Problems Lectures on Flavour Physics
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1. Show that in the Standard Model one can always choose a weak basis, without loss of generality, where the mass matrices of the up and the down quarks are Hermitian matrices.

2. In the Standard Model, Z mediated flavour changing neutral currents (ZFCNC) are absent due to the GIM mechanism. If we include one down (Q $= -\frac{1}{3}$) isosinglet vector-like quark (D), i.e., D_L and D_R are singlets of SU(2), and one up (Q $= -\frac{2}{3}$) isosinglet vector-like quark (U) in the Lagrangian, ZFCNC's and deviations from unitarity of V_{CKM} arise.

a) Write down the most general quark mass terms which are obtained in the framework of this model.

b) Derive the structure of the charged currents, showing that there are deviations from unitarity of the 3×3 V_{CKM} matrix

c) Derive the structure of the neutral currents, showing that there are Z mediated FCNC in this model.

d) Show how the Z mediated FCNC are closely related to the deviations from unitarity in V_{CKM}

See, for instance, Ref: G. C. Branco, T. Morozumi, P. A. Parada and M. N. Rebelo, "CP asymmetries in B0 decays in the presence of flavor changing neutral currents," Phys. Rev. D **48** (1993) 1167. doi:10.1103/PhysRevD.48.1167

3. Consider for definiteness an extension of the Standard Model with one single down vectorial quark added. Show that deviations from unitariry, and therefore ZFCNC, are naturally suppressed in this model, provided that the isosinglet quark D is much heavier than the standard quarks.

4. The fact that quark masses are strongly hierarchical suggests that these could be obtained by a perturbation starting from the extreme chiral limit, where only the third generation of quarks acquires mass. In fact, it is the mass of the third generation of each quark sector that is much higher than the other two, therefore we are close to the extreme chiral limit.

Show that in the extreme chiral limit we are left with one mixing angle that we cannot rotate away.

Comment: This mixing is, in general, expected to be of order one and the fact that $|V_{13}|^2 + |V_{23}|^2 \approx 1.6 \times 10^{-3}$ implies the existence of a novel fine-tuning problem in the SM. In fact, the smallness of $|V_{13}|^2 + |V_{23}|^2$ may be interpreted as a hint from experiment, indicating that one should find a symmetry or a principle accounting for this fact.

See, for instance, Ref: F. J. Botella, G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, "What if the masses of the first two quark families are not generated by the standard model Higgs boson?," Phys. Rev. D **94** (2016) no.11, 115031 doi:10.1103/PhysRevD.94.115031 [arXiv:1602.08011 [hep-ph]].

5. The standard way of testing the compatibility of the SM with the existing data consists of adopting the Wolfenstein parametrisation and plotting in the ρ, η plane the constraints derived from various experimental inputs. Then, the challenge of the Standard Model is to find a region in the ρ, η plane where all the constraints are simultaneously satisfied. An alternative way is to choose a complete set of rephasing invariants of the V_{CKM} matrix and use 3×3 unitarity of V_{CKM} to derive a set of exact relations written in terms of measurable quantities. These relations can than be tested experimentally.

Obtain the set of exact relations involving these quantities corresponding to Eqs. (7) to (15) of Ref F. J. Botella, G. C. Branco, M. Nebot and M. N. Rebelo, "Unitarity triangles and the search for new physics," Nucl. Phys. B **651** (2003) 174 doi:10.1016/S0550-3213(02)01089-1 [hep-ph/0206133].

6. Physically meaningful quantities must be invariant under a rephasing of the fields. The simplest rephasing invariants of V_{CKM} matrix are its moduli and the next simplest are the "quartets" $Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}$. Invariants of higher order may be written as functions of quartets and moduli, barring singular cases in which some of the elements of V_{CKM} are zero.

a) Show that

$$V_{\alpha i} V_{\beta j} V_{\gamma k} V^*_{\alpha j} V^*_{\beta k} V^*_{\gamma i} = \frac{Q_{\alpha i \beta j} Q_{\beta i \gamma k}}{|V_{\beta i}|^2} \tag{0.1}$$

b) Show that the imaginary part of all quartets is equal to J up to a sign, with J defined as:

$$J \equiv \mathrm{Im} Q_{uscb} = \mathrm{Im}(V_{us} V_{cb} V^*_{ub} V^*_{cs}) \tag{0.2}$$

This implies that the area of all unitarity triangles of V_{CKM} is the same even though some of the unitarity triangles are squashed.

7. In models with a single Higgs doublet there are no Higgs mediated flavour changing neutral currents. Explain why. What is the structure of the HFCNCs in models with two Higgs doublets, when one singles out the standard model-like Higgs by choosing a basis where only one of the doublets acquires vacuum expectation value?