



JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ

# Exploring Synthetic Quantum Matter with Cold Atoms & Tensor Networks



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Universität zu Köln & Forschungszentrum Jülich

***Benasque, 27.02.2019***

# The Big Picture

Quantum Many-Body Systems:  
usually very difficult problems  
for classical math / algorithms

Classical Hardware reaching limit  
& exploding Big Data:  
demand for a new information age

new understanding of  
entanglement properties:

**Tensor Networks**

**HPC Numerical Methods**  
for strongly-correlated systems

## Quantum Information Science

- Q-communication & Q-cryptography [ready!]
- Q-Computer (fully digital) [long-time!]
- Q-Simulator (analog) [near-term!]

## Condensed Matter Physics

Among many recipes for qubits:

**topological states of matter**

Still difficult to access & control ...

## Quantum Optics & Atomic physics

Control of single quantum objects,  
long coherence times, ...

(background)  
gauge fields

Geometrical  
constraints

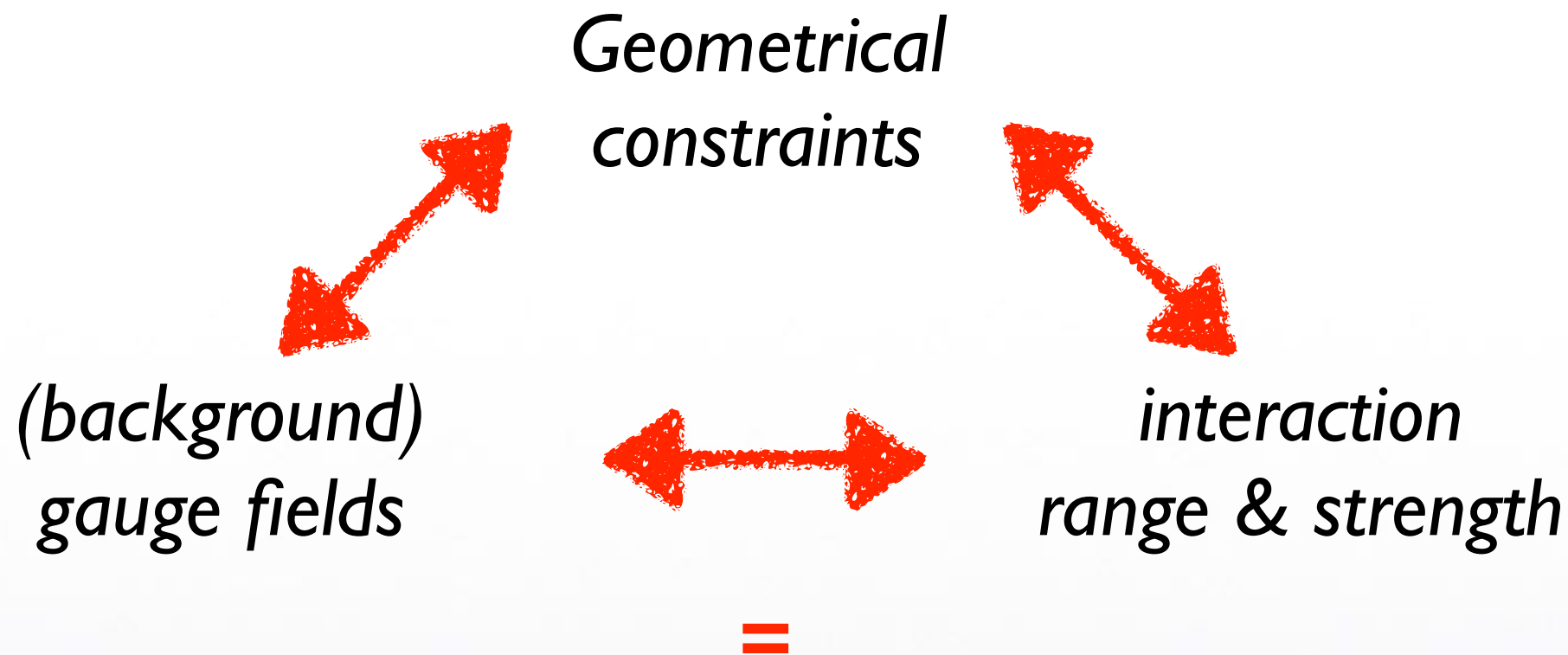
interaction  
range & strength

**Synthetic Quantum Matter**



# The Big Picture

## Quantum Simulation with Cold Atomic Gases



**Access to synthetic topological matter & more !**

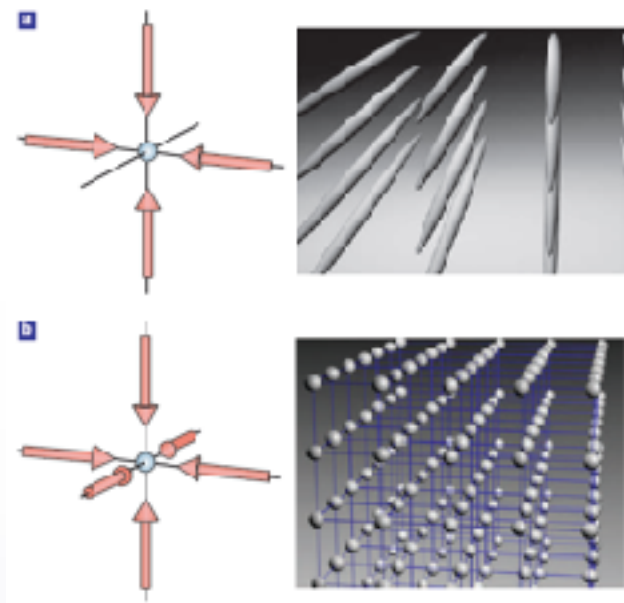
Here:

**Fermionic Models: studies via Mappings & Tensor Networks**

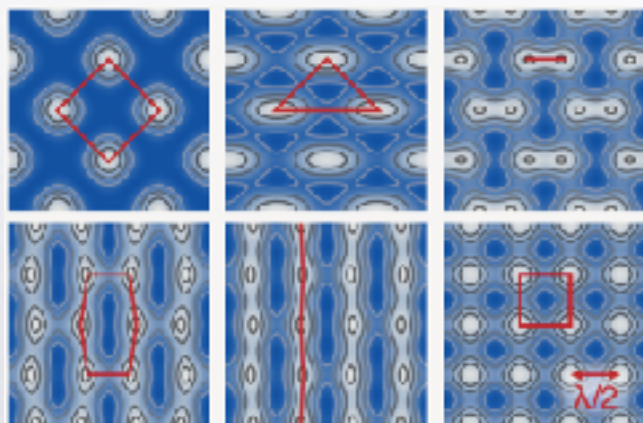
# Synthetic Quantum Matter

## Bottom-up engineering

trapped ions, superconducting qubits, quantum dots, NV centers, etc.



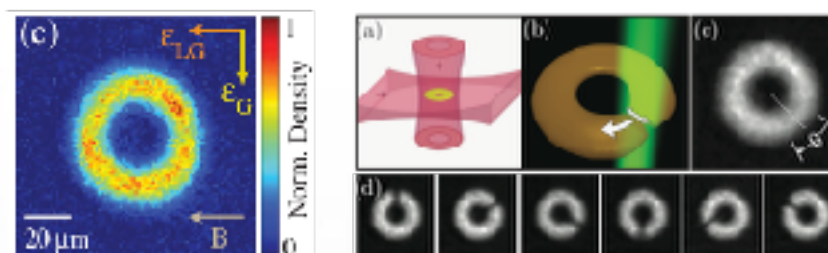
I. Bloch, et al. RMP **80**, 885 (2008)



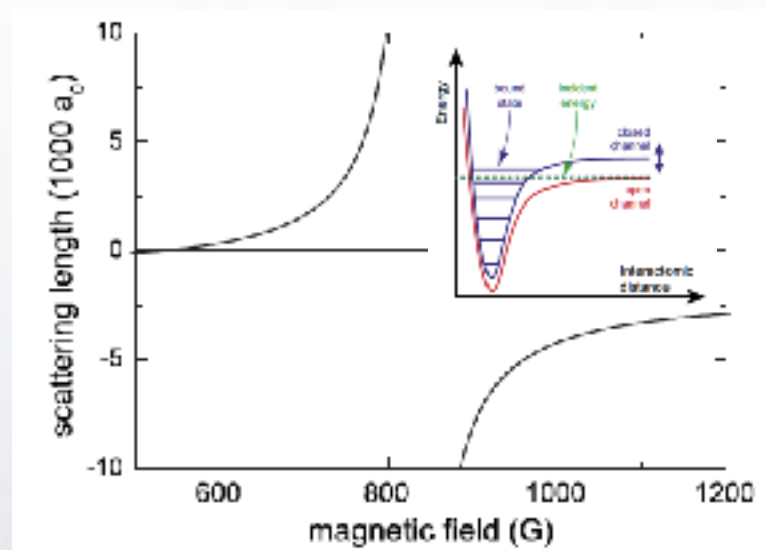
L. Tarruell, et al., Nature **483**, 302 (2012)

## Hamiltonian engineering

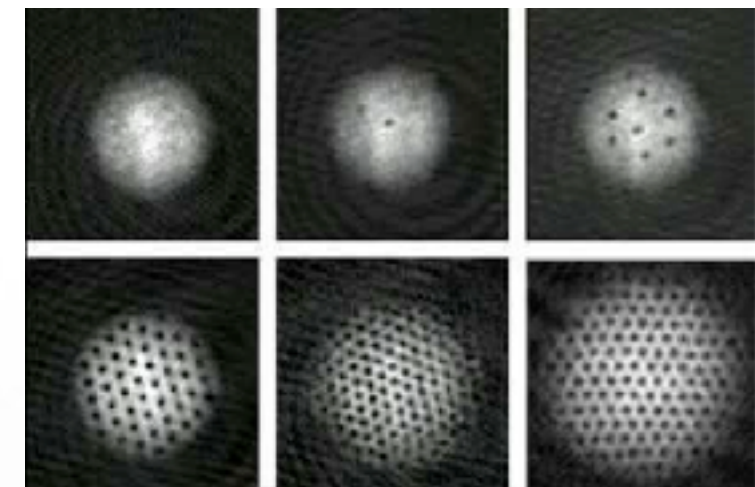
cold atomic gases, photonic systems, ...



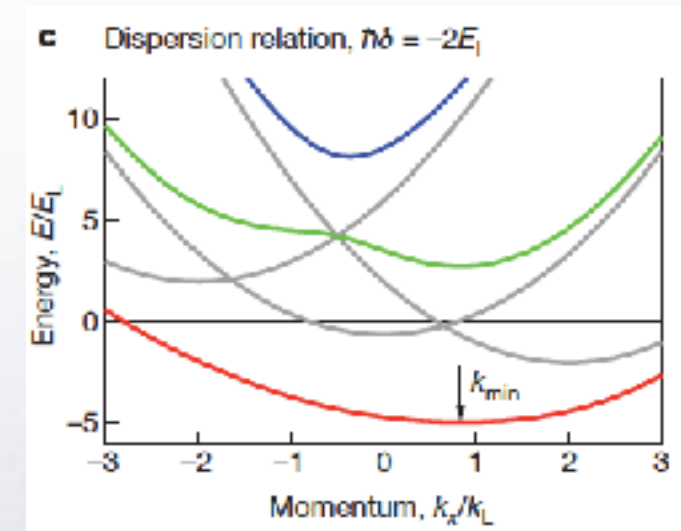
Ramanathan et al., PRL **106**, 130401 (2011);



C. Chin, et al., RMP **82**, 1225 (2010)



A. Fetter. RMP **81**, 647 (2009)



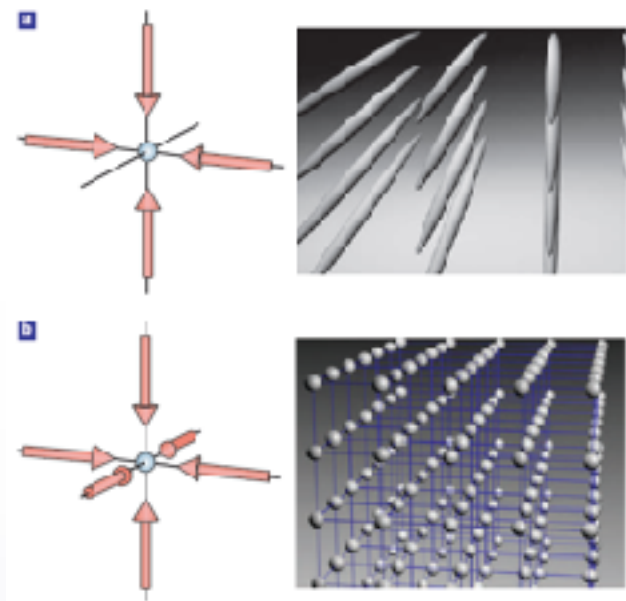
J. Dalibard, et al., RMP **83**, 1523 (2011)  
N. Goldman, et al., RPP **77**, 126401 (2014)



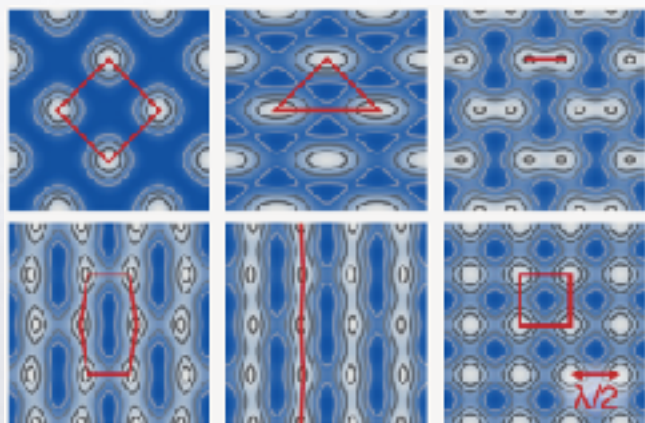
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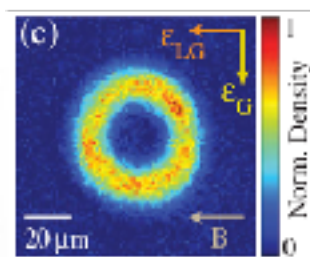
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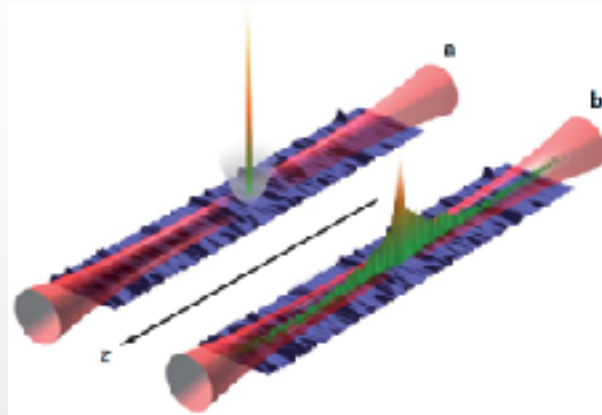
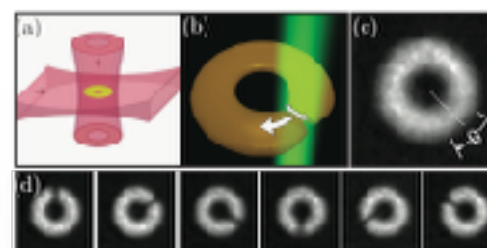
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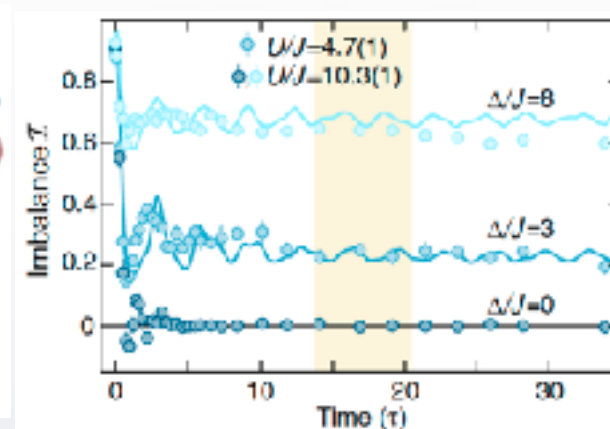
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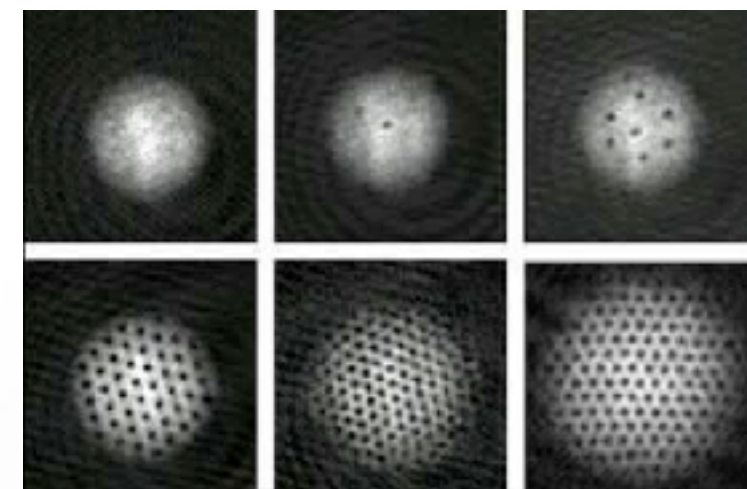
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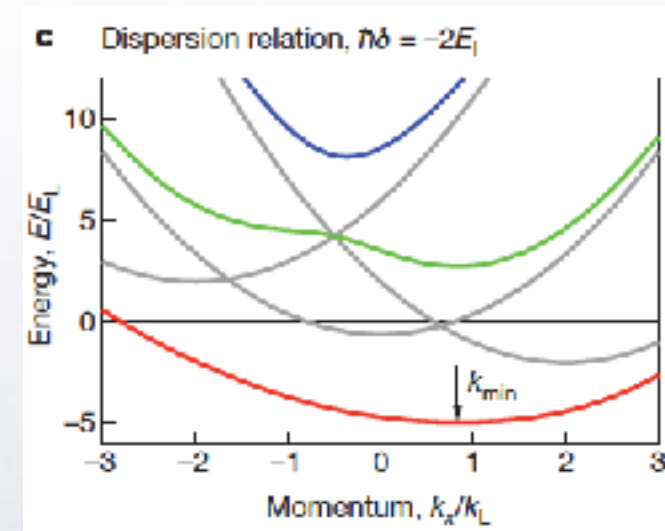
J. Billy, et al.,  
Nature **453**, 891 (2008)



M. Schreiber, et al.,  
Science **349**, 842 (2015)



A. Fetter. RMP **81**, 647 (2009)



J. Dalibard, et al., RMP **83**, 1523 (2011)  
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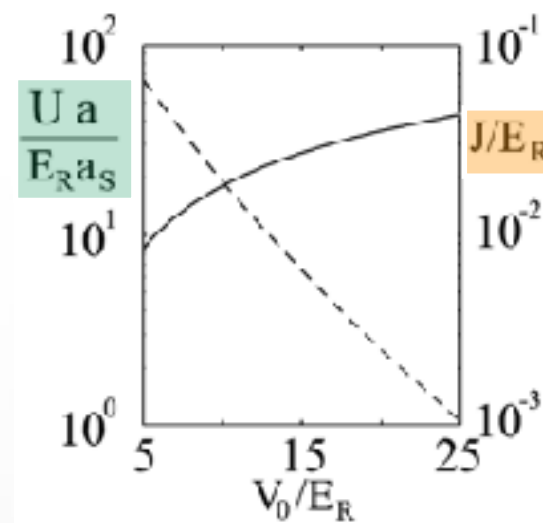
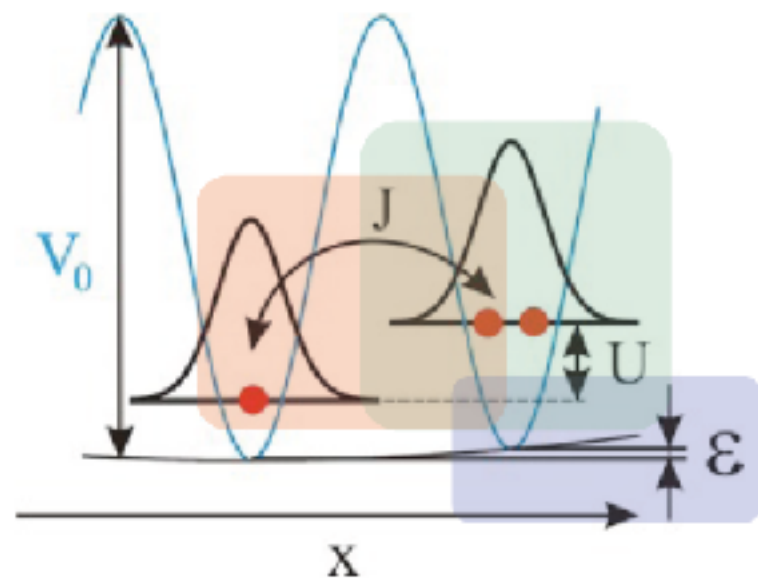


# Hubbard models

Tight-binding model  
out of localized  
(single particle)  
Wannier functions

$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

D. Jaksch et al., PRL **81** 3108 (1998)

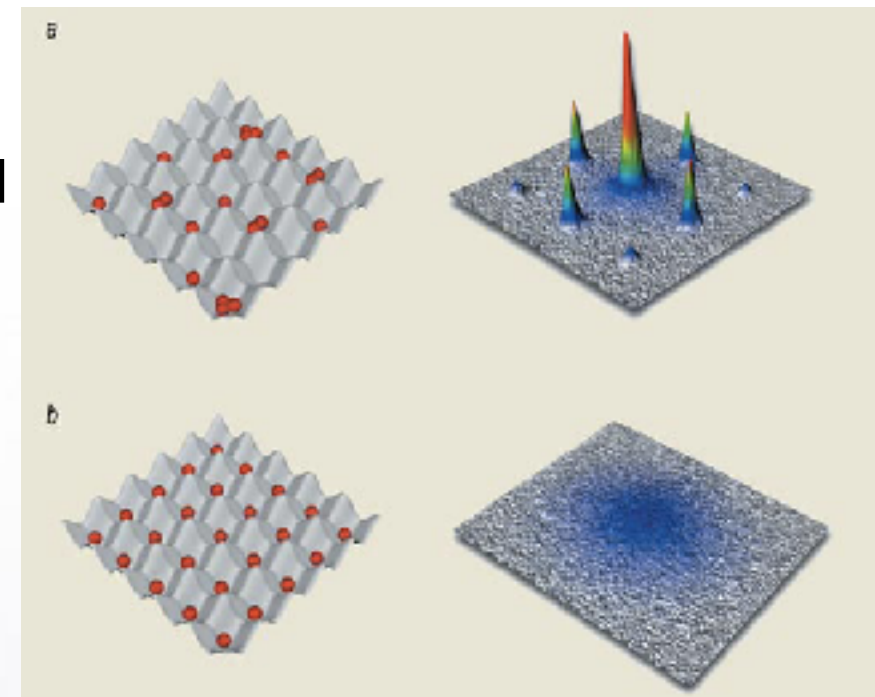


Superfluid

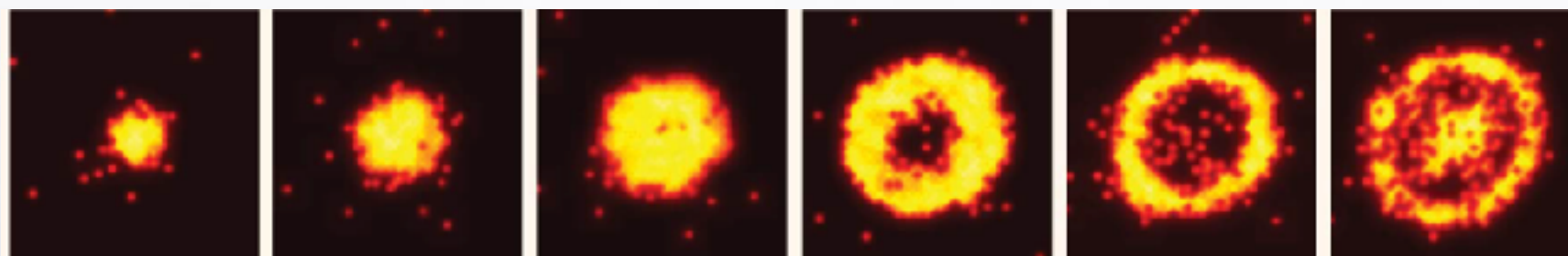
Mott  
Insulator

real space

momentum space



M. Greiner, et al., Nature **415**, 39 (2002)



Sherson, et al. Nature **467**, 68 (2010).

nowadays: spectacular progresses with 2D Fermi-Hubbard!

e.g., Greiner & Bloch & Esslinger labs

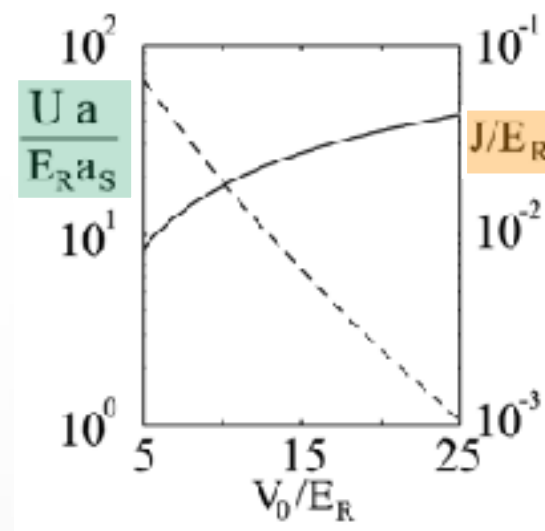
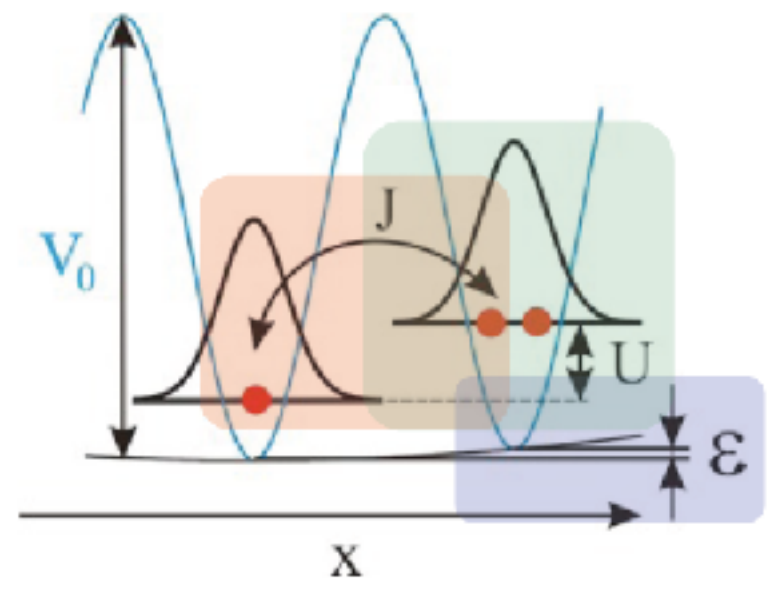


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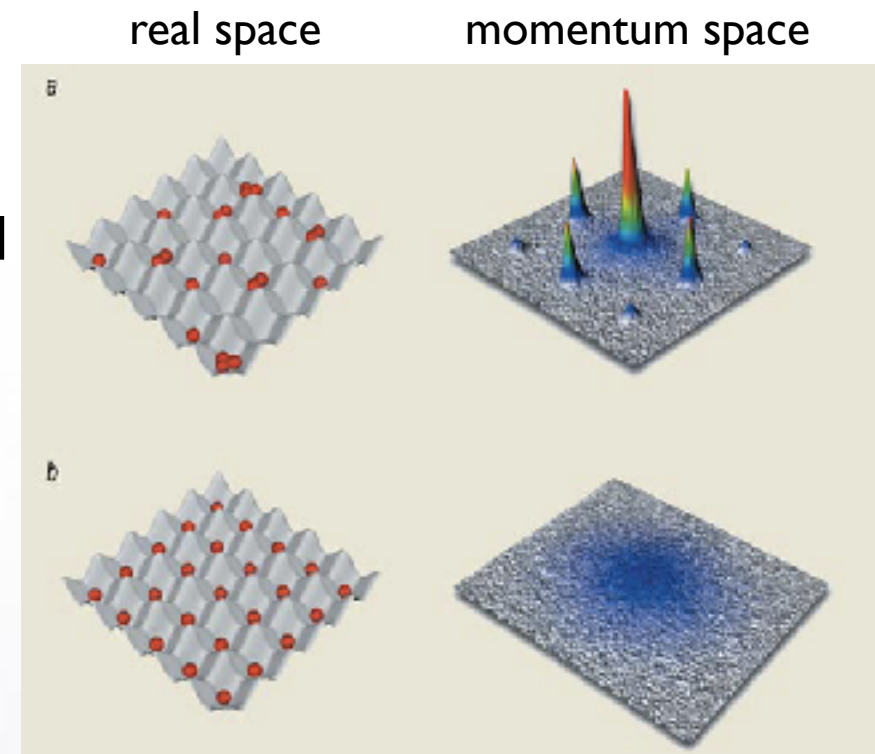
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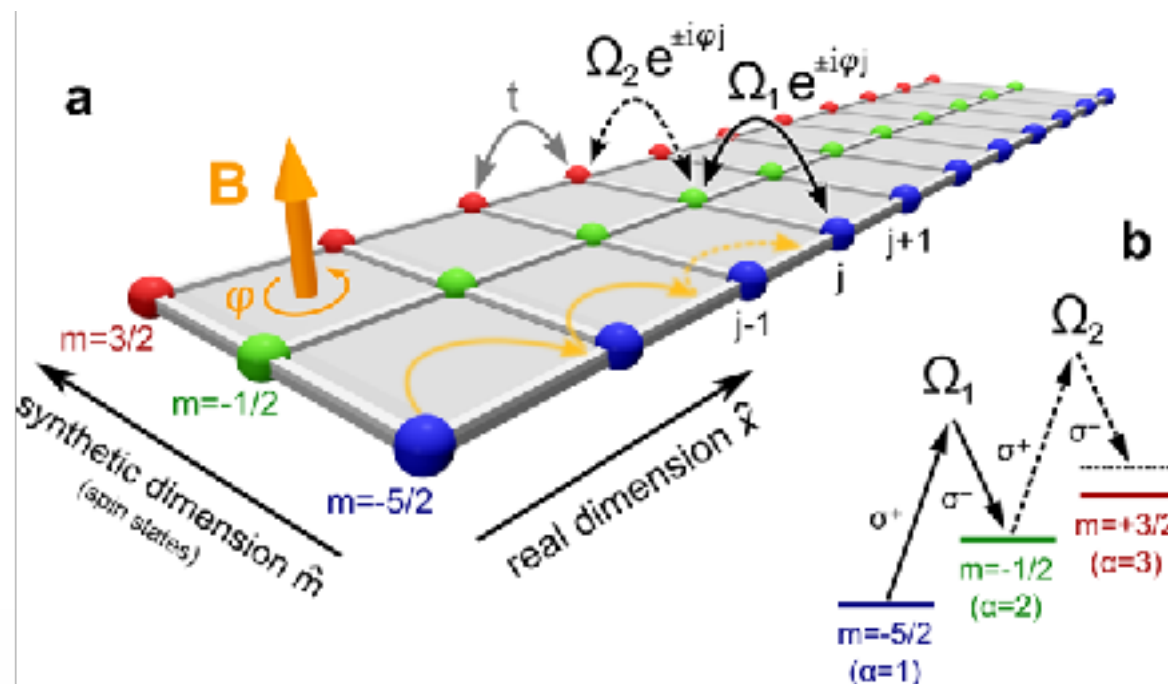


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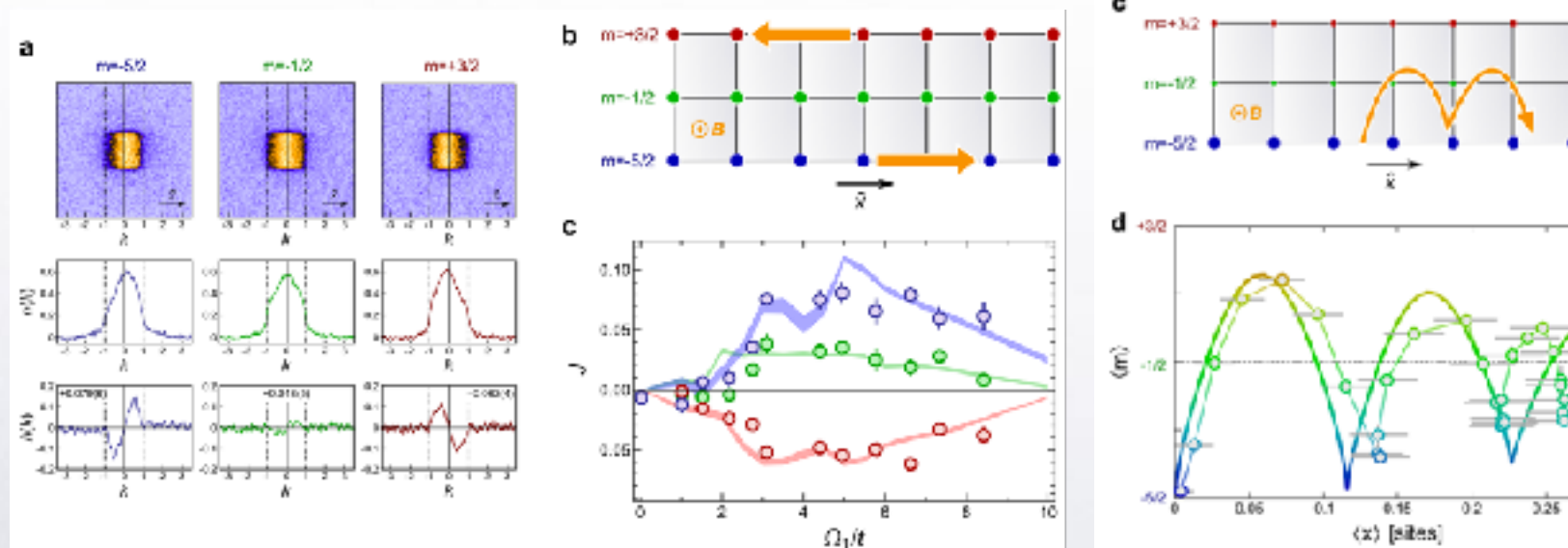
nuclear / hyperfine levels  $\begin{cases} \text{multi-flavor / multi-orbital Hubbard models} \\ \text{synthetic (extra-)dimension} \end{cases}$

A. Celi et al., PRL **108**, 133001 (2012) & **112**, 043001 (2014)

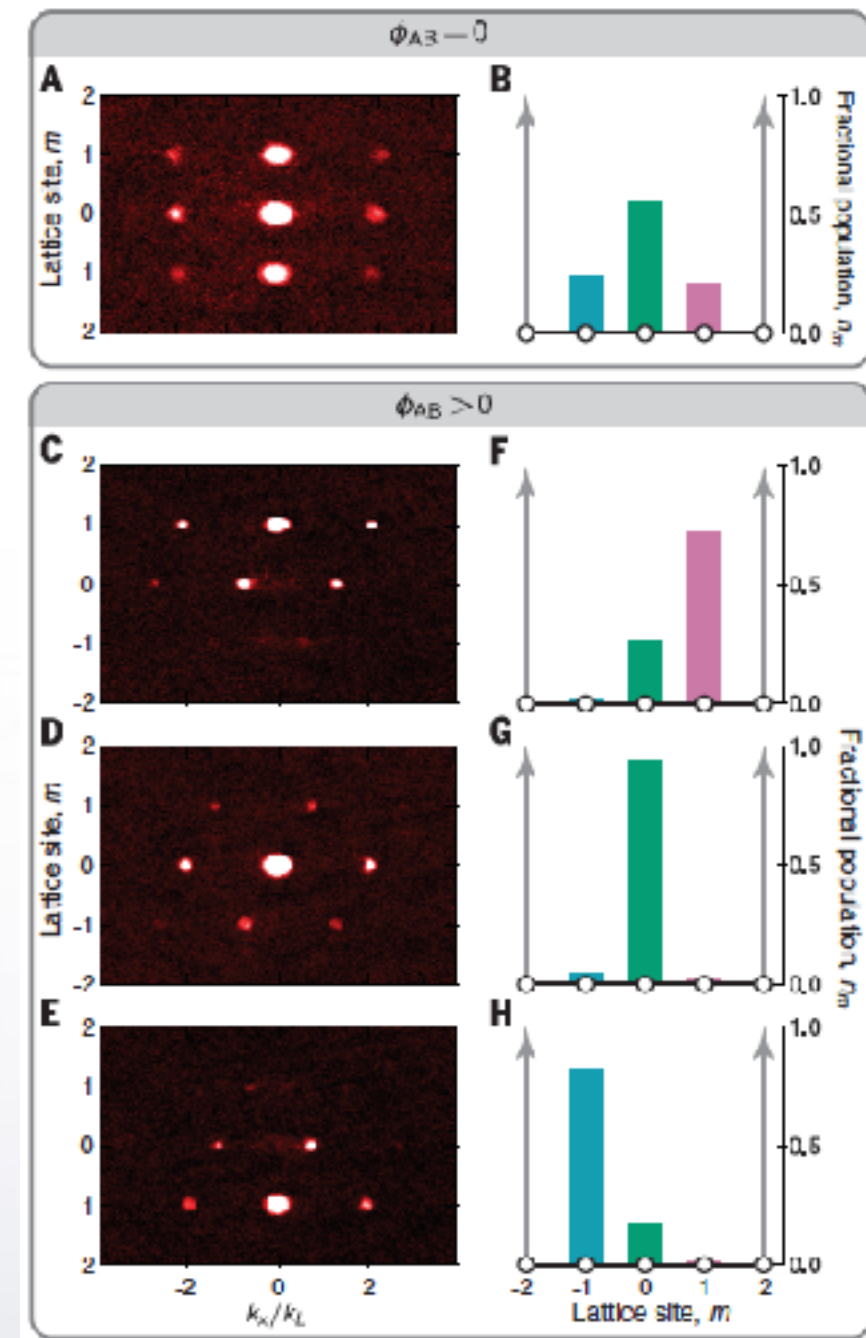
# Synthetic ladders



## Chiral edge currents & skipping orbits



M. Mancini, et al., Science **349**, 1510 (2015)



B.K. Stuhl, et al., Science **349**, 1514 (2015)

+ many more related experiments @ Harvard, MIT, Munich, Paris, etc.



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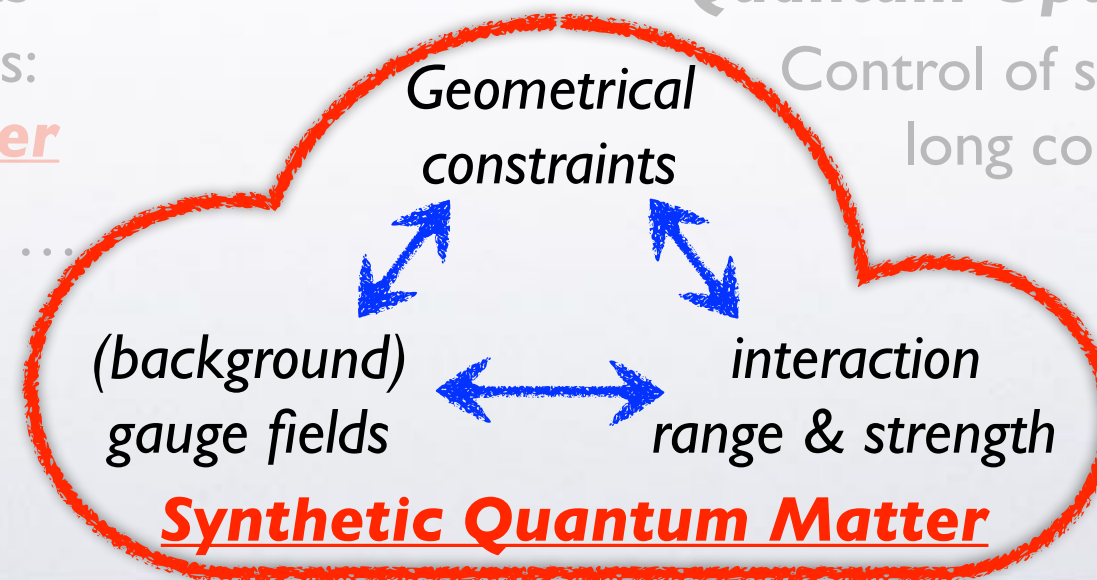
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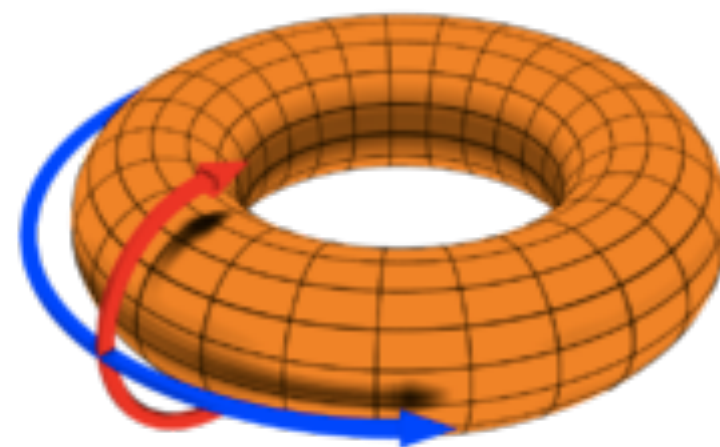
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## Optics & Atomic physics

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(background) gauge fields ↔ interaction range & strength

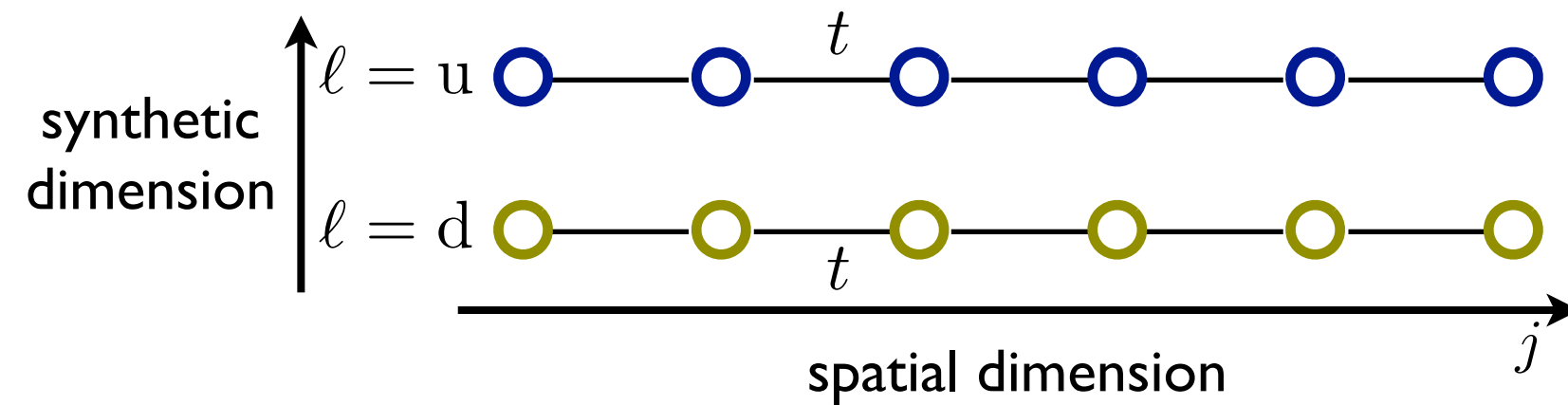
**Synthetic Quantum Matter**



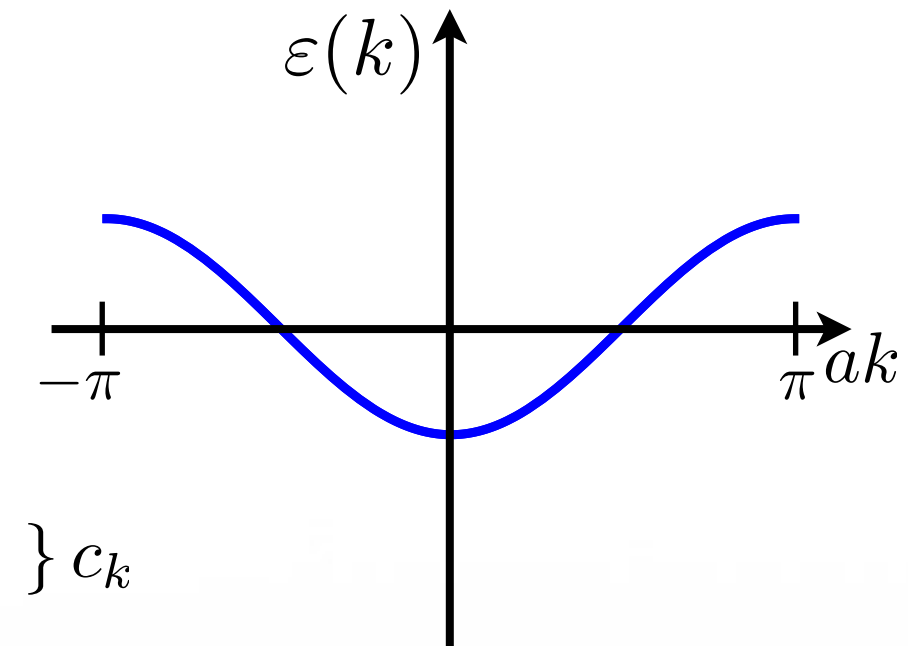
# OUTLINE

- The Creutz-Hubbard Ladder: general features
- Topology & Interactions
  - SPT vs orbital magnetism at half-filling
  - relation to high-energy models
  - interacting SPT phases at fractional filling
- Tuning the Drude Weight of Dirac fermions
- Other related works & plans

# From standard to flat bands

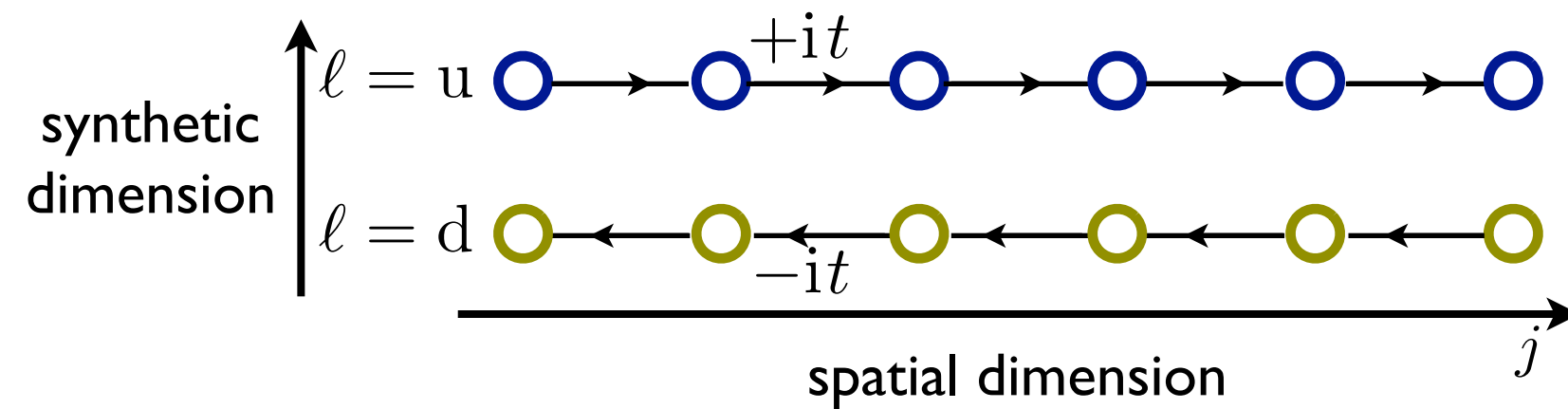


$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ -2t \cos(ak) \sigma_0 +$$

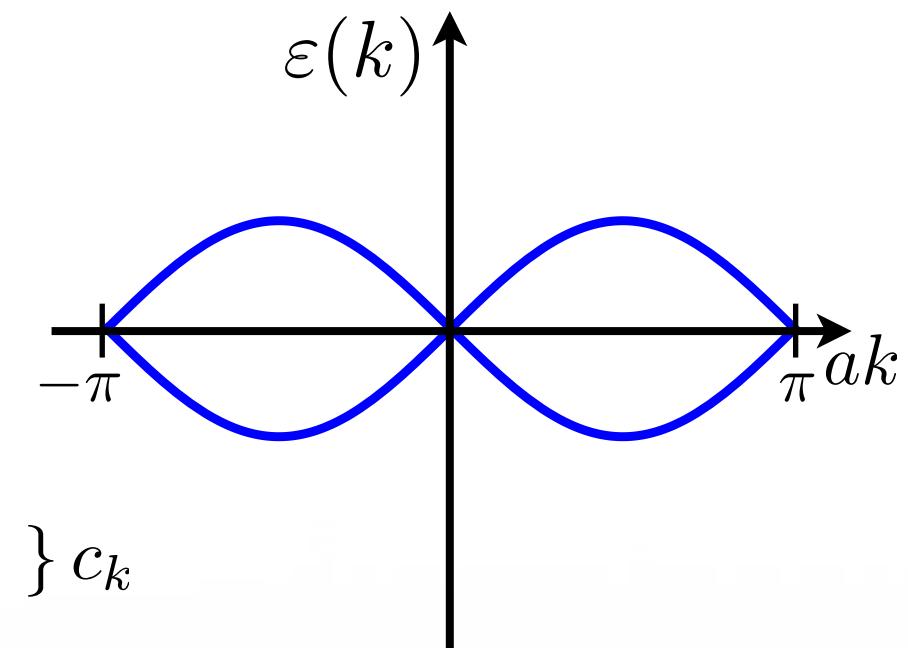




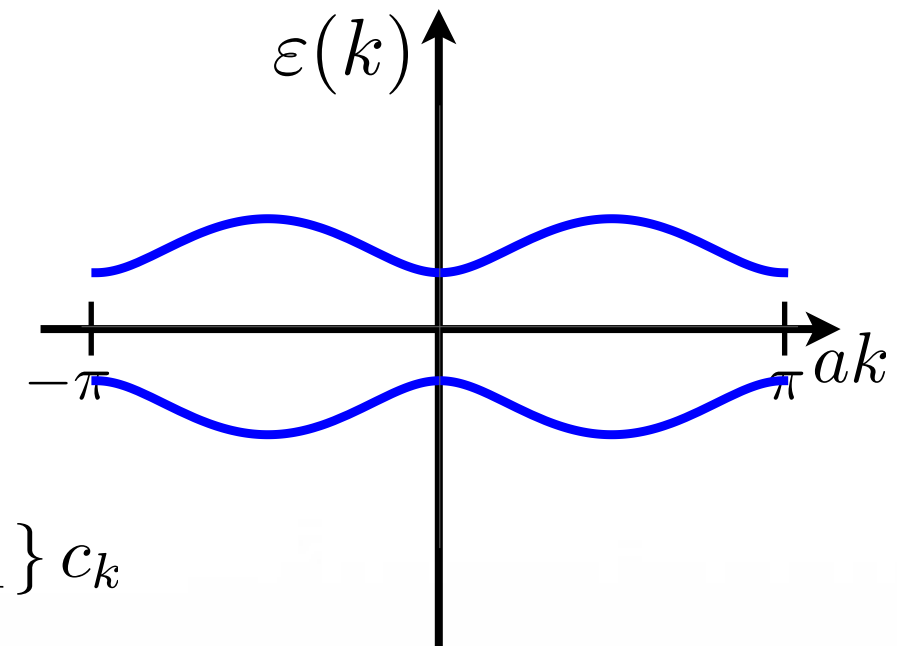
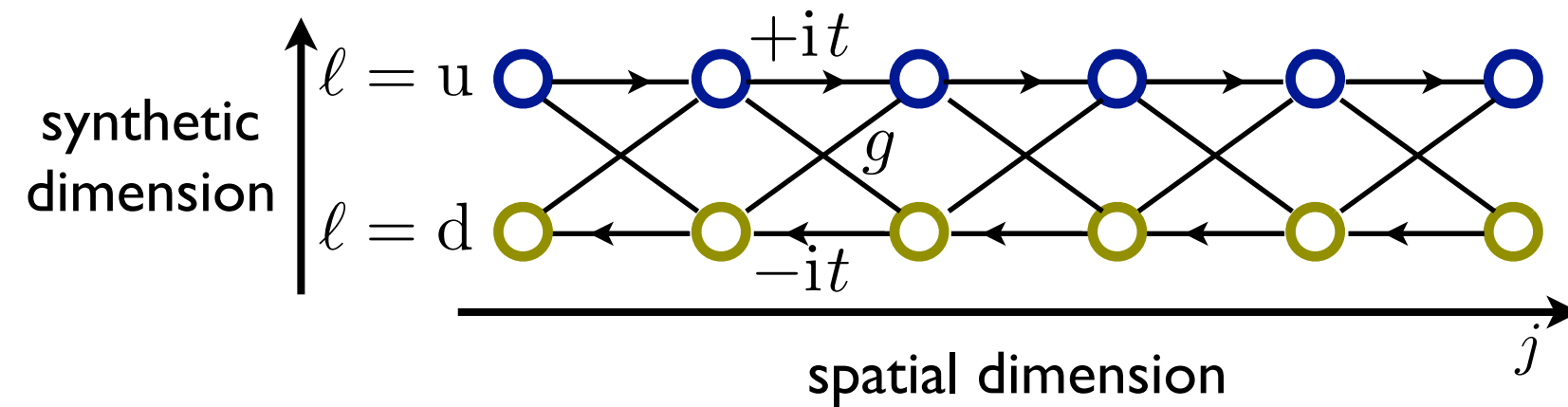
# From standard to flat bands



$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ [ + 2t \sin(ak) ] \sigma_3 +$$



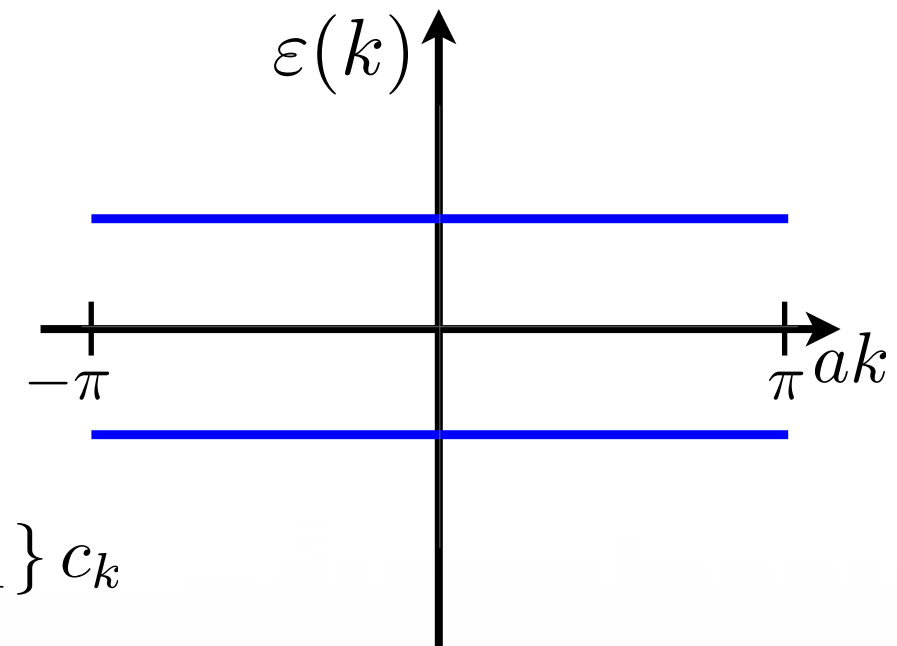
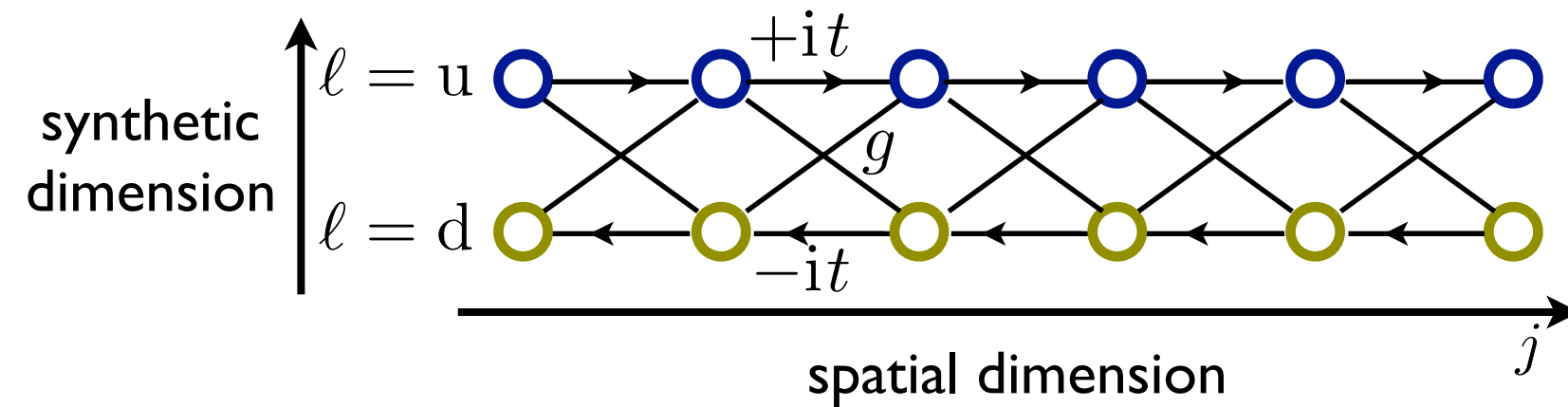
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$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ [ + 2t \sin(ak) ] \sigma_3 + [ - 2g \cos(ak) ] \sigma_1 \} c_k$$



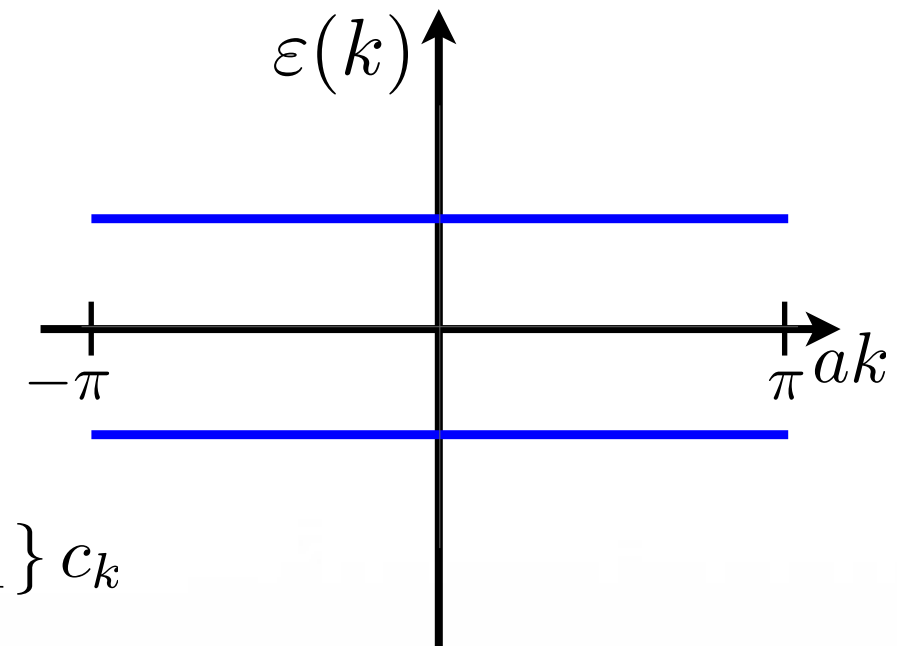
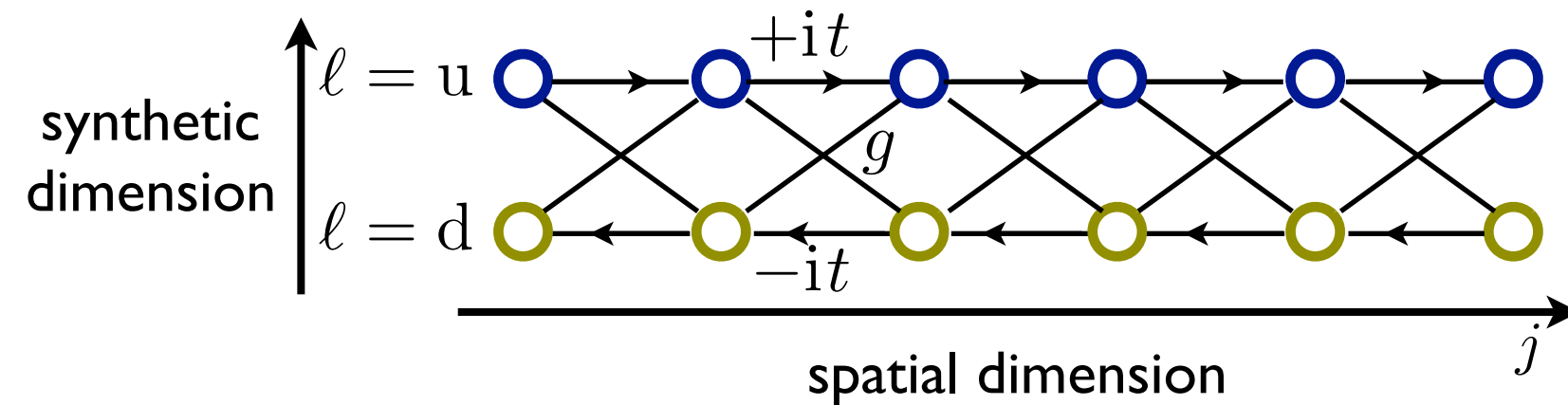
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$g = t$  flat bands

# Flat topological bands



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$$\mathcal{H}_0 = \sum_{k \in \text{BZ}} c_k^\dagger \{ \mathbf{B}(k) \cdot \boldsymbol{\sigma} \} c_k$$

constrained in  $(B_1, B_3)$  plane by chiral symmetry:

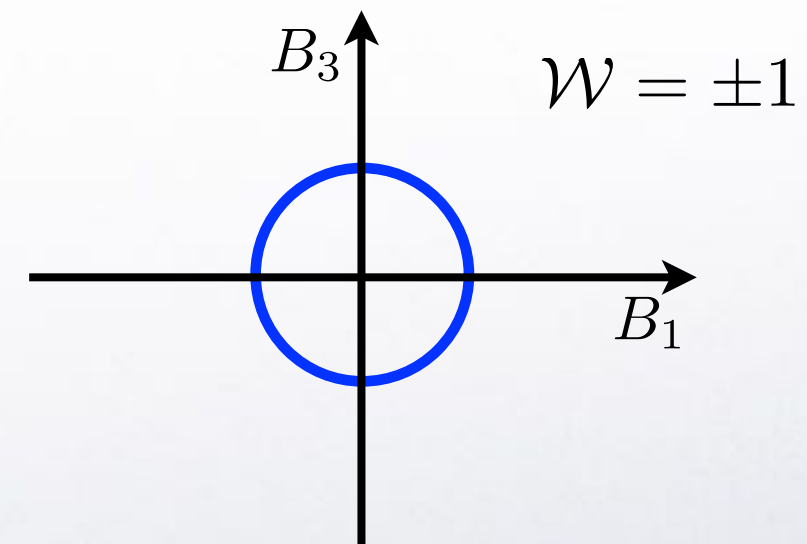
$$\sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k)$$

Symmetry Protected Topological (SPT) Order

$$\mathcal{A}_\pm(k) = \langle \varepsilon_\pm(k) | i\partial_k | \varepsilon_\pm(k) \rangle \quad \varphi_{\text{Zak}, \pm} = \int_{\text{BZ}} dk \mathcal{A}_\pm(k) = \pi$$

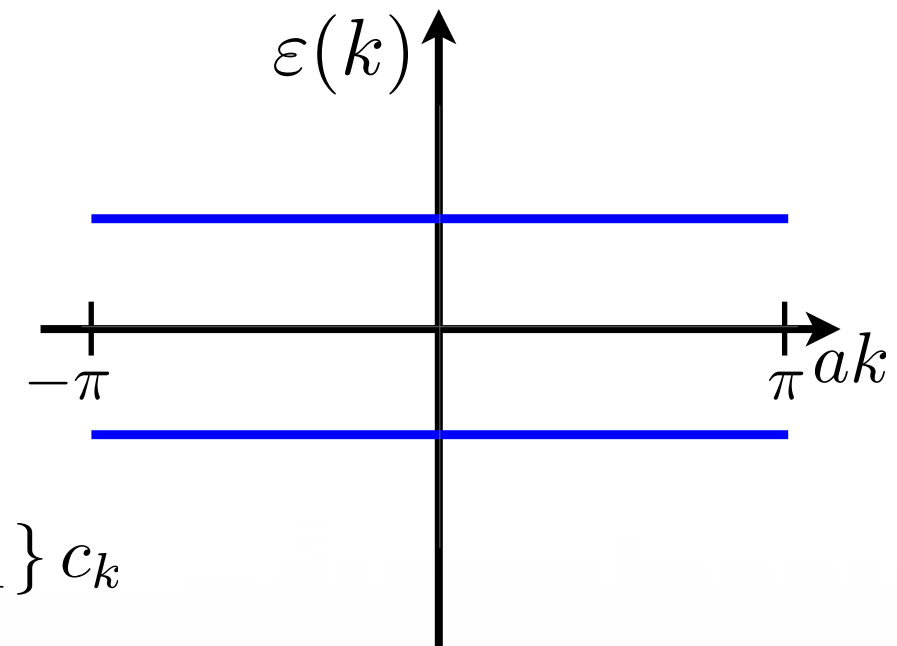
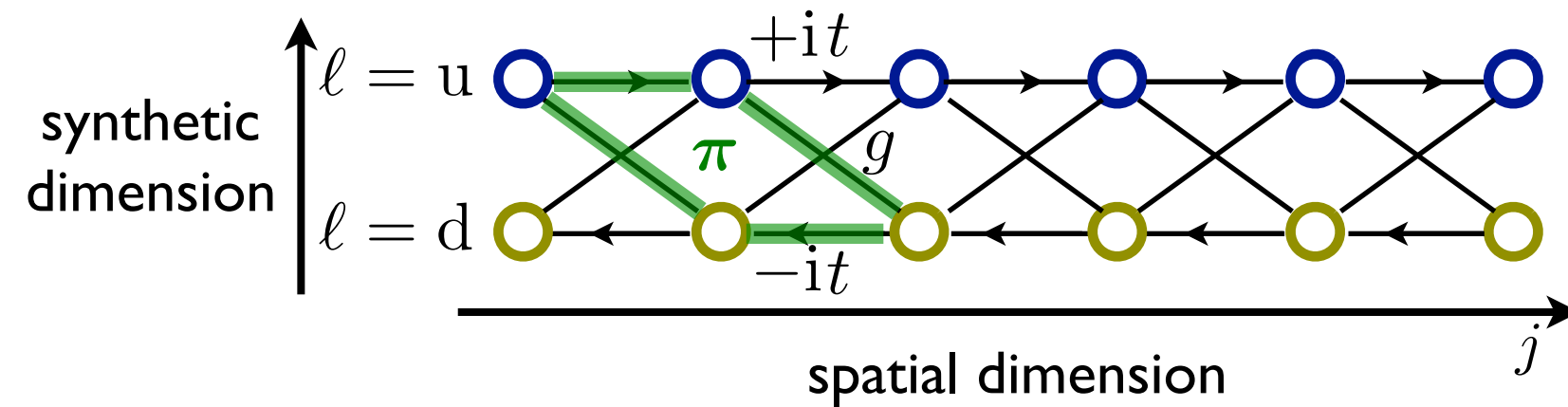
e.g., S. Ryu, et al., NJP **12**, 065010 (2010), D. Xiao, et al., RMP **82**, 1959 (2010)

$\varphi_{\text{Zak}}$  measured in cold gases  
via Ramsey interferometry  
M. Atala, et al., Nat. Phys. **9**, 795 (2013)





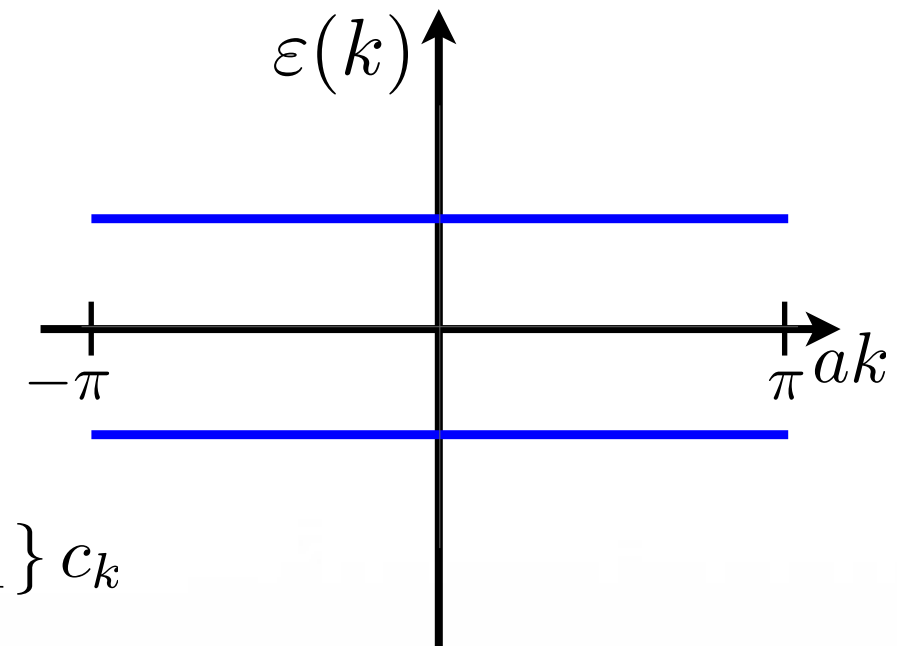
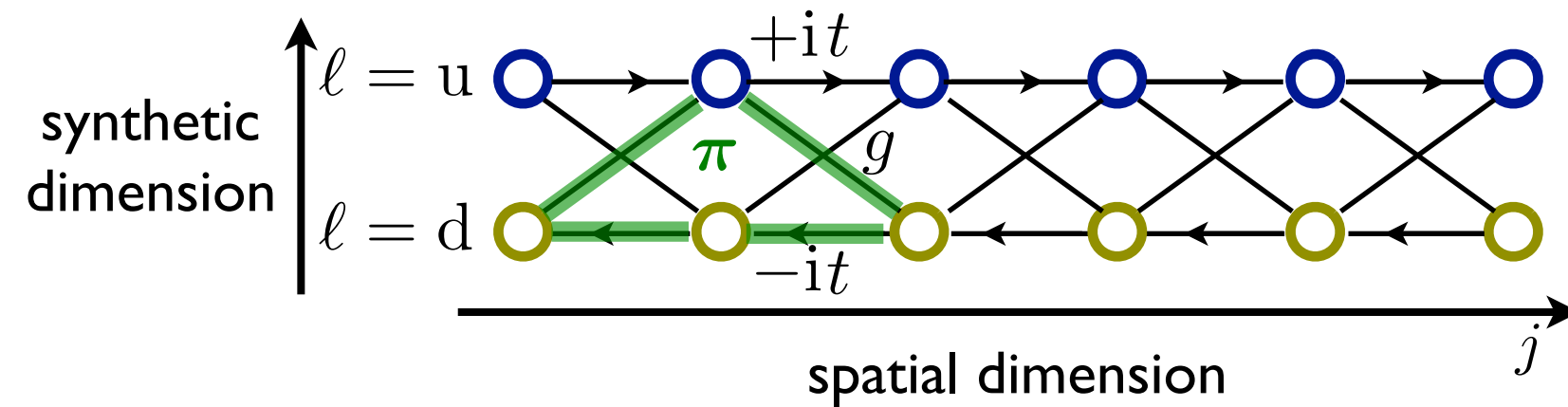
# Aharonov-Bohm cages



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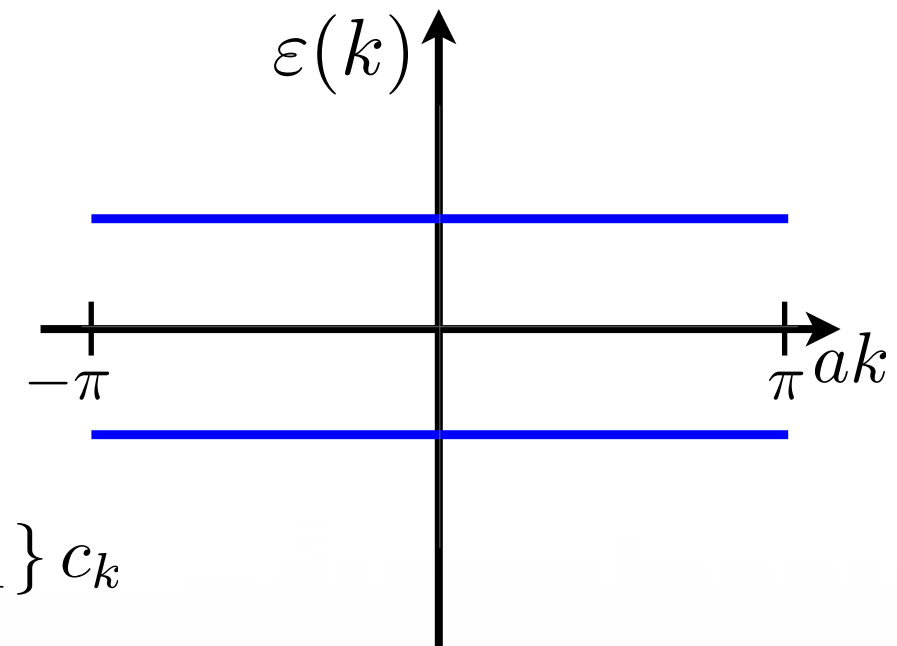
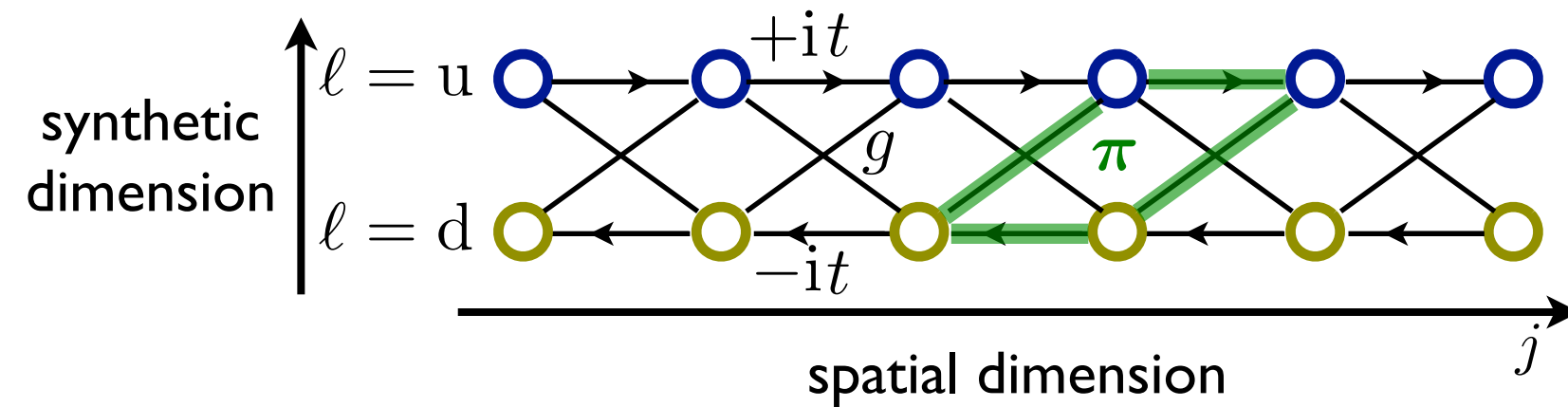


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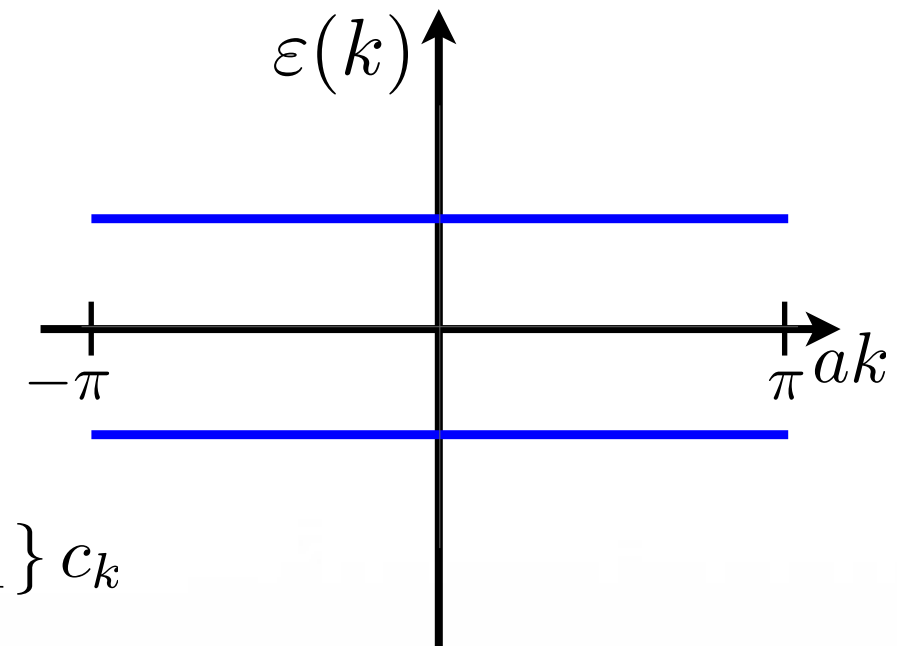
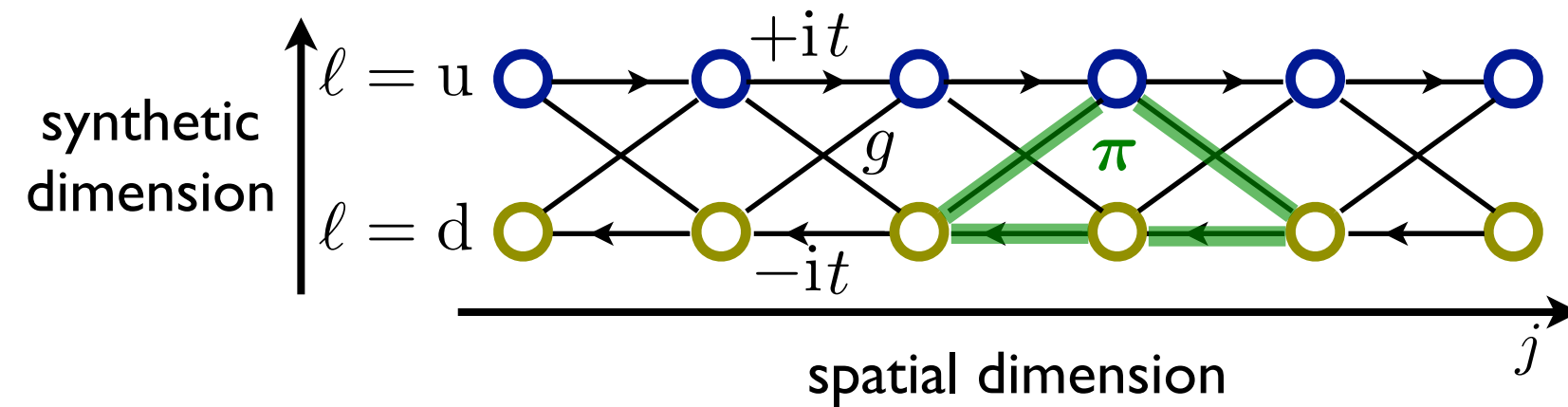
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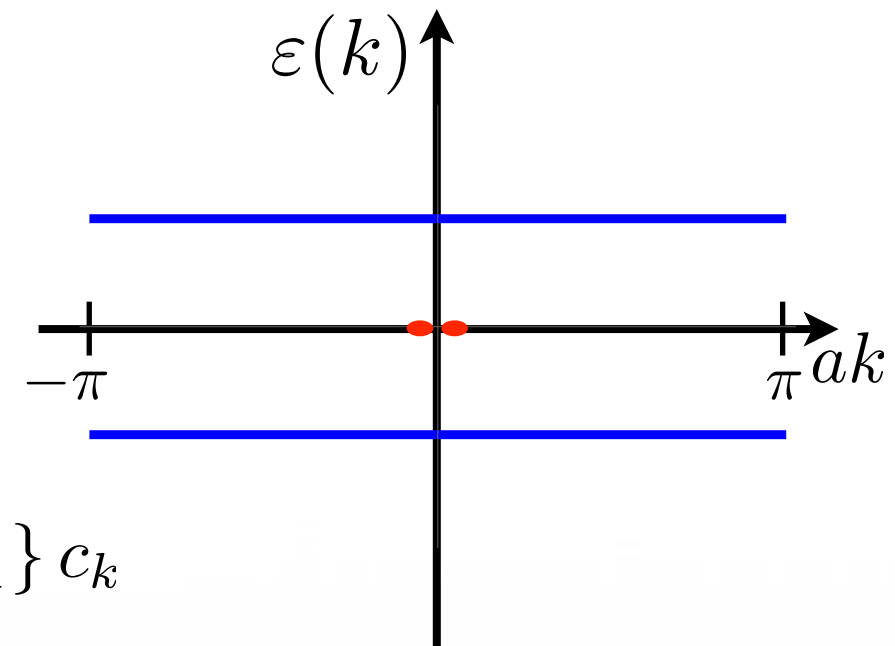
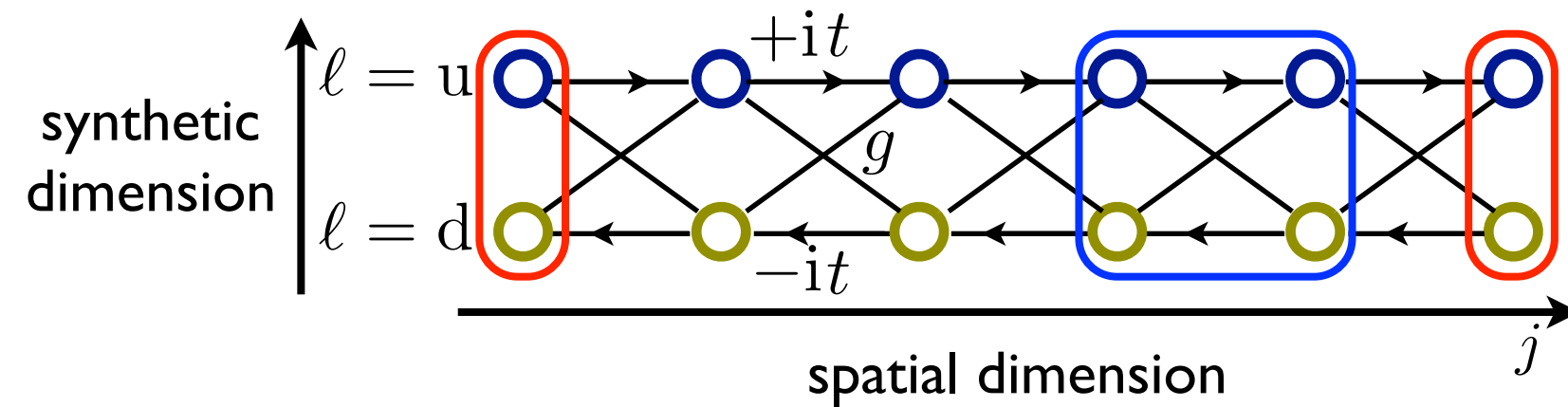


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Basis of localized states  $w_{j,\pm}^\dagger$ ,  
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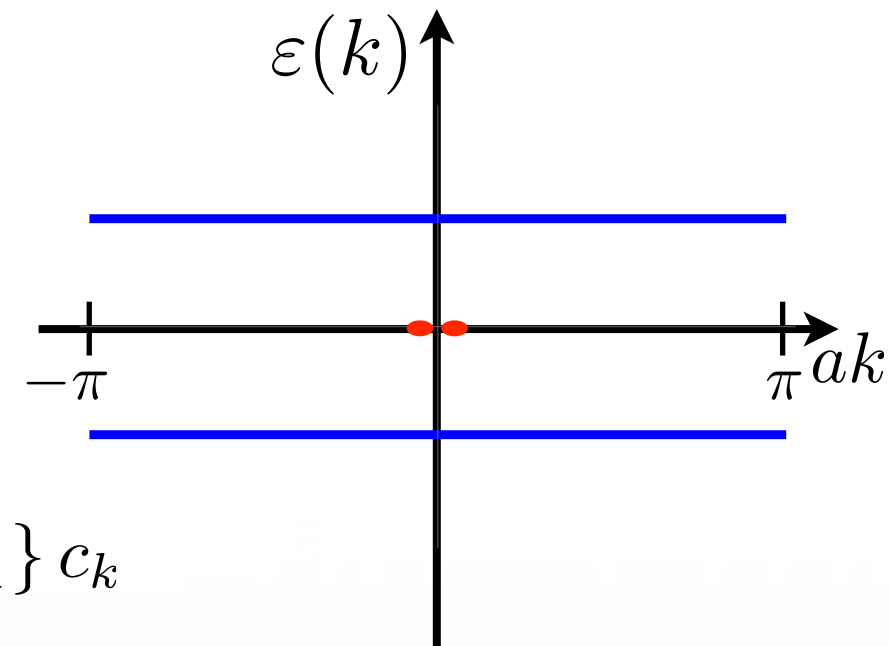
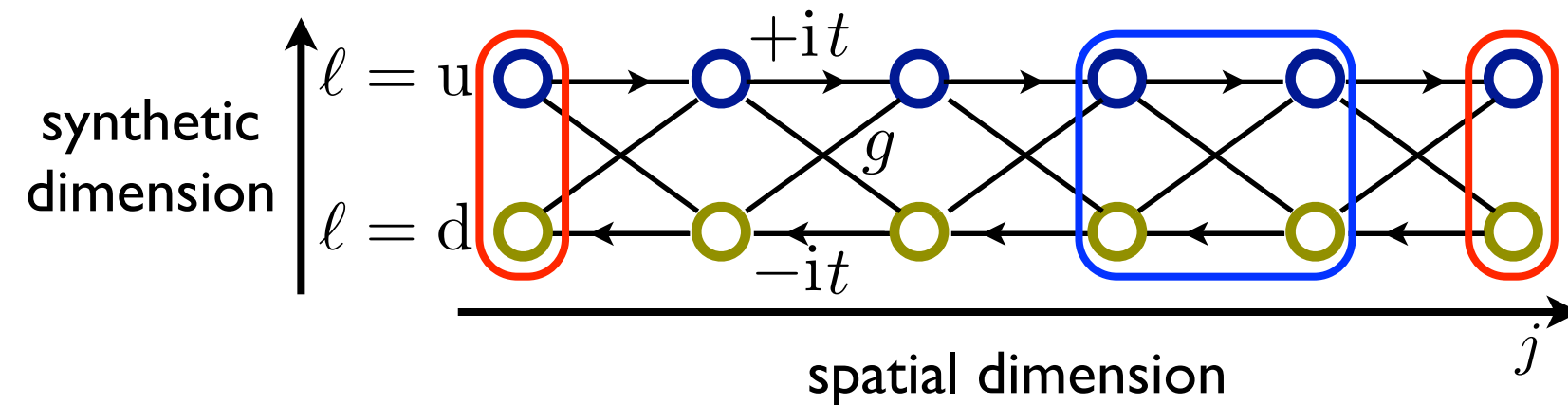
Vidal, Mosseri, & Doucot, PRL **81**, 5888 (1998)

OBC: Mid-gap zero-energy edge states ( $l^\dagger, r^\dagger$ ),  
[bulk-edge correspondence]

S. Ryu and Y. Hatsugai, PRL **89**, 077002 (2002)

P. Delplace, et al., PRB **84**, 195452 (2011)

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a *workhorse* for flat-band & SPT physics:

Tovmasyan, van Nieuwenburg, & Huber, PRB **88**, 220510(R) (2013)

Takayoshi, Katsura, Watanabe, & Aoki, PRA **88**, 063613 (2013)

Huber & Altman, PRB **82**, 184502 (2010)

Tovmasyan, Peotta, Törmä, & Huber, PRB **94**, 245149 (2016)

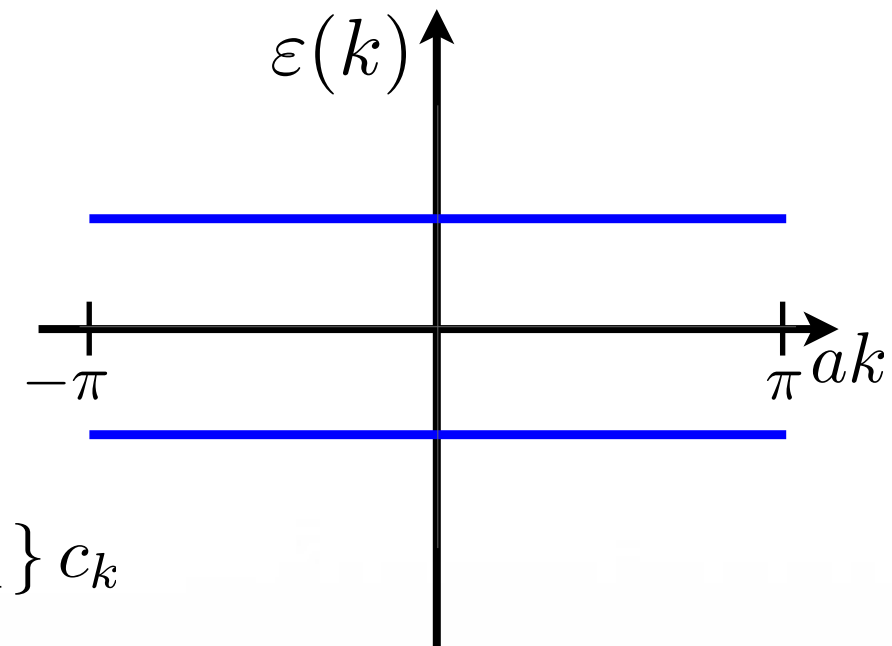
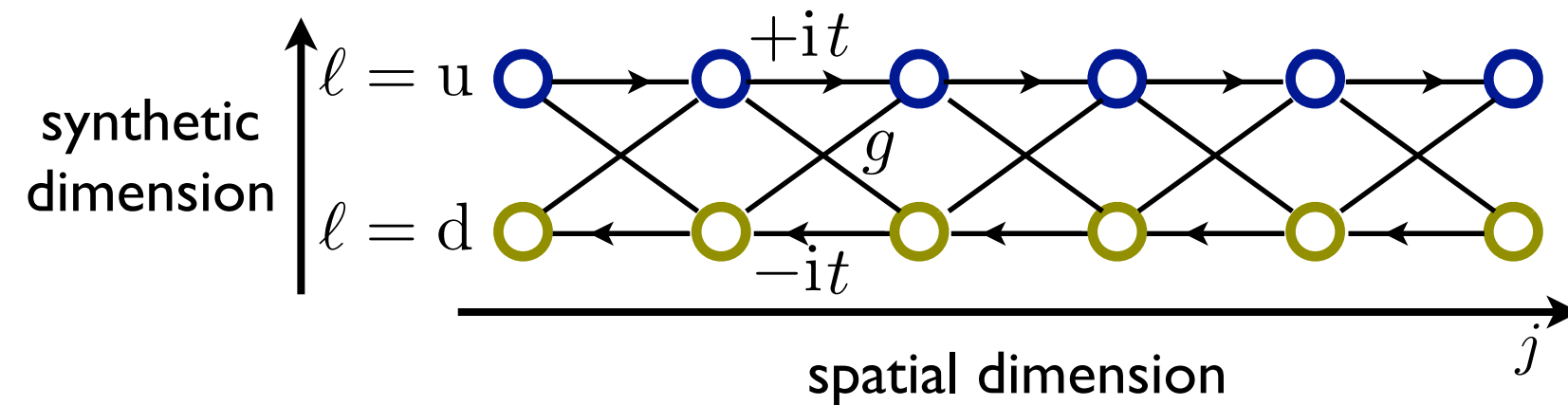
Sticlet, Seabra, Pollmann, & Cayssol, PRB **89**, 115430 (2014)

Bragg techniques  
to measure edge states  
in ultracold cold atoms

Goldman, Beugnon, Gerbier,  
PRL **108**, 255303 (2012)



# SPT - Trivial transition



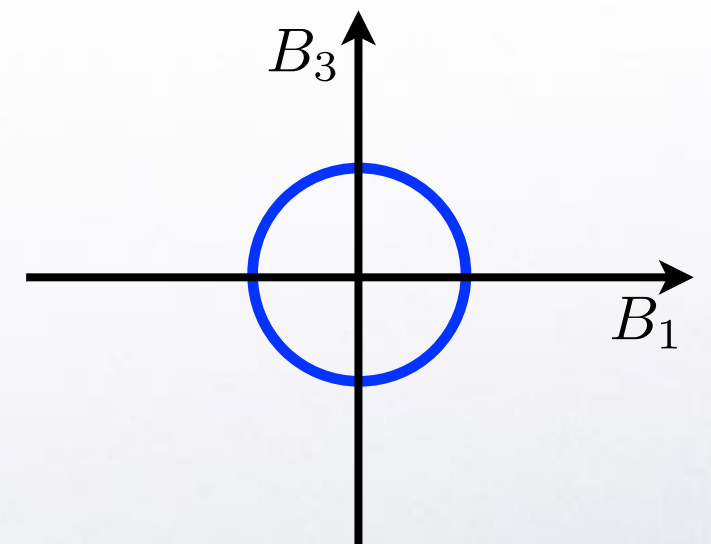
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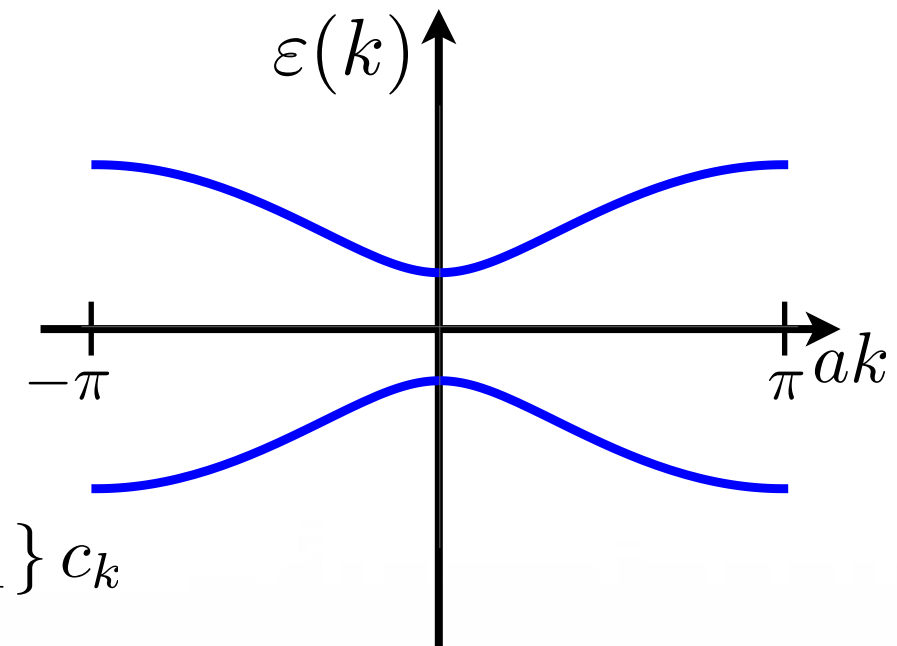
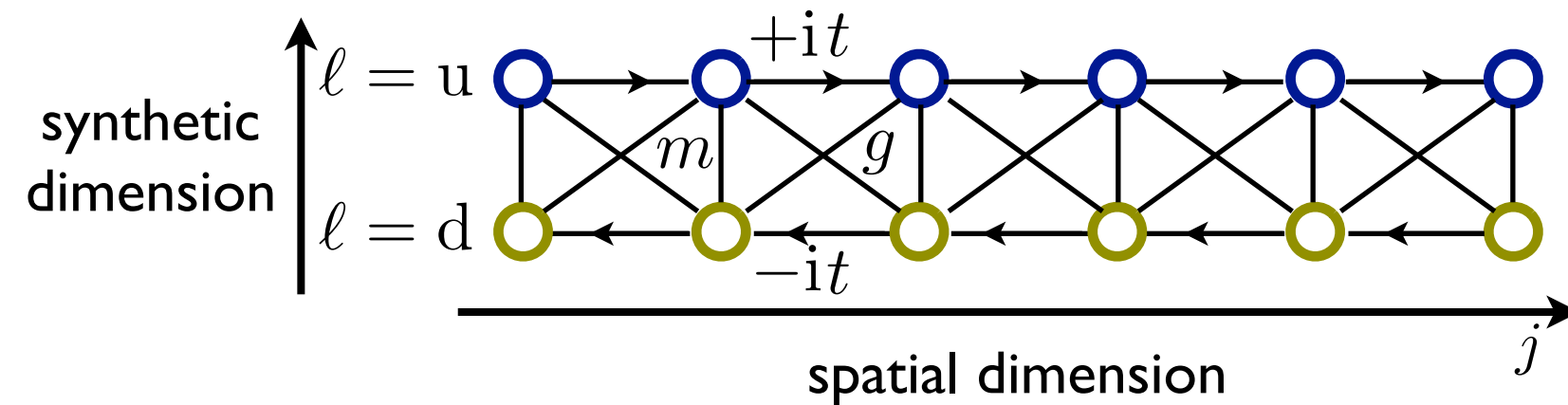
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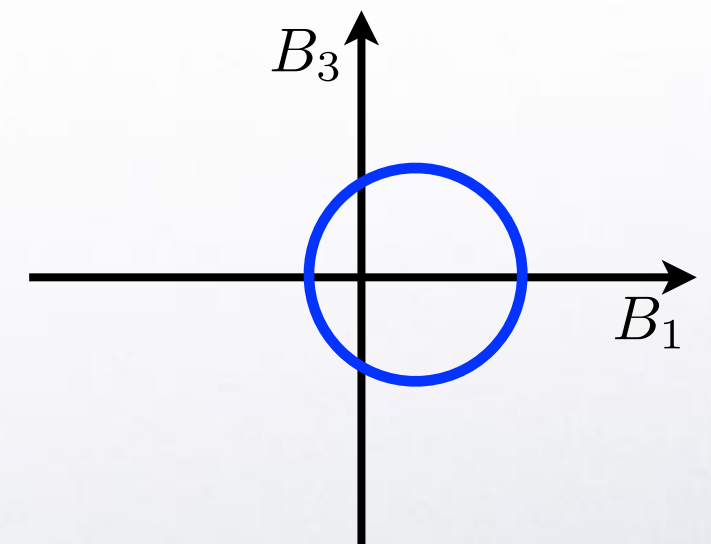
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$$\mathcal{H}_0 = \sum_{k \in \text{BZ}} c_k^\dagger \{ \mathbf{B}(k) \cdot \boldsymbol{\sigma} \} c_k$$

constrained in  $(B_1, B_3)$  plane by chiral symmetry:

$$\sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k)$$



$$T : \sigma_1 \mathcal{H}_0^*(-k) \sigma_1 = +\mathcal{H}_0(k)$$

$$C : \sigma_3 \mathcal{H}_0^*(-k) \sigma_3 = -\mathcal{H}_0(k)$$

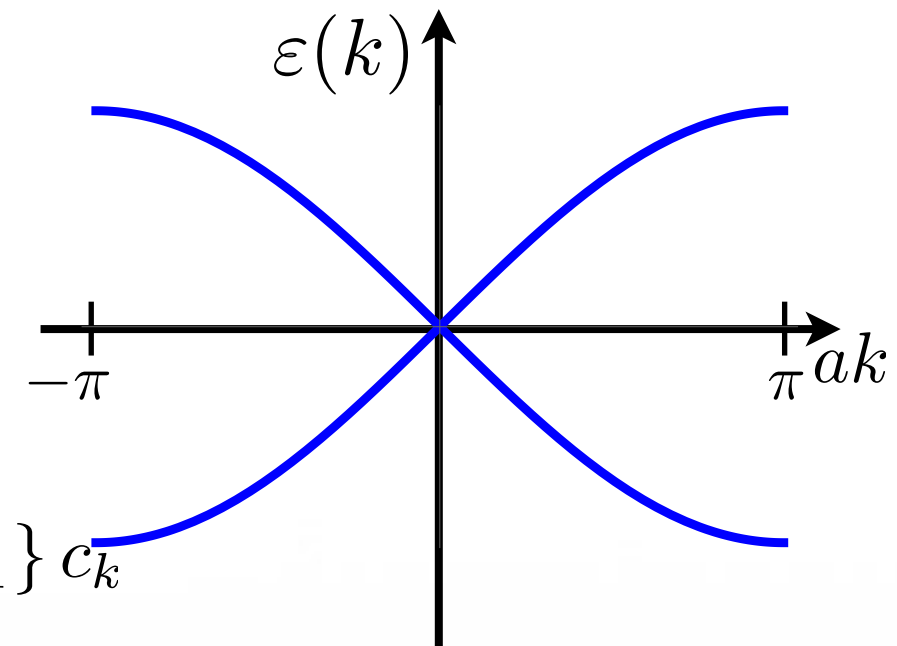
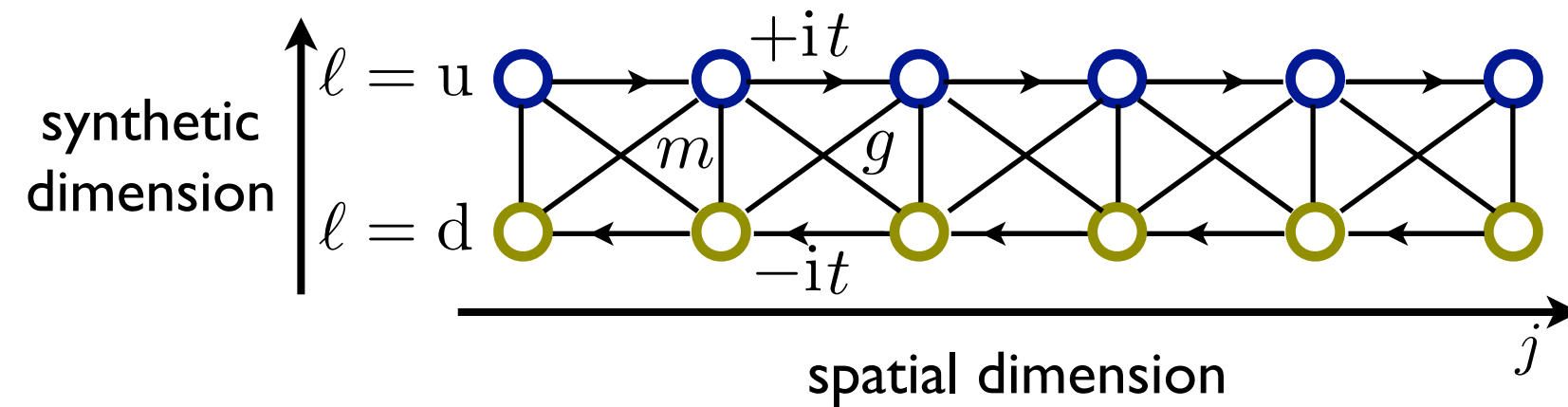


**Class BDI** of AZ table

Altland, Zirnbauer, PRB **55**, 1142 (1997)



# SPT - Trivial transition



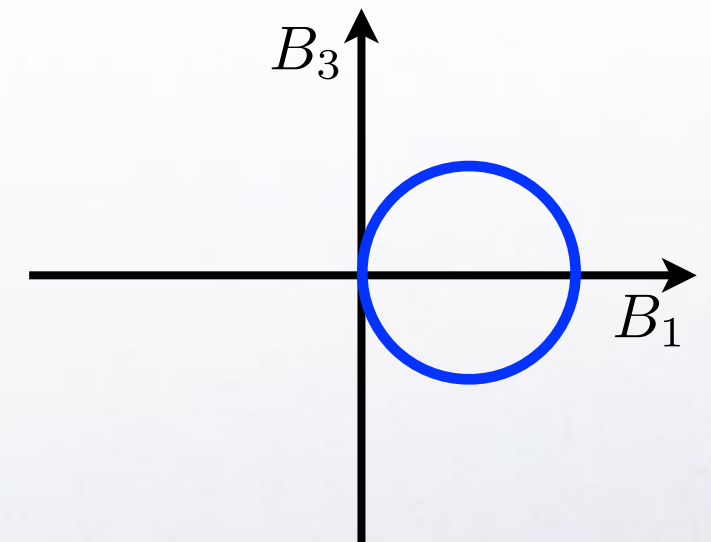
$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ [ + 2t \sin(ak) ] \sigma_3 + [ 2m - 2g \cos(ak) ] \sigma_1 \} c_k$$

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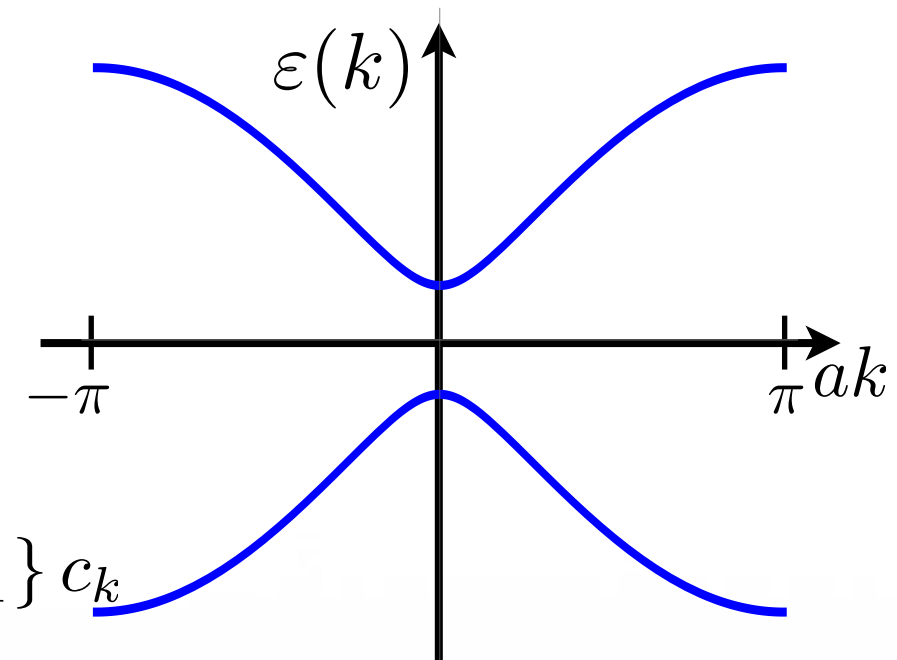
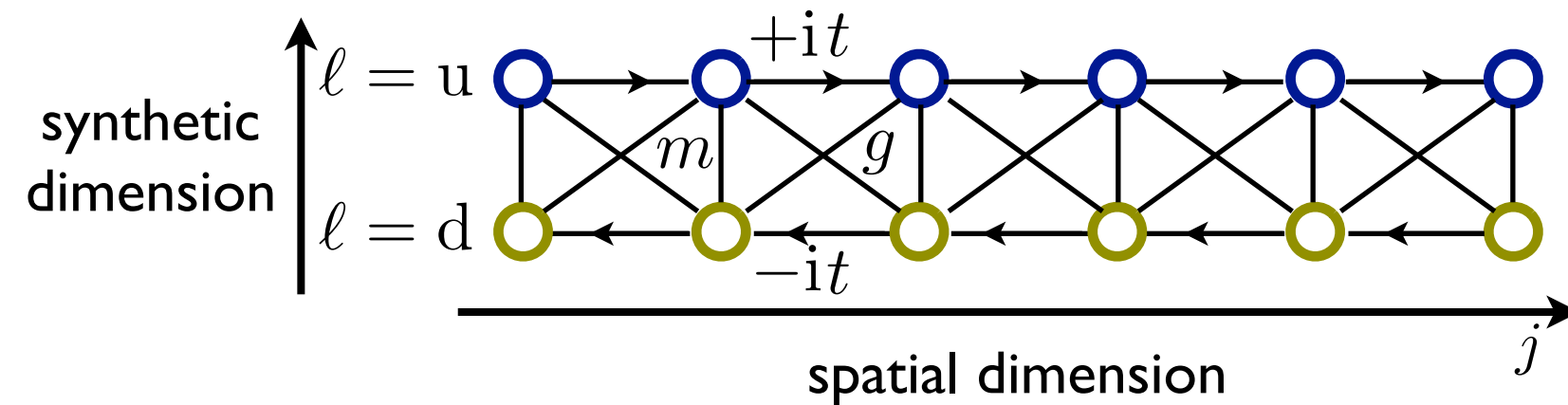


$m = g = t$  isolated Dirac cone  
[no Fermion doubling]

Creutz, Horváth, PRD **50**, 2297 (1994)

Creutz, PRL **83**, 2636 (1999)

# SPT - Trivial transition



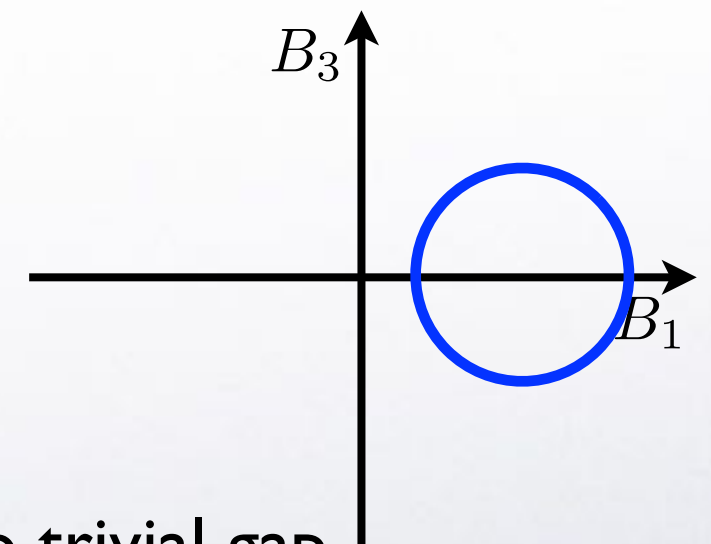
$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ [ + 2t \sin(ak) ] \sigma_3 + [ 2m - 2g \cos(ak) ] \sigma_1 \} c_k$$

$g = t$  flat bands, with topological character! (SPT)

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$$\sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k)$$



$m = g = t$  isolated Dirac cone  
[no Fermion doubling]  $m > g = t$  transition to trivial gap

$$\mathcal{W} = 0 \quad \varphi_{\text{Zak}} = 0$$

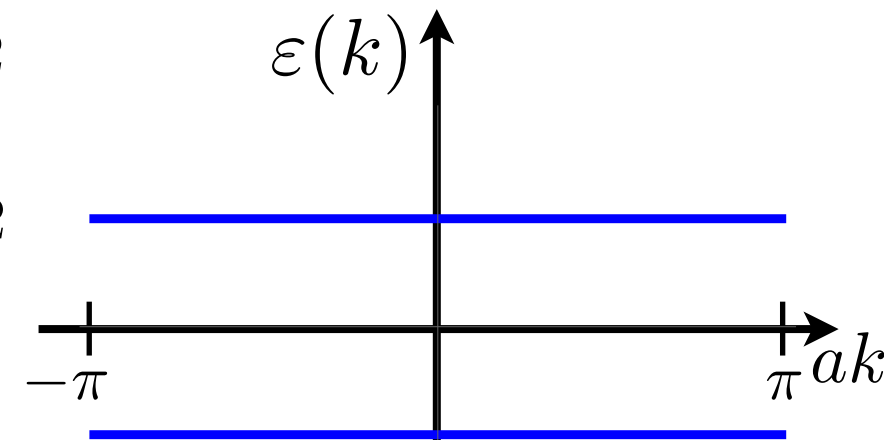
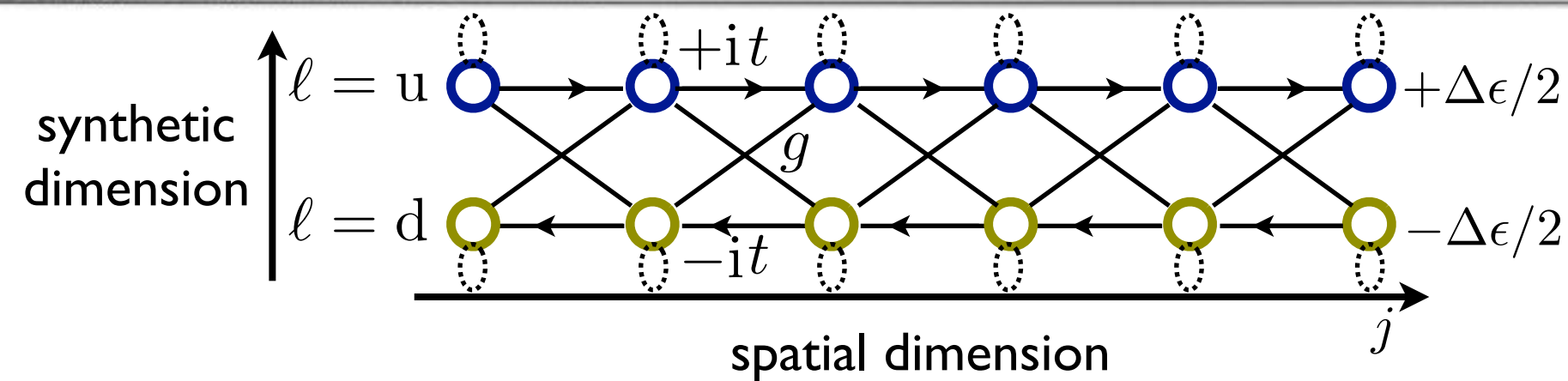
no more edge states

Creutz, Horváth, PRD **50**, 2297 (1994)

Creutz, PRL **83**, 2636 (1999)



# AIII SPT phase



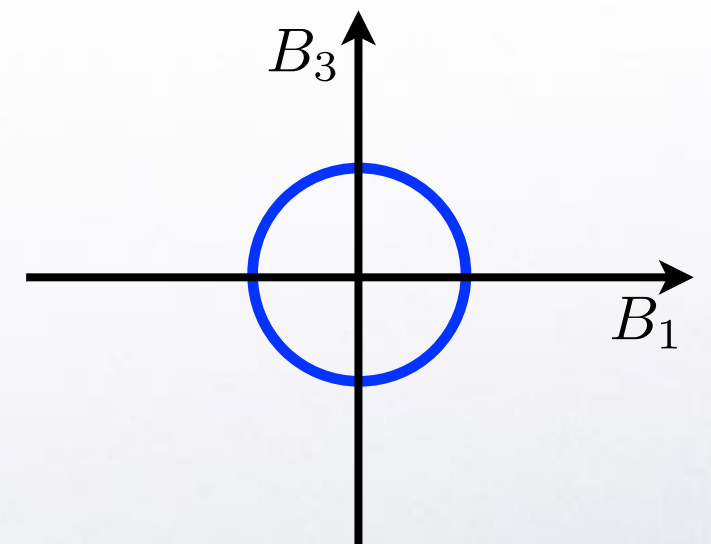
$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ [\Delta\epsilon/2 + 2t \sin(ak)] \sigma_3 + [-2g \cos(ak)] \sigma_1 \} c_k$$

$g = t$  flat bands, with topological character! (SPT)

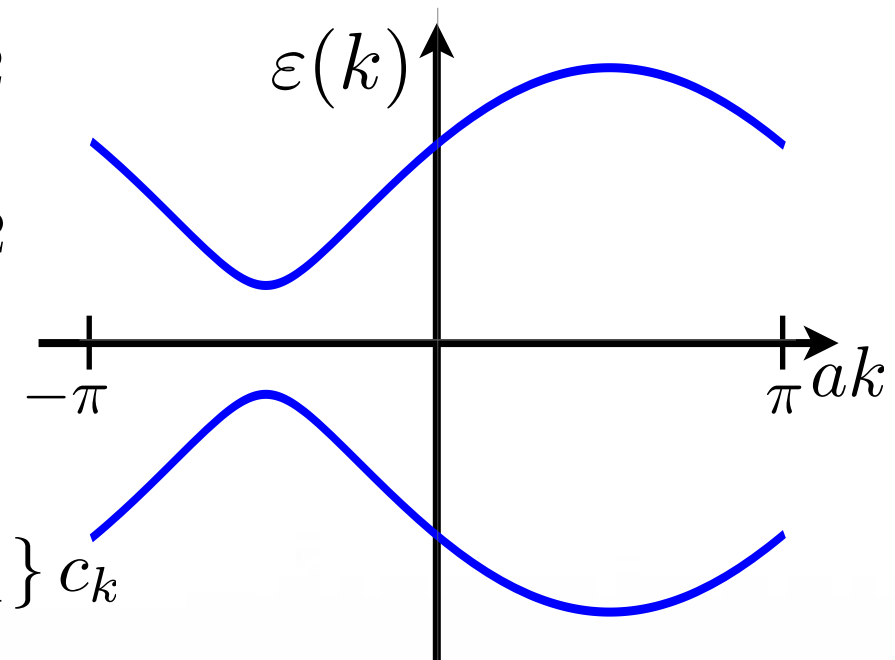
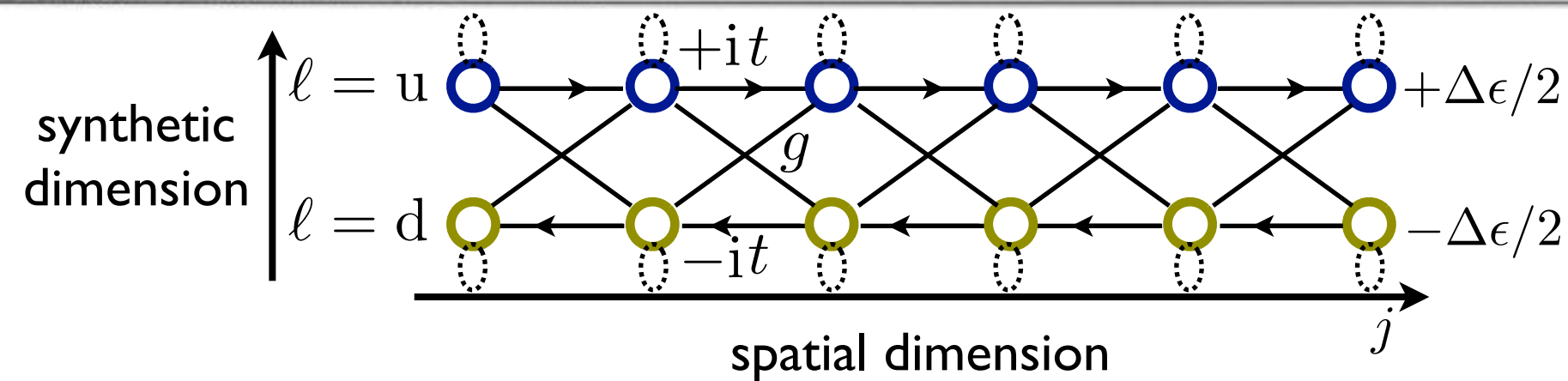
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$$\sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k)$$



# AIII SPT phase



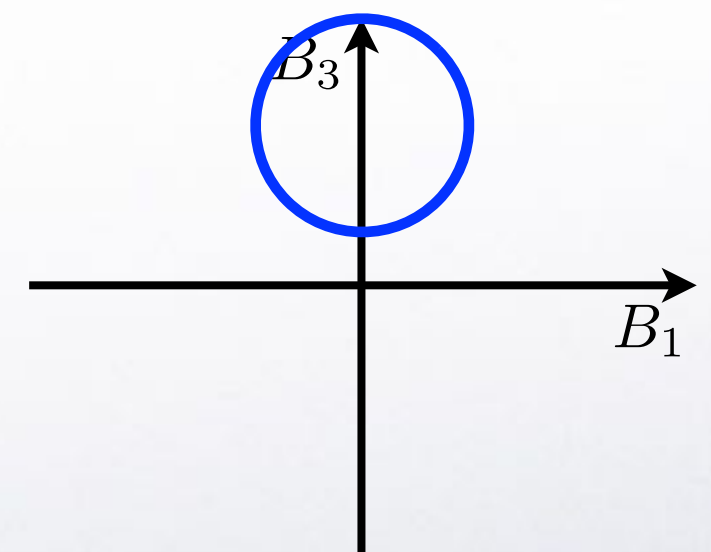
$$\mathcal{H}_0 = \sum_k c_k^\dagger \{ [\Delta\epsilon/2 + 2t \sin(ak)] \sigma_3 + [-2g \cos(ak)] \sigma_1 \} c_k$$

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$$\sigma_2 \mathcal{H}_0(k) \sigma_2 = -\mathcal{H}_0(k)$$



Zeeman  $\Delta\epsilon$  breaks T & C symmetry!

$$\nexists U_{T/C} \text{ s.t. } U_\alpha \mathcal{H}_0^*(-k) U_\alpha^\dagger = \pm \mathcal{H}_0(k)$$



**Class AIII** of AZ table

Altland, Zirnbauer, PRB **55**, 1142 (1997)

[another scheme  
Velasco, Paredes,

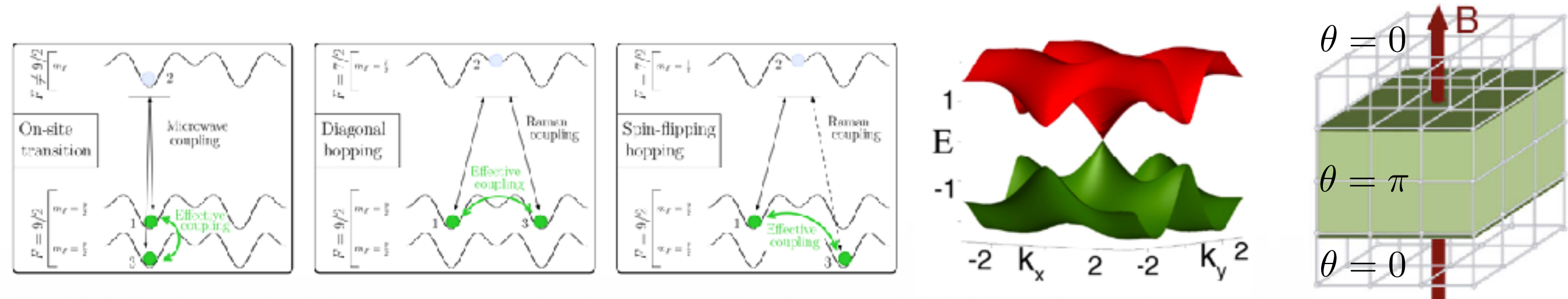
PRL **119**, 115301 (2017)]



# Experimental schemes

An optical-lattice-based quantum simulator for relativistic field theories and topological insulators

A. Bermudez, et al., PRL **105**, 190404 (2010);  
L. Mazza, et al., NJP **4** 015007 (2012);  
MR, PoS - SISSA **193**, 036 (2014)

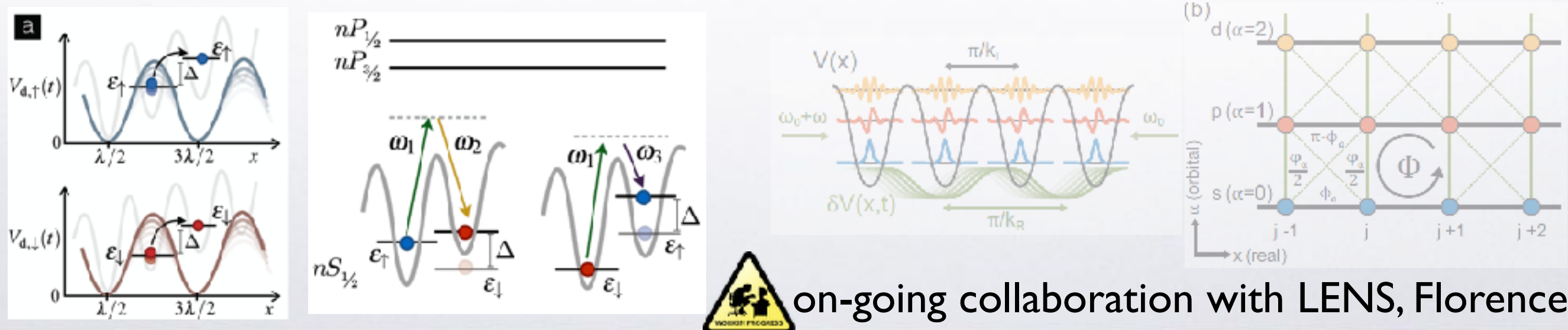


A shaken lattice proposal for the Creutz ladder

J. Jünemann, et al., PRX **7**, 031057 (2017)

Realization of a cross-linked chiral ladder by orbital-momentum coupling

J.H. Kang, J.H. Han, & Y. Shin, arXiv:1807.01444



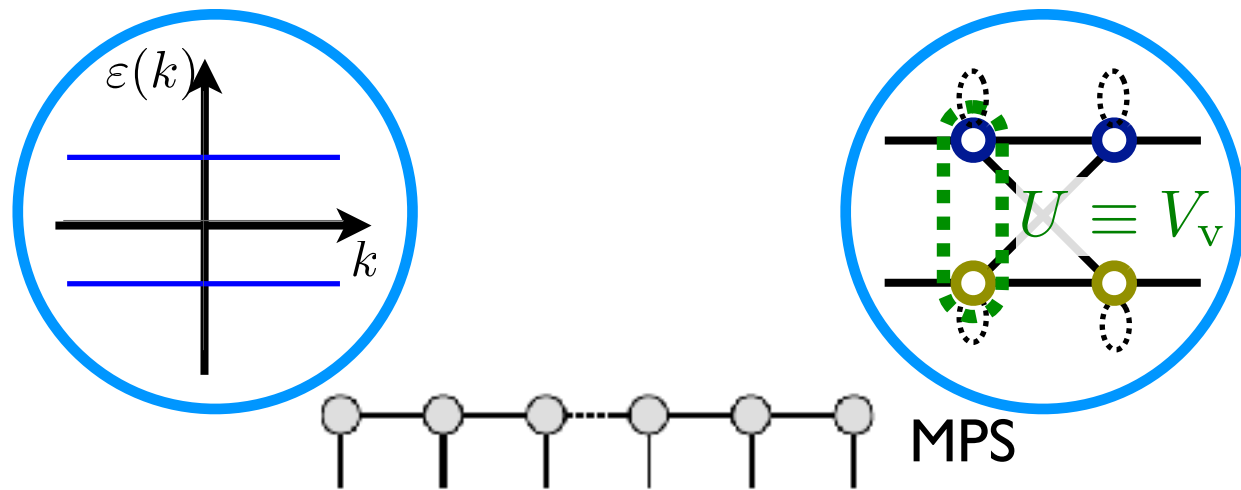
on-going collaboration with LENS, Florence

Entanglement in Strongly Correlated Systems  
Benasque 2019

[matteo.rizzi@fz-juelich.de](mailto:matteo.rizzi@fz-juelich.de)

Creutz Hubbard Ladder

# Interaction effects



half-filling

Topological Phase vs. Orbital Magnetism

Jünemann, et al., PRX **7**, 031057 (2017)

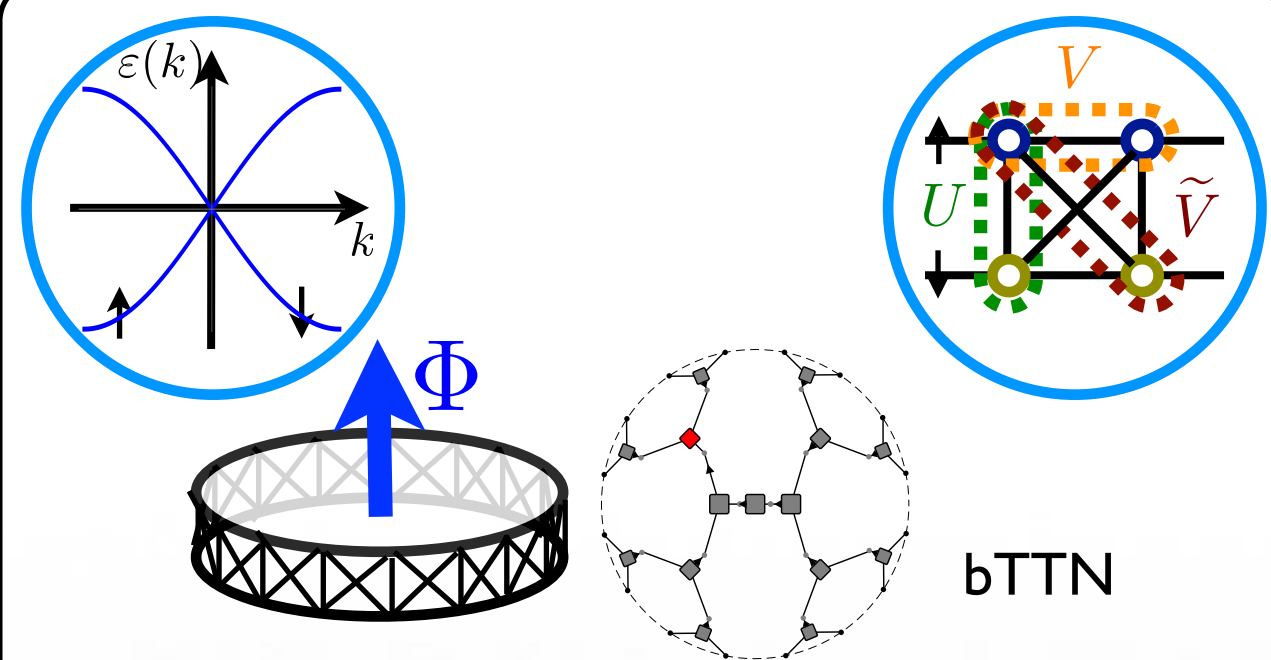
Gross-Neveu-Wilson model

Bermudez, et al., Ann. Phys. **339**, 149 (2018)

quarter-filling

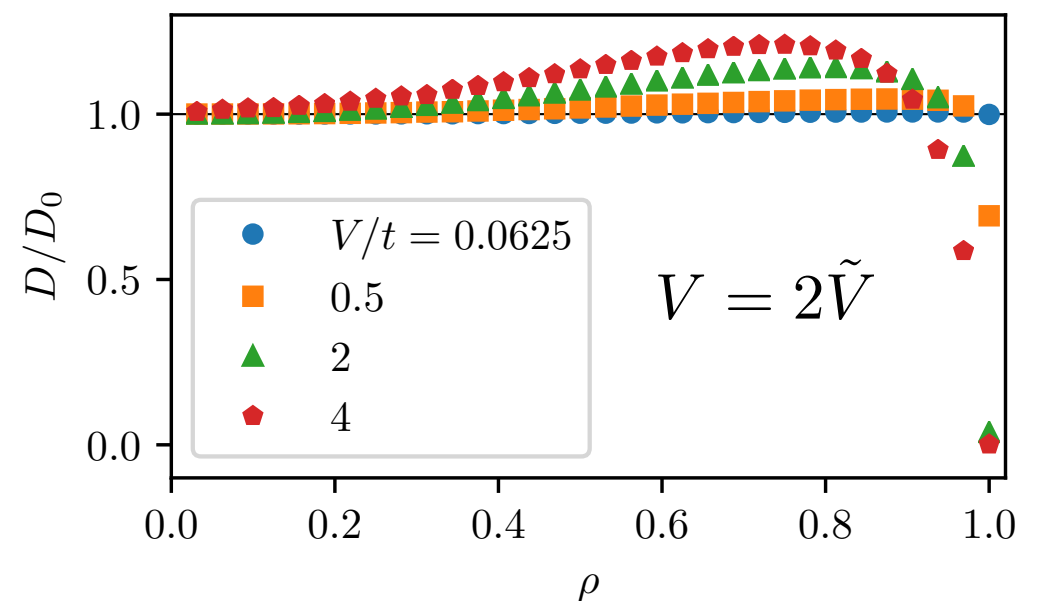
Fractional topological phases

Barbarino, et al., arXiv:1810.02337



Tunable Drude Weight of Dirac Fermions

Bischoff, et al. (MR), PRB **96**, 241112(R) (2017)

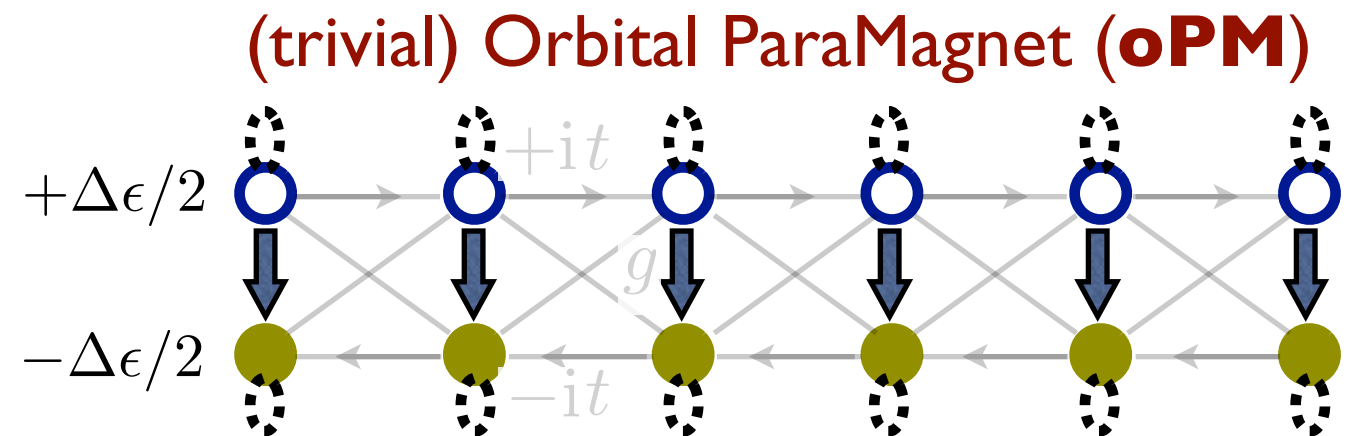
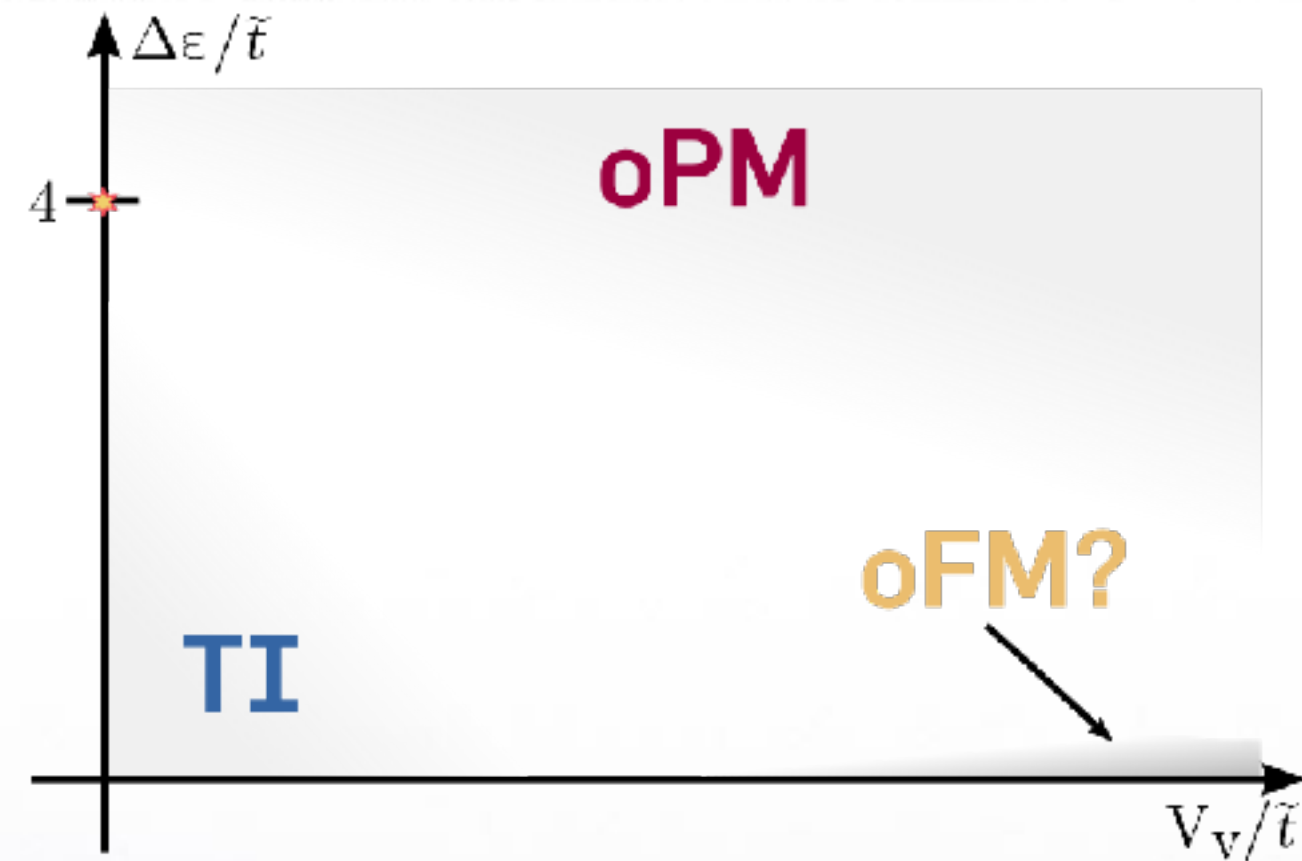




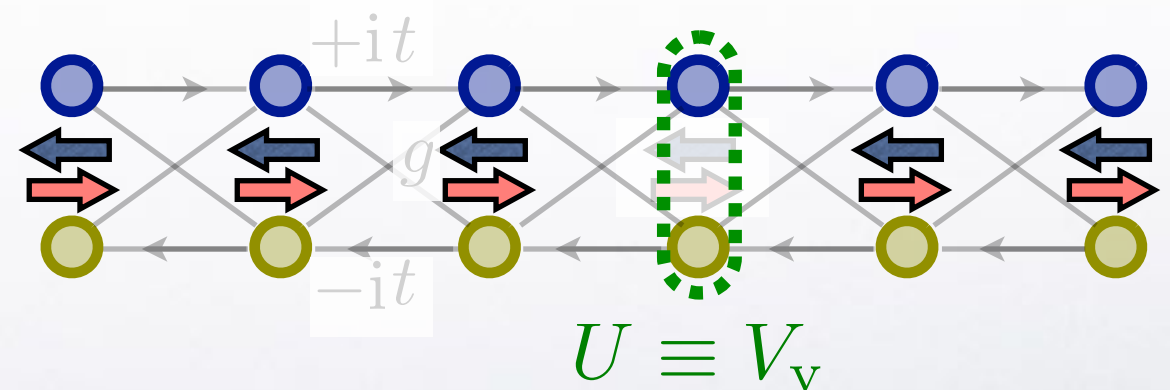
# OUTLINE

- The Creutz-Hubbard Ladder: general features
- Topology & Interactions
  - SPT vs orbital magnetism at half-filling
  - relation to high-energy models
  - interacting SPT phases at fractional filling
- Tuning the Drude Weight of Dirac fermions
- Other related works & plans

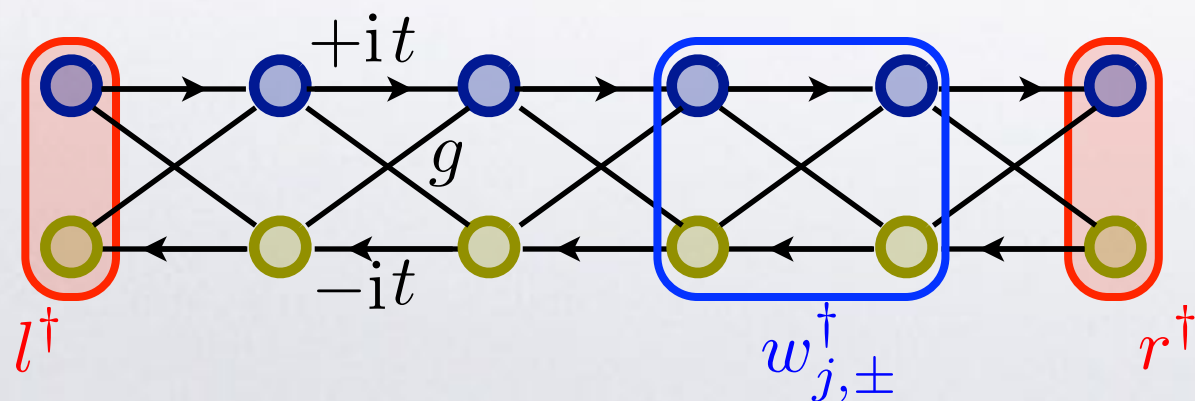
# Phase diagram (half-filling)



Orbital FerroMagnet (**oFM**)

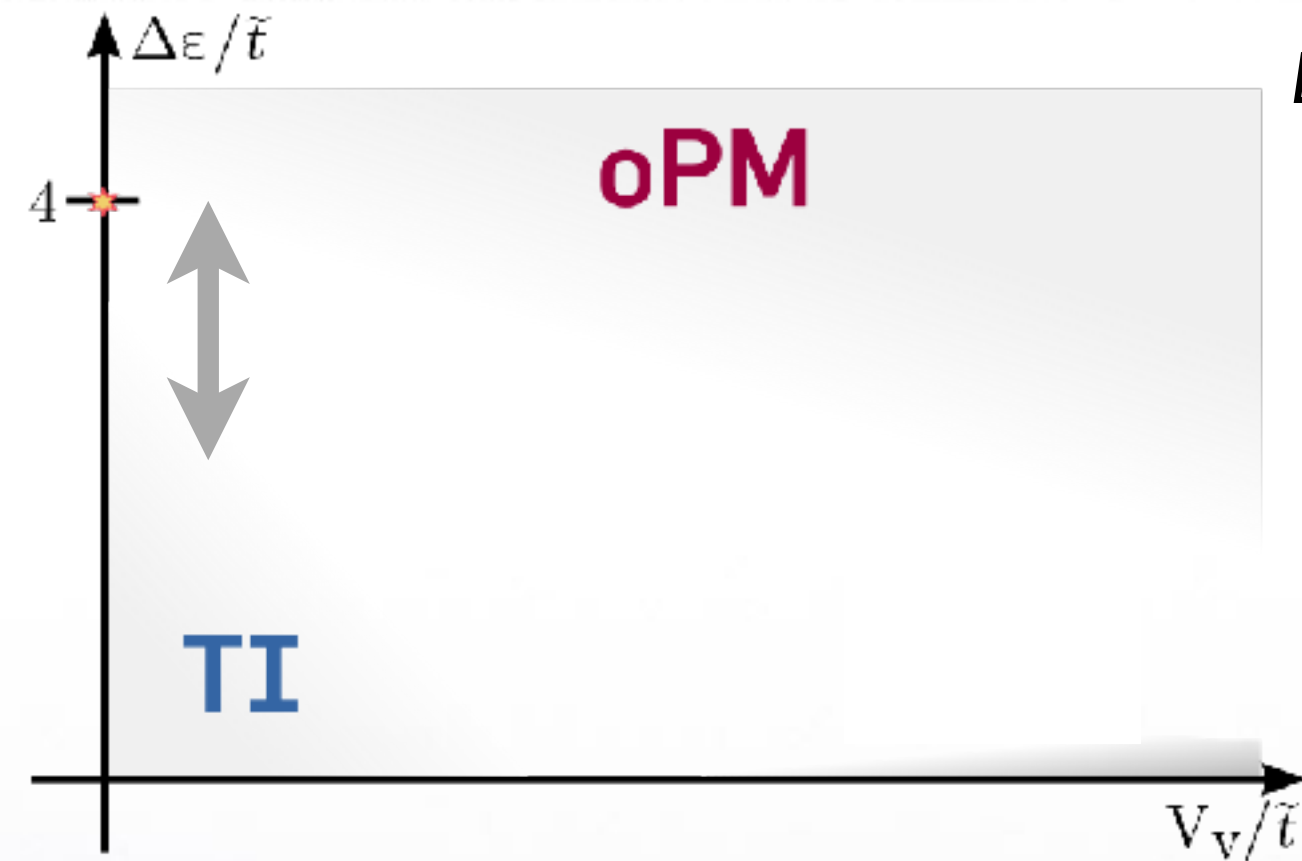


Flat-band SPT topological insulator (**TI**)





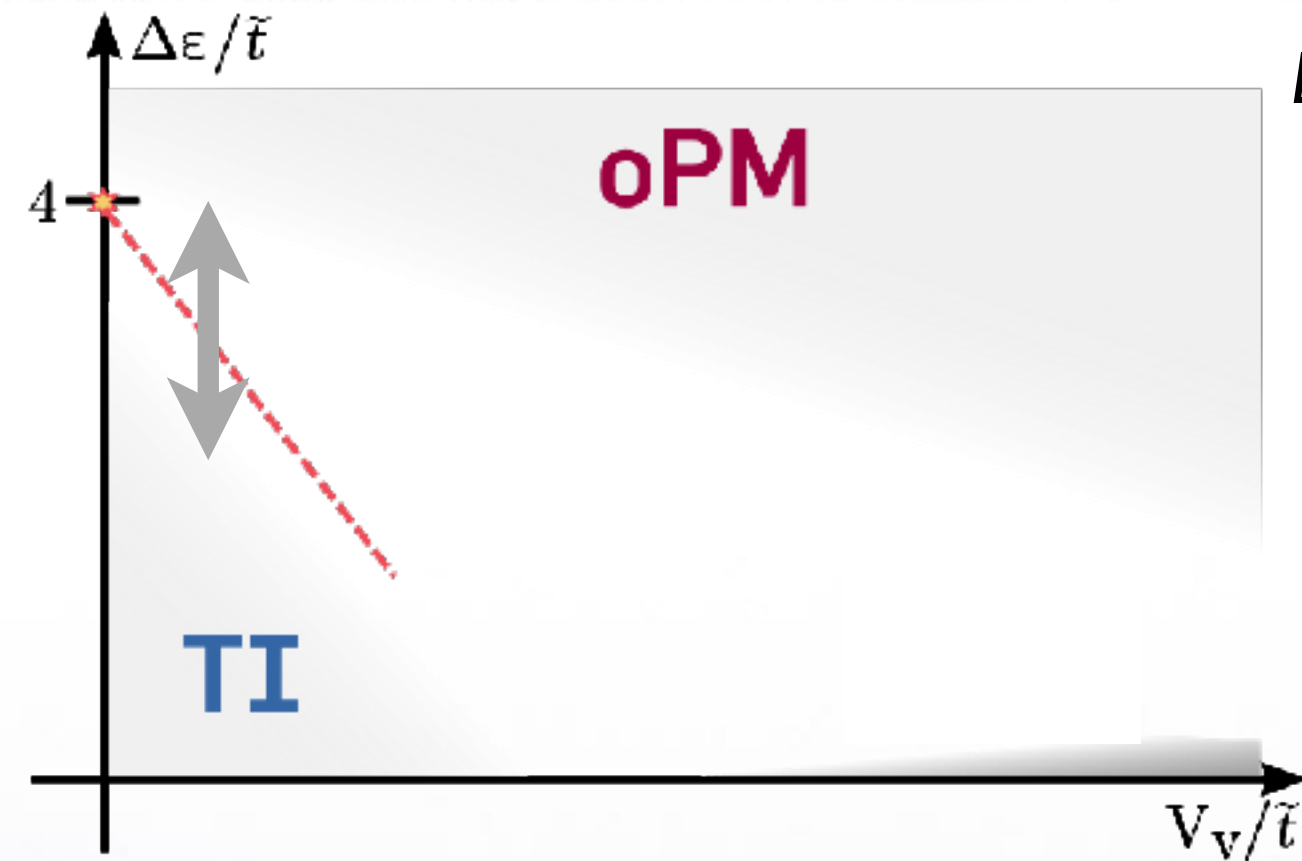
# Weak interactions



Exact (non-local) mapping of non-interacting  $H$  onto **two Ising models**

$$H_{\pi C} = \sum_j \sum_{n=1,2} \left( -\tilde{t} \tau_{j,n}^x \tau_{j+1,n}^x + \frac{\Delta\epsilon}{4} \tau_{j,n}^z \right)$$

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$$V_{\text{Hubb}} = -\frac{V_v}{4} \sum_j \tau_{j,1}^z \tau_{j,2}^z + \text{const.}$$

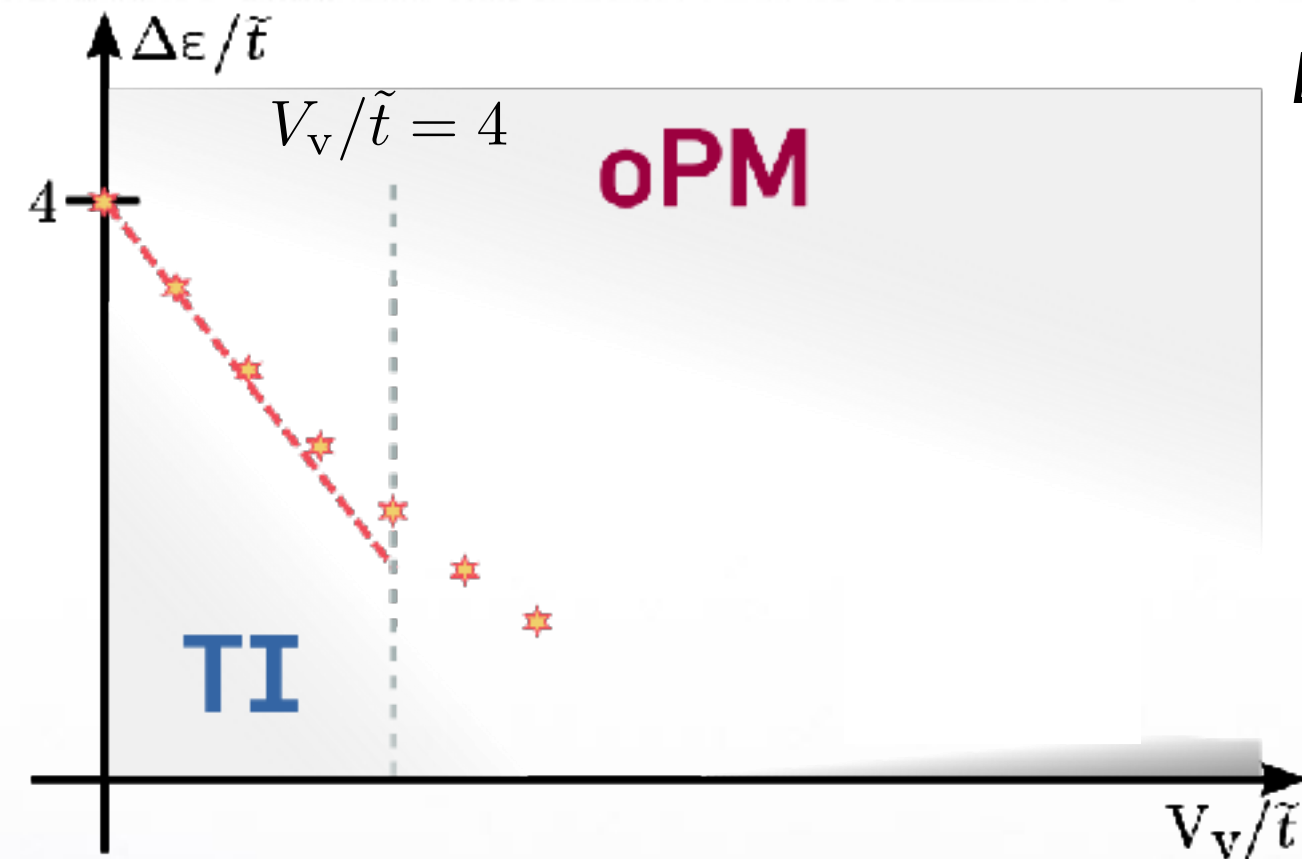
$$\frac{\Delta\epsilon}{\tilde{t}} = 4 - \frac{2}{\pi} \frac{V_v}{\tilde{t}} + \mathcal{O}\left(\frac{V_v^2}{\tilde{t}^2}\right)$$

For a more complete **RG treatment**  
(possibly extensible to more than 1D)

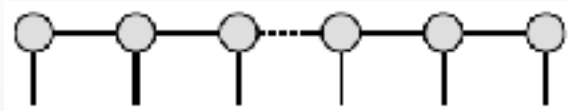
Tirrito, et al. (MR), arXiv:1812.05973 - PRB in press



# Weak interactions



MPS calculations  $N = 8 \dots 256$



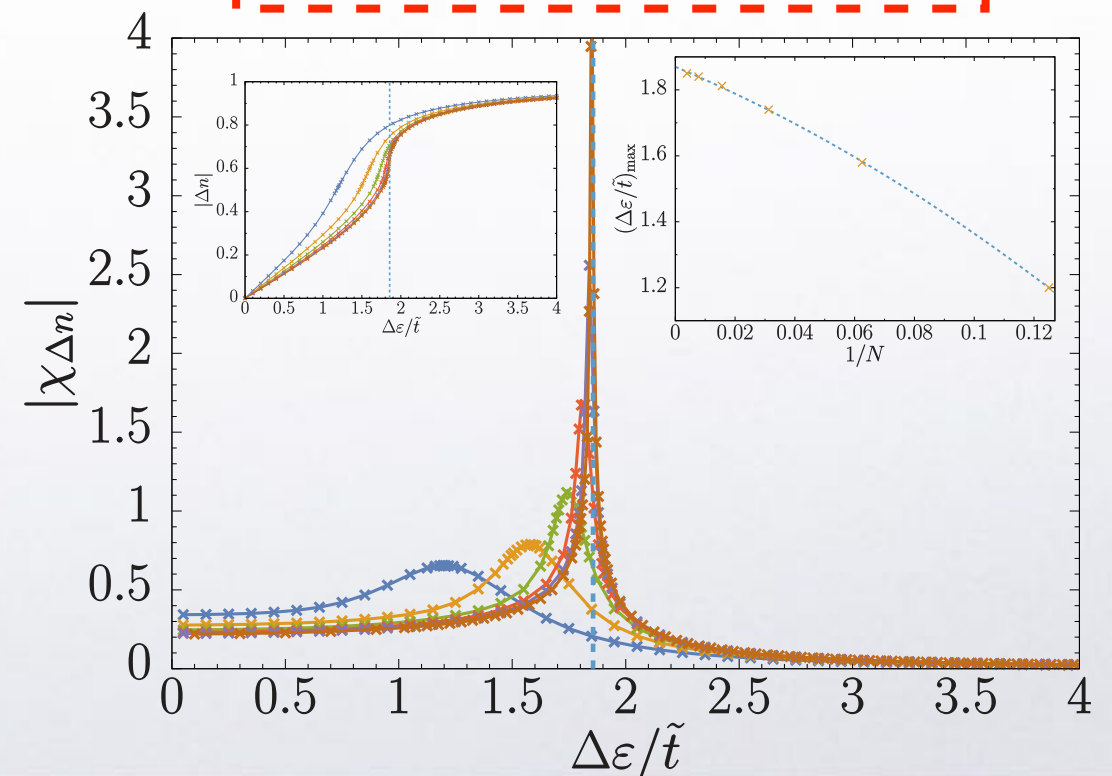
$$T^z \equiv \Delta n \equiv n_u - n_d = \frac{1}{2} \langle \tau_1^z + \tau_2^z \rangle$$

Exact (non-local) mapping of non-interacting H onto **two Ising models**

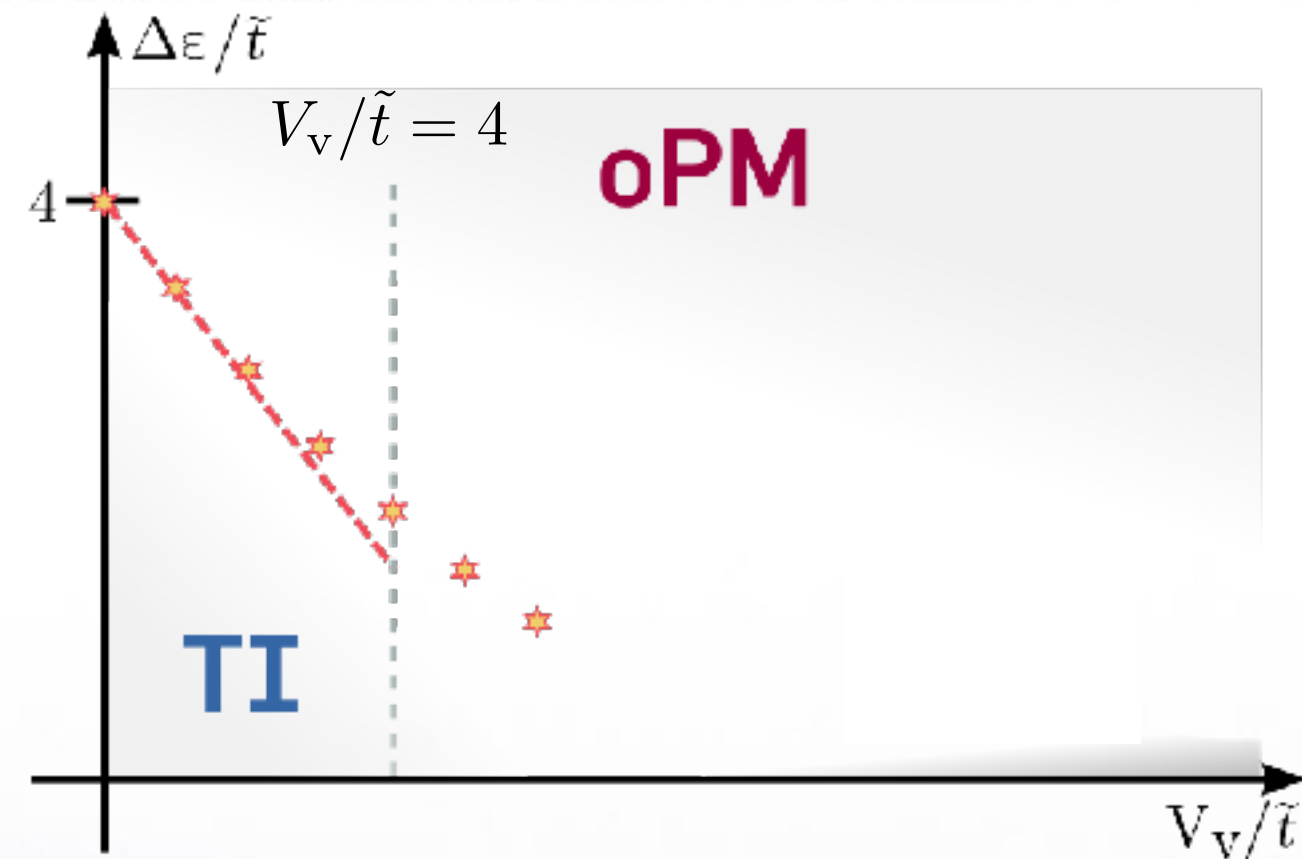
$$H_{\pi C} = \sum_j \sum_{n=1,2} \left( -\tilde{t} \tau_{j,n}^x \tau_{j+1,n}^x + \frac{\Delta \epsilon}{4} \tau_{j,n}^z \right)$$

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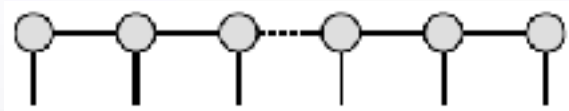
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# Weak interactions



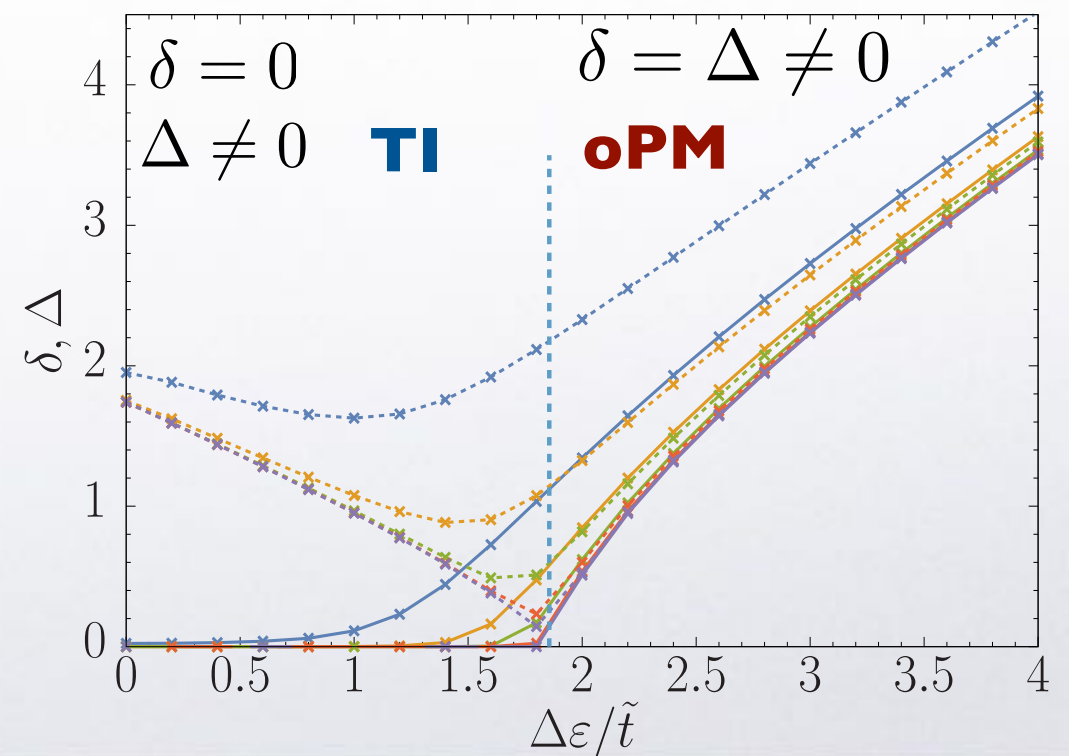
MPS calculations  $N = 8 \dots 256$



**zero-energy edge modes**  
vs. compressibility gap

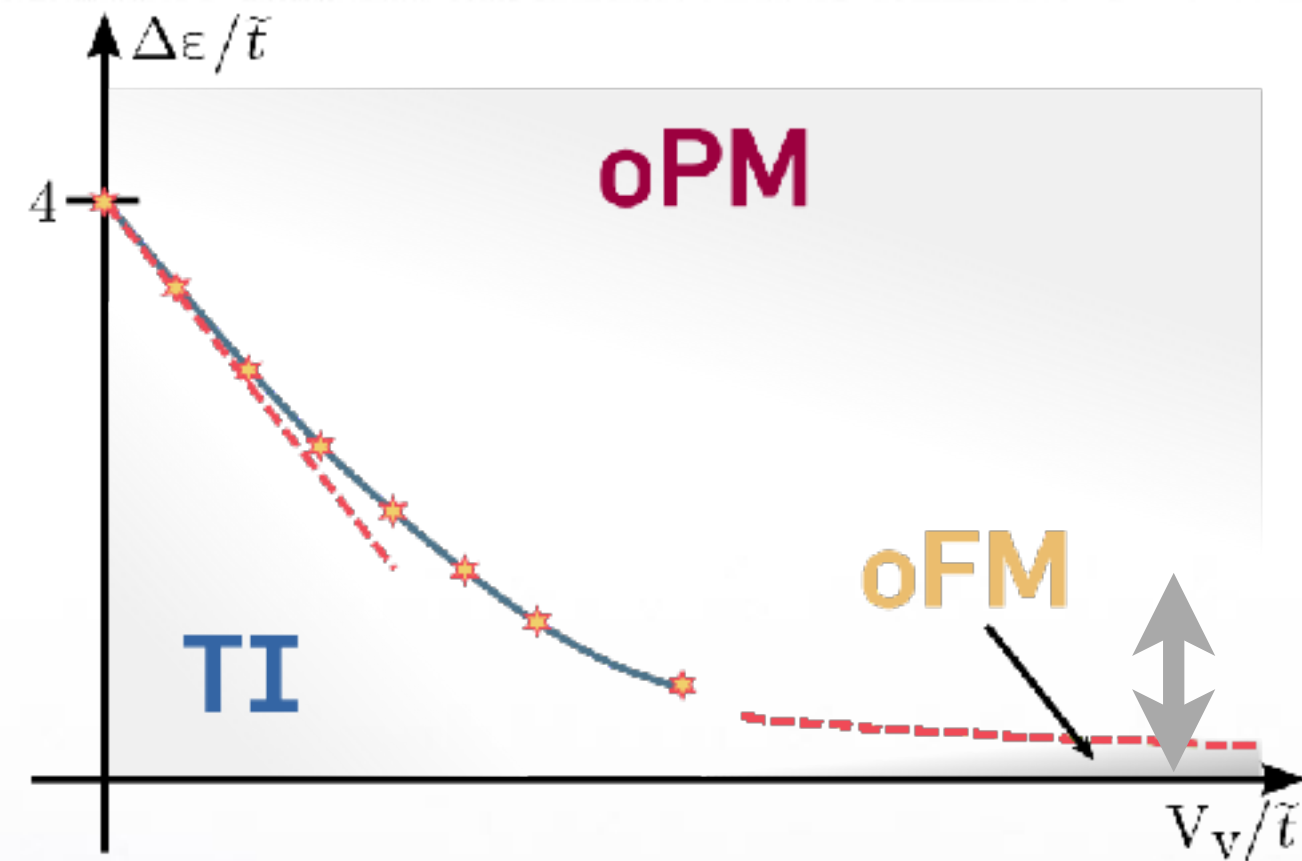
$$\delta = \lim_{N \rightarrow \infty} [E(N+1) + E(N-1) - 2E(N)]$$

$$\Delta = \lim_{N \rightarrow \infty} \frac{1}{2} [E(N+2) + E(N-2) - 2E(N)]$$





# Strong interactions



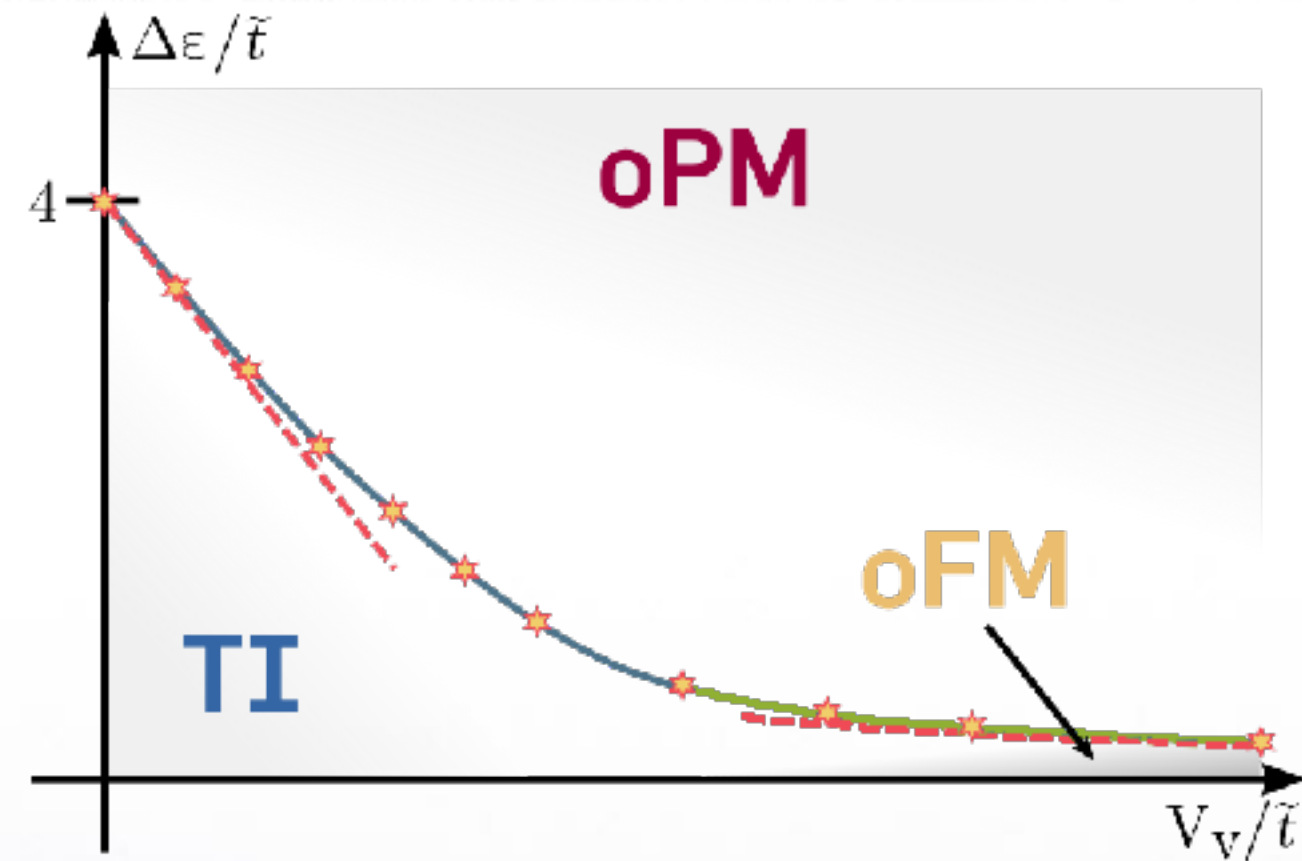
2<sup>nd</sup>-order *perturbation theory* [singly occ. rungs]  
gives **single Ising model**

$$\mathcal{P}_r H_{\pi\text{CH}} \mathcal{P}_r = \frac{1}{4} J N + J \sum_i T_i^y T_{i+1}^y + \Delta\epsilon \sum_i T_i^z$$

$$J = -8\tilde{t}^2/V_v \quad T_j^\alpha = \frac{1}{2} c_j^\dagger \sigma^\alpha c_j$$

critical line  $\frac{2\Delta\epsilon}{|J|} = 1 \iff \frac{\Delta\epsilon}{\tilde{t}} = \frac{4\tilde{t}}{V_v}$

# Strong interactions



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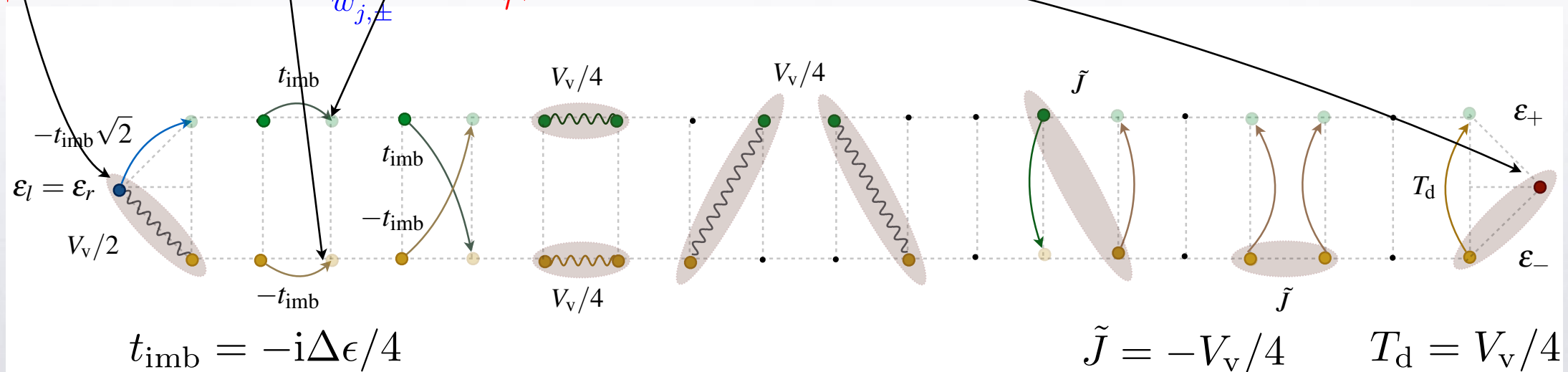
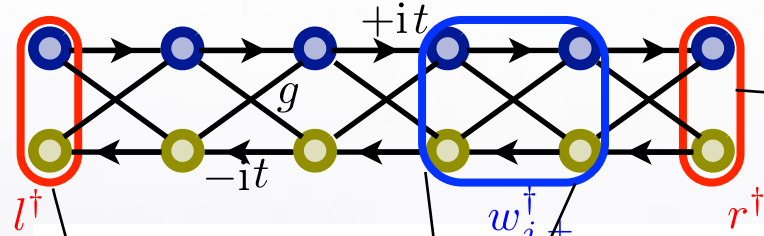
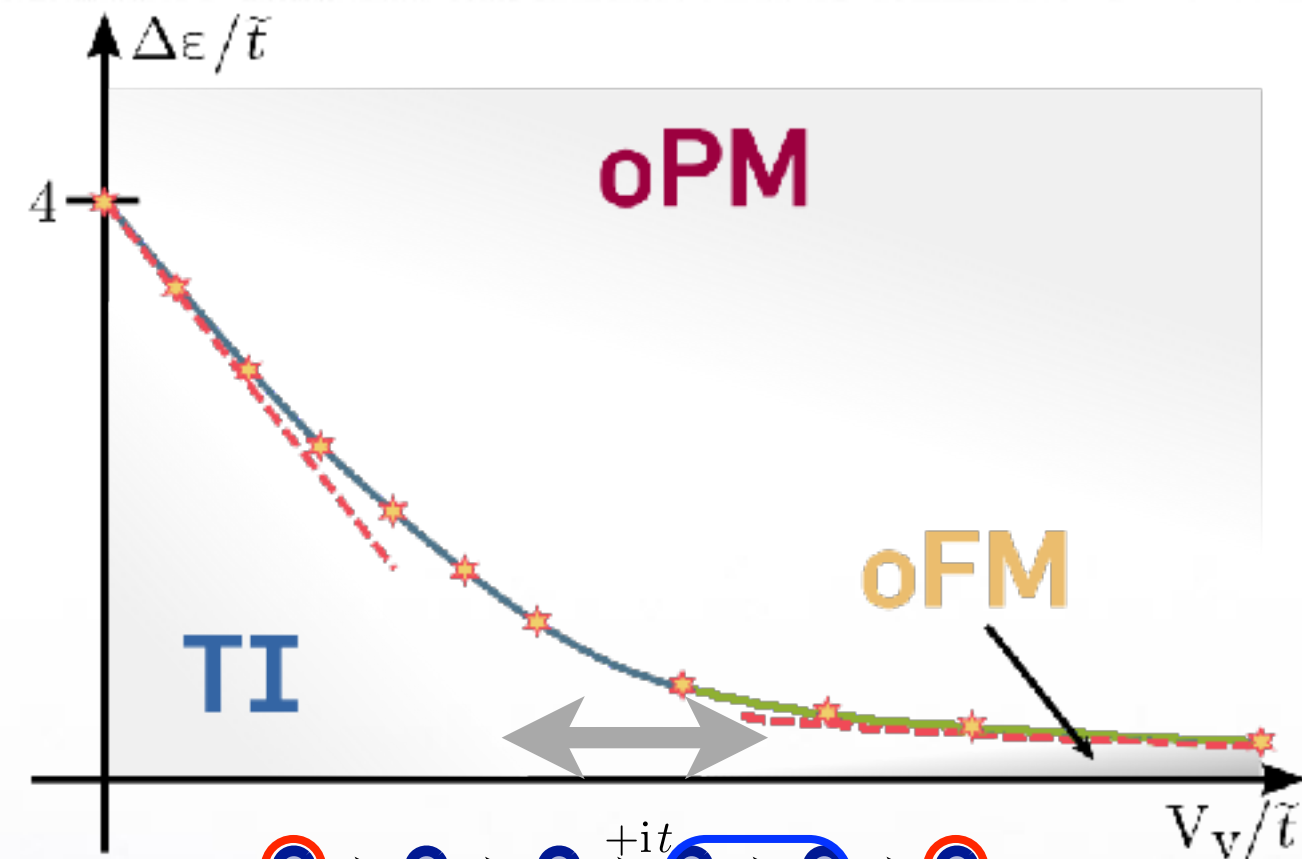
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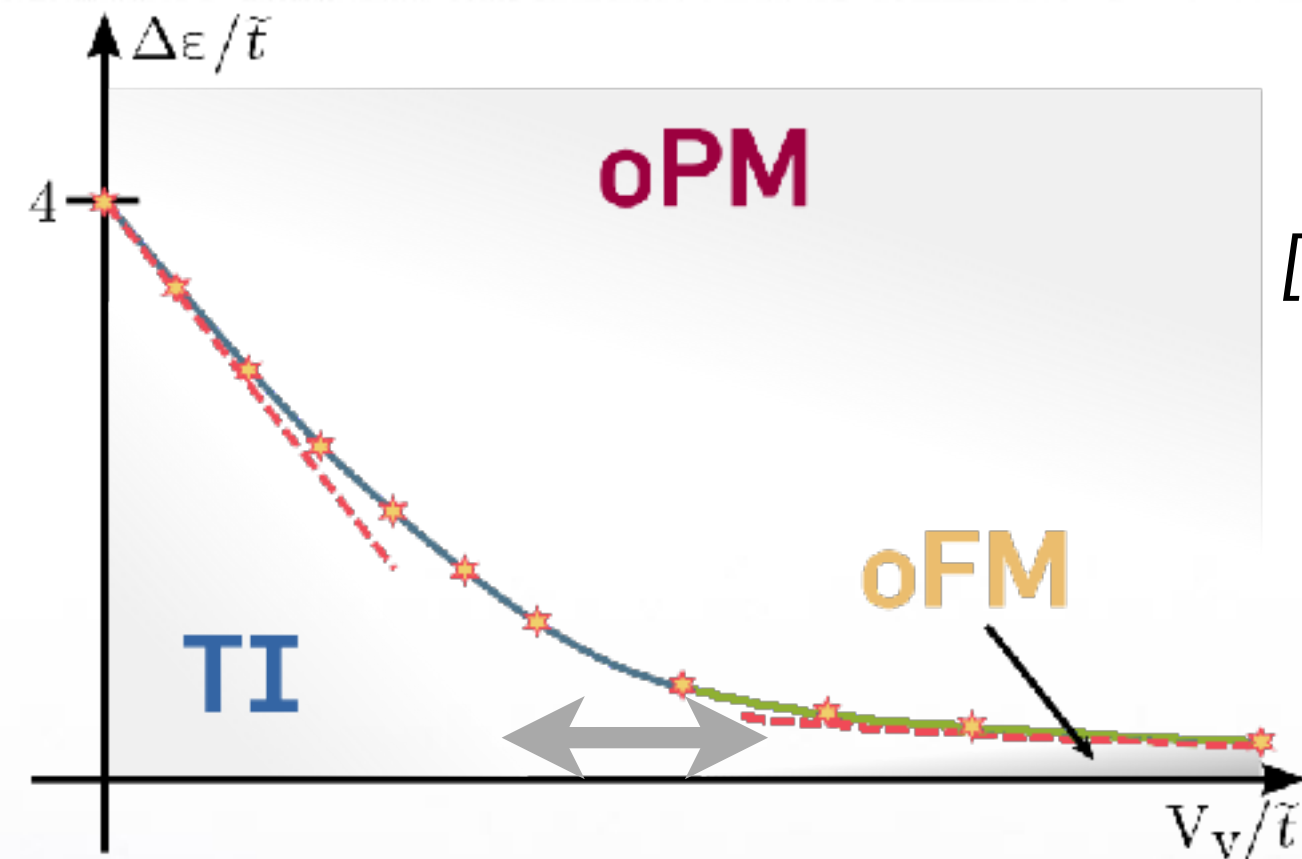


# Weak imbalance

on the basis of Aharonov-Bohm cages

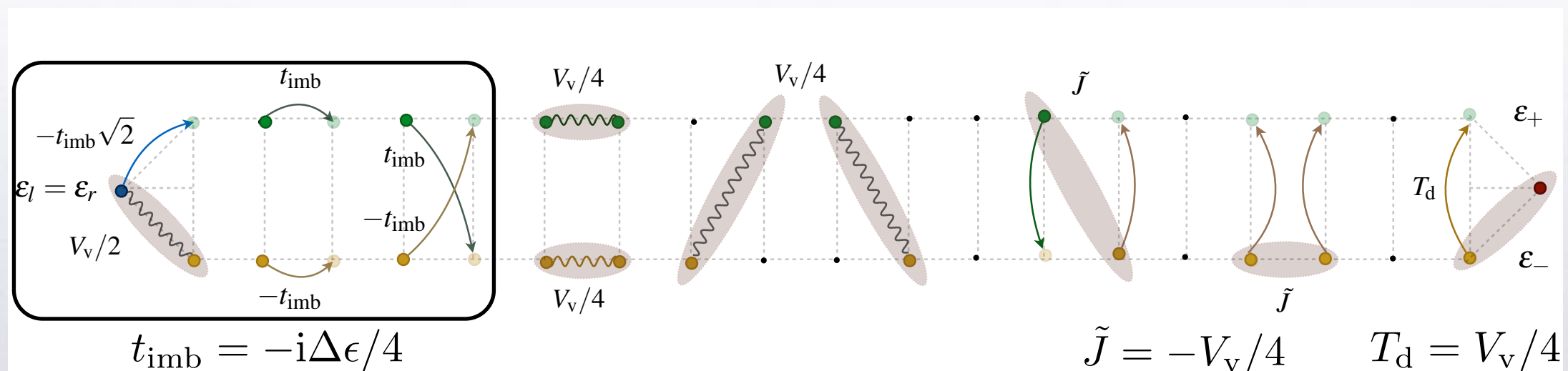


# Weak imbalance



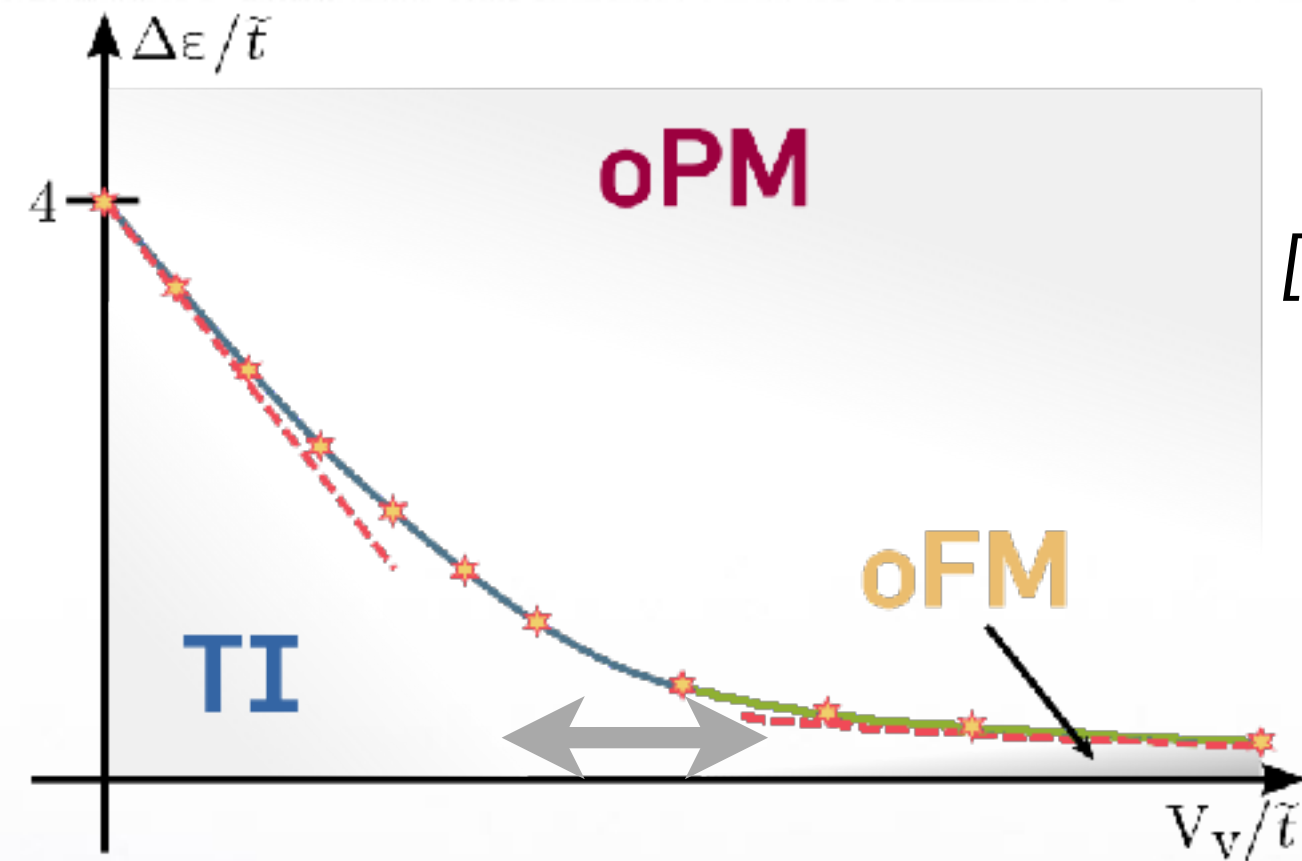
on the basis of Aharanov-Bohm cages  
**exotic Hubbard model**  
*[without dipolar atoms or other “strange” schemes]*

- imbalance induced hopping
- n.n. interactions
- pair tunnelling
- density-assisted tunnelling





# Weak imbalance

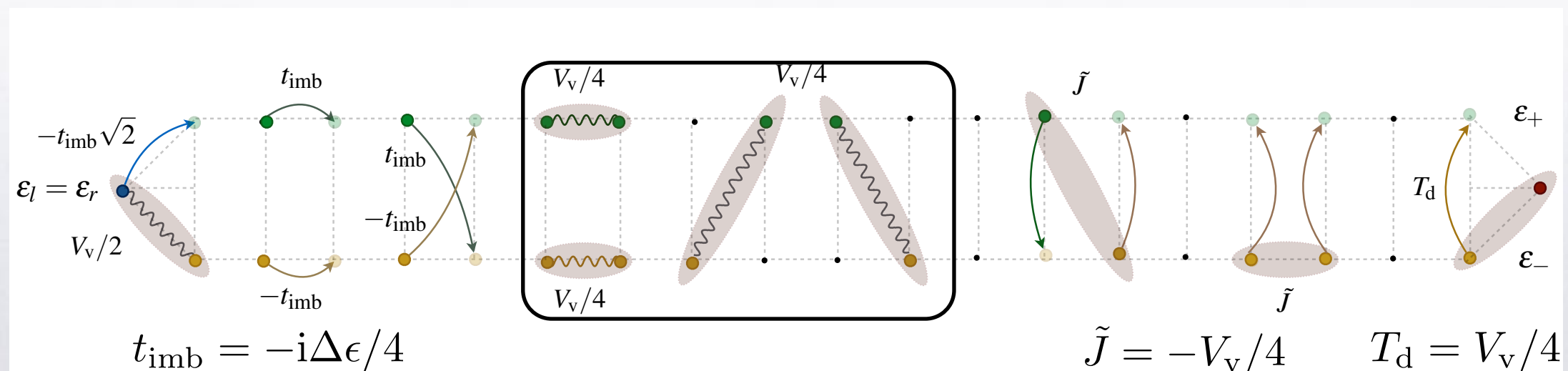


on the basis of Aharanov-Bohm cages

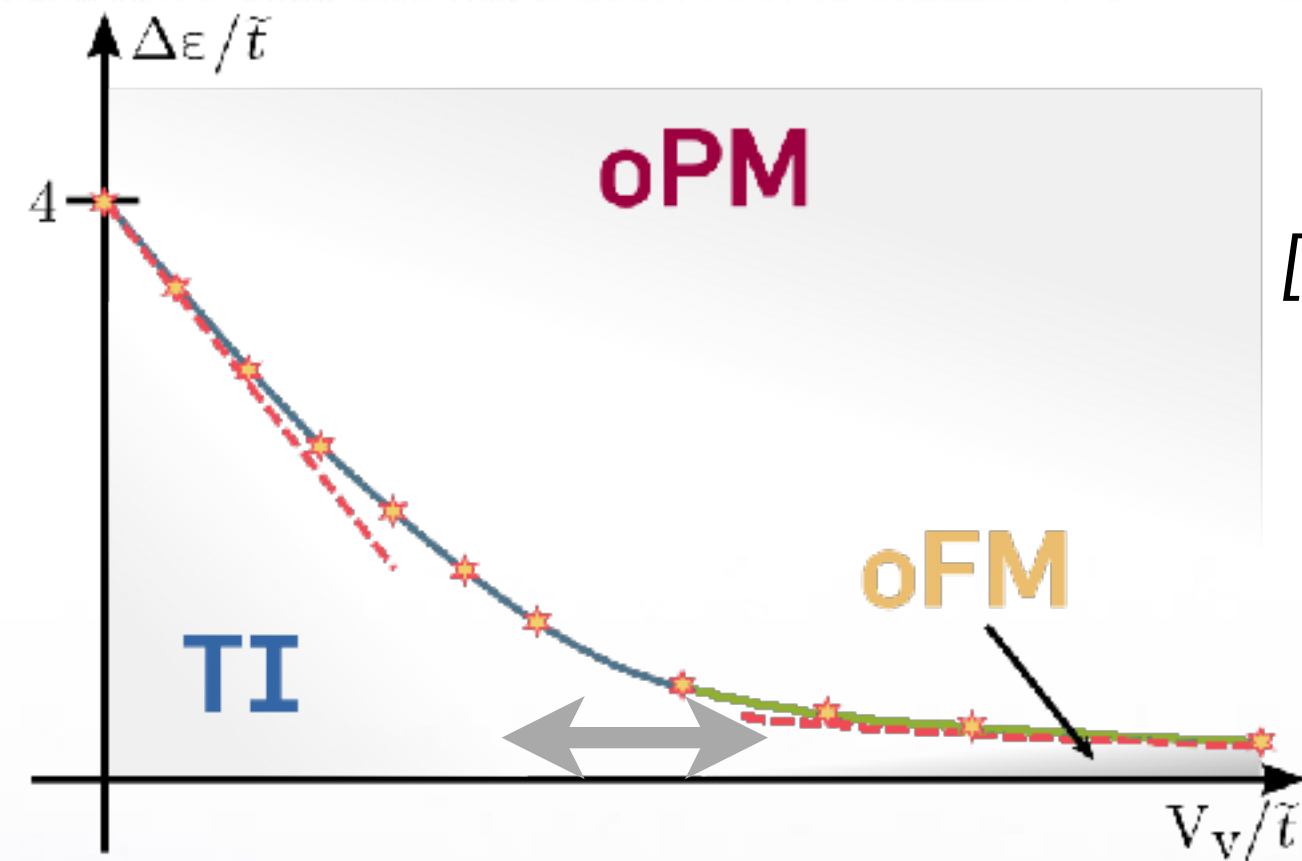
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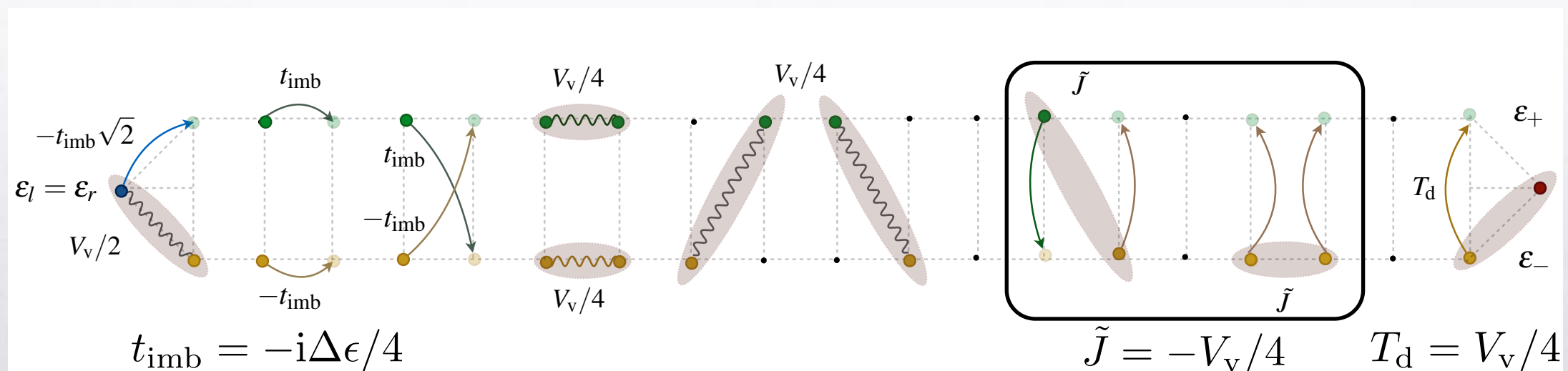


on the basis of Aharanov-Bohm cages

## exotic Hubbard model

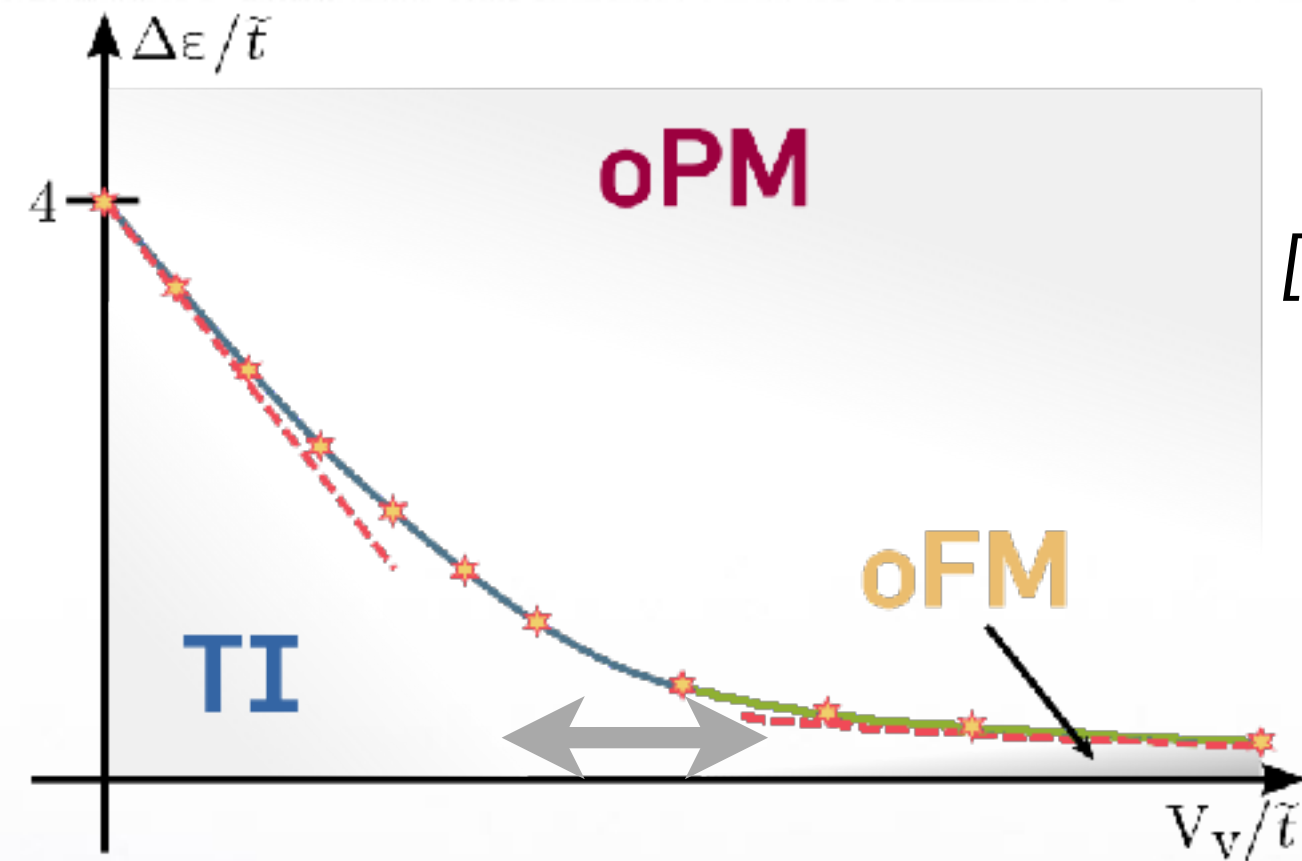
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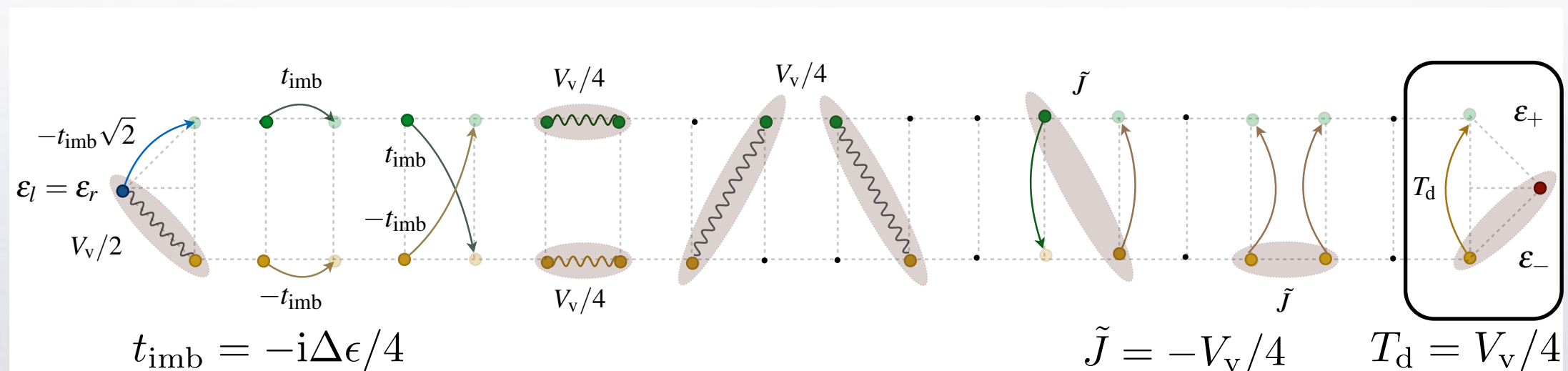


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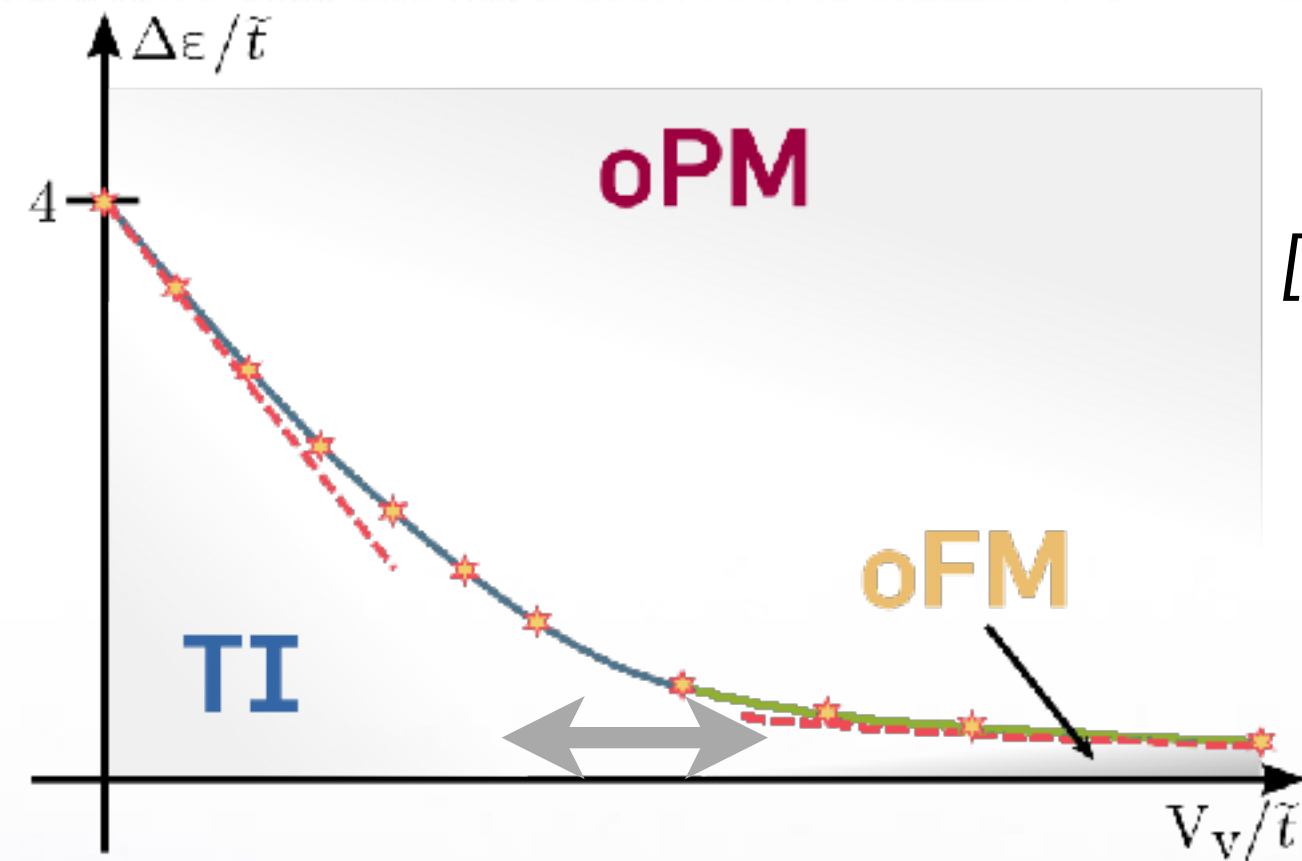
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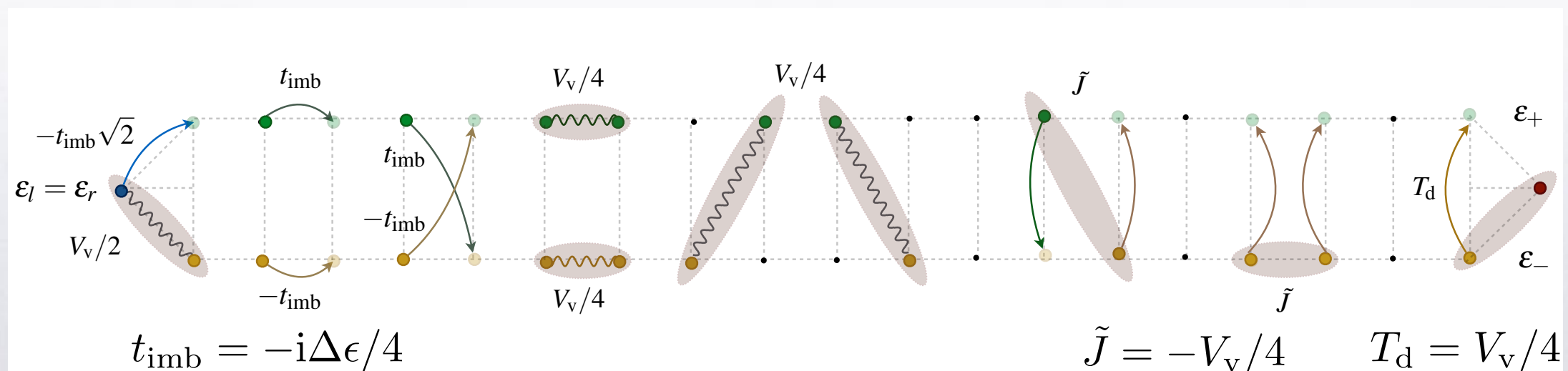


on the basis of Aharanov-Bohm cages

## exotic Hubbard model

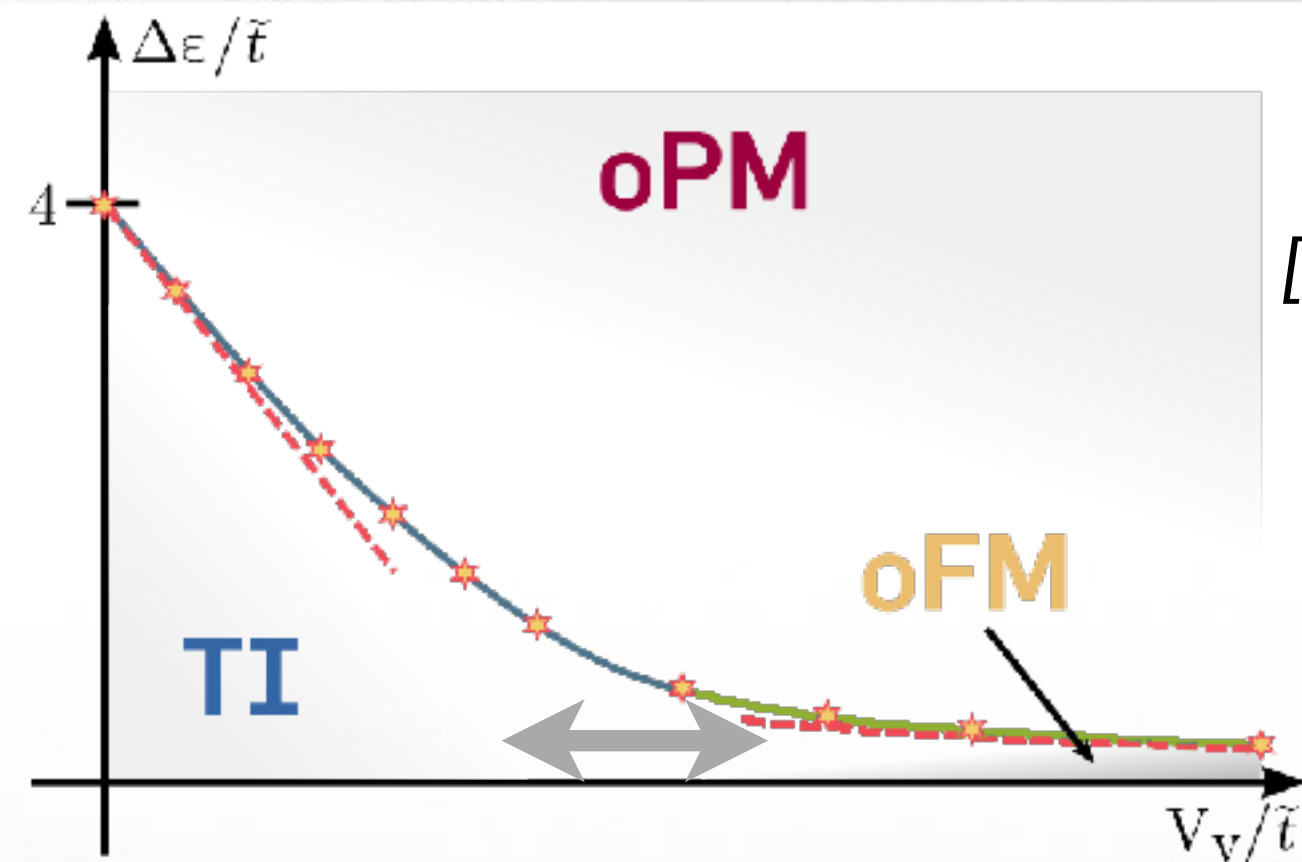
[without dipolar atoms or other “strange” schemes]

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# Weak imbalance



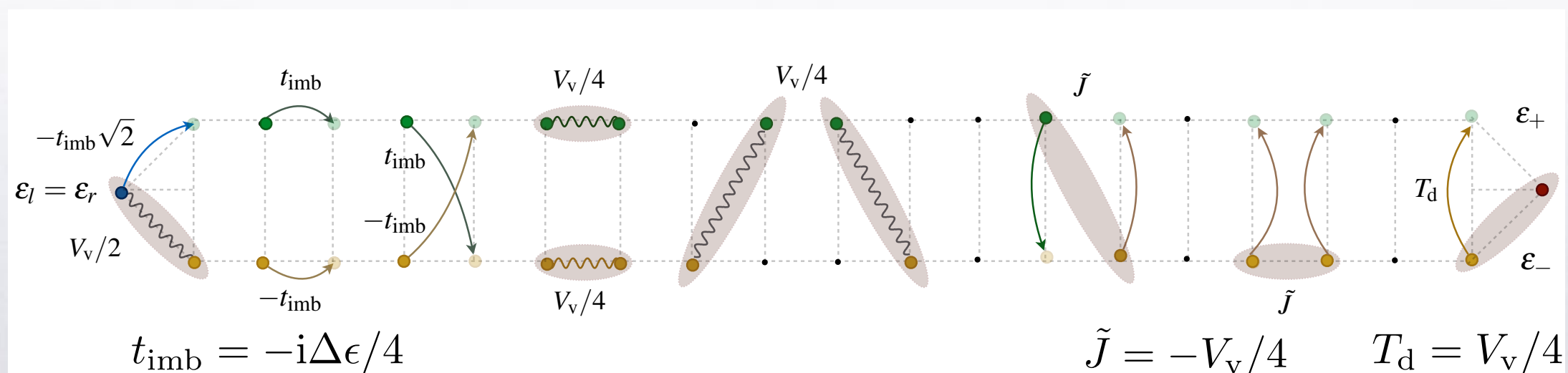
on the basis of Aharonov-Bohm cages

## exotic Hubbard model

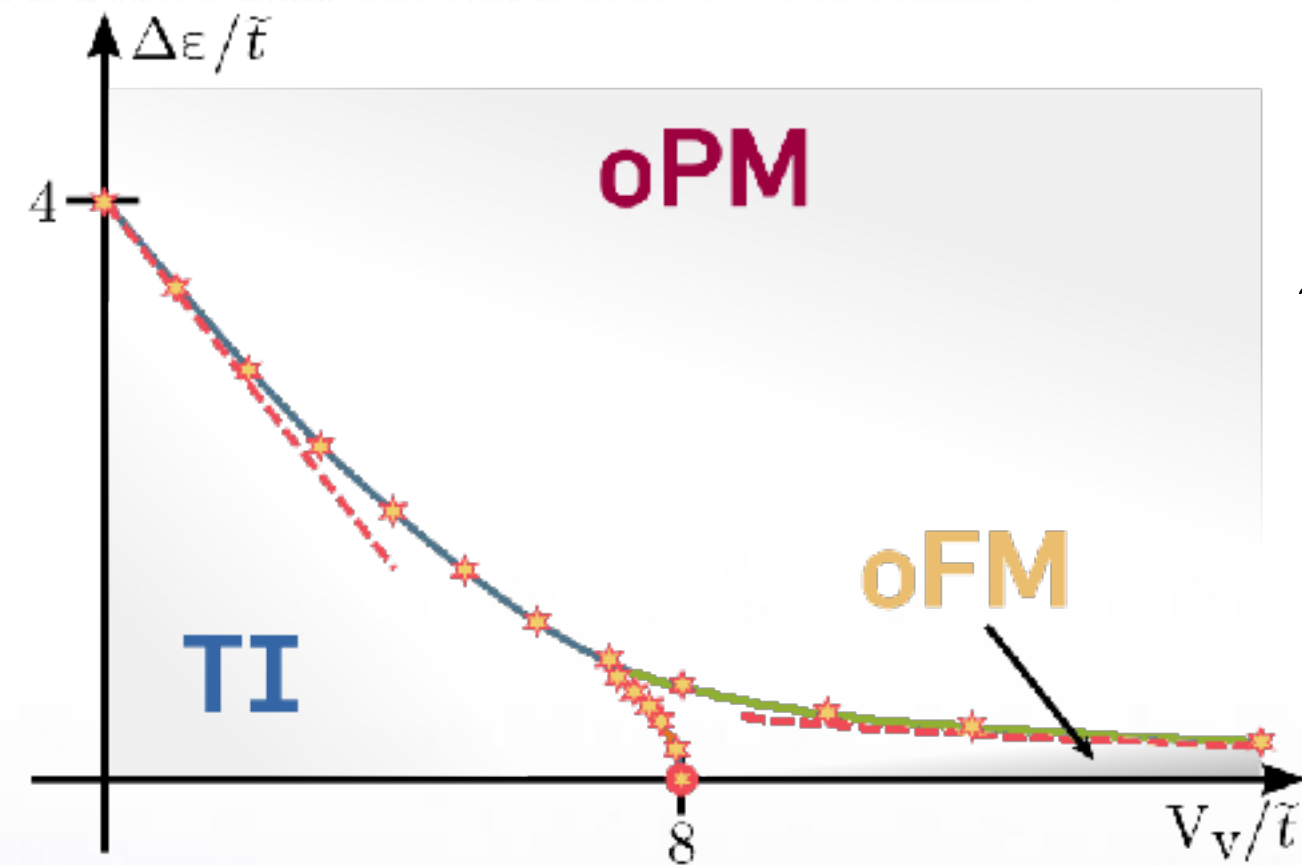
[without dipolar atoms or other “strange” schemes]

- imbalance induced hopping
- n.n. interactions
- pair tunnelling
- density-assisted tunnelling

bulk-mediated [*à la* Fano-Anderson]  
**edge-edge interactions**



# Weak imbalance



rewriting on singly-occupied AB cages  
gives **single Ising model**

$$\mathcal{P}_c H_{\text{bulk}} \mathcal{P}_c = \frac{V_v}{4} N + 4\tilde{t} \sum_i \tilde{T}_i^z + 4\tilde{J} \sum_i \tilde{T}_{i-1}^x \tilde{T}_i^x$$

$$\tilde{J} = -V_v/4$$

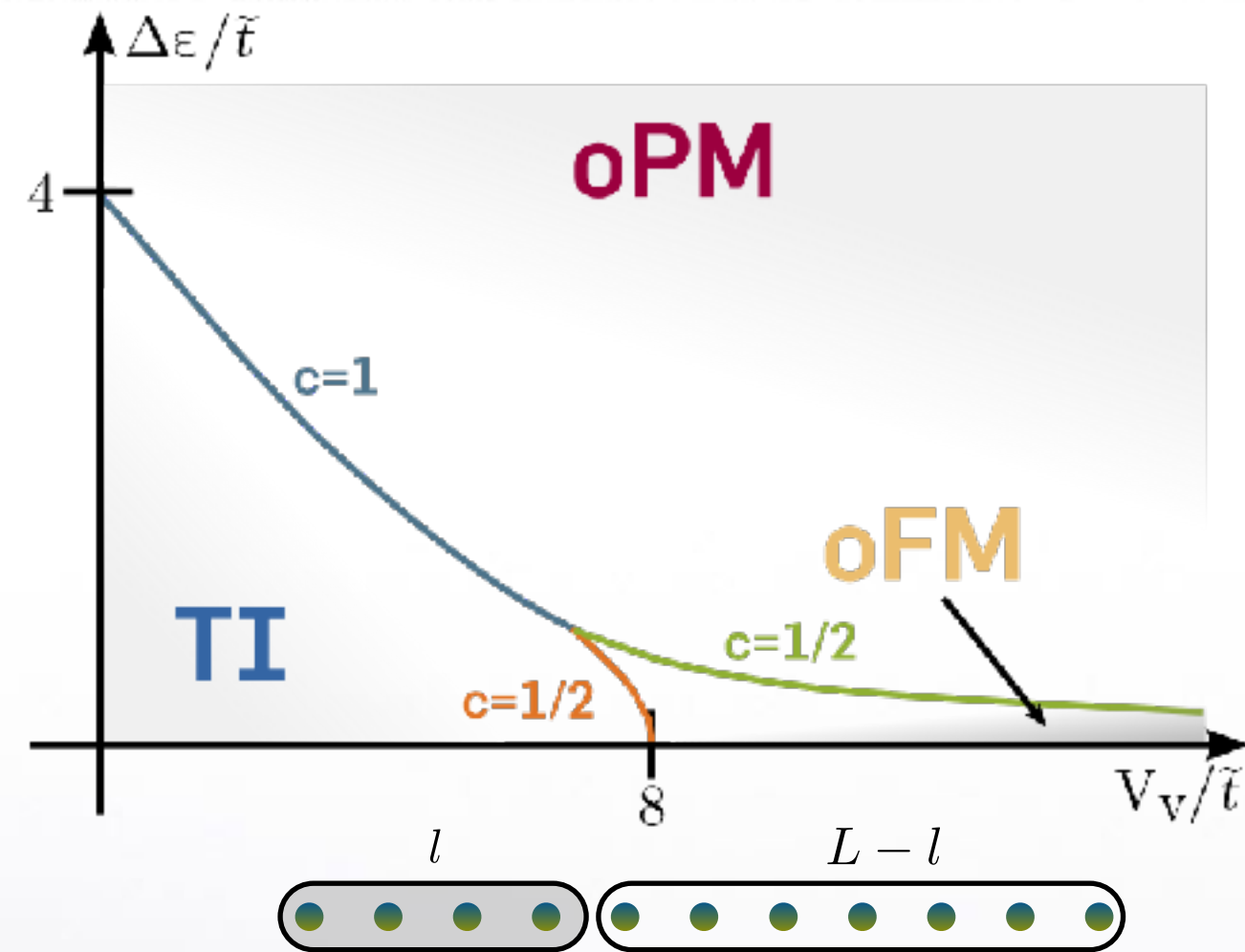
$$\tilde{T}_i^\alpha = \frac{1}{2} w_i^\dagger \tilde{\sigma}^\alpha w_i$$

critical point

$$\frac{2\tilde{t}}{|\tilde{J}|} = 1 \iff \frac{V_v}{\tilde{t}} = 8$$



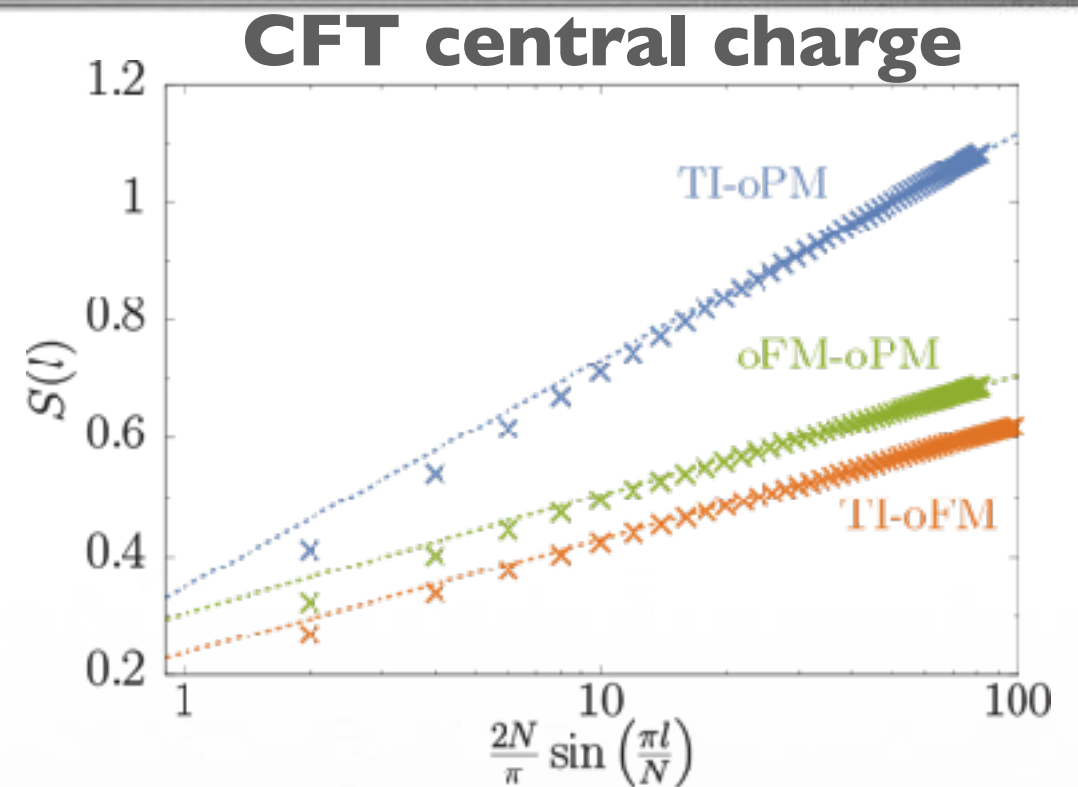
# Entanglement analysis via MPS



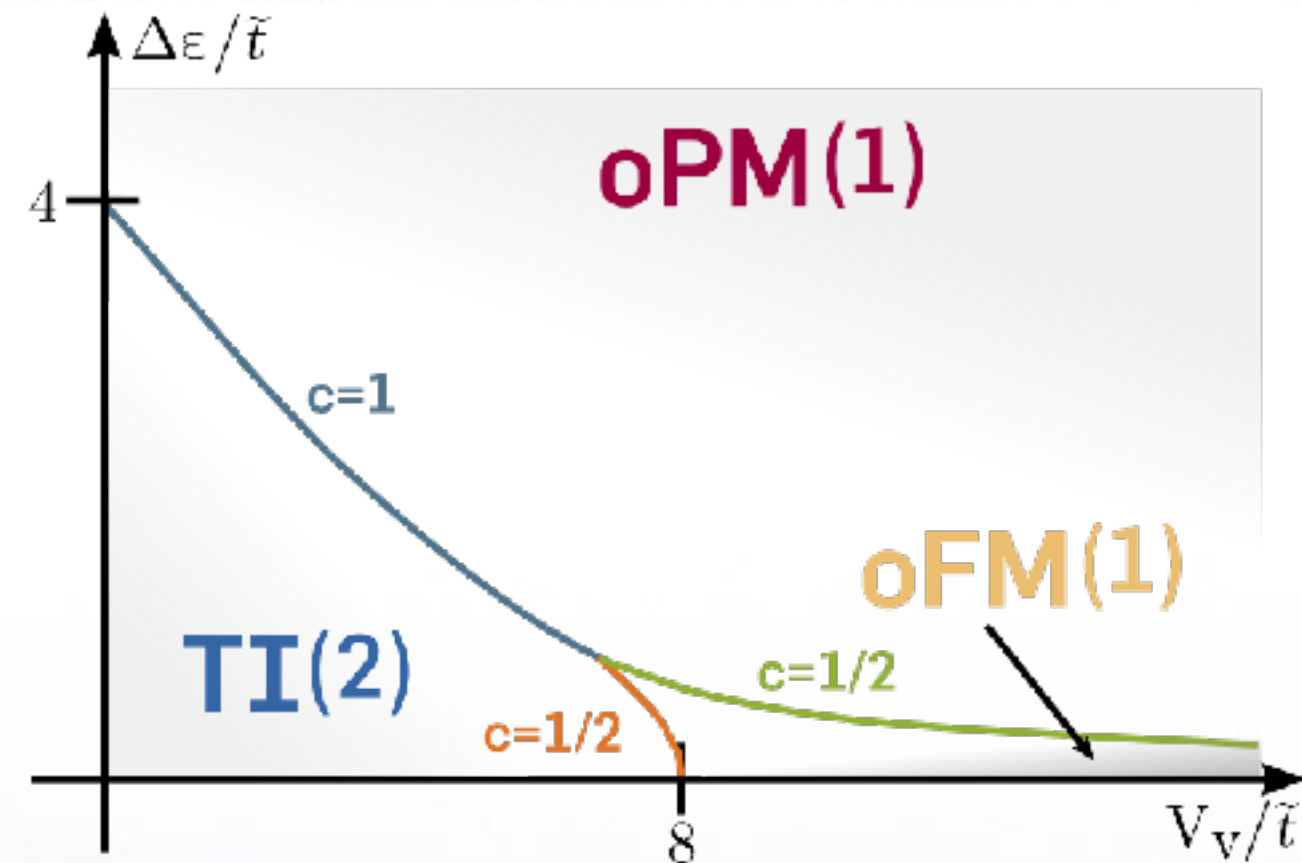
$$S(l) = -\text{Tr}\{\rho_l \log \rho_l\} \simeq \frac{c}{6} \ln \left( \frac{2L}{\pi} \sin \frac{\pi l}{L} \right) + a$$

Vidal, Latorre, Rico & Kitaev, PRL **90**, 227902 (2003)

Calabrese & Cardy, JSTAT P06002 (2004)



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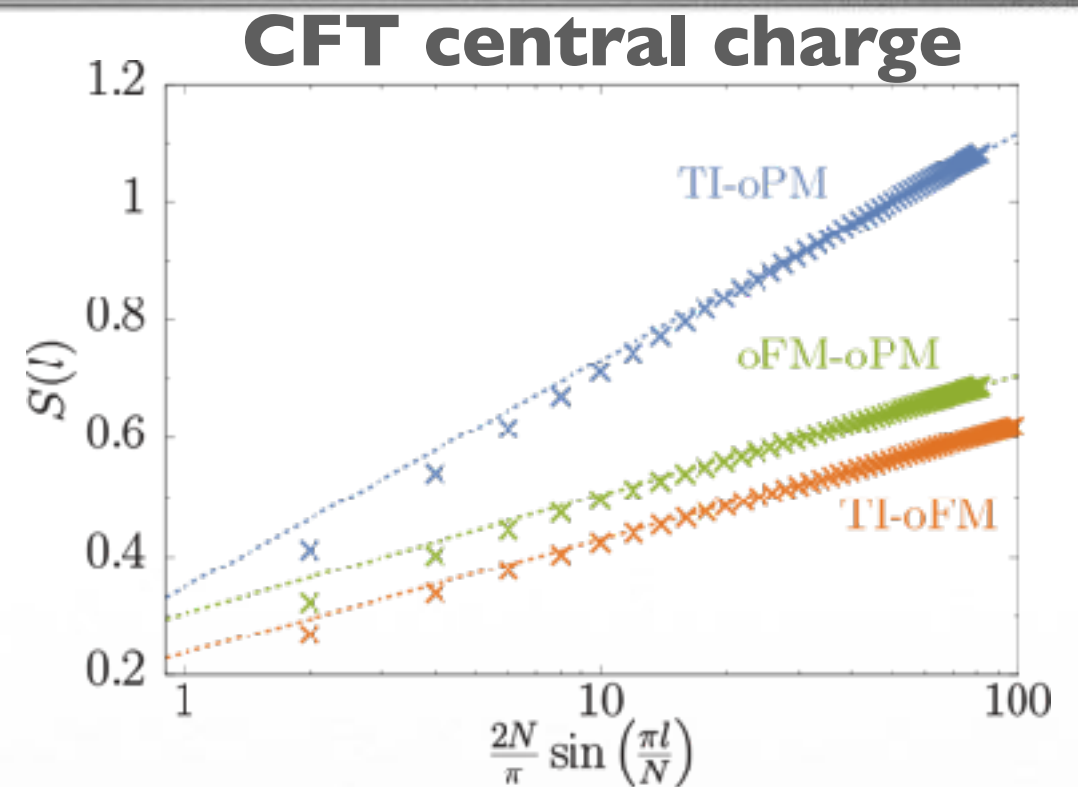
Vidal, Latorre, Rico & Kitaev, PRL **90**, 227902 (2003)

Calabrese & Cardy, JSTAT P06002 (2004)

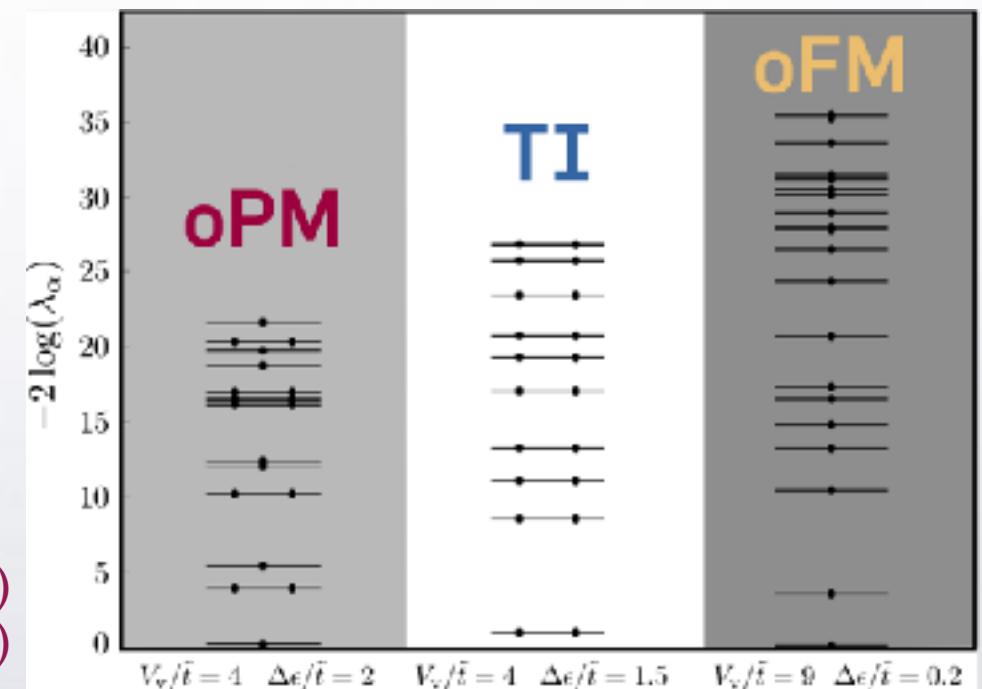
$$|\psi\rangle = \sum_i \lambda_i |\psi_l^i\rangle \otimes |\psi_{L-l}^i\rangle$$

Li & Haldane, PRL **101** 010504 (2015)

F. Pollmann, et al, PRB **81** 064439 (2010)

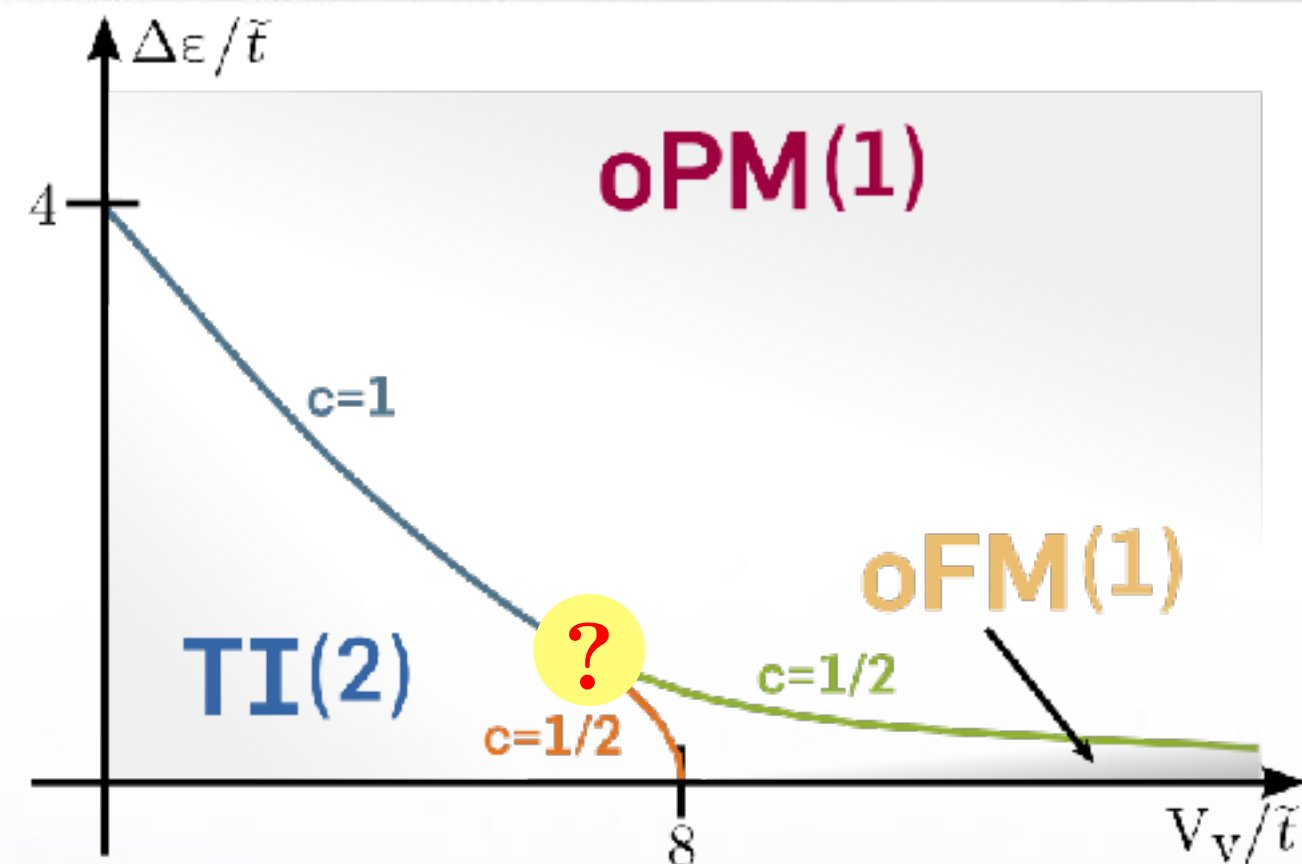


entanglement spectrum (SPT)





# Entanglement analysis via MPS



$$S(l) = -\text{Tr}\{\rho_l \log \rho_l\} \simeq \frac{c}{6} \ln \left( \frac{2L}{\pi} \sin \frac{\pi l}{L} \right) + a$$

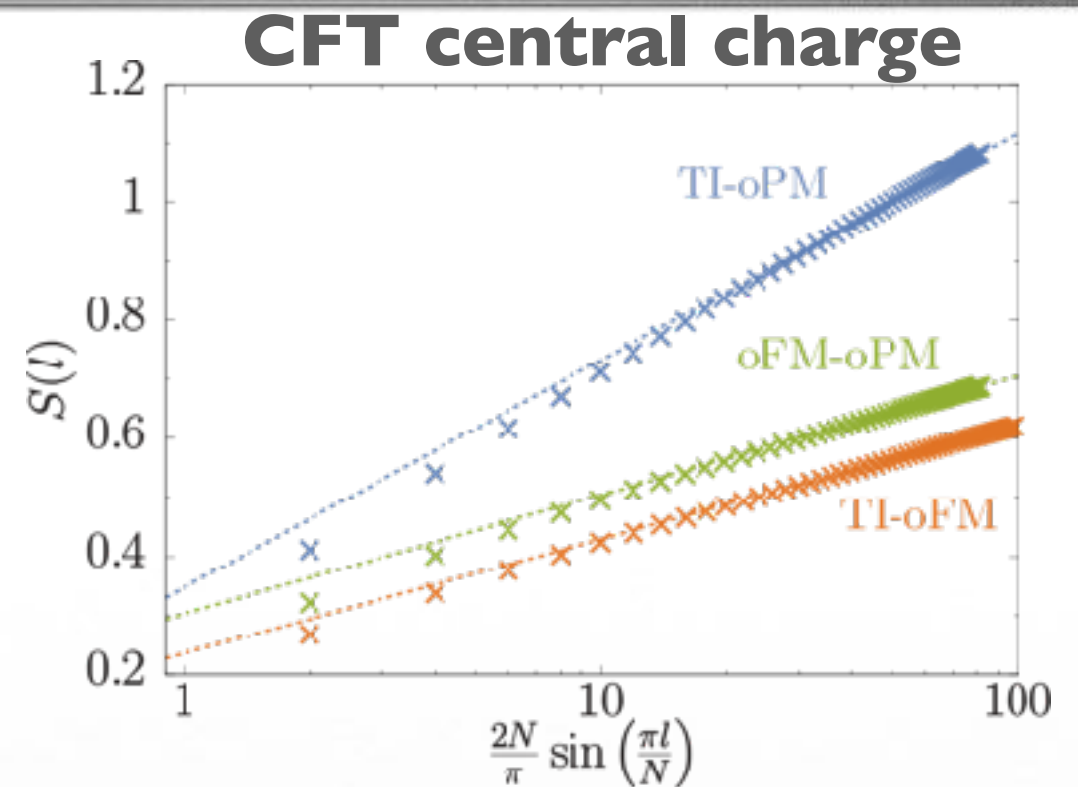
Vidal, Latorre, Rico & Kitaev, PRL **90**, 227902 (2003)

Calabrese & Cardy, JSTAT P06002 (2004)

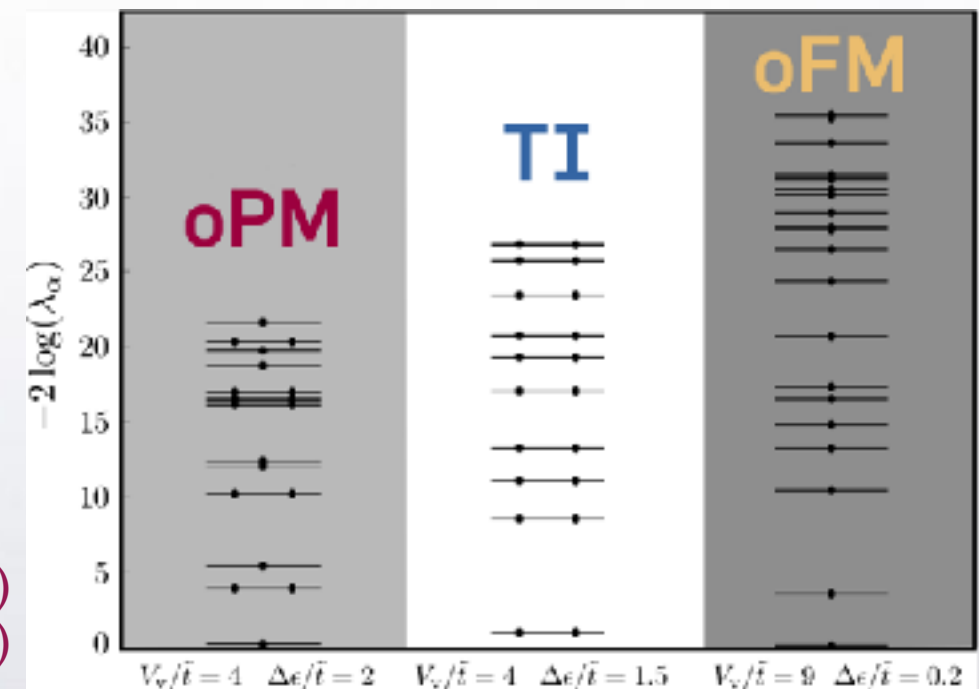
$$|\psi\rangle = \sum_i \lambda_i |\psi_l^i\rangle \otimes |\psi_{L-l}^i\rangle$$

Li & Haldane, PRL **101** 010504 (2015)

F. Pollmann, et al, PRB **81** 064439 (2010)



entanglement spectrum (SPT)



# OUTLINE

- The Creutz-Hubbard Ladder: general features
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# Gross-Neveu model

(1+1)D toy model for QCD

D.J. Gross and A. Neveu, PRD **10**, 3235 (1974)

$$\gamma^0 = \sigma^z \quad \gamma^1 = i\sigma^y \quad \gamma^5 = \gamma^0\gamma^1 = \sigma^x$$

$$\Psi(x) = (\psi_1(x) \dots \psi_N(x))^t \quad \bar{\psi}_n = \psi_n^\dagger \gamma^0$$

$$\mathcal{H} = -\bar{\Psi}(x) (\mathbb{I}_N \otimes i\gamma^1) \partial_x \Psi(x) - \frac{g^2}{2N} (\bar{\Psi}(x) \Psi(x))^2$$

asymptotic freedom & dynamical mass generation *without non-Abelian gauge fields!*

RG treatment exact in  $N \rightarrow \infty$  limit: what about finite  $N$ ?

Wilson lattice discretization, to avoid doublers(!)  $h_k = \left(m + \frac{1 - \cos ka}{a}\right) \gamma^0 - \frac{\sin ka}{a} \gamma^5$

Kogut & Susskind, PRD **11**, 395 (1975); K.Wilson (1977); Nielsen & Ninomiya, Nuc. Phys. B **185**, 20 & **193**, 173 (1981)

is essentially a Creutz-ladder

up to a rotation  $\Psi(x) \rightarrow e^{-i\frac{\pi}{2a}x} \Psi(x)$

$$h_k^{\text{CL}} = \left(\frac{\Delta\epsilon}{2} + 2\tilde{t} \sin k\right) \gamma^0 - 2\tilde{t} \cos k \gamma^5$$

N. Kawamoto & J.Smit, Nuc. Phys. B **192**, 100 (1981)

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Gross & Neveu,  
PRD **10**, 3235 (1974)

chiral condensate  
 $\sigma_0 \equiv \langle \bar{\Psi}(x) \Psi(x) \rangle \neq 0$

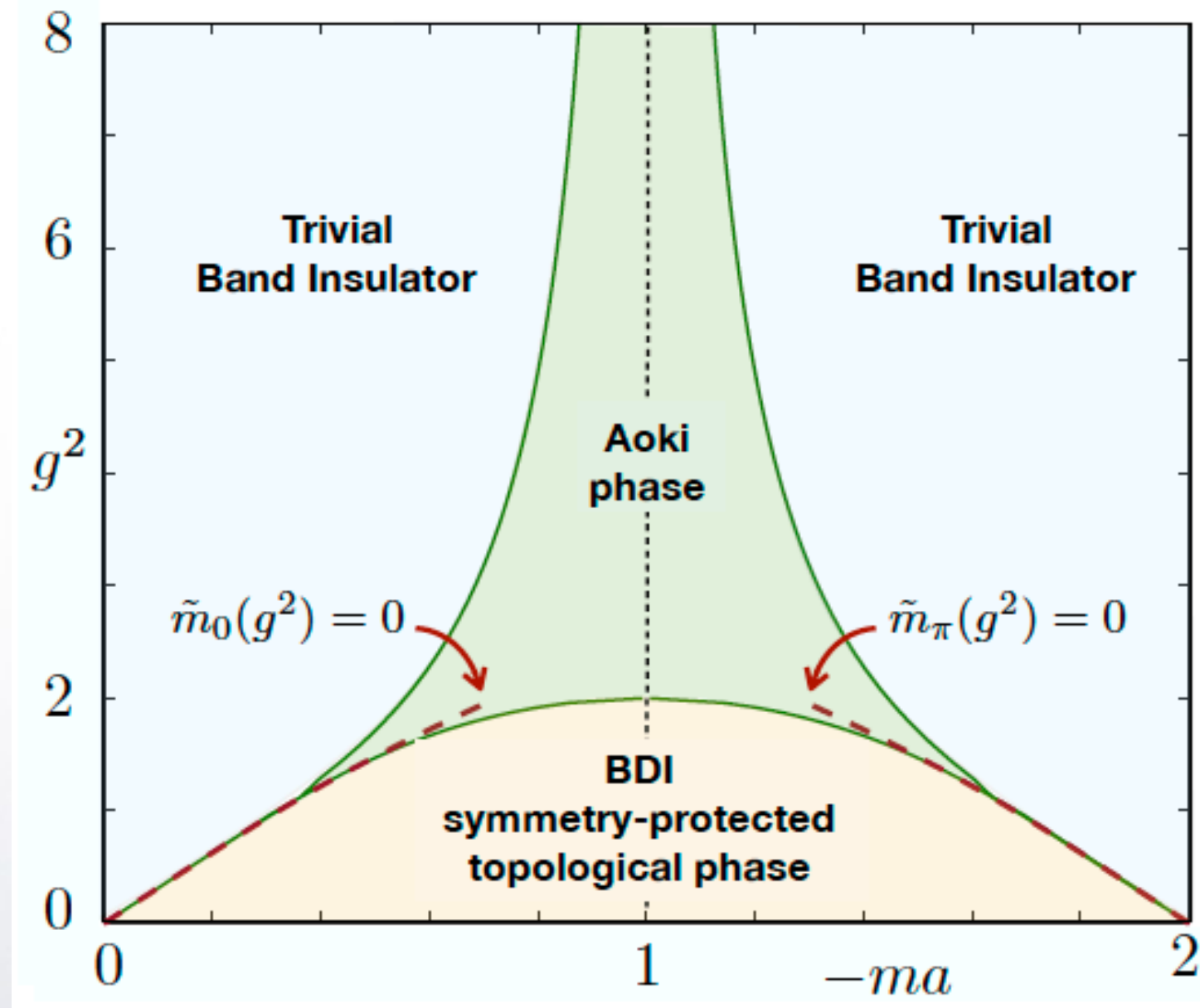
pseudo-scalar cond.  
 $\Pi_0 \equiv \langle \bar{\Psi}(x) i\gamma^5 \Psi(x) \rangle \neq 0$

S.Aoki, PRD **30**, 2653 (1984);  
Izubuchi, Noaki, Ukawa,  
PRD **58**, 114507 (1998)

# Gross-Neveu model

Various methods at hand, nowadays:

lattice QMC, large-N expansion, MPS for small N, quantum simulation with atoms, ...



$$\tilde{m}_0(g^2) = m + \sigma_0$$

$$\sigma_0 \simeq \frac{g^2}{\pi a} + \frac{8}{a} e^{-2\pi/g^2}$$

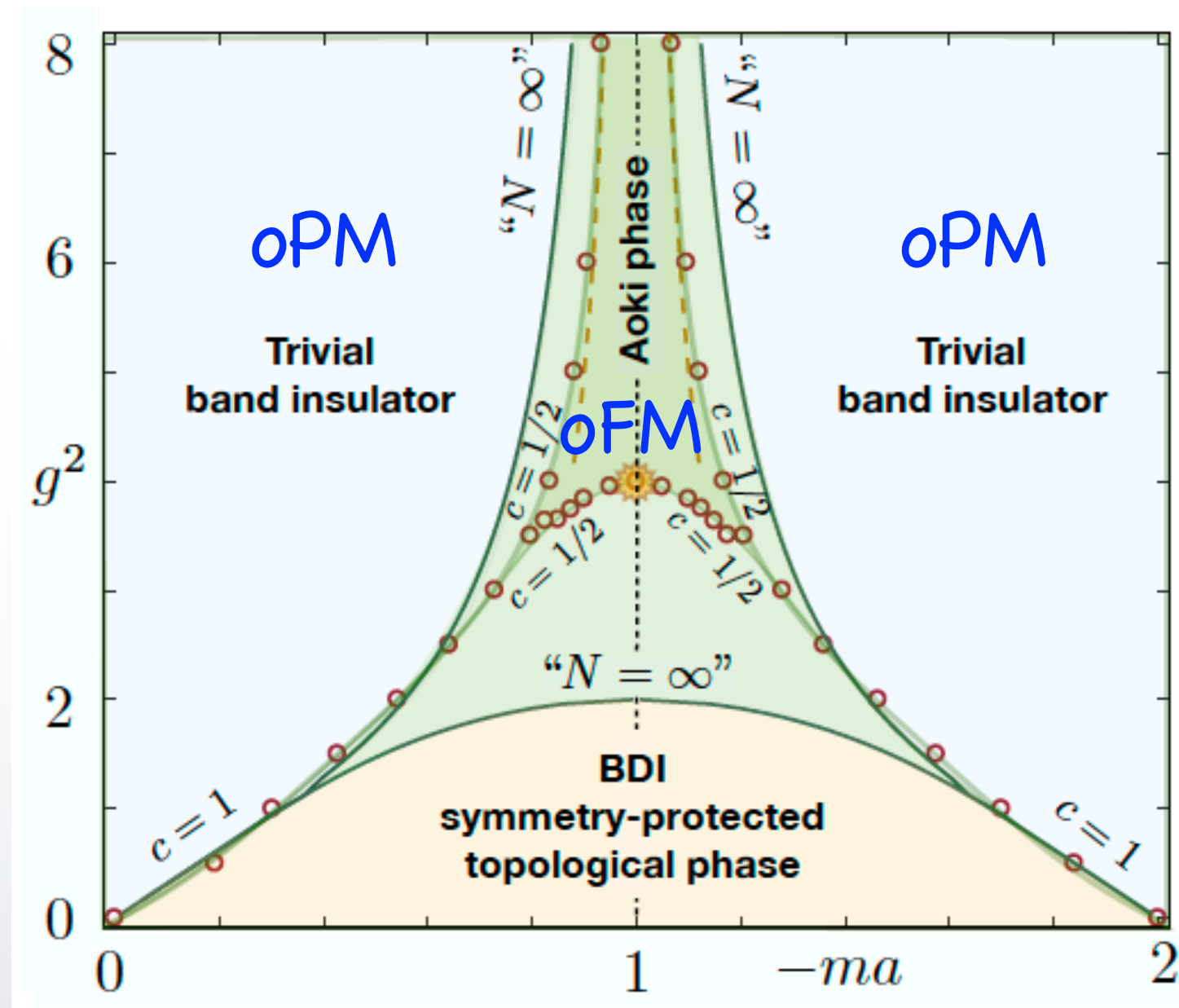
$$ma = \frac{\Delta\epsilon}{4\tilde{t}} - 1$$

$$\tilde{h}_k = \left( m + \sigma + \frac{1 - \cos ka}{a} \right) \gamma^0 - \frac{\sin ka}{a} \gamma^5 + i\Pi \gamma^1$$



# Gross-Neveu model

Various methods at hand, nowadays:  
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Explanation  
of cond-mat results  
by hep-th language  
+  
New perspective  
on quant. simul.  
of hep-th models  
with low-N

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# Topological CDW

Can intrinsically interacting SPT phases arise  
at  $\nu = N/L \neq 1$  for which non-interacting system is trivial?

## Creutz-Hubbard ladder

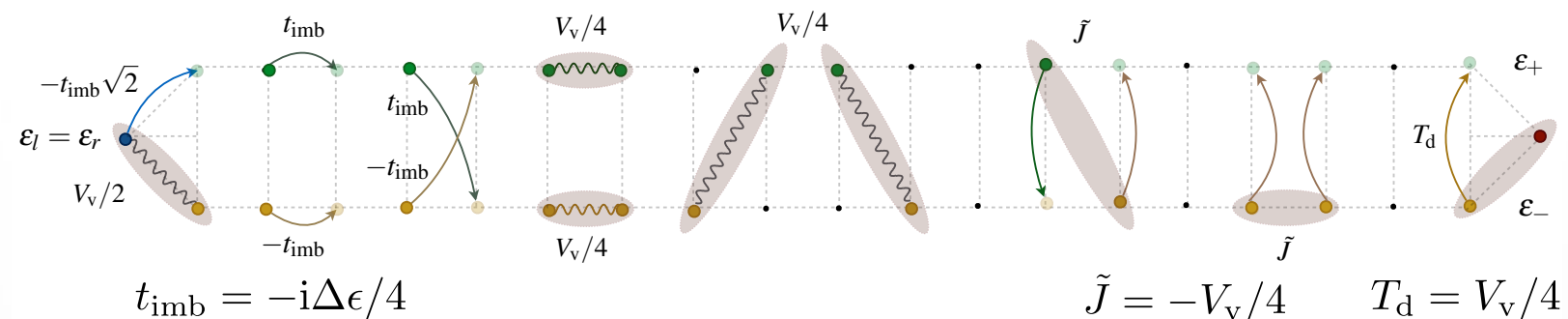


Projection on lowest band  
& approximate cage basis:  
... n.n. interactions ...



gapped phase at  $\nu = \frac{1}{2}$  & critical  $U/t$

$$\delta_{\text{charge}} = E_1(N) - \frac{1}{2} [E_1(N+1) + E_1(N-1)]$$



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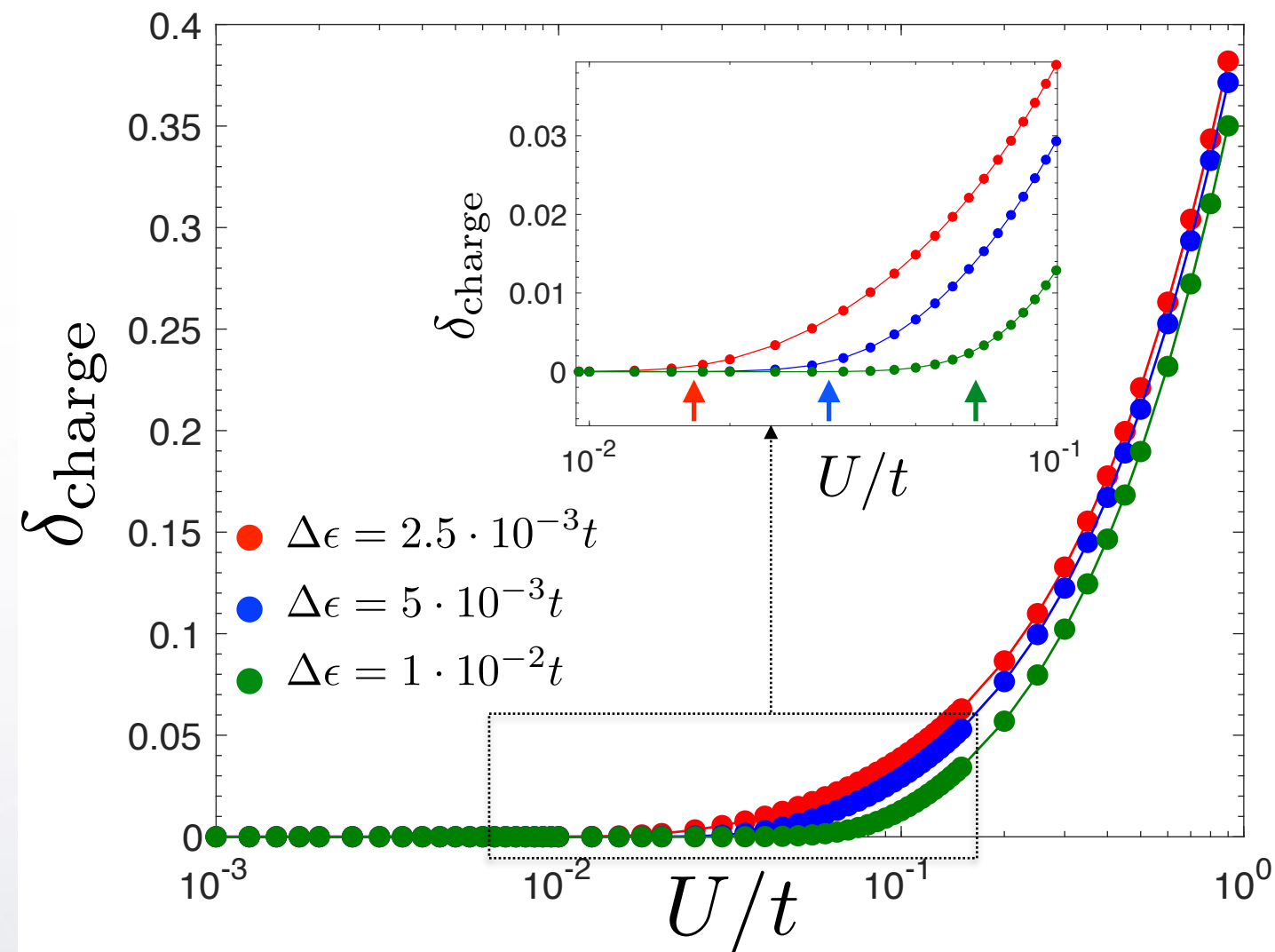
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&

double degeneracy with PBC

$$\delta_{\text{spin},\alpha} = E_{\alpha}(N) - E_1(N)$$





# Topological CDW

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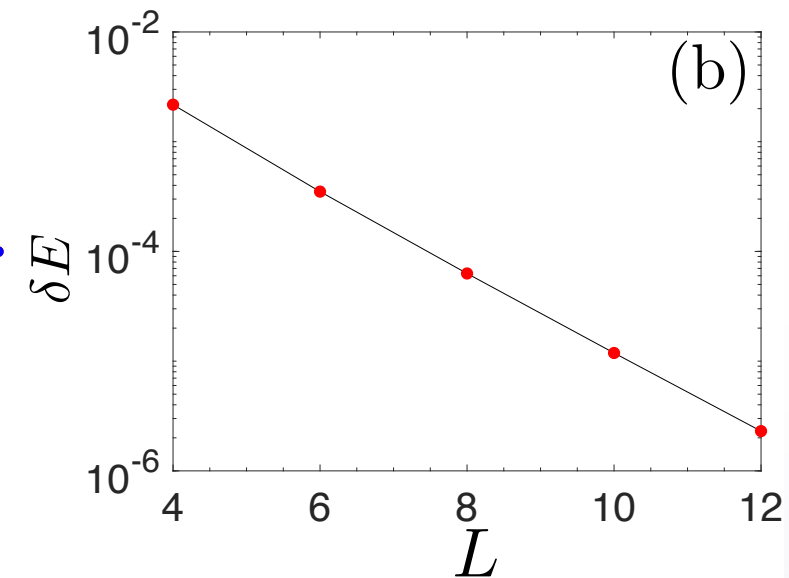
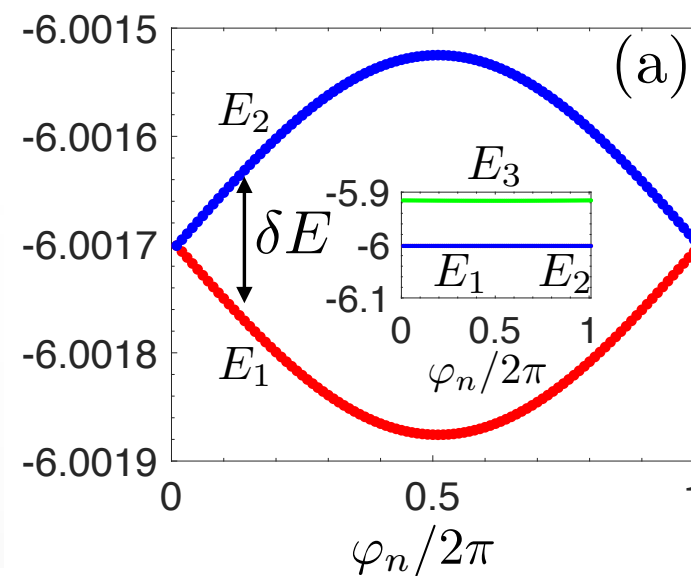
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Twisted PBC  $t \rightarrow t \exp(i\varphi/L)$



# Topological CDW

generalized SPT invariant

destroyed by

$$\hat{H}_{\text{SB}} = iM \sum_j (\hat{c}_{j,\uparrow}^\dagger \hat{c}_{j,\downarrow} - \text{H.c.}) \propto \sigma_y$$

explicitly breaking chiral symm.

$$U_S = \sigma_y \quad \begin{aligned} U_S H_0(k) U_S^\dagger &= -H_0(k) \\ U_S H_{\text{SB}} U_S^\dagger &= +H_{\text{SB}} \end{aligned}$$

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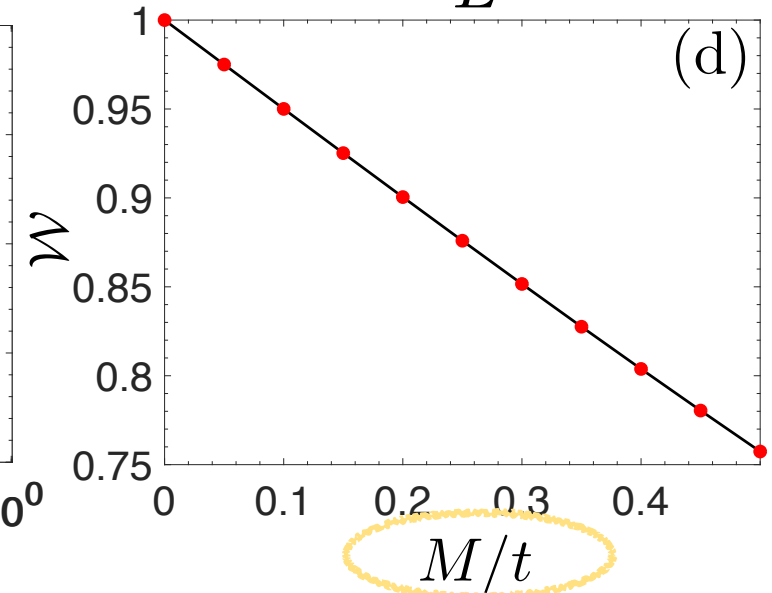
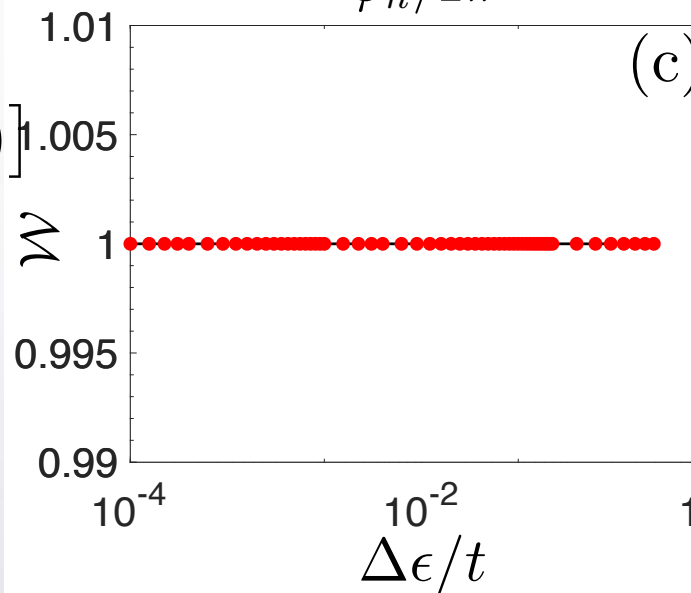
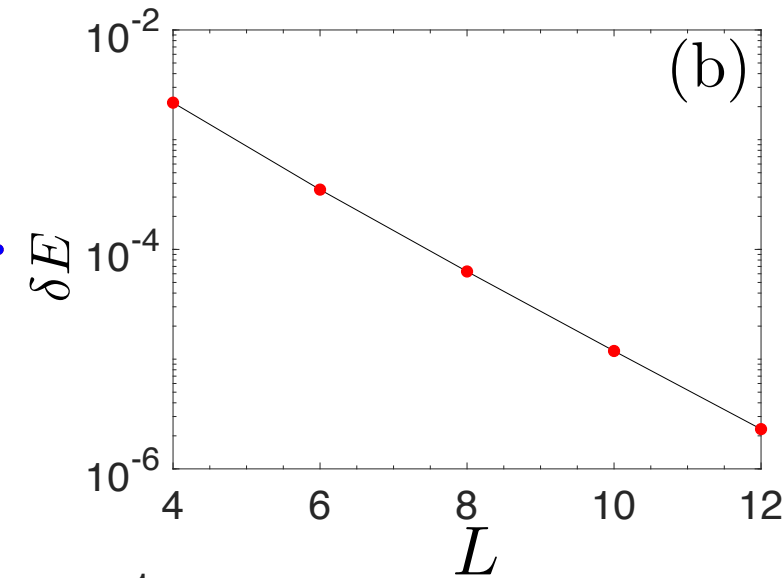
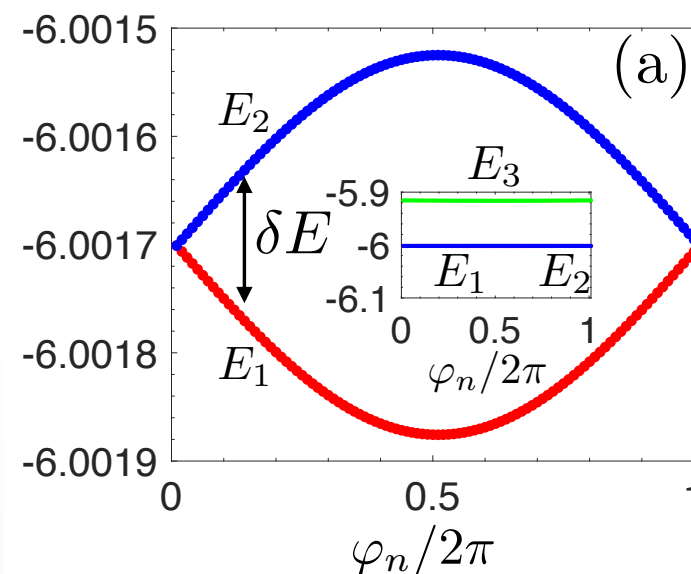
double degeneracy with PBC

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Twisted PBC  $t \rightarrow t \exp(i\varphi/L)$

$$\mathcal{W} = \frac{i}{\pi} \int_0^{2\pi} d\varphi \text{Tr} [\langle \Psi_\alpha(\varphi) | \partial_\varphi | \Psi_\beta(\varphi) \rangle]$$

F. Wilczek & A. Zee., PRL **52**, 2111 (1984)





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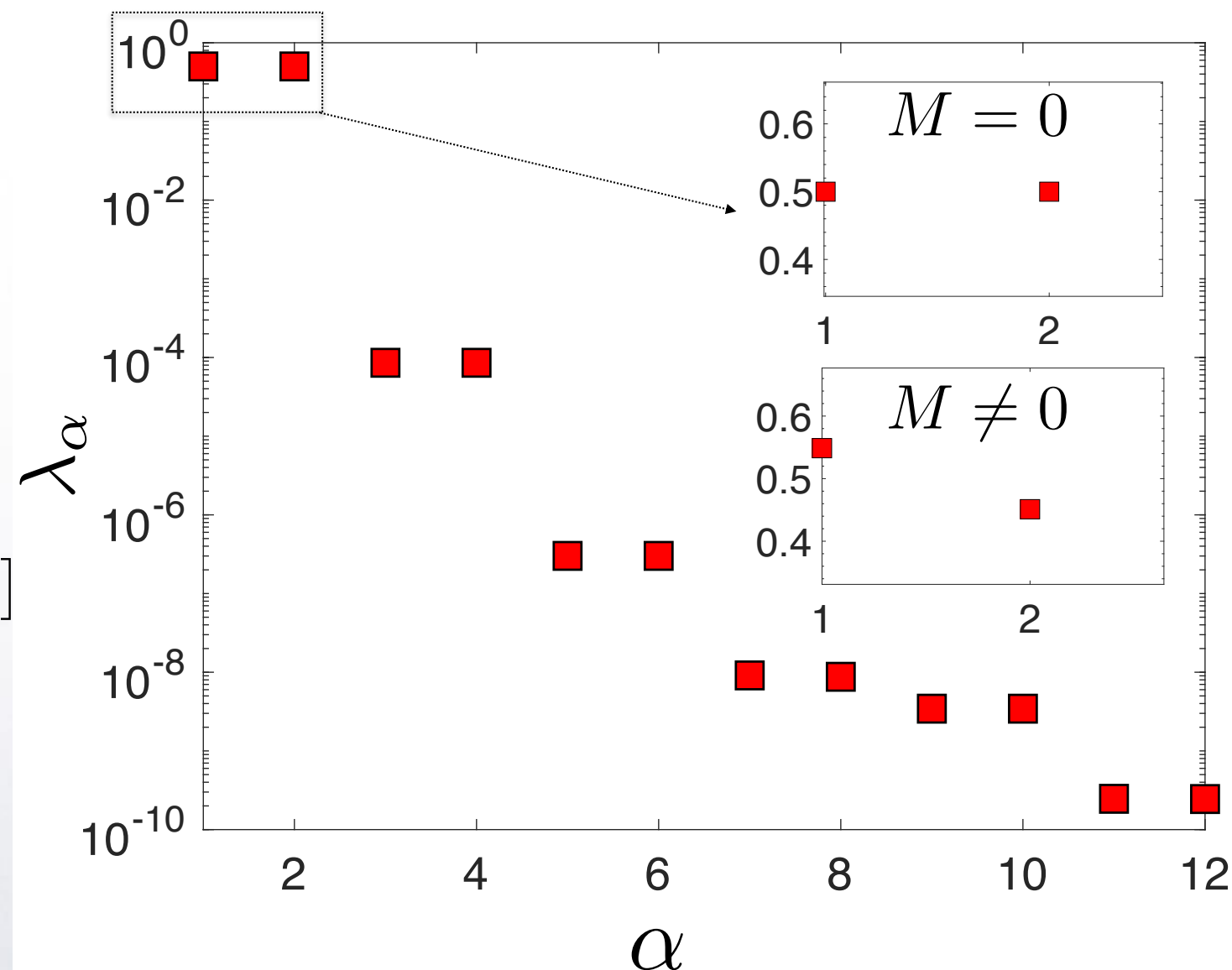
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Same happens for  
for entanglement spectrum!



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# Dirac Fermions

For graphene (i.e., 2D Dirac Fermions), interactions give **positive** correction to magnetic susceptibility  $\chi_{\text{orb}}$ !

McClure, PR **104**, 666 ('56) & PR **119**, 606 ('60)  
Sharma, Johnson, and McClure, PRB **9**, 2467 ('74)

Principi, et al., PRL **104**, 225503 ('10)  
& many more!!!

## 1D Systems:

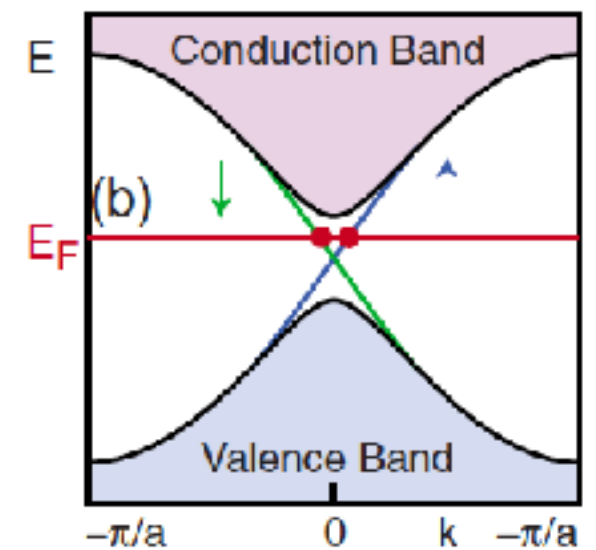
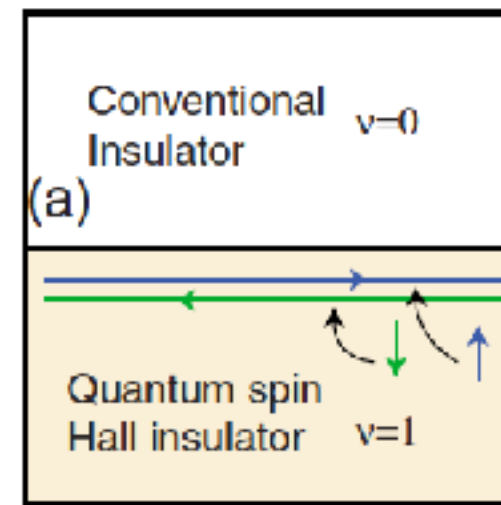
- Dirac-like dispersions possible, too

M.Z. Hasan and C.L. Kane, RMP **82**, 3045 ('10)

X.-L. Qi and S.-C. Zhang, RMP **83**, 1057 ('11)

- no *true* electric/magnetic field distinction

$$D \simeq \chi_{\text{orb}} \quad \sigma_1(\omega) = D \delta(\omega) + \sigma_1^{\text{reg}}(\omega)$$

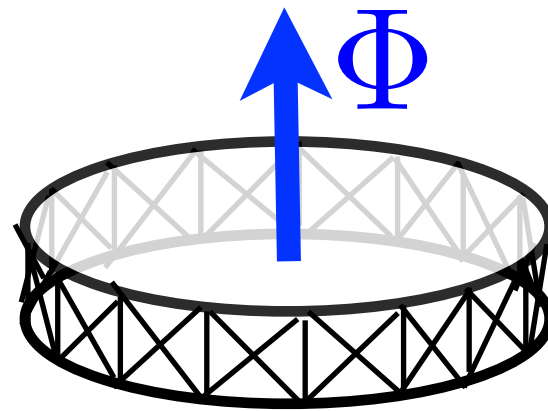
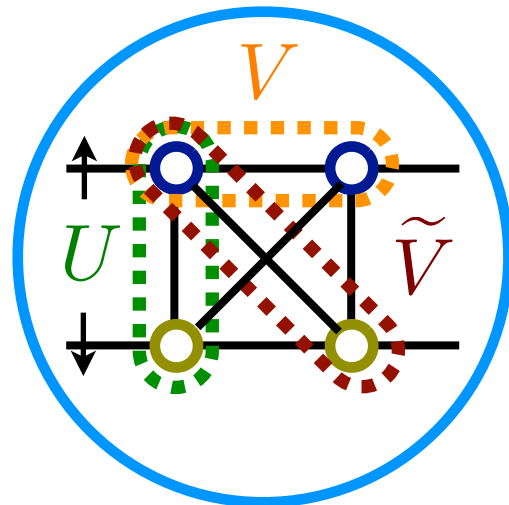
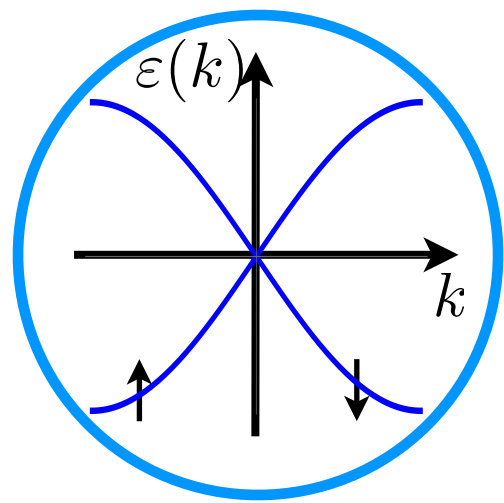


## QUESTION HERE:

do many-body effects (i.e., interactions)

lead to a **qualitative** change of the transport properties of 1D Dirac Fermions?

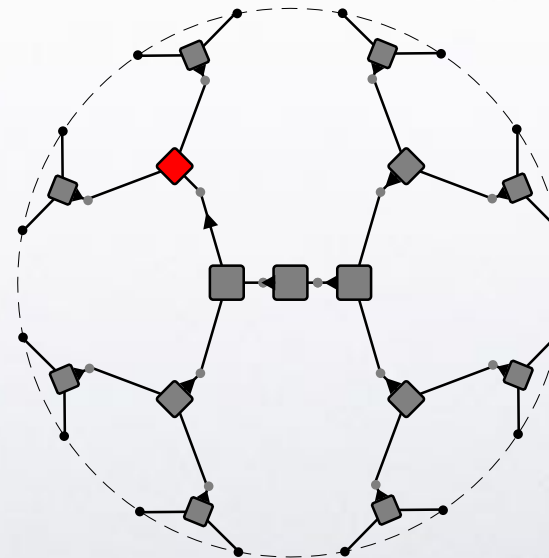
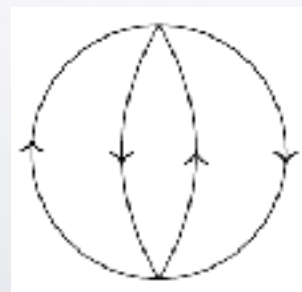
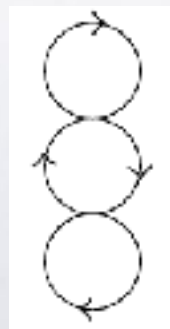
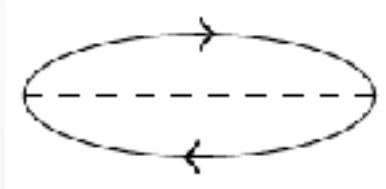
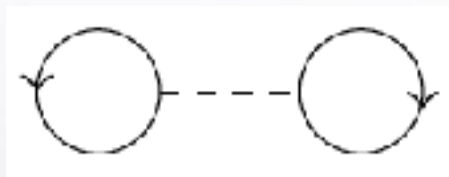
# Model & Methods



Rings are not essential to measure transport, e.g., D from quenches

C. Karrasch, et al.,  
PRB **95**, 060406(R) (2017)

$$\varphi = \frac{2\pi}{L} \frac{\Phi}{\Phi_0} \quad \text{Persistent Current} \quad I(\Phi) = -\frac{\partial E}{\partial \Phi} \quad \text{Drude Weight} \quad D = \frac{L}{2\pi} \left. \frac{\partial^2 E}{\partial \Phi^2} \right|_{\Phi=0}$$



M. Gerster, MR, et al.,  
PRB **90**, 125154 ('15)  
NJP **18**, 015015 ('16)  
PRB **96**, 195123 (2017)

P. Silvi, MR, et al.,  
arXiv:1710.03733v1

Perturbative calculations

Tree Tensor Networks DMRG calculations

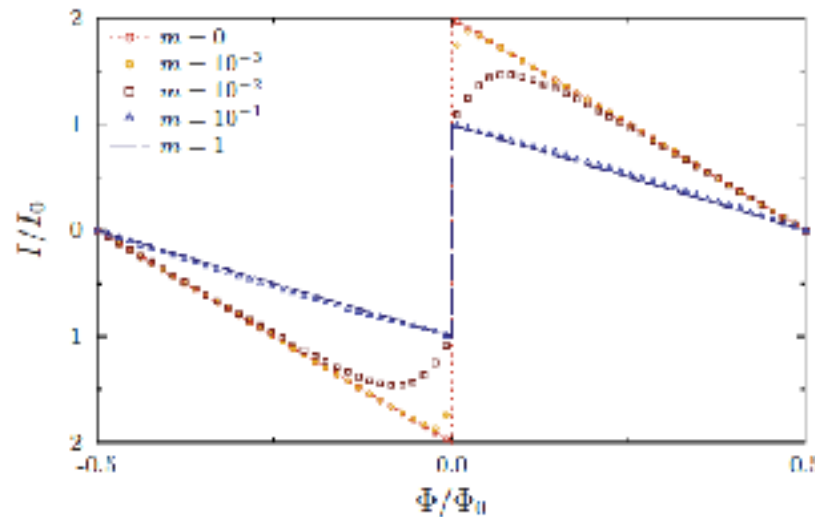
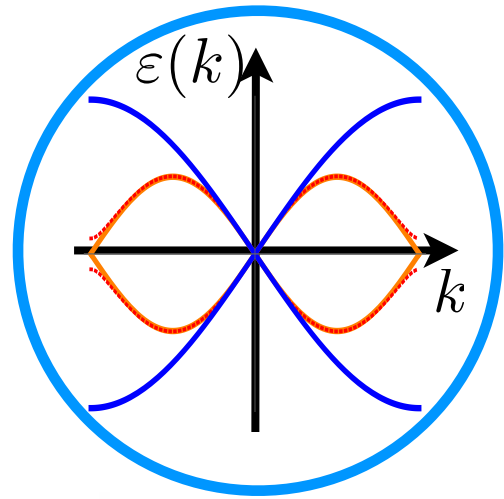
Entanglement in Strongly  
Correlated Systems  
Benasque 2019

[matteo.rizzi@fz-juelich.de](mailto:matteo.rizzi@fz-juelich.de)  
PRB **96**, 241112(R) (2017)

Tuning the Drude Weight  
of 1D Dirac fermions



# Key Results

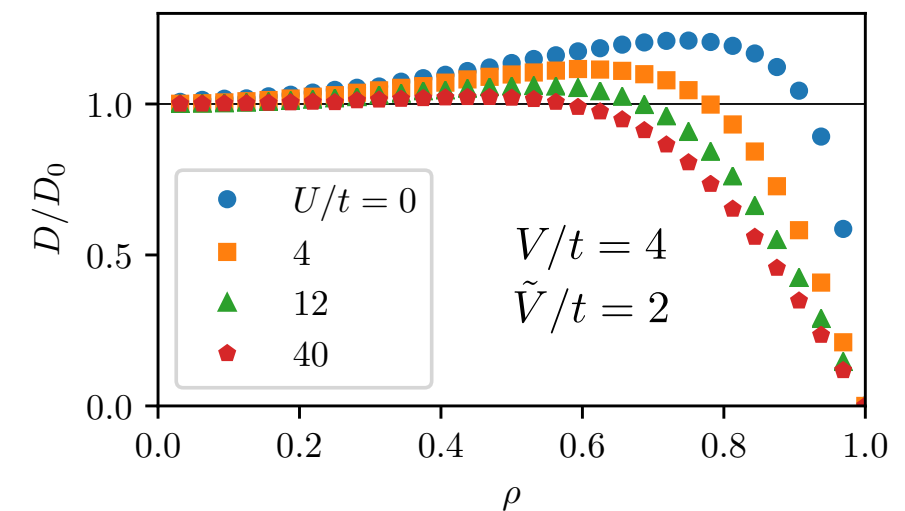
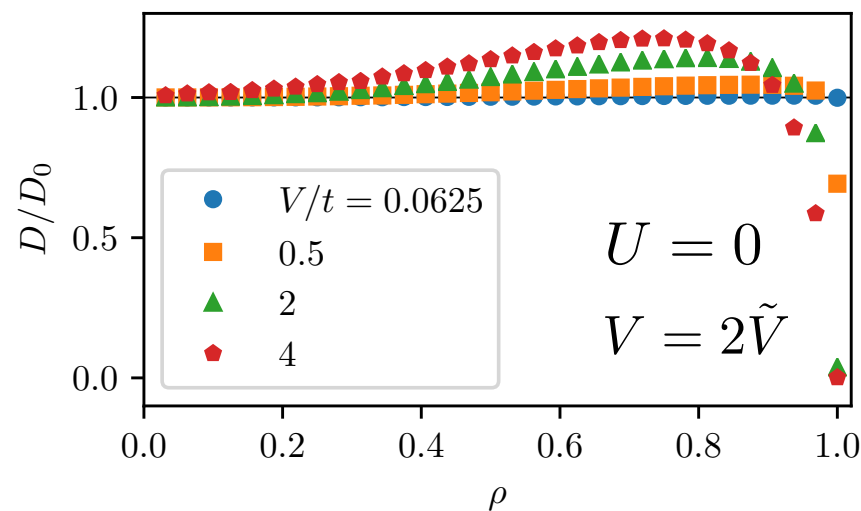
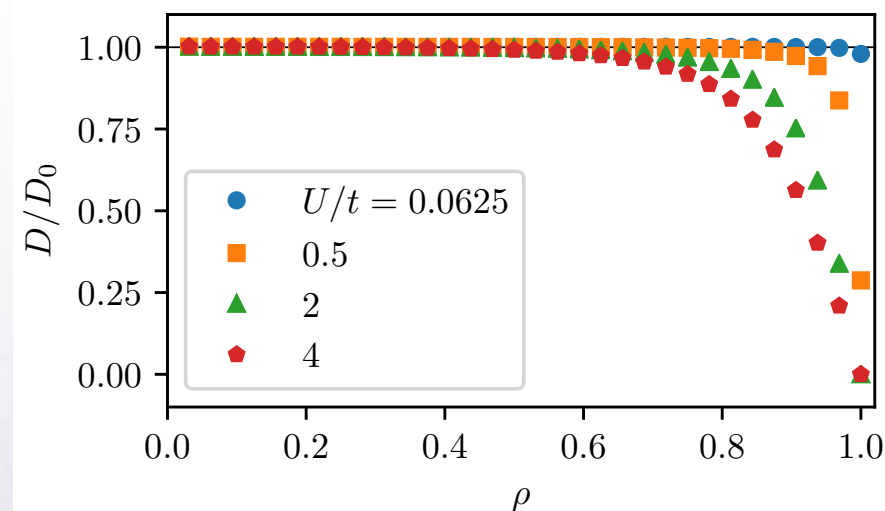


Non-interacting:  
just a factor 2 (# lin. disp.)  
w.r.t. Hubbard  $U=0$

Sticlet, Dóra, Cayssol, PRB 88, 205401 ('13)

$$D/D_0 = \left( 1 + \frac{4 \sin^3 \left( \frac{\pi \rho}{2} \right)}{3\pi} (V - \tilde{V}) - \left[ U^2 \frac{\sin^4 \left( \frac{\pi \rho}{2} \right)}{32\pi^2 \cos \left( \frac{\pi \rho}{2} \right)} \int_{\pi(1-\rho)}^{\pi} \frac{\cos(k) + 3}{\sin \left( \frac{k}{2} \right)} dk + \tilde{V}^2 \dots + \dots \right] \right)$$

[on-going bosonization  
with M. Filippone  
@ UniGe.ch]



Onsite-interacting:  
*analogue* to Hubbard,  
Drude Weight suppression!

Longer-range & Spin-dep.:  
**qualitative** difference,  
tunable Drude Weight !

**Robust** effect  
against on-site terms!

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Jünemann, et al., PRX 7, 031057 (2017)
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Bermudez, et al., Ann. Phys. 339, 149 (2018)
  - interacting SPT phases at fractional filling  
Barbarino, et al., arXiv:1810.02337
- Tuning the Drude Weight of Dirac fermions  
Bischoff, et al., PRB 96, 241112(R) (2017)
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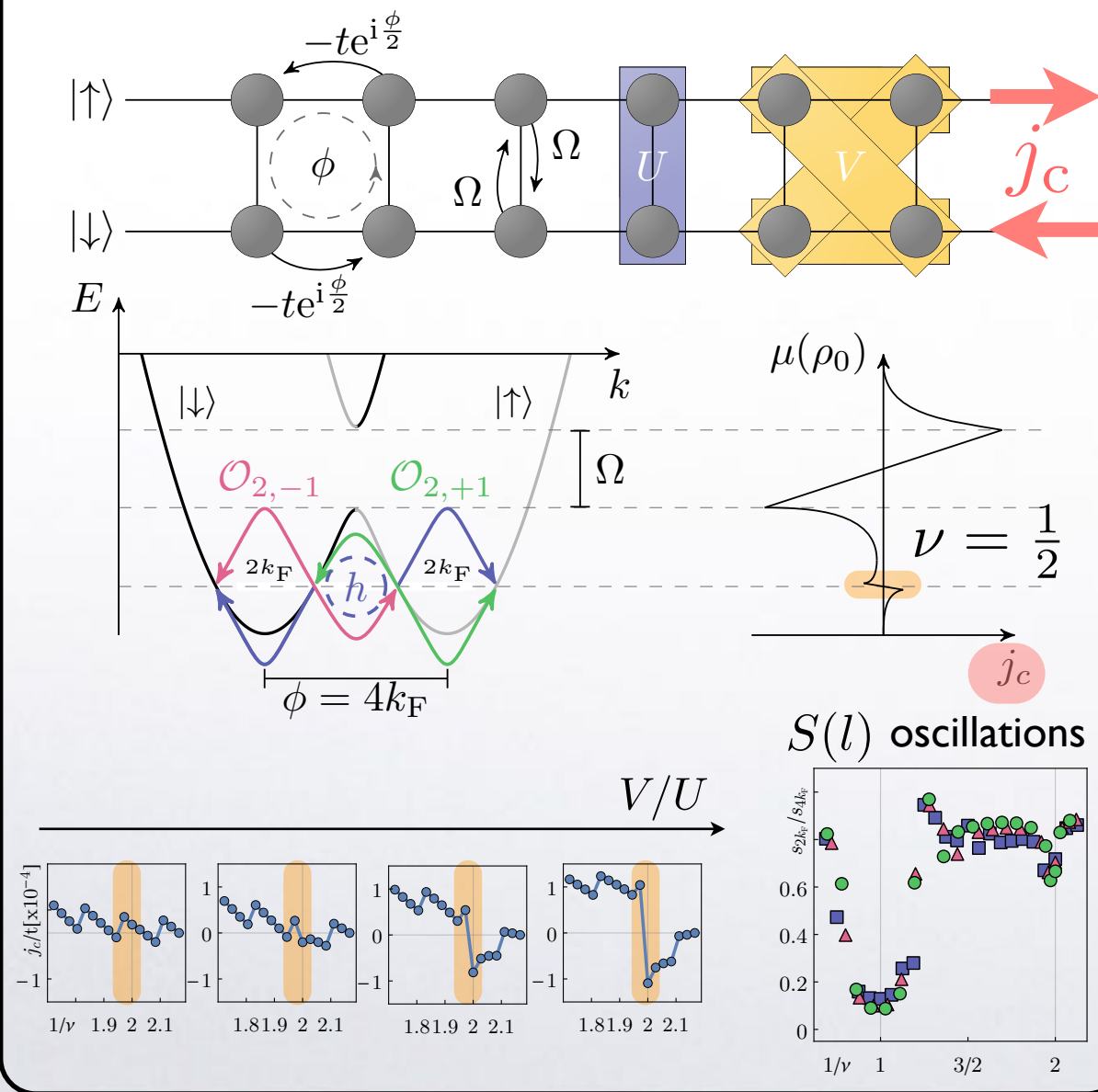


# Other related works

## topological states

... in quantum ladders

A. Haller, MR, M. Burrello,  
New J. Phys. **20**, 053007 (2018)

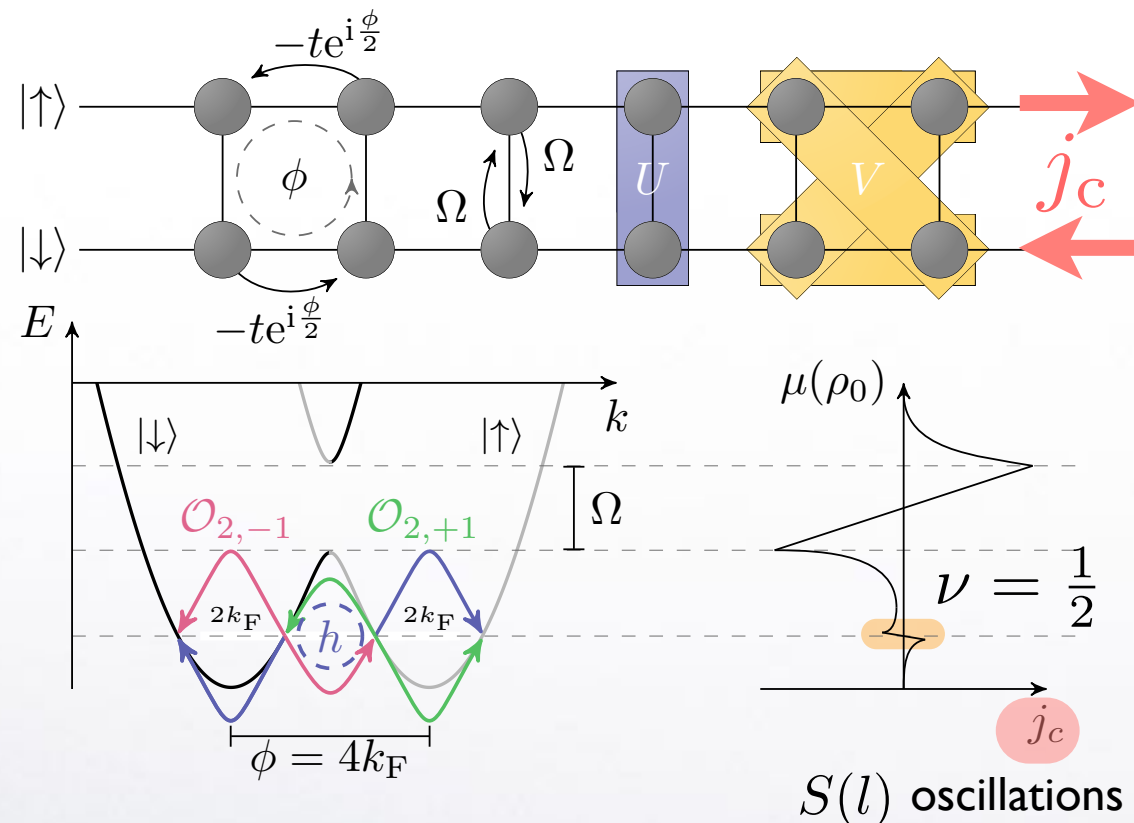


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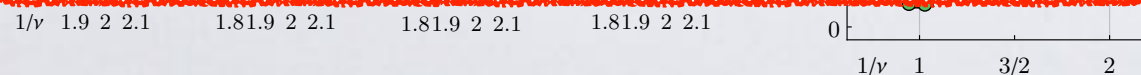
... in quantum ladders

A. Haller, MR, M. Burrello,  
New J. Phys. **20**, 053007 (2018)



multi-leg ladders + density-assisted hoppings  
to go towards Majorana & “parafermions”

with M. Burrello (Copenhagen) & L. Mazza (Paris)



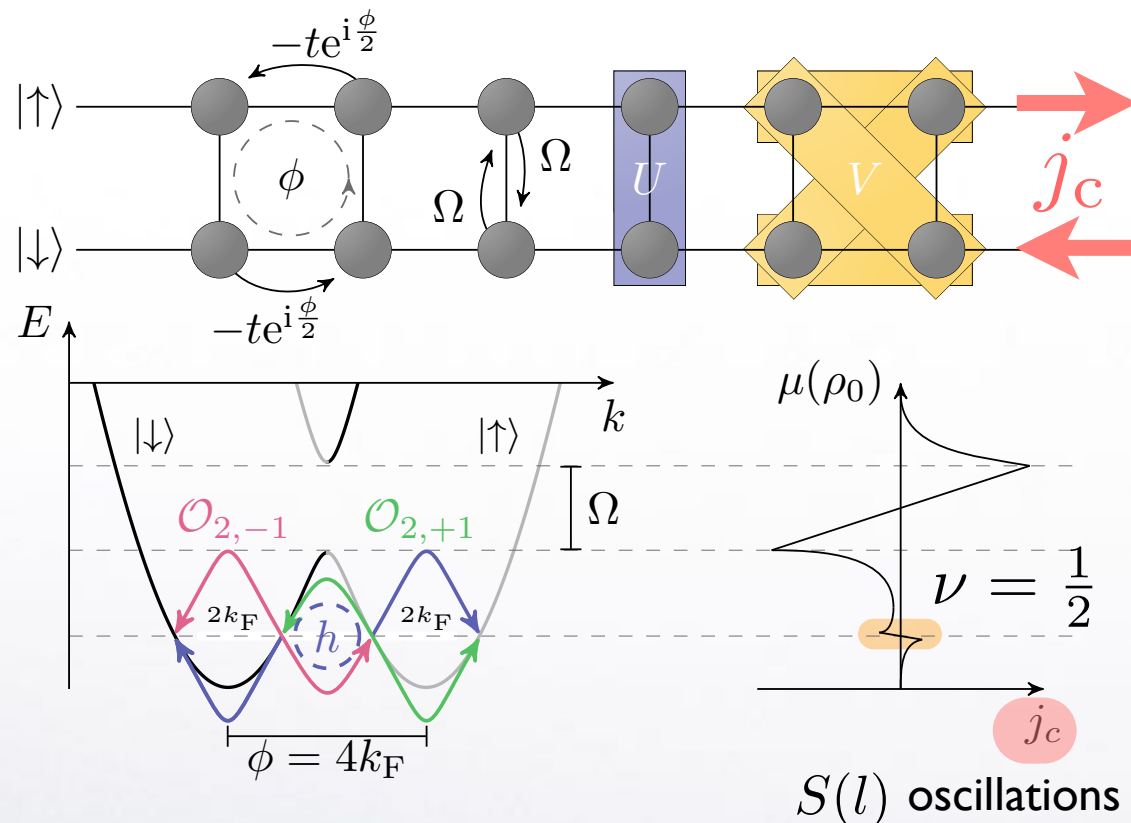


# Other related works

## topological states

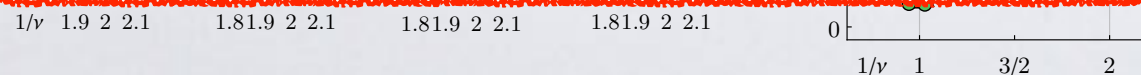
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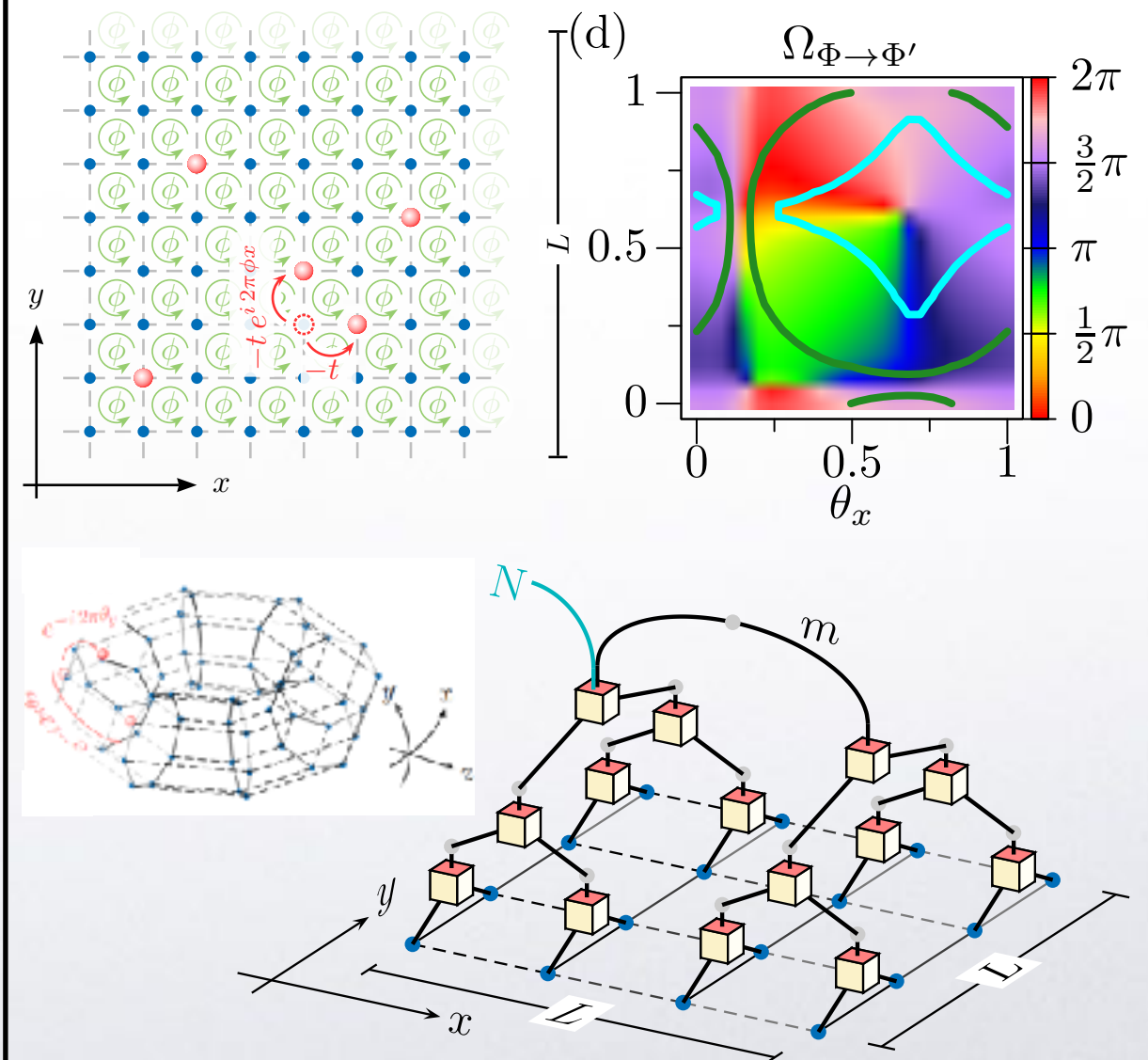
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... in 2D Harper-Hofstadter

M. Gerster, et al. (MR) PRB **96**, 195123 (2017)

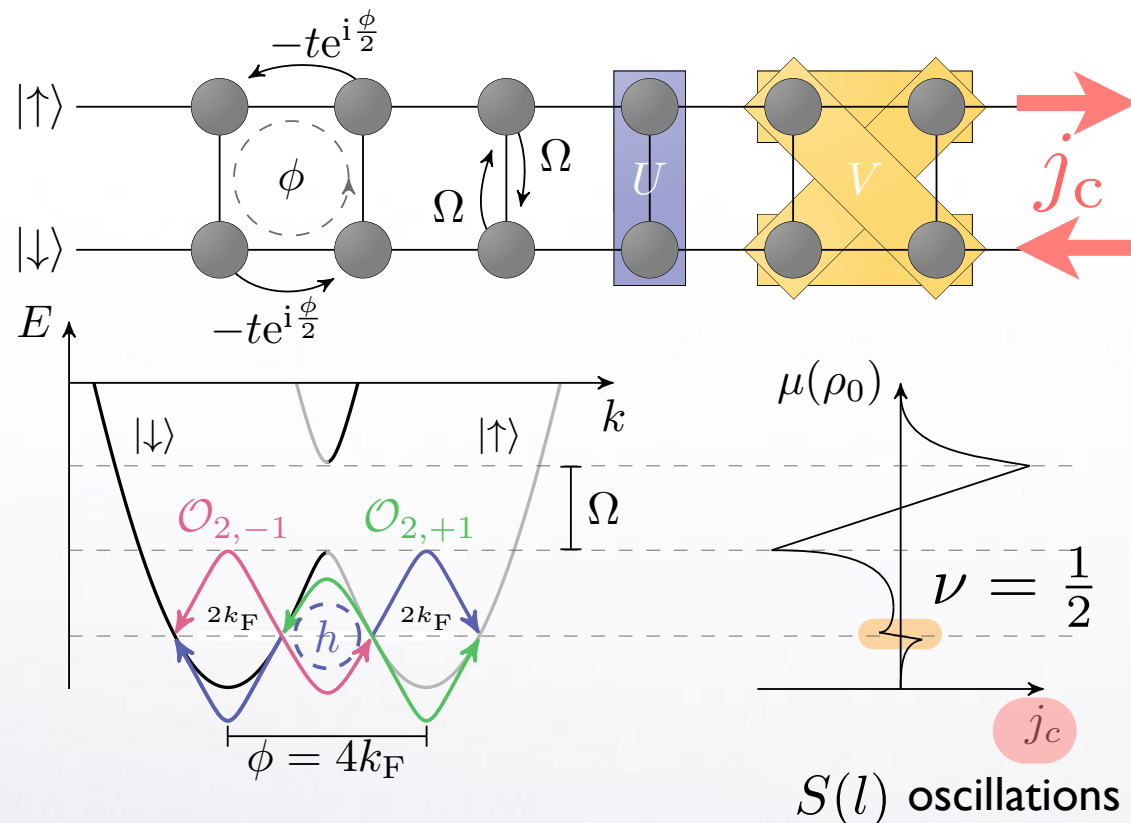


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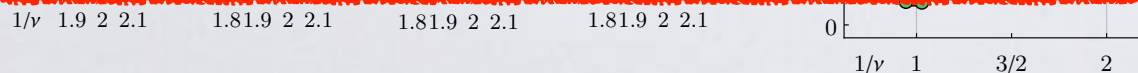
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New J. Phys. **20**, 053007 (2018)



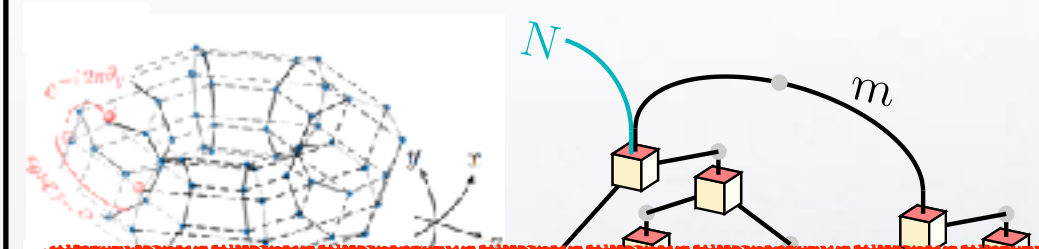
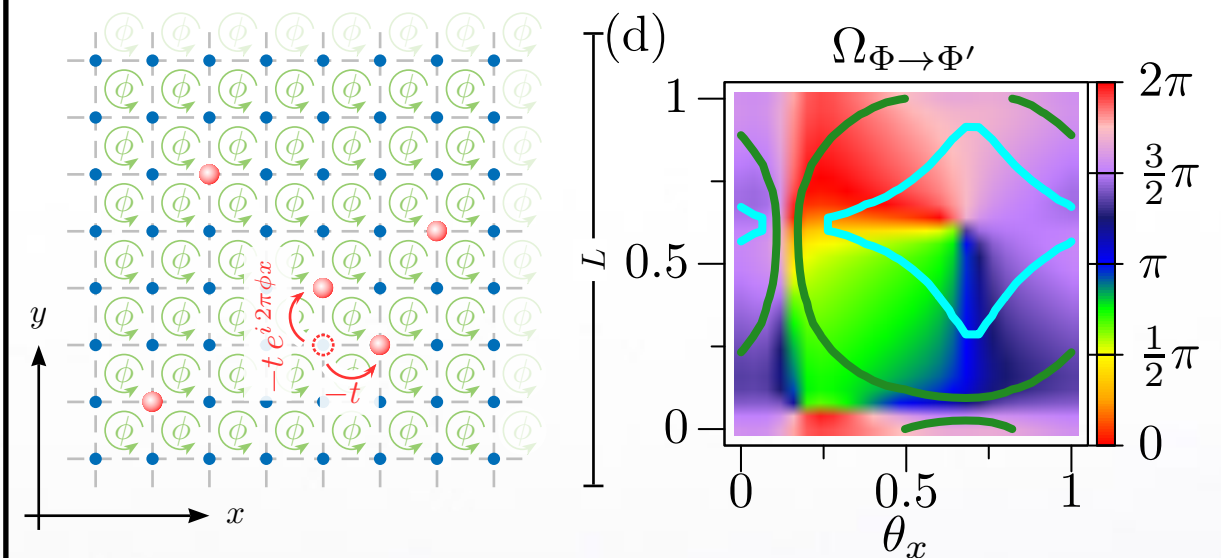
multi-leg ladders + density-assisted hoppings  
to go towards Majorana & “parafermions”

with M. Burrello (Copenhagen) & L. Mazza (Paris)



... in 2D Harper-Hofstadter

M. Gerster, et al. (MR) PRB **96**, 195123 (2017)



detection schemes for cold atoms?  
pinning strategies of quasi-holes?

with I. Carusotto (Trento)



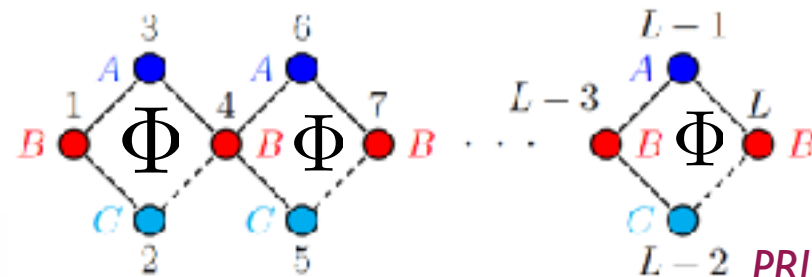
# Other related works

## flat-band bosonic systems

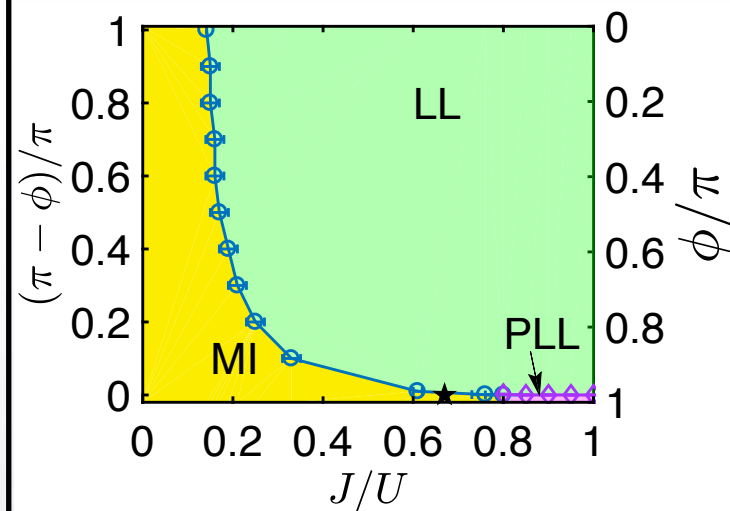
### Robustness(?) of Pair Luttinger Liquid

MR, Cataudella, Fazio PRB **73**, R100502 & 144511 ('06)

C. Cartwright, G. deChiara, MR PRB **98**, 184508 ('18)

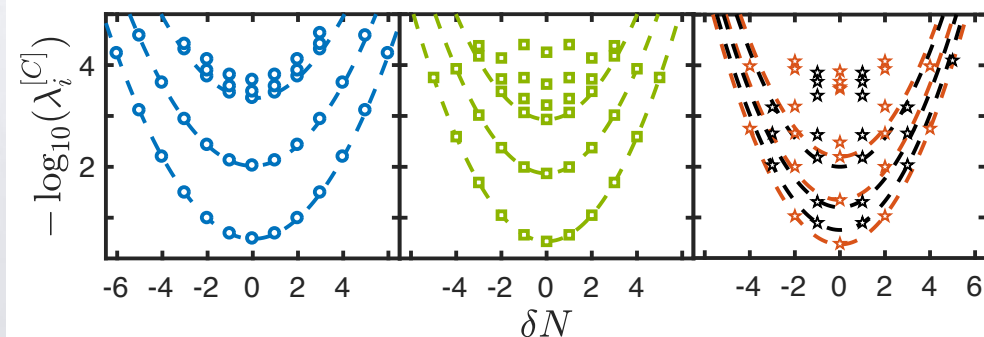


B. Douçot, J. Vidal,  
PRL **88**, 227005 (2002)



AB caging  
+  
emerging  
 $Z_2$  local sym.

signature in ES ?



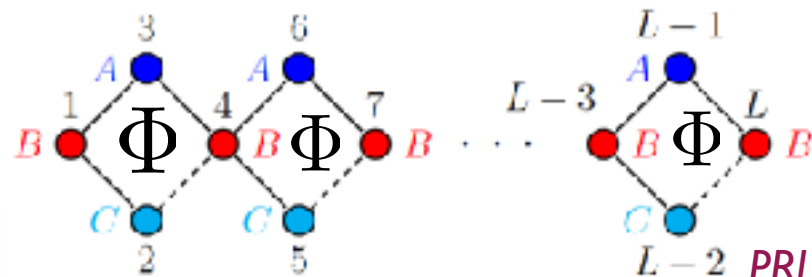
# Other related works

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### Robustness(?) of Pair Luttinger Liquid

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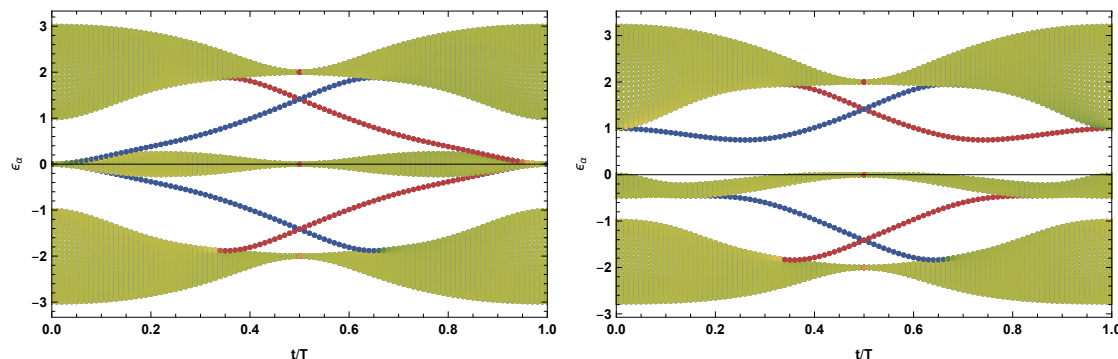
### Square-Root TI: quantized pumping?



$$\mathcal{H}_{\diamond\pi} \longrightarrow \gamma_{\text{Zak}} \notin \{0, \pi\}$$

$$\mathcal{H}_{\diamond\pi}^2 = \mathcal{H}_{\text{trivial}} \oplus \mathcal{H}_{\text{SSH}}$$

M. Kremer, et al., 1805.05209





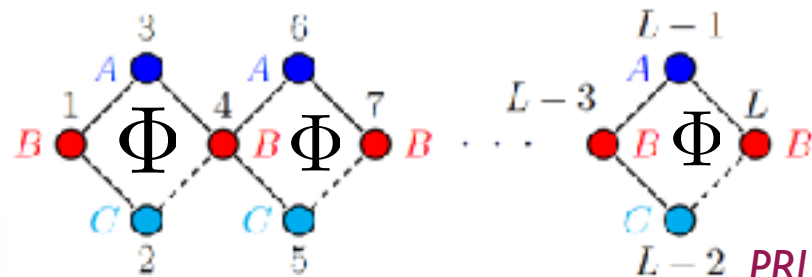
# Other related works

## flat-band bosonic systems

### Robustness(?) of Pair Luttinger Liquid

MR, Cataudella, Fazio PRB **73**, R100502 & I44511 ('06)

C. Cartwright, G. deChiara, MR PRB **98**, I84508 ('18)



B. Doucot, J. Vidal,  
PRL **88**, 227005 (2002)

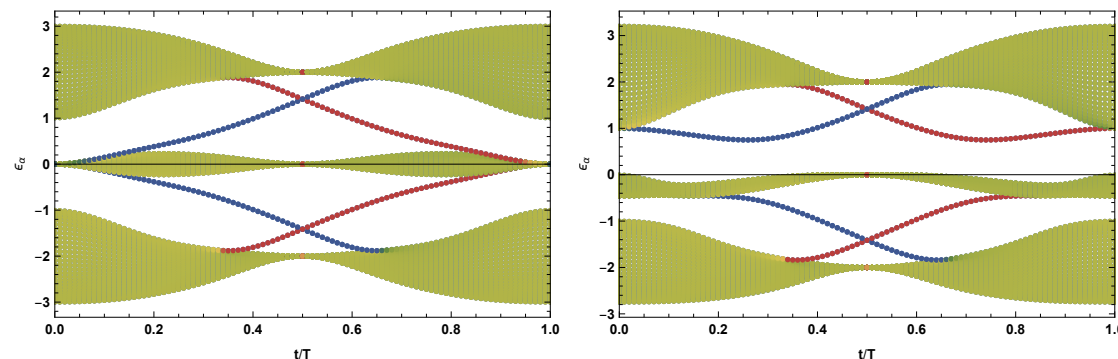
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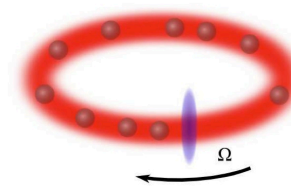
$$\mathcal{H}_{\diamond\pi}^2 = \mathcal{H}_{\text{trivial}} \oplus \mathcal{H}_{\text{SSH}}$$

M. Kremer, et al., I805.05209



## persistent currents

### “optimal” interactions vs quantum fluctuations

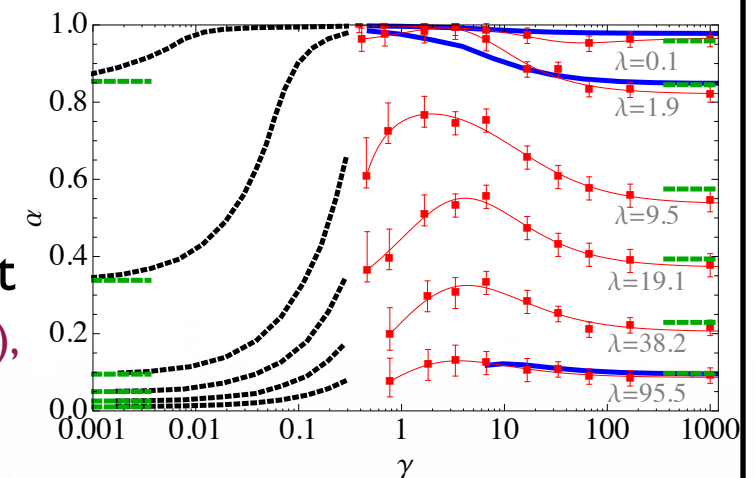


### “atomtronics” qubit

D. Aghamalyan, et al. (MR),  
NJP **17**, 045023 (2015)

### cMPS study

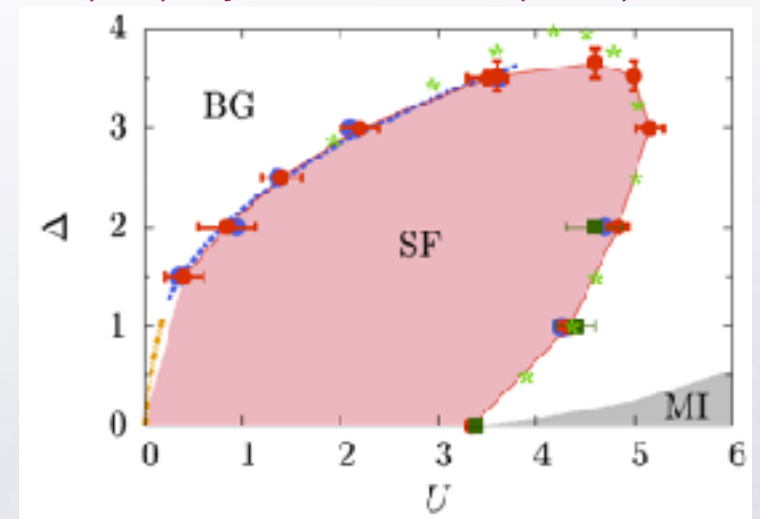
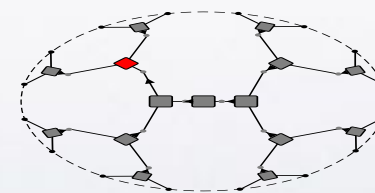
D. Draxler, et al. (MR),  
PRB **95**, 045145 (2017)



M. Cominotti, et al. (MR),  
PRL **113**, 025301 (2014)

### 1D disordered Bose-Hubbard: Bose Glass...

M. Gerster, et al. (MR) NJP **18**, 015015 (2016)





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Nathan Goldman  
Université libre de  
Bruxelles



Matteo Rizzi  
University of Cologne  
& Forschungszentrum  
Jülich



Ingmar Swart  
University of Utrecht



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D. Rossini (Uni Pisa)

M. Dalmonte (ICTP Trieste)

R. Fazio (ICTP Trieste)

S. Barbarino (SISSA Trieste)

M. Filippone (Uni Geneva)

L. Mazza (Paris-Sud)

R. Orus (Mainz > Donostia)

## My team @ JGU



J. Jünemann



M. Bischoff



A. Haller

+ N. Tausendpfund & J. Nothhelfer

# all of you for your attention!