Standing theoretical issues in ν physics

When the SM was built there was no evidence of ν mass

Moreover, ν only have weak interacctions $\Rightarrow$ only the left-handed part is seen $\Rightarrow$ ν_R were not included

$\Rightarrow$ the usual mechanism for giving fermions a mass in the SM (Yukawas) are not present

$$Y_\nu \overline{L_L} \tilde{\phi} \nu_R + h.c. \xrightarrow[\langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}]{\text{selects } \nu} Y_\nu \frac{v}{\sqrt{2}} \overline{\nu_L} \nu_R + h.c.$$

Dirac mass term

However, for ν there is a second option:

$$\frac{m_M}{2} \overline{\nu_L^c} \nu_L \qquad \text{"Majorana mass term"}$$

$$\nu_L^c = C \overline{\nu_L}^t \qquad \overline{\nu_L^c} = \nu_L^t C$$

$\Rightarrow$ Violates <u>all symetries</u> the ν_L is charged under

- Violates lepton number L since ν_L and $\overline{\nu_L^c}$ both have $L = 1$ and $\overline{\nu_L^c} \nu_L = \nu_L^t C \nu_L = \overline{\nu_L} \nu_L^c$

$L = -2$

- Does not violate QED or QCD since ν_L is neutral and colorless

But violates hipercharge $U(1)$ and $SU(2)_L$

$\Rightarrow$ Can't write it like that in the Lagrangian

Need a trick similar to the Yukawas for m_D

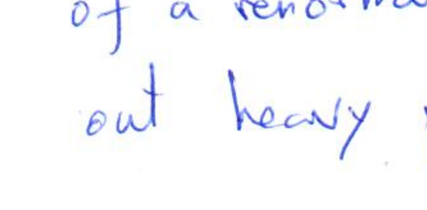
$$\frac{2}{v^2}\frac{m_M}{2}\underbrace{\left(\overline{L^c_L}\,\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger L_L\right)}_{d=5 \text{ operator Weinberg}} \xrightarrow{\text{selects } \nu} \frac{m_M}{2}\,\overline{\nu_L^c}\,\nu_L$$

$$\langle\tilde{\phi}\rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}$$

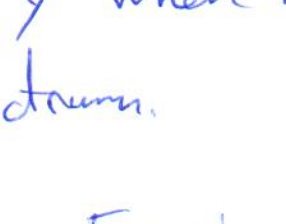
$\rightarrow$ coefficient with $\sim \frac{1}{M}$ dimension

A $d=5$ operator is not renormalizable but it can be induced effectively as a low energy effect of a renormalizable ($d \leq 4$) theory when integrating out heavy particles from the spectrum.

Just like the $d=6$ effective Fermi interactitions when the W and Z are removed



$$\xrightarrow{p \ll M_W^2}$$



$G_F/\sqrt{2}$

$\pi^- \to l\bar{\nu}$ $\quad -(\xi-1)\frac{p^\mu p^\nu}{p^2-M_W^2}$

$$\frac{g}{\sqrt{2}}(\bar{u}\gamma_\mu P_L d)\frac{g^{\mu\nu}}{p^2-M_W^2}(\bar{l}\gamma_\nu P_L \nu)\frac{g}{\sqrt{2}}$$

$$\frac{-g^2}{8M_W^2}(\bar{u}\gamma_\mu(1-\gamma_5)d)(\bar{l}\gamma^\mu(1-\gamma_5)\nu)$$

The Weinberg $d=5$ operator can in the same way be the remnant of heavy particles that have been integrated out like the W's and Z's

In fact we expect the SM to be completed with extra high E physics which at low E is encoded in a series of $d>4$ operators

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\mathcal{L}^{d=6} + \dots$$

Λ new physics scale like M_W in G_F

With the SM particle content and respecting the SM symmetries 59 + 5 (B and L violating) $d=6$ operators exist (without flavour structure)

Only one $d=5$ operator Weinberg

It is suspicious that the evidence for physics Beyond SM shows from the effective op that we expect to be least suppressed!

Two ways to induce ν masses

① add ν_R

$$Y_\nu \overline{L_L} \tilde{\phi} \nu_R$$

Complete gauge singlet under the SM $\Rightarrow$ Majorana mass allowed

$$M_R \overline{\nu_R^c} \nu_R$$

Completely new physics scale unrelated to EWSB y unlike all other scales in the SM (except Λ_{QCD})

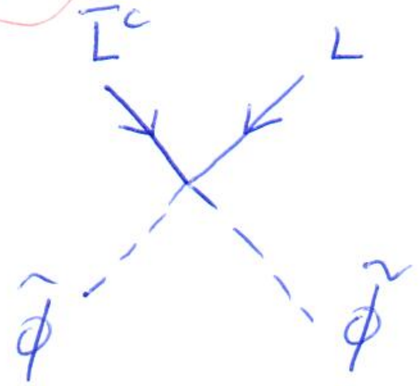
Either forbid it promoting L (or rather $B-L$) to an exact SM symetry or new scale M_R at which $\cancel{L}$ is broken

Are they so different?

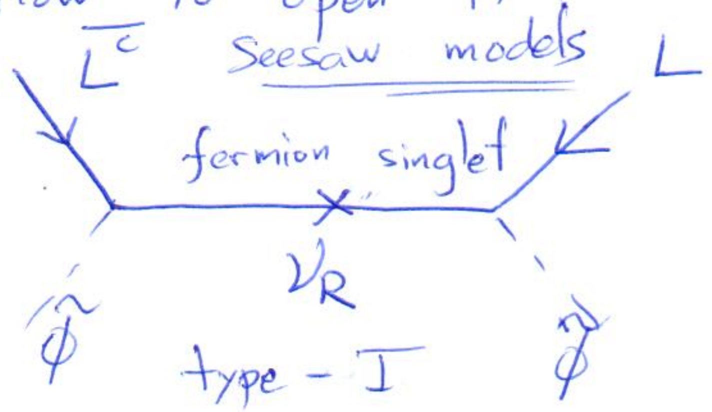
② $d=5$ op

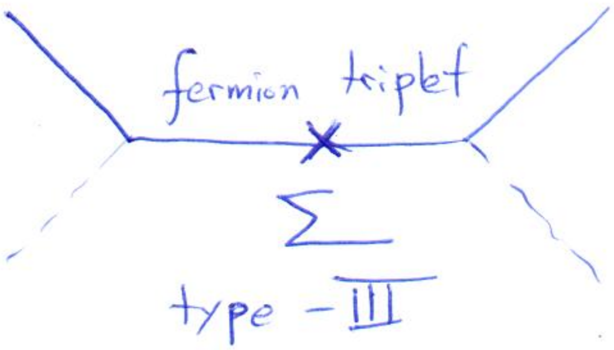
Again new scale at which L is broken

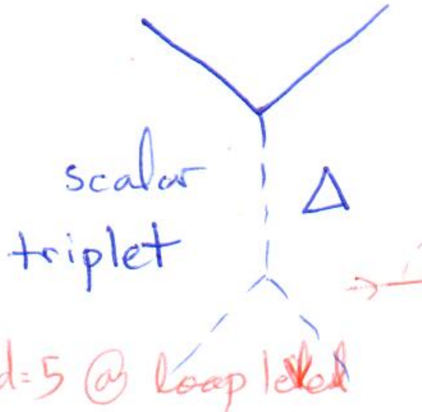
$$\frac{1}{\Lambda} \frac{1}{2} \left(\overline{L_L^c} \tilde{\phi}^*\right)\left(\tilde{\phi}^+ L_L\right)$$



How to open it
Seesaw models

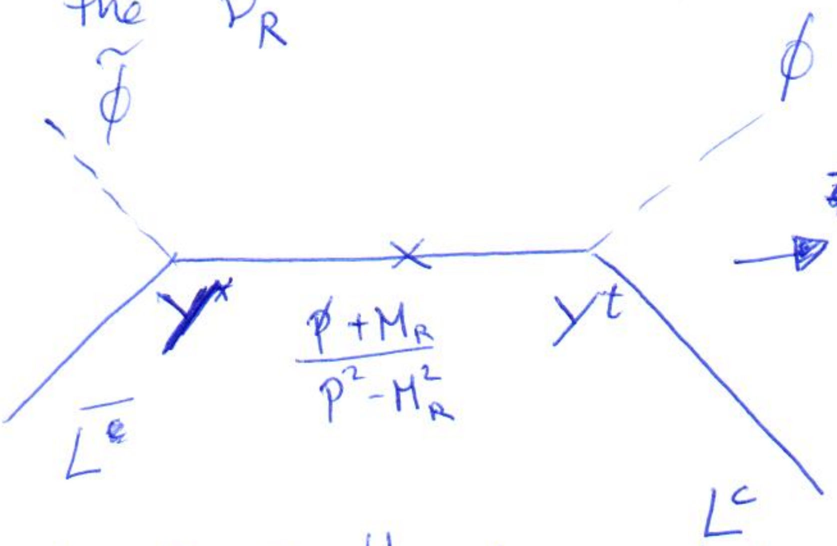






Can also generate $d=5$ @ loop level

In fact if $M_R \gg v$ I can integrate out the ν_R



$\longrightarrow \mp Y M_R^{-1} Y^t (\overline{L^c}\tilde{\phi}^*)(\tilde{\phi}^\dagger L)$

$\langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$

$\frac{v^2}{2} Y M_R^{-1} Y = m_D \, \overline{\nu_L^c}\nu_L$

So if $M_R \gg v$ the two solutions are the same

There will be a $d=6$ op from the $\frac{\not{p}}{M_R^2}$ term

$$Y M_R^{-2} Y^\dagger (\overline{L}\tilde{\phi}) i\not{\partial} (\tilde{\phi}^\dagger L)$$

$$\downarrow$$

$$\frac{v^2}{2}(Y M_R^{-2} Y)^\dagger_{\alpha\beta} = \varepsilon_{\alpha\beta} \overline{\nu_L^\alpha} \not{\partial} \nu_L^\beta$$

contribution to ν kinetic term

$\Rightarrow$ Low energy $\mathcal{L} = (\delta_{\alpha\beta} + \varepsilon_{\alpha\beta}) \overline{\nu_{L\alpha}} i\not{\partial} \nu_{L\beta} + m_{\alpha\beta} \overline{\nu_L^c}\nu_L$

To go to the mass basis with canonical normalized states:

1. Diagonalize the Kinetic terms $\delta_{\alpha\beta} + \varepsilon_{\alpha\beta}$ with unitary rotation U_1
2. Normalize the Kinetic terms with diagonal rescaling d
3. Diagonalize mass matrix with unitary rotation U_2

From the original flavour basis to the mass basis we have: $U^{PMNS} = U_1 \, d \, U_2$

$\nu_\alpha = U_1 \nu_\alpha' = U_1 \, d \, \nu_\alpha'' = U_1 d U_2 \nu_i$

Completely general (non-unitary) matrix

To leading order in ε

$$U^{PMNS} = \overbrace{\left(\delta_{\alpha\beta} - \varepsilon_{\alpha\beta}/2\right)}^{U_1 d} \overbrace{U}^{U_2}$$

$$\varepsilon_{\alpha\beta} = \frac{v^2}{2}\left(y M^{-2} y^\dagger\right)_{\alpha\beta} \Rightarrow U^{PMNS} \simeq U$$

Easier way to see it:

$$\mathcal{L} = m_D\left(\overline{\nu_L}\,\nu_R + \overline{\nu_R}\,\nu_L\right) + \frac{M_R}{2}\,\overline{\nu_R^c}\,\nu_R =$$

after EWSB

$$= \frac{1}{2}\left(\overline{\nu_L^c}\ \ \overline{\nu_R}\right)\begin{pmatrix} 0 & m_D \\ m_D^t & M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

$$\equiv \frac{1}{2}\,\overline{N^c}\,\mathcal{M}\,N \quad \text{with } N \equiv \left(\nu_L\ \ \nu_R^c\right)$$

just diagonalize $\mathcal{M}$ to go to the mass basis

$$\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\operatorname{tg} 2\theta = \frac{2 m_D}{M_R}\ ; \quad m_{1,2} = \frac{M_R}{2} \mp \sqrt{\left(\frac{M_R}{2}\right)^2 + m_D^2}$$

$\to \theta \sim m_D / M_R$

$$\simeq \frac{M_R}{2} \mp \left(\frac{M_R}{2} + \frac{m_D^2}{M_R}\right)$$

$$\Rightarrow \begin{cases} m_1 \simeq \dfrac{m_D^2}{M_R} \\ m_2 \simeq M_R \end{cases} \to d=5 \text{ op } \ Y^* M_R\, Y^t \frac{v^2}{2}$$

$\to$ "See-saw" if $M_R \gg m_D \Rightarrow m_1 \ll M_R$, m_2

ex: $Y \sim 1$, $M_R \sim 10^{16}$ $\quad m_1 \sim 10^{-3}\,\text{eV}$

$Y \sim 10^{-6}$, $M_R \sim 1\,\text{TeV}$ $\quad m_1 \sim 10^{-2}\,\text{eV}$

and $\nu_1 = \cos\theta\ \nu_L + \sin\theta\ \nu_R$

$$\to 1 - \frac{\theta^2}{2} + \ldots = 1 - \frac{m_D^2}{2 M_R^2}$$

$$\varepsilon = Y M_R^{-2}\, Y^t \frac{v^2}{2}$$

$\nu_2 = \cos\theta\ \nu_R - \sin\theta\ \nu_L$

$$\nu_1 = (1 - \varepsilon/2)\, \nu_L$$

If we have 3 families after blck diagonalizing

$$\begin{pmatrix} 0 & m_D \\ m_D^t & M_R \end{pmatrix} \longrightarrow \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

Need U to diagonalize m_1

$$\Rightarrow U^{PMNS} = (1 - \varepsilon/2)\, U$$

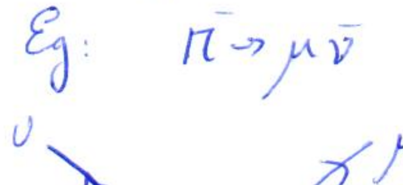
Full 6×6 matrix is unitary

$$\mathcal{U} = \begin{pmatrix} (1-\varepsilon/2)\, U & \sqrt{\varepsilon} \\ \sqrt{\varepsilon}\, U & (1-\varepsilon/2) \end{pmatrix}$$

But I dont have enough energy to see the rest

Testing for "non-unitarity"

Eg: $\pi \to \mu\bar{\nu}$ vs $\pi \to e\bar{\nu}$



$U^{PMNS}_{\mu i} = (\delta_{\mu\alpha} - \varepsilon_{\mu\alpha}/2)\, U_{\alpha i}$

$\left(\delta_{e\alpha} - \frac{\varepsilon_{e\alpha}}{2}\right) U_{\alpha i}$

I don't measure $\bar{\nu}_i \Rightarrow \Gamma_{\pi\to\mu\nu} \propto \sum_{i,\alpha} \left|\left(\delta_{\mu\alpha} - \frac{\varepsilon_{\mu\alpha}}{2}\right) U_{\alpha i}\right|^2 = 1 - \varepsilon_{\mu\mu}$

while $\Gamma_{\pi\to e\nu} \propto 1 - \varepsilon_{ee} \Rightarrow$ C.C. Non-universal

Also since U^{PMNS} is <u>NOT</u> unitary, GIM cancellation not effective $\Rightarrow$ enhancement of Flavour Changing Neutral Currents

However $\varepsilon \sim \dfrac{m_D^2}{M_R^2} \sim$ 10^{-28} for $Y \sim 1$ $M_R \sim 10^{16}$ GeV
$\qquad\qquad\qquad\qquad\quad$ 10^{-14} for $Y \sim 10^{-6}$ $M_R \sim 1$ TeV

Too small to be seen if smallness of ν mass is explained through

However there can be a different explanation for the smallness of ν masses.

$d=5$ op $\qquad\qquad\qquad\qquad\qquad\qquad$ $d=6$ op

$$Y M^{-1} Y^{t} \left(\overline{L^c_L}\, \tilde{\phi}^{*}\right)\left(\tilde{\phi}^{+} L\right) \qquad\qquad Y M^{-2} Y^{+} \left(\overline{L}_L\, \tilde{\phi}\right) \partial \left(\tilde{\phi}^{+}_L L\right)$$

violates L $\qquad\qquad\qquad\qquad\qquad\qquad$ doesn't violate L

Maybe L is aproximately conserved $\Rightarrow$ $d=5$ small but $d=6$ big

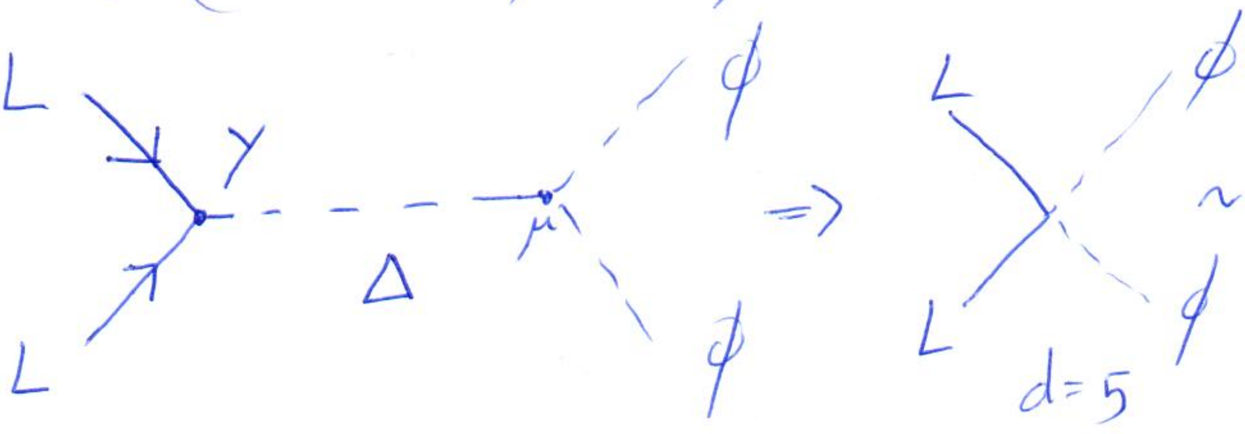
there are L-conserving choices of Y so that

$$Y M^{-1} Y^{t} = 0 \qquad \text{while} \qquad Y M^{-2} Y^{+} \neq 0 \text{ and large}$$

then break L by a small parameter $\Rightarrow$ $d=5$ small
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $d=6$ large

More evident in type-II see-saw:

$$Y\left(\overline{L^c}\,\vec{\tau}\,L\right)\vec{\Delta} + \mu\left(\phi^\dagger\vec{\tau}\,\phi\right)\vec{\Delta}$$

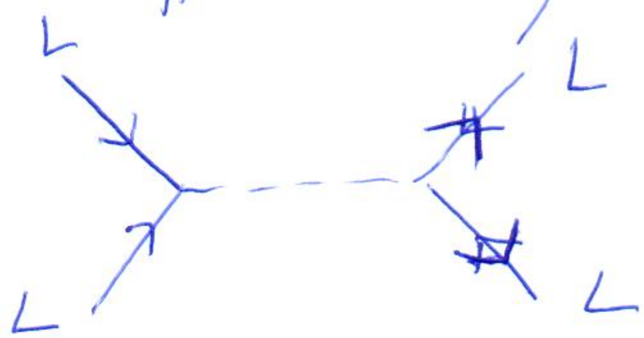


$\sim m_\nu \sim \frac{v^2}{2}\frac{Y\mu}{M_\Delta^2}$

If $\mu = 0 \Rightarrow m_\nu = 0$ and L conserved

(I can asign $L=2$ to Δ)

but I still have interesting $d=6$ operators not suppressed by μ



$$\propto \frac{YY^\dagger}{M_\Delta^2}$$

Interesting type-I SS aplication: Leptogenesis

Sakharov conditions to create B:

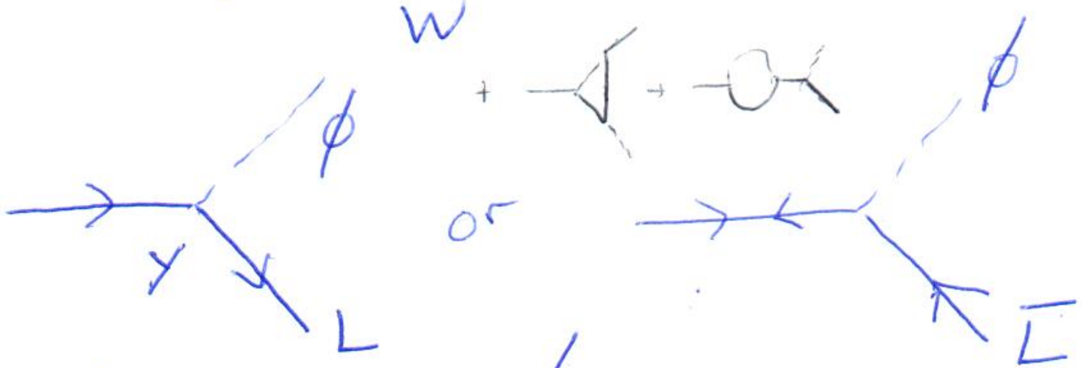
1. Violation of B! → Sphalerons
2. Violation of C and CP → Complex Yukawas
3. Departure from thermal equilibrium → decay of N

Sphalerons are transitions between 2 configurations of W bosons that induce violation of B and L (but conserve B-L).

$$\Delta B = \Delta L = \int dx^\mu \, \underbrace{\bar{f} \gamma_\mu f}_{\text{B or L current}} = n_g \Delta n_{cs}$$

$$\alpha \int W_{\mu\nu} \tilde{W}^{\mu\nu}$$

N decays



$\neq$

if $\cancel{CP}$

Imagine $\Gamma^1_{N \to \bar{L}} > \Gamma^1_{N \to L}$

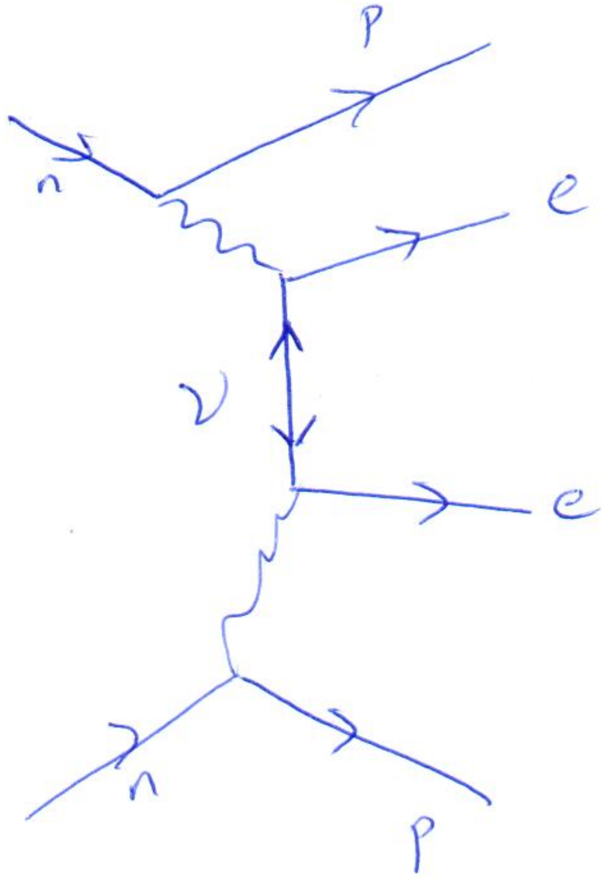
L	B
$-L_0$	0
+3	+3
+6	+6
$\sim -L_0/2$	$+L_0/2$

Baryogenesis via Leptogenesis

How to test the Majorana nature of ν?

Have to look for L-violating process

Most promising $0\nu\beta\beta$



$$\propto \quad m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

$$m_{\beta\beta} = m_{ee}$$

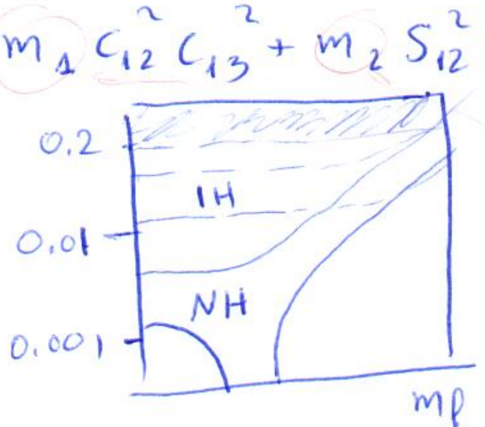
$$\bar e\, U_{ei} \left(\gamma_\mu P_L\right)^t C \frac{\not p + m_i}{p^2 - m_i^2} U_{ei} \gamma_\nu P_L e =$$

$$P_L \; \gamma_\mu^t$$

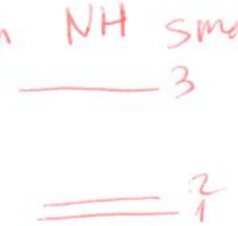
$$= \bar e^c \gamma_\mu P_R \frac{\not p + m_i}{p^2 - m_i^2} \gamma_\nu P_L e\, U_{ei}^2 \propto \sum_i m_i U_{ei}^2$$

$$P_R$$

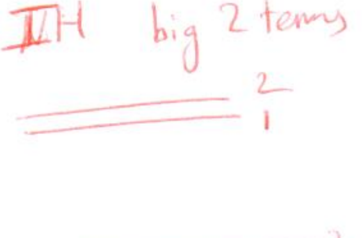
$$m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha_2} + m_3 s_{13}^2 e^{2i\alpha_3}$$



in NH small



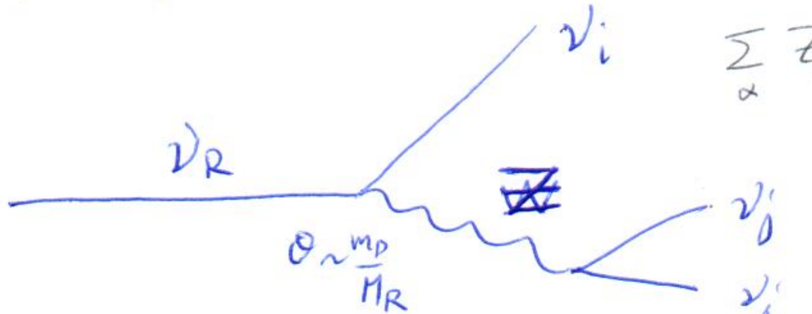
IH big 2 terms



What if M_R is not $\gg m_D$?

Then we dont have a See-Saw and we might need very small y to accomodate ν masses but there can be interesting pheno from the extra sterile neutrinos

If $M_R \sim$ KeV ν_R is a viable DM candidate
It's neutral and can be stable over the age of the Universe. Hower it decays



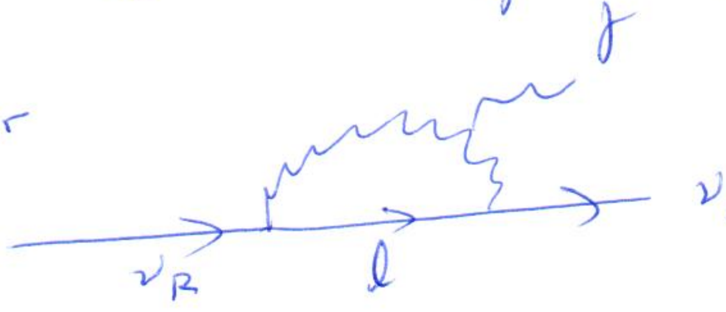
$$\sum_\alpha Z^\mu \bar{\nu}_\alpha \gamma_\mu \nu_\alpha =$$

$$= Z^\mu \sum_\alpha \bar{\nu}_i U^{PMNS*}_{\alpha i} \gamma^\mu U^{P-}_{\alpha j} \nu_j$$

but $\sum_{\alpha = e,\mu,\tau} U^*_{\alpha i} U_{\alpha j} \neq \delta_{ij}$

since U^{PMNS} is 6×6

or



$T^1 \propto m_\nu^5$ if m_ν too light $\Rightarrow$ hot Dark Matter
if m_ν too heavy $\Rightarrow$ too fast decay

there is a claime at $\sim 3\sigma$ for a 3.6 KeV line that coul come from the decay of a 7.2 KeV ν DM.

If $M_R \sim$ keV it could help to explain some anomalies in ν oscillation data that do not fit the 3-family scenario since the seem to have signal for L/E too small

$\Rightarrow \Delta m^2$ too big to be Δm^2_{31} or Δm^2_{21}

<u>LSND</u> $\bar{\nu}_\mu \to \bar{\nu}_e$ 3.8σ

<u>Miniboone</u> $\sim 2\sigma$ low E excess in ν_e and $\bar{\nu}_e$

<u>Reactors</u> $\sim 1.8\sigma$ $\bar{\nu}_e$ disapp

<u>Gallium solar ν callibration</u> $2-3\sigma$ ν_e disap

But no evidence of ν_μ disap $\Rightarrow$ strong tension between datasets $\Rightarrow$ future experimets to test