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# The thermal history of the Universe And the Cosmic Microwave Background

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**[CEFCA]**

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# OUTLINE

- The inflationary epoch
- Nucleosynthesis, and the release of cosmological backgrounds of neutrinos and radiation
- The information contained in the angular anisotropies of the Cosmic Microwave Background (CMB). The meaurement of polarization anisotropies
- Useful codes for computing power spectra and other cosmological quantities: **CAMB, MGCAMB, EFTCAMB, CLASS**

# The inflationary epoch

Motivation:

- It solves the “homogeneity” problem: why regions so far apart are *so* similar ...
- It explains why we find no unwanted relics, like gravitino, moduli from superstring theories, topological defects, magnetic monopoles, domain walls, etc …
- It explains the fact that, at early times, all FLRW metric are so flat …

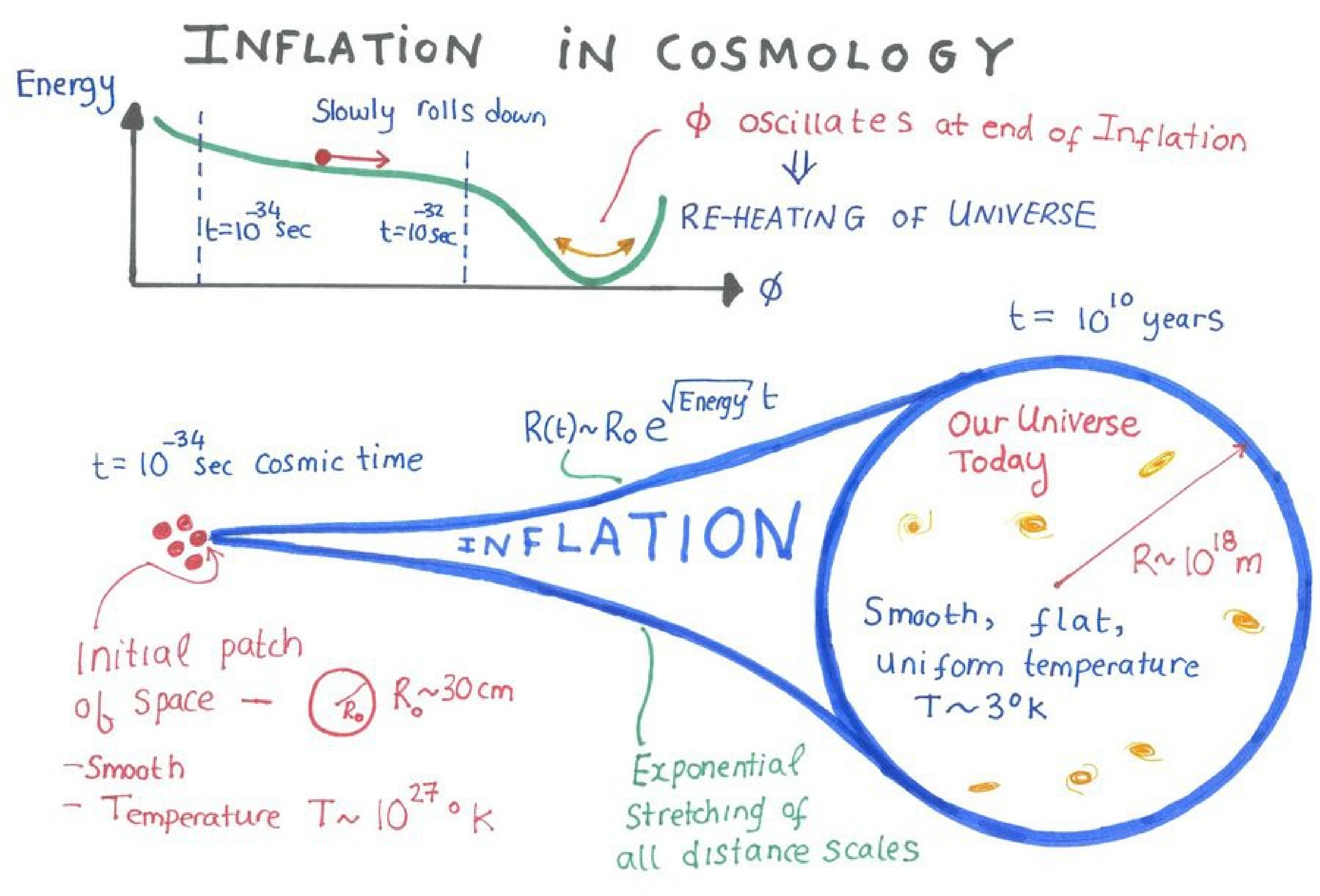
Definition:

- **d [H/a]/ dt < 0** or **ρ+3p<0 [== ä > 0]**

# The inflationary epoch

Mechanism:

- A ***scalar*** field $\Phi$ (named *inflaton*) is driving the dynamics of the universe, introducing an accelerated expansion
- Quantum (vacuum) fluctuations of the inflaton field are stretched into macroscopic scales, and are projected into curvature fluctuations
- These curvature fluctuations constitute the seeds of all other posterior fluctuations that grow at later stages




Credit: somewhere from the web …

# The inflationary epoch

Mechanism:

- A ***scalar*** field Φ (named *inflaton*) is driving the dynamics of the universe, introducing an accelerated expansion
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**Write *a(t)* for a fluid with *p=-ρ***

Predictions:

- Curvature fluctuations are mostly ***Gaussian,*** with some room for *non-Gaussianities* depending upon the model
- A **power law** description of the scale dependence of the power spectrum of curvature fluctuations given by an spectral index $n_s$ which must be slightly **< 1**
- Some level of tensor metric fluctuations should be introduced (Gravitational Waves – GW), parametrized by the tensor-to-scalar ratio $r = C_l^{tensor} / C_l^{scalar}$ .

# Abstract from *Planck* 2018 parameter paper ...



**ABSTRACT**

We present cosmological parameter results from the final full-mission *Planck* measurements of the cosmic microwave background (CMB) anisotropies, combining information from the temperature and polarization maps and the lensing reconstruction. Compared to the 2015 results, improved measurements of large-scale polarization allow the reionization optical depth to be measured with higher precision, leading to significant gains in the precision of other correlated parameters. Improved modelling of the small-scale polarization leads to more robust constraints on many parameters, with residual modelling uncertainties estimated to affect them only at the $0.5\,\sigma$ level. We find good consistency with the standard spatially-flat 6-parameter ΛCDM cosmology having a power-law spectrum of adiabatic scalar perturbations (denoted "base ΛCDM" in this paper), from polarization, temperature, and lensing, separately and in combination. A combined analysis gives dark matter density $\Omega_c h^2 = 0.120 \pm 0.001$, baryon density $\Omega_b h^2 = 0.0224 \pm 0.0001$, scalar spectral index $n_s = 0.965 \pm 0.004$, and optical depth $\tau = 0.054 \pm 0.007$ (in this abstract we quote 68 % confidence regions on measured parameters and 95 % on upper limits). The angular acoustic scale is measured to 0.03 % precision, with $100\theta_* = 1.0411 \pm 0.0003$. These results are only weakly dependent on the cosmological model and remain stable, with somewhat increased errors, in many commonly considered extensions. Assuming the base-ΛCDM cosmology, the inferred (model-dependent) late-Universe parameters are: Hubble constant $H_0 = (67.4\pm0.5)\,\mathrm{km\,s^{-1}Mpc^{-1}}$; matter density parameter $\Omega_m = 0.315\pm0.007$; and matter fluctuation amplitude $\sigma_8 = 0.811\pm0.006$. We find no compelling evidence for extensions to the base-ΛCDM model. Combining with baryon acoustic oscillation (BAO) measurements (and considering single-parameter extensions) we constrain the effective extra relativistic degrees of freedom to be $N_{\mathrm{eff}} = 2.99 \pm 0.17$, in agreement with the Standard Model prediction $N_{\mathrm{eff}} = 3.046$, and find that the neutrino mass is tightly constrained to $\sum m_\nu < 0.12$ eV. The CMB spectra continue to prefer higher lensing amplitudes than predicted in base ΛCDM at over $2\,\sigma$, which pulls some parameters that affect the lensing amplitude away from the ΛCDM model; however, this is not supported by the lensing reconstruction or (in models that also change the background geometry) BAO data. The joint constraint with BAO measurements on spatial curvature is consistent with a flat universe, $\Omega_K = 0.001 \pm 0.002$. Also combining with Type Ia supernovae (SNe), the dark-energy equation of state parameter is measured to be $w_0 = -1.03 \pm 0.03$, consistent with a cosmological constant. We find no evidence for deviations from a purely power-law primordial spectrum, and combining with data from BAO, BICEP2, and Keck Array data, we place a limit on the tensor-to-scalar ratio $r_{0.002} < 0.07$. Standard big-bang nucleosynthesis predictions for the helium and deuterium abundances for the base-ΛCDM cosmology are in excellent agreement with observations. The *Planck* base-ΛCDM results are in good agreement with BAO, SNe, and some galaxy lensing observations, but in slight tension with the Dark Energy Survey's combined-probe results including galaxy clustering (which prefers lower fluctuation amplitudes or matter density parameters), and in significant, $3.6\,\sigma$, tension with local measurements of the Hubble constant (which prefer a higher value). Simple model extensions that can partially resolve these tensions are not favoured by the *Planck* data.

**Key words.** Cosmology: observations – Cosmology: theory – Cosmic background radiation – cosmological parameters

# Nucleosynthesis

# Nucleosynthesis

We start at **T~0.1 GeV**, below which we are *sure* of our Physics, although we have to make **assumptions** based upon our knowledge of today's universe

The Universe is **Radiation Dominated** (RD), and **photons**, **neutrinos**, plus **electrons** and **positrons** must be *relativistic.* Their abundance is dictated by the corresponding weak interaction reactions, which will take place as long as the interaction rate **Γ** is **higher** than the Hubble rate ***H(T)=da/(adt)*** **.** Protons and neutrons must already be in place, in equilibrium according to the reaction connecting these two species.

Since dynamics is dominated by *radiation* (RD), **all species interacting with radiation** are assumed to be in thermal equilibrium (at least, *ab initio)*, and due to the assumed *homogeneity* and *isotropy* of the universe (Copernican Principle), all regions in the universe have the same temperature (and eventually same abundance of elements/nucleii). **As a consequence, we can write *distribution functions f(x,p)=f(T,p)* for those species, either bosonic or fermionic.**

# Nucleosynthesis

- At T~0.1 GeV (~1e11K) dynamics is dominated by relativistic species: [**e,positron==e+**], **three** flavors of **neutrinos** and **photons**.

- At **T~1 MeV**, neutrinos stop interacting with [e,e+] since reaction rate **Γ=nσv** falls below *H(T)* at that epoch ==> **neutrinos** thus **decouple** from the rest of matter and free stream until present

$$\Gamma = n\sigma|v| \simeq 1.3 G_F^2 T^5, \qquad H(T) \simeq 1.66 g^{1/2}\left(\frac{T^2}{M_{pl}}\right)$$

$$\frac{\Gamma}{H} \simeq 0.24\, T^3 \left(\frac{M_{Pl}}{G_F^{-2}}\right) \simeq \left(\frac{T}{1.3\,\mathrm{MeV}}\right)^3$$

- But at **T~0.5 MeV**, **e and e+ annihilate** and leave an (observed) residual fraction of electrons, at the expense of ***increasing*** the **number of photons** (and thus increasing its temperature). From arguments of entropy conservation (that depends upon the number of relativistic degrees of freedom and temperature), one can easily derive that **(Prove it!)**

$$\left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3} \simeq 0.71$$

# Nucleosynthesis

The binding energies for the lightest nuclei are [**2.22, 6.92, 7.72, 28.3**] **MeV** for [$^{2}$**H**,$^{3}$**H**,$^{3}$**He**,$^{4}$**He**], respectively. However, the actual nucleosynthesis takes place at significantly lower temperatures (**~10x**) due to the high entropy (high number of photons per baryon). It can be found that for an species of mass/atomic number A/Z,

$$\frac{n_A}{n_B} = X_A = F(A)\left(\frac{T}{m_A}\right)^{3(A-1)/2} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(B_A/T_\gamma\right)$$

with $\boldsymbol{\eta = n_{photons} / n_B \sim 1e\text{-}9}$, F(A)~O[1], and $B_A = Zm_p + (A-Z)m_n - m_A$) is the **binding** energy of the nucleus

# Nucleosynthesis

But before the **lighest nuclei** can be **formed**, we have to understand how many **neutrons** are **available**: protons and neutrons are in equilibrium (via weak interactions) for as long the rate of those reactions rates are higher than *H(T).* During those times,

$$\frac{n_n}{n_p} = \exp - \frac{m_n - m_p}{T} := \exp - \frac{Q}{T}$$

.

But at T~0.7—0.8 MeV, those rates become similar, and for **T<0.7 MeV only the reaction n--> p+e+nu occurs**. At that moment,

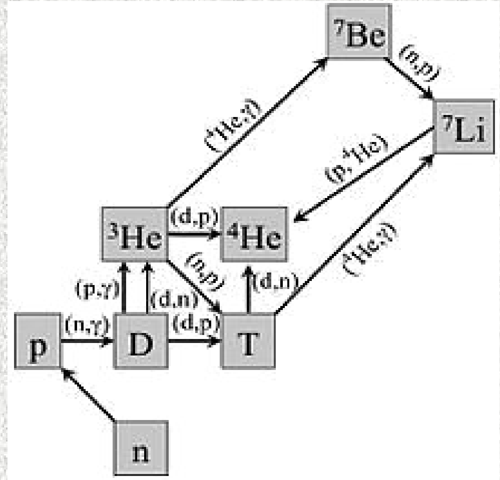
$$\frac{n_n}{n_p} = \exp - \frac{m_n - m_p}{0.7\,\mathrm{MeV}} \sim \frac{1}{6}$$

$^4$He is built upon D, $^3$He, $^3$H, whose abundances are negligible until T~0.1 MeV. The synthesis of $^4$He follows thereafter.

# Nucleosynthesis

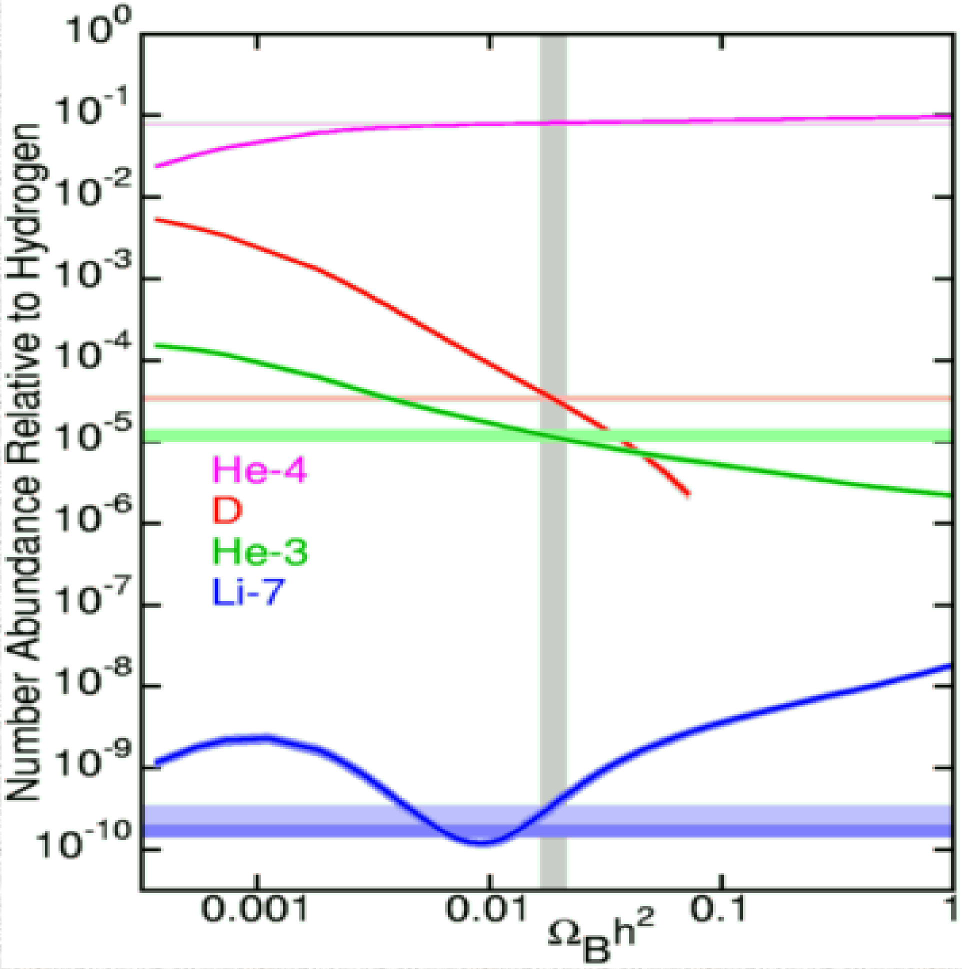
- Under the assumption that all neutrons end up in $^4$He, and after accounting for the fact that **from T~0.7 MeV down to T~0.1 MeV about 20% of the neutrons have decayed** ( exp(-t[0.1 MeV]/915 s) ~ exp(-3mins/915s) ~ 0.8 ), one can compute the abundance of $^4$He: **Y ~ 0.24** ***(do it!)***

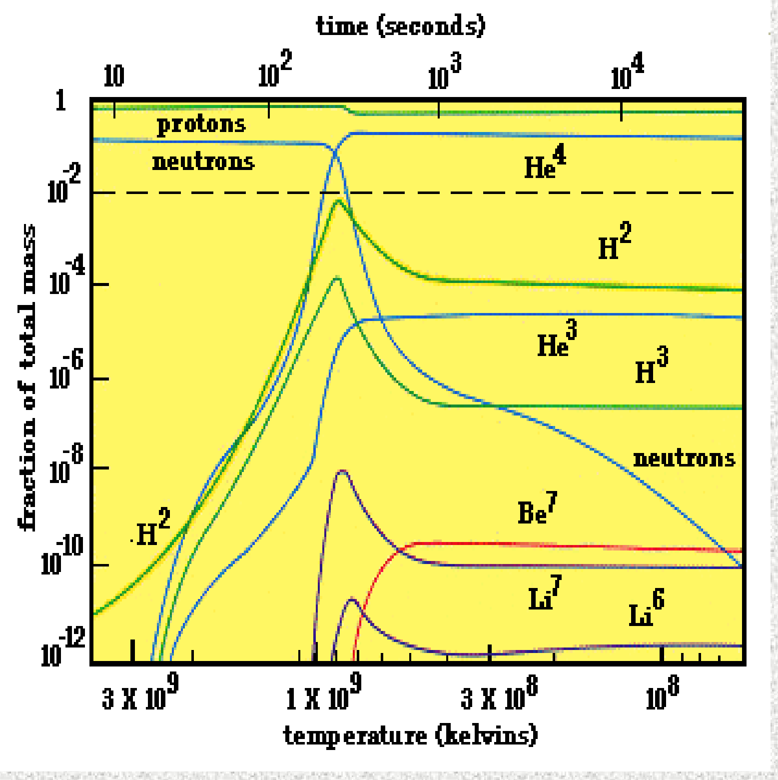
- Some **residual fractions** of **D, $^3$H, and $^3$He are left**, and (very small) traces of **$^7$Li** and **$^7$Be** are produced from reactions involving **$^4$He**, **$^3$H, n,** and **$^3$He**




- The synthesis of **heavier nuclei** is **supressed** since $^4$He + $^4$He or $^4$He + H **don't give rise to stable nuclei**, and participating nucleii must overcome the Coulomb barrier

# Nucleosynthesis




From astro.ucla.edu (Ned Wright)

# Basics of cosmological linear perturbation theory

# Basics of cosmological linear perturbation theory

**Synchronous gauge**

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

$$h_{ij} = h\delta_{ij}/3 + h_{ij}^{\parallel} + h_{ij}^{\perp} + h_{ij}^{T}.$$

$$h_{ij}^{\parallel} = (\partial_i \partial_j - \tfrac{1}{3}\delta_{ij}\nabla^2)\mu ,$$

$$h_{ij}^{\perp} = \partial_i A_j + \partial_j A_i , \quad \partial_i A_i = 0 .$$

$$h_{ij}^{\parallel}(\boldsymbol{x}, \tau) = \int d^3k e^{i\boldsymbol{k}\cdot\boldsymbol{x}}(\hat{\boldsymbol{k}}_i \hat{\boldsymbol{k}}_j - \tfrac{1}{3}\delta_{ij})[h(\boldsymbol{k}, \tau) + 6\eta(\boldsymbol{k}, \tau)] ,$$

$$\boldsymbol{k} = k\hat{\boldsymbol{k}} .$$

# Basics of cosmological linear perturbation theory

**Newtonian gauge (no tensor nor vector modes observed)**

$$ds^2 = a^2(\tau)[-(1 + 2\psi)d\tau^2 + (1 - 2\phi)dx^i dx_i]$$

In the absence of **stress** and in the **Newtonian** limit, the **scalar field ψ** is equivalent to the **Newtonian gravitational potential**, and ψ = Φ

**Connection between the two gauges:**

$$\psi(\boldsymbol{k}, \tau) = \frac{1}{2k^2}\left\{\ddot{h}(\boldsymbol{k}, \tau) + 6\ddot{\eta}(\boldsymbol{k}, \tau) + \frac{\dot{a}}{a}[\dot{h}(\boldsymbol{k}, \tau) + 6\dot{\eta}(\boldsymbol{k}, \tau)]\right\},$$

$$\phi(\boldsymbol{k}, \tau) = \eta(\boldsymbol{k}, \tau) - \frac{1}{2k^2}\frac{\dot{a}}{a}[\dot{h}(\boldsymbol{k}, \tau) + 6\dot{\eta}(\boldsymbol{k}, \tau)] .$$

# Basics of cosmological linear perturbation theory

## Distribution functions and energy-momentum tensor

$$T_{\mu\nu} = \int dP_1\, dP_2\, dP_3 (-g)^{-1/2} \frac{P_\mu P_\nu}{P^0} f(x^i, P_j, \tau)$$

$$f_0 = f_0(\epsilon) = \frac{g_s}{h_P^3} \frac{1}{e^{\epsilon/k_B T_0} \pm 1}$$

$$f(x^i, P_j, \tau) = f_0(q)[1 + \Psi(x^i, q, n_j, \tau)]$$

# Basics of cosmological linear perturbation theory

$$T^{\mu\nu}{}_{;\mu} = \partial_\mu T^{\mu\nu} + \Gamma^{\nu}{}_{\alpha\beta} T^{\alpha\beta} + \Gamma^{\alpha}{}_{\alpha\beta} T^{\nu\beta} = 0$$

**Conservation of energy-momentum tensor**

*Synchronous gauge*:

$$\dot{\delta} = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) - 3\frac{\dot{a}}{a}\left(\frac{\delta P}{\delta\rho} - w\right)\delta\,,$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{\delta P/\delta\rho}{1+w}k^2\delta - k^2\sigma\,.$$

*Conformal Newtonian gauge*:

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) - 3\frac{\dot{a}}{a}\left(\frac{\delta P}{\delta\rho} - w\right)\delta\,,$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{\delta P/\delta\rho}{1+w}k^2\delta - k^2\sigma + k^2\psi$$

They are required for solving for the sound speed *w*

# Basics of cosmological linear perturbation theory

*Synchronous gauge*:

$$k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} = 4\pi G a^2\,\delta T^0{}_0(\mathrm{Syn})\,,$$

$$k^2\dot{\eta} = 4\pi G a^2(\bar{\rho} + \bar{P})\theta(\mathrm{Syn})\,,$$

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta = -8\pi G a^2\,\delta T^i{}_i(\mathrm{Syn})\,,$$

**Linearised Einstein eqs.**

*Conformal Newtonian gauge*:

$$k^2\phi + 3\frac{\dot{a}}{a}\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) = 4\pi G a^2\,\delta T^0{}_0(\mathrm{Con})\,,$$

$$k^2\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) = 4\pi G a^2(\bar{\rho} + \bar{P})\theta(\mathrm{Con})\,,$$

$$\ddot{\phi} + \frac{\dot{a}}{a}(\dot{\psi} + 2\dot{\phi}) + \left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\psi + \frac{k^2}{3}(\phi - \psi) = \frac{4\pi}{3}G a^2\,\delta T^i{}_i(\mathrm{Con})\,,$$

$$k^2(\phi - \psi) = 12\pi G a^2(\bar{\rho} + \bar{P})\sigma(\mathrm{Con})\,,$$

# Basics of cosmological linear perturbation theory

$$\frac{Df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = \left(\frac{\partial f}{\partial \tau}\right)_C ,$$

**Boltzmann equations**

$$f(x^i, P_j, \tau) = f_0(q)[1 + \Psi(x^i, q, n_j, \tau)]$$

*Synchronous gauge:*

$$\frac{\partial \Psi}{\partial \tau} + i\,\frac{q}{\epsilon}(\boldsymbol{k}\cdot\hat{\boldsymbol{n}})\Psi + \frac{d\ln f_0}{d\ln q}\left[\dot{\eta} - \frac{\dot{h}+6\dot{\eta}}{2}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}})^2\right] = \frac{1}{f_0}\left(\frac{\partial f}{\partial \tau}\right)_C , \tag{40}$$

*Conformal Newtonian gauge:*

$$\frac{\partial \Psi}{\partial \tau} + i\,\frac{q}{\epsilon}(\boldsymbol{k}\cdot\hat{\boldsymbol{n}})\Psi + \frac{d\ln f_0}{d\ln q}\left[\dot{\phi} - i\,\frac{\epsilon}{q}(\boldsymbol{k}\cdot\hat{\boldsymbol{n}})\psi\right] = \frac{1}{f_0}\left(\frac{\partial f}{\partial \tau}\right)_C . \tag{41}$$

# Basics of cosmological linear perturbation theory

**We need however to keep track of the physics of the gas, regardless how simple it is:**

$$\dot{T}_b = -2\frac{\dot{a}}{a}T_b + \frac{8}{3}\frac{\mu}{m_e}\frac{\bar{\rho}_\gamma}{\bar{\rho}_b}an_e\sigma_T(T_\gamma - T_b)\,.$$

Temperature of gas (baryons)

$$\frac{n_e x_{n+1}}{x_n} = \frac{2g_{n+1}}{g_n}\left(\frac{m_e k_B T_b}{2\pi\hbar^2}\right)^{3/2} e^{-\chi_n/k_B T_b}$$

Ionization fraction of He and He+

$$\frac{dx_H}{d\tau} = aC_r[\beta(T_b)(1-x_H) - n_H\alpha^{(2)}(T_b)x_H^2]$$

Ionization rate

Recombination rate

Ionization fraction of hydrogen $x_H = n_e / n_H$

# Basics of cosmological linear perturbation theory

*Conformal Newtonian gauge*:

$$\dot{\delta}_c = -\theta_c + 3\dot{\phi}\,, \quad \dot{\theta}_c = -\frac{\dot{a}}{a}\,\theta_c + k^2\psi$$

**Cold Dark Matter (CDM)**

*Conformal Newtonian gauge*:

$$\dot{\delta}_b = -\theta_b + 3\dot{\phi}\,,$$

$$\dot{\theta}_b = -\frac{\dot{a}}{a}\,\theta_b + c_s^2\,k^2\delta_b + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b}\,an_e\,\sigma_T(\theta_\gamma - \theta_b) + k^2\psi\,.$$

**Baryons (CDM)**

*Conformal Newtonian gauge*:

$$\dot{\delta}_\gamma = -\frac{4}{3}\,\theta_\gamma + 4\dot{\phi}\,,$$

$$\dot{\theta}_\gamma = k^2\left(\frac{1}{4}\,\delta_\gamma - \sigma_\gamma\right) + k^2\psi + an_e\,\sigma_T(\theta_b - \theta_\gamma)\,,$$

$$\dot{F}_{\gamma 2} = 2\dot{\sigma}_\gamma = \frac{8}{15}\,\theta_\gamma - \frac{3}{5}\,kF_{\gamma 3} - \frac{9}{5}\,an_e\,\sigma_T\,\sigma_\gamma + \frac{1}{10}\,an_e\,\sigma_T(G_{\gamma 0} + G_{\gamma 2})\,,$$

**Photons (CMB)**

# Basics of cosmological linear perturbation theory

**Results:**

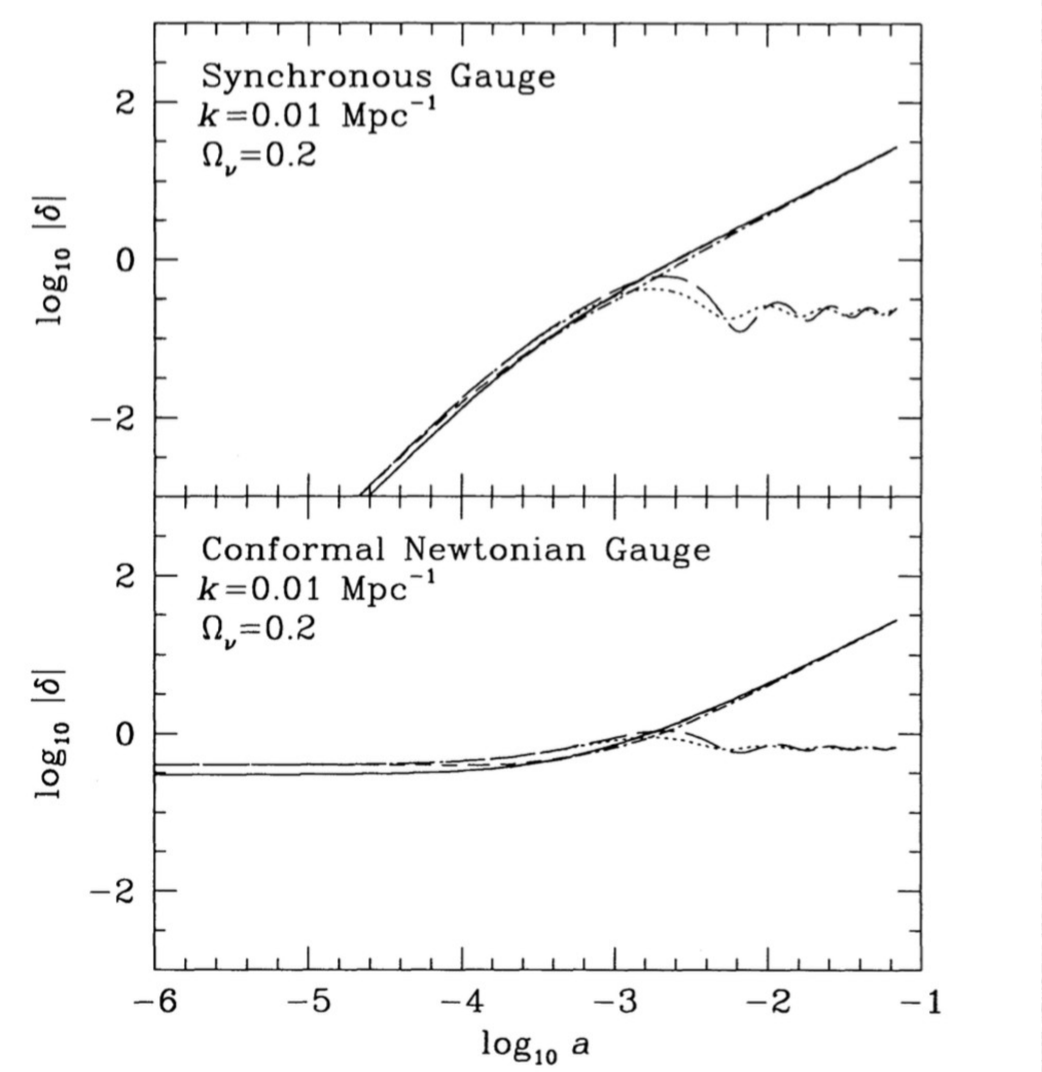



FIG. 3a

Low $k$, large scales

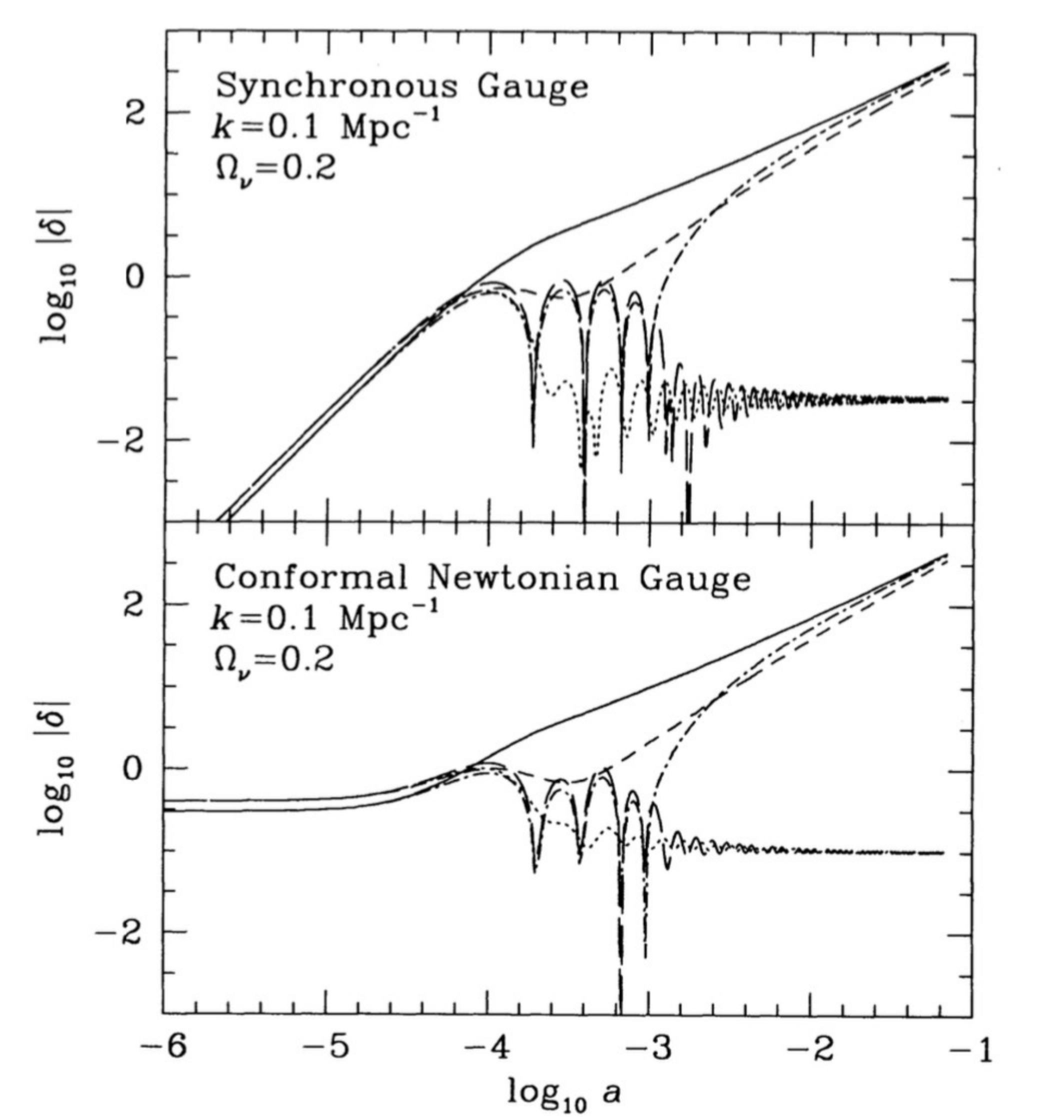
**Solid line == CDM**
**Dash-dotted == baryons**
**Long-dashed == CMB**
**Dotted == massless nu**
**Short-dashed == massive nu**

Ma & Bertschinger 95

# Basics of cosmological linear perturbation theory

**Results:**

Mid *k*, middle scales

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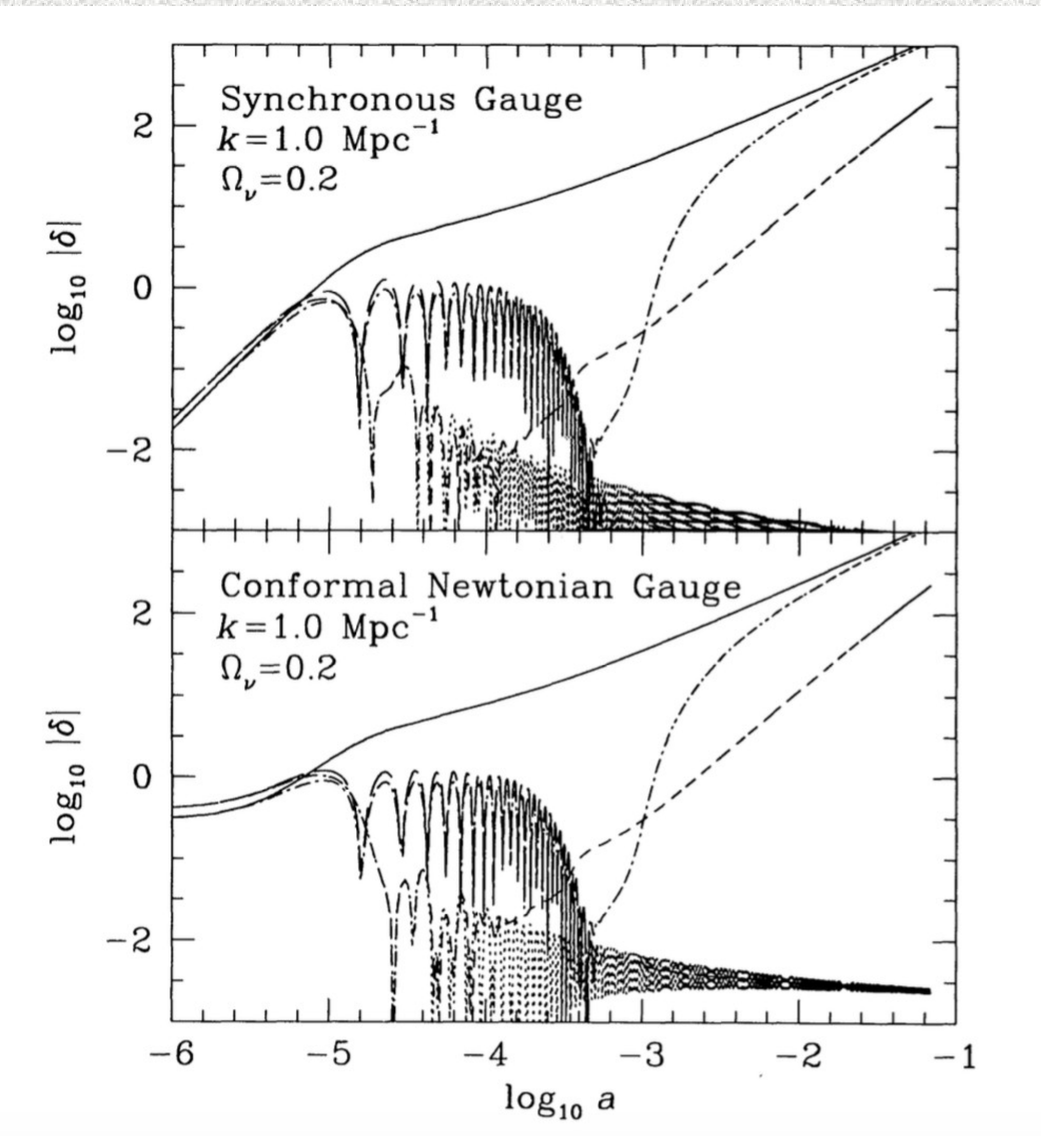
# Basics of cosmological linear perturbation theory

**Results:**

High *k*, small scales

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Ma & Bertschinger 95

# The particular case of CMB anisotropies:

$$B_\nu[T_{CMB}(1+\Delta)] = \frac{2h\nu^3/c^2}{\exp\frac{h\nu}{k_B T_{CMB}(1+\Delta)} - 1}; \quad \Delta \equiv \frac{\delta T_{CMB}}{T_{CMB}}$$

We work with brightness temperature units

$$\dot{\Delta}_T^{(S)} + ik\mu\,\Delta_T^{(S)} = \dot{\phi} - ik\mu\psi + \dot{\kappa}[-\Delta_T^{(S)} + \Delta_{T0}^{(S)} + i\mu v_b + \tfrac{1}{2}P_2(\mu)\Pi]$$

$$\dot{\Delta}_P^{(S)} + ik\mu\,\Delta_P^{(S)} = \dot{\kappa}\{-\Delta_P^{(S)} + \tfrac{1}{2}[1 - P_2(\mu)]\Pi\},$$
$$\Pi = \Delta_{T2}^{(S)} + \Delta_{P2}^{(S)} + \Delta_{P0}^{(S)}$$

For **each *k*** or Fourier mode (that evolve independently of other modes in linear theory), one can write an **integral** solution:

The **linearised** Boltzmann equation for the CMB brightness temperature has this form (at **linear** level of perturbations, all **changes** in the CMB distribution function are **independent** of **frequency**, i.e., the resulting spectrum is **Planckian**, but with a slightly different temperature

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$$\Delta_{T,P}^{(S)} = \int_0^{\tau_0} d\tau\, e^{ik\mu(\tau-\tau_0)} S_{T,P}^{(S)}(k,\tau)$$

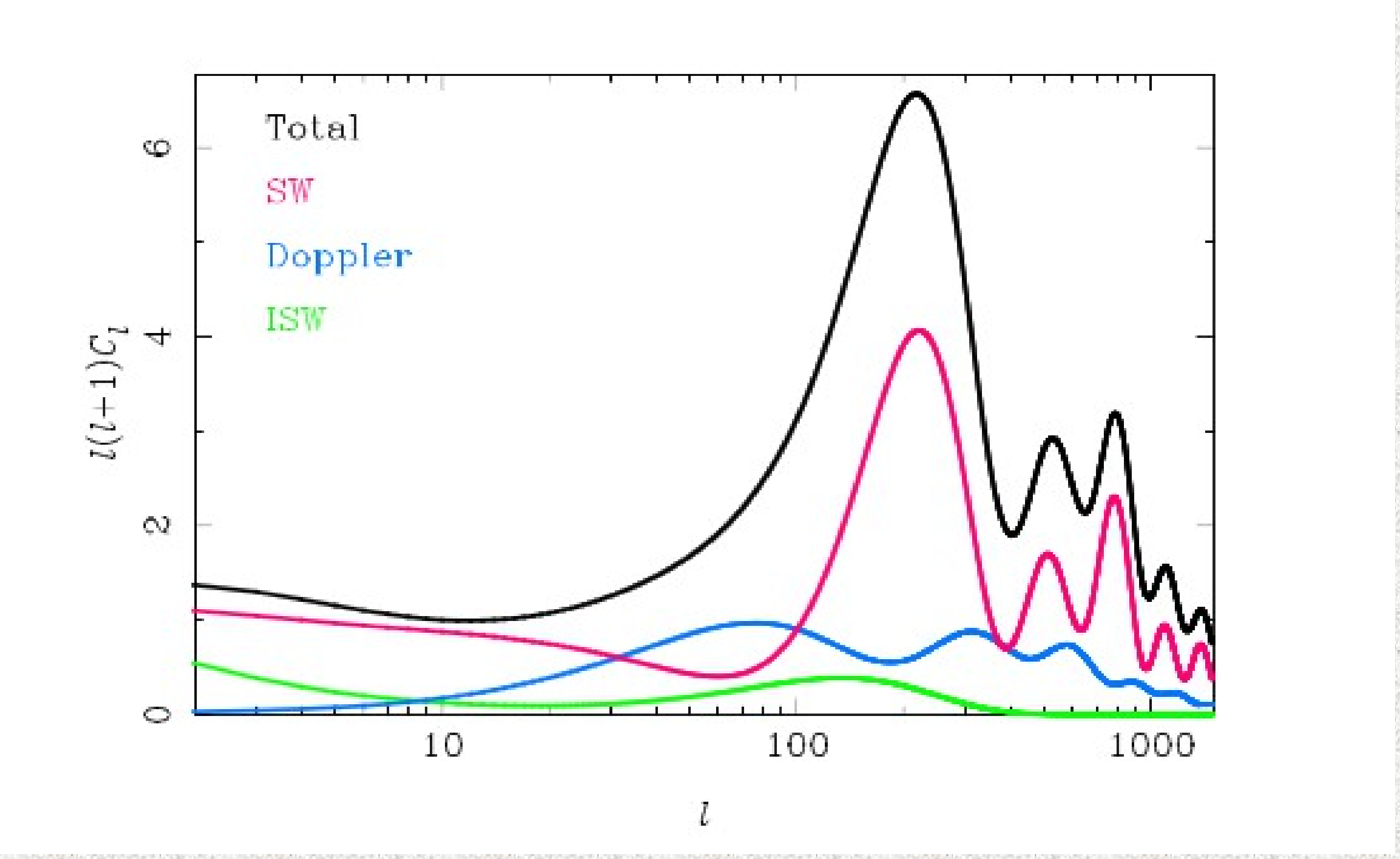
$$S_T^{(S)}(k,\tau) = g\left(\Delta_{T0} + \psi + \frac{\dot v_b}{k} + \frac{\Pi}{4} + \frac{3\ddot\Pi}{4k^2}\right) + e^{-\kappa}(\dot\phi + \dot\psi) + \dot g\left(\frac{v_b}{k} + \frac{3\dot\Pi}{4k^2}\right) + \frac{3\ddot g\Pi}{4k^2}\,,$$

Local monopole + gravitational potential: **Sachs Wolfe** effect

Time varying potentials: **integrated Sachs Wolfe** effect

Doppler or kSZ term

# The particular case of CMB anisotropies:




A. Challinor

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$$\Delta_{T,P}^{(S)} = \int_0^{\tau_0} d\tau e^{ik\mu(\tau-\tau_0)} S_{T,P}^{(S)}(k,\tau)$$

$$S_T^{(S)}(k,\tau) = g\left(\Delta_{T0} + \psi + \frac{\dot{v}_b}{k} + \frac{\Pi}{4} + \frac{3\ddot{\Pi}}{4k^2}\right) + e^{-\kappa}(\dot{\phi} + \dot{\psi}) + \dot{g}\left(\frac{v_b}{k} + \frac{3\dot{\Pi}}{4k^2}\right) + \frac{3\ddot{g}\Pi}{4k^2}$$

$$\Delta_{(T,P)l}^{(S)}(k, \tau = \tau_0) = \int_0^{\tau_0} S_{T,P}^{(S)}(k,\tau) j_l[k(\tau_0 - \tau)]d\tau$$

$$C(\theta) \equiv \langle \Delta(\hat{\boldsymbol{n}}_1)\Delta(\hat{\boldsymbol{n}}_2)\rangle = \frac{1}{4\pi}\sum_{l=0}^{\infty}(2l+1)C_l P_l(\hat{\boldsymbol{n}}_1 \cdot \hat{\boldsymbol{n}}_2)$$

$$C_l = 4\pi \int d^3k P_\psi(k)\Delta_l^2(k,\tau).$$

# The particular case of CMB anisotropies:

$$\dot{\Delta}_T^{(S)} + ik\mu\,\Delta_T^{(S)} = \dot{\phi} - ik\mu\psi + \dot{\kappa}\left[-\Delta_T^{(S)} + \Delta_{T0}^{(S)} + i\mu v_b + \tfrac{1}{2}P_2(\mu)\Pi\right]$$

$$\dot{\Delta}_P^{(S)} + ik\mu\,\Delta_P^{(S)} = \dot{\kappa}\left\{-\Delta_P^{(S)} + \tfrac{1}{2}[1 - P_2(\mu)]\Pi\right\},$$

$$\Pi = \Delta_{T2}^{(S)} + \Delta_{P2}^{(S)} + \Delta_{P0}^{(S)}$$

$$\Delta_{T,P}^{(S)} = \int_0^{\tau_0} d\tau\, e^{ik\mu(\tau-\tau_0)} S_{T,P}^{(S)}(k,\tau)$$

$$S_T^{(S)}(k,\tau) = g\left(\Delta_{T0} + \psi + \frac{\dot{v}_b}{k} + \frac{\Pi}{4} + \frac{3\ddot{\Pi}}{4k^2}\right) + e^{-\kappa}(\dot{\phi} + \dot{\psi}) + \dot{g}\left(\frac{v_b}{k} + \frac{3\dot{\Pi}}{4k^2}\right) + \frac{3\ddot{g}\Pi}{4k^2}$$

$$\Delta_{(T,P)l}^{(S)}(k,\tau=\tau_0) = \int_0^{\tau_0} S_{T,P}^{(S)}(k,\tau)\, j_l[k(\tau_0-\tau)]\,d\tau$$

**Prove this!**

$$C(\theta) \equiv \langle \Delta(\hat{\boldsymbol{n}}_1)\Delta(\hat{\boldsymbol{n}}_2)\rangle = \frac{1}{4\pi}\sum_{l=0}^{\infty}(2l+1)C_l P_l(\hat{\boldsymbol{n}}_1\cdot\hat{\boldsymbol{n}}_2)$$

$$C_l = 4\pi\int d^3k P_\psi(k)\Delta_l^2(k,\tau).$$

# Handling polarization in all-sky CMB maps

The Q & U stokes parameters are not invariant wrt rotations of the reference frame by an angle ψ (they are "*spin-2*" quantities)

$$Q' = Q\cos 2\psi + U\sin 2\psi$$
$$U' = -Q\sin 2\psi + U\cos 2\psi$$

One must define eigen-vectors of the spin ...

$$(Q \pm iU)'(\hat{\boldsymbol{n}}) = e^{\mp 2i\psi}(Q \pm iU)(\hat{\boldsymbol{n}}).$$

… which can be made spin-0 or scalar quantities via spin-raising and spin-lowering operations ...

$$\bar{\eth}^{\,2}(Q+iU)(\hat{\boldsymbol{n}}) = \sum_{lm}\left[\frac{(l+2)!}{(l-2)!}\right]^{1/2} a_{2,lm}Y_{lm}(\hat{\boldsymbol{n}})$$
$$\eth^{\,2}(Q-iU)(\hat{\boldsymbol{n}}) = \sum_{lm}\left[\frac{(l+2)!}{(l-2)!}\right]^{1/2} a_{-2,lm}Y_{lm}(\hat{\boldsymbol{n}}).$$

… allowing to define E and B modes of the polarization, with opposite parity properties ...

$$a_{E,lm} = -(a_{2,lm} + a_{-2,lm})/2$$
$$a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2.$$

$$C_{Tl} = \frac{1}{2l+1}\sum_m \langle a^*_{T,lm}a_{T,lm}\rangle$$
$$C_{El} = \frac{1}{2l+1}\sum_m \langle a^*_{E,lm}a_{E,lm}\rangle$$
$$C_{Bl} = \frac{1}{2l+1}\sum_m \langle a^*_{B,lm}a_{B,lm}\rangle$$
$$C_{Cl} = \frac{1}{2l+1}\sum_m \langle a^*_{T,lm}a_{E,lm}\rangle$$

# The impring of gravitational waves (GW) on all-sky CMB maps

The two tensor polarization of GWs are re-defined and made orthogonal …

$$\xi^1 = (\xi^+ - i\xi^\times)/\sqrt{2}$$
$$\xi^2 = (\xi^+ + i\xi^\times)/\sqrt{2}$$

$$\langle \xi^{1*}(\boldsymbol{k_1})\xi^1(\boldsymbol{k_2})\rangle = \langle \xi^{2*}(\boldsymbol{k_1})\xi^2(\boldsymbol{k_2})\rangle = \frac{P_h(k)}{2}\delta(\boldsymbol{k_1}-\boldsymbol{k_2}),\ \langle \xi^{1*}(\boldsymbol{k_1})\xi^2(\boldsymbol{k_2})\rangle = 0$$

.. so that they can be easily plugged into the Boltzmann equation for CMB intensity and polarization anisotropies …

$$\Delta_T^{(T)}(\tau_0,\hat{\boldsymbol{n}},\boldsymbol{k}) = \left[(1-\mu^2)e^{2i\phi}\xi^1(\boldsymbol{k}) + (1-\mu^2)e^{-2i\phi}\xi^2(\boldsymbol{k})\right]\int_0^{\tau_0} d\tau S_T^{(T)}(\tau,k)\, e^{ix\mu}$$
$$\Delta_{\tilde{E}}^{(T)}(\tau_0,\hat{\boldsymbol{n}},\boldsymbol{k}) = \left[(1-\mu^2)e^{2i\phi}\xi^1(\boldsymbol{k}) + (1-\mu^2)e^{-2i\phi}\xi^2(\boldsymbol{k})\right]\hat{\mathcal{E}}(x)\int_0^{\tau_0} d\tau S_P^{(T)}(\tau,k)\, e^{ix\mu}$$
$$\Delta_{\tilde{B}}^{(T)}(\tau_0,\hat{\boldsymbol{n}},\boldsymbol{k}) = \left[(1-\mu^2)e^{2i\phi}\xi^1(\boldsymbol{k}) - (1-\mu^2)e^{-2i\phi}\xi^2(\boldsymbol{k})\right]\hat{\mathcal{B}}(x)\int_0^{\tau_0} d\tau S_P^{(T)}(\tau,k)\, e^{ix\mu}.$$

# The impring of gravitational waves (GW) on all-sky CMB maps

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.. so that they can be easily plugged into the Boltzmann equation for CMB intensity and polarization anisotropies …

$$C_{Tl}^{(T)} = \frac{4\pi}{2l+1}\int k^2 dk P_h(k) \sum_m \left| \int d\Omega Y_{lm}^*(\hat{\boldsymbol{n}}) \int_0^{\tau_0} d\tau S_T^{(T)}(k,\tau)\,(1-\mu^2)e^{2i\phi}e^{ix\mu} \right|^2$$

# The impring of gravitational waves (GW) on all-sky CMB maps

The two tensor polarization of GWs are re-defined and made orthogonal …

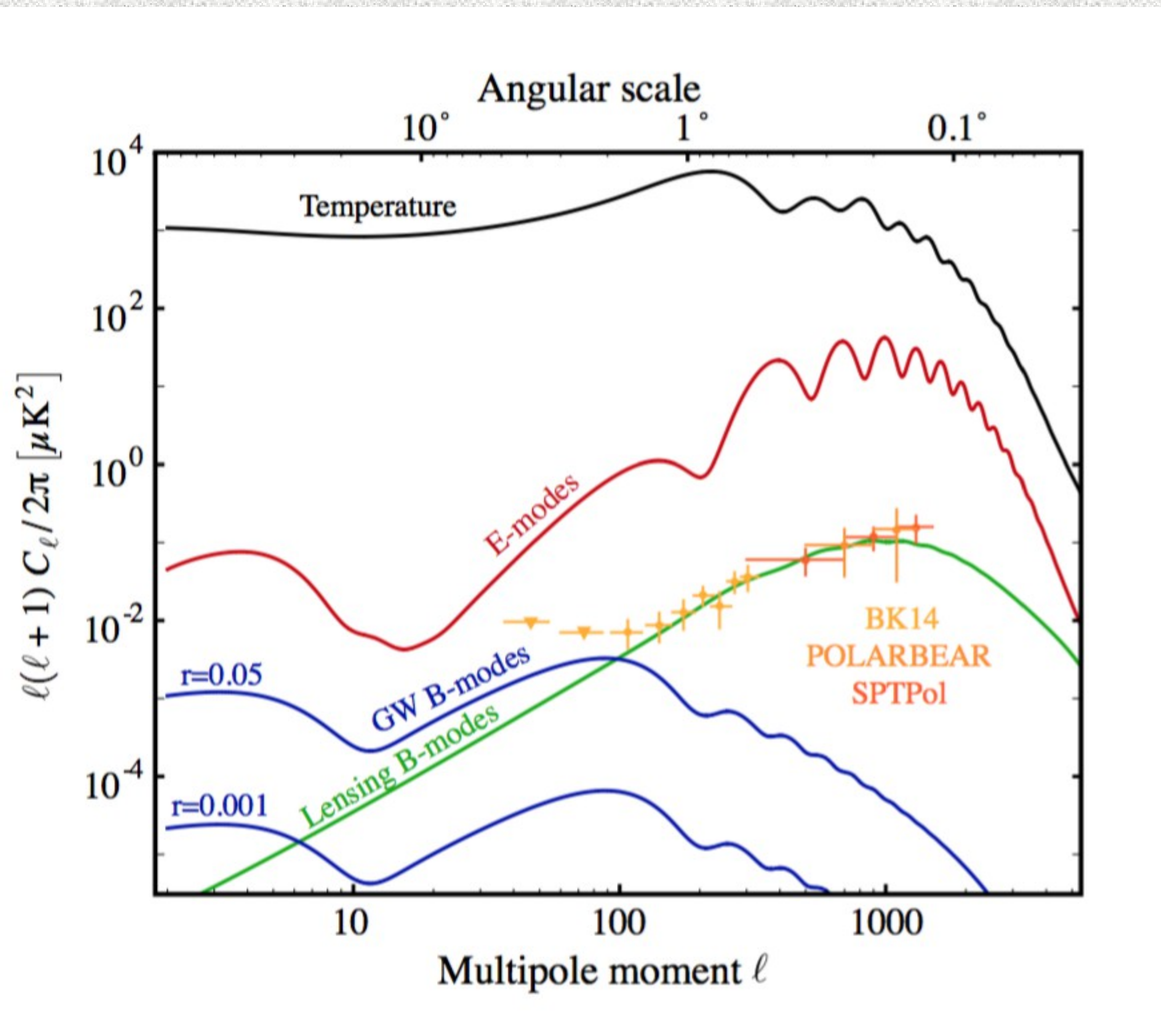
$$\xi^1 = (\xi^+ - i\xi^\times)/\sqrt{2}$$
$$\xi^2 = (\xi^+ + i\xi^\times)/\sqrt{2}$$

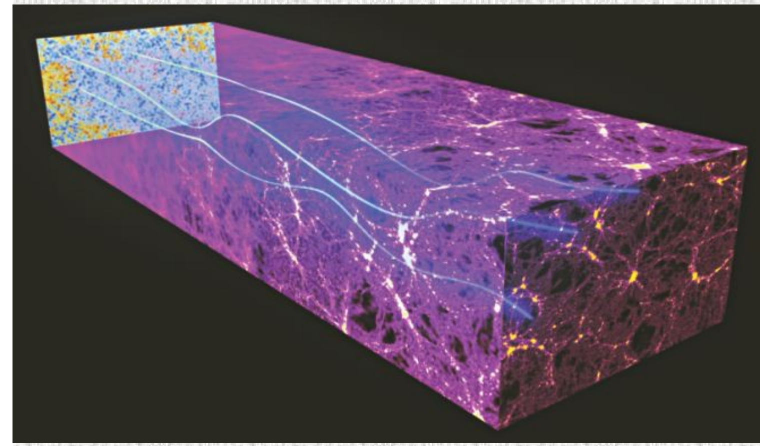
$$\langle \xi^{1*}(\boldsymbol{k_1})\xi^1(\boldsymbol{k_2})\rangle = \langle \xi^{2*}(\boldsymbol{k_1})\xi^2(\boldsymbol{k_2})\rangle = \frac{P_h(k)}{2}\delta(\boldsymbol{k_1}-\boldsymbol{k_2}),\ \langle \xi^{1*}(\boldsymbol{k_1})\xi^2(\boldsymbol{k_2})\rangle = 0$$

.. so that they can be easily plugged into the Boltzmann equation for CMB intensity and polarization anisotropies …

$$\begin{aligned} C_{El}^{(T)} &= (4\pi)^2 \int k^2 dk P_h(k) \left| \int_0^{\tau_0} d\tau S_P^{(T)}(k,\tau)\hat{\mathcal{E}}(x)\frac{j_l(x)}{x^2} \right|^2 \\ &= (4\pi)^2 \int k^2 dk P_h(k) \left( \int_0^{\tau_0} d\tau S_P^{(T)}(k,\tau) \Big[ -j_l(x) + j_l''(x) + \frac{2j_l(x)}{x^2} + \frac{4j_l'(x)}{x} \Big] \right)^2 \\ C_{Bl}^{(T)} &= (4\pi)^2 \int k^2 dk P_h(k) \left| \int_0^{\tau_0} d\tau S_P^{(T)}(k,\tau)\hat{\mathcal{B}}(x)\frac{j_l(x)}{x^2} \right|^2 \\ &= (4\pi)^2 \int k^2 dk P_h(k) \left( \int_0^{\tau_0} d\tau S_P^{(T)}(k,\tau) \Big[ 2j_l'(x) + \frac{4j_l}{x} \Big] \right)^2 \end{aligned}$$

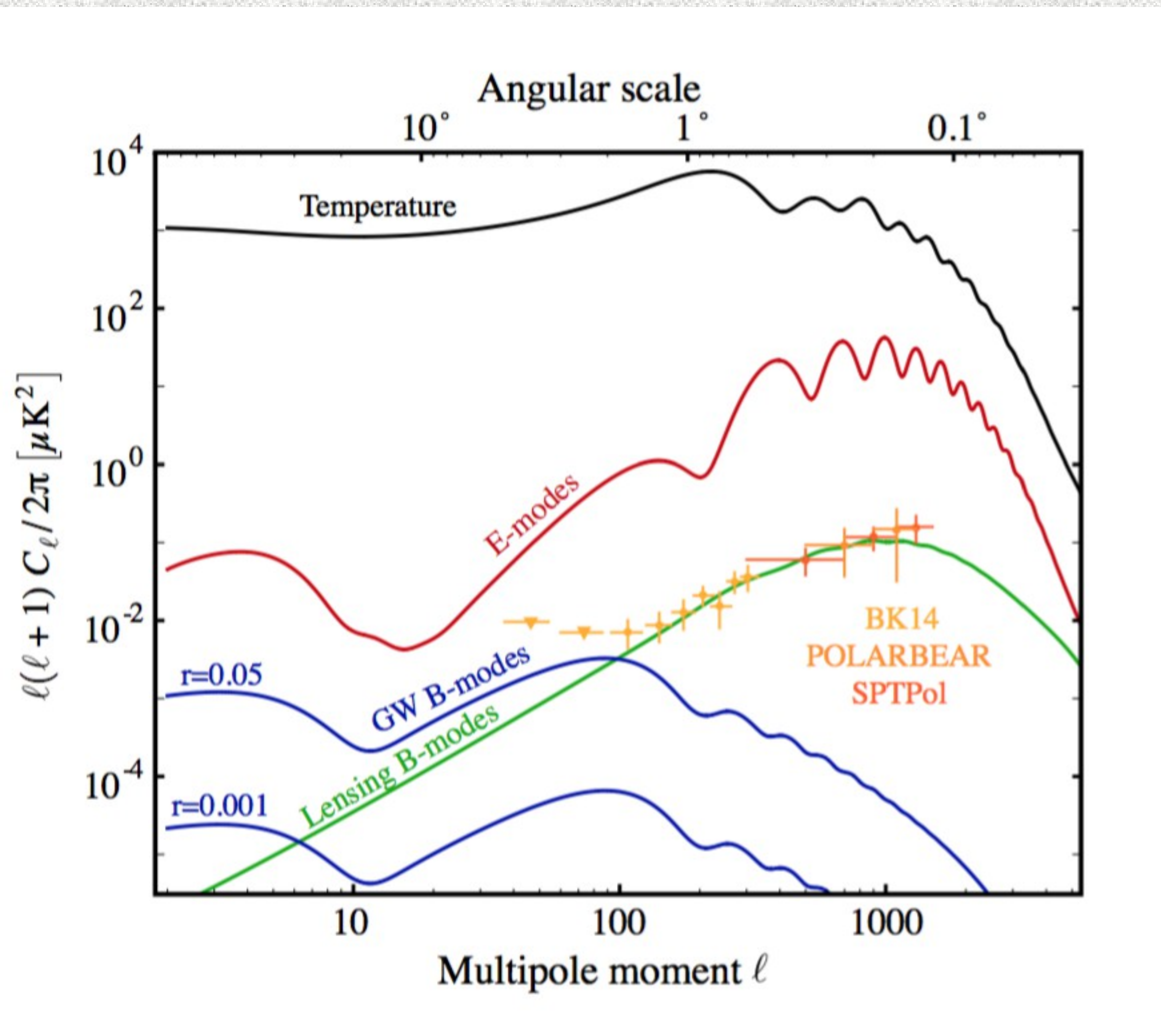
# Stage IV CMB experiments

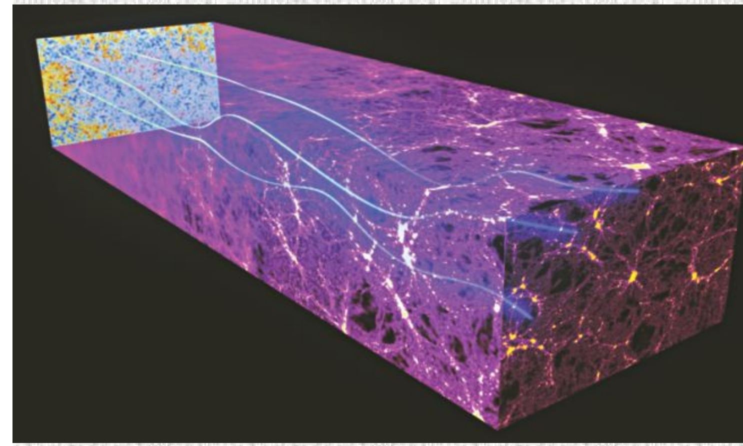


**Science Book, Stage IV**

# Stage IV CMB experiments



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