A New Type of Symmetry Discovered in Nature

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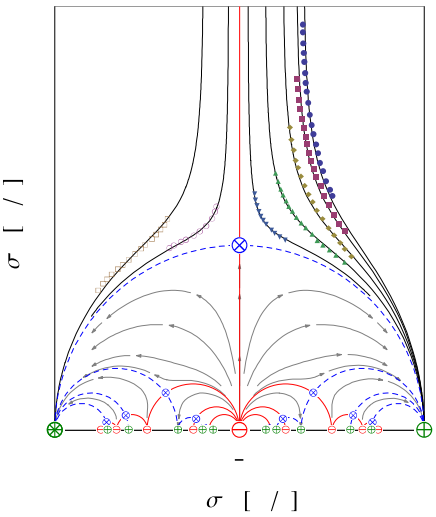
Although the QHE is one of the corner stones of modern metrology, four decades and several Nobel prizes after its discovery it continues to strain our understanding of quantum mechanics.

A comprehensive review of experiments probing the quantum Hall effect (QHE) provides substantial support for a conjecture that the morass of Hall data conceals a new type of symmetry of great utility and beauty, which is called *modular*.



The mathematical roots of these *infinite discrete symmetries* date back to Archimedes and Apollonius. Twenty-three centuries later they are central to many recent developments in string theory and mathematics, including the proof of Fermat's last theorem. Also familiar from some of M.C. Escher´s finest prints of nested fractal structures, like his famous "Angels and Demons" motif, this type of symmetry has not previously been seen in Nature.

By leveraging this symmetry an effective field theory capable of modeling all universal aspects of the QHE is developed to a point where it can be compared with scaling data. In order to exhibit the modular symmetry seen in experiments, the target space of this emergent sigma model is a torus.

Toroidal geometry permits a quantum equivalence known as mirror symmetry, which together with other results from string theory enables construction of the topological part of the partition function. This gives us access to the non-perturbative structure of the theory, including a phase diagram that automatically unifies the fractional and integer Hall effect.

While the location of quantum critical points and critical exponents derived from this model agree with numerical experiments at the *per mille* level, a reliable comparison with real experimental data awaits better finite size scaling experiments.

Application of the operator of invariant differentiation to integrating systems of ordinary differential equations

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We discuss the algorithm for integrating systems of ordinary differential equations (ODEs) of nth order admitting n-dimensional Lie algebras of operators. This algorithm is based on representation of given systems by differential invariants of admitted Lie algebra and on using operator of invariant differentiation (OID). We consider the conditions for constructing OID with special form coefficient for admitted Lie algebra. Also we present the modification of proposed algorithm for ODEs with small parameter admitting approximate Lie algebras of operators. We study on the structure of admitted approximate Lie algebra and conditions for constructing OID with special form coefficient for this Lie algebra. The applicability of described algorithms is demonstrated in several examples.

Abstract, Symmetry2019, 1-7 September 2019, Benasque

Asymmetric Electromagnetic Energy-Momentum Tensor: Remarks on the Spacelike Minkowski Four-Momentum

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Minkowski's energy-momentum tensor in phenomenological electrodynamics may be considered to have its origin in the breaking of Planck's principle of inertia of energy, usually expressed as $\mathbf{g} = \mathbf{S}/c^2$, where $\mathbf{g}$ is the momentum density and $\mathbf{S}$ is Poynting's vector. Instead, this relation in Minkowski's theory (superscript M) reads $\mathbf{g}^M = n^2\mathbf{S}/c^2$, n being the refractive index. This implies for a pure radiation field in matter that the energy-momentum tensor is divergence-free, $\partial_\nu T^M_{\mu\nu} = 0$, leading to the important property that the total field energy and momentum components constitute a four-vector. A characteristic property of this quantity is that it is spacelike, thus giving a negative radiation energy in some inertial frames. A typical case showing this phenomenon is the anomalous Doppler effect.

Asymptotic approximations of the hyperbolic umbilic diffraction catastrophe

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Different asymptotic approximations of the hyperbolic umbilic diffraction catastrophe, which describes part of the structurally diffraction patterns produced by optical diffraction, are presented. Asymptotic techniques are used in the derivation of these expansions valid for large values of some of the control variables. Numerical experiments illustrate the accuracy of the approximations.

Baby-skyrmions and spectral flow of the fermions in a thinner layer

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Topological solitons play an important role for understanding the physics of many branches, such as hadrons, condensed matter systems, and gravity. It is known that several notable structures of two-dimensional skyrmions are emerged in magnetic configurations or in liquid crystals, with special interaction so-called Dzyaloshinskii-Moriya type interaction (DMI). A mathematical theorem tells us that in standard nonlinear field theory models, the static solutions (the soliton solutions) possess the scale invariance and are unstable against a change of size moduli (Derrick-Hobart theorem), which is successfully evaded via the DMI. Another possible resolution is to rely on the special form of a quartic derivative term called Skyrme term. The model is sometimes called the baby-Skyrme model or the Skyrme-Faddeev model. As in the DMI, the Skyrme term is also genuine interaction in a condensed-matter system, for example known as a ring-exchange interaction in the Heisenberg model.

We consider a baby-Skyrme model in complex projective target space CP^N. These models possess the planar soliton solutions which can be considered as a certain realization of the magnetic excitation of the material. We also solve the fermionic model coupled with the model. Basic property of localizing mode of fermions on a topological soliton can be understood in terms of basis of Atiyah-Singer Index theorem, which may be realized numerically via spectral flow analysis (Fig.1).

Three-dimensional extension of the solutions may be realized from several nontrivial dynamics, such as a dipolar interaction of magnetization or a surface energy anchoring of a material. It certainly creates spontaneous symmetry breaking and then the spectral flow drastically is modified (Fig.2), which clearly indicates the breaking of the symmetry and emergence of the Majorana fermions. We shall present our current understandings for mathematical implications of our model.

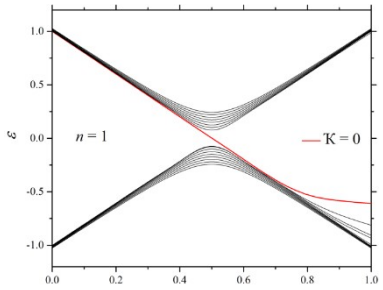

Fig.1 The typical spectral flow of the fermions in 2D skyrmions.

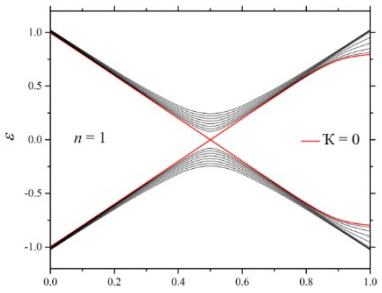

Fig.2 The typical spectral flow of the fermions in 3D skyrmions.

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We consider special classes of linear bounded operators in Banach spaces and suggest certain operator variant of symbolic calculus. It permits to formulate an index theorem and to describe Fredholm properties of elliptic pseudo-differential operator on manifolds with non-smooth boundaries.

Bouncing cosmology from warped extra dimensional scenario

(Eur.Phys.J. C77 (2017) no.12, 813)

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From the perspective of four dimensional effective theory on a two brane warped geometry model, we examine the possibility of “bouncing phenomena”on our visible brane. Our results reveal that the presence of a warped extra dimension lead to a non-singular bounce on the brane scale factor and hence can remove the “big-bang singularity”. We also examine the possible parametric regions for which this bouncing is possible.

BRS Symmetry and Cosmological Constant Problem

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In order to solve one of the problems related with the cosmological constant, we propose a topological field theory with an infinite numbers of the BRS symmetries. The BRS symmetries are, however, spontaneously broken in general but there is one and only one BRS symmetry which is not broken and the unitarity can be guaranteed. In the model, the quantum problem of the vacuum energy, which may be identified with the cosmological constant, reduces to the classical problem of the initial condition.
In this talk, after the brief review of the BRS symmetry, which exclude the negative norm states, we explain how one of the cosmological constant can be solved in this model.

Casimireffect and Lorentz invariance violation(LIV)

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The Casimir effect is one of the most direct manifestations of the existence of the vacuum quantum fluctuations, discovered by H. B. Casimir in 1948. On the other hand, Lorentz invariance is one of the main and basic concepts in physics for more than one century. In this work, using the corrections due to LIV on electric and magnetic fields, we calculate the corrections imposed by LIV on Casimir effect(force) and obtain an upper bound on the LIV parameter.

Casimir effect in interacting system

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Casimir effect stems from a vacuum fluctuation in a presence of boundaries. For non-interacting systems, Casimir effect has been well investigated and applictions are highly-diversified, from biology to high energy physics. We invesigate the Casimir effect in the presence of the interaction in an interval . In the case of the fermionic system, we found the sign change of the Casimir force. The case of the bosonic case is also investigated and qualitatively different behaviour is found. We also found that the Casimir force can be used to detect the phase transition. We consider a inhomogeneous superconducting state in which amplitude of the order parameter modulates, so-called Larkin-Ovchinnikov state. As a result, the discontinuous changes on the Casimir force at the phase transition points are found.

Casimir forces in inhomogeneous media

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Abstract. We consider Casimir and Casimir-Polder forces between bodies in inhomogeneous media, ones in which the permittivity and permeability vary with position as well as with frequency. First, we analyze the electromagnetic stress tensor and energy density within such a medium, and, using point-splitting, isolate both universal (Weyl) divergences and singularities as edges are approached. Then, we consider dielectric or conducting bodies immersed in such a medium. Unlike in a homogeneous background, the expressions for such forces are divergent. We can extract finite forces between bodies by subtracting the forces that would be present if a single body were immersed in the same inhomogeneous medium. This "renormalized" interaction between dielectric or conducting bodies, or between atoms or atoms and surfaces, satisfies the principle of virtual work, in that the change of the energy with the position of the bodies gives the force on the bodies. Explicit examples are given to illustrate the general ideas. The meaning of the background subtraction, and of the "bulk" and "scattering" contributions will be discussed.

Casimir forces on actuation dynamics between real materials towards chaos

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Understanding the vacuum state of a system is a challenge of fundamental physics and associated important technologies. If confined within boundaries, quantum vacuum fluctuations manifest themselves by the generation of Casimir forces. However, boundaries between interacting bodies possess in many cases nanoscale surface roughness, which is both difficult to avoid and control. At short separations of less than 200 nm, nanoscale roughness starts to play an important role on the Casimir interaction between the bodies and their adhesion upon contact. Indeed, control of this short-distance interaction is crucial for MEMS/NEMS, and adhesion technologies. Although the Casimir forces for flat bodies can be described partly by Lifshitz theory that takes into account the actual measured optical properties of the interacting materials, for rough surfaces the problem is more complicated by the nonadditivity of the dispersion forces. In my talk, I will discuss the current state of the problem between real materials with attention to metals, phase change materials, and poor conductors with respect to actuation dynamics of devices towards chaotic behavior in terms of Melnikov and Poincare map analysis. Finally, the consequences of the Drude-Plasma model uncertainty on the emergence of chaotic behavior will be also addressed.

Chiral dualism as a physical symmetry factor of hierarchical structure formation in biological systems

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Abstract. An introduction of a common symmetry factor – the chirality – offers a unifying approach in molecular biology when considering the physical nature of molecular hierarchies as the periodic system of chiral structures. This report presents the discussion of new phenomenon of an alternation of the chirality sign in the structural hierarchies of the intramolecular and supramolecular structural levels L-D-L-D in proteins and D-L-D-L in DNA, as well as in homochiral model systems. We substantiate the concept of the chiral dualism of carbon compounds being the physical base of the protein folding – directional structure formation. The homochirality of amino acids, riboses and deoxyriboses is a resource of free energy and a symmetry tool for stratification. The molecular chiral hierarchy extends to the scale of the cytoskeleton and its interaction with chromosome and membrane structures. A sign-alternating chiral hierarchy of structures in proteins and nucleic acids serves as the basis of the selected, but interconnected macroscopic/mechanical degrees of freedom in the constructions of macromolecular machines.

An important feature of complex natural systems is their hierarchy. We believe that the reproduction of characteristic patterns in self-organizing structures of different levels with changing physical mechanisms of structural formation is based on common symmetrical principles. As we have shown, the evolutionary formation of a basic system of clearly stratified levels of macromolecular and supramolecular biological structures was based on the principle of chiral dualism. In biological systems, a general systemic principle of spontaneous formation of discrete hierarchical structures in initial homohiral systems has been found. Its characteristic feature is manifested in the formation of a sequence of nested or parallel developing structures with a similar or changing type of symmetry, increasing scale and, importantly, changing sign of chirality. As we will explain below, homohiral systems are, in principle, non-equilibrium and capable of structural transformations in the course of free energy dissipation.

In macromolecular systems, sign-alternating hierarchies of chiral structures in sequences from the "lower" asymmetric carbon atom in sp3-hybridization to superhelices and supramolecular structures are identified as chiral invariants. The alternation of the chirality sign D-L-D-L has been noted in the transition to a higher level of structural and functional organization of DNA for the most widespread B-form. Deoxyribose molecules connected by phosphodiester bonds are D-isomers. The nucleotides including them are mainly located in the left gosh-conformation, which allows, due to their complementary connection by hydrogen bonds, to form the right-handed DNA double helix, which is the next level of organization. The subsequent super-spiralization of semi-flexible polymeric DNA chains manifests itself in the left-handed wraping of the right-handed double helixes, which can be observed in prokaryotes. It should be emphasized that we consider the linear sequence of deoxyriboses as the primary structure for B-DNA. This is due to the need to compare similar structural levels of DNA and proteins by chirality signs.

The sequence of change of the chirality sign in the structure-functional hierarchy of protein structures is similar to that observed for DNA, but it starts with the left enantiomer: L-D-L-D. Proteins are known to be linear polymers composed of L-amino acid residues. To form a secondary structure, the polypeptide chain make the right-handed α-helix or a folded β-layer.

The tertiary structure of proteins is represented by interacting α-helixes. Practically in all cases when the intramolecular close overlap of α-helixes is visually observed, an unambiguous tendency of their binding to the left superhelix is revealed. The quaternary structure of proteins is represented by supramolecular structures formed mainly by the right stacking of left superhelixes. The periodic table of the chiral primary-quaternary structures of DNA and proteins as a whole is an achiral invariant - a symmetric nucleus of molecular biology.

The stability of the sign-alternating chiral hierarchies is conditioned by the rigid conjugation of formation and degradation of their organization levels. In this respect, the principle of alternation of the chirality sign serves as a stabilizing latch for the multilayer structure. The type of symmetry in the sequence of chirality sign alternation can change: we see it in the transition from primary to secondary structures of protein or DNA (tetrahedral, spiral, gosh). Inversions in sign change are possible: for example, left-handed polyproline helix in collagens (not right-handed α-helix). In this case, the chiral dislocation is stabilized by the thermodynamic profitability of the integrating structure - the right-handed superhelix.

Note that chiral sign-alternating hierarchies are not necessarily limited to the internal structures of biomolecules, but can extend to their complexes, in which the elements of systemic recognition of the «friend or foe» and affinity will be determined by the sign of chiral correspondence of the corresponding levels of organization of molecules. In this connection, it is also necessary to note an important feature of the developed approach: chiral hierarchy can cover the space close to macromolecules in the water environment that "organizes" such information communications. Such properties of complex heterochiral structures can be directly related to the peculiarities of systems with high dilution and their long-term structural and functional effects.

From a physical point of view, proteins, nucleic acids, and membranes are molecular machines that perform useful work and transform the type of energy, substance, or information. Primary, secondary, etc. structures of macromolecules stratified by different symmetries and chirality sign are the details of hierarchical structures of molecular machines that implement the allocated "mechanical" degrees of freedom, kinetically separated from thermal degrees of freedom. Principally, that cyclically working machines with the need are dynamic chiral objects.

A well-known and unsolved problem of self-assembly of unique tertiary and quaternary structures of proteins and nucleic acids is the choice of optimal folding trajectories. The symmetrical nature of structural transformations in the folding processes determines the task of each subsequent step in the folding of one-dimensional chains of macromolecules. From the point of view of thermodynamics, their conformational motion to a minimum on the surface of the energy configuration funnel is just the motion along the chain of sign-alternating chiral states corresponding to the decrease of system free energy. The essential feature of any initial homohiral system is that it has a free energy reserve on the entropic component, so the whole system tends to racemize. In this case, there is not a horizontal racemization with the preservation of the type of symmetry, but a vertical hierarchical racemization. In artificial non-biological homohiral systems we have found numerous examples of spontaneous formation of sign-alternating structural chiral hierarchies too.

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CHRISTOFFEL DEFORMATIONS OF DISCRETE ENSEMBLES

PIERRE LAZAG

1. Abstract

In this talk, I will present Christoffel deformations of some discrete orthogonal polynomial ensembles and their scaling limits. Discrete orthogonal polynomial ensembles are probability measures on N-points configurations on a lattice, determined by a discrete weight. Multiplying the weight by a positive polynomial leads to the so-called Christoffel deformation of the system. I will focus on the Charlier and Meixner ensembles, and prove limit theorems leading to deformations of the discrete Bessel point process and of the process with the Gamma kernel introduced by Borodin and Olshanski.

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Classical Orthogonal Polynomials. Orthogonality and Duality

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In this contribution we show the who classification of classical orthogonal polynomials, continuous, discrete and q-discrete, and we will describe some of the duality relations among them, as well as some of the orthogonality relations in the complex plane. Some details about the symmetry about their parameters will be considered.

Classification of metric-affine geometries by spacetime symmetries

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While general relativity is most conventionally formulated in terms of the Levi-Civita connection of a metric, there exist alternative, so-called teleparallel formulations based either on the torsion or nonmetricity of an affine connection. The underlying geometries may be defined in terms of different objects - tetrads, spin connections, metrics and / or affine connections. In addition, theories of gravity are considered in which more than one of the aforementioned tensorial quantities is present. In order to find particular, exact or perturbative solutions to these theories, one usually considers an ansatz for the fundamental fields which possesses a certain amount of symmetry. I present a classification of such geometries based on their spacetime symmetries and the vanishing / non-vanishing of tensorial quantities, and show how they lead to a simplification of the field equations for a generic class of theories.

Compact boson stars and charged black holes in the gravitating CP^N nonlinear sigma model

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Q-balls are stationary soliton solutions of certain self-interacting, complex scalar field theories. The $U(1)$ invariance of the scalar field leads to the conserved charge Q which is identified with the electric charge of the constituents for theories coupled to the electromagnetic field. Compactons are extended objects of scalar fields emerging via finite support. The field configurations approach the vacuum at a finite distance and do not have infinitely extended tails. The scalar field models with standard kinetic terms and V-shaped potential gives rise to such compact-type Q-balls. The solutions coupled with the electric field, in terms of the balance of the forces, tend to be shell-like, which are called Q-shells. In the Q-shell solutions, the scalar field vanishes identically both inside a certain radius $r_{¿}$ and outside a certain radius r_{out}, and the matter exists only in the $r_{¿} \leq r \leq r_{out}$.

When the Q-balls (-shells) are coupled with gravity, boson stars (or boson shell stars) are obtained. For the boson shells, probably the most likely (or the interesting) is the case that the shells hold a flat Minkowski-space in the interior $r<r_{¿}$, while the exterior region $r>r_{out}$ becomes a Reissner-Nordstr$\acute{o}$m solution.

We consider a $U(1)$ gauged nonlinear sigma model in CP^Ntarget space coupled with Einstein gravity. There exist compact solutions for all odd numbers N, i.e., $N=2n+1, n=0,1,2,\ldots$. For $n=0,1$ the solutions form Q-ball, while $n\geq 2$ they always have shell structures. Furthermore, such Q-shells can involve a Schwarzschild black hole in the interior (empty) region of the shells (Fig.1).

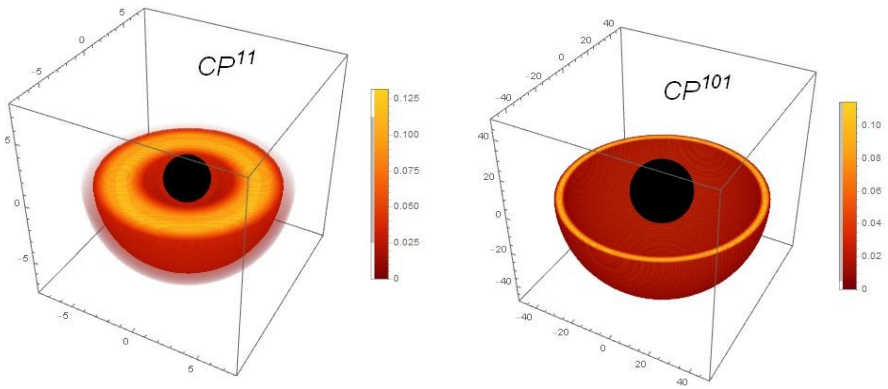

Fig.1 The density of the matter field with the black hole (black ball) for CP^{11} and CP^{101}.

CONFORMATIONAL DESTABILIZATION OF IFNg AND S100 IN THE PRESENCE OF HIGH DILUTIONS OF ANTIBODIES

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A specific innovative way, which possesses modifying properties towards structure and functioning of biological target, is the use of high dilutions (HD) of antibodies (released-active antibodies) [1].

Two facts have been established:

- conformational transition temperature of IFNg significantly ($p<0.05$) decreases in the presence of HD of antibodies to IFNg to 57.3C (vs 58.5C or 58.3C in the presence of water or HD of water, respectively);
- from the beginning to the end of conformational transition process of S100, temperature range significantly ($p<0.05$) increases to 5.9C in the presence of HD of antibodies to S100 (vs 4.7C and 3.2C for water and HD of water, respectively).

Thus, we have confirmed that HD of antibodies exert modifying effect on conformation of the respective target, which can underlie the mechanism of HD's regulating effect on corresponding biological process and their use in basic and applied medicine.

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Cosmological model of F(T) gravity with fermion fields via Noether symmetry

Myrzakulov Nurgissa

In this article, we investigate the modified F(T) gravity, which is non-minimally coupled with the Dirac (fermionic) field in Friedman-Robertson-Walker space-time. Point-like Lagrangian is derived and modified Friedmann equations and Dirac equations for the fermion field are obtained by using the Lagrange multiplier. The Noether symmetry method related to differential equations is a useful tool for studying conserved quantities. In addition, this method is very useful for determining the unknown functions that exist in the point-like Lagrangian. Using this method, the form of the coupling between gravity and matter, the self-consistent potential, the symmetry generators, the form of F(T) gravity and the first integral (Noether charge) or a conserved quantity for this model are determined. Cosmological solutions that have a power-law form and describe the late time accelerated expansion of the Universe are obtained. The Hubble parameter, pressure and energy density for the Dirac (fermion) field and the equation of state parameter are found.

Cosmological quantum entanglement : A possible testbed for the existence of Kalb-Ramond field

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In the present paper, we explore the possible effects of a second rank antisymmetric tensor field, known as Kalb-Ramond (KR) field, on cosmological particle production as well as on quantum entanglement for a massive scalar field propagating in a four dimensional FRW spacetime. For this purpose, the scalar field is considered to be coupled with the KR field and also with the Ricci scalar via the term $\sim \xi R\Phi^2$ (with ξ be the coupling). The presence of KR field spoils the conformal symmetry of a massless scalar field even for $\xi = 1/6$ in four dimensional context, which has interesting consequences on particle production and consequently on quantum entanglement as we will discuss. Moreover due to the presence of KR field, the upper bound of the entanglement entropy becomes larger in comparison to the case when the KR field is absent. This may provide an interesting testbed for the existence of Kalb-Ramond field in our universe.

Dual symmetries (dualities) of the QCD phase diagram

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Abstract

In this talk the phase structure of the dense quark matter has been investigated in the presence of baryon μ_B, isospin μ_I, chiral μ_5 and chiral isospin μ_{I5} chemical potentials in the framework of Nambu–Jona-Lasinio model. It has been shown that in the large-Nc limit (Nc is the number of colors of quarks) there exist duality correspondences (symmetries) at the phase portrait, which are the symmetries of the thermodynamic potential and the phase structure itself. The first one is a duality (symmetry) between the chiral symmetry breaking and the charged pion condensation phenomena. (This symmetry is, for example, called this way for it connects different phenomena when in addition one changes the matter content of the system.) And there are two other dualities that hold only for chiral symmetry breaking and charged pion condensation phenomena separately. For example, we have shown that charged pion condensation does not feel the difference between chiral and isospin imbalances of the medium. The duality between the chiral symmetry breaking and the charged pion condensation phases has been established for the first time in low-dimensional toy model for QCD, then it has been checked to take place in a more realistic effective model for QCD. They were shown to exist in the matter with chiral imbalance that can be produced in compact stars or heavy ion collisions. One of the key conclusions of these studies is the fact that chiral imbalance generates charged pion condensation in dense baryonic/quark matter.

It was also shown that our results in particular cases are consistent with the simulation of lattice QCD, which is possible in these cases. Moreover, in a number of works, catalysis of chiral symmetry breaking by chiral imbalance was predicted, but there were also works that predict the opposite effect. After lattice calculations that confirmed the effect of catalysis of chiral symmetry breaking, we can, of course, say that the problem has been solved, but it would be interesting to find out the reasons for the discrepancy between the results of effective models, moreover lattice QCD simulations had unphysically large pion mass. We have made a small step in this direction and showed that, based on the dualities and the well-studied properties of the phase structure of QCD with only isotopic imbalance, one can conclude that there should be an effect of catalysis of chiral symmetry breaking. It was also shown that the fact that the chiral imbalance generates the phenomenon

of charged pion condensation in dense baryonic/quark matter also remains true for the case of charge neutral matter, which is interesting in the context of the astrophysics of neutron stars.

It is known that chiral imbalance can occur in high energy experiments of the collision of heavy ions, due to temperature and sphaleron transitions. Our studies show that different types of chiral imbalance can occur in the cores of neutron stars or in heavy ion experiments, where large baryon densities can be reached, due to another phenomena - the so-called chiral separation and chiral vortical effects.

There have been also a lot of studies and hints that the ground state could be characterized by spatially inhomogeneous condensates, so the question arises if duality is a rather deep property of the phase structure or just accidental property in the homogeneous case. We have shown that even if the phase diagram contains phases with spatially inhomogeneous condensates, it still possesses the duality (symmetry) between the chiral symmetry breaking and the charged pion condensation phenomena. From various earlier studied aspects of QCD phase diagram of isospin asymmetric matter with possible inhomogeneous condensates, the unified picture and full phase diagram of isospin imbalanced dense quark matter have been assembled. Acting on this diagram by a dual transformation, we obtained, in the framework of an approach with spatially inhomogeneous condensates and without any calculations, a full phase diagram of chirally asymmetric dense medium. This example shows that the duality is not just entertaining mathematical property but an instrument with very high predictivity power. The obtained phase diagram is quite rich and contains various spatially inhomogeneous phases.

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Eliptic scenario of evolution of the universe

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In the framework of the Starobinsky model in homogeneous and isotropic space-time described by the Friedman-Robertson-Walker metric, the evolution of the universe described by a special-purpose scale factor with an anzatz in the form $\dot{H}|_{t=\infty} \to const$. Cosmological solutions from reductions of elliptic functions leading to hyperbolic functions are investigated. The inflation model of the universe was reconstructed in terms of a scalar field. For the selected model, obtained are of slow rollparameters, perturbation of curvatureand scalar to tensor ratio characterizing the era of inflation. Also obtained are the equation of state parameterdescribing the evolution of the universe at present time and deceleration parameter. The energy conditions are calculated for these models. Graphical dependencies of these parameters on cosmological time are constructed.

Enabling Semantics in Digital Mathematical Libraries

M. Schubotz

This talk summarizes recent developments in the field of digital mathematical libraries that are applicable for OPSF. We describe the shift from classical document-centric libraries to libraries of various forms of mathematical research data. We present the software database swMATH and the Wikimedia Community User Group Math as examples for initiatives that collect curate mathematical research data, without a direct link to individual publications. In the second part of the talk, we focus on the NIST Digital Repository of Mathematical Formulae (DRMF), which considers mathematical formulae as first-class objects. In particular, we outline how a one-time semantification effort using semantic LaTeX macros enables several math services such as search, and translation on different software platforms. In this context, we report on the strength and weaknesses of manual versus artificial-intelligence-driven methods and outline the potential of a hybrid approach.

Equilibrium measures under weakly admissible external fields

Joaquín F Sánchez-Lara

An admissible external field in $\mathbb{R}$ is a function \varphi:\mathbb{R}\ longrightarrow \mathbb{R}\cup\{+\infty\}$ which satisfies \begin{equation}\label{admis} \liminf_{|x|\to \infty}\frac{\varphi(x)}{\log|x|}=+\infty\,\end{equation} together with some other mild conditions. The theory of logarithmic potentials in an external field has been developed under assumption \eqref{admis} and establishes the existence of a unique measures μ_t minimizing the energy $$I^{\varphi}(\sigma)=\iint \log\frac{1}{|x-y|}d\ sigma(x)d\sigma(y)+2\int \varphi(x)d\sigma(x)\,,$$ among all the measures σ of total mass $t\in\mathbb{R}$. The support of μ_t is compact.

A weakly admissible external field is an external field where condition \eqref{admis} is replaced by a weaker version: there exists $T>0$ such that \begin{equation*} \liminf_{|x|\to \infty}\varphi(x)-T\log|x|\in\mathbb{R}\,.\end{equation*}
Now the equilibrium measures μ_t exists only for $t\leq T$, but $\Supp(\mu_T)$ is not necessarily bounded.

Our aim is to study the equilibrium measure in a weakly admissible external field with special interest in the external field created by several fixed charges. For this purpose we have to generalize the tools and results of logarithmic potentials to the case of measures with unbounded support.

Finally we will see some applications to the study of the asymptotics of some orthogonal polynomials.

Finite size effect on the chiral symmetry breaking

Tomohiro Inagaki and Hiromu Shimoji

Chiral symmetry breaking is one of the fundamental concepts to construct the standard model of particle physics. It is known that the broken symmetry can be restored under some extreme conditions. A four-fermion interaction model is a simple model in which the chiral symmetry breaking takes place. To find the critical phenomena at the phase boundary the model is often studied under extreme conditions.

In this presentation we employ the Gross-Neveu type model and investigate the finite size effect on the chiral symmetry breaking in arbitrary spacetime dimensions. For a homogeneous state the effective potential of the model is calculated in a cylindrical space with an U(1) valued boundary condition. The explicit expression of the effective potential is shown in the leading order of the 1/N expansion. A ground state is found by observing the minimum of the effective potential. The Casimir force is derived as the first derivative of the effective potential at the minimum. Evaluating the effective potential, we find the phase diagram for the chiral symmetry and the sign flip condition for the Casimir force.

Free Form Building Structures Arranged Regularly on Smooth Surfaces with Polyhedral Nets

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The paper is a new insight into interdisciplinary topics related to searching for regular parametric, innovative complex free form building structures roofed with multi-segment shell structures made up of nominally plane thin-walled folded steel sheets transformed into spatial shapes is discussed. Each such structure is created on the basis of a regular reference polyhedral structure formed from many complete reference tetrahedrons connected by common flat faces and arranged in a three-dimensional space on a smooth reference surface. The development of a parametric description of the reference structure enabled the implementation of the author's algorithms in computer programs.

Two subsequent straight lines k_i and k_{i+1} belonging to the same face of a reference structure, and being its side edges, must intersect. The position of k_i and k_{i+1} is defined based on the properties of the reference surface as follows. A finite number of points N_i arranged regularly on the reference surface is adopted. Straight lines t_i normal to the reference surface are passed through N_i. Since each two subsequent k_i and k_{i+1} must intersect, but two subsequent normals t_i and t_{i+1} are mutually skew, the author has developed an algorithm for the process of looking for the directions of k_i and k_{i+1} passing through N_i and optimally close to the directions of t_i and t_{i+1}. The specific properties, including symmetries of the reference surface and the manner of selection of points N_i affect the regularity of the shape and position of the reference tetrahedrons forming the reference structure.

Other way of shaping the complex free forms of buildings relies on setting together subsequent reference tetrahedrons, so that each pair of two adjacent tetrahedrons had one wall contained in the same plane. Parameterization of the shape and mutual position of all reference tetrahedrons forming the reference structure allowed to develop a method for shaping an innovative multi-wall spatial reference structures defined by means of a small number of independent variables so that each structure became regular, curved and consistent with the properties of almost any smooth surface characterized by negative, positive or zero Gaussian curvature.

Owing to the proposed innovative flat-walled spatial reference structure regularity and specific order in determining the shape and mutual position of the oblique interior and façade walls of building free forms can be obtained. Additional parameterization enables the implementation of the developed algorithms in computer programs employed to define the shape, position and orientation of these walls by means of regular reference structures.

The presented original method allows all reference tetrahedrons of a whole reference structure to be arranged, and thus also the whole composite building free forms roofed with multi-segment shell structures, in relation to various regular surfaces. The

benefits of the parameterization of the reference structure include the possibility of adopting appropriate values of selected independent variables, and specifying the relations between the dependent and independent variables in order to obtain the regularity and curvature while the shell segments of a roof structure and building walls in three dimensional space are being placed.

Parameterization of the flat-walled reference structure enables the determination of the flat roof directrices contained in the selected planes of the reference structure or spatial directrices based on these planes in a relatively simple manner. In addition, the structure gives the possibility for the forms of structural systems intended for the investigated complex building free forms to be shaped easily.

The use of regularity and symmetries in shaping the forms of: a) single tetrahedrons and the entire polyhedral structures, b) individual walls and the entire complex buildings, c) single folds and the entire transformed roof shells as well as their structures allows a creative search for rational, optimal solutions of individual building elements, including transformed roof shells and structural systems.

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$f(R,T)$ theory of gravity with f -essence

Y. Myrzakulov

In this work we have investigated a model of $f(R,T)$ theory of gravity with f -essence in flat, isotropy and homogeneous Friedmann-Robertson-Walker spacetime. The corresponding field equations are obtained. By using analytical method solution for scale factor a was obtained. It can be seen that this solution corresponds to the behaviour of late time evalution of Universe.

F(R,X,ϕ) cosmology solution involving one arbitrary function

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To describe modern observational data, we investigate one of the forms of modified gravity F(R, X, ϕ) — gravity model. On the one hand, this model generalizes the k-essence, that is the scalar field model ϕ with a kinetic term X. On the other hand, this model generalized F(R) -gravity, where the Ricci R scalar [1-2].

The action of F(R, X, ϕ) gravity has the following form [3-4]:

$$S_{43}=\int d^4x\sqrt{-g}\left[F(R,X,)+L_m\right], \tag{1}$$

where

$$R=\varepsilon_1 g^{\mu} R_{\mu}+u(a,\acute{a}), \tag{2}$$

$$X=\frac{1}{2}\acute{\varphi}^2 \tag{3}$$

Here L_m is the matter Lagrangian. The action of F(R,X, ϕ) gravity for the FRW metric has the following form:

$$S=2\pi^2\int dt\,a^3\left\{F(R,X,\phi)-\square_1\left[R-u+6\left(\frac{\acute{a}}{a}+\frac{\acute{a}^2}{a^2}\right)\right]-\square_2\left[X-\frac{1}{2}\acute{\varphi}^2\right]\right\}. \tag{4}$$

From the symmetry properties we have:

$$F(R,T,X,\phi)=e^{C_1R}+C_2R+C_3X+C_4\varphi^2+C_5 \tag{5}$$

were $C_1\ldots C_5$- constants.

Here considered the F(R,X,ϕ) gravity model and found an expression for an action with one arbitrary function.

The result coincides with observational data [5-6].

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Fundamental spin fields in conformal field theories

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Conformal field theory (CFT) is a unified framework for studying various critical phenomena. Modern developments such as those in the Schramm-Loewner evolution and in the numerical conformal bootstrap point to the importance of the one-parameter family of CFTs with varying central charges. In particular, the unitarity bounds in the conformal bootstrap for the Z_2 sum rule, which applies to the standard CFTs with Virasoro symmetry, is almost saturated by the curve of the critical O(n) models with $-2 \le n \le 1$, where the key step is identifying the fundamental spin field at rational Kac indices (1/2, 0).

Using this identification, it is possible to observe the $\Gamma(2)$ congruence subgroup symmetry on the Poincare disk in the analytic structure of the operator product coefficients for infinitely many primary operators. Recently, we have identified the fundamental spin field at certain rational Kac indices also in generic CFT with W_3 symmetry. We single it out by requiring the conditions that the full set of operators in a given charge sector is generated upon fusion and obtained a generalization of the Z_3 fundamental spin field of the three-state Potts model. By relaxing the conditions partly, we obtained two other versions of the spin field. As vertex operators, these three spin fields have the charge vectors plotted in Figure 1. As in the Virasoro case, the obtained exact curves for conformal dimensions are expected to pass very close to the unitarity bounds. The talk is based on arXiv: 1906.01323 with Y. Ikhlef and 1803.06938.

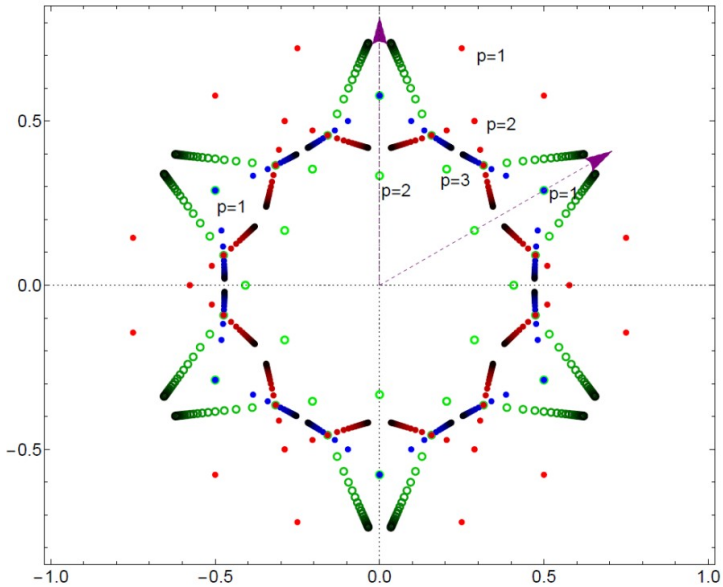

Figure 1:
The vertex charges of the three spin fields for generic W_3 CFT with central charge $c=2-24/(p(p+1))$. They coincide with each other at $p=4$, and tend to the face center (blue), the mid-edge (red), and the vertex (green) of the fundamental triangle spanned by the weight vectors

(purple) as $p \to \infty$.

Generalized symmetry structures for differential equations

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Abstract

One important and very useful application of continuous symmetry groups is in the study of differential equations [1,2,3], especially for finding group-invariant solutions and for mapping a given solution into a family of solutions. The generators of a continuous symmetry group have a natural geometrical formulation as vector fields that are tangent to the solution space of a differential equation in the jet space of the independent variables, dependent variables, and their derivatives.

In this setting, an interesting generalization of symmetries is given by adjoint-symmetries [3,4], which have a direct connection to conservation laws. Adjoint-symmetries have a geometric formulation as covector fields defined through a natural pairing with symmetry vector fields from the Frechet derivative of a differential equation.

These two structures can be generalized by studying tensor fields in the same geometrical setting, which will be presented in this talk. Some known examples [1] are symmetry recursion operators, which can viewed as tensors of type (1,1); symplectic (Hamiltonian) operators, which can viewed as tensors of type (0,2); cosymplectic operators, which can viewed as tensors of type (2,0).

New applications of these tensors to finding solutions of a differential equation will be discussed. Other types of tensors can be expected to have a variety of interesting applications as well, which will be outlined.

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Inflationary parameters for the general $F(R)$ gravity

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The thermal equilibrium of the cosmic microwave background (CMB) inside the curre nt horizon requires the horizon and the flatness problems of the universe. The inflatio n, accelerating expansion of the universe at the beginning, is proposed to solve thes e cosmological problems. It is expected that the quantum fluctuations grow up during the inflation and to be seed of the galaxy, and it generates the thermal fluctuations of CMB. Hence, there is a possibility to test the model of the inflation in the fluctuations of CMB.

Planck satellite observed details of the fluctuations of CMB and found some in flationary parameters, the amplitude of the scalar power spectrum A_s, the scalar spe ctral index *ns* and running one α_s, the tensor-to-scalar ratio*r*, and non-Gaussianity f_{NL} [1, 2]. The model of the inflation should derive the small fluctuations of the CMB a nd should be consistent with these observational values. The one of the motivation t o research the modified gravity is to reveal the phenomenon for the expansion of uni verse. Due to the development of observation technology, we have reached to consi der the concrete model and its classification. Among the various modified gravity mo del, $F(R)$ gravity, which is a general form of the Ricci scalar R, is the popular and an extra degree of freedom plays a role of the scalar field called inflaton after the transfo rmation to ordinary gravity theory [3].

In general case of the scalar field theory, the kinetic term can be written by t he function$X = G_{ij}\partial_\mu \varphi^i \partial^\mu \varphi^j$ where G_{ij} is the metric of the coordinate φ. Applying the co variant formalism [4] for the inflaton to the $F(R)$ gravity, we can deal with non-canoni cal kinetic term of the scalar field, so that the inflaton field can be chosen as Ricci sc alar. The advantage of this formalism is that the inflaton potential is clear and the infl ationary parameters are described only by the function F and its derivatives. If the rel ation between the e-folding N and the inflaton, we can found the inflationary paramet ers quantitatively and access to the gravity theory at early stage of the universe.

The future research is

1. to study the unification model of dark energy and dark matter,
2. no-ghost higher derivative quantum field theory,
3. the connection to the standard model of the particle physics.

We hope to report some results to these researches in the future.

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Infrared problems of QED and Lorentz Invariance

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Abstract

One of the striking features of QED is the appearance of infrared divergences in the presence of charged particles. In fact, the infrared properties of the dressed electronic state are in conflict with Lorentz symmetry. These infrared pathologies may be very relevant for understanding the black hole information paradox, but also raise some questions about the consistency of the whole theory.

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Lorentz invariance violation(LIV) in atomic physics

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Lorentz symmetry is one of the cornerstone of both general relativity and the standard model of particle physics . We study the violation of Lorentz symmetry in atomic physics. using the Green's function, and the source 4-current, the differential equation of 4-vector of electromagnetic potential is solved and the modified coulomb potential is obtained. We find the corrections due to LIV on the spectrum of Hydrogen and the Helium atoms, we also investigate the consequence of LIV on Stark, Zeeman and Spin orbit effects and obtain an upper bound for the LIV coefficient.

Mirror Breaking Symmetry of Non-Rigid Luminophores in Solutions by Circularly Polarized Luminescence and Circular Dichroism Spectroscopy

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We investigated systematically whether nearly 50 kinds of non-rigid π-conjugated luminophores in the photoexcited (S_1) and ground (S_0) states dissolved in achiral solvents with various viscosity [η] are mirror symmetrical by means of circularly polarized luminescence (CPL) and circular dichroism (CD) spectroscopy.[1,2] The luminophores involves oligofluorenes, linear/cyclic oligo-*p*-arylenes, fused aromatics, binaphthyls, scintillators, coumarins, BODIPY, rhodamine B, and DCM. Without exception, all non-rigid luminophores showed negative-sign CPL signals in the UV-visible region, suggesting temporal generation of energetically non-equivalent non-mirror image structures as far-from equilibrium open-flow systems at the S_1 state. When the fluorene ring number increased, the disymmetry ratio of CPL (g_{lum}) extrapolated at [η] = 0 reached -0.8×10^{-3} at 420 nm, leading to (–)-CPL signals predicted in the vacuum state. For comparison, unsubstituted achiral rigid planar luminophores, including naphthalene, anthracene, tetracene, and pyrene, did not show obvious CPL/CD signals. Moreover, a rigid chiral *D*/*L*-camphor showed mirror-image CPL/CD signals. Our comprehensive CPL and CD experimental tests should provide a possible answer to the molecular parity violation hypothesis by weak neutral current at molecular level mediated by Z_0-boson.

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Modified Dirac equation with Lorentz invariance violation and some its consequences

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Using Dirac equation in the presence of Lorentz invariance violation (LIV) we derive the effects of LIV on the spectrum of relativistic Hydrogen atom (four components formalism). On the other hand, there is a difference between theoretical and experimental values of muon anomalous magnetic moment (g- factor). It can be affected by physics beyond the Standard Model, so it has been measured very precisely . We also investigate the effects of LIV on muon g factor and using the accuracy of recent g measurement we obtain an upper bound on LIV parameter.

More Precise Symmetric Descriptions for Properties of the Askey-Wilson Polynomials and their Symmetric Sub-Families

Authors: Howard S. Cohl, Roberto S. Costas-Santos, Linus Ge

We give a more precise symmetric parametric description for various properties of the Askey-Wilson polynomials including hypergeometric and q-integral representations. We study the symmetric q-inverse sub-families of the Askey-Wilson polynomials. We also study the continuous q-inverse ultraspherical/q-inverse Rogers polynomials. We examine basic hypergeometric representation and transformation formulae, limit transitions, connection relations, and generating functions and corresponding q-integrals for these families. We have also focused on the q-inverse Rogers generating function. This generating function has the intriguing property in that it is able to cross the natural boundary at q=1. Using this generating function, we compute a q-inverse analogue of the Ismail-Simeonov expansion for the q-inverse Rogers generating function. This leads to a new terminating quadratic transformation for basic hypergeometric functions.

New metrics of a spherically symmetric gravitational field

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Abstract

A good model of a physical problem preserves all of the symmetries inherent in the problem. However, if the model introduces additional symmetries not present in the problem, the consequences should be tested to check whether the model fits the physical reality.

Most tests of General Relativity (GR) are based on the Schwarzschild solution of Einstein's field equations for a spherically symmetric, non-rotating body. This solution preserves all the symmetries of the problem, but also predicts the equality of the speed of light propagating away from and toward the gravitating body, a symmetry which is not inherent in the problem.

We found an infinite series of metrics, all of which preserve the spherical symmetry and pass all the tests of GR. However, these metrics predict different velocities for light propagating away from and toward the gravitating body. One of these metrics is analytic everywhere and predicts the speed of light to be constant in the direction of the gravitating body. We present possible experiments to check these predictions.

Our model, called *Relativistic Newtonian Dynamics*, passes all tests of GR. Moreover, it is also applicable to non-gravitational forces and has the potential to describe also the quantum region.

Introduction

In GR, the gravitational force curves spacetime. GR is a direct application of Riemann's ``force equals geometry.“ Relativistic Newtonian Dynamics (RND)[1] assumes that for any given particle, there is a metric, defined in a preferred inertial frame, which is a function of the total potential of the forces which affect the particle. Since an inanimate object has no internal mechanism with which to change its velocity, the object moves along a *geodesic* with zero acceleration.

Most tests of GR are based on the Schwarzschild solution [2] of Einstein's field equations for a spherically symmetric, non-rotating body. The RND model was applied [3[to such a field and the results are presented here.

Results

Consider the motion of an object in a static, spherically symmetric gravitational field. We use spherical coordinates (ct, r, θ, φ) in our preferred inertial frame and

place the origin of our frame at the center of the symmetry of the field. It was shown that the metric of this field must be of the form

$$ds^2=(1-u(r))c^2dt^2-\frac{1-g_{01}(r)}{1-u(r)}dr^2-2c\,g_{01}(r)dtdr-r^2(d\theta^2+\sin^2\theta\,d\varphi^2)$$

with $u(r)=\frac{2GM}{c^2r}$, in order to satisfy all the symmetries of the problem, preserve angular momentum for geodesic motion (following from the symmetry), satisfy the Newtonian limit and pass all classical tests of GR.

In case $g_{01}(r)=0$, we recover the Schwarzschild metric, for which the speed of light propagating away from and toward the gravitating body is the same. Surprisingly, this implies that light *slows down* as it approaches the gravitating body. If we take $g_{01}(r)=u(r)$, the metric becomes

$$ds^2=d\eta^2-u(r)(cdt+dr)^2,$$

where $d\eta^2$ is the Minkowski metric in spherical coordinates. This metric is analytic everywhere except the origin and implies that the speed of light approaching the gravitating body remains c - the speed of light in vacuum, as expected.

An experiment measuring the one-way speed of light both toward and from the gravitating body can reveal the true value of $g_{01}(r)$. This can also be revealed by accurate measurments of the orbital velocities of stars with high eccentricity near black holes.

RND extends to multiple non-static forces, each of which obeys an inverse square law and whose field propagates with the speed of light. Our approach is also applicable to non-gravitational fields, in particular, the electromagnetic field. Application of this model to the dynamics of an atom predicts some quantum behavior.

Conclusions

Any model of a physical problem must preserve the symmetries present in the problem, but one must also test additional symmetries that are present in the model to see if they actually exist in reality. The consequences of the additional symmetries should be explored and tested for the validity of the model.

We have shown that the additional symmetry of the Schwarzschild solution for a spherically symmetric gravitational field of GR can be removed preserving all the symmetries of the physical problem and passing all the tests of GR. The new predictions of this model are reasonable, and they can and should be tested to be able to identify the true model of such a field.

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Noncommutative covariant symmetric spacetimes and quantum observers

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In this talk I will discuss two recent important results concerning quantum group defor- mations of the groups of isometries for the three maximally symmetric spacetimes of constant curvature, Poincaré and (anti-)de Sitter. These two results have in common its construction using techniques from Poisson homogeneous spaces, followed by their quantization.

The first result is the construction of the generalization of the well-known κ-Minkowski spacetime to the case of a non-vanishing cosmological constant [1], an important result both theoretically and from the point of view of quantum gravity phenomenology.

The second result is a recently proposed idea to describe quantum observers from quan- tum group symmetries [2]. This idea heavily relies on the fact that for simply connected Lorentzian spaces of constant curvature, as the ones previously mentioned, their space of oriented time-like geodesics is itself a homogeneous space, so the same Poisson homogeneous space techniques can be applied. The important example of the κ-Poincaré quantum group will be explicitly constructed, and some ongoing research relating it to quantum gravity effects will be mentioned.

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Observation-based symmetry breaking measures

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The study of symmetry is central in physics. Symmetry considerations are among the most general statements that we can make about physical systems. Given a system and a symmetry, the question *is the system symmetric?* leads to a binary outcome. A positive answer implies quantitative constrains like the conservation laws in Noether's theorem [1]. A negative answer precludes the same constraints. Often, symmetry breaking is just considered in this binary way. An object is chiral if it cannot be superimposed onto its mirror image, and otherwise it is achiral. In the ferromagnetic phase transition, the alignment of the spins breaks the rotational symmetry. Questions like *how chiral is an object?* or, h*ow much broken is the rotational symmetry?* are typically not considered. The potential physical significance and uses of their answers are then left untapped.

In a recent contribution [2], I have introduced the following measure of symmetry breaking for a system interacting with external fields (particles).

$$M(S,X)=(1/2)||S-XSX^{-1}||_F^2/(||S||_F^2+||XSX^{-1}||_F^2), \qquad (1)$$

where S is the scattering operator of the system, X the linear operator representing the symmetry under consideration, and $||(.)||_F$ represents the Frobenius norm for operators. Despite appearances, the measure in Eq. (1) can be experimentally obtained without having to measure the full operator S. In my talk, I will explain the physical motivation behind Eq. (1), provide examples, and present preliminary result for the connection between Eq. (1) and physical effects using the example of electromagnetic back-scattering efficiency.

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ON CLASSIFICATION OF SYMMETRY REDUCTIONS FOR THE EULER-LAGRANGE-BORN-INFELD EQUATION

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Symmetry reduction is one of the most powerful tools for the investigation of PDEs with non-trivial symmetry groups.

According to the classical group analysis approach, the main classification of symmetry reductions and invariant solutions for PDEs with nontrivial symmetry groups should be performed by using ranks of nonconjugate subalgebras of Lie algebras of symmetry groups of the equations under investigations.

However, it turned out that the reduced equations, obtained with the help of nonconjugate subalgebras of the same ranks of the Lie algebras of the symmetry groups of some PDEs, were of different types.

Recently, we suggested to use the structural properties of nonconjugate subalgebras of the same ranks of the Lie algebras of the symmetry groups of the equations under consideration in order to try to explain some of the differences in the properties of the reduced equations.

In our talk, we plan to present some of the results concerning the relationship between the structural properties of three-dimensional nonconjugate subalgebras of the Lie algebra of the generalized Poincaré group P(1,4) and the types of symmetry reductions for the Euler-Lagrange-Born-Infeld Equation.

More details on this theme can be found in:

1. Fedorchuk V., Fedorchuk V. On Classification of Symmetry Reductions for the Eikonal Equation. Symmetry, 2016, 8, Art. 51, 32pp; doi:10.3390/sym8060051.

2. Fedorchuk V., Fedorchuk V. Classification of Symmetry Reductions for the Eikonal Equation. - Lviv: Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of National Academy of Sciences of Ukraine, 2018, 176 pp.

On Classification of Symmetry Reductions for fhe Euler-Lagrange-Born-Infeld equation

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Abstract

One important and very useful application of continuous symmetry groups is in the study of differential equations [1,2,3], especially for finding group-invariant solutions and for mapping a given solution into a family of solutions. The generators of a continuous symmetry group have a natural geometrical formulation as vector fields that are tangent to the solution space of a differential equation in the jet space of the independent variables, dependent variables, and their derivatives.

In this setting, an interesting generalization of symmetries is given by adjoint-symmetries [3,4], which have a direct connection to conservation laws. Adjoint-symmetries have a geometric formulation as covector fields defined through a natural pairing with symmetry vector fields from the Frechet derivative of a differential equation.

These two structures can be generalized by studying tensor fields in the same geometrical setting, which will be presented in this talk. Some known examples [1] are symmetry recursion operators, which can viewed as tensors of type (1,1); symplectic (Hamiltonian) operators, which can viewed as tensors of type (0,2); cosymplectic operators, which can viewed as tensors of type (2,0).

New applications of these tensors to finding solutions of a differential equation will be discussed. Other types of tensors can be expected to have a variety of interesting applications as well, which will be outlined.

On one-loop divergences in six-dimensional supersymmetric gauge theories

I.L. Buchbinder (TSPU, Russia)

We study the 6D, N=(1,0) and N=(1,1) supersymmetric gauge theories formulated in harmonic superspace. These theories are non-renormalizable on power counting. The superfield background field method is developed which allows us to preserve the manifest gauge invariance and N=(1,0) supersymmetry at all steps of calculating the effective action. Structure of the one-loop divergences is studied and it is shown that the N=(1,1) gauge theory is off-shell finite.

On some Fredholm operator families

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We consider special classes of linear bounded operators in Banach spaces and suggest certain operator variant of symbolic calculus. It permits to formulate an index theorem and to describe Fredholm properties of elliptic pseudo-differential operator on manifolds with non-smooth boundaries.

ON THE INTEGRABLE CHAPLYGIN TYPE HYDRODYNAMIC SYSTEMS AND THEIR GEOMETRIC STRUCTURE

YAREMA A. PRYKARPATSKY, DENIS BLACKMORE, AND ANATOLIJ K. PRYKARPATSKI

Abstract. We investigate geometric structures of a class of spatially one-dimensional completely integrable Chaplygin type hydrodynamic systems, which proved to be deeply connected with differential systems on the complexified torus and the related diffeomorphism group orbits on them. These geometric structures made it possible to find an additional relationship between seed differential forms on the torus and describe an new infinite hierarchy of integrable Chaplygin type hydrodynamic systems.

It is well known that investigation of integrable models or solvable nonlinear partial differential equations is an active area [5, 7, 15] of research since the discovery of the inverse scattering method. In our report we are mainly interested in studying geometrical structures, which characterize the classical Chaplygin type hydrodynamical flows, being important for describing both their exact solutions and related mathematical structures, responsible for their properties.

The report is organized as follows: In Part II we review or at least introduce some basic notions and mathematical constructions, which lie in a background of the Lie differantial-geometric approach to studying integrable Lax-Sato type dispersionless differential equations. In Part III we study geometric structures of the one-dimensional completely integrable Chaplygin hydrodynamic system, which proved to be deeply connected with differential systems on the complexified torus and the related diffeomorphism group orbits on them. This geometric structure made it possible to find an additional relationship between seed differential forms on the torus and describe a new related infinite hierarchy of integrable Chaplygin type hydrodynamic systems, to which there is devoted Part IV. These systems, as it was demonstrated in [4], are closely related with a class of completely integrable Monge type equations, whose geometric structure was also recently analyzed in [6], using a different approach, based on the Grassman manifold imbedding properties of general differential systems defined on jet-submnifolds. The latter poses an interesting problem of finding relationships between different geometric approaches to describing completely integrable dispersionless differential systems.

We have studied the geometric structure of the following Chaplying [1, 9, 14] hydrodynamic system

$$u_t = -uu_x - kv_x v^{-3}, \quad v_t = -(uv)_x, \tag{0.1}$$

where $k \in \mathbb{R}$ is a constant parameter, $(u, v) \in M \subset C^\infty(\mathbb{R}/2\pi\mathbb{Z}; \mathbb{R}^2)$ are 2π-periodic dynamical variables on the functional manifold M with respect to the evolution parameter $t \in \mathbb{R}$. To describe the geometric structure of the h system (0.1), let us define the loop Lie algebra $\tilde{\mathcal{G}} := \widetilde{diff}(\mathbb{T}^1_{\mathbb{C}})$ on the one-dimensional complexified torus $\mathbb{T}^1_{\mathbb{C}}$ and take a seed element $\tilde{l} \in \tilde{\mathcal{G}}^*$ in the following form:

$$\tilde{l} = \left[(\frac{1}{8}\alpha_x + uu_x)\lambda + \frac{1}{2}u_x\lambda^3\right]dx + \left[\frac{3}{8}(\alpha + 4u^2) \ + \frac{5}{2}u\lambda^2 + \lambda^4\right]d\lambda, \tag{0.2}$$

where we have put $\alpha := kv^{-2} + u^2$, and calculate gradients of some Casimir functionals $h^{(y)}, h^{(t)}$and $h^{(s)} \in \mathrm{I}(\tilde{\mathcal{G}}^*)$:

$$\nabla h^{(t)}(l) = \lambda^2 \nabla h^{(2)}(l), \nabla h^{(y)}(l) = \lambda^2 \nabla h^{(4)}(l), \nabla h^{(s)}(l) = \lambda^2 \nabla h^{(6)}(l), \tag{0.3}$$

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where

$$\begin{aligned}(0.4)\qquad \nabla h^{(2)}(l) &= \begin{pmatrix} -2 \\ 0 \end{pmatrix}\lambda^0 + \begin{pmatrix} 0 \\ u_x \end{pmatrix}\lambda^{-1} + \begin{pmatrix} u \\ 0 \end{pmatrix}\lambda^{-2} + O(\lambda^{-3}) \\ \nabla h^{(4)}(l) &= \begin{pmatrix} -8 \\ 0 \end{pmatrix}\lambda^2 + \begin{pmatrix} 0 \\ 4u_x \end{pmatrix}\lambda^1 + \begin{pmatrix} -4u \\ 0 \end{pmatrix}\lambda^0 + \\ &+ \begin{pmatrix} 0 \\ \alpha_x \end{pmatrix}\lambda^{-1} + \begin{pmatrix} \alpha \\ 0 \end{pmatrix}\lambda^{-2} + O(\lambda^{-3})\end{aligned}$$

and

$$\begin{aligned}(0.5)\qquad \nabla h^{(6)}(l) &= \begin{pmatrix} -2 \\ 0 \end{pmatrix}\lambda^4 + \begin{pmatrix} 0 \\ u_x \end{pmatrix}\lambda^3 + \begin{pmatrix} -3u \\ 0 \end{pmatrix}\lambda^2 + \\ &+ \begin{pmatrix} \\ \alpha_x/4 + uu_x \end{pmatrix}\lambda^1 + \begin{pmatrix} -\alpha/4 - 1/2u^2 \\ 0 \end{pmatrix}\lambda^0 + \\ &+ \begin{pmatrix} 0 \\ -(u\alpha)_x/8 \end{pmatrix}\lambda^{-1} + \begin{pmatrix} u\alpha/8 \\ 0 \end{pmatrix}\lambda^{-2} + O(\lambda^{-3}).\end{aligned}$$

as $\lambda \to \infty$. The corresponding Lax-Sato vector field generators are, by definition, equal to the expressions

$$\begin{aligned}(0.6)\qquad \nabla h^{(t)}_+(l) &: = (\lambda^2 \nabla h^{(2)}(l))|_+ = \begin{pmatrix} -2 \\ 0 \end{pmatrix}\lambda^2 + \\ &+ \begin{pmatrix} 0 \\ u_x \end{pmatrix}\lambda^1 + \begin{pmatrix} u \\ 0 \end{pmatrix}\lambda^0, \\ \nabla h^{(y)}_+(l) &: = (\lambda^2 \nabla h^{(4)}(l))|_+ = \begin{pmatrix} -8 \\ 0 \end{pmatrix}\lambda^4 + \\ &+ \begin{pmatrix} 0 \\ 4u_x \end{pmatrix}\lambda^3 + \begin{pmatrix} -4u \\ 0 \end{pmatrix}\lambda^2 + \\ &+ \begin{pmatrix} 0 \\ \alpha_x \end{pmatrix}\lambda^1 + \begin{pmatrix} \alpha \\ 0 \end{pmatrix}\lambda^0,\end{aligned}$$

and

$$\begin{aligned}(0.7)\qquad \nabla h^{(s)}_+(l) &: = (\lambda^2 \nabla h^{(6)}(l))|_+ = \begin{pmatrix} -2 \\ 0 \end{pmatrix}\lambda^6 + \begin{pmatrix} 0 \\ u_x \end{pmatrix}\lambda^5 + \\ &+ \begin{pmatrix} -3u \\ 0 \end{pmatrix}\lambda^4 + \begin{pmatrix} 0 \\ \alpha_x/4 + uu_x \end{pmatrix}\lambda^3 + \\ &+ \begin{pmatrix} -\alpha/4 - 1/2u^2 \\ 0 \end{pmatrix}\lambda^2 + \begin{pmatrix} 0 \\ -(u\alpha)_x/8 \end{pmatrix}\lambda^1 + \begin{pmatrix} u\alpha/8 \\ 0 \end{pmatrix}\lambda^0\end{aligned}$$

for any $\lambda \in \mathbb{C}$.

Based now on the gradient expressions (0.6) and (0.7), one can calculate successively the following evolution flows:

$$(0.8)\qquad \partial\tilde{l}/\partial t = -ad^*_{\nabla h^{(t)}_+(l)}\tilde{l} \;\sim\; \left.\begin{array}{c} u_t = -uu_x - kv_xv^{-3}, \\ v_t = -(uv)_x, \end{array}\right\}$$

with respect to the evolution parameter $t \in \mathbb{R}$, being equivalent to the hydrodynamical system (0.1),

$$(0.9)\qquad \partial\tilde{l}/\partial y = -ad^*_{\nabla h^{(y)}_+(l)}\tilde{l} \;\sim\; \left.\begin{array}{c} u_y = -u^2u_x + k(uv^{-2})_x \\ v_y = -\left((kv^{-2} + u^2)v\right)_x \end{array}\right\}$$

with respect to the evolution parameter $y \in \mathbb{R}$, and

$$(0.10)\qquad \partial\tilde{l}/\partial s = -ad^*_{\nabla h^{(s)}_+(l)}\tilde{l} \;\sim\; \left.\begin{array}{c} u_s = \left(6ku^2v^{-2} + 3k^2v^{-4} - u^4\right)_x/12 \\ v_s = -\left(kuv^{-1} + u^3v)\right)_x/3 \end{array}\right\}$$

with respect to the evolution parameter $s \in \mathbb{R}$. As by construction, all these flows are commuting to each other, that can be rewritten as the following set of commuting to each other Lax-Sato type vector fields on the complexified torus $\mathbb{T}^1_{\mathbb{C}}$ for all parameters t, y and $s \in \mathbb{R}$, giving rise to three diverse systems of integrable heavenly type dispersionless differential equations. The obtained above result can be formulated as the following theorem.

Theorem 0.1. *The Chaplygin hydrodynamic system (0.8) is equivalent to the completely integrable Hamiltonian system (0.10), generated by the seed-element (0.2) on the adjoint space $\tilde{\mathcal{G}}^*$ to the loop Lie algebra $\tilde{\mathcal{G}} \simeq \widetilde{diff}(\mathbb{T}^1_{\mathbb{C}})$ of vector fields on the complexified torus $\mathbb{T}^1_{\mathbb{C}}$. The related Casimir functionals on $\tilde{\mathcal{G}}^*$ generate an infinite hierarchy of commuting to each other additional both Hamiltonian systems, like (0.9) and (0.10), and Lax-Sato type vector fields on $\mathbb{T}^1_{\mathbb{C}}$, resulting in some new heavenly type dispersionless equations.*

As it was demonstrated in [4] Chaplygin hydrodynamic system (0.8) is closely related with a class of completely integrable Monge type equations, whose geometric structure was also recently analyzed in [6], using a different approach, based on the Grassman manifold imbedding properties of general differential systems defined on jet-submnifolds. The latter poses an interesting problem of finding relationships between different geometric approaches to describing completely integrable dispersionless differential systems.

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PERIODS OF CONTINUOUS MAPS ON SOME COMPACT SPACES

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The objective of this talk is to provide information on the set of periodic points of a continuous self-map defined in the following compact spaces: S^n (the n-dimensional sphere), S^n×S^m (the product space of the n-dimensional with the m-dimensional spheres), CP^n (the n-dimensional complex projective space) andHP^n (the n-dimensional quaternion projective space). We use as main tool the action of the map on the homology groups of these compact spaces.

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Preliminary group classification of the boundary layer equations

Tarik Amtout, Mustapha Er-Riani, Mustapha El Jarroudi

Abstract: In this talk, the preliminary group classification is used for boundary layer equations of a thermo-dependent fluid around any profile. The equations involve three parameters which are the stress, the conductivity and the pressure gradient. In this work by using optimal system of one-, two-dimensional of sub-algebras, we demonstrate the existence of constitutive laws that give rise to self-similar solutions compatible with the boundary conditions.

Keywords: Lie group, Equivalence transformation, Lie algebra, Similarity solution, PDE system, Boundary layer.

Radion tunneling in modified theories of gravity

(Eur.Phys.J. C78 (2018) no.4, 338)

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We consider a five dimensional warped spacetime where the bulk geometry is governed by higher curvature F(R) gravity. In this model, we determine the modulus potential originating from the scalar degree of freedom of higher curvature gravity. In the presence of this potential, we investigate the possibility of modulus (radion) tunneling leading to an instability in the brane configuration. Our results reveal that the parametric regions where the tunneling probability is highly suppressed, corresponds to the parametric values required to resolve the gauge hierarchy problem.

Randomness, non-randomness, and return to player of gambling slot machines

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In many countries gambling in casinos is a well-organized market that produces huge income to national gross domestic product. One of the most popular games organized by casinos use slot machines, also known as "one-armed bandits", "fruit machines", or simply "slots". These machines constitute even 70% of average casino's income. Slot machines according to national regulations must accomplish rigorous requirements in terms of return to player (RTP), randomness of the machine, permanent recording system, tax calculations, etc.

Cyber Security Laboratory is one of the units that is allowed by Polish Minister of Finance to carry out technical inspections (investigations) to verify compliance of slot machines with polish law. In our surveys we have found out in slots multiple symmetries and asymmetries in randomness and RTP. These findings are especially interesting in terms of avoiding randomness and RTP requirements. The conclusion of our research are universal and might be used in all countries where gambling is regulated by national law.

One of the basic requirements to slot machines is their randomness. In majority of cases current "one-armed bandits" are computers designed to imitate rotations of mechanical reels. These machines are equipped with pseudo-random generators (PRNG) that generate a sequence of numbers which determine the final position of the reels and eventually either winning or losing combination. The verification of statistical properties of PRNG is realized with Monte Carlo methods and simulations, however RTP must be carefully mathematically calculated. Especially exact predictability of return to player could be a challenge.

In our manuscript as well as during the conference poster session we will present our techniques and procedures to verify return to player and randomness of slot machines. The major part of the paper will be focused on modifications provided by machines' producers to avoid randomness and to control return to player.

Reheating and dark matter production in quintessential inflation

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Quintessential inflation is an attempt to unify the early and late time acceleration of the universe, through the introduction of a single scalar field. One of its more important issues is the reheating of the universe after inflation, and as we will see, it is completely different from standard inflation were the scalar field oscillate to release its energy. In quintessential inflation the reheating mechanism consists in the gravitational production of non-conformally coupled super-heavy particles, whose light decay products will be the responsible to match the end of inflation with the hot big bang universe. Moreover, we will show that the creation of other kind of conformally coupled super-heavy particles, which only interacts gravitationally, could be the responsible for the abundance of dark matter in our universe.

Scale Invariance in Cosmology and Particle Physics using metric independent measures of integrations in the action

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The use of a metric independent measure of integration in the action opens new possibilities for constructing globally scale invariant theories, since the new measure can be assigned a different scaling transformation than the usual metric dependent measure sqrt(-g). There are various ways to construct a density that can serve as a metric independent measure of integration, from the derivatives of 4 scalar fields or the derivative of a three index tensor field contracted with the alternating symbol. The integration of the equations of motion of these "measure fields" leads to the spontaneous breaking of the scale invariance. A dilaton field with exponential potentials is added and coupled to the different measures. In the effective Einstein frame, potentials for the dilaton with flat regions appear, if curvature square terms are introduced, two flat regions appear, one capable of describing inflation and the other describing the slowly accelerated phase of the present universe. These models allow non singular cosmologies of the emergent type. In the context of the late universe, it is shown that the scale invariance is responsible for the avoidance of the 5th force problem that could have appeared in connection with the nearly massless dilaton. Also a see saw cosmological mechanism that could explain the smallness of the present vacuum energy can be formulated. Finally these techniques have . been used to formulate scale invariant extensions of the Standard Model

Sign-alternating chirality as a basis of stratification in protein and artificial structures

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Abstract. A new fundamental physicochemical phenomenon has been discovered, described and systematized, which is associated with the phenomenon of hierarchical structure formation in initially homochiral systems of living and non-living nature. One of the most important fundamental problems of molecular biophysics is the elucidation of the physical mechanisms of the formation of structural hierarchies in proteins, nucleic acids, supramolecular formations, as well as the mechanisms of specific intermolecular interactions in cells. For the first time, we substantiated the concept that the chiral dualism of carbon compounds is the physical symmetry basis of structure formation in molecular biology, and the homochirality of amino acids, ribose and deoxyribose is a free energy resource and a key tool for folding, as well as intramolecular and supermolecular stratification structural levels. Theoretical analysis of the phenomenon will make it possible to approach innovative developments in the field of pharmacology (chiral drugs), functional bionanostructures and nanoelectronics from a general perspective.

For the first time, a systemic molecular biological pattern has been identified and described in the world scientific literature: starting from the level of asymmetric carbon in deoxyribose and amino acids, there is a tendency to alternate the sign of the chirality of the intramolecular structural levels as L-D-L-D for proteins and D-L-D-L for DNA [1-4]. The primary polymer structures of proteins and nucleic acids, amino acid residues and (deoxy)ribose, are homochiral in asymmetric carbon (L and D), and in the secondary and higher levels of the structure, the sign of chirality is determined by the helical and supercoiled structures. It justifies the point of view according to which the sign-alternating chiral hierarchy of conjugated levels of macromolecular structures in proteins and nucleic acids has a general biological significance: it causes their discreteness, serves as a folding tool, the structural basis of “distinguished mechanical” degrees of freedom in the structures of macromolecular machines.

Formation of chiral sign-alternating hierarchical structures in the initially homochiral systems is not an exceptional feature of living systems, but is reliably reproduced in physical, physicochemical, polymeric, liquid-crystal systems [1, 4]. A thermodynamically governed process of racemization of any macroscopic homochiral system leads to the elimination of the chiral polarization of substances in non-living nature, and, as a result, a spontaneous formation of a succession of chiral structures is suppressed. At the same time, artificially created homochiral systems demonstrate the same effect of spontaneous formation of hierarchies of molecular and supramolecular structures with an alternating sign of chirality, characteristic for living systems.

Let consider physicochemical systems demonstrating the formation of sign-alternating chiral levels. It was shown that synthetic tris[3(3'-carbamoylamino)-2,2'-bipyridyl]-benzene-1,3,5-tricarbonamide derivatives containing three chiral bis[(R) or (S)-2-methylbutylthio]-tetrathiafulvalenyl units at the periphery can assemble into twisted fibers with the right-handed helix being assembled from the left enantiomers, and the left-handed helix from the right enantiomers [5].

Super amphiphiles consisting of isocyanopeptides (hydrophilic block) and polystyrene (hydrophobic block) can assemble in water into rod micelles, vesicles, and helical aggregates [6]. Micellar rods consisting of a polystyrene core and a surface formed by the right-handed polyisocyanides self-assemble into left supercoiled fibers.

A possibility of hierarchical self-assembly of disk-shaped molecules with chiral tails into single helical structures was demonstrated in [7]. A crown ether appended phthalocyanine with chiral aliphatic tails self-assembles into right-handed helical stacks in chloroform, which in turn self-assembles into left-handed coiled-coil aggregates.

Given examples illustrate the rule for changing the chirality sign during the transition to the next level of the structural hierarchy.

Conclusions. To summarize, any chiral system, possessing free energy, tends to spontaneous formation of a new, higher structural level with the same type of symmetry but with an opposite sign of chirality and on an enlarged scale. The resulting hierarchy of conjugated sign-alternating chiral structures kinetically stabilizes them by inhibiting spontaneous racemization; forms a conjugated system with selected degrees of freedom, which makes the work of biological machines possible; in living systems macromolecular and autowave chiral structures are conjugated as a consequence of their channeling through the structures of chiral biological machines; defines the vector of the general development of a system in the direction of an upper, “open-ended” hierarchical level [1].

The important stage in the study of this phenomenon, characterized by the spontaneous formation of sign-alternating hierarchies of chiral structures is a creation of a theoretical model of the physical mechanisms of their formation, as well as the development of the fundamentals of promising technologies for 3D design of materials with desired properties for the needs of nanomaterial science, nanoelectronics, nanooptics, chiral pharmaceuticals and pharmacology. Conjugate multi-scale hierarchical structures may in principle be highly sensitive to weak physical influences and small concentrations of effectors.

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Silhouette Based Face Recognition Using Deep Network

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Face detection is integral to any automatic face recognition system in the field of pattern recognition. The goal of our proposed research is to develop a system based on deep network model performing the recognition task of human face using silhouette. Our novel approach based on the minimum features helps to recognize the human face with higher accuracy. A system to correctly locate and identify human faces will find several applications, some examples are criminal identification and authentication in secure systems. In order to reduce computation time and resources, face silhouettes are used instead of intensity images. A set of basis face silhouettes is also generated by capturing 10 pictures of human front face of each of 100 volunteers. These images are used for feature extraction and fed to deep neural network in order to recognize face. The results show that the approach is robust, accurate and reasonably fast.

Spontaneous Symmetry Breaking in Self-assembled Systems and Top-bottom Chirality Transfer: A Plausible Scenario for the Emergence of Biological Homochirality

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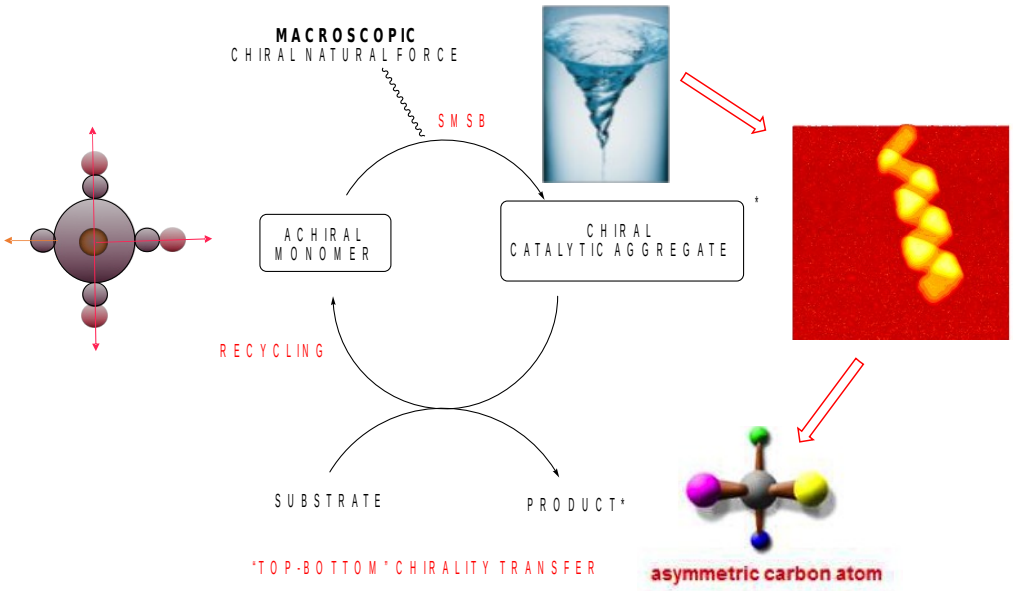
Abstract: Enantioselective chemical reactions that take place under spontaneous mirror-symmetry breaking (SMSB) conditions ("absolute asymmetric synthesis") may have played a key role in the origin of biomolecular homochirality. However, the achievement of SMSB in chemical reactions taking place in solution requires of highly specific non-linear reaction networks based on enantioselective autocatalysis, and examples of its experimental realization are very rare. On the other hand, emergence of net supramolecular chirality by SMSB in the self-assembly of achiral molecules has been seen to occur in several instances, and the chirality sign of the resulting supramolecular system can be controlled by the action of macroscopic chiral forces. We will present a hitherto unregarded mechanism for the emergence of net chirality in molecular systems, in which the SMSB takes place in the formation of chiral supramolecular dissipative structures, leading to asymmetric imbalances in their composition that are subsequently transferred to a standard enantioselective catalytic reaction (absolute asymmetric catalysis). This "top-bottom" chirality transfer could be relevant for the emergence of biological homochirality in connection recently proposed scenarios in which chiral supramolecular aggregates of achiral prebiotic molecules arise by the action of hydrodynamic torques in the chiral microvortices created in the rock micropores near to deep-sea hydrothermal vents on prebiotic Earth.



Recent results in our laboratory show that the purely supramolecular chirality of amphiphilic porhyrin J-aggregates (generated by SMSB during the aggregation process) can be transferred down to the molecular level in absolute asymmetric synthesis conditions, leading to small but measurable deviations from the racemic composition in the chiral products arising from a Diels–Alder cycloaddition. The sense of the asymmetric induction in this reaction can be correlated with the supramolecular chirality sign of the catalytic heteroaggregates, as measured by the CD signals of the bisignated exciton coupling bands.

Symmetric Shape Transformations of Folded Shell Roofs Determining Creative and Rational Shaping of Building Free Forms

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The paper presents an innovative approach to solving interdisciplinary problems emerging in the design process of building free forms roofed with elastically transformed corrugated shells, their structural systems and regular elevation patterns. Effectiveness and rationality of shaping such free forms and creativeness in parametric searching for these forms require applying regularity and symmetry of their models. In addition, the simplified smooth models used for engineering developments and accurate folded models implemented for scientific research have to be created by means of unconventional methods different from those employed so far.

Elastic shape transformations of nominally flat thin-walled folded steel sheets are examined. They allow both attractive shell roof forms to be obtained while maintaining the ability to carry live loads despite large deformations of cross-sections and mutual displacements of the subsequent folds in a roof. These transformations are effected by spreading sheets on mutually skew roof directrices. They are called effective transformations since the sheets are provided with freedom to adjust their shape to the shape and mutual position of the roof directrices.

Despite the complicated form and static-strength work of the transformed folds, their longitudinal axes and edges remain straight lines, and are modeled with pairs of skew straight lines. For engineering developments, individual shell folds are modeled with axis-symmetric central sectors of ruled surface.
Rationality of the transformed roof shells, revealed in the limitation of the level of the initial fold’s effort resulting from the transformation, and their attractiveness are achieved by axial symmetry and contraction of each shell fold at its half-length. The symmetries adopted in the process of modeling such roof shells are also used to obtain coherent unconventional general forms of entire buildings shaped with regular reference tetrahedrons. Owing to the variety of the forms of tetrahedrons and their parametric description the developed algorithms can be implemented in computer programs supporting the search for innovative and rational building forms.

As a result of the spreading of flat folded sheeting on mutually skew directrices, its subsequent folds are subjected to diversified shape transformations and take various directions and different inclinations to the horizontal plane. The imposition of symmetry on the shape of each fold and the entire transformed shell leads to the ordering of the shell form and its integration with the entire building free form.

The adoption of appropriate shape and inclination proportions between the walls and edges of the façade as well as the eaves and roof shell allows to increase the cohesion and attractiveness of the free form. The employment of the previously assumed symmetry of the roof shell for shaping the free form of the entire building is an additional positive factor affecting its visual attractiveness.

The use of any kind of symmetry may help increase the rationality of the adopted solutions of the form and construction of the building. This rationality reveal in: a) relatively simple shaping the structural system intended for the free form based on flat walls and roof directrices contained in the wall’s planes, b) using repeated building elements, c) defining effective shape transformations allowing the deformed folding to maintain the ability to carry live roof loads.

Symmetries of fractional differential equations: automated computation

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Some of the modern mathematical models includes fractional derivatives to describe non-local behavior or memory effects. The talk is devoted to procedure of finding Lie point symmetries of fractional differential equations. As shown in multiple works, the symmetries can be used to reduce the number of independent variables, construct exact solutions or conservation laws.

Finding admitted operators for an equation or a system with fractional derivatives requires a lot of calculations, because the algorithm needs solving infinite overdetermined systems of partial differential equations (PDEs) with some fractional terms. However, the resulting symmetry group is usually generated by operators that have rather simple form.

Here we present the computer algebra code (based on open source SymPy Python library) that finds symmetries as a combination of some known basis infinitesimal operators for equations with Riemann-Liouville fractional derivatives. Using the fixed form of operator allows one to avoid symbolical solving of PDEs and do the final steps numerically (solving the resulting linear overdetermined algebraic system by SVD decomposition, for example). The only steps to be done symbolically are differentiation and substitution to construct the determining equations. This allows the program to work with rather complicated forms of equations.

The code can be used to check the manually obtained results or as an express-analysis tool to study nonlinear systems of FDEs. The program also supports finding approximate symmetries of equation with a small parameter. Some examples of using the code to analyze symmetries of fractional filtration models are presented.

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Symmetry and Special Relativity

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Abstract

We explore the role of symmetry in the theory of Special Relativity. Using the symmetry of the principle of relativity, we derive the Lorentz transformations and an invariant metric, *without assuming the constancy of the speed of light*.

Since the velocity ball D_V is a *bounded symmetric domain* with respect to the group of affine automorphisms of D_V, we can use the Newtonian limit and the invariance of the metric to derive a relativistic dynamics equation. However, it is not possible to obtain explicit solutions for the important case of the motion of a charged particle in uniform, constant and perpendicular electric and magnetic fields. Taking advantage of an additional symmetry, the *symmetric velocities*, the corresponding bounded domain is symmetric with respect to the conformal maps. This leads to a spin-half representation of the Lorentz group and explicit, analytic solutions for the motion of the charged particle.

Symmetry of the interior of isotropic Black Holes

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Schwarzschild (S) and Reissner-Nordström (RN) space-times are static and spherically symmetric geometries described by the following line element:

$$ds^2 = f_{(i)}dt^2 - f_{(i)}^{-1}dr^2 - r^2 d\Omega^2 \qquad (1)$$

where, $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ and θ, φ denote angular coordinates. $f_S(r) = 1 - \frac{2M}{r}$ and $f_{RN}(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$ where M and Q are the mass and charge of a source of gravitational field. The black hole (BH) horizon, defined as a (geometric) singularity of a metric tensor, $f_{(i)}(r_g) = 0$, is a well-known result, defining an event horizon in the former case (S), at $r_g = 2M$, and outer/inner (+/-) horizons in the other case (RN), at $r_\pm = M \pm \sqrt{M^2 - Q^2}$. The static and isotropic character of the space-times results in conservation of energy and angular momentum manifested by the existence of two Killing vectors, a time-like one, $\xi = \partial_t$ and a space-like one $\eta = \partial_\varphi$, respectively. It is less well-known, but well-established in S space-time [1] that one can describe the interior of the BH by applying expression (1) directly. In this case, one has to accept an important consequence of the fact that for this space-time, $f_{(i)} < 0$ is characterized by an interchange the roles of the coordinates t and r: the former becomes a spatial coordinate and the latter a temporal one. Moreover, the Killing vector ξ, inside the horizon becomes space-like representing translational symmetry, hence homogeneity along the $t-$¿axis. The corresponding conserved quantity is t-component of linear momentum. Originating from these observations one may go on to describe the properties of the BH's interior – that is the subject of our considerations.

By using a simple method based on the observations of the Doppler frequency shift of signals exchanged by the hypothetical observers within the horizon one finds that the interior of these spherically symmetric static space-times is a dynamically changing space-time (analogous to a"cosmological" space-time). Applying coordinates (1), it turns out that the homogeneity axis (the t-coordinate axis) is the direction of expansion; this expansion is accompanied by a characteristic frequency red-shift, analogous to a cosmological Doppler redshift. However, the dynamics inside the horizon are strongly anisotropic: there is a contraction perpendicular to the homogeneity axis accompanied by a corresponding Doppler blue-shift. This is how the anisotropic, $R \times S^2$, cylindrically shaped interior of isotropic, static Black Holes evolves: expanding in R and contracting in the S^2 directions. The

interesting feature is that this dynamic behaviour continues to the very end in the case of the S BH while it is ceases in the case of RN space-time.

Finally our considerations are illustrated with the example of the Black Hole in M87. If such a supermassive BH, the only one imaged so far [2], at a distance of 55 Mly from the Solar System is both isotropic and static, hence either of type S or RN, then one can describe in detail the journey to its horizon and beyond, and the eventual fate of the unfortunate astronaut. Free fall from a distance of one light year under the gravitational pull of the BH would last for a couple of years but the rest of this dramatic journey inside the horizon would continue for an additional fifteen hours only. In some circumstances the last stage may be slightly extended but no more than up to 30 hours. These and other specific features of such a hypothetical journey will complete our discussion.

[1] R. Doran, F. S. Lobo, and P. Crawford, "*Interior of a Schwarzschild black hole revisited*" Foundations of Physics, vol. 38, no. 2, pp. 160–187, 2008

[2] E. Landau, *Black Hole Image Makes History*, 2019, https://www.jpl.nasa.gov/news/news.php?feature=7372

The Casimir force between Dirac lattices at nonzero temperature and the heat kernel expansion

Irina Pirozhenko

The Casimir energy for Dirac lattices is calculated at nonzero temperature. The heat kernel expansion for periodic singular background is discussed in relation with the high temperature asymptote of the free energy.

The inflationary universe in F(R) gravity with antisymmetric tensor fields and their suppression during the universe evolution

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The intriguing question, why the present scale of the universe is free from any perceptible footprints of rank-2 antisymmetric tensor fields? (generally known as Kalb-Ramond fields) is addressed. A quite natural explanation of this issue is given from the angle of higher-curvature gravity, both in four and in five-dimensional spacetime. The results here obtained reveal that the amplitude of the Kalb-Ramond field may be actually large and play a significant role during the early universe, while the presence of higher-order gravity suppresses this field during the cosmological evolution, so that it eventually becomes negligible in the current universe. Besides the suppression of the Kalb-Ramond field, the extra degree of freedom in F(R) gravity, usually known as scalaron, also turns out to be responsible for inflation. Such F(R) gravity with Kalb-Ramond fields may govern the early universe to undergo an inflationary stage at early times (with the subsequent graceful exit) for wider range of F(R) gravity than without antisymmetric fields.. Furthermore, the models—in fourand five-dimensional spacetimes—are linked to observational constraints, with the conclusion that the corresponding values of the spectral index and tensor-to-scalar ratio closely match the values provided by the Planck survey 2018 data.

The universe acceleration in modified gravity: an overview

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Abstract

In this talk, a follow up of previous presentations, I will start by discussing the role of each of the contributors to the discovery of the Universe expansion, namely Edwin Hubble, Georges Lemaître and Vesto Slipher (concentrating in particular on the seminal role of the last), discoveries that led to the even more important finding that our Universe had an origin. I will then address hot issues concerning the explanation in simple terms of the Universe acceleration and the concept of dark energy, in particular, the possible contribution of quantum vacuum fluctuations.

Things We Are Still Learning about Uniform Acceleration

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Radiation by an accelerated charge, especially a uniformly accelerated one, has been the subject of debate for almost a century. It obviously must have some connection with the Unruh-Wald radiation from an accelerated detector, and with the Moore-DeWitt radiation from an accelerated mirror, but the details of those relationships have never been entirely clear. In the early '90s, Higuchi, Matsas, and Sudarsky, and then Ren and Weinberg, studied the quantum radiation from a prescribed c-number charge by perturbation theory in both inertial and uniformly accelerated frames; they found that consistency requires that in the latter case a contribution of the Unruh thermal bath (via absorption and stimulated emission) must be included. Recently, we have reopened the investigation, using the basis of "Unruh modes" that factors the Bogolubov transformation between Minkowski and Rindler modes into a unitary transformation and a diagonal one. This analysis makes the consistency between classical radiation and quantum transition amplitudes more manifest. An important observation is that the radiation is visible as such only outside the Rindler wedge, in the sense that inside the wedge the retarded and advanced solutions coincide; this is the classical counterpart of the quantum observation that the transition probability vanishes except for Rindler zero-frequency modes, which are localized on the horizon. The final state is a coherent state whose field expectation value equals the classical retarded solution. If time permits, I will review "ancient" and modern developments concerning accelerated mirrors, accelerated detectors, and atoms falling into or supported near black holes.

Uniform convergent expansions of the Appell F_1 function

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Abstract

We consider the first Appell function F_1 and we derive two kinds of expansions which are uniform and convergent. In the first case, we have uniformity in both variables x and y plus an explicit, accurate bound for the remainder, but the approximation is given in terms of the hypergeometric ${}_2F_1$ function. In the second case the approximation is given in terms of elementary (rational) functions but these expansions are only uniformly valid in one variable. Thanks to simmetry, an expansion for each variable is available. The accuracy of the expansions is illustrated with some numerical experiments.

Keywords & Phrases: First Appell function; convergent expansions; uniform expansions.