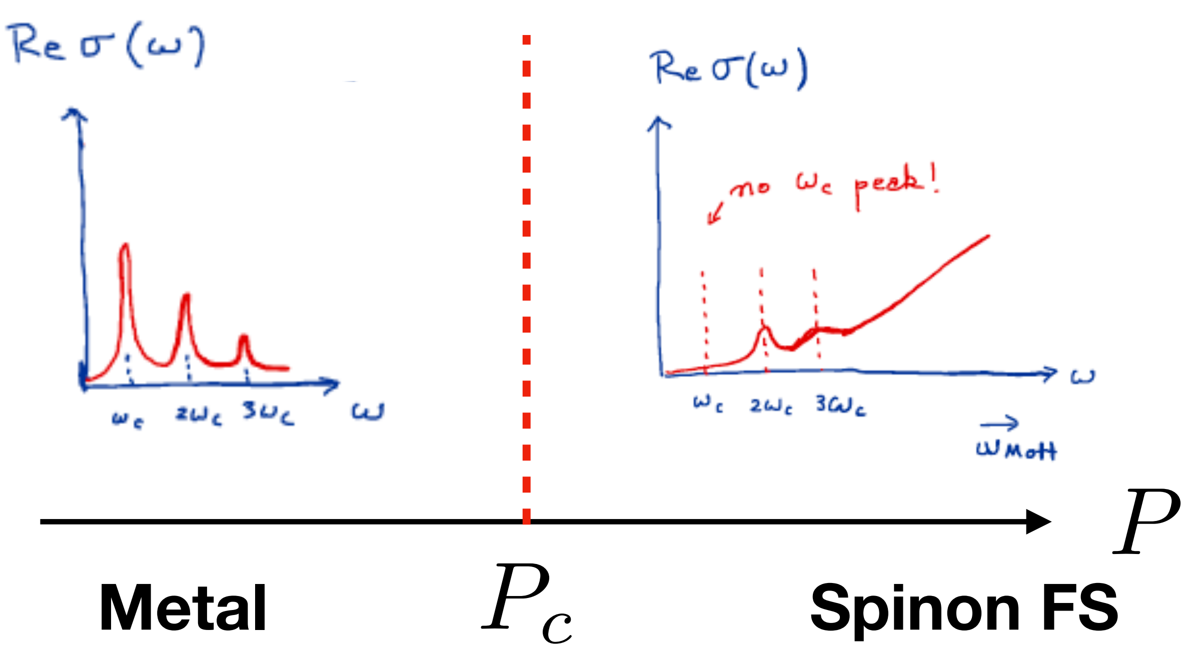


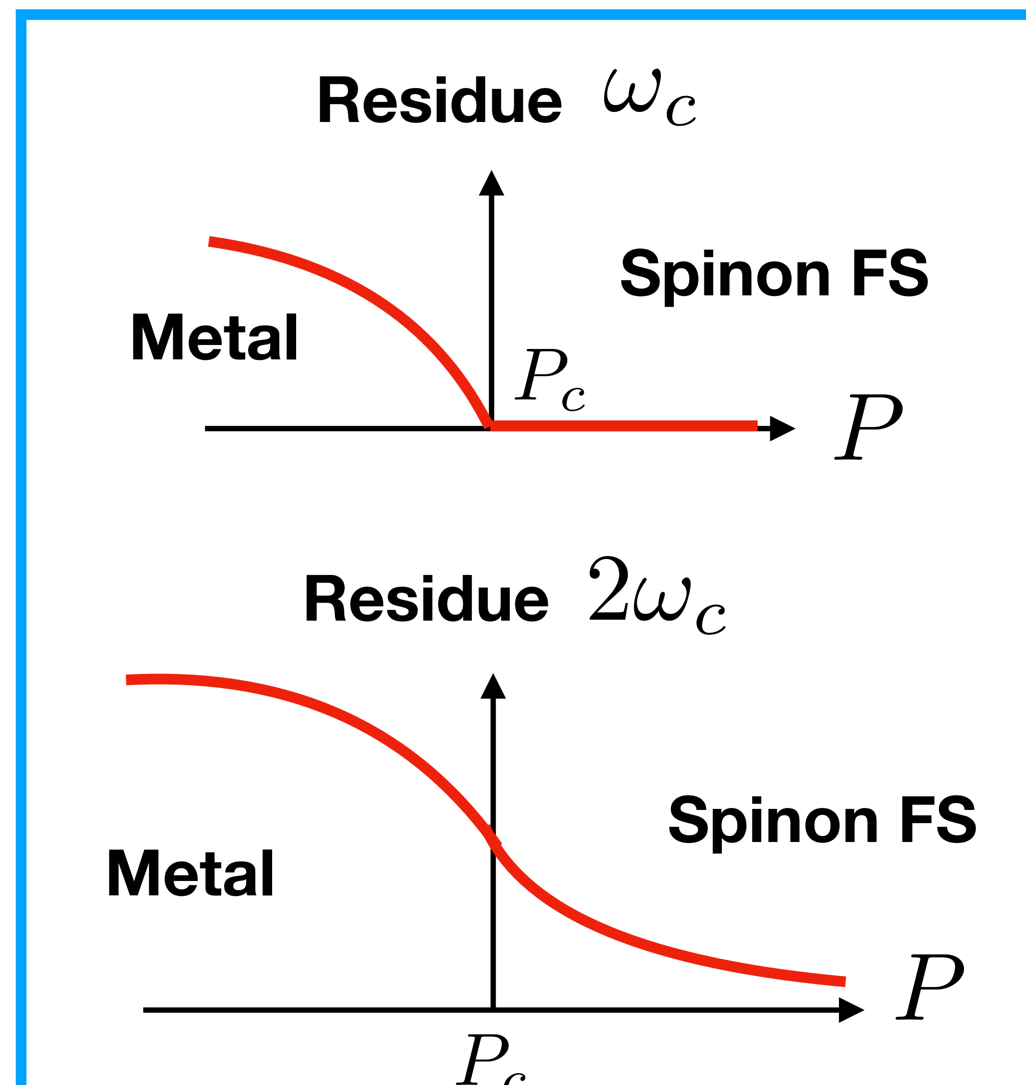


Residue of peak

$$\mathrm{Res}(\sigma_{\mathrm{spinon}}) \sim \left(\frac{\omega_b}{\omega_{\mathrm{Mott}}}\right)^4 \mathrm{Res}(\sigma_{\mathrm{electron}})$$

Width of peak $\frac{1}{\tau} \approx \omega \left(\frac{\omega}{\Lambda}\right)^{1/3}$



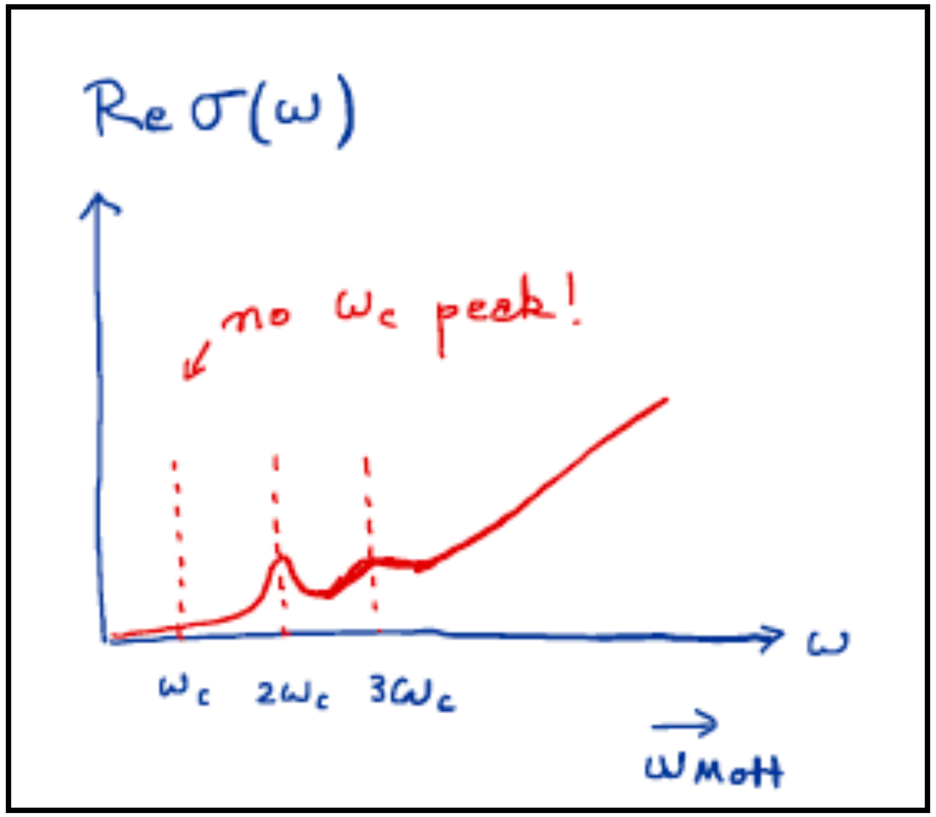


Summary Part I
“Smoking gun” probes for U(1) spin liquids with gapless fermions

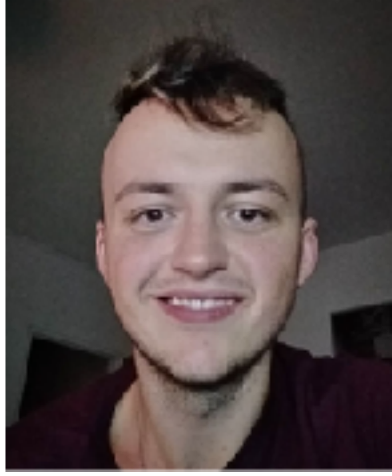
Peng Rao

1) Spinons in U(1) spin liquids are electrically neutral particles but they develop Landau levels under physical magnetic fields.

2) These spin liquids have cyclotron resonance and magnetisation oscillations (de-Haas van alphen) even though they are insulators.

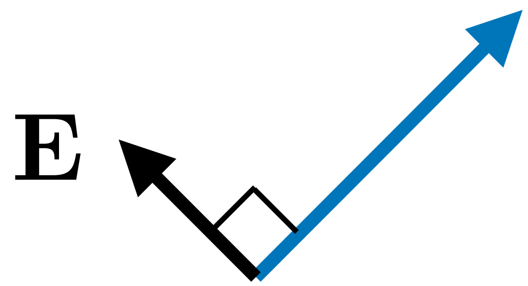


Outline Part II
Berry curvature dipole of metals



Oles Matsyshyn

1) Non-linear Hall effect in time reversal invariant materials controlled by the Berry curvature dipole



$$\mathbf{j}_{\mathrm{NLHE}} \propto \mathbf{E}^2$$

2) Berry curvature dipole measures a non-Newtonian and non-linear acceleration

$$\frac{d^2\mathbf{r}}{dt^2} \sim (\text{Berry dipole})\ \mathbf{E}^2$$

Berry curvature in crystals

Vacuum

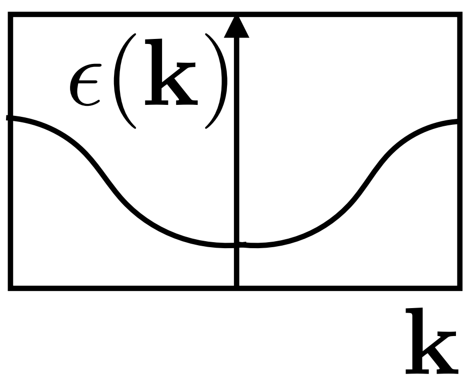
$$\mathbf{r}_\alpha = i\frac{\partial}{\partial \mathbf{p}_\alpha} \qquad [\mathbf{r}_\alpha, \mathbf{r}_\beta] = 0 \qquad \alpha, \beta \in \{x, y, z\}$$

Berry curvature in crystals

Vacuum

$$\mathbf{r}_\alpha = i\frac{\partial}{\partial \mathbf{p}_\alpha} \qquad [\mathbf{r}_\alpha, \mathbf{r}_\beta] = 0 \qquad \alpha, \beta \in \{x, y, z\}$$

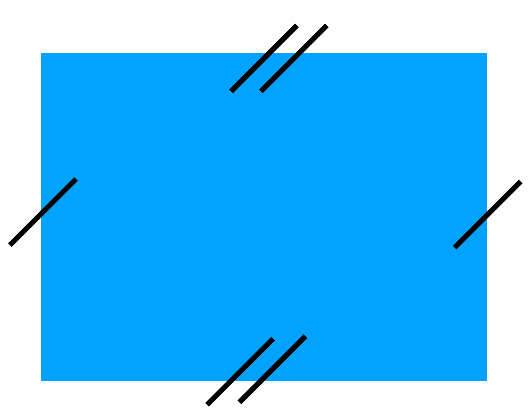
Crystals



$|u(\mathbf{k})\rangle$

Berry connection

$$\mathbf{r} = i\frac{\partial}{\partial \mathbf{k}} + \mathbf{A}(\mathbf{k}) \qquad \mathbf{A}(\mathbf{k}) = i\langle u(\mathbf{k})|\partial_{\mathbf{k}}|u(\mathbf{k})\rangle$$



$\mathbf{k} \in$ Brillouin zone

E. Blount, in Solid state physics, Vol. 13 (Elsevier, 1962)

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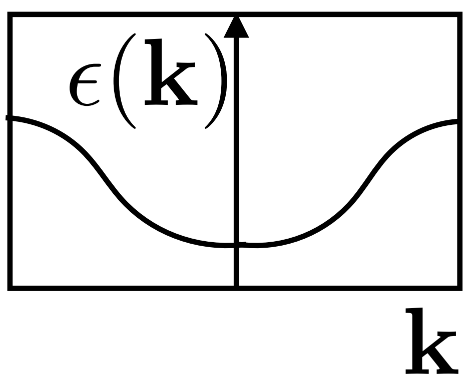
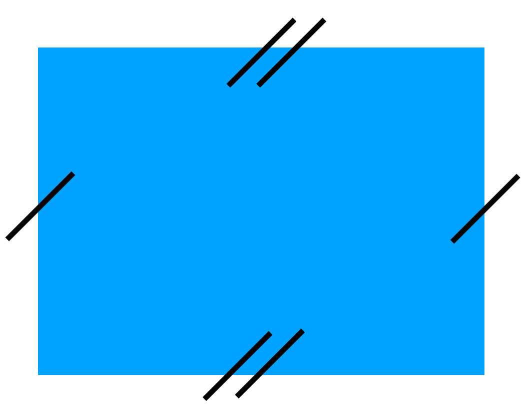


$|u(\mathbf{k})\rangle$



$\mathbf{k} \in \text{Brillouin zone}$

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Berry curvature

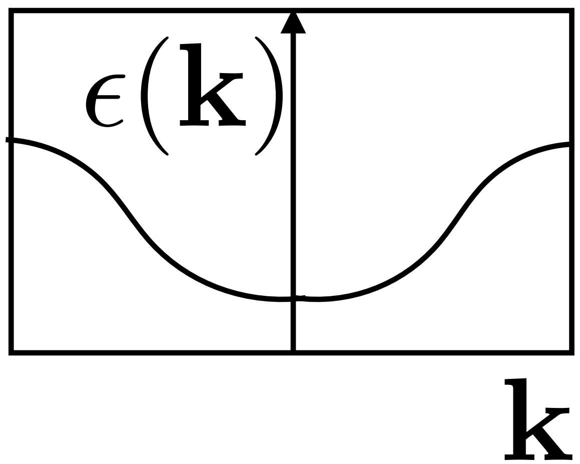
$$[\mathbf{r}_\alpha, \mathbf{r}_\beta] = i\epsilon_{\alpha\beta\gamma}\boldsymbol{\Omega}_\gamma$$

$$\boldsymbol{\Omega}(\mathbf{k}) = \partial_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$$

E. Blount, in Solid state physics, Vol. 13 (Elsevier, 1962)

“Anomalous” velocity

Berry connection = momentum-locked dipole



$$H = \epsilon(\mathbf{k}) + e\mathbf{E}(t)\cdot\mathbf{r} = \epsilon(\mathbf{k}) + e\mathbf{E}\cdot\partial_{\mathbf{k}} + e\mathbf{E}\cdot\mathbf{A}(\mathbf{k})$$

$$\mathbf{r} = i\frac{\partial}{\partial\mathbf{k}} + \mathbf{A}(\mathbf{k}) \qquad \frac{d\mathbf{k}}{dt} = i[H,\mathbf{k}] = -e\mathbf{E}$$

Karplus and Luttinger, Phys. Rev. ***95****, 1154 (1954).*

Chang and Niu, Phys. Rev. Lett. ***75****, 1348, (1995).*

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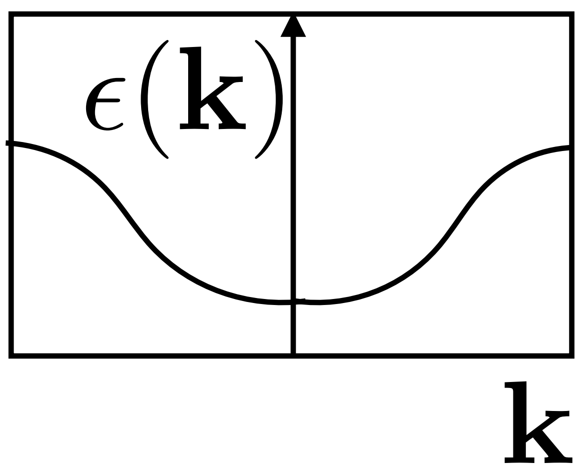


$$H = \epsilon(\mathbf{k}) + e\mathbf{E}(t)\cdot\mathbf{r} = \epsilon(\mathbf{k}) + e\mathbf{E}\cdot\partial_{\mathbf{k}} + e\mathbf{E}\cdot\mathbf{A}(\mathbf{k})$$

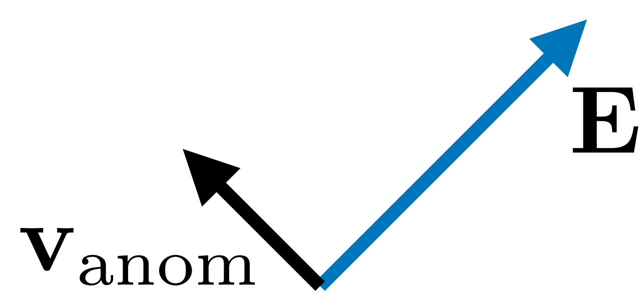
$$\mathbf{r} = i\frac{\partial}{\partial\mathbf{k}} + \mathbf{A}(\mathbf{k}) \qquad \frac{d\mathbf{k}}{dt} = i[H,\mathbf{k}] = -e\mathbf{E}$$

The velocity has an extra piece besides the group velocity:

$$\mathbf{v}_\alpha = \frac{d\mathbf{r}_\alpha}{dt} = i[H,\mathbf{r}_\alpha] = \partial_{\mathbf{k}_\alpha}\epsilon - ei[\mathbf{r}_\alpha,\mathbf{r}_\beta]\mathbf{E}_\beta$$

$$[\mathbf{r}_\alpha,\mathbf{r}_\beta] = i\epsilon_{\alpha\beta\gamma}\mathbf{\Omega}_\gamma$$

$$\mathbf{v} = \frac{\partial\epsilon}{\partial\mathbf{k}} + e\Omega\times\mathbf{E}(t)$$



*Karplus and Luttinger, Phys. Rev. **95**, 1154 (1954).*

*Chang and Niu, Phys. Rev. Lett. **75**, 1348, (1995).*

Linear response

$$\mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{k}} + e\Omega \times \mathbf{E}$$

$$\frac{d\mathbf{k}}{dt} = -e\mathbf{E}$$

$$\mathbf{j}_\alpha = -e \int \frac{d^d k}{(2\pi)^d} \mathbf{v}_\alpha(\mathbf{k}) f_0(\mathbf{k} + e\tau\mathbf{E})$$

$$\Delta\mathbf{k} = -e\tau\mathbf{E}$$

Linear response

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$$\Delta\mathbf{k} = -e\tau\mathbf{E}$$

$$= \underbrace{e^2\tau\langle\partial^2_{\mathbf{k}_\alpha \mathbf{k}_\beta}\epsilon\rangle}_{\text{Drude weight}}\mathbf{E}_\beta - \underbrace{e^2\epsilon_{\alpha\beta\gamma}\langle\boldsymbol{\Omega}_\beta\rangle}_{\text{Hall conductivity}}\mathbf{E}_\gamma + \mathcal{O}(\mathbf{E}^2)$$

Drude weight **Hall conductivity**

$$\langle \cdots \rangle \equiv \int_{\mathrm{BZ}} \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \cdots$$

Linear response

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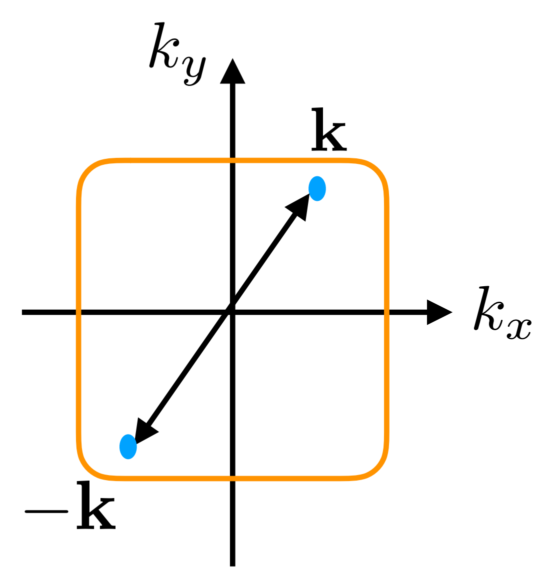
$$\langle \cdots \rangle \equiv \int_{\text{BZ}} \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \cdots$$

Time reversal:

$$\mathbf{\Omega}(\mathbf{k}) = -\mathbf{\Omega}(-\mathbf{k}) \qquad \sigma_{\text{Hall}} = 0$$



Non-linear Hall effect

$$\mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{k}} + e\Omega \times \mathbf{E}$$

$$\langle \cdots \rangle \equiv \int_{\mathrm{BZ}} \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \cdots$$

$$\mathbf{j}_\alpha = -e \int \frac{d^d k}{(2\pi)^d} \mathbf{v}(\mathbf{k}) f_0(\mathbf{k} + e\tau\mathbf{E}) = \text{linear response} +$$

$$- \frac{e^3\tau^2}{2} \underbrace{\langle \partial^3_{\mathbf{k}_\alpha \mathbf{k}_\beta \mathbf{k}_\gamma} \epsilon \rangle}_{\text{“Jerk”}} \mathbf{E}_\beta \mathbf{E}_\gamma + e^3 \tau \epsilon_{\alpha\beta\gamma} \langle \partial_{\mathbf{k}_\delta} \mathbf{\Omega}_\beta \rangle \mathbf{E}_\gamma \mathbf{E}_\delta$$

J. E. Moore and J. Orenstein, Phys. Rev. Lett. 105, 026805 (2010)

Inti Sodemann and Liang Fu, Phys. Rev. Lett. 115, 216806 (2015)

Non-linear Hall effect

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Time reversal:

$$\partial_{\mathbf{k}_\alpha} \mathbf{\Omega}_\beta |_{\mathbf{k}} \to \partial_{\mathbf{k}_\alpha} \mathbf{\Omega}_\beta |_{-\mathbf{k}}$$

$$\langle \partial^3_{\mathbf{k}_\alpha \mathbf{k}_\beta \mathbf{k}_\gamma} \epsilon \rangle = 0$$

Berry curvature dipole

$$D_{\alpha\beta} = \langle \partial_{\mathbf{k}_\alpha} \mathbf{\Omega}_\beta \rangle$$

$$\chi^{\mathrm{NLH}}_{\alpha\gamma\delta} \equiv e^3 \tau \epsilon_{\alpha\beta\gamma} D_{\delta\beta}$$

J. E. Moore and J. Orenstein, Phys. Rev. Lett. 105, 026805 (2010)

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Inversion :

$$D_{\alpha\beta} = 0$$

Insulator :

$$D_{\alpha\beta} = 0$$

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Order parameter for broken inversion symmetry in metals

Non-linear Hall effect

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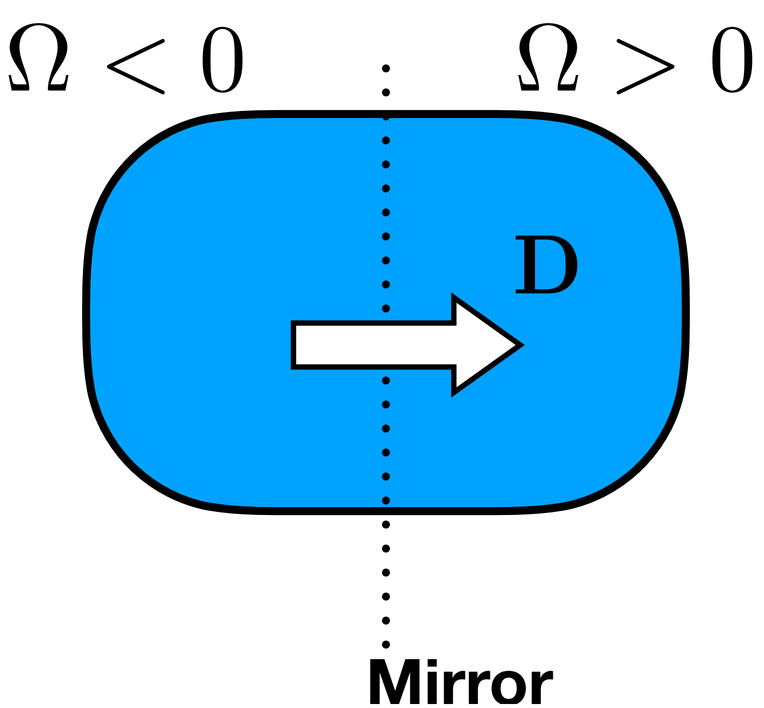
Order parameter for broken inversion symmetry in metals

Example: 2D metal

$$\mathbf{D} \equiv \int \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \frac{\partial \Omega(\mathbf{k})}{\partial \mathbf{k}}$$

$$\mathbf{J} = e^3 \tau (\mathbf{D} \cdot \mathbf{E}) \hat{\mathbf{z}} \times \mathbf{E}$$



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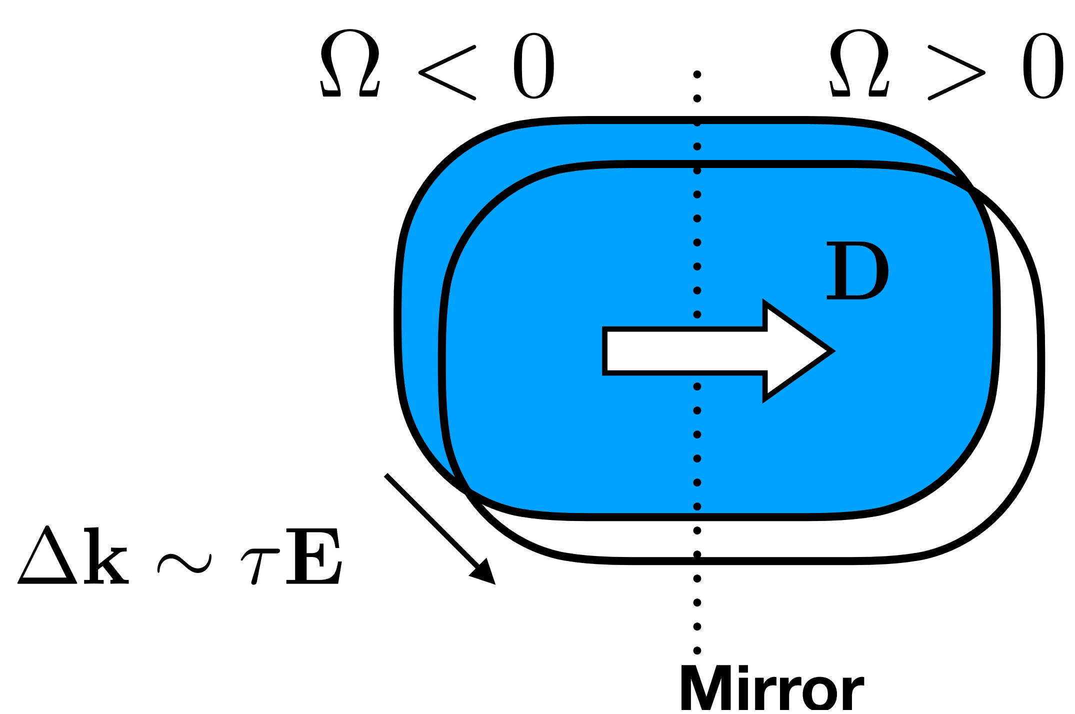
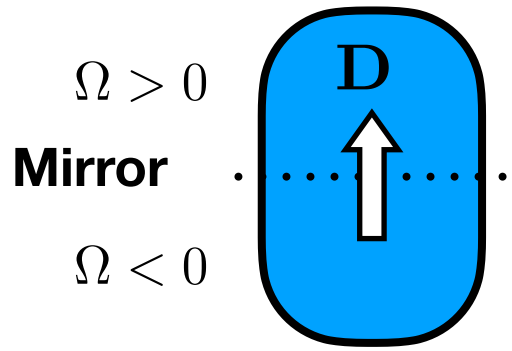
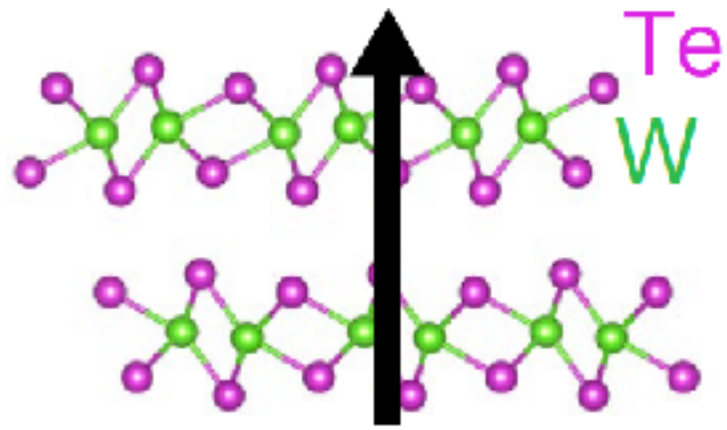


Inti Sodemann and Liang Fu, Phys. Rev. Lett. 115, 216806 (2015)

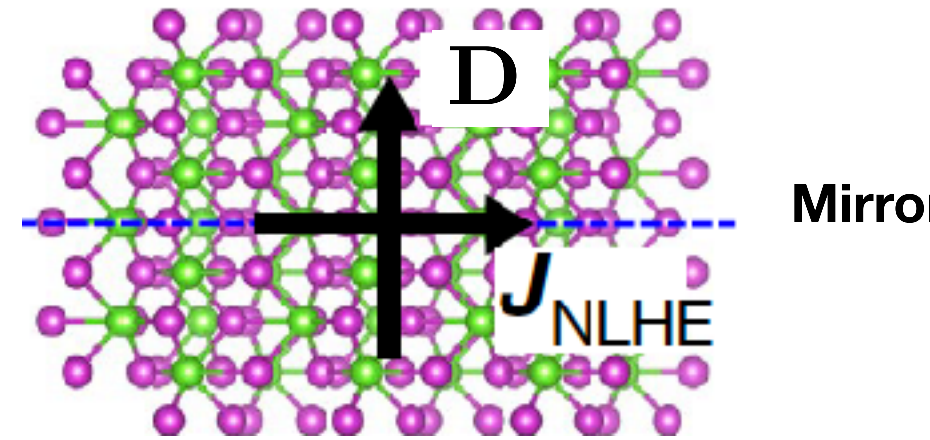
Experimental realisation

Td-WTe2 bilayer:





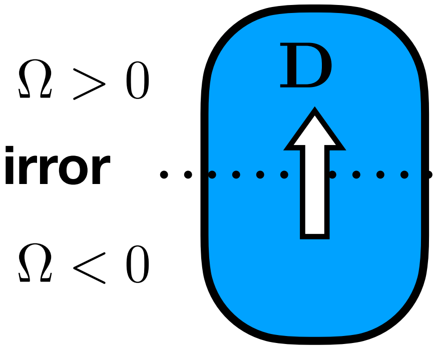
Side view

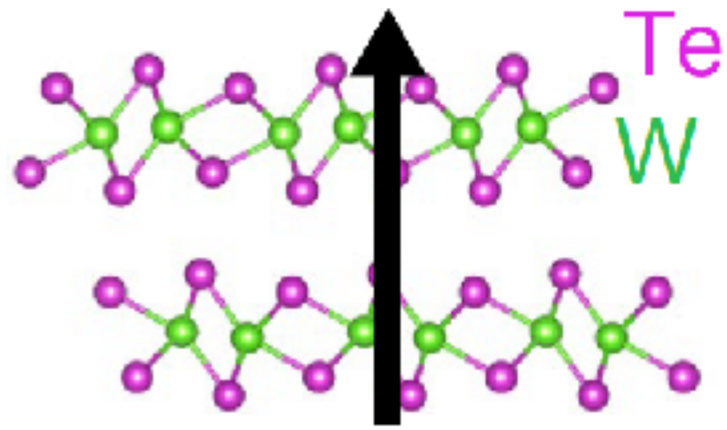


Top view

Experimental realisation

Td-WTe2 bilayer:





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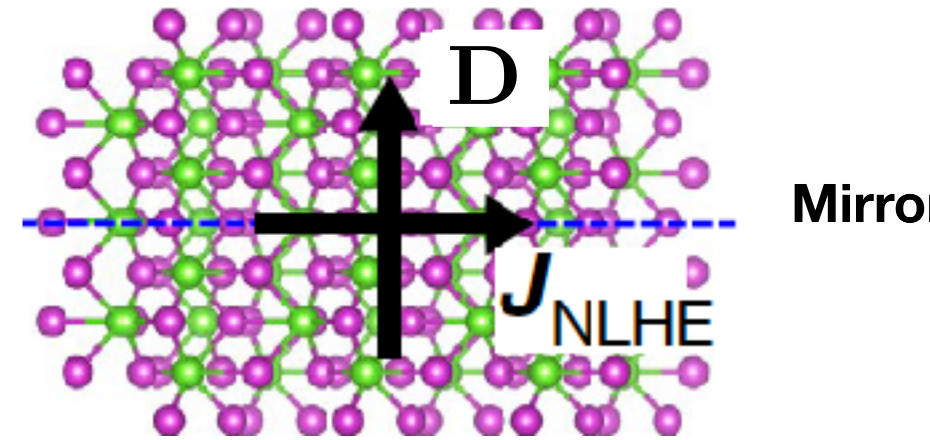


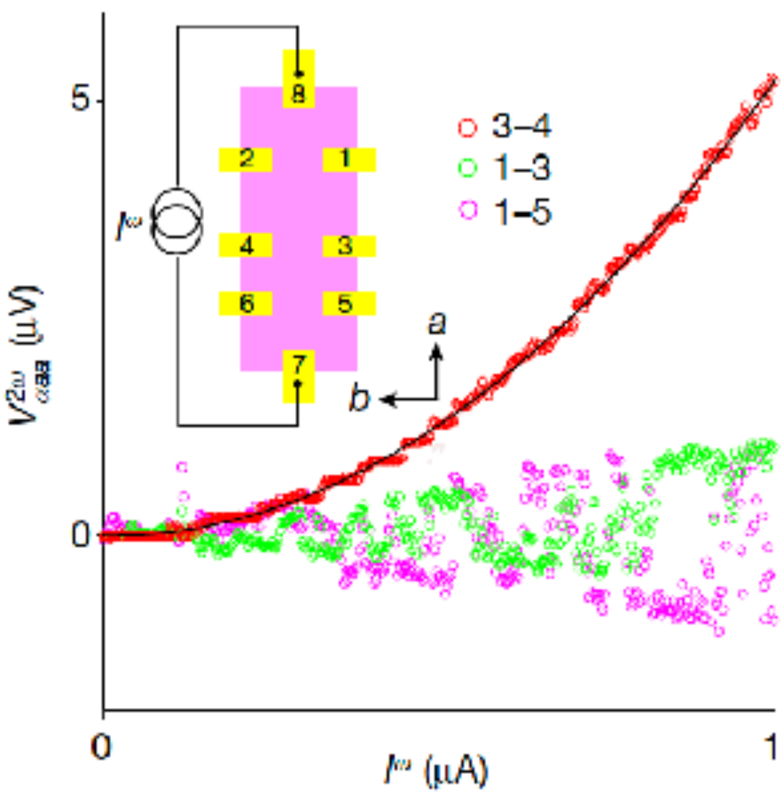
Top view

$$\mathbf{E}_\omega \to \{\mathbf{j}_0, \mathbf{j}_{2\omega}\}$$

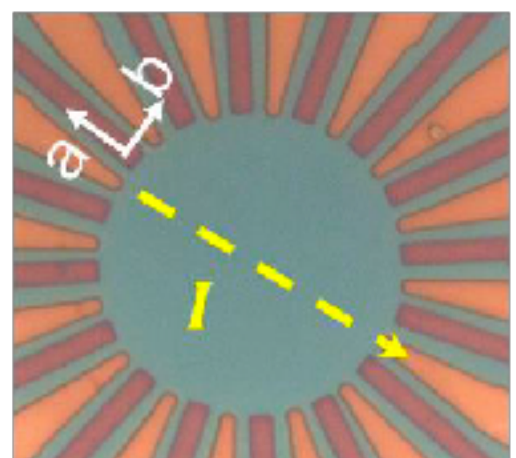
$$\mathbf{J} = e^3\tau(\mathbf{D}\cdot\mathbf{E})\hat{\mathbf{z}}\times\mathbf{E}$$

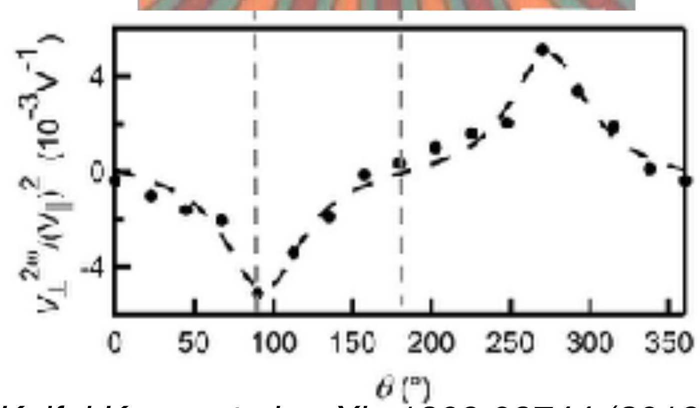
No scattering time:

$$\mathbf{E_x} = -\frac{e\langle\partial_{\mathbf{k_y}}\boldsymbol{\Omega}\rangle}{\langle\partial^2_{\mathbf{k_x}}\epsilon\rangle}\mathbf{E_y^2}$$

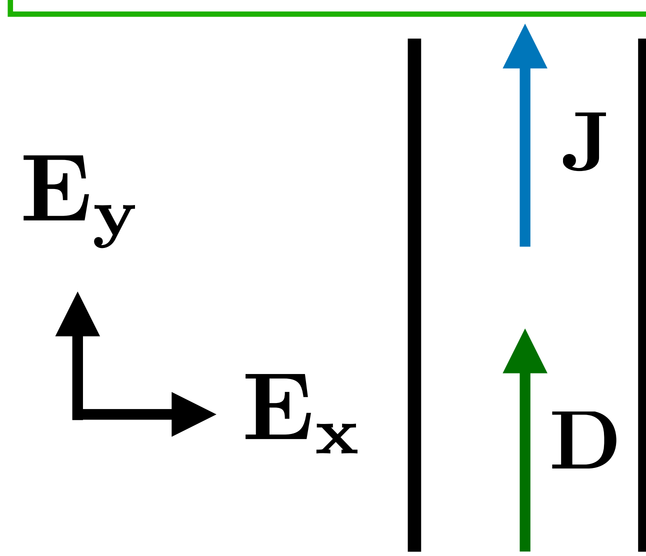


Q. Ma et al. Nature 565, 337 (2019)



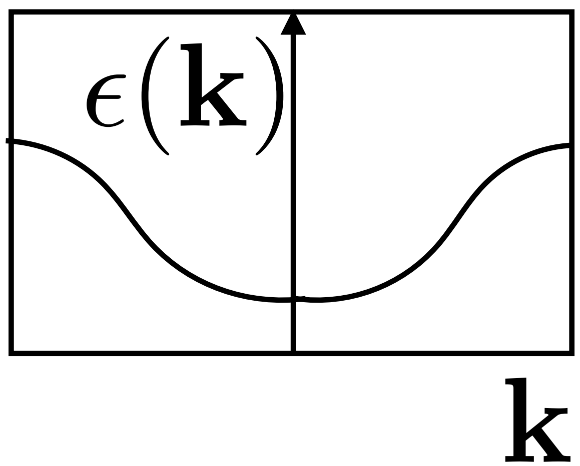


Kaifei Kang, et al. arXiv:1809.08744 (2018)



“Anomalous” non-Newtonian acceleration

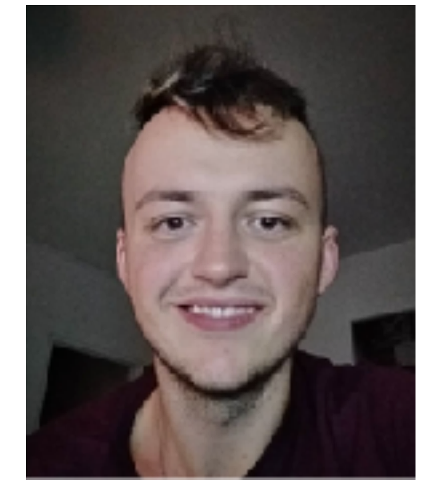
Consider electron in electric field:



$$H = \epsilon(\mathbf{k}) - e\mathbf{r}\cdot\mathbf{E}(t)$$

$$\mathbf{r} = i\partial_{\mathbf{k}} + \mathbf{A}_{\mathbf{k}}$$

$$\mathbf{v} = \partial_{\mathbf{k}}\epsilon + e\mathbf{\Omega}\times\mathbf{E}$$



Oles Matsyshyn

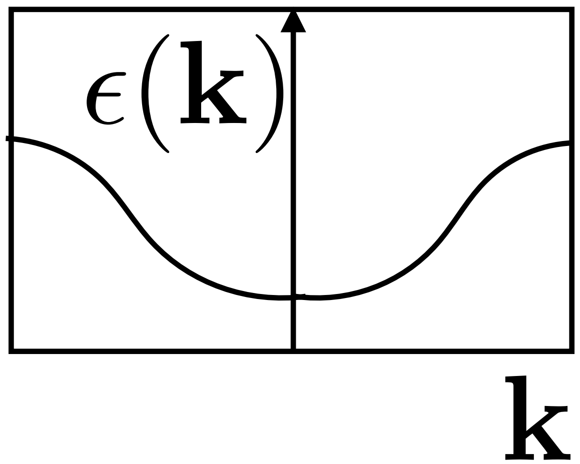
Non-linear inertia:

$$\mathbf{a}_\alpha = \frac{d\mathbf{v}_\alpha}{dt} = \underbrace{\partial^2_{\mathbf{k}_{\alpha\beta}}\epsilon}\,(-e\mathbf{E}_\beta) + \epsilon_{\alpha\beta\gamma}\underbrace{\partial_{\mathbf{k}_\delta}\mathbf{\Omega}_\beta}(e^2\mathbf{E}_\gamma\mathbf{E}_\delta)$$

Drude weight **Berry curvature dipole**

“Anomalous” non-Newtonian acceleration

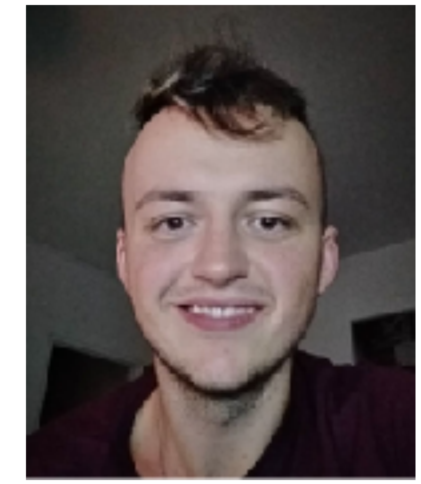
Consider electron in electric field:



$$H = \epsilon(\mathbf{k}) - e\mathbf{r} \cdot \mathbf{E}(t)$$

$$\mathbf{r} = i\partial_{\mathbf{k}} + \mathbf{A}_{\mathbf{k}}$$

$$\mathbf{v} = \partial_{\mathbf{k}}\epsilon + e\mathbf{\Omega} \times \mathbf{E}$$



Oles Matsyshyn

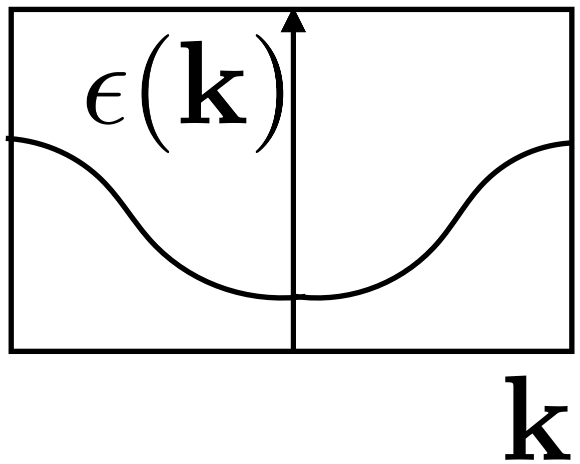
Non-linear inertia:

$$\mathbf{a}_\alpha = \frac{d\mathbf{v}_\alpha}{dt} = \underbrace{\partial^2_{\mathbf{k}_{\alpha\beta}}\epsilon}_{\text{Drude weight}}\,(-e\mathbf{E}_\beta) + \epsilon_{\alpha\beta\gamma}\underbrace{\partial_{\mathbf{k}_\delta}\mathbf{\Omega}_\beta}_{\text{Berry curvature dipole}}(e^2\mathbf{E}_\gamma\mathbf{E}_\delta) \; \underbrace{-\frac{\mathbf{v}_\alpha}{\tau}}_{\text{Friction}}$$

Drude weight **Berry curvature dipole** **Friction**

“Anomalous” non-Newtonian acceleration

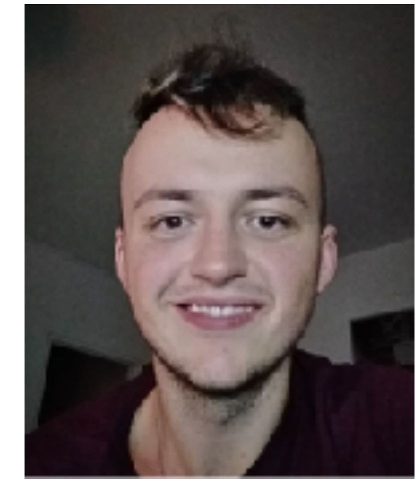
Consider electron in electric field:



$$H = \epsilon(\mathbf{k}) - e\mathbf{r}\cdot\mathbf{E}(t)$$

$$\mathbf{r} = i\partial_{\mathbf{k}} + \mathbf{A}_{\mathbf{k}}$$

$$\mathbf{v} = \partial_{\mathbf{k}}\epsilon + e\mathbf{\Omega}\times\mathbf{E}$$



Oles Matsyshyn

Non-linear inertia:

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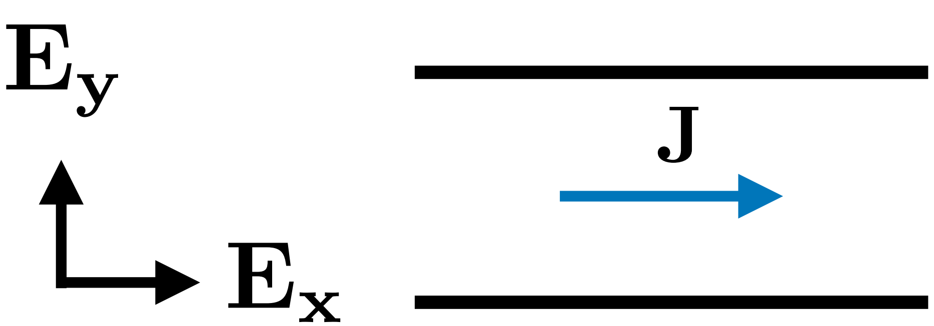


$$D_{\alpha\beta} = \langle\partial_{\mathbf{k}_\alpha}\mathbf{\Omega}_\beta\rangle$$

$$\langle\partial^2_{\mathbf{k_y}}\epsilon\rangle\mathbf{E_y} = -e\langle\partial_{\mathbf{k_x}}\mathbf{\Omega}\rangle\mathbf{E}^2_{\mathbf{x}}$$

$$\chi^{\text{NLH}}_{\alpha\gamma\delta} \equiv \frac{\tau}{1+i\omega\tau}e^3\epsilon_{\alpha\beta\gamma}D_{\delta\beta}$$

Unified theory of insulators and metals

$$H = \delta_{nm}\epsilon_m(\mathbf{k}) - e\mathbf{r}_{nm}\cdot\mathbf{E}_t$$

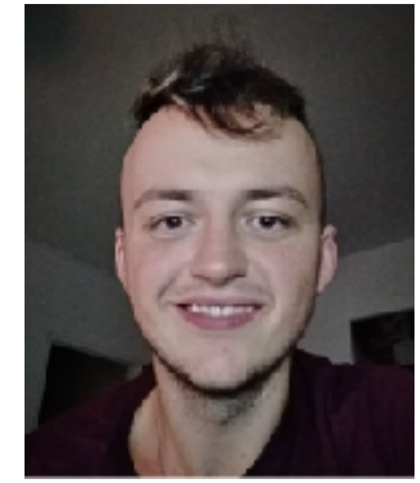
$$\mathbf{r}_{nm} = i\delta_{nm}\partial_{\mathbf{k}} + \mathbf{A}_{nm}(\mathbf{k})$$

$$\mathbf{A}_{nm}(\mathbf{k}) = i\langle u_{n\mathbf{k}}|\partial_{\mathbf{k}}|u_{m\mathbf{k}}\rangle$$

Aversa & Sipe, PRB 52, 14636 (1995)

Quantum Drude model

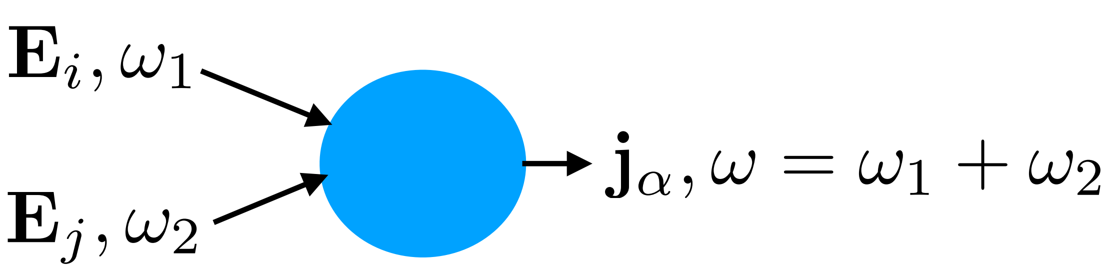
$$\frac{d\rho}{dt} + i[H,\rho] = \frac{\rho_0 - \rho}{\tau}$$

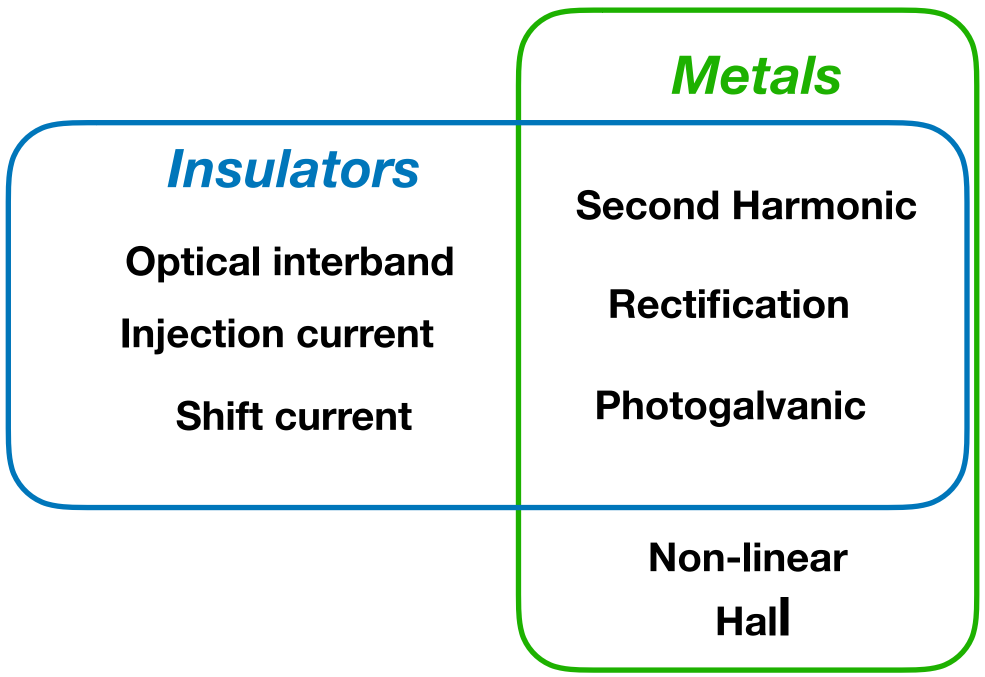


Oles Matsyshyn

Non-linear conductivity

$$\sigma_{(2)} = -\langle[\mathbf{r}(t_2),[\mathbf{r}(t_1),\mathbf{v}]]\rangle$$





$$\langle j^\mu(\omega)\rangle_2 = e\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_1 d\omega_2 E^j(\omega_2)E^i(\omega_1)\sigma_{(2)}^{ji\mu}(-\omega,\omega_1,\omega_2)$$

Quantum rectification sum rule

Only two low-frequency divergences:

"Jerk" **Berry-curvature dipole**

$$\frac{\sigma^{ji\mu}_{(2)}(-\omega,\omega_1,\omega_2)}{2\pi} = \delta(\omega-(\omega_1+\omega_2))\sum_a \left\{ f_a \frac{i\partial^i i\partial^j v^\mu_{aa}}{(\omega+i\Gamma)(\omega_2+i\Gamma)} + \sum_b f_{ab}\left[\frac{i\partial^j}{\omega_2+i\Gamma}\left(\frac{A^i_{ab}v^\mu_{ba}}{\omega-\varepsilon_{ab}+i\Gamma}\right) + \frac{A^j_{ab}}{\omega_2-\varepsilon_{ab}+i\Gamma}\, i\partial^i \frac{v^\mu_{ba}}{\omega-\varepsilon_{ab}+i\Gamma}\right] + \sum_b f_{ab}A^j_{ab}\sum_c\left[\frac{A^i_{bc}v^\mu_{ca}}{(\omega-\varepsilon_{ac}+i\Gamma)(\omega_2-\varepsilon_{ab}+i\Gamma)} - \frac{v^\mu_{bc}A^i_{ca}}{(\omega-\varepsilon_{cb}+i\Gamma)(\omega_2-\varepsilon_{ab}+i\Gamma)}\right]\right\}$$

Interband resonances

Metals

Non-linear Hall

Second Harmonic

Rectification

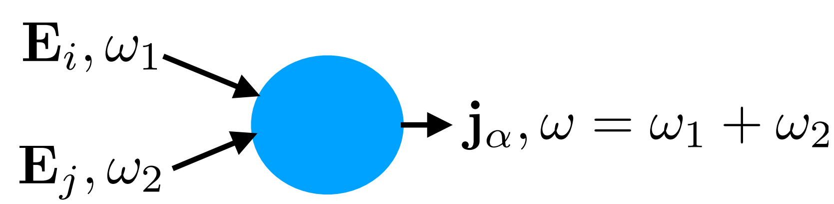
Photogalvanic

Insulators

Optical interband

Injection current

Shift current



Quantum rectification sum rule

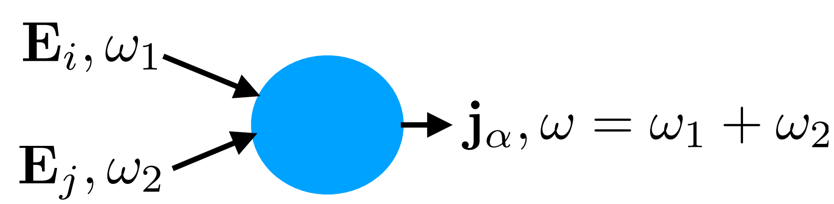
Only two low-frequency divergences:

"Jerk" **Berry-curvature dipole**

Metals	**Insulators**
Non-linear Hall	Optical interband
Second Harmonic	Injection current
Rectification	Shift current
Photogalvanic	

$$\frac{\sigma^{ji\mu}_{(2)}(-\omega,\omega_1,\omega_2)}{2\pi} = \delta(\omega-(\omega_1+\omega_2))\sum_a \Big\{ f_a \frac{i\partial^i i\partial^j v^\mu_{aa}}{(\omega+i\Gamma)(\omega_2+i\Gamma)} + \sum_b f_{ab}\Big[\frac{i\partial^j}{\omega_2+i\Gamma}\Big(\frac{A^i_{ab}v^\mu_{ba}}{\omega-\varepsilon_{ab}+i\Gamma}\Big) + \frac{A^j_{ab}}{\omega_2-\varepsilon_{ab}+i\Gamma} i\partial^i \frac{v^\mu_{ba}}{\omega-\varepsilon_{ab}+i\Gamma}\Big] + \sum_b f_{ab}A^j_{ab}\sum_c\Big[\frac{A^i_{bc}v^\mu_{ca}}{(\omega-\varepsilon_{ac}+i\Gamma)(\omega_2-\varepsilon_{ab}+i\Gamma)} - \frac{v^\mu_{bc}A^i_{ca}}{(\omega-\varepsilon_{cb}+i\Gamma)(\omega_2-\varepsilon_{ab}+i\Gamma)}\Big]\Big\}$$

Interband resonances



Net rectification in TRI systems purely quantum geometric:

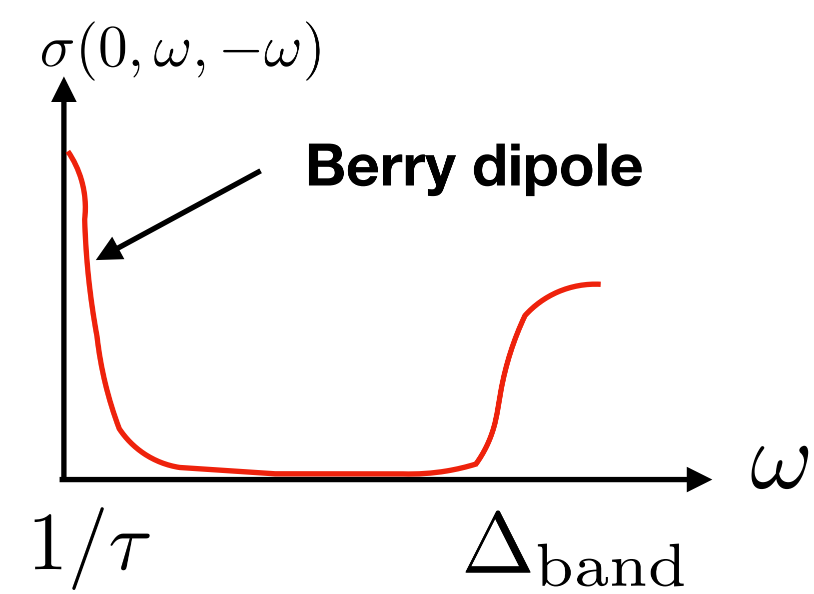
$$\int \frac{d\omega}{(2\pi)^2\delta(0)}\sigma^{(2)}(0,-\omega,\omega) = \sum_a f_a[r^\beta,[r^\alpha,\bar{A}^\mu]]_{aa} =$$

$$\epsilon^{\alpha\mu\gamma}\langle\partial^\beta\Omega^\gamma\rangle + i\langle[A^\beta,\partial^\alpha\bar{A}^\mu]\rangle + \langle[A^\beta,[A^\alpha,\bar{A}^\mu]]\rangle$$

Berry dipole

$$\mathbf{A}_{nm}(\mathbf{k}) = i\langle u_{n\mathbf{k}}|\partial_{\mathbf{k}}|u_{m\mathbf{k}}\rangle$$

Berry curvature takes all intra-band spectral weight:



Spectator of quantum geometry

Measurable quantity that depends solely on $\mathbf{A}(\mathbf{k})$

- Polarisation (insulators)

$$\mathbf{P} = e\langle \mathbf{A}(\mathbf{k})\rangle$$

- Hall conductivity

$$\sigma = e^2\langle \boldsymbol{\Omega}(\mathbf{k})\rangle$$

- Magneto-electric coefficient of time reversal invariant 3D insulators

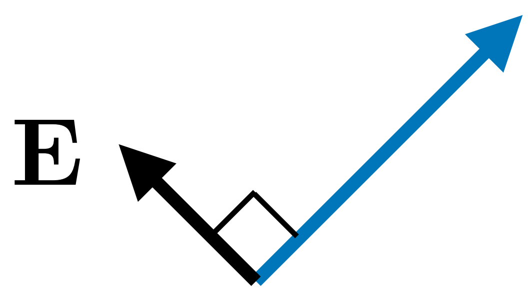
$$\theta = -\frac{e^2}{2}\epsilon^{\alpha\beta\gamma}\langle A^\alpha \partial^\beta A^\gamma - (2i/3)A^\alpha A^\beta A^\gamma\rangle$$

- Non-linear rectification weight of time reversal metals or insulators

$$\int \frac{d\omega}{(2\pi)^2}\sigma^{(2)}(0,-\omega,\omega) = \epsilon^{\alpha\mu\gamma}\langle \partial^\beta \Omega^\gamma\rangle + i\langle [A^\beta, \partial^\alpha \bar{A}^\mu]\rangle + \langle [A^\beta, [A^\alpha, \bar{A}^\mu]]\rangle$$

Summary Part II
Berry curvature dipole
of metals

1) Non-linear Hall effect in time reversal invariant materials controlled by the Berry curvature dipole



$$\mathbf{j}_{\mathrm{NLHE}} \propto \tau \ (\mathrm{BerryDipole}) \ \mathbf{E}^2$$

$$\text{Berry dipole} = \langle \partial_k \mathbf{\Omega} \rangle$$

2) Berry curvature dipole measures a non-Newtonian and non-linear acceleration

$$\frac{d^2\mathbf{r}}{dt^2} \sim (\text{Berry dipole}) \ \mathbf{E}^2$$

Bonus: quantum rectification sum rule

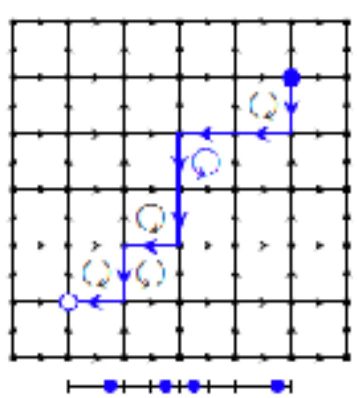
Outline Part II
The quantum dimer and six vertex models one electric field line at a time

J. Herzog-Arbeitman, S. Mantilla, I. Sodemann, arXiv:1902.01858

1) Quantum dimer and six vertex models have a conservation law for “strings” = “electric-field lines”.

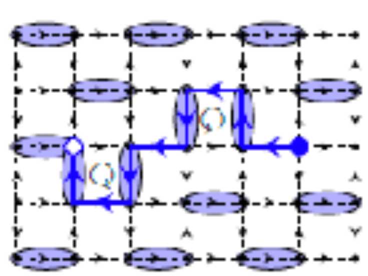
2) The single “strings” subspace maps to 1D spin chains

Quantum 6 vertex



1D spin 1/2 XXZ chain

Quantum Dimer

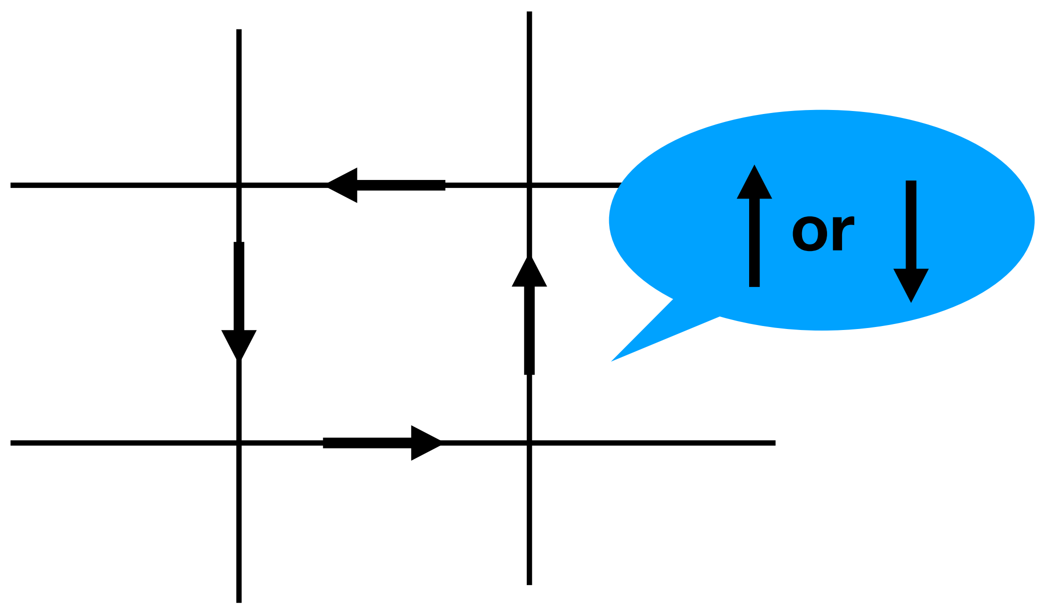


Two-leg 1/2 ladder

Quantum 6 vertex model

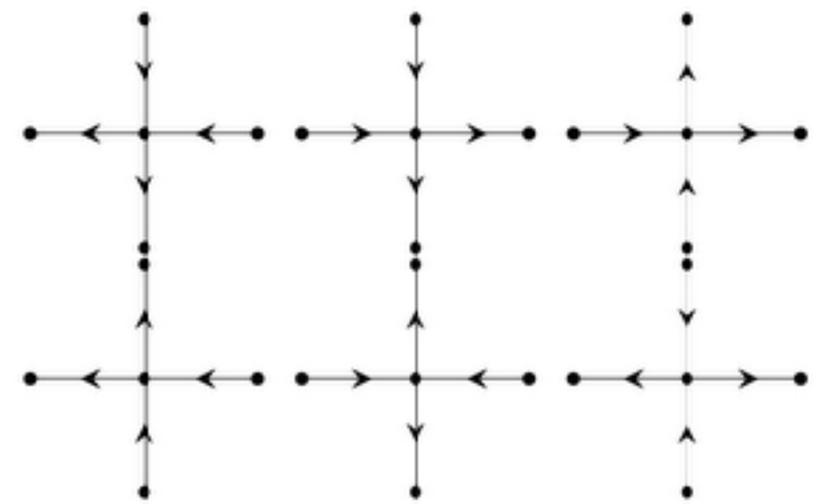
Hilbert space

Square lattice with arrows on links



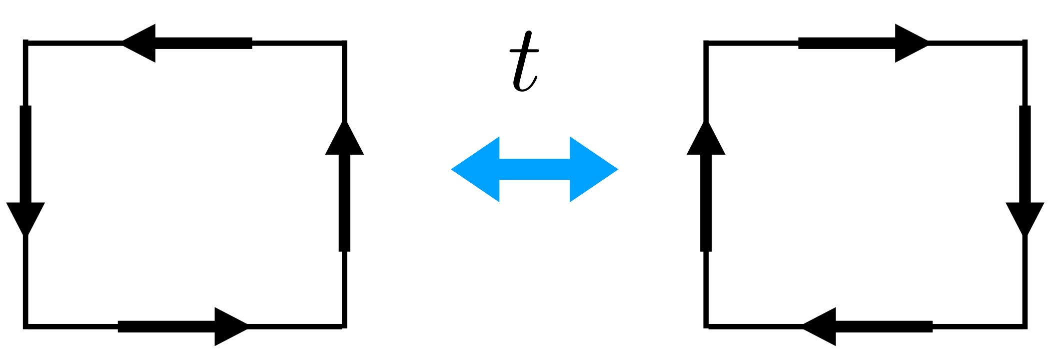
Constraints

Every site has
as many going in as out

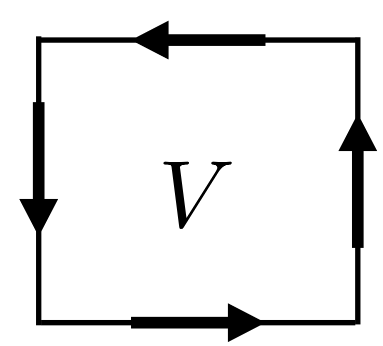


Hamiltonian $$H = \sum_{\square} -t\left(|\circlearrowleft\rangle\langle\circlearrowright| + |\circlearrowright\rangle\langle\circlearrowleft|\right) + V\left(|\circlearrowleft\rangle\langle\circlearrowleft| + |\circlearrowright\rangle\langle\circlearrowright|\right)$$

Flipping plaquette:



Counting Flippable

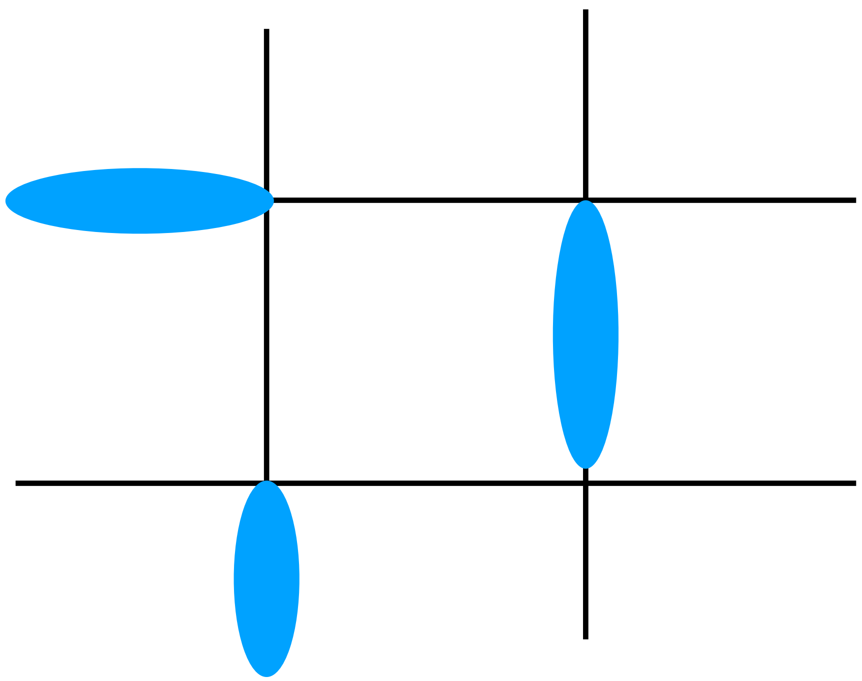


$$v = \frac{V}{t}$$

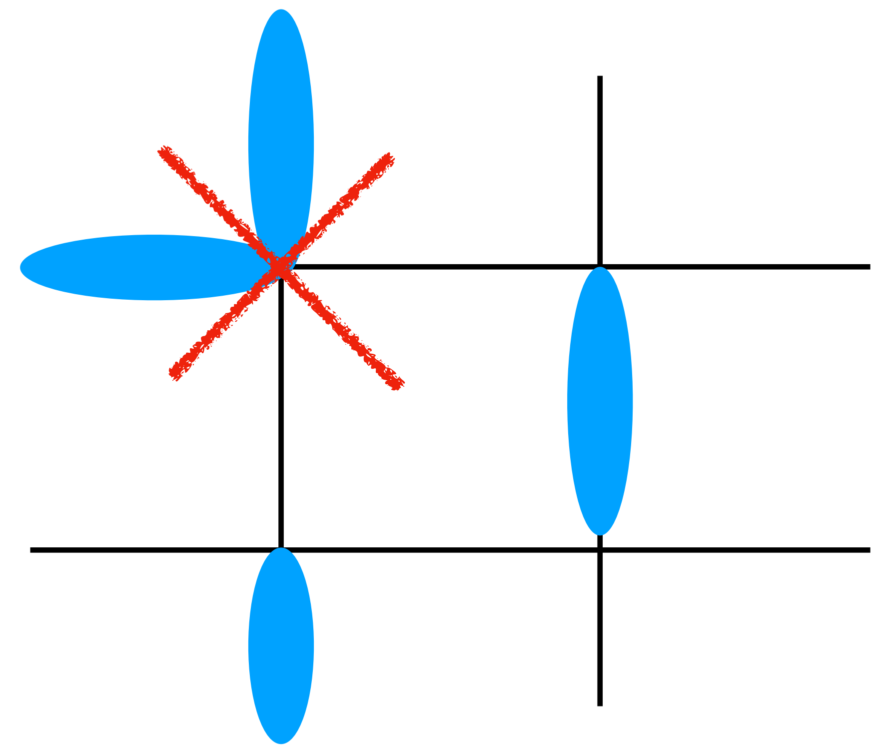
Quantum Dimer model (Rokhsar-Kivelson)

Hilbert space

Dimers on links touch every sites only once

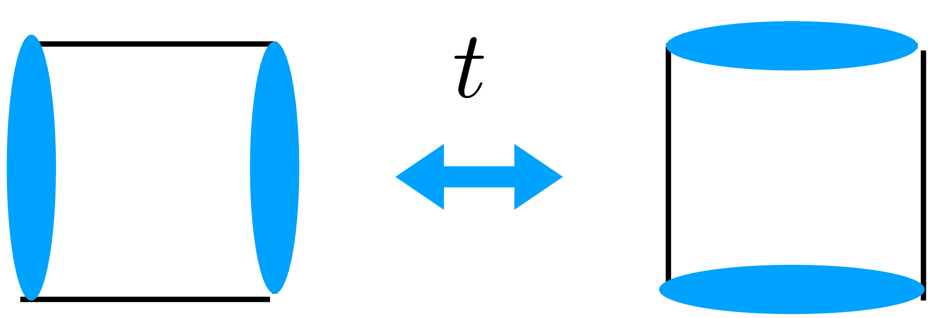


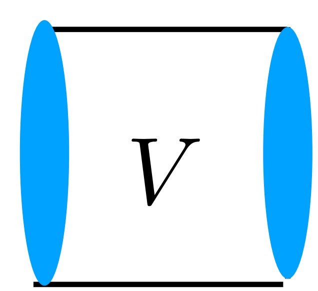
Constraints



Hamiltonian

$$H = \sum_{\square} -t\left(|\circlearrowleft\rangle\langle\circlearrowright| + |\circlearrowright\rangle\langle\circlearrowleft|\right) + V\left(|\circlearrowleft\rangle\langle\circlearrowleft| + |\circlearrowright\rangle\langle\circlearrowright|\right)$$

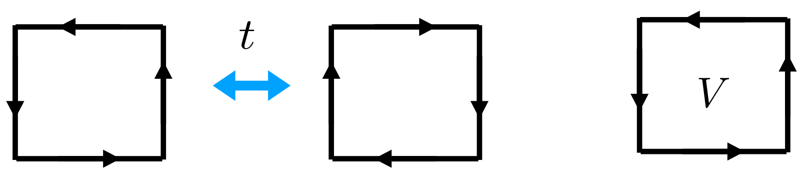




$$v = \frac{V}{t}$$

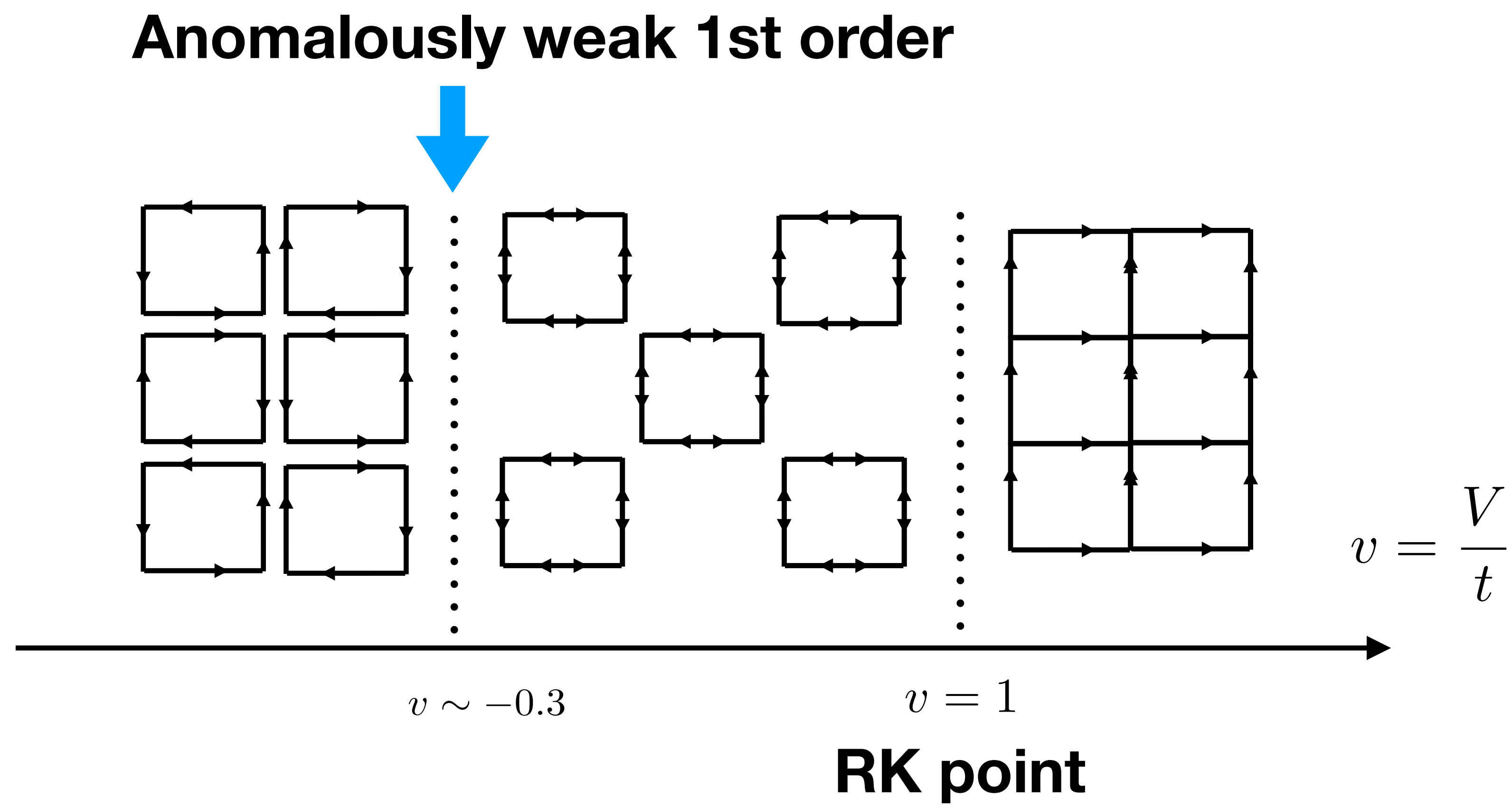
Phase diagrams

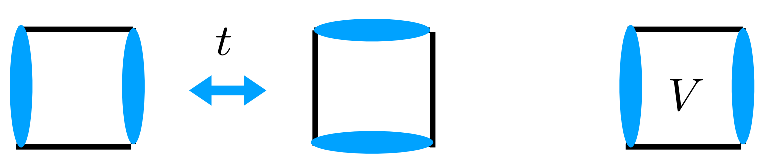
Hamiltonian



Shannon, Misguich, Penc, PRB (2004)

Banerjee, Jiang, Widmer, Wiese, J. Stat. Mec. (2013).



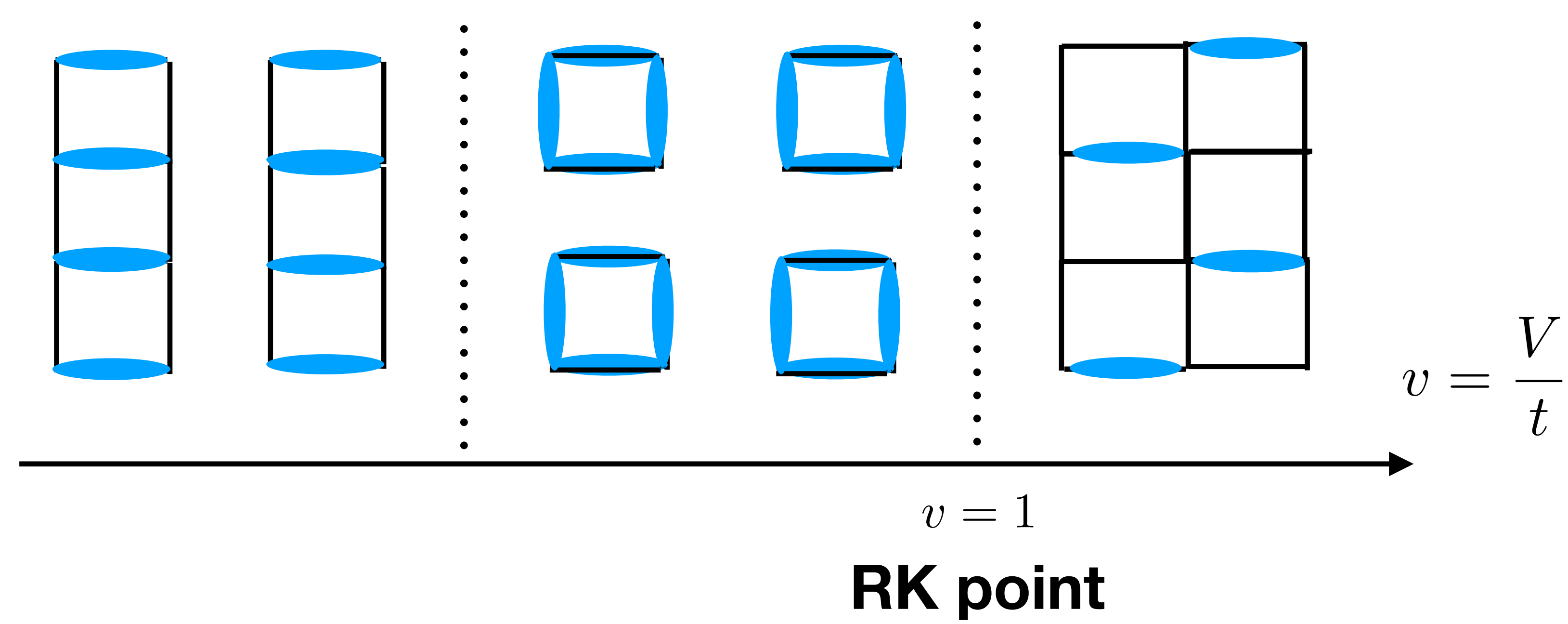


Sachdev PRB (1989)

Leung, Chiu, Runge, PRB (1996)

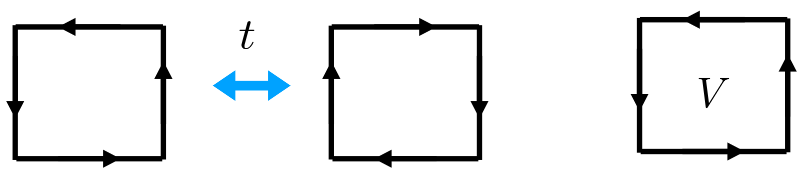
Syljuasen, PRB (2006)

Ralko, Poilblanc, Moessner, PRL (2008)



Phase diagrams

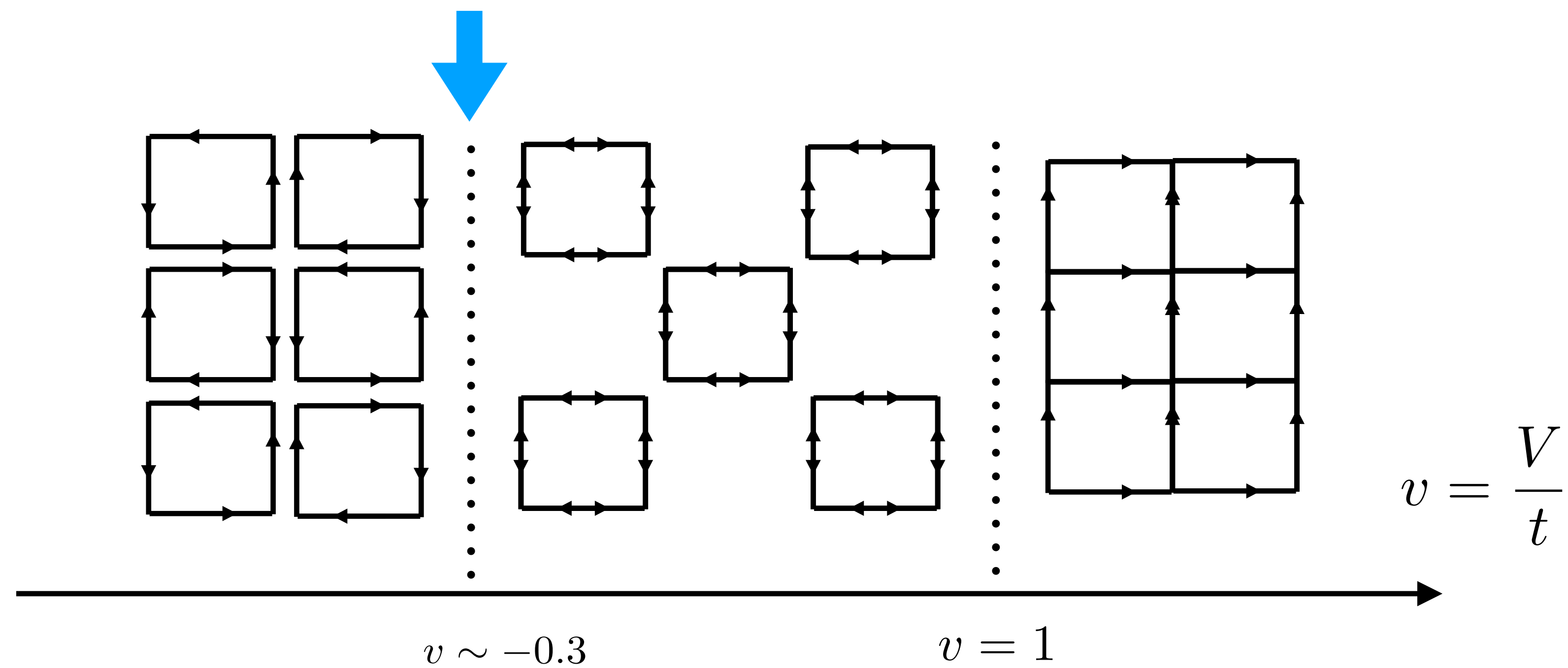
Hamiltonian



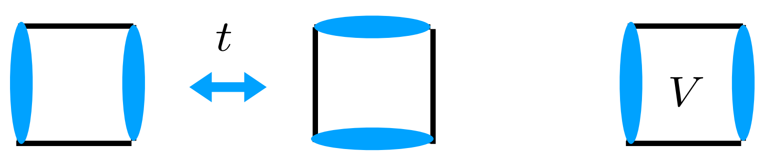
Shannon, Misguich, Penc, PRB (2004)

Banerjee, Jiang, Widmer, Wiese, J. Stat. Mec. (2013).

Anomalously weak 1st order



RK point



Sachdev PRB (1989)

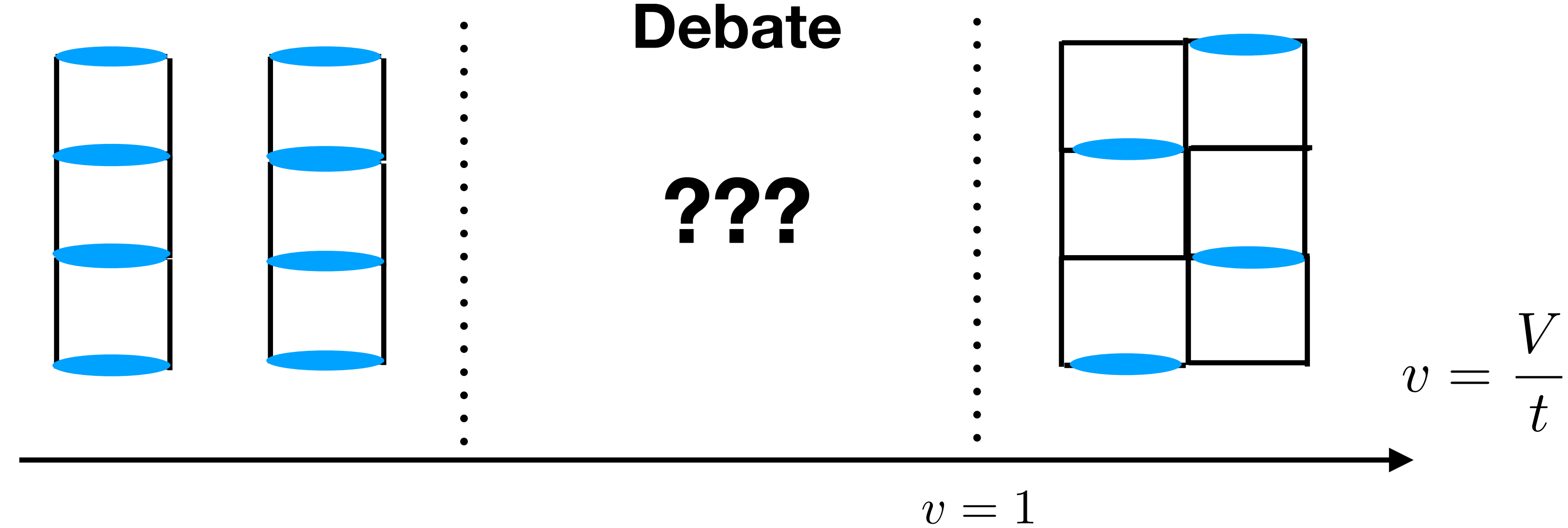
Leung, Chiu, Runge, PRB (1996)

Syljuasen, PRB (2006)

Ralko, Poilblanc, Moessner, PRL (2008)

Zeng, Henley, PRB (1997)

Under Debate

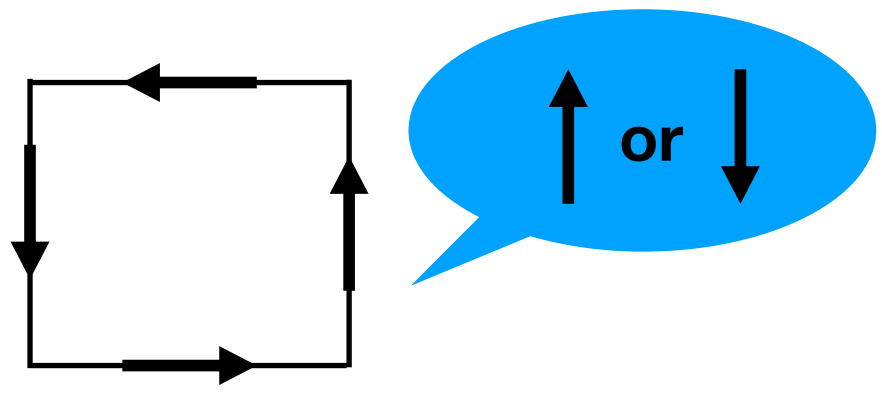


RK point

<u>Banerjee, et al., PRB 90, 245143 (2014).</u>

<u>Oakes, Powell, Castelnovo, Lamacraft, Garrahan, PRB 98, 064302 (2018).</u>

Lattice U(1) gauge theory



$$E_{\mathbf{r},x} = \sigma^z_{\mathbf{r},x}$$

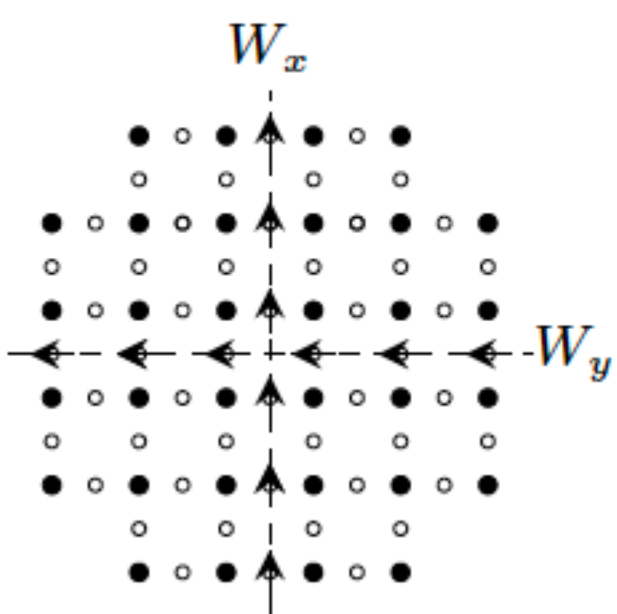
$$E_{\mathbf{r},y} = \sigma^z_{\mathbf{r},y}$$

$$\nabla \cdot \mathbf{E}_{\mathbf{r}} = E_{\mathbf{r},x} - E_{\mathbf{r}-\hat{\mathbf{x}},x} + E_{\mathbf{r},y} - E_{\mathbf{r}-\hat{\mathbf{y}},y} \equiv Q_{\mathbf{r}}$$

Quantum 6 vertex $\quad Q_{\mathbf{r}} = 0$

Quantum dimer $\quad Q_A = 2 \qquad Q_B = -2$

t'Hooft operators

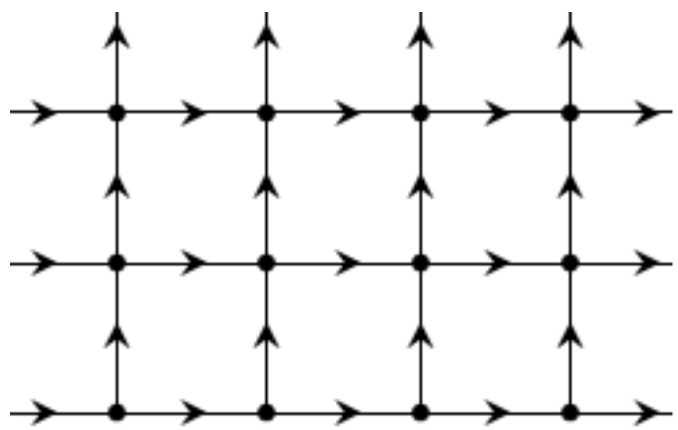


$$W_x = \oint +d\ell_y E_{\mathbf{r},x} = \sum_{\uparrow} E_{\mathbf{r},x} = \sum_{\uparrow} \sigma^z_{\mathbf{r},x}$$

$$W_y = \oint -d\ell_x E_{\mathbf{r},y} = \sum_{\leftarrow} E_{\mathbf{r},y} = \sum_{\leftarrow} \sigma^z_{\mathbf{r},y}$$

Conservation of strings

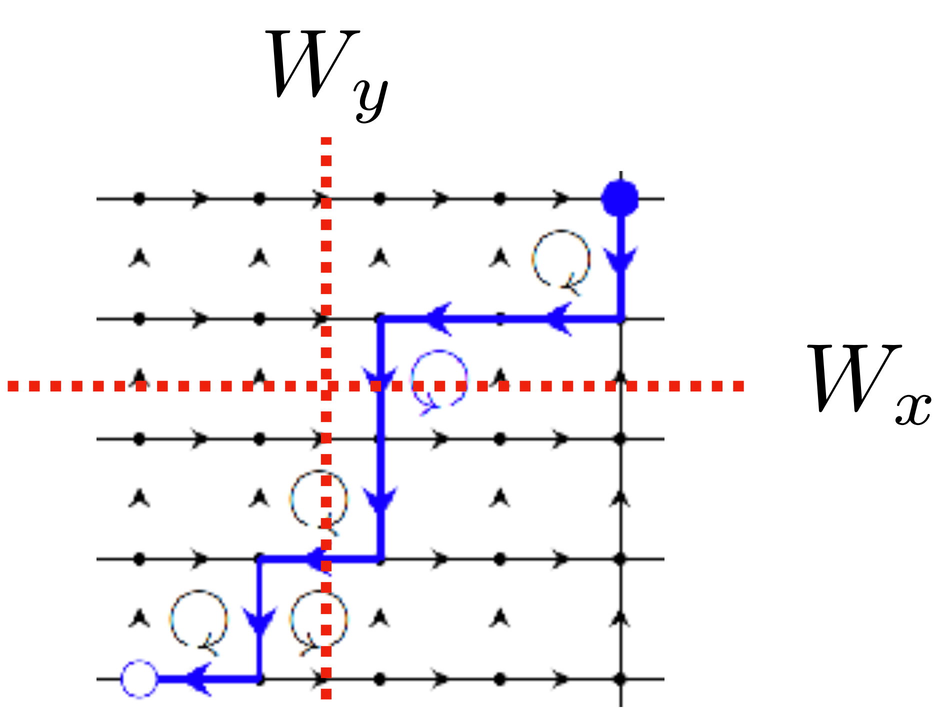
Reference vacuum with zero strings ⟷ **No flippable Plaquettes**



$$H = \sum_{\square} -t(|\circlearrowleft\rangle\langle\circlearrowright| + |\circlearrowright\rangle\langle\circlearrowleft|) + V(|\circlearrowleft\rangle\langle\circlearrowleft| + |\circlearrowright\rangle\langle\circlearrowright|)$$

$$H|O\rangle = 0$$

One string sector:



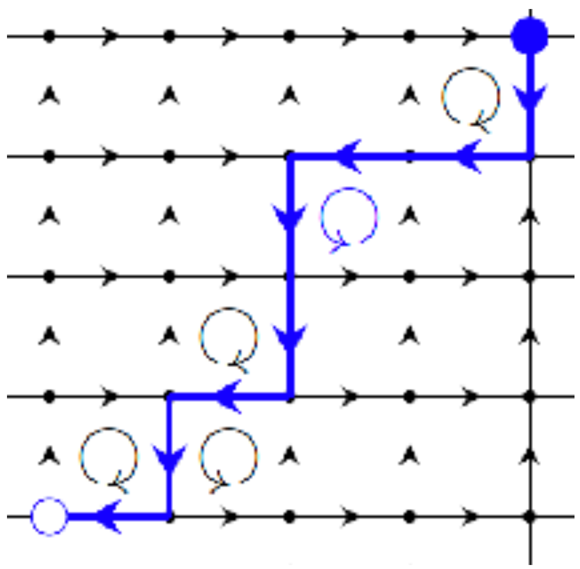
$$W_x = \oint +d\ell_y E_{\mathbf{r},x} = \sum_{\uparrow} E_{\mathbf{r},x} = \sum_{\uparrow} \sigma^z_{\mathbf{r},x}$$

$$W_y = \oint -d\ell_x E_{\mathbf{r},y} = \sum_{\leftarrow} E_{\mathbf{r},y} = \sum_{\leftarrow} \sigma^z_{\mathbf{r},y}$$

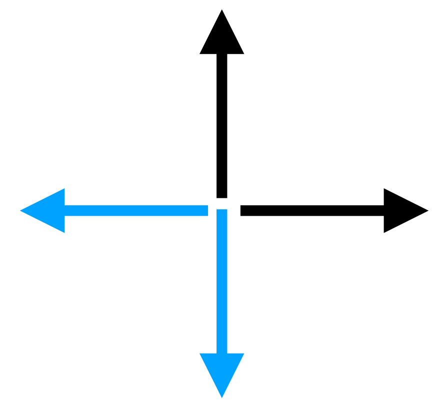
$$[H, W_{x,y}] = 0$$

One string problem

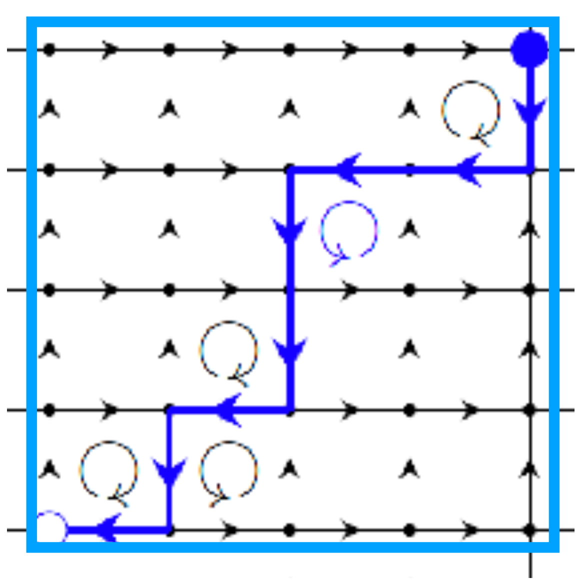
String always goes right or up



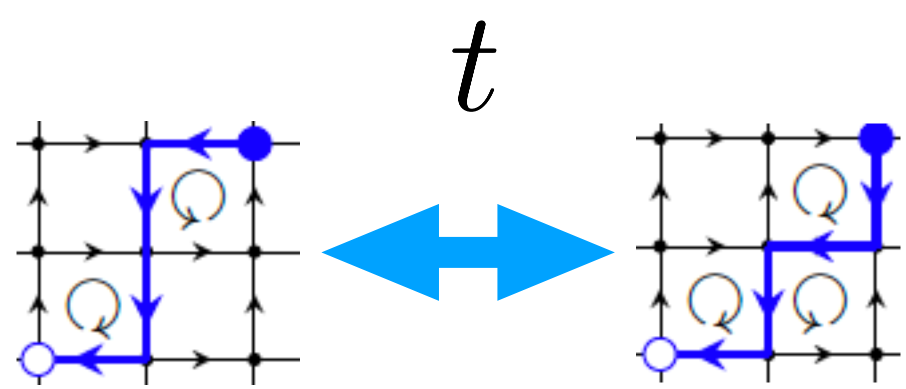
Charges are created otherwise



String moves within square



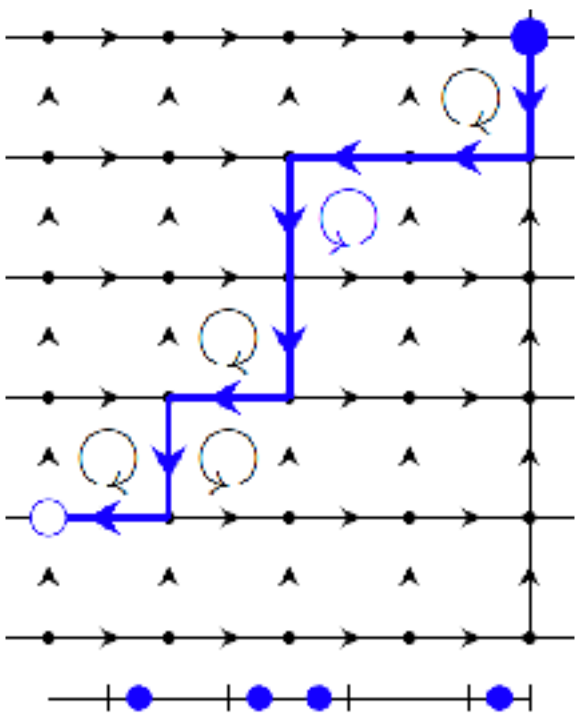
$$H = \sum_{\square} -t\left(|\circlearrowleft\rangle\langle\circlearrowright| + |\circlearrowright\rangle\langle\circlearrowleft|\right) + V\left(|\circlearrowleft\rangle\langle\circlearrowleft| + |\circlearrowright\rangle\langle\circlearrowright|\right)$$



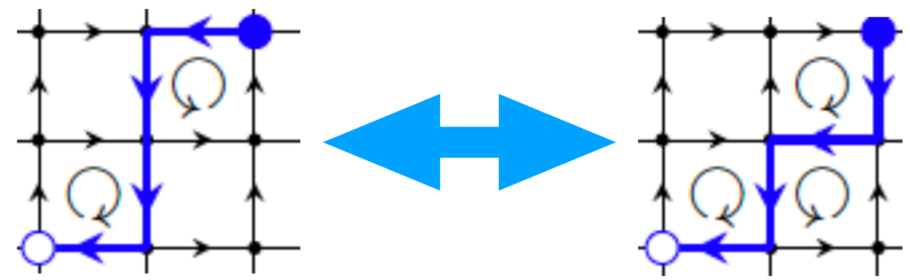
Plaquette flipping

One string problem = XXZ spin 1/2 chain

String can be represented as spin 1/2 chain



t



$$H = \sum_{\square} -t\left(|\circlearrowright\rangle\langle\circlearrowleft| + |\circlearrowleft\rangle\langle\circlearrowright|\right) + V\left(|\circlearrowright\rangle\langle\circlearrowright| + |\circlearrowleft\rangle\langle\circlearrowleft|\right)$$

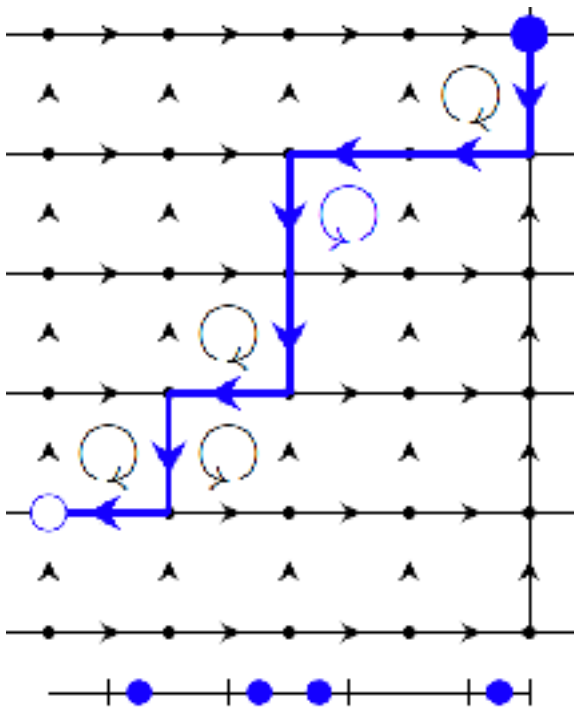
XXZ spin 1/2

$$H_{6v} = -J\sum_{i=1}^{L}\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + v S_i^z S_{i+1}^z - \frac{v}{4}\right)$$

$$N_b = \sum_i b_i^\dagger b_i = \ell_y \qquad \nu = \frac{N_b}{L} = \frac{1}{1+\ell_x/\ell_y}$$

Solving one string problem

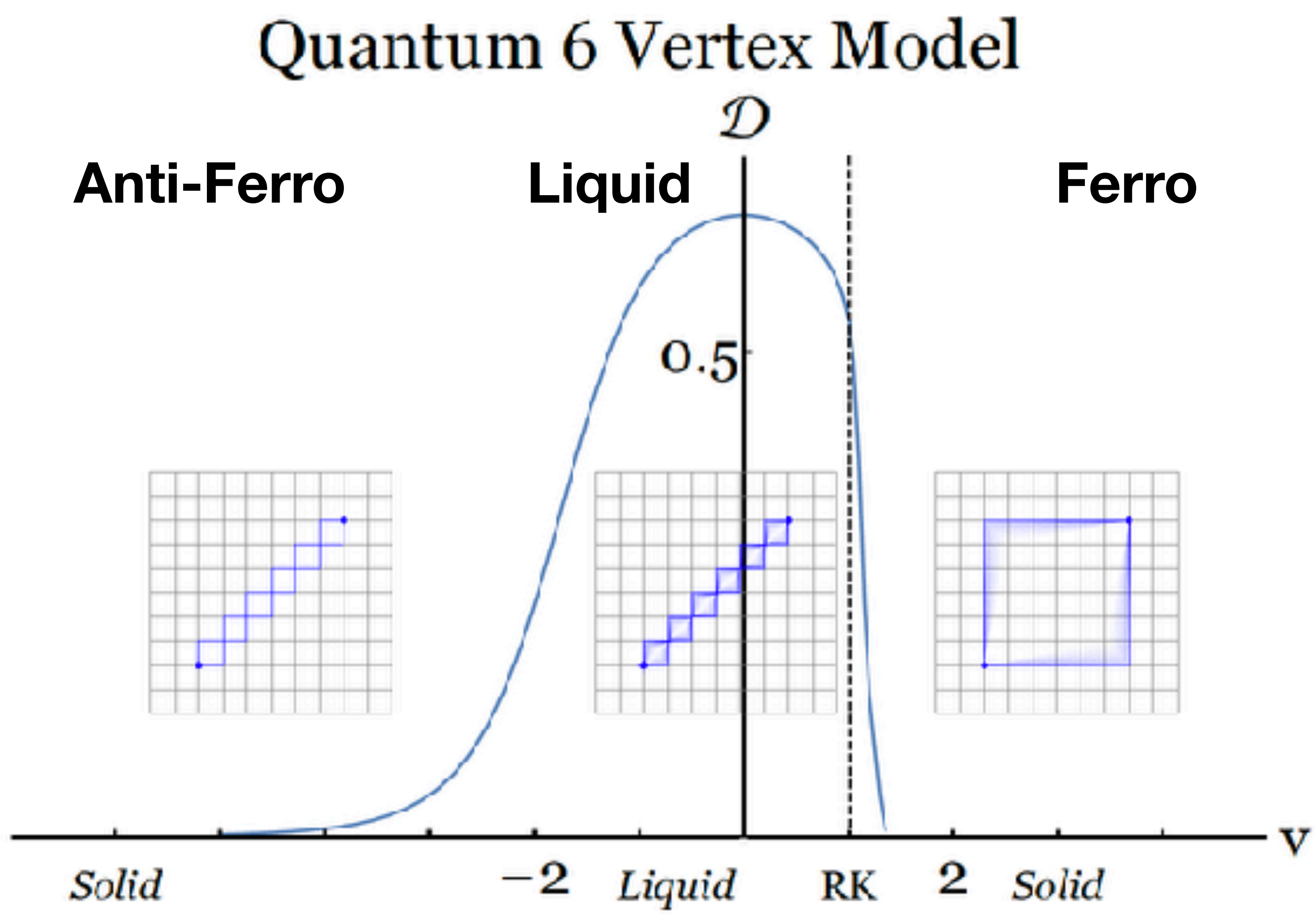
String can be represented as spin 1/2 chain



XXZ spin 1/2

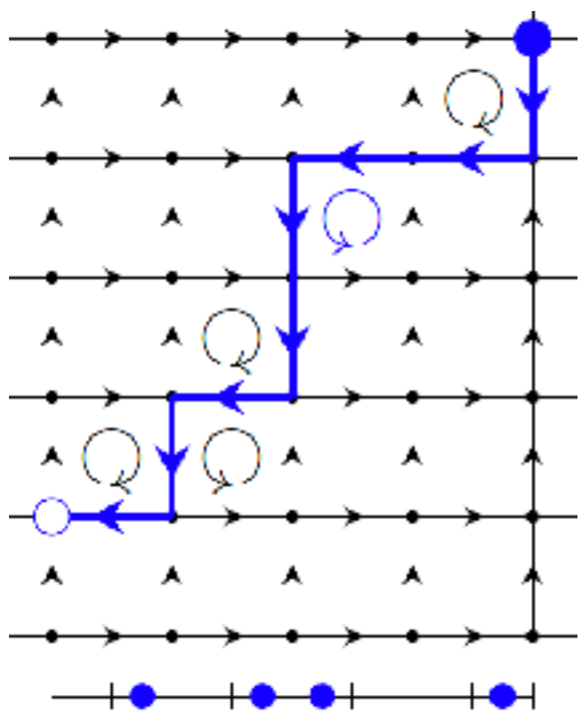
$$H_{6v} = -J \sum_{i=1}^{L} \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + v S_i^z S_{i+1}^z - \frac{v}{4} \right)$$

Drude weight for L=7



Solving one string problem

String can be represented as spin 1/2 chain



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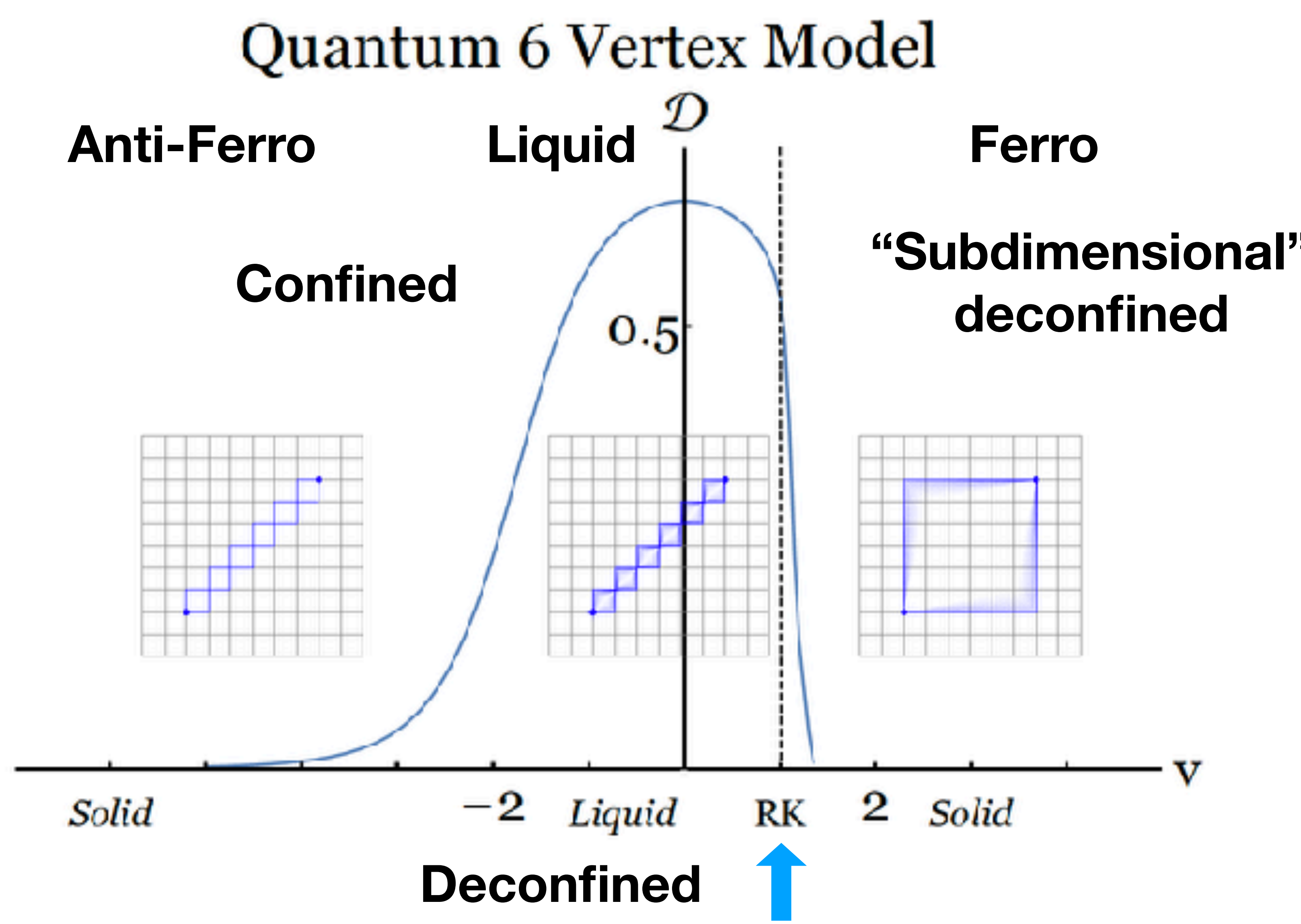
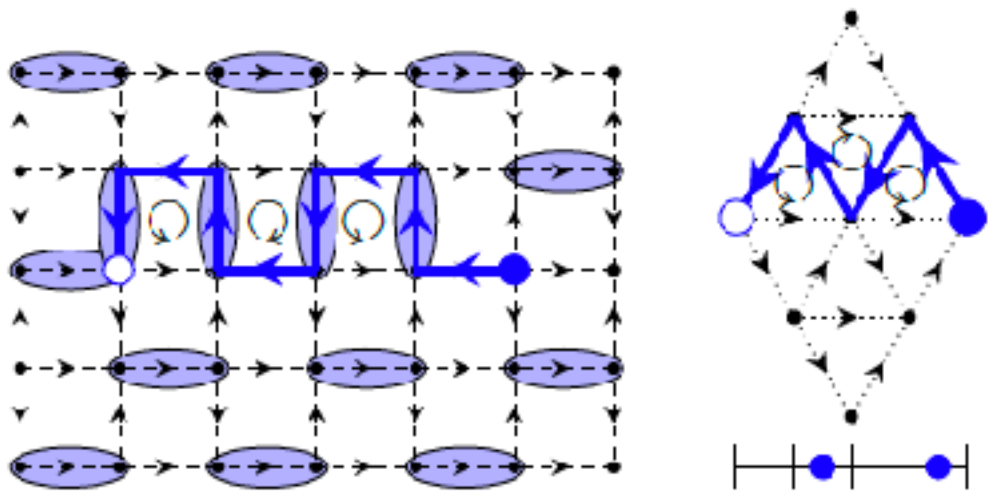
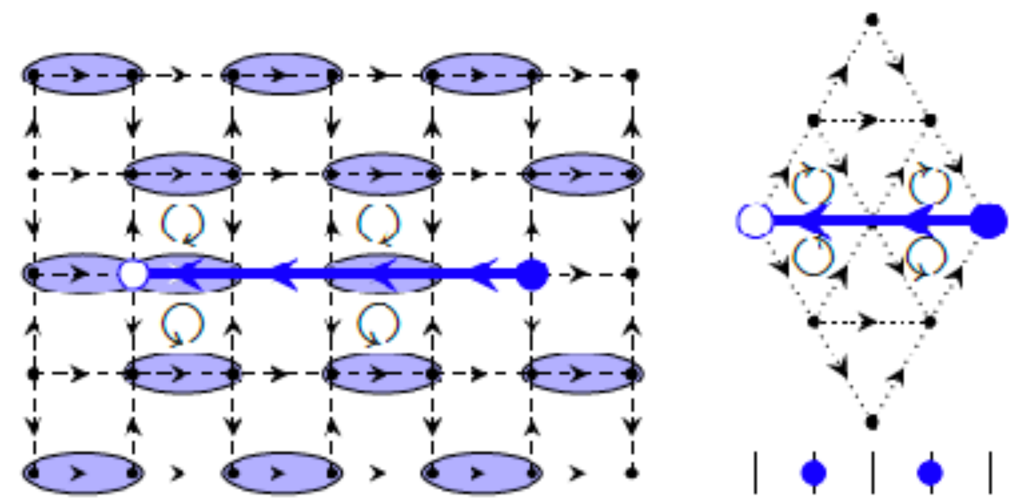


RK point has "hidden" SU(2) symmetry

Strings in quantum dimer

String moves in a triangular lattice





Mapping requires two site basis

Two leg ladder

$$H_{hop} = -\sum_i t b_i^\dagger b_{i+1} + h.c.$$

$$H_{pot,odd} = V \sum_{i\ odd} (n_i - n_{i+2})^2$$

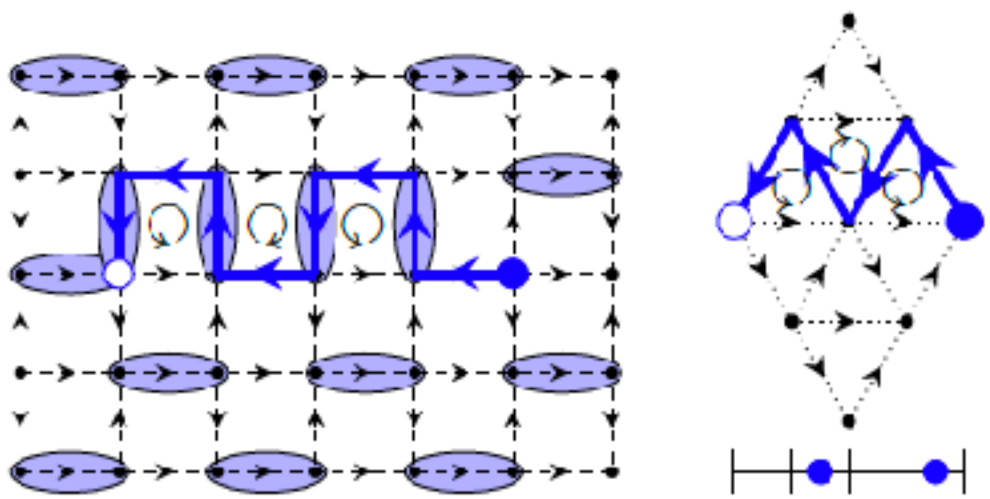
$$H_{pot,even} = 2V \sum_{i\ even} n_i$$

$$H_{pot,sub} = -V \sum_i n_i n_{i+3}$$

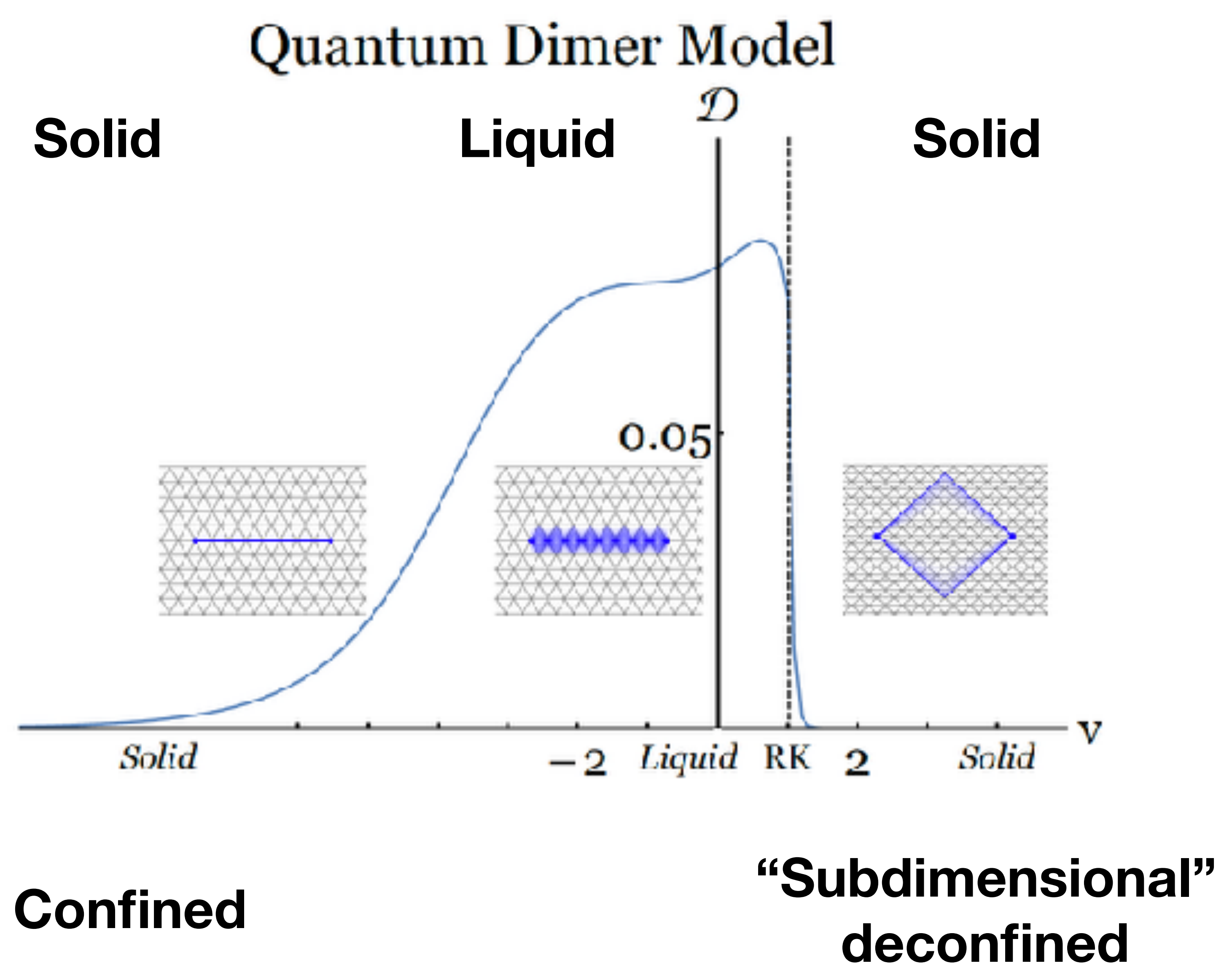
$$H_{con} = U \sum_i n_i n_{i+1} + U \sum_{i\ even} n_i n_{i+2}, \quad U \to \infty$$

One string problem in quantum dimer

String can be represented as spin 1/2 2-leg ladder

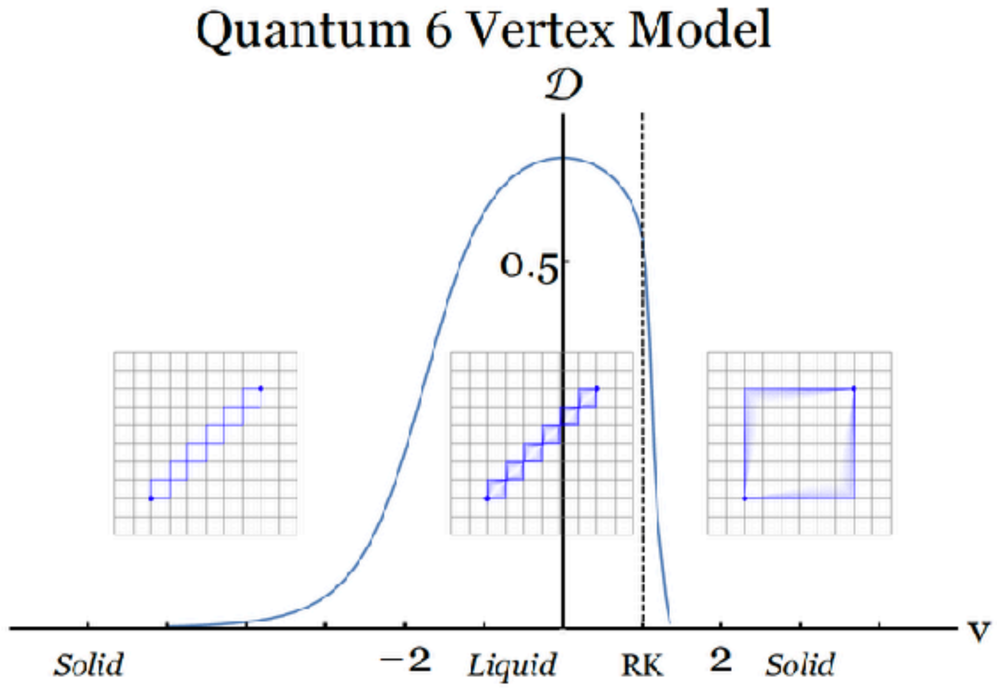


Drude weight for L=7

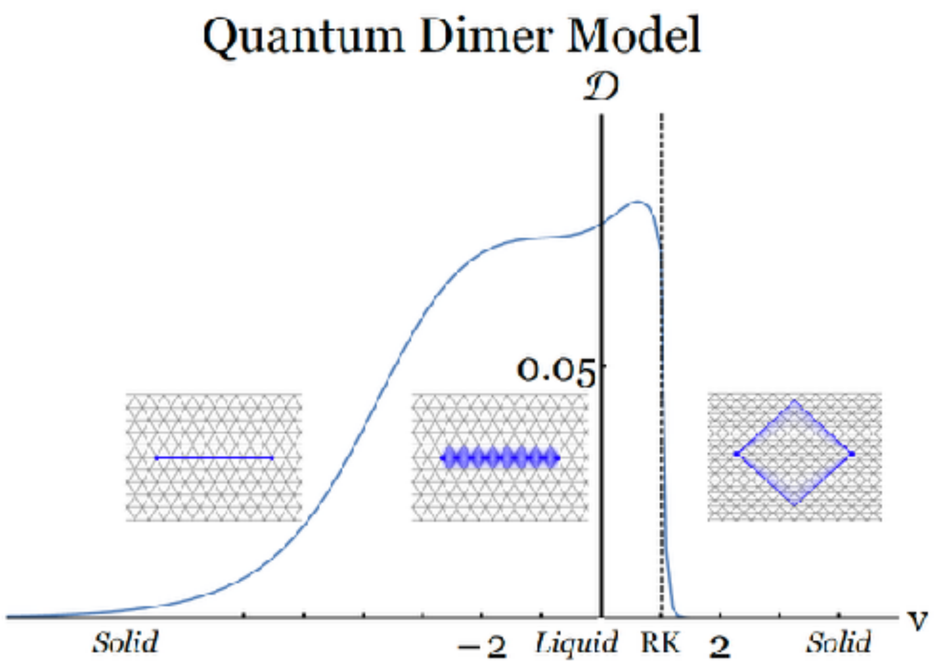


Back to multi-strings

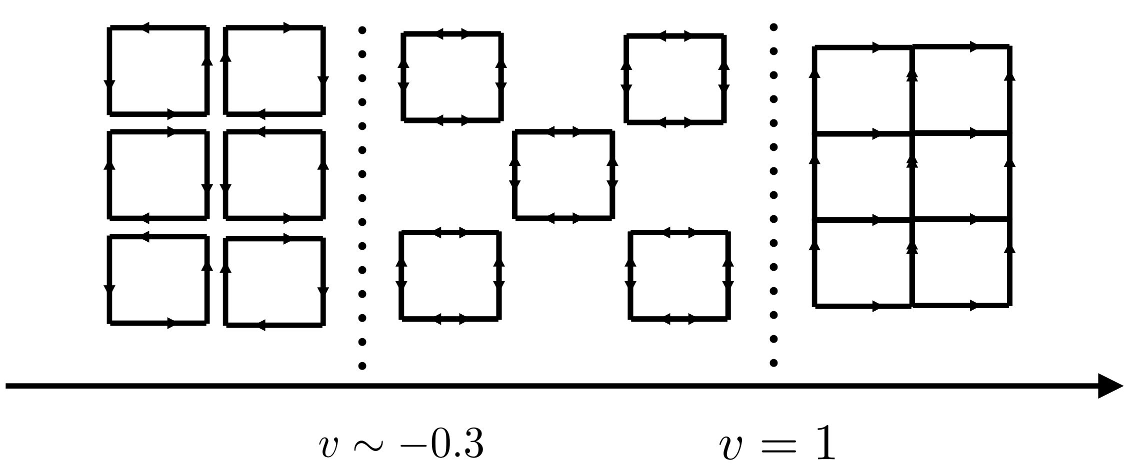
One string



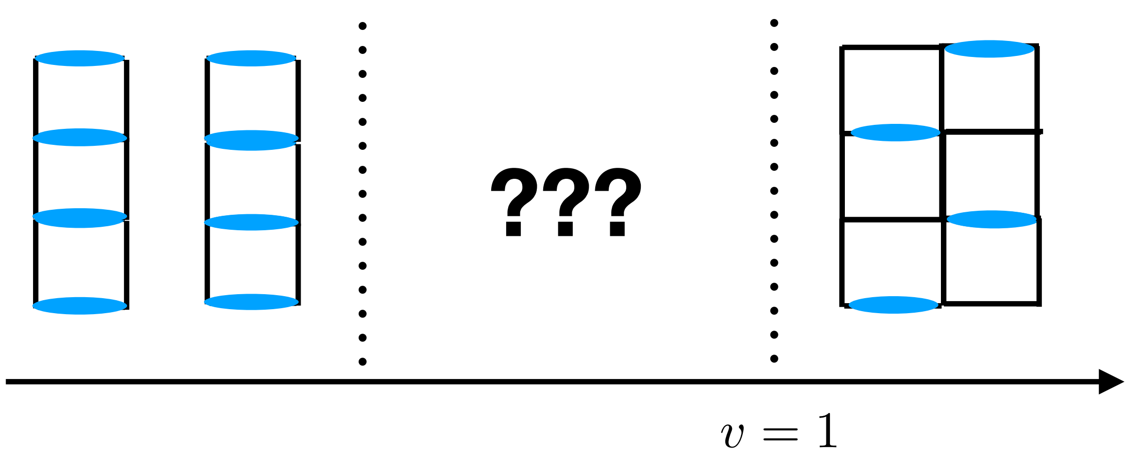
One string



Many strings



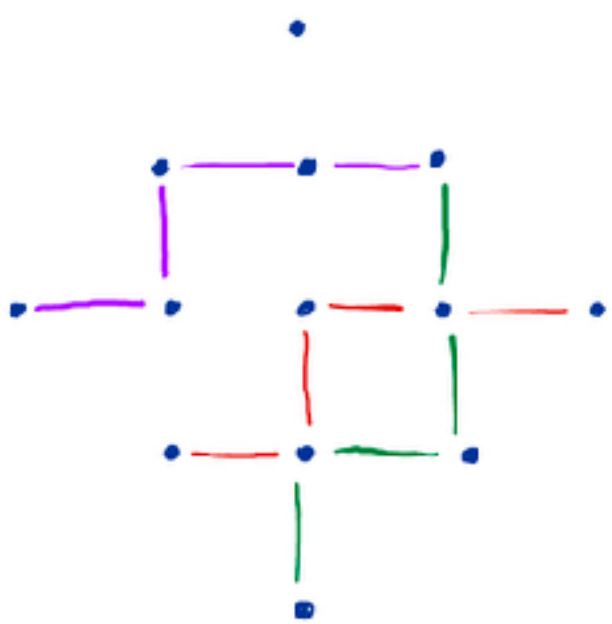
Many strings



RK point

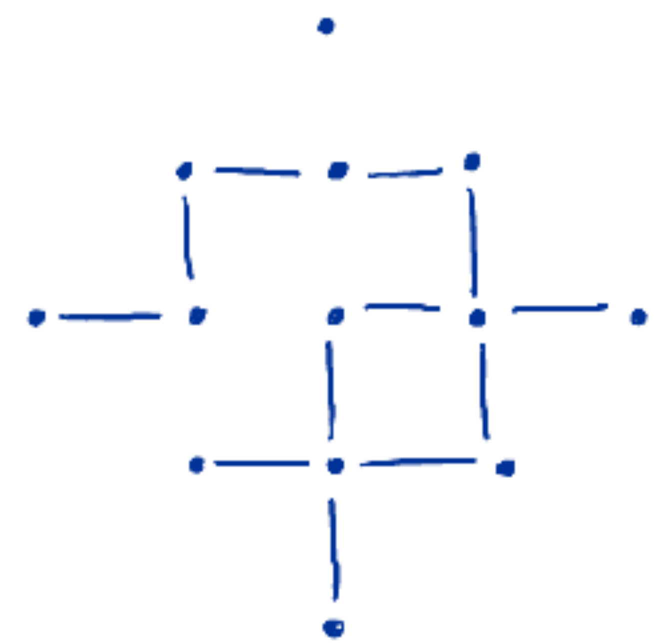
Ideal BEC of strings?

Many string problem: $H = H_1^{\text{one string}} + H_2^{\text{one string}} + \cdots$



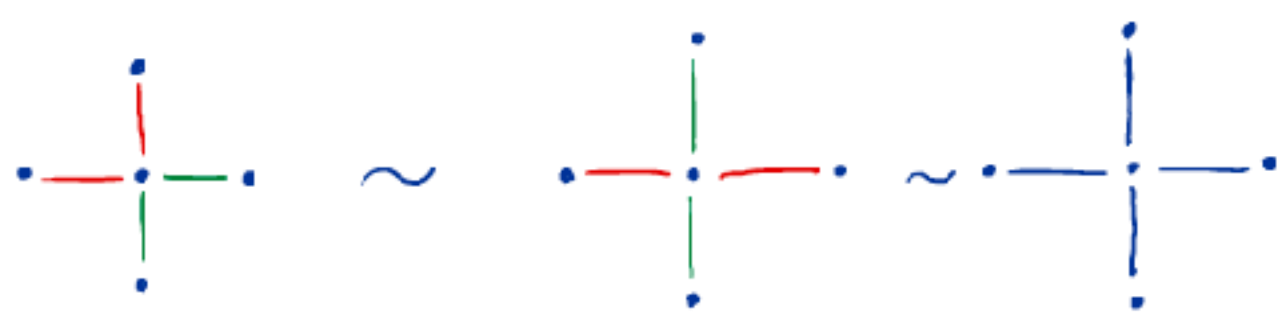
$\equiv$

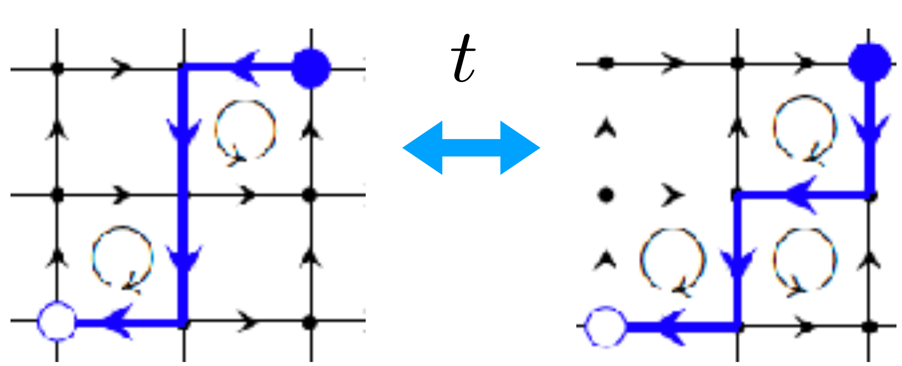
Symmetrize strings

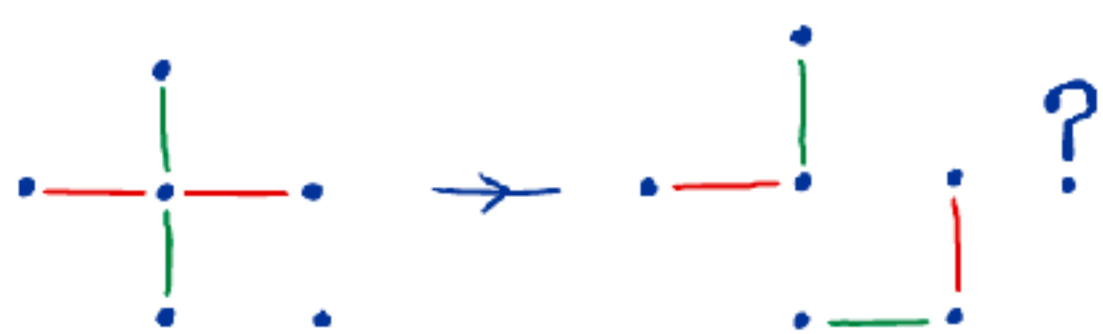


Soft bosons
On links
$n \in \mathbb{Z}$

Inconsistency:







$C \propto T$

Emergent fermi surface??

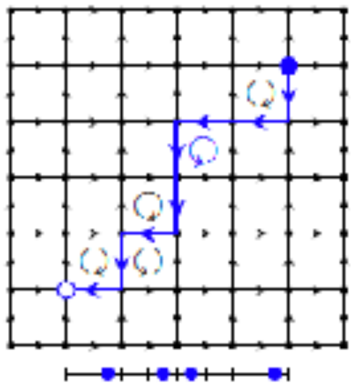
Summary Part II
The quantum dimer and six vertex models one electric field line at a time

J. Herzog-Arbeitman, S. Mantilla, I. Sodemann, arXiv:1902.01858

1) Quantum dimer and six vertex models have a conservation law for “strings” = “electric-field lines”.

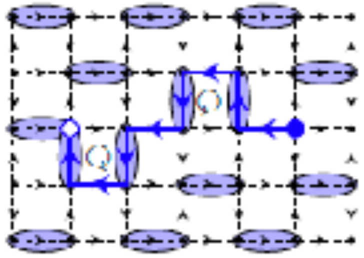
2) The single “strings” subspace maps to 1D spin chains

Quantum 6 vertex



1D spin 1/2 XXZ chain

Quantum Dimer



Two-leg 1/2 ladder