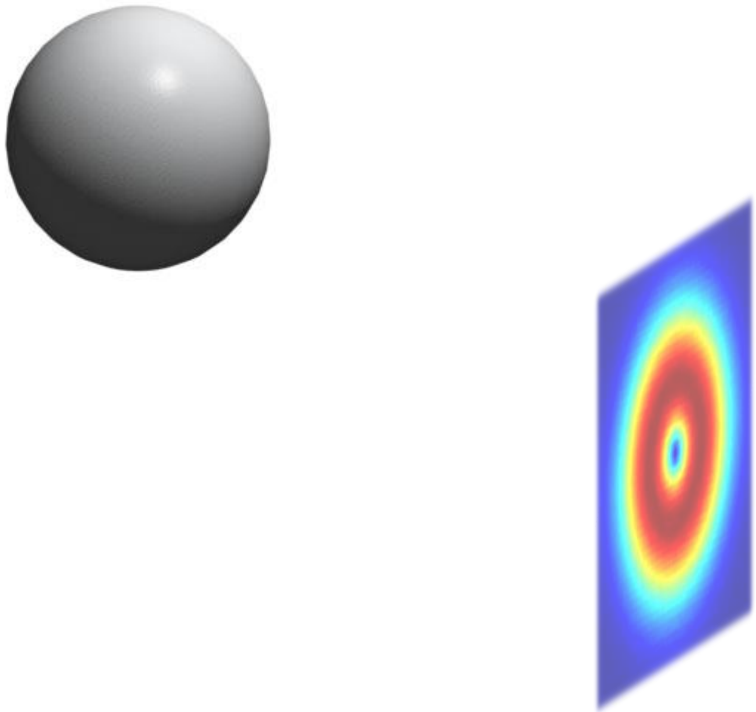

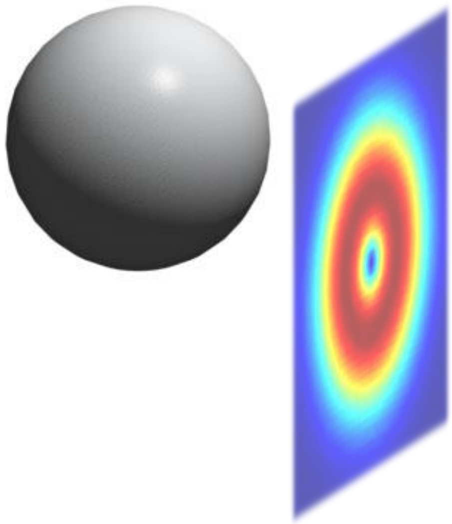

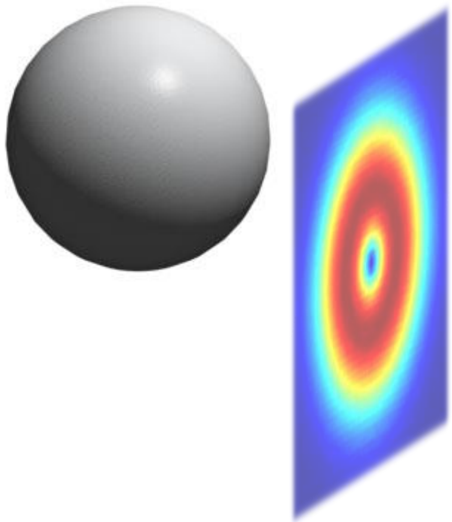

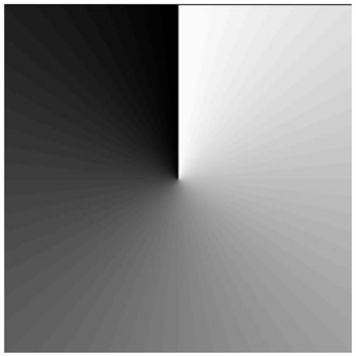

$q = 1$

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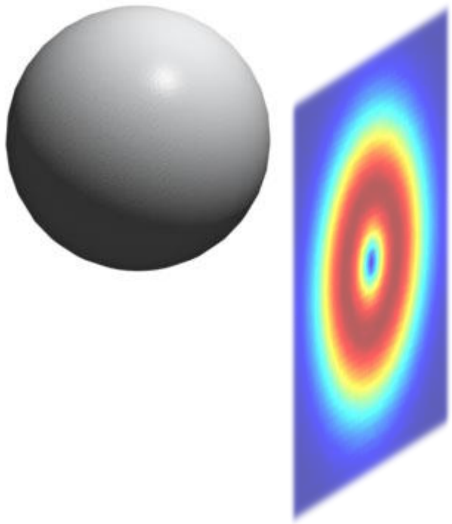

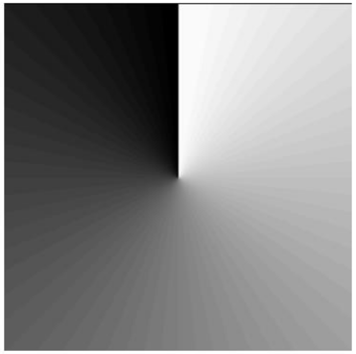

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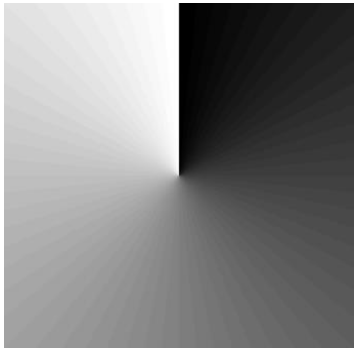

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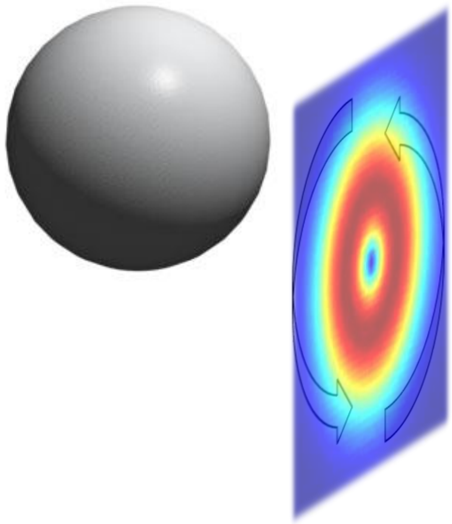

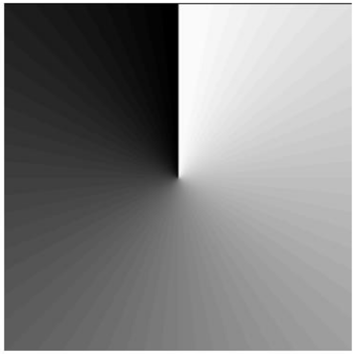

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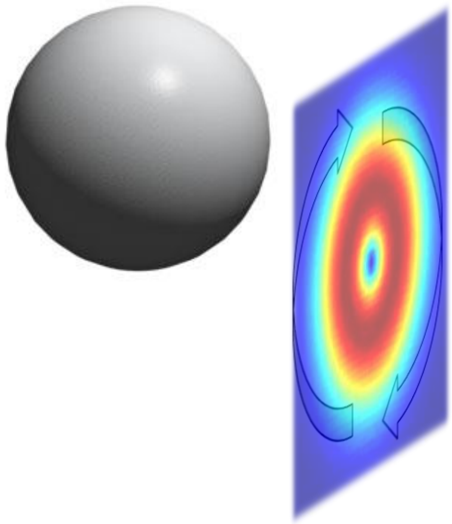

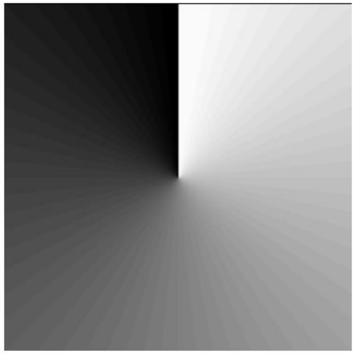

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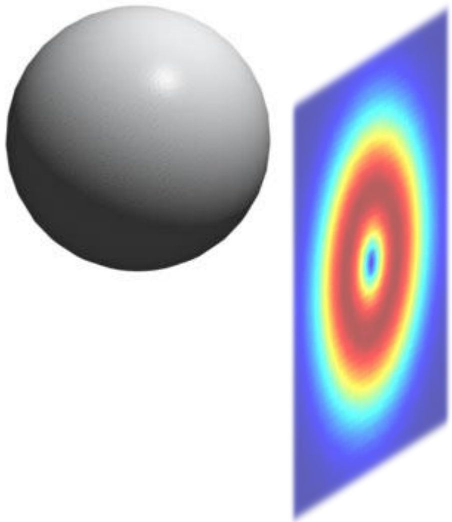

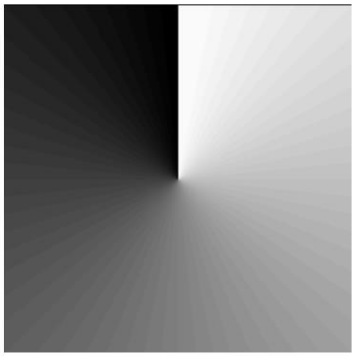

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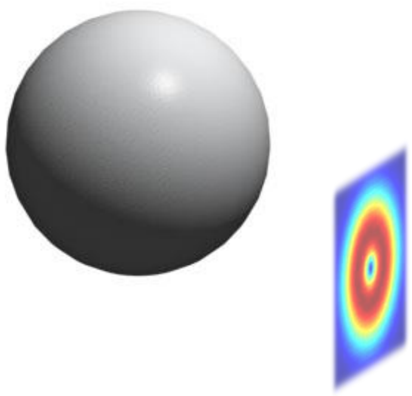

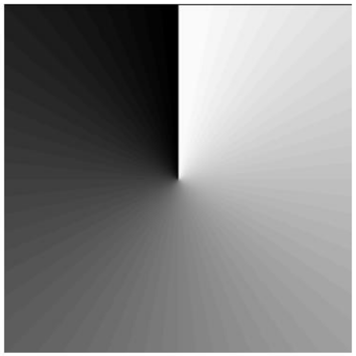

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Tailoring Mie scattering with Helicity and Angular Momentum

Xavier Zambrana-Puyalto

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 795838

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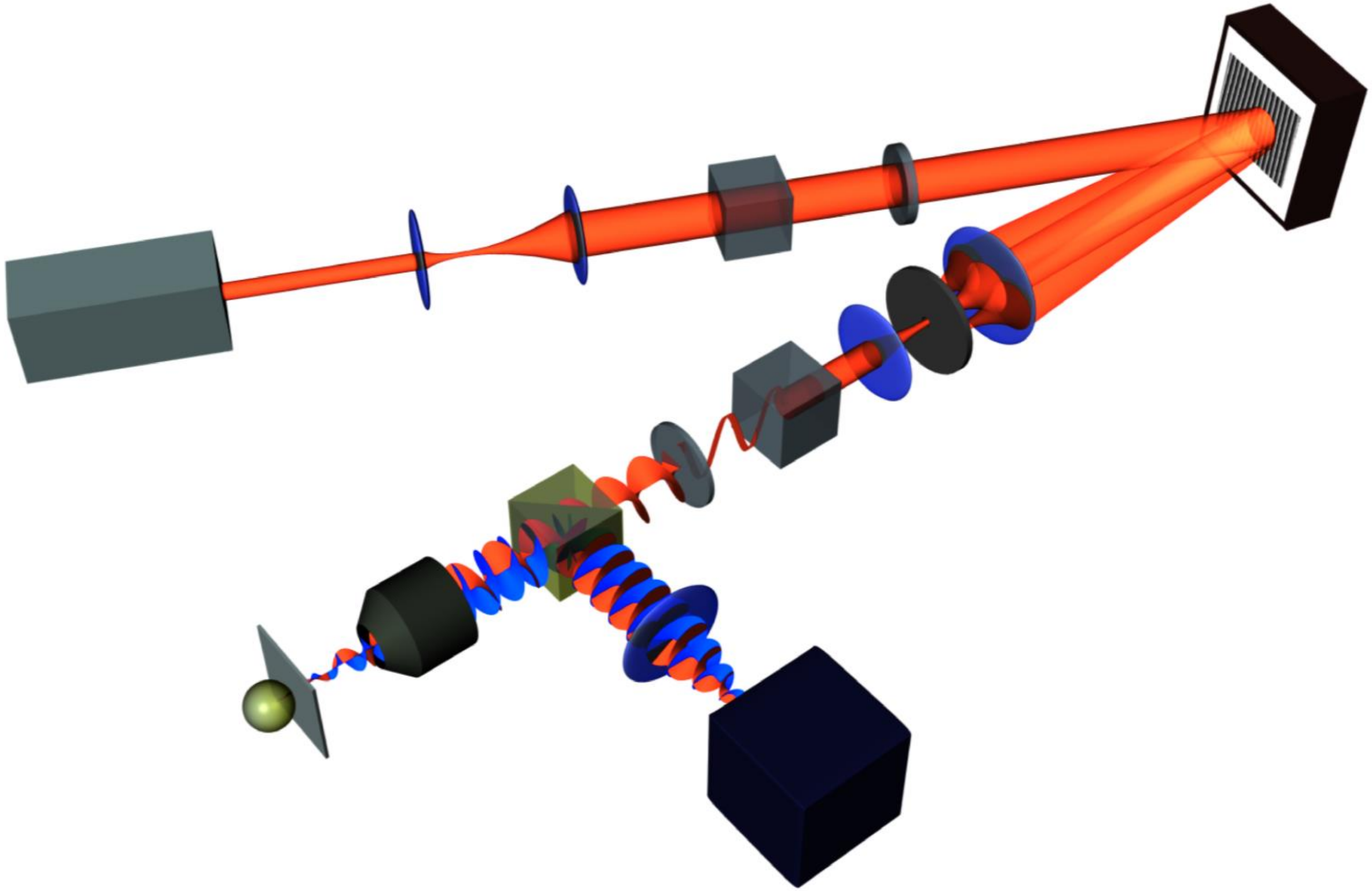
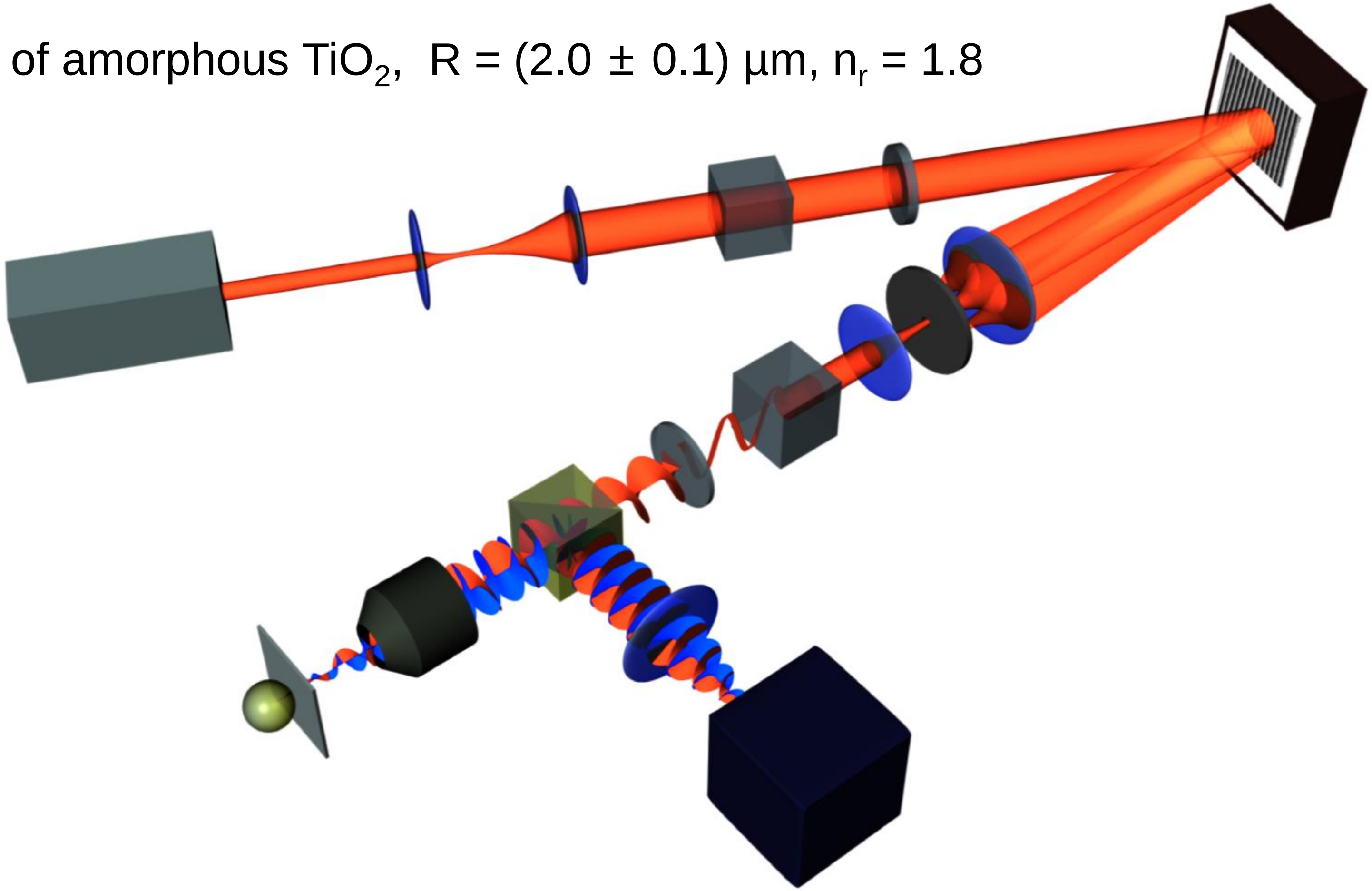
Experimental details



Experimental details

- ✓ Particles made of amorphous TiO_2, $R = (2.0 \pm 0.1)$ µm, $n_r = 1.8$



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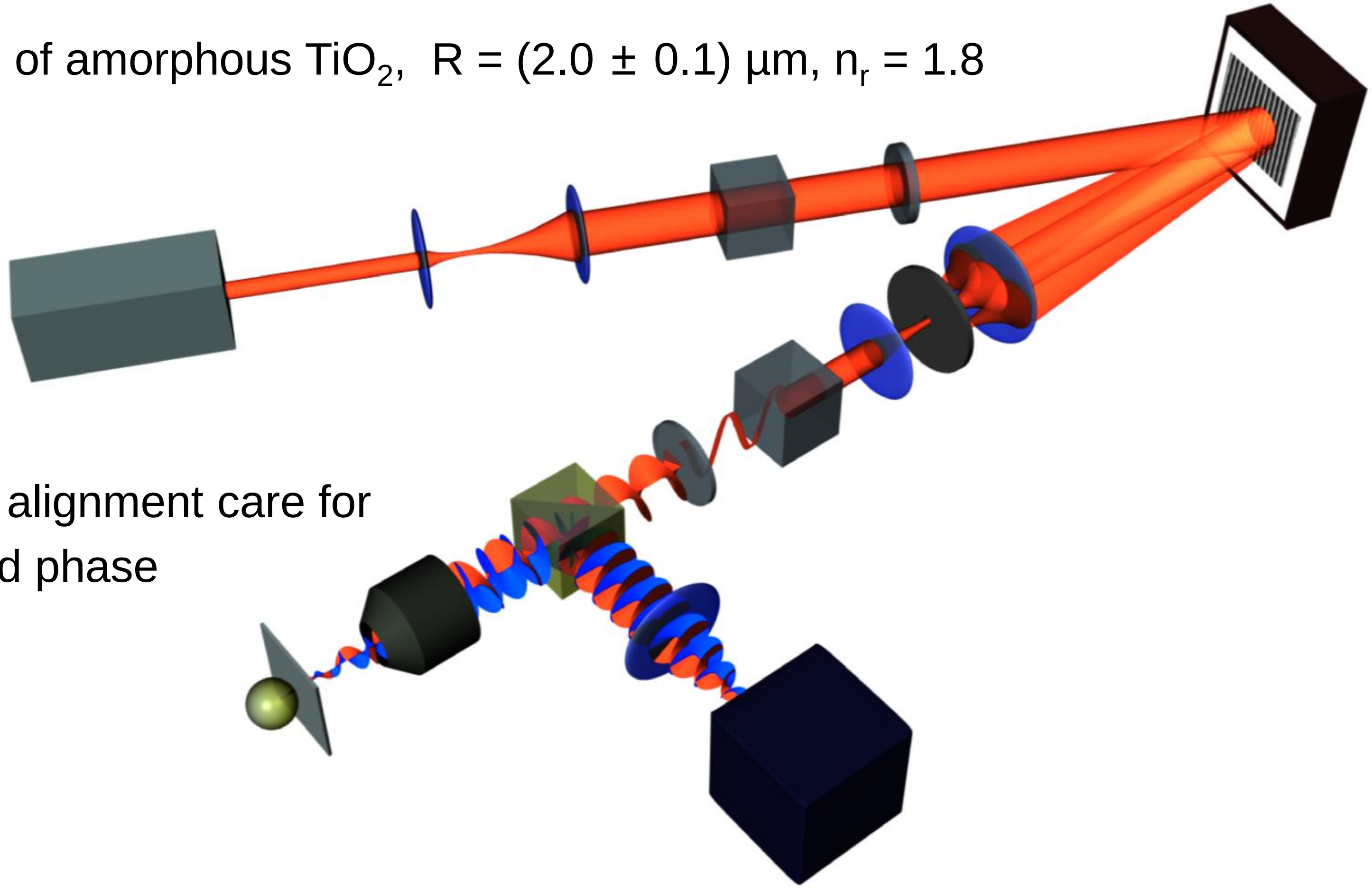

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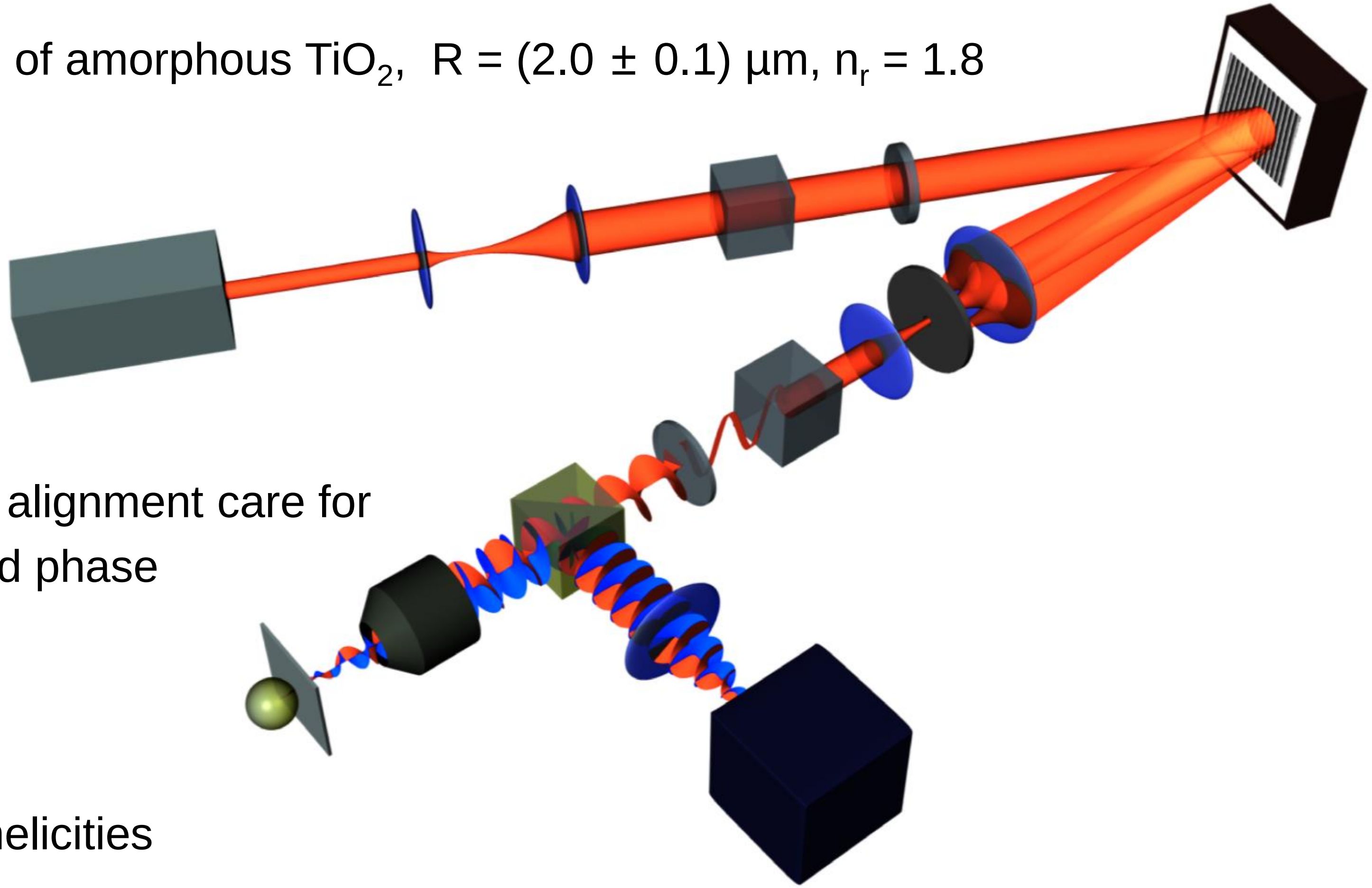

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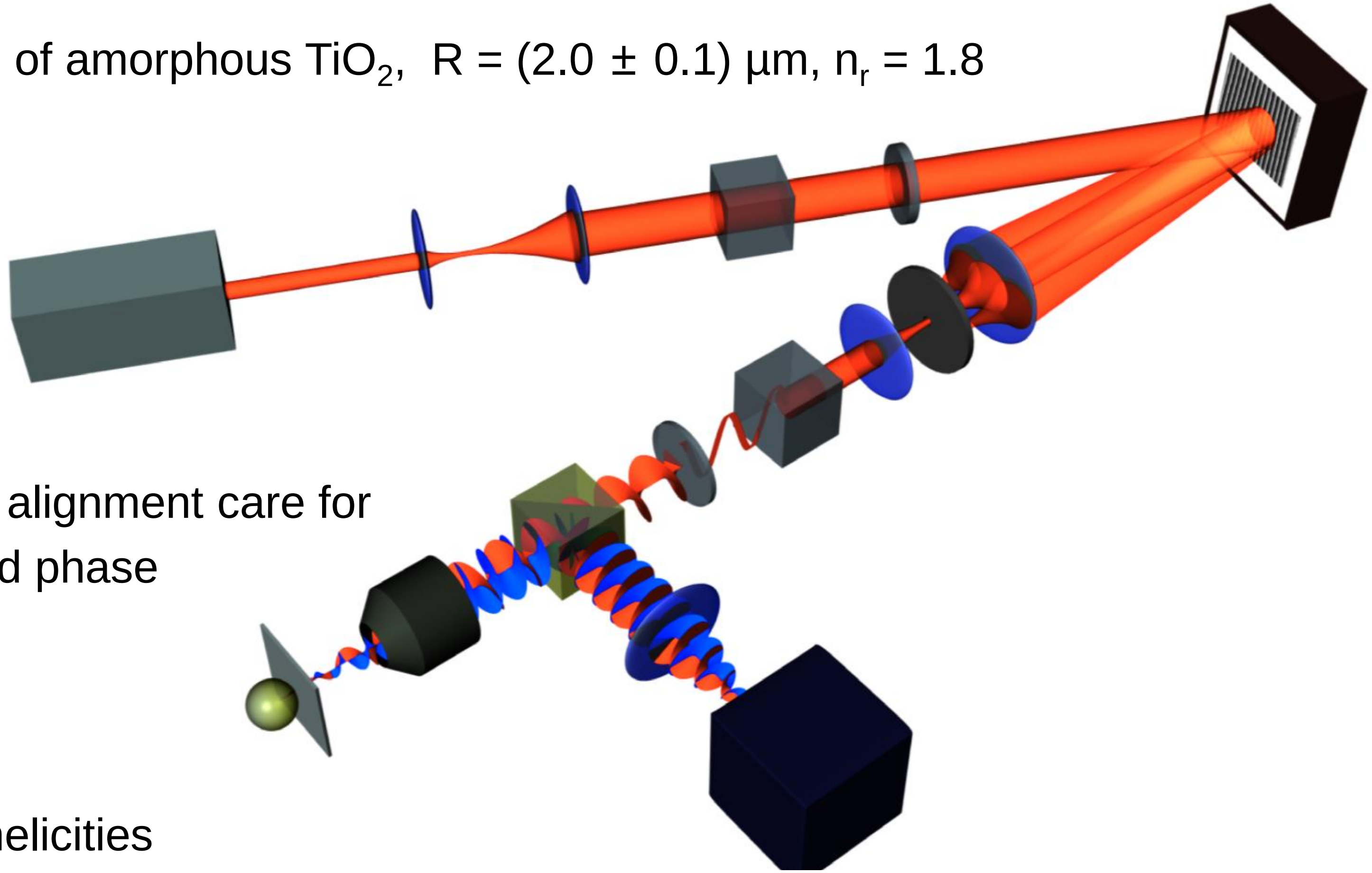

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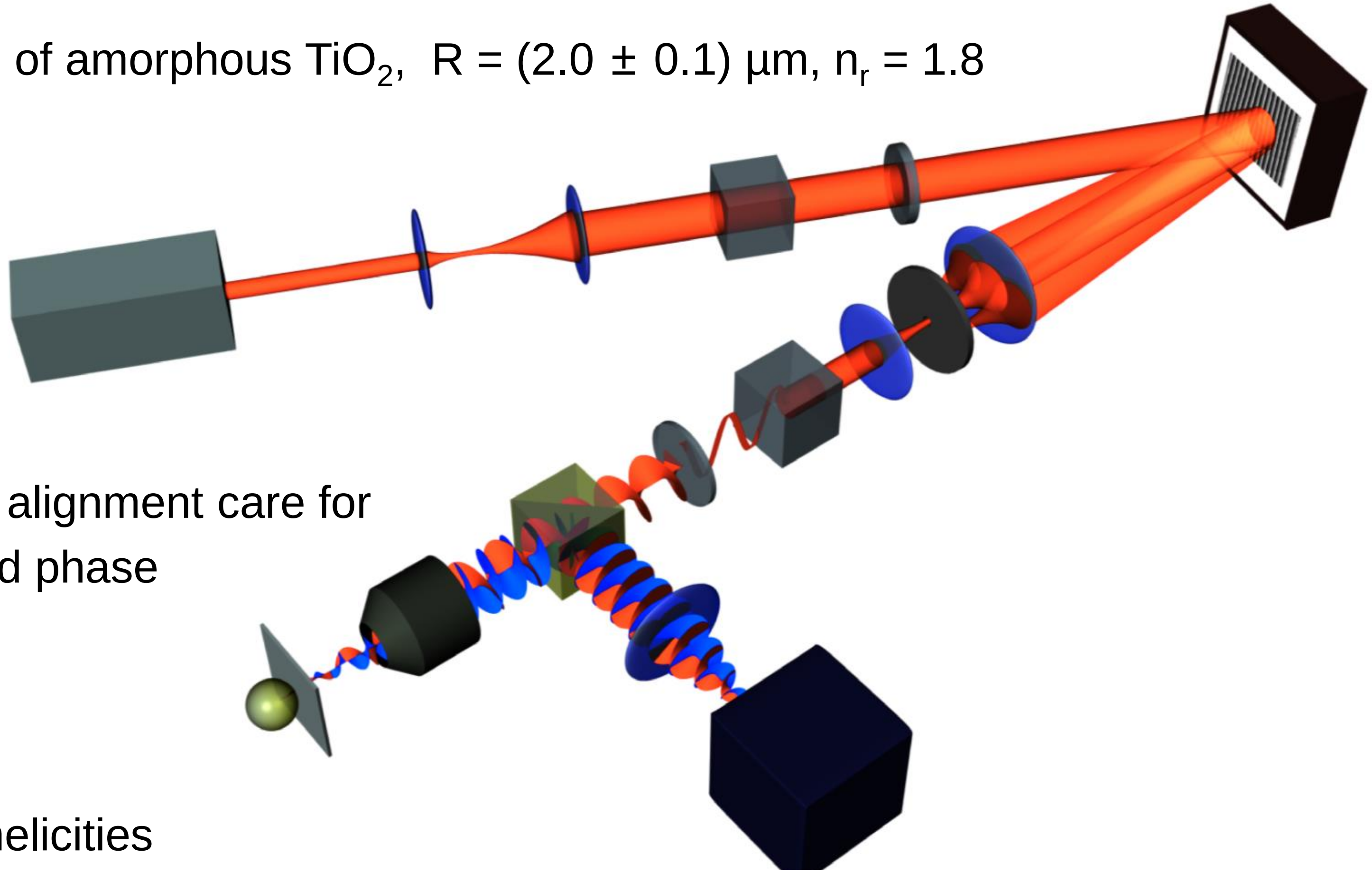

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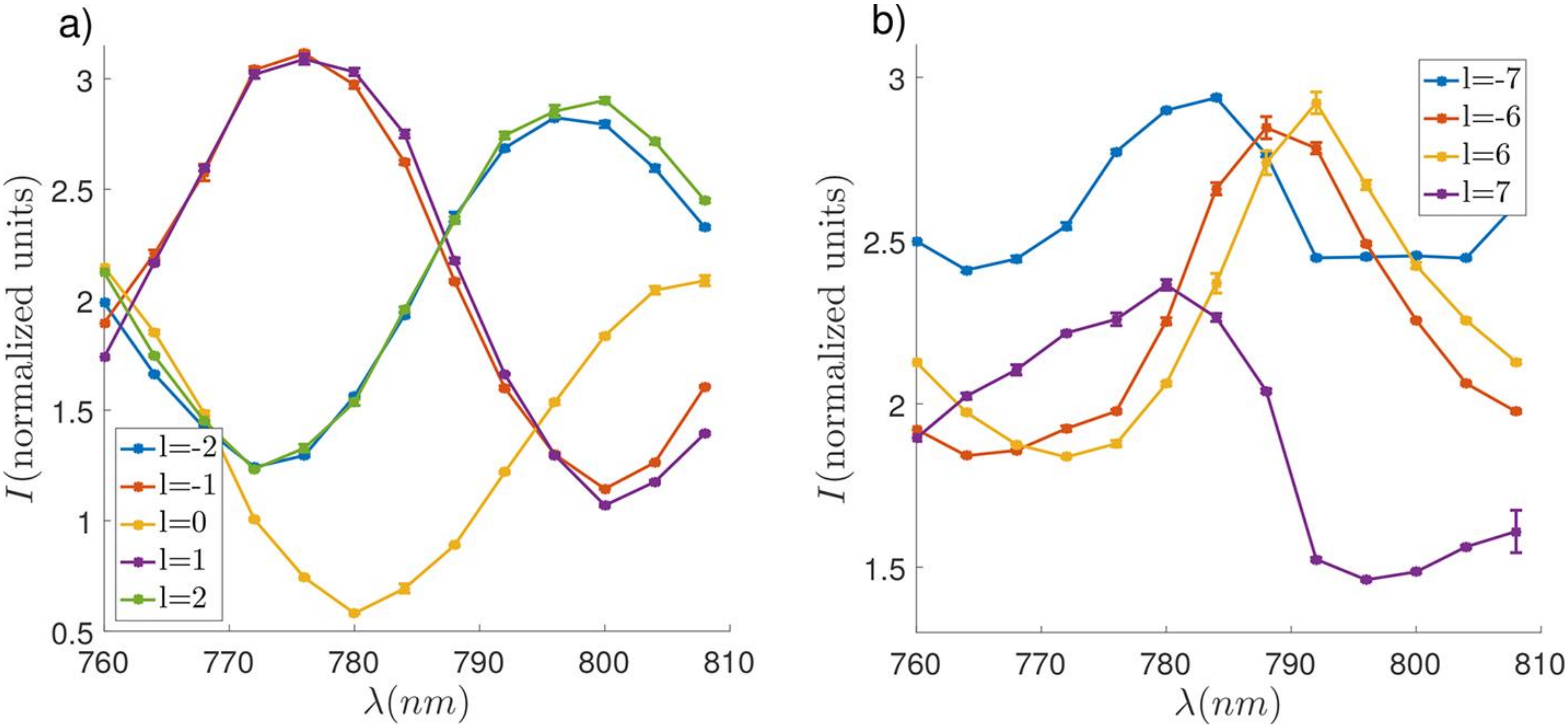

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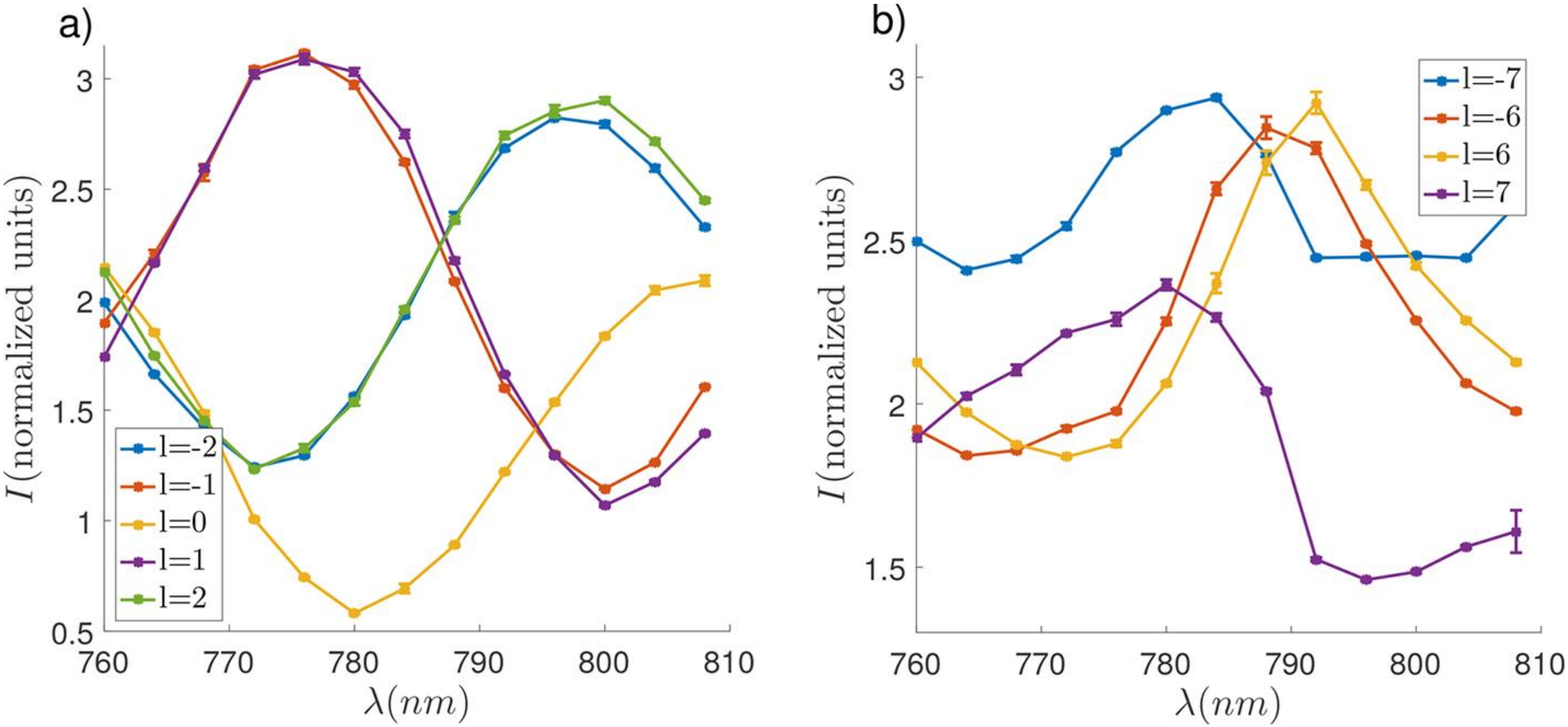

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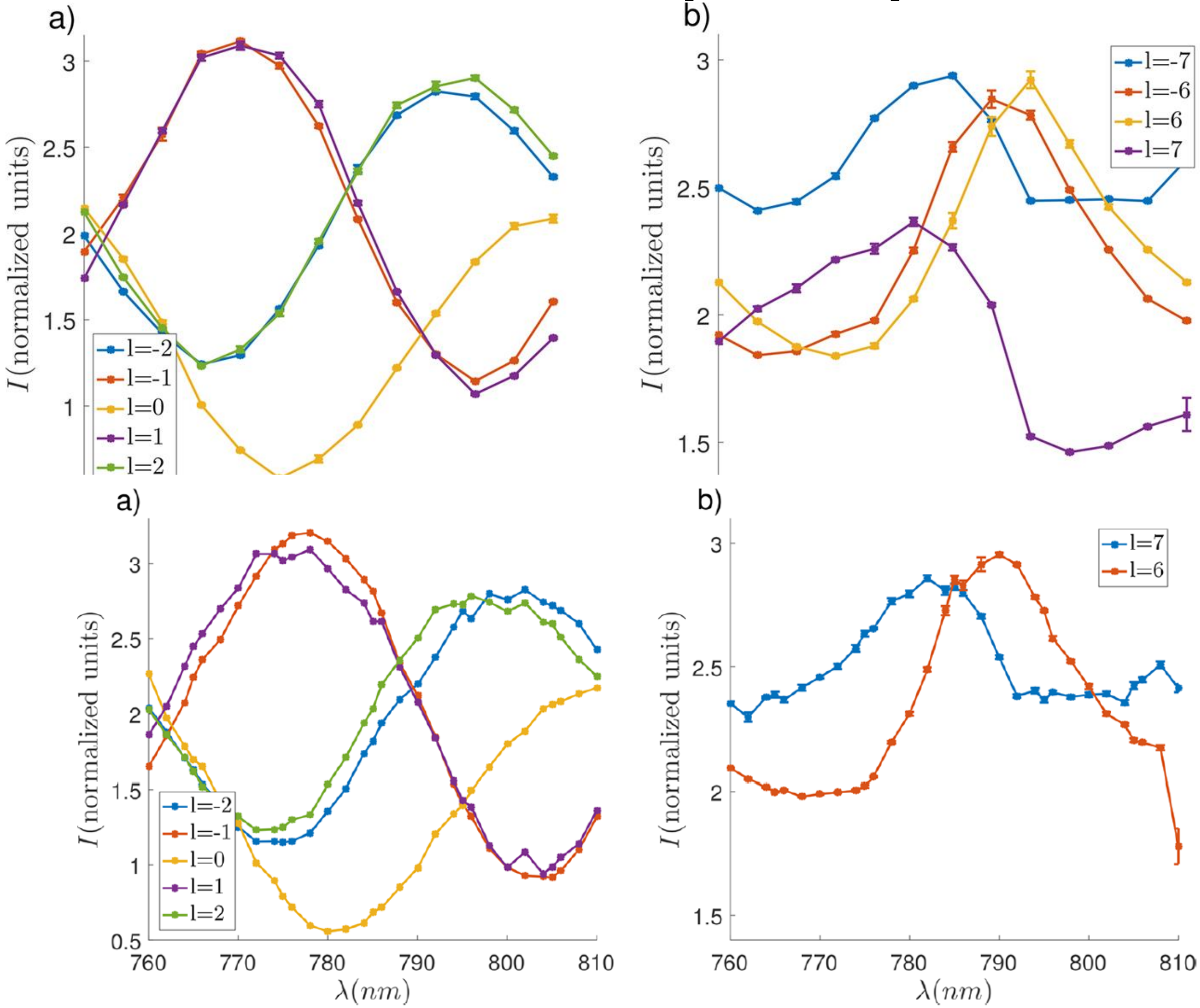

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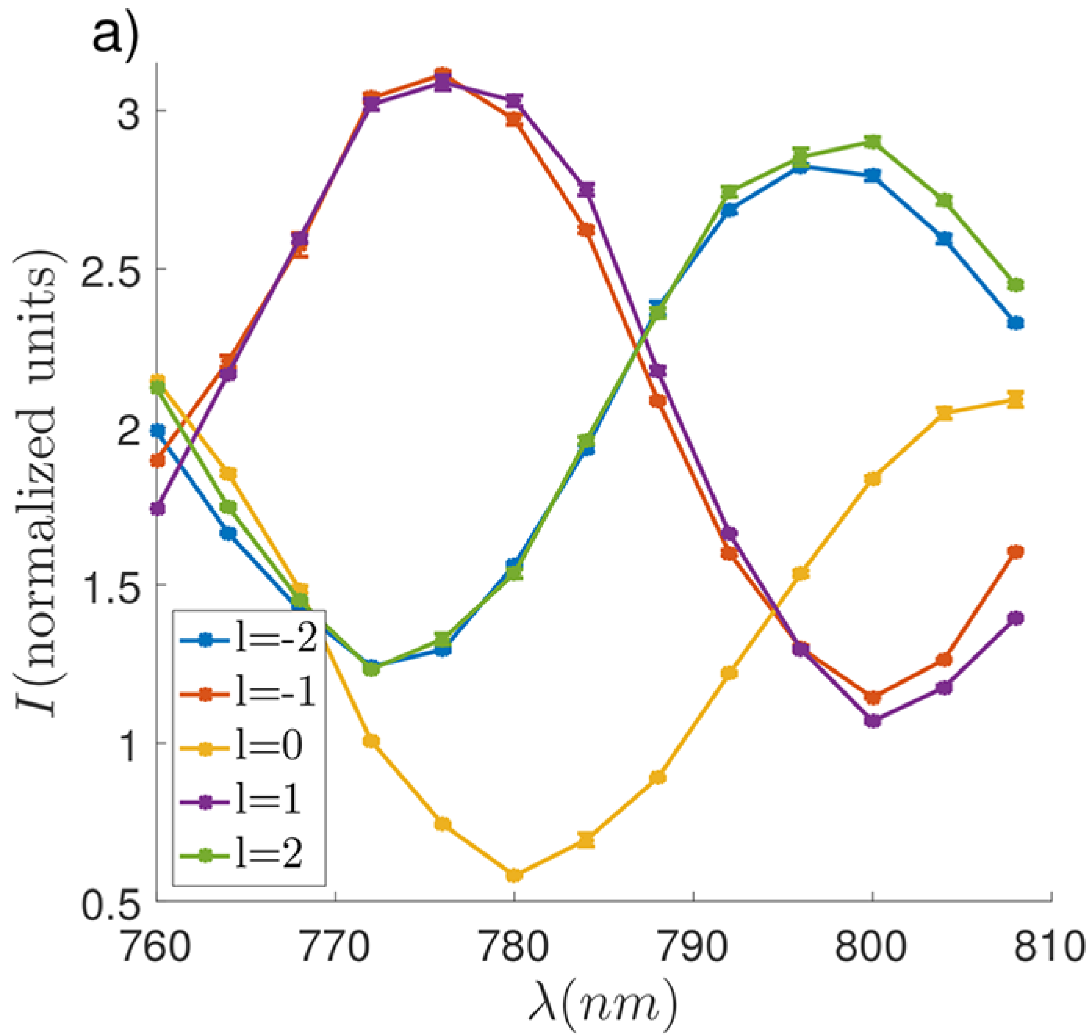

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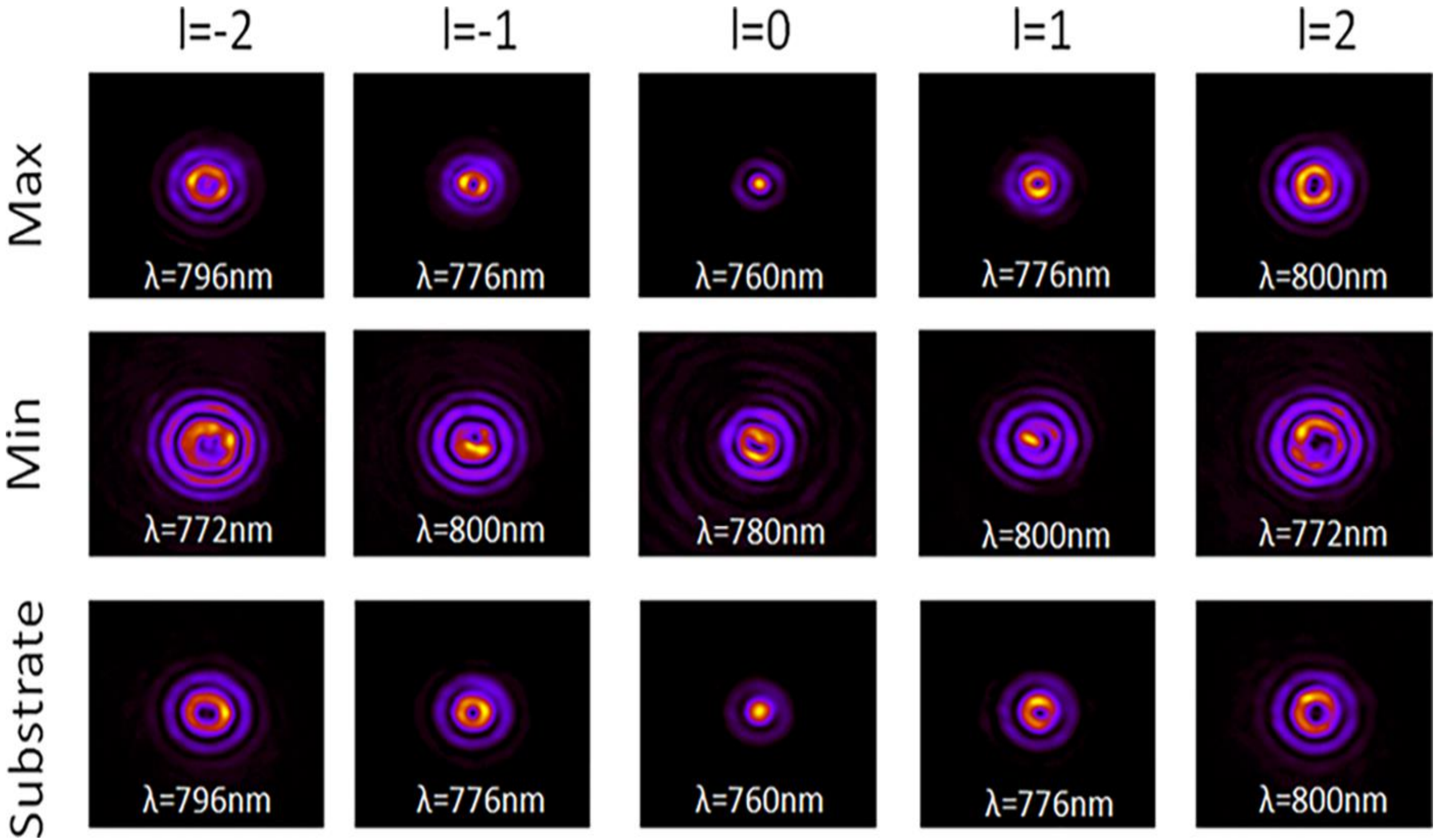
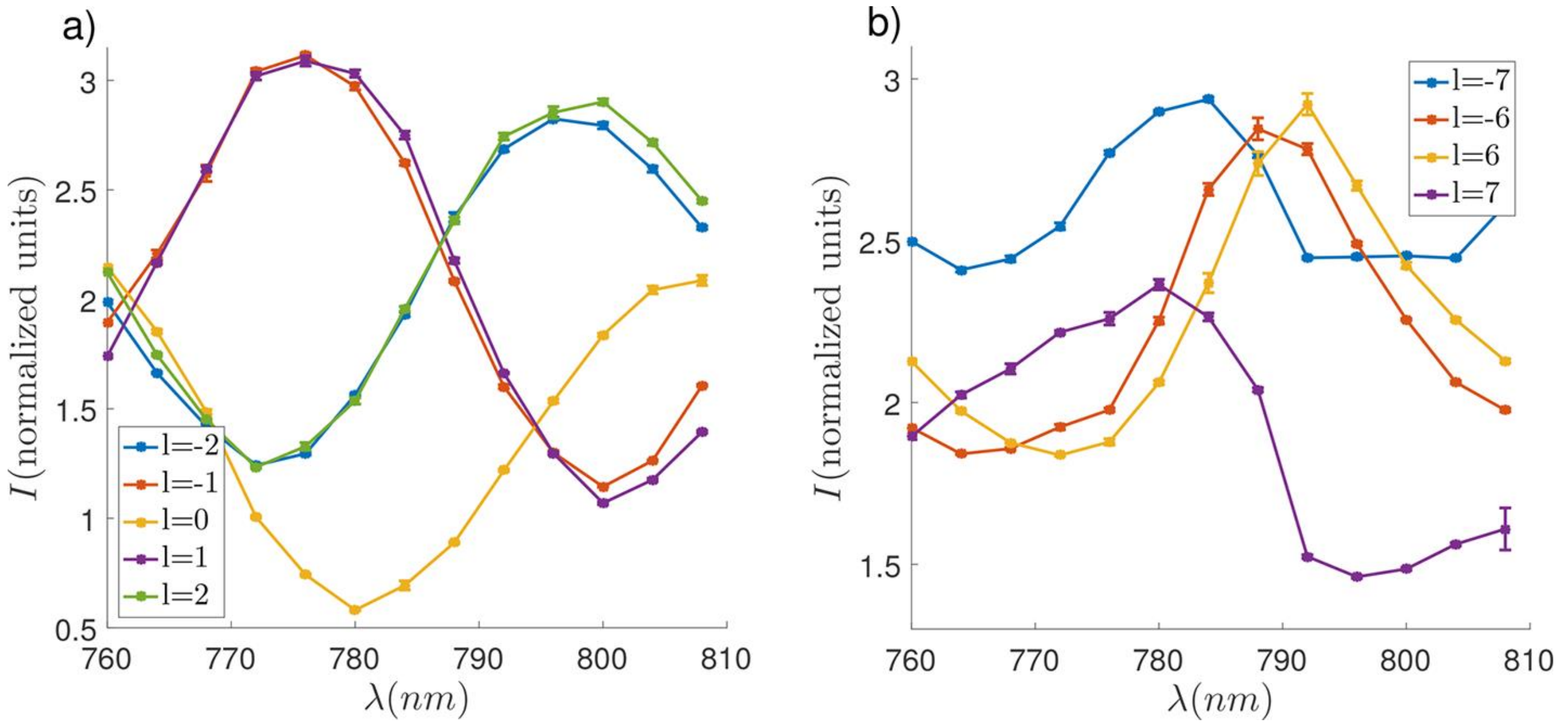
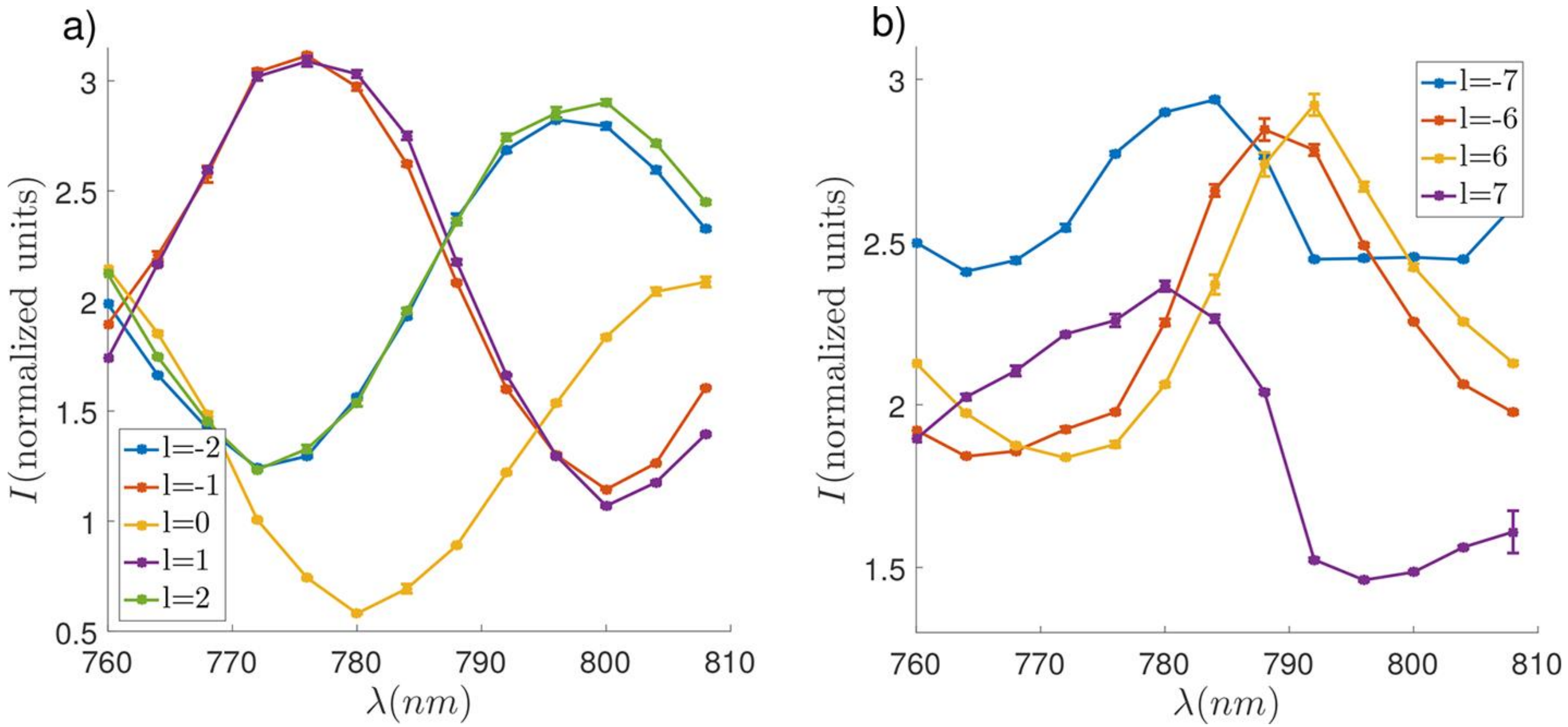
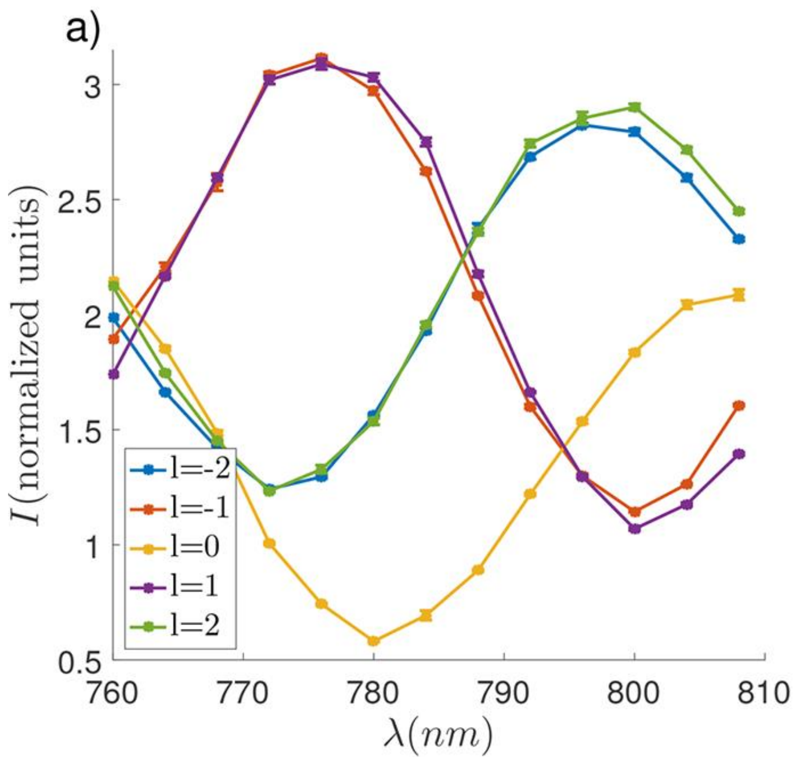


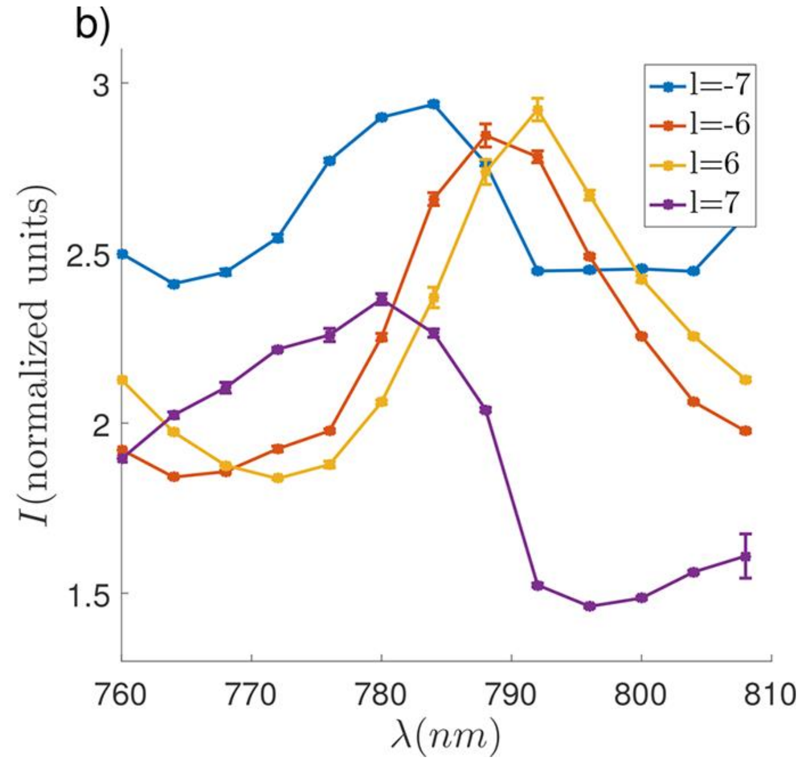

a)
b)
I(normalized units)
$\lambda(nm)$
l=-2
l=-1
l=0
l=1
l=2
l=-7
l=-6
l=6
l=7
760
770
780
790
800
810
0.5
1
1.5
2
2.5
3

Issues



- 1) Why do these resonances move around?





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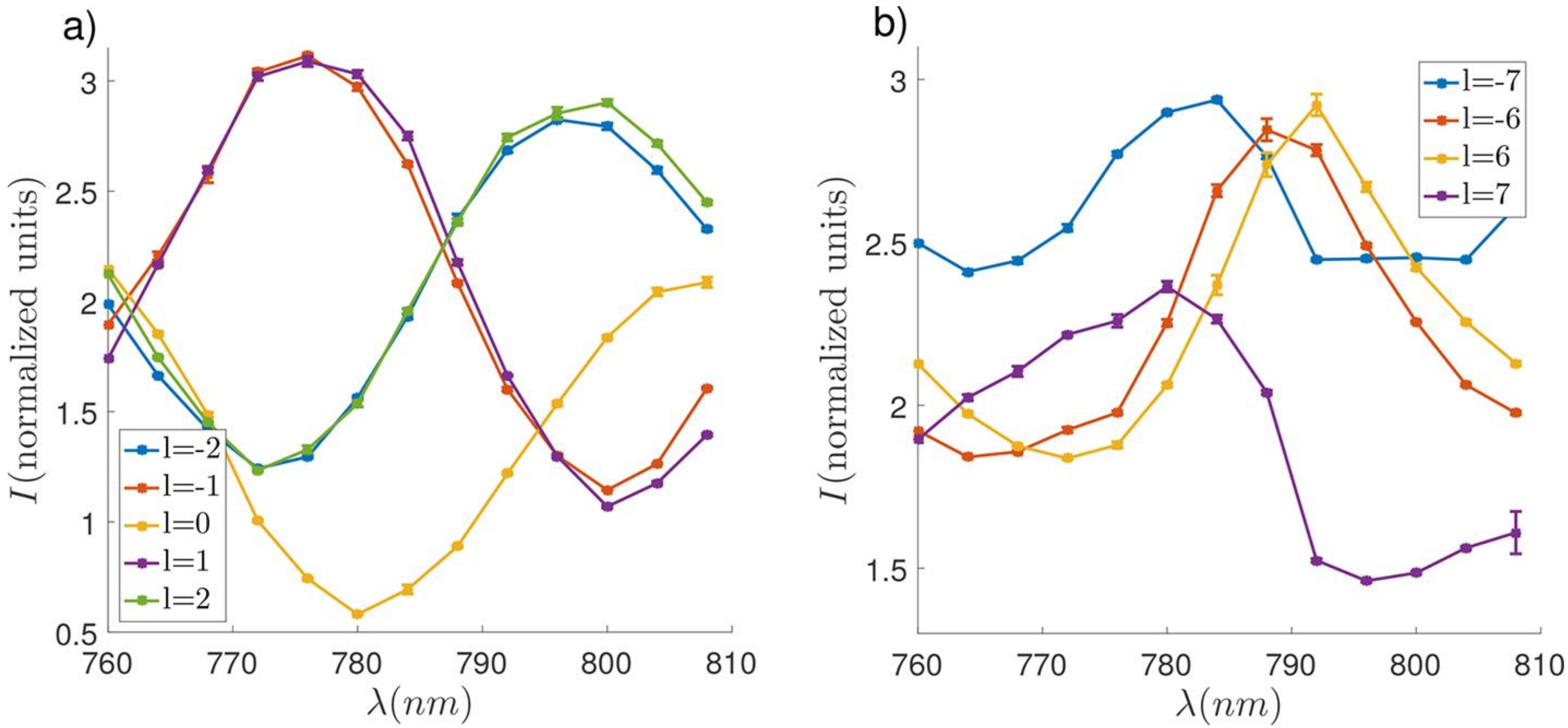

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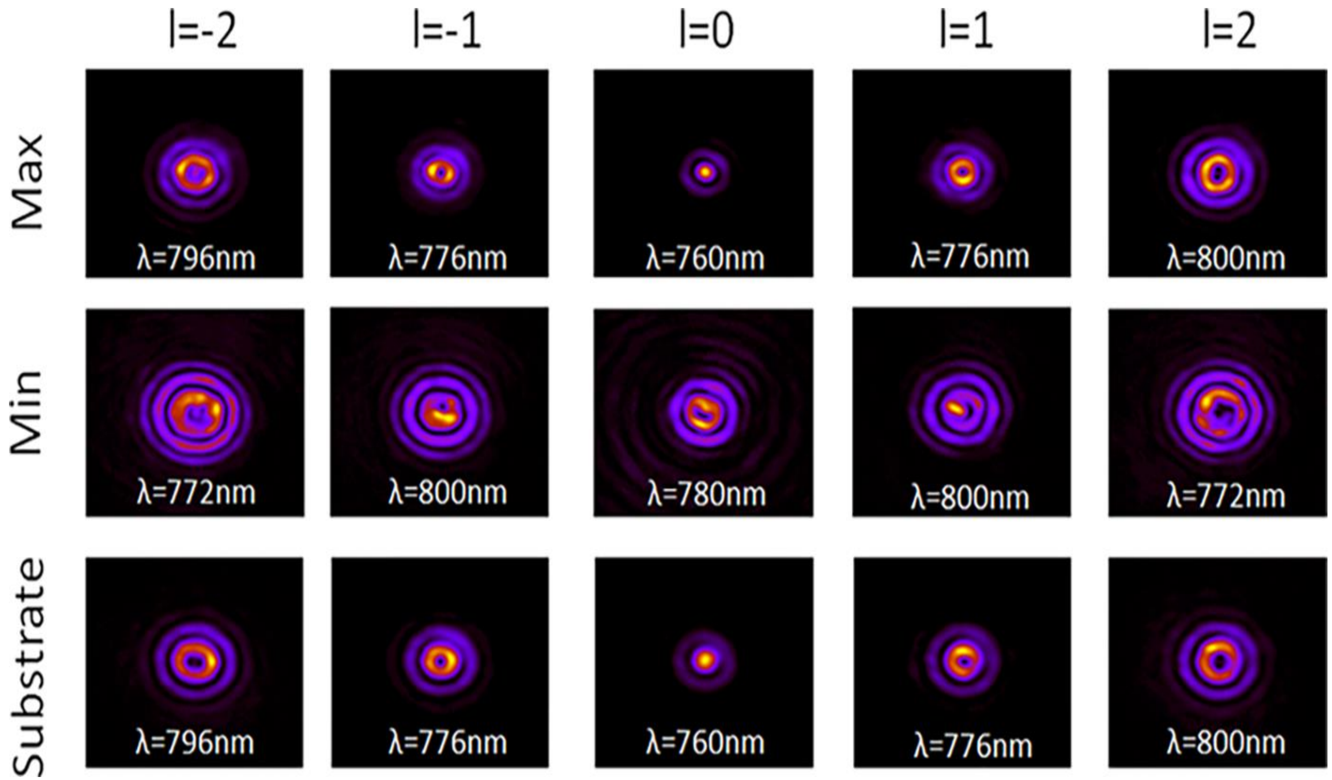

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- 4) Why do we see different intensity patterns?

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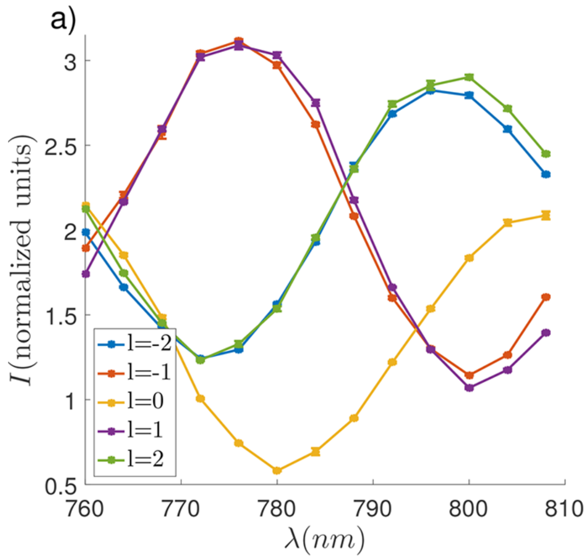

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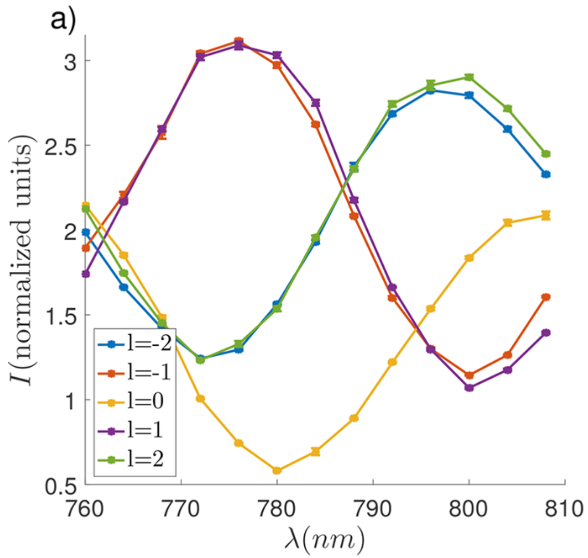

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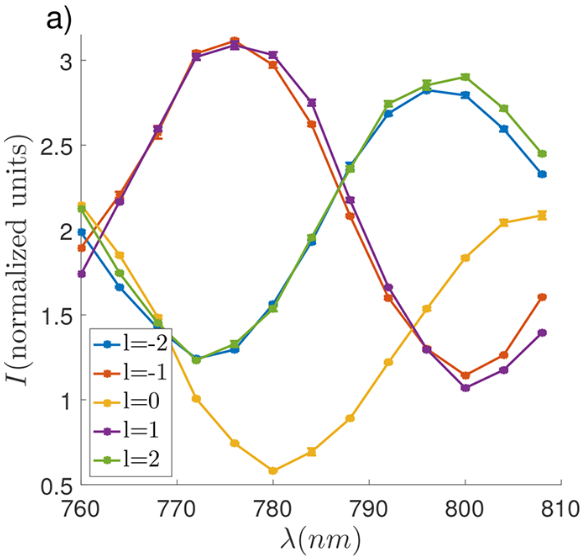

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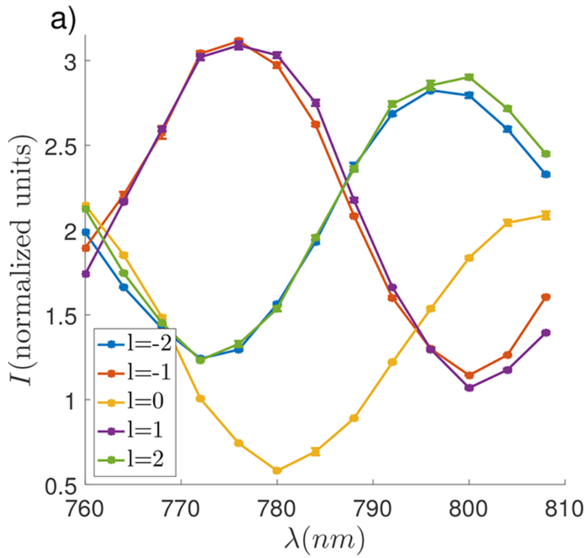

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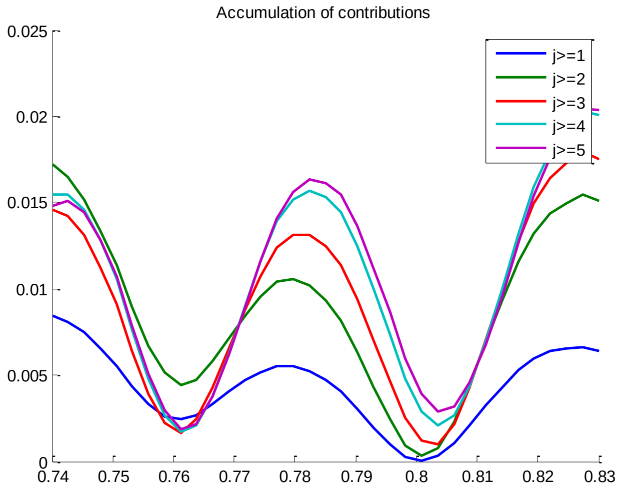

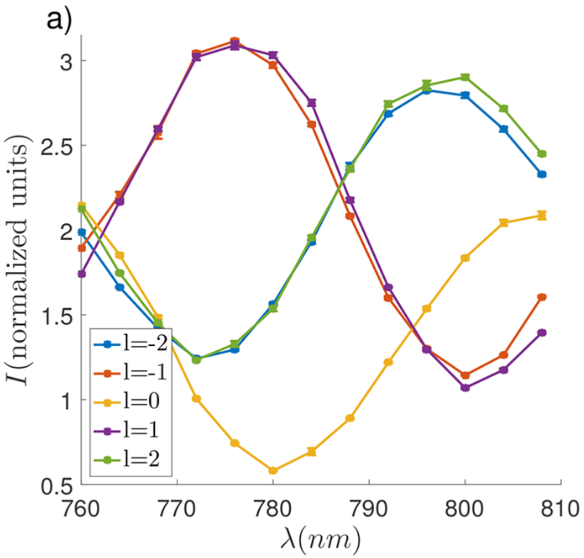

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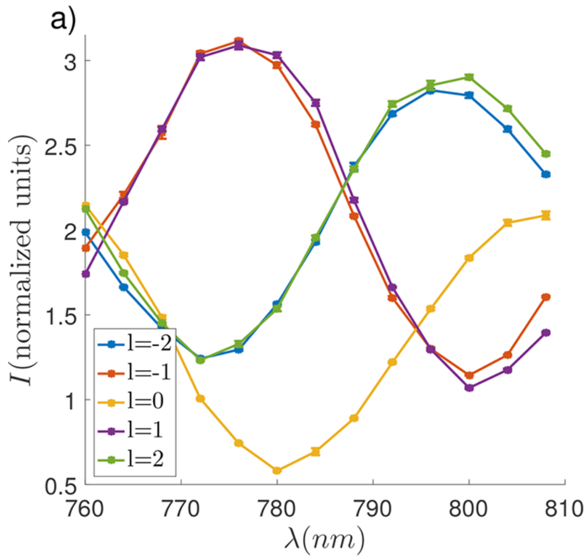

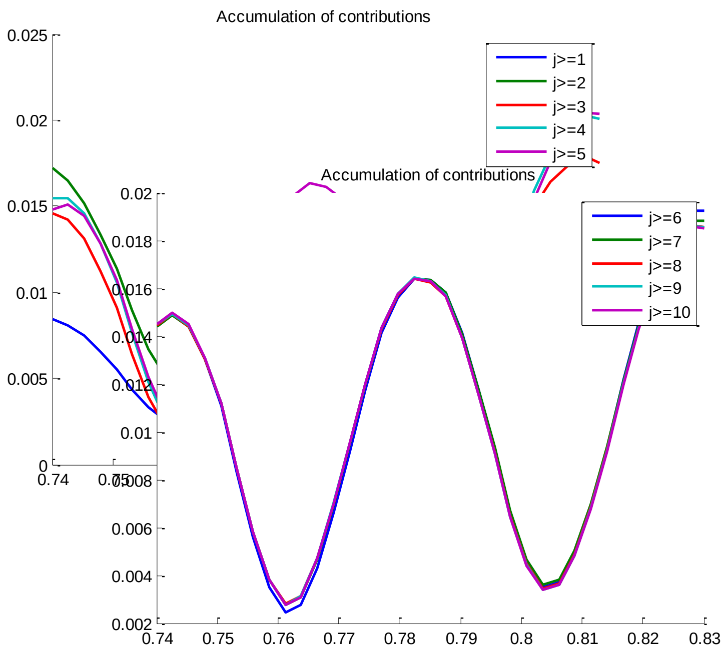

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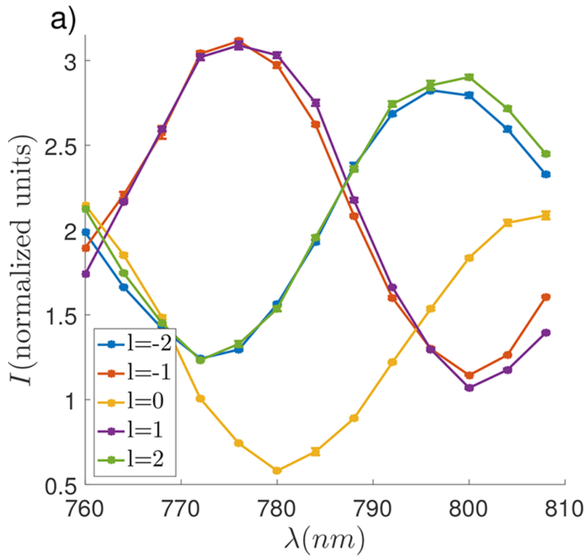

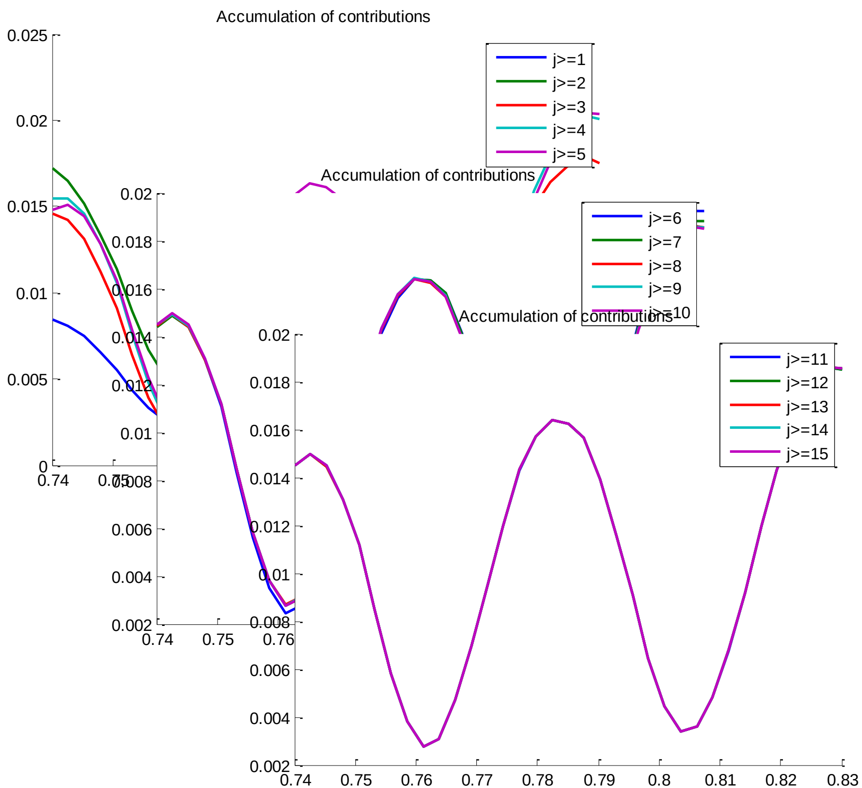

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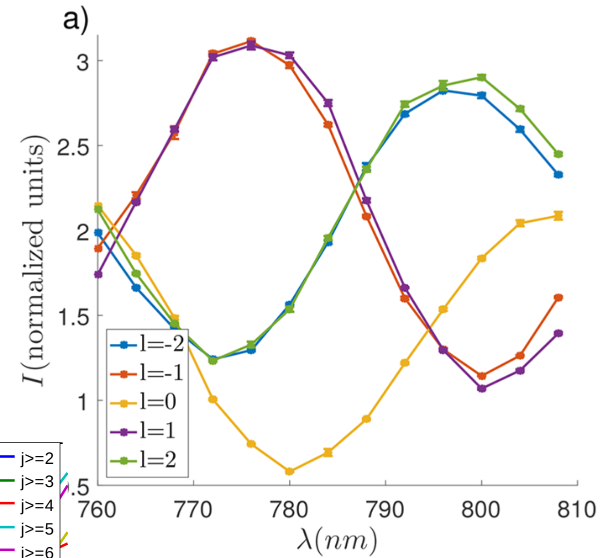

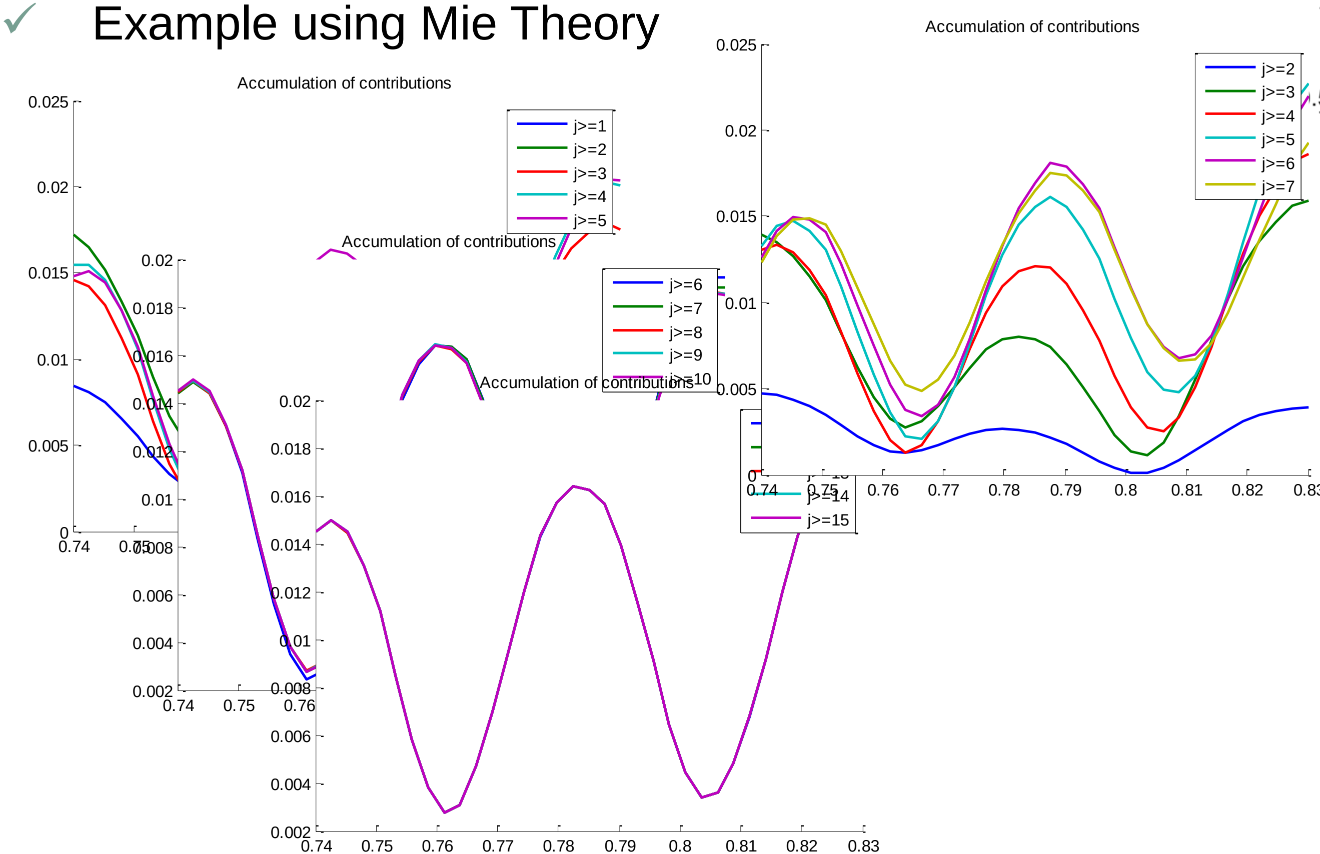

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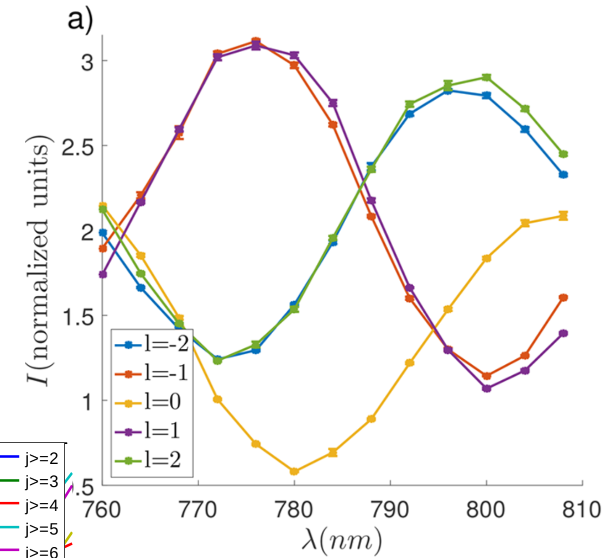

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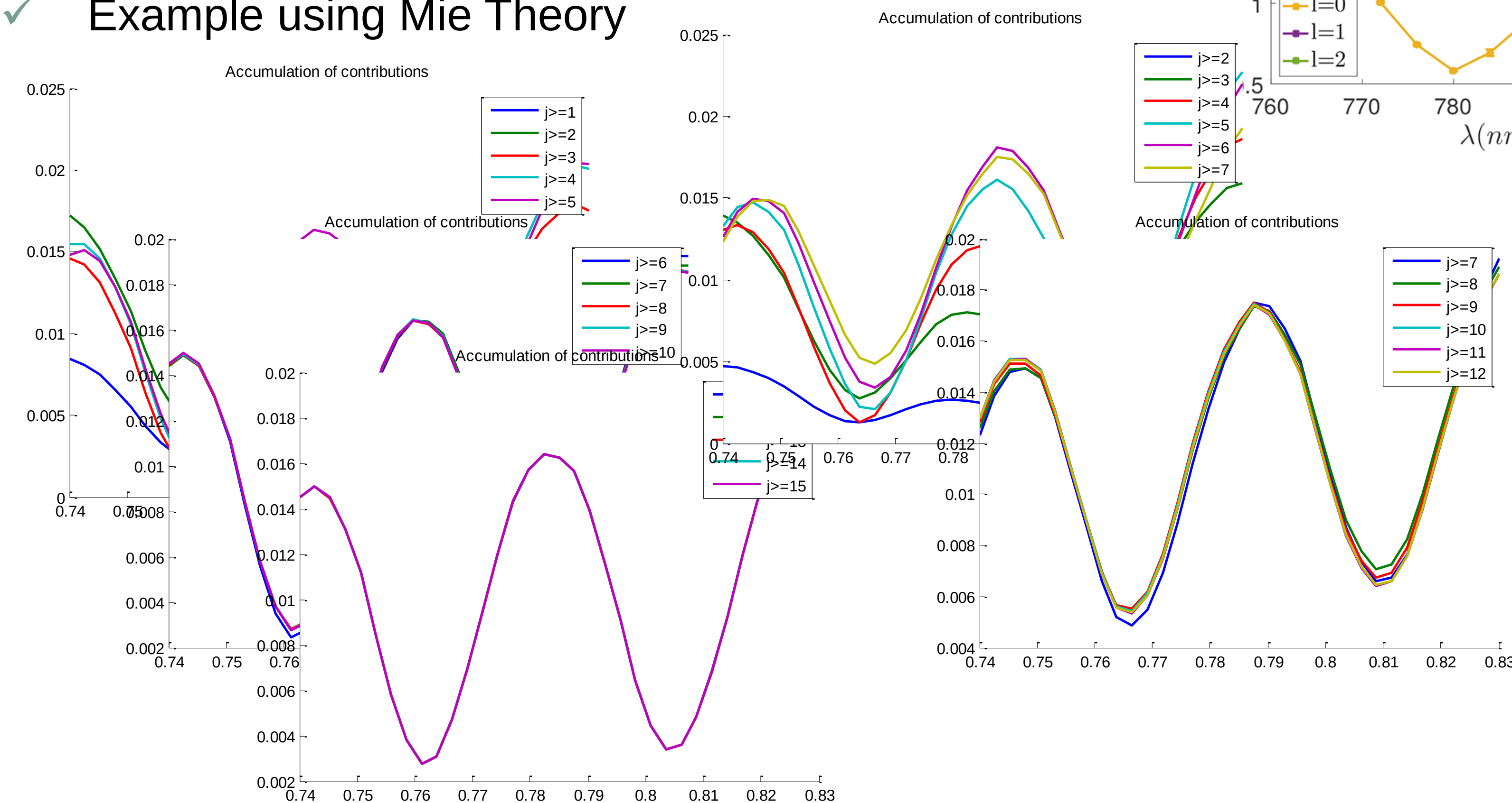

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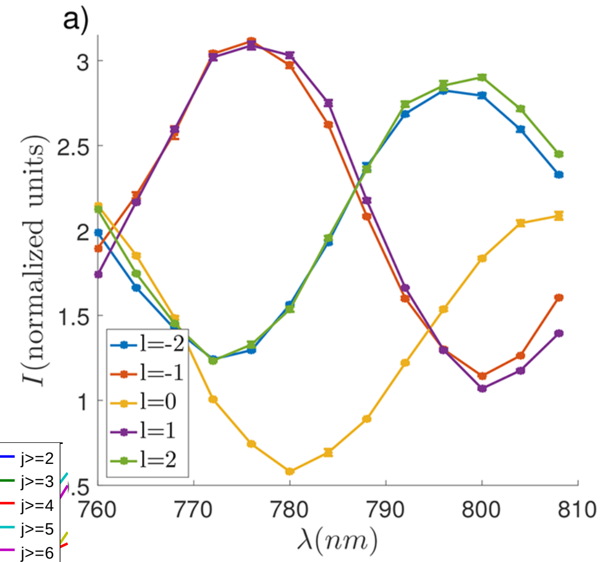
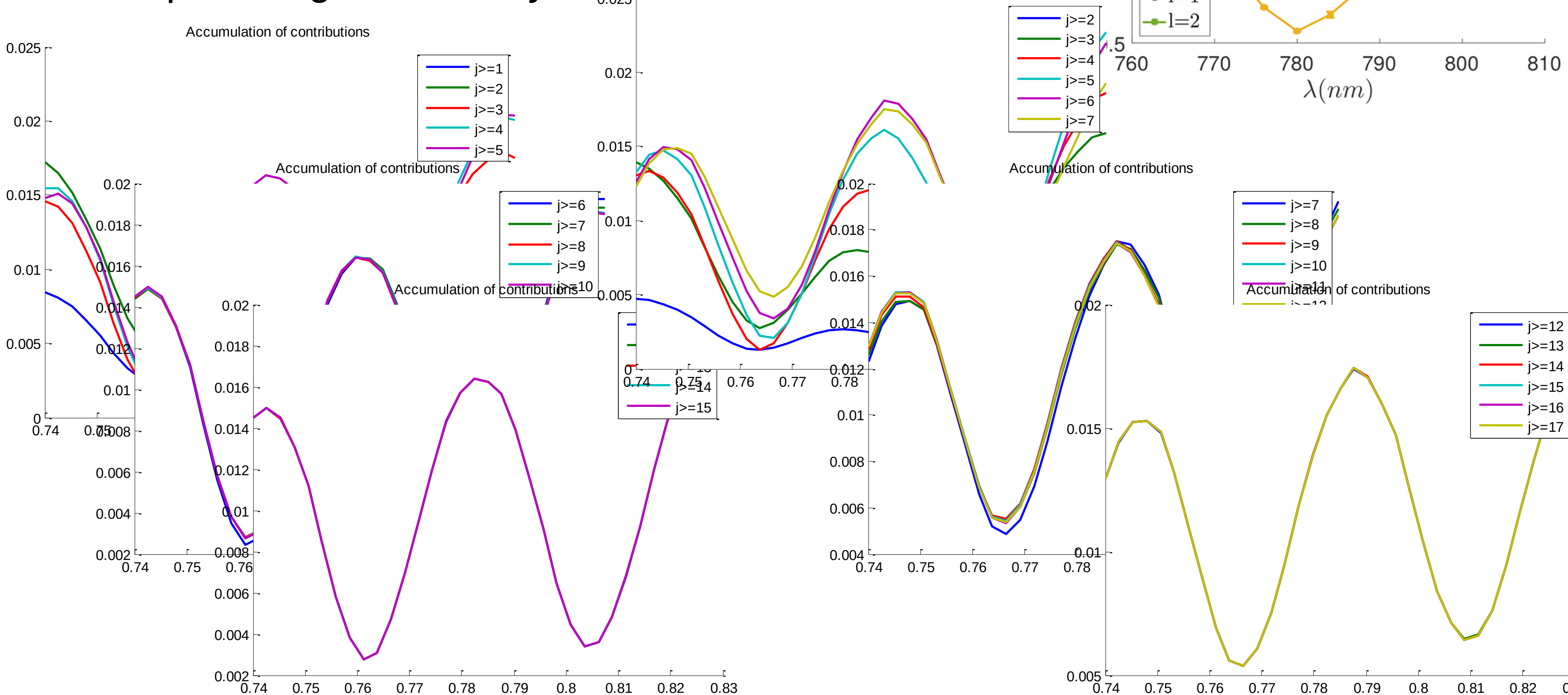
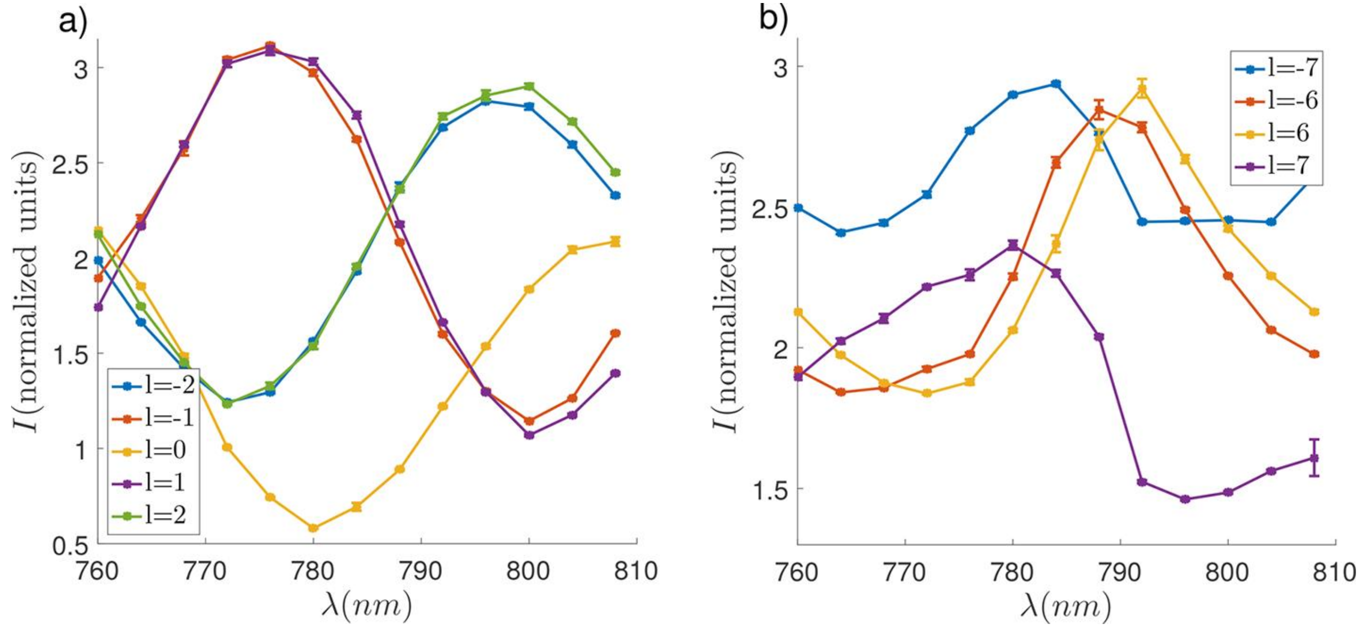




Why do l = ±1 behave equally, while l =±7 don’t? (I)



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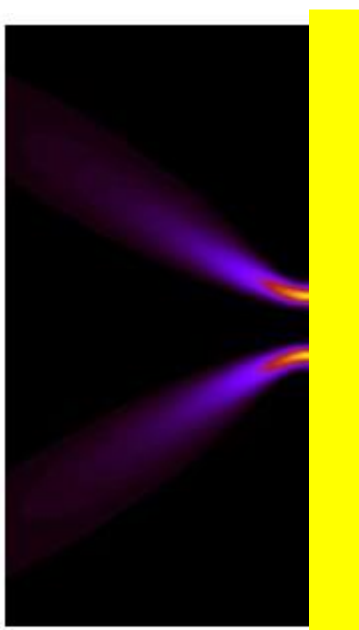
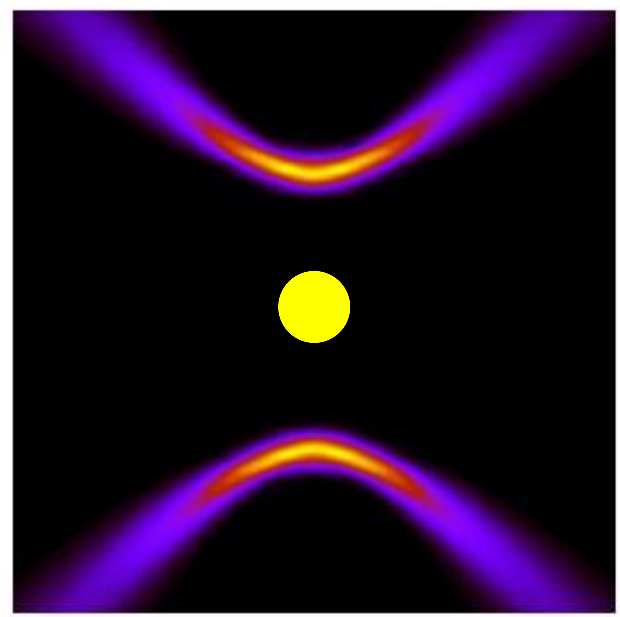
- 2 extreme cases:
  - i) Beam width is much smaller than the scatterer
  - ii) Beam width is much larger than the scatterer





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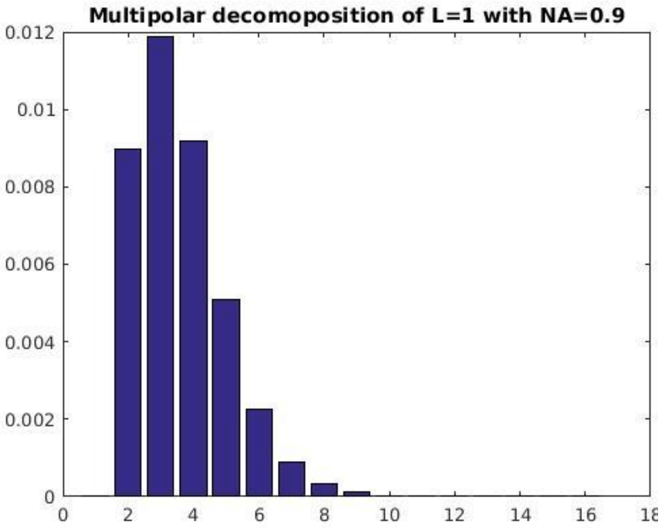

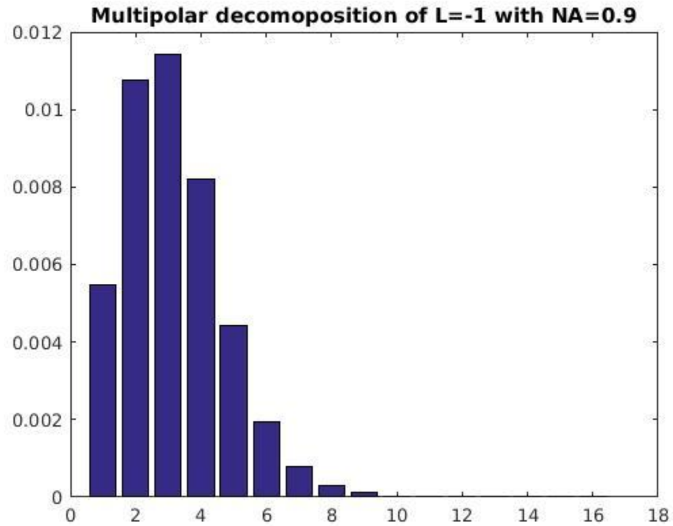

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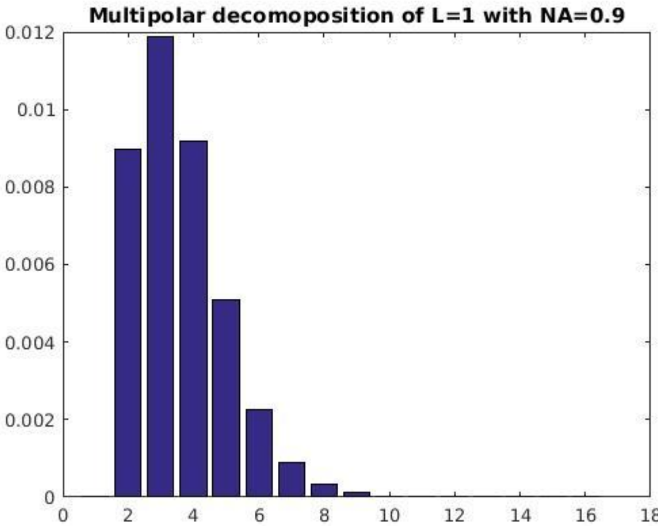
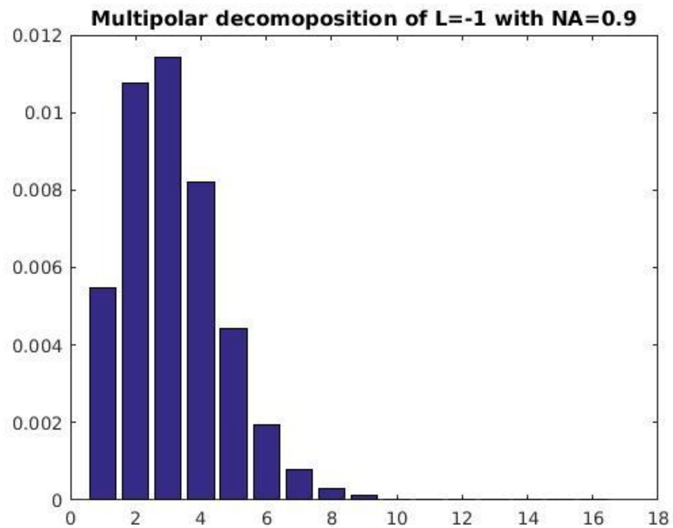


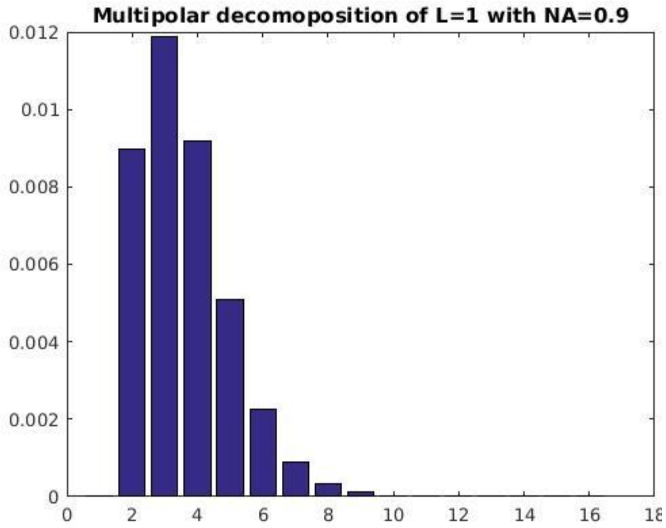


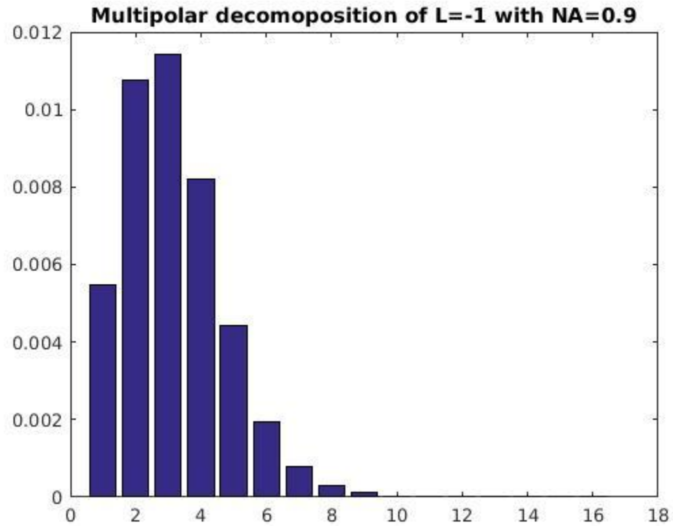
Differences in j=1,2,3 then it's almost equal!

Why do l = ±1 behave equally, while l =±7 don't? (II)

- ✓ Rigorous picture: Simulation | Mie Theory

- ✓ **<u>Mie Theory</u>**: Given a particle (R, n_r), then
  - ✓ The scattering is composed of j* modes, where j* ≈ 2πR/λ ≈ 16
  - ✓ The 'fine details' (narrow resonances) about scattering are found in the higher order modes
  - ✓ What's the difference between the multipolar decomposition for l = ±1?





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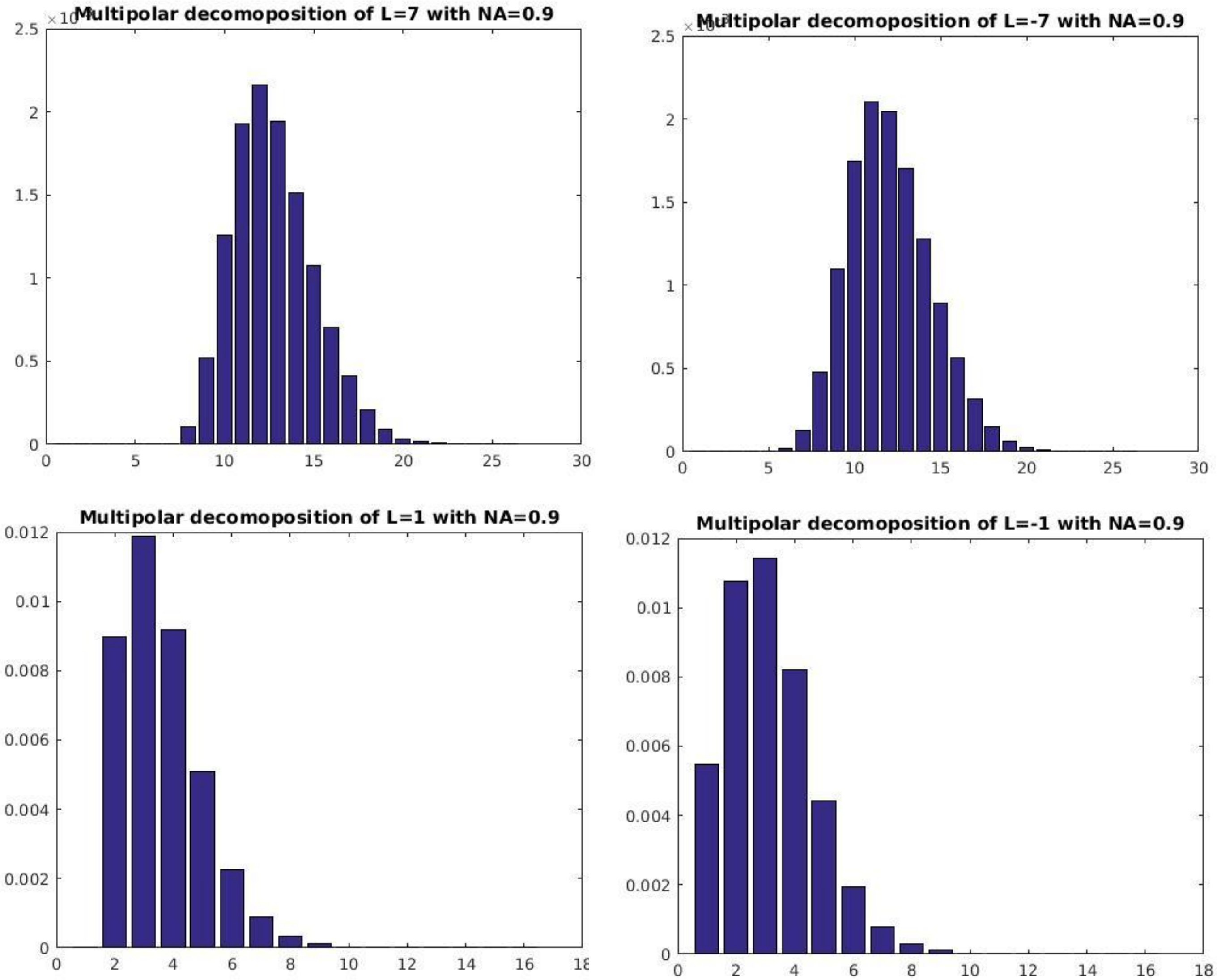

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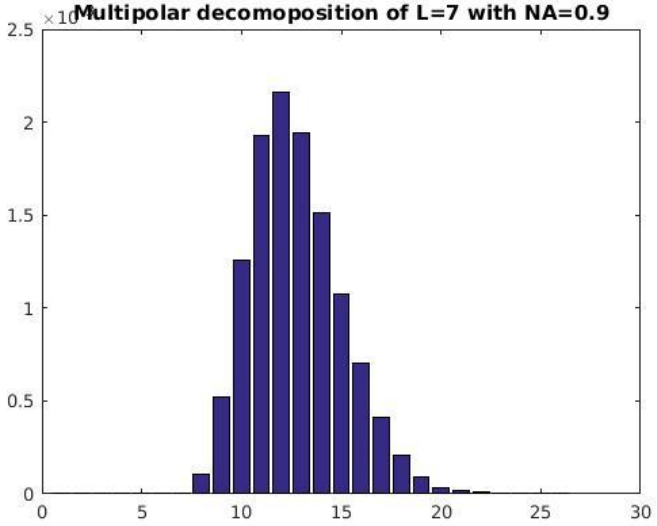
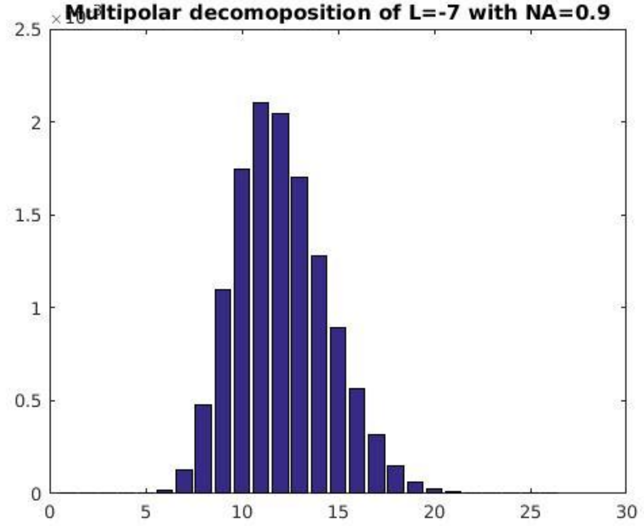
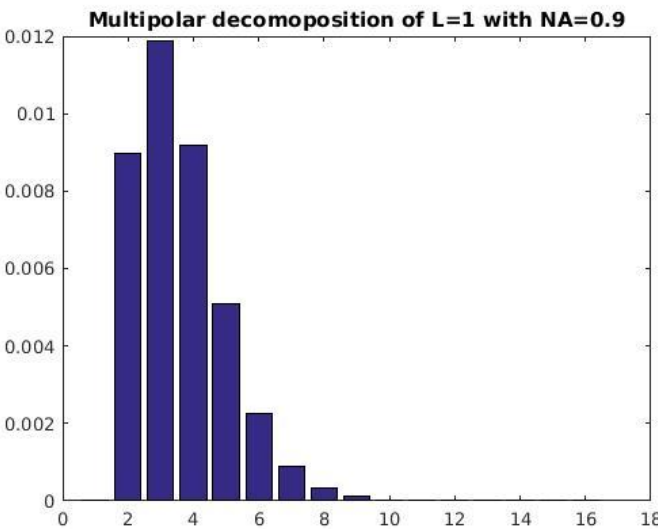
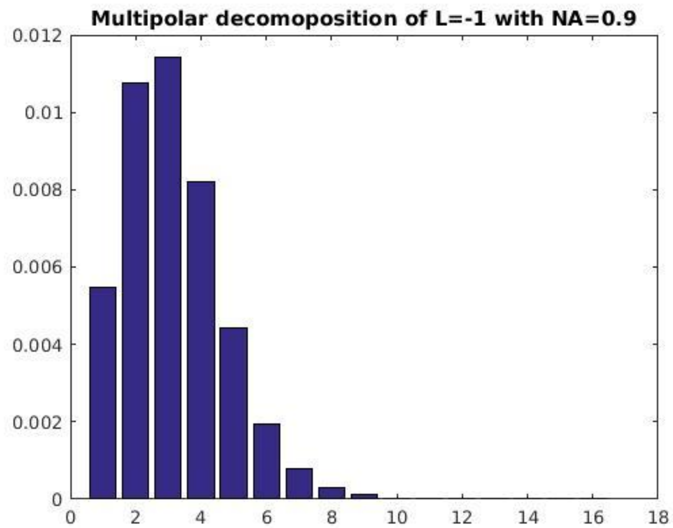




The decomposition is different in modes where j ~ j*





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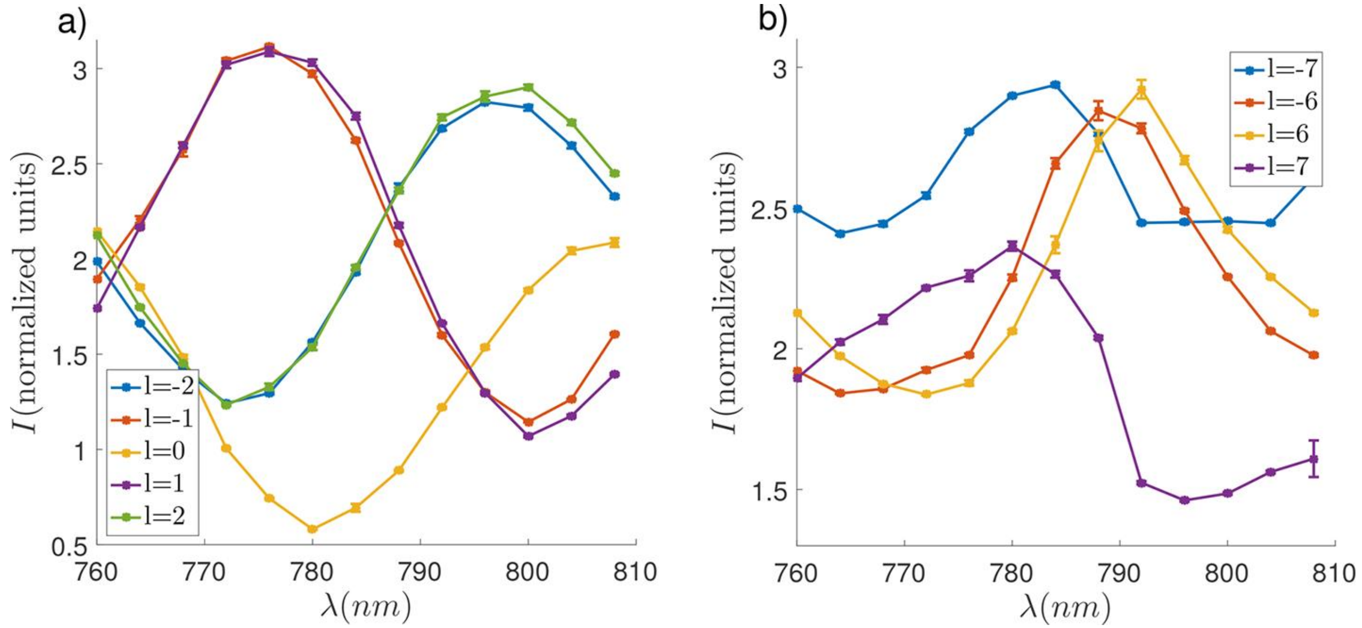

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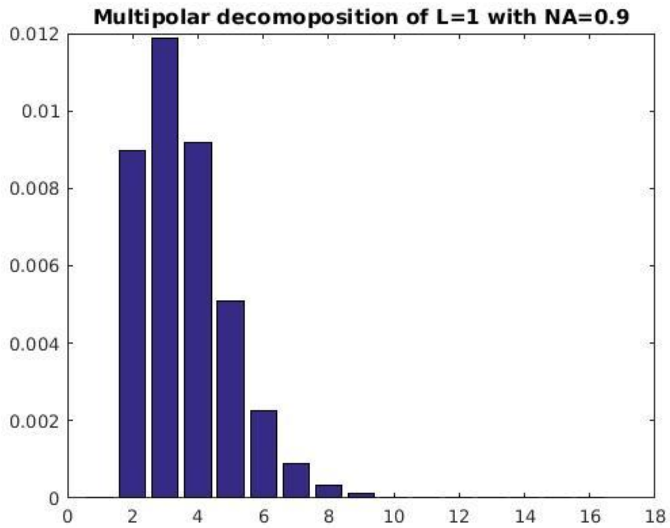

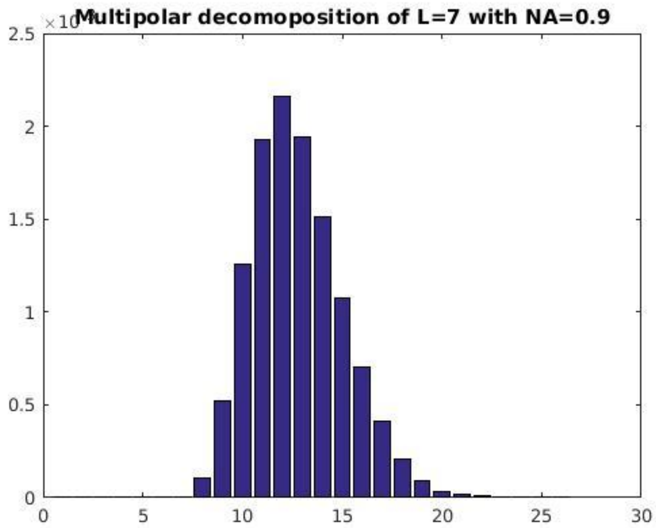

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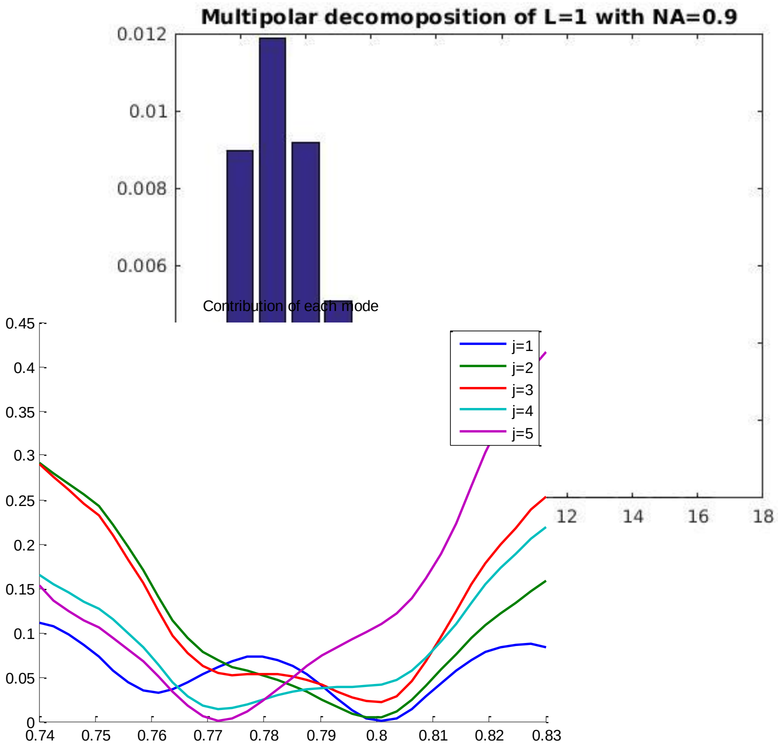

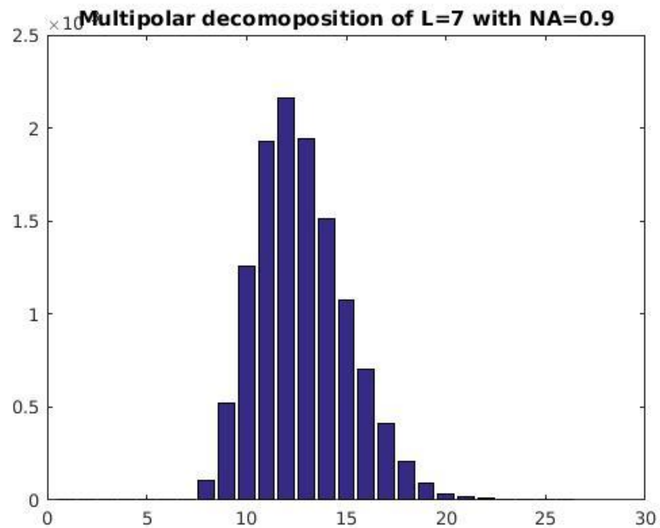

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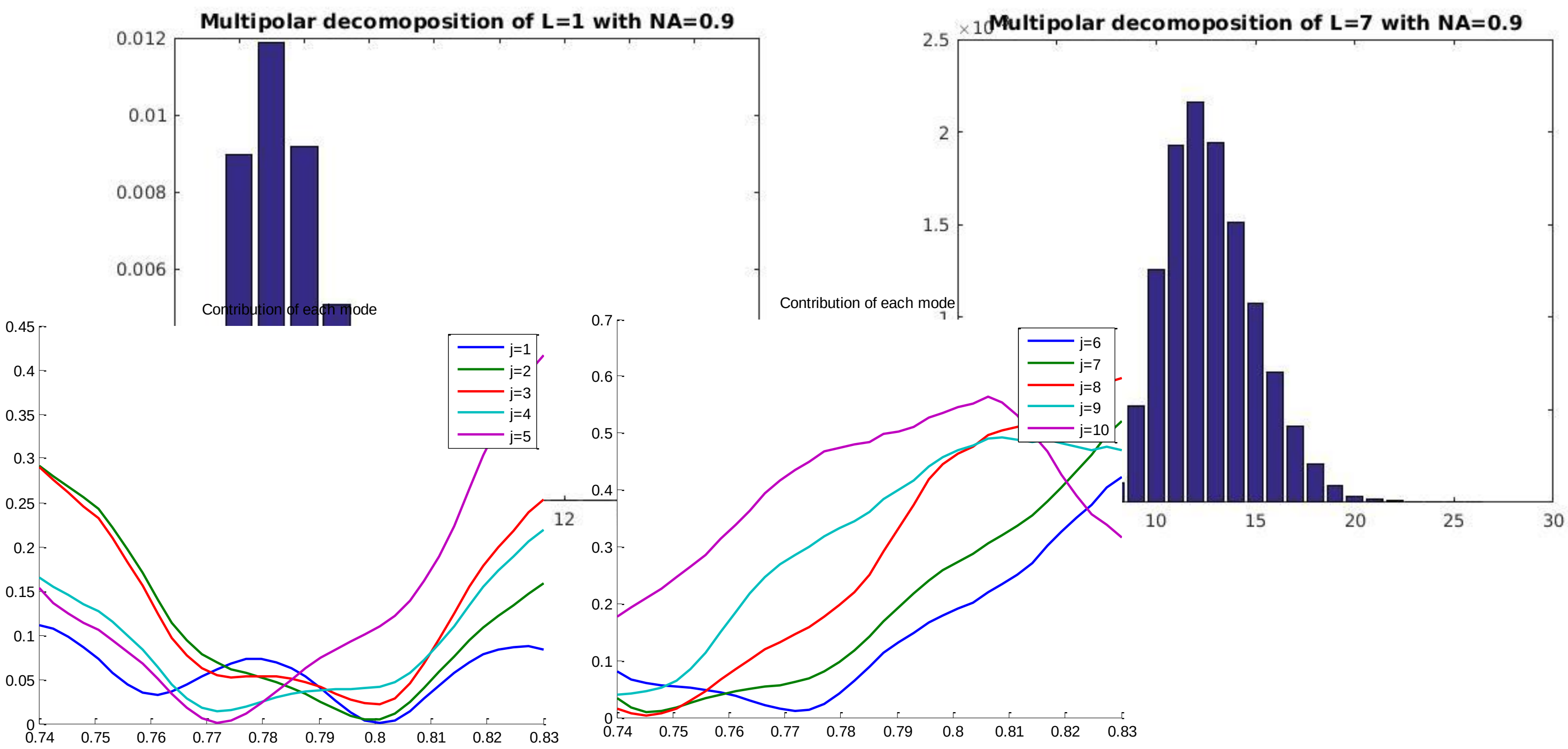

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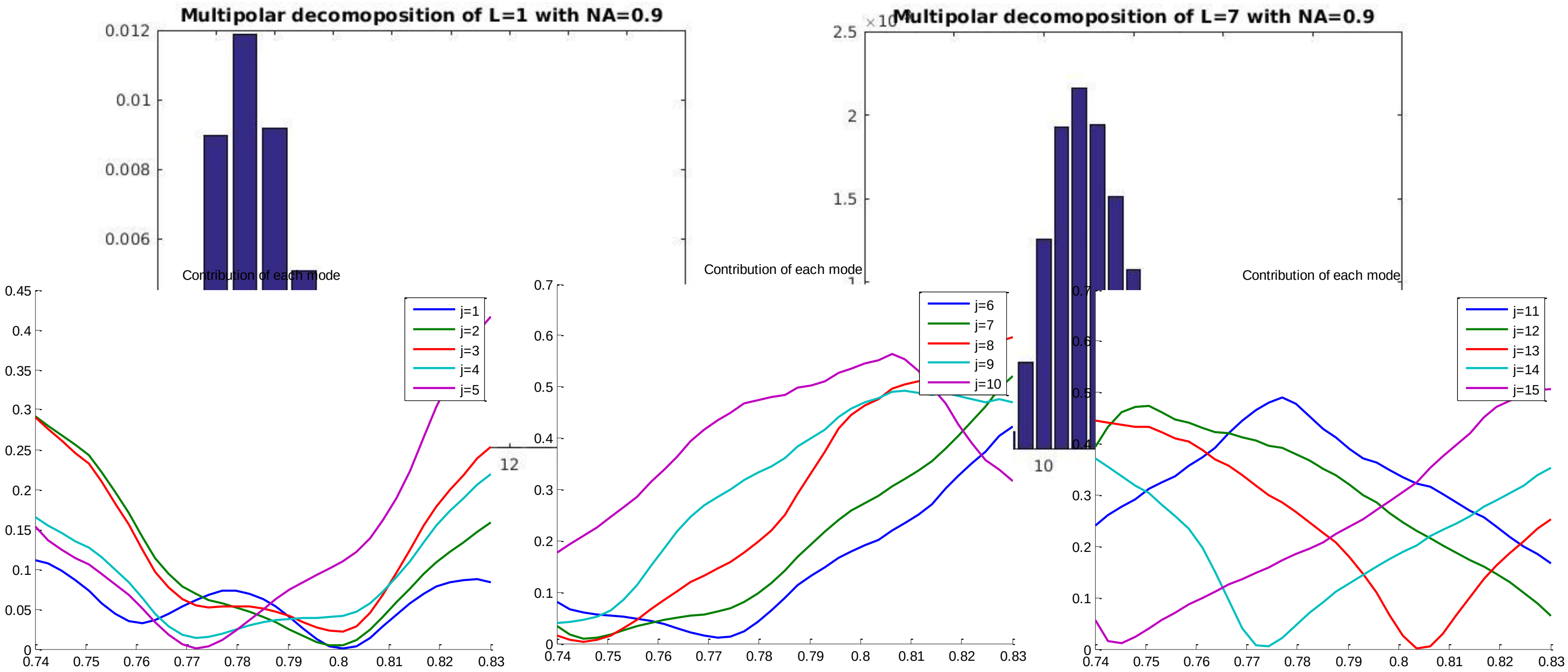

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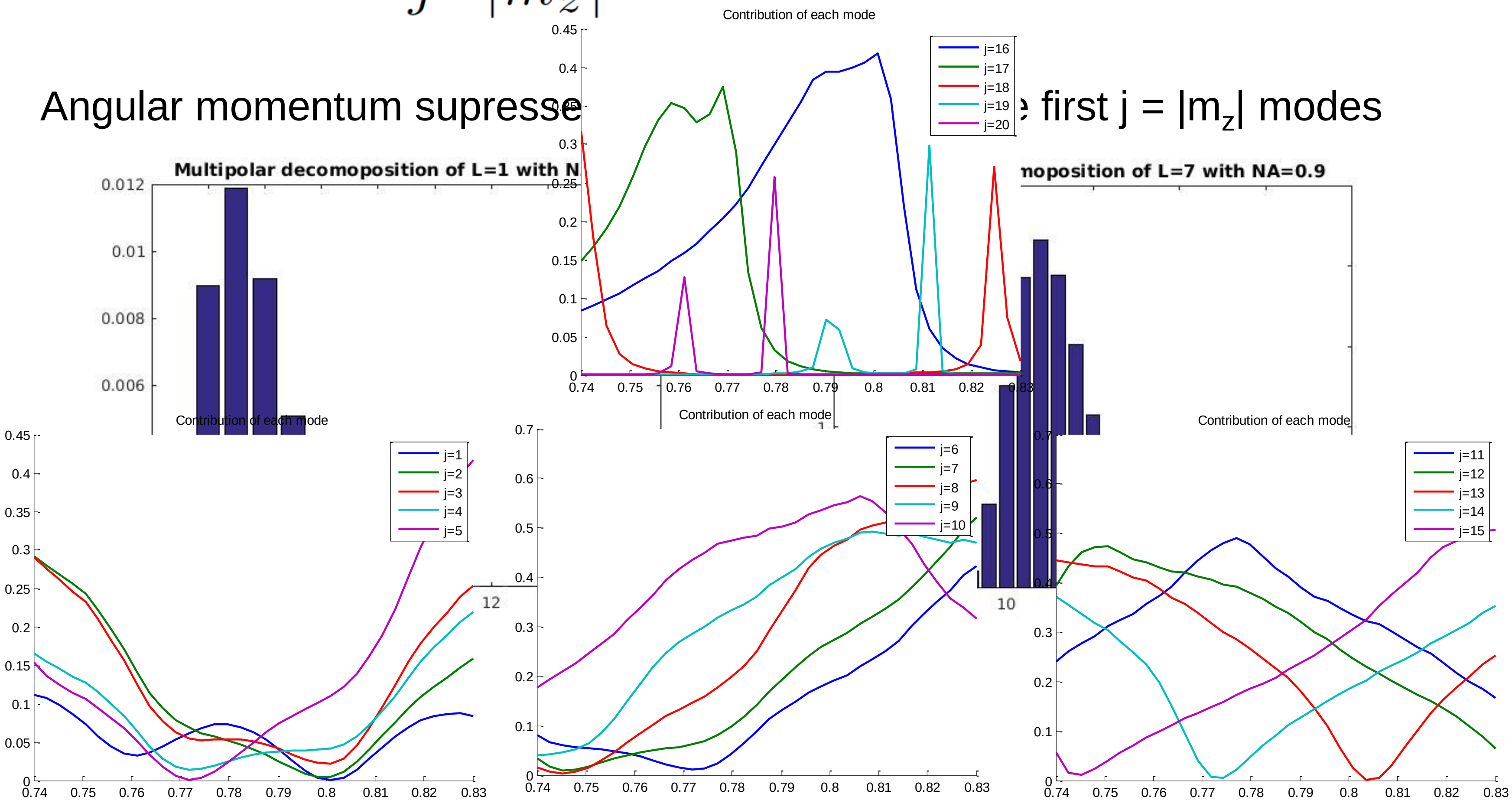

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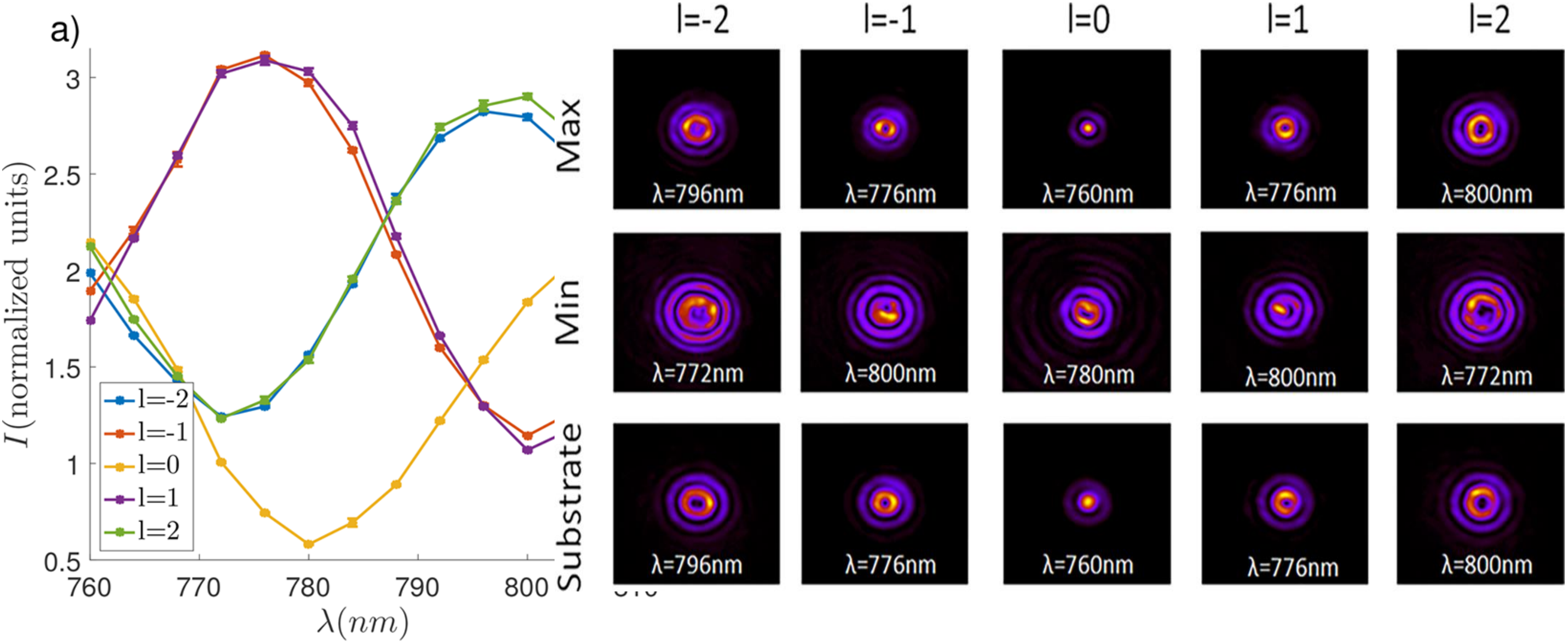

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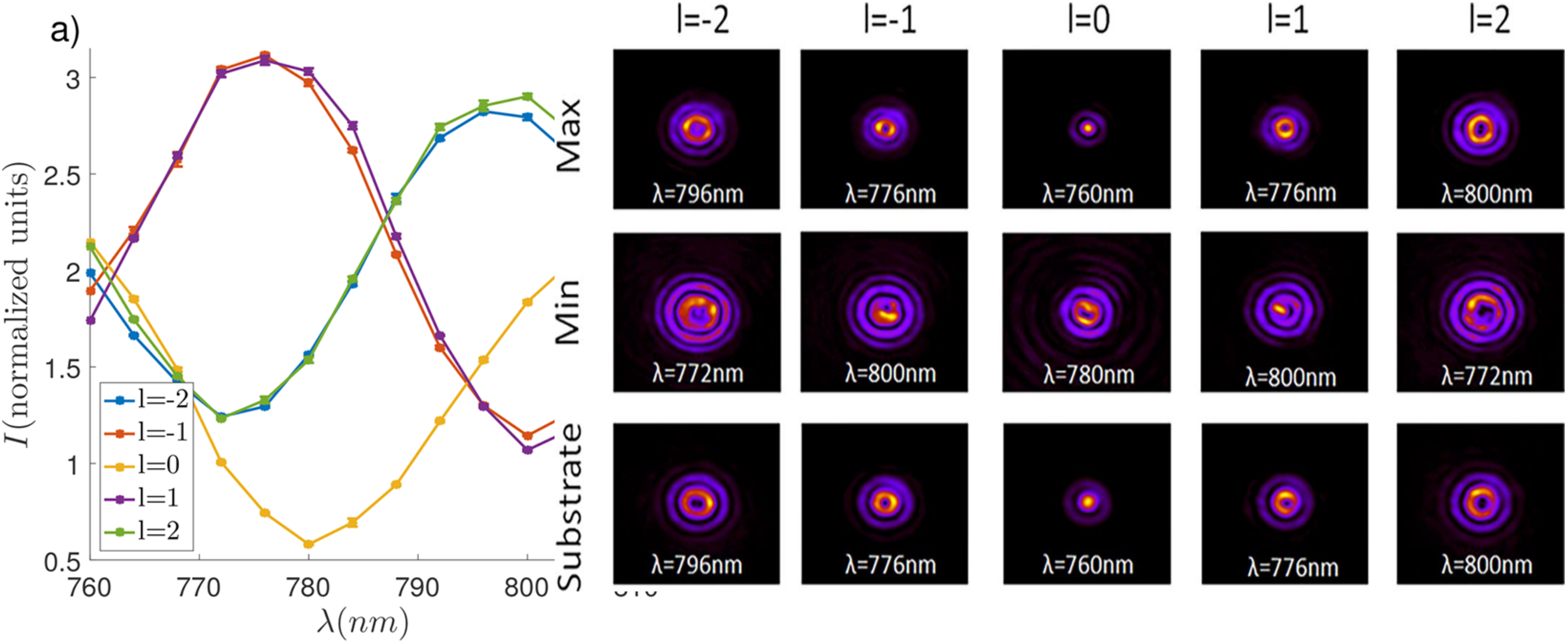

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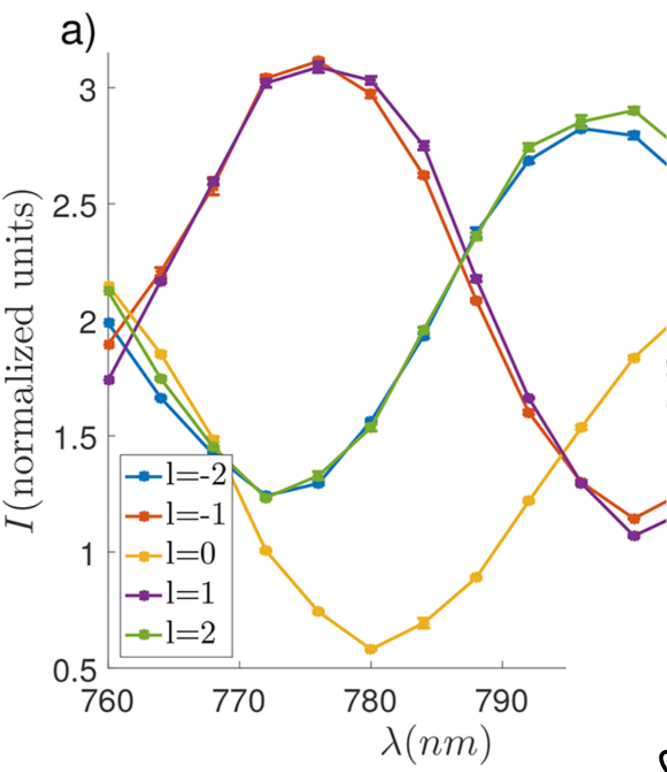

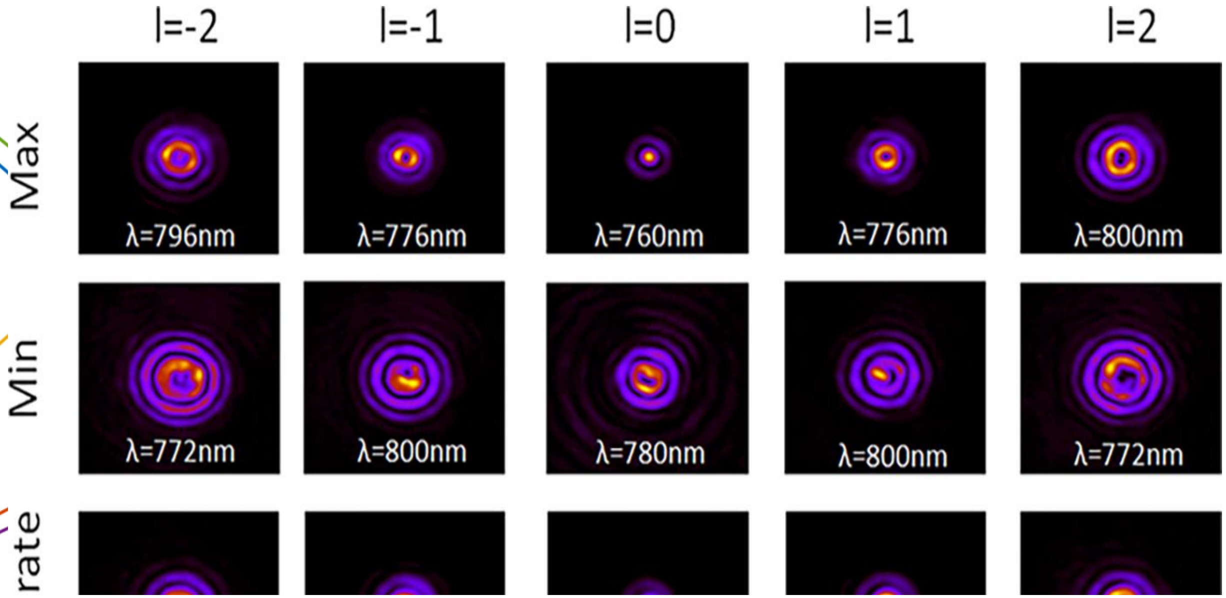

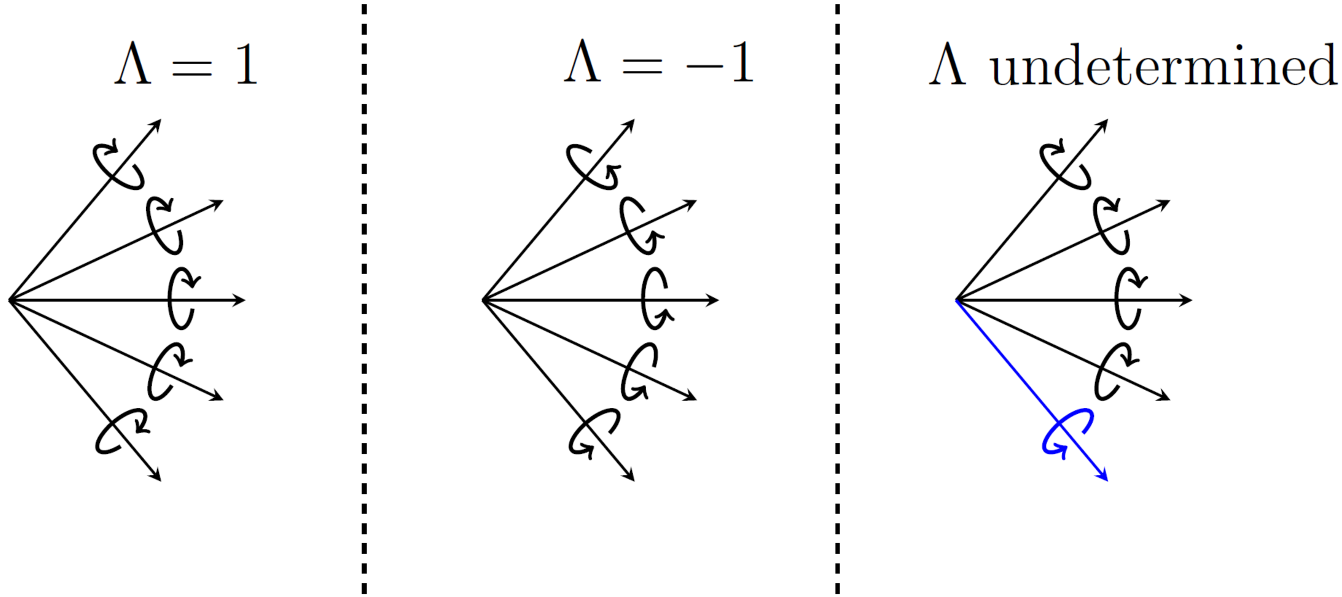

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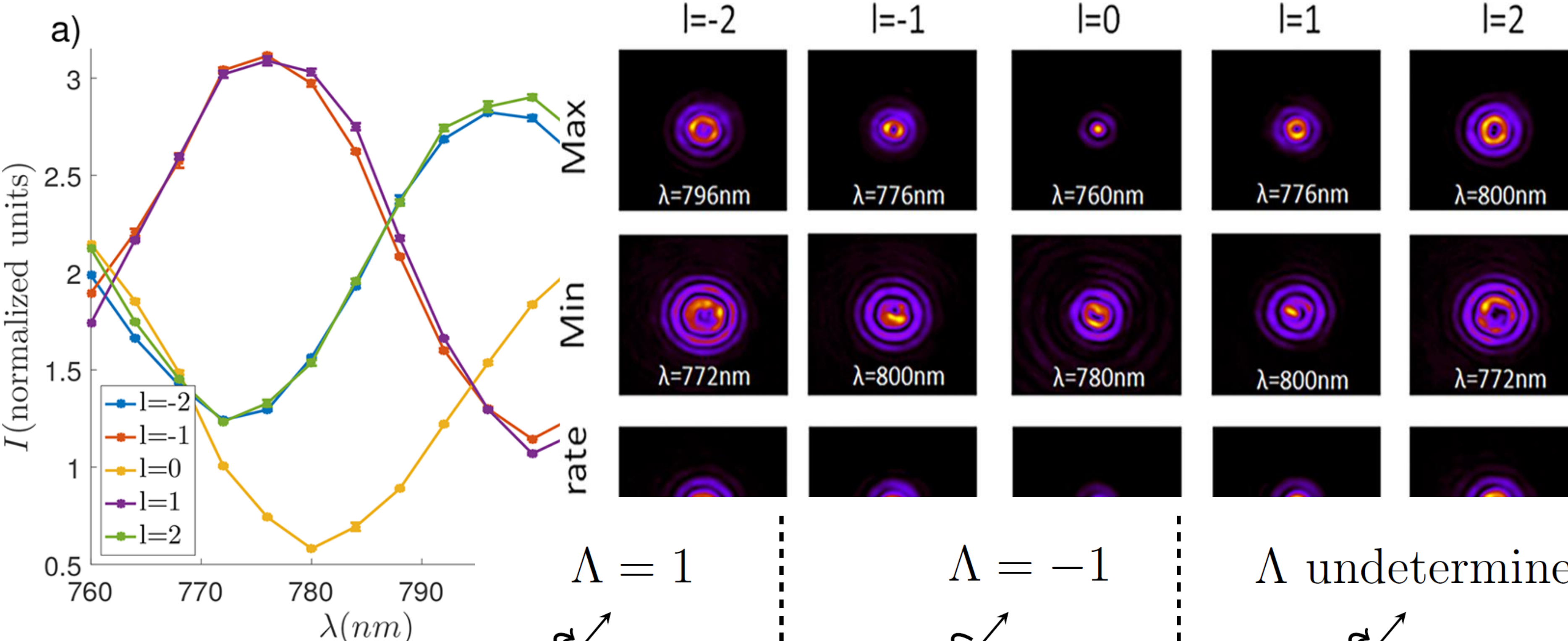

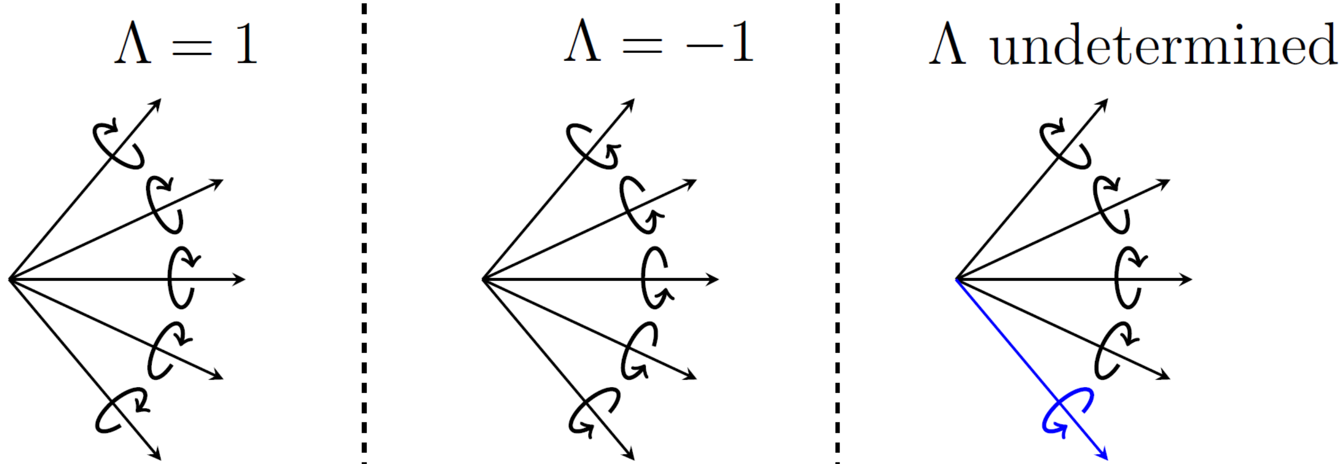

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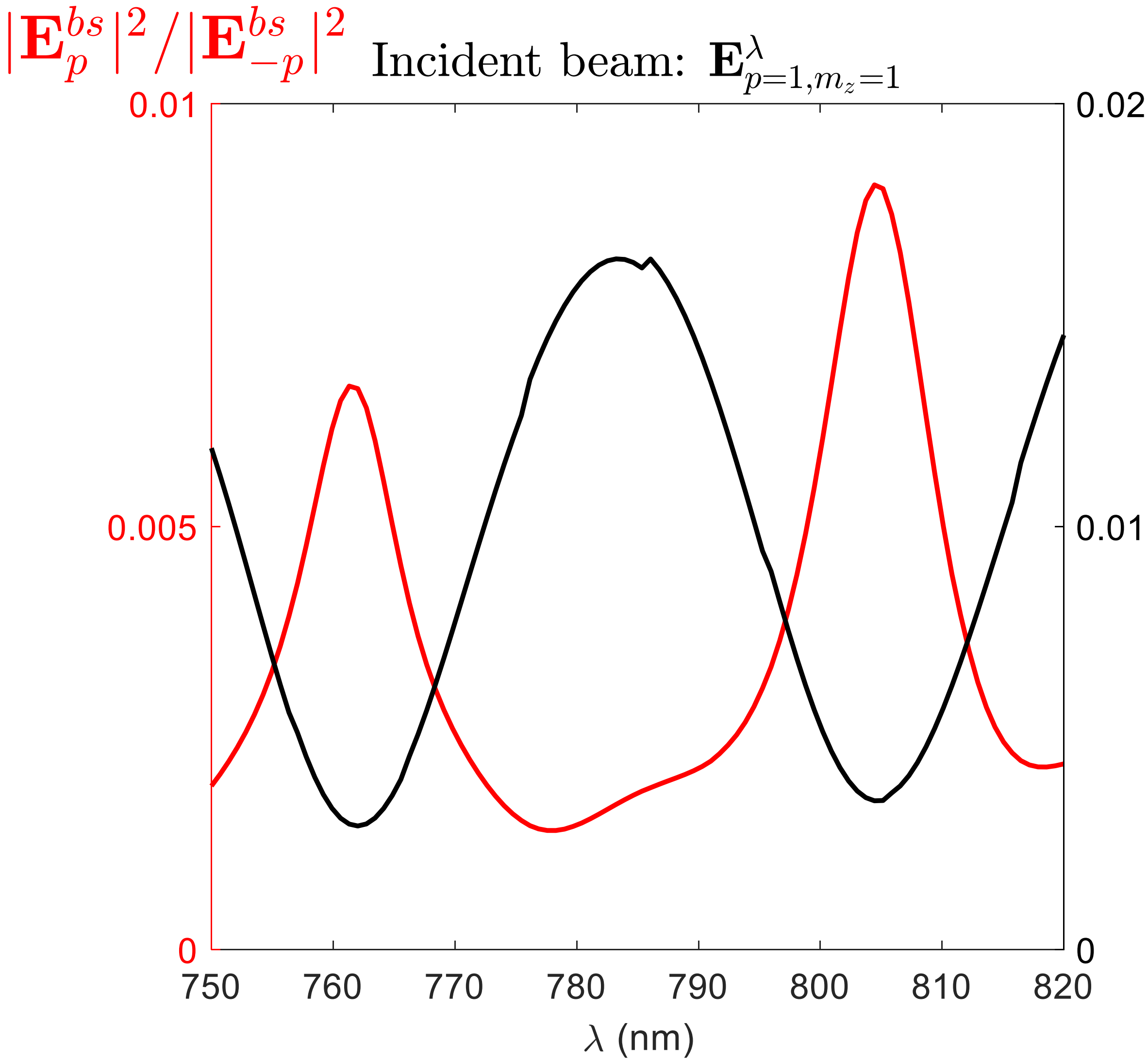

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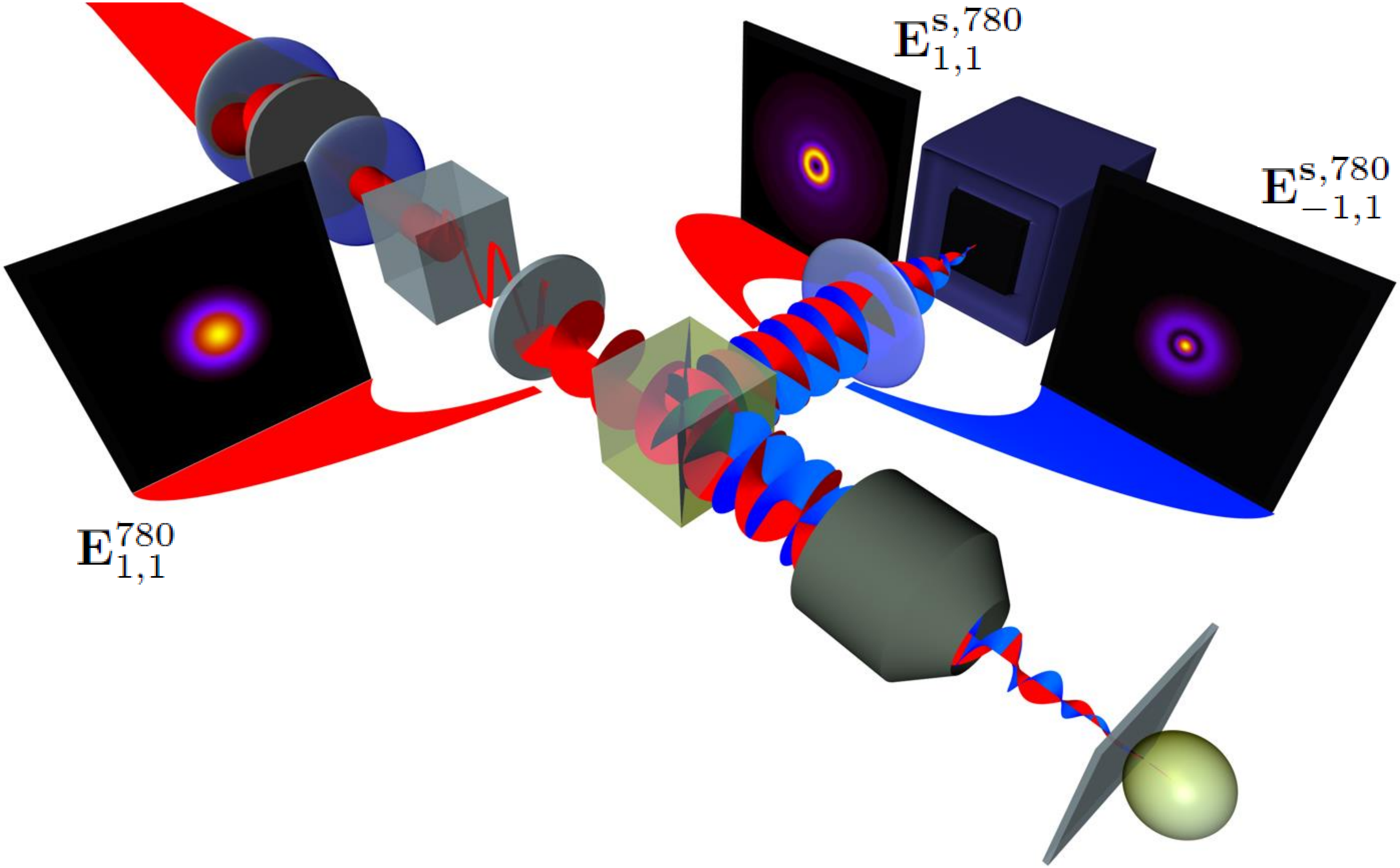

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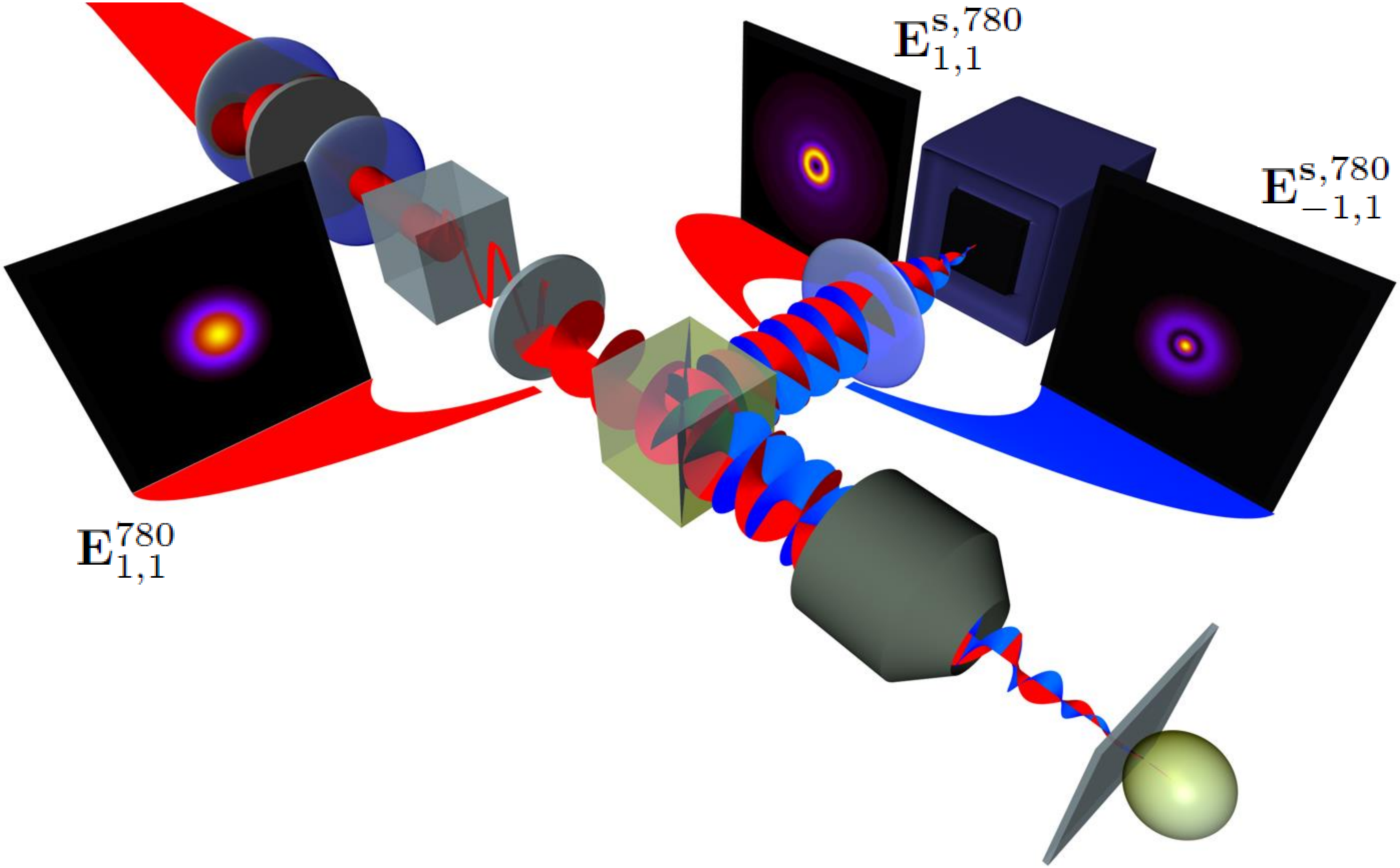

- ✓ Multipoles with opposite helicity behave similarly in opposite semispaces

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- ✓ Q_3: Does the focusing regime change the answers to Q_1 and Q_2? Symmetries of the problem don't change
- ✓ A_3: Yes

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