Mean-field approach for topological phases and beyond

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in collaboration with:

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- N. Schuch (Garching)
- J. Carrasco (Vienna)

Outline

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Topological quantum order and its robustness

Topological quantum order in condensed matter in three dates

- 1989 : High-T_c superconductors, FQHE (X.-G. Wen, F. Wilczek, A. Zee)
- 1997 : Fault-tolerant quantum computation (A. Kitaev, J. Preskill)
- 2005 : String-net condensation (M. Levin, X.-G. Wen)

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Main features

2D gapped quantum systems at $T = 0$ with:

- Ground-state degeneracy depends on the system topology
- Anyonic excitations
- Long-range entanglement, nonvanishing topological entropy
- Robustness against local perturbations

Topological quantum order and its robustness

Why do we care?

Topological order is stable under local perturbations

S. Bravyi, M.B. Hastings, S. Michalakis, J. Math. Phys. 51 093512 (2010)

↓

Encode informations protected against decoherence

↓

Topologically protected qubits (memories + computation)

Topological quantum order and its robustness

Is it really a recent idea?

No!

Quipus were already used in Andean South America more than 45 centuries ago...

Topological quantum order and its robustness

However, as early noticed...

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Condensed-matter issues

- Nature of phase transitions
- Low-energy excitations
- New universality classes ?

Topological quantum order and its robustness

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Condensed-matter issues

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Tools

- Field-theorical approaches (but no local order parameter)
- Monte-Carlo simulations (but sign problem)
- Exact diagonalizations (but small system sizes)
- High-order perturbative expansions (but resummation)
- Variational approaches (but variational!)

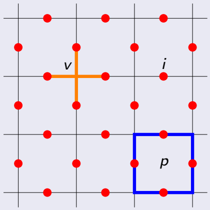
Outline

Mean-field Ansatz

The toric code

$$H_{\mathrm{TC}} = -J\left(\sum_v A_v + \sum_p B_p\right)$$

$$A_v = \prod_{i\in v}\sigma_i^x,\, B_p = \prod_{i\in p}\sigma_i^z$$

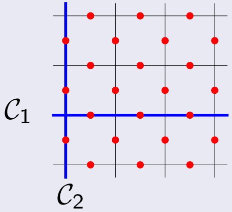


A. Y. Kitaev Ann. Phys. **303**, 2 (2003)

Conserved quantities on a torus

- $[H_{\mathrm{TC}}, A_v] = [H_{\mathrm{TC}}, B_p] = [A_v, B_p] = 0$
- ${A_v}^2 = {B_p}^2 = \mathbb{1}$
- $\prod_v A_v = \prod_p B_p = \mathbb{1}$
- $N_v + N_p = N_b$
- 2 $\mathbb{Z}_2$ operators conserved, e.g.,

$$Z_1 = \prod_{i\in\mathcal{C}_1}\sigma_i^z,\, Z_2 = \prod_{i\in\mathcal{C}_2}\sigma_i^z$$



Mean-field Ansatz

Four ground states on a torus

$$Z_1 = \prod_{i\in\mathcal{C}_1}\sigma_i^z, Z_2 = \prod_{i\in\mathcal{C}_2}\sigma_i^z, A_v = \prod_{i\in v}\sigma_i^x, B_p = \prod_{i\in p}\sigma_i^z$$

- $|\psi_0, z_1, z_2\rangle = \mathcal{N}\left(\frac{\mathbb{1}+z_1 Z_1}{2}\right)\left(\frac{\mathbb{1}+z_2 Z_2}{2}\right)\prod_v\left(\frac{\mathbb{1}+A_v}{2}\right)\prod_p\left(\frac{\mathbb{1}+B_p}{2}\right)|\mathrm{Ref.}\rangle,\ z_{1,2} = \pm 1$
- $Z_1|\psi_0, z_1, z_2\rangle = z_1|\psi_0, z_1, z_2\rangle,\ Z_2|\psi_0, z_1, z_2\rangle = z_2|\psi_0, z_1, z_2\rangle$
- $A_v|\psi_0, z_1, z_2\rangle = +|\psi_0, z_1, z_2\rangle,\ B_p|\psi_0, z_1, z_2\rangle = +|\psi_0, z_1, z_2\rangle$
- $H_{\mathrm{TC}}|\psi_0, z_1, z_2\rangle = -J(N_v + N_p)|\psi_0, z_1, z_2\rangle$

Special choice: $|\psi_0\rangle = \mathcal{N}\prod_p\left(\frac{\mathbb{1}+B_p}{2}\right)|+X\rangle$

Mean-field Ansatz

The toric code on a compact surface of genus g

- Sphere: $g = 0$, torus $g = 1$; double torus $g = 2$...
- $\prod_v A_v = \prod_p B_p = \mathbb{1}$
- $2g$ $\mathbb{Z}_2$ operators conserved
- $N_v + N_p - 2 + 2g = N_b$ independent conserved quantities
- 2^{2g} ground states $\rightarrow$ Topologically-ordered system

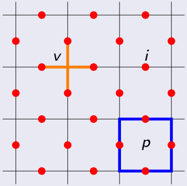
Mean-field Ansatz

A simple example: the toric code in a uniform magnetic field

$$H = -J\Big(\sum_v A_v + \sum_p B_p\Big) - h_x \sum_i \sigma_i^x$$

Vertex (charge) operator: $A_v = \prod_{i\in v} \sigma_i^x$

Plaquette (flux) operator: $B_p = \prod_{i\in p} \sigma_i^z$



- $J = 0$: 1 ground state + bosonic excitations (trivial phase)
- $h_x = 0$: 4 ground states + anyonic excitations (topological phase)

Mean-field Ansatz

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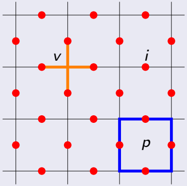

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Phase diagram

- Mapping onto the transverse-field Ising model (square lattice)
  $\rightarrow$ 2$^{\text{nd}}$ order transition at $h_x/J \simeq 0.328$

S. Trebst, P. Werner, M. Troyer, K. Shtengel, C. Nayak, Phys. Rev. Lett. **98**, 070602 (2007)

A. Hamma, D. A. Lidar, Phys. Rev. Lett. **100**, 030502 (2008)

Mean-field Ansatz

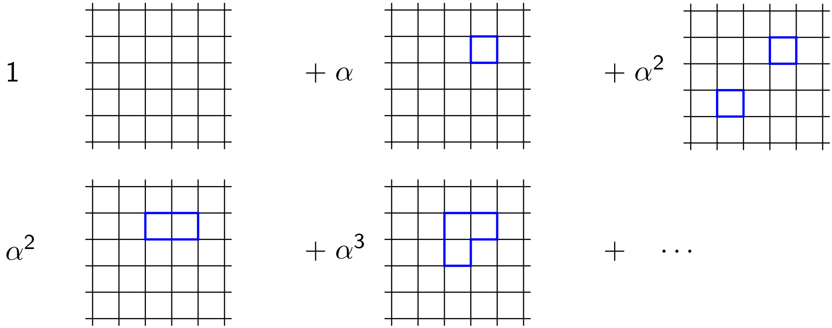
Mean-field description of the ground state

- Simple Ansatz: $|\alpha\rangle = \mathcal{N}_\alpha \prod_p \left(\frac{\mathbb{1}+\alpha B_p}{2}\right) |+X\rangle$
- $\alpha = 0$: exact ground state for $J = 0$
- $\alpha = 1$: exact ground state for $h_x = 0$
- Topological order only for $\alpha = 1 \rightarrow \alpha$ is an order parameter

S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

Mean-field Ansatz

Pictorial representation of the mean-field Ansatz $|\alpha\rangle$



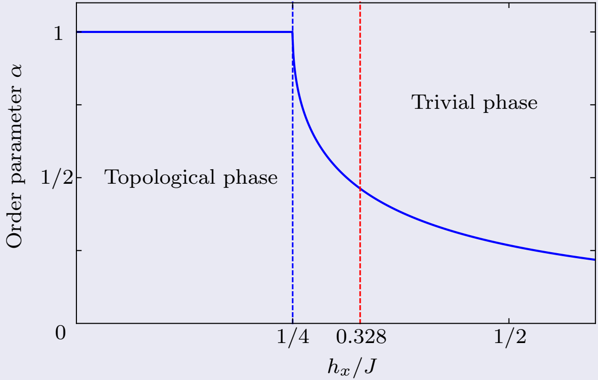
black line $= |+X\rangle_i$

blue line $= |-X\rangle_i$

Weight of a loop configuration: α^{area}

Mean-field Ansatz

Mean-field phase diagram (Toric code + X-field)

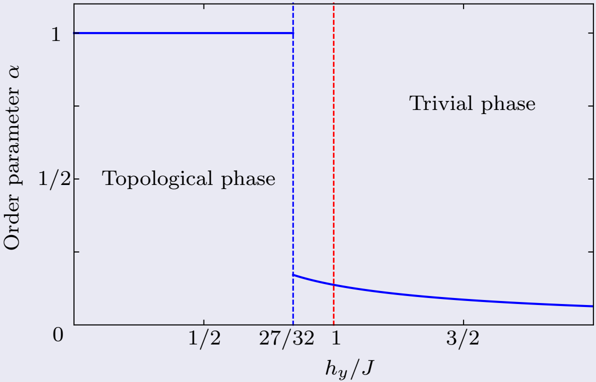


- 2^{nd} order transition (α is a continuous function of h_x/J)
- Critical point at $h_x/J = 1/4$ instead of $h_x/J \simeq 0.328$ ($\sim 24\,\%$ off)

S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

Mean-field Ansatz

Mean-field phase diagram (Toric code + Y–field)



- 1^{st}-order transition (α is a discontinuous function of h_y/J)
- Transition point at $h_y/J = 27/32$ instead of $h_y/J = 1$ ($\sim 16\,\%$ off)

J. Vidal, R. Thomale, K. P. Schmidt, S. Dusuel, Phys. Rev. B **80**, 081104 (2009)

S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

Mean-field Ansatz

A brief summary

- Natural order parameter α
- Good qualitative description of phase transitions

but...

Mean-field Ansatz

A brief summary

- Natural order parameter α
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but...

What about Wilson loops ?

- $\mathcal{L}$: Closed loop made of connected plaquettes (area $\mathcal{A}$ and perimeter $\mathcal{P}$)
- Wilson-loop operator: $W_{\mathcal{L}} = \prod_{p \in \mathcal{L}} B_p$
- Topological phase: perimeter law $\rightarrow \langle W_{\mathcal{L}} \rangle \sim \exp\left(-\mathcal{P}/\mathcal{P}_0\right)$
- Non-topological phase: area law $\rightarrow \langle W_{\mathcal{L}} \rangle \sim \exp\left(-\mathcal{A}/\mathcal{A}_0\right)$

Fingerprint of topological phases!

K. G. Wilson, Phys. Rev. D **10**, 2445 (1974)

Mean-field Ansatz

Wilson-loop behavior with the mean-field Ansatz

- Factorization property: $\langle W_n \rangle = \langle \alpha | \prod_{p=1}^{n} B_p | \alpha \rangle = \left(\prod_{p=1}^{n} \langle \alpha | B_p | \alpha \rangle \right)^n = \left(\frac{2\alpha}{1+\alpha^2} \right)^n$
- Topological phase: $\alpha = 1 \rightarrow \langle W_n \rangle = 1$
  Trivial perimeter law (diverging characteristic perimeter)
- Non-topological phase: $0 \leqslant \alpha < 1 \rightarrow \langle W_n \rangle \sim \exp(-n/\lambda)$
  Non-trivial area law with a dimensionless characteristic area $\lambda = -\frac{1}{\log\left(\frac{2\alpha}{1+\alpha^2}\right)}$

Mean field = Uncorrelated-flux approximation

Outline

Beyond the mean-field approximation

What is missing ?

- Correlations between fluxes
- Better approximation in the topological phase

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- Better approximation in the topological phase

Adding a string tension

- Example: Toric code + X-field

  Improved Ansatz: $|\alpha, \beta\rangle = \mathcal{N}_{\alpha,\beta} \; \mathrm{e}^{\,\beta \sum_i \sigma_i^x} \; \prod_p \left(\frac{\mathbb{1} + \alpha B_p}{2} \right) |+X\rangle$

  $\rightarrow$ Exact ground state for $h_x \ll J$ (order 1: $\alpha = 1$, $\beta = \frac{h_x}{4J}$)

  $\rightarrow$ Exact ground state for $h_x \gg J$ (order 1: $\alpha = \frac{J}{8h_x}$, $\beta = 0$)

  $\rightarrow$ Nontrivial perimeter law (penalty to large-loop configurations)

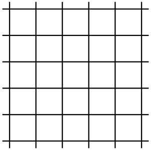
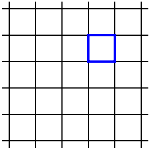
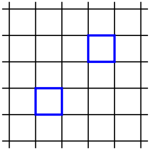
J. Haegeman, K. Van Acoleyen, N. Schuch, J. I. Cirac, F. Verstraete, Phys. Rev. X **5**, 011024 (2015)

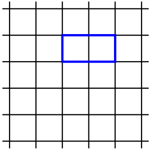
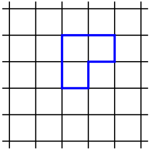
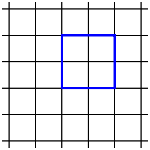
J. Haegeman, V. Zauner, N. Schuch, F. Verstraete, Nat. Commun. **6**, 8284 (2015)

L. Vanderstraeten, M. Mariën, J. Haegeman, N. Schuch, J. Vidal, F. Verstraete, Phys. Rev. Lett. **119**, 070401 (2017)

Beyond the mean-field approximation

Pictural representation of the improved Ansatz $|\alpha, \beta\rangle$

1  $+ \alpha\, e^{-4\beta}$  $+ \alpha^2 e^{-8\beta}$  $+$

$\alpha^2 e^{-6\beta}$  $+ \alpha^3\, e^{-8\beta}$  $+ \alpha^4 e^{-8\beta}$  $+ \cdots$

— $= |+X\rangle_i$

— $= |-X\rangle_i$

Weight of a loop configuration: $\alpha^{\text{area}}\ e^{\,\text{perimeter}\times\beta}$

Beyond the mean-field approximation

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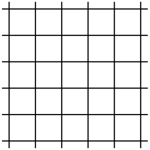
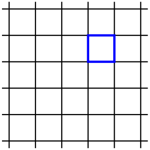
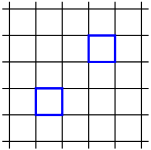
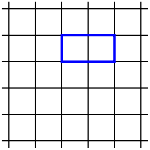
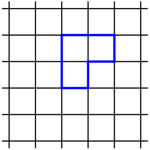
1  $+\, \alpha\, e^{-6\beta}$  $+\, \alpha^2 e^{-8\beta}$ 

$\alpha^2 e^{-6\beta}$  $+\, \alpha^3 e^{-8\beta}$  $+ \quad \cdots$

black line $= |+X\rangle_i$
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Weight of a loop configuration: $\alpha^{\text{area}}\ e^{-\,\beta \times \text{perimeter}}$

Beyond the mean-field approximation

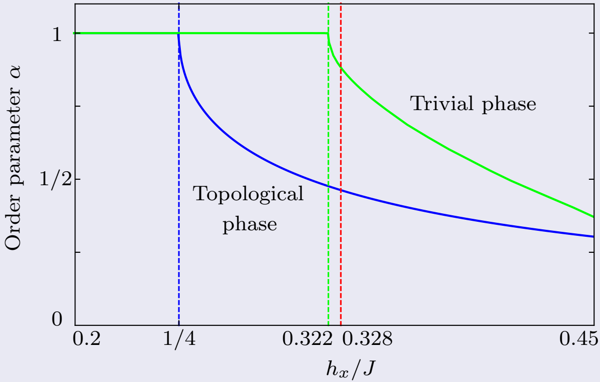
Perturbative Projected Entangled Paired State

- Numerical implementation using tensor networks
- Ex: Toric code+ X-field $\to |\alpha, \beta\rangle$ is a PEPS with bond dimension $D = 2$
- α is still an order parameter

L. Vanderstraeten, M. Mariën, J. Haegeman, N. Schuch, J. Vidal, F. Verstraete, Phys. Rev. Lett. **119**, 070401 (2017)

Beyond the mean-field approximation

Improved Ansatz phase diagram (Toric code + X-field)



- 2^{nd} order transition (α is a continuous function of h_x/J)
- Critical point at $h_x/J \simeq 0.322$ instead of $h_x/J \simeq 0.328$ ($\sim 2\,\%$ off)

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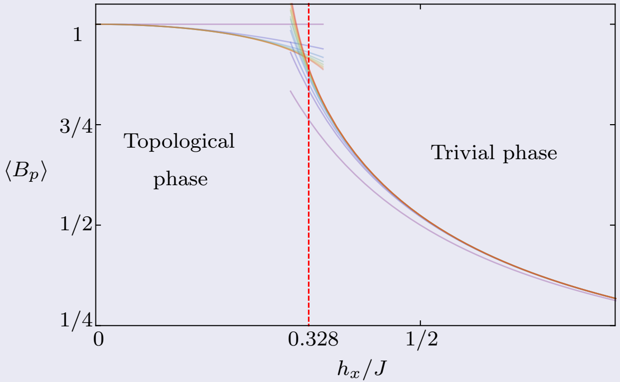
Beyond the mean-field approximation

Elementary Wilson loop (Toric code + X-field)

Series $h_x \ll J$ (up to order 16) + Series $h_x \gg J$ (up to order 26)



H.-X. He, C. J. Hamer, J. Oitmaa, J. Phys. A **23**, 1775 (1990)

C. J. Hamer, J. Oitmaa, Z. Weihong, J. Phys. A **25**, 1821 (1992)

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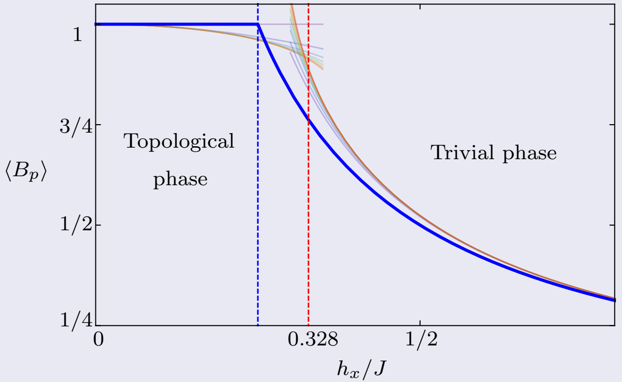

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S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

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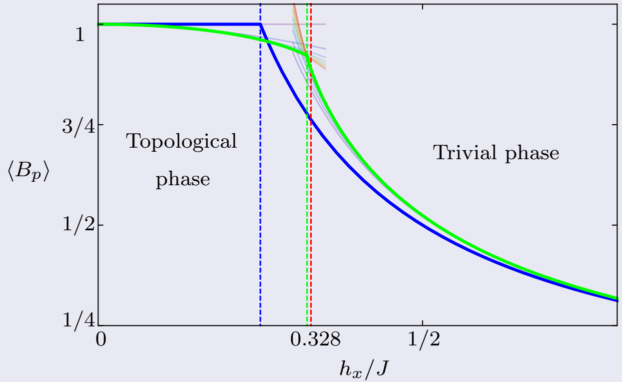

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L. Vanderstraeten, M. Mariën, J. Haegeman, N. Schuch, J. Vidal, F. Verstraete, Phys. Rev. Lett. **119**, 070401 (2017)

Outline

String-net models with tension

Model

$$H = -J\Big(\sum_v Q_v + \sum_p P_p\Big) - h\sum_i V_i, \ (J > 0 \text{ and } h > 0)$$

- Degrees of freedom defined on the links of the honeycomb lattice
- Each link j can be in N different states $|1\rangle_j, |2\rangle_j, ..., |N\rangle_j$
- Restricted Hilbert space: $Q_v|\psi\rangle = +|\psi\rangle$ (no charge excitation)
- P_p: Projector onto the vacuum in the plaquette p
- Tension term: $[Q_v, V_i] = 0$ (charge conserving)
- $V_j = \delta_{j,1}$: projector onto state $|1\rangle_j$
- Mean-Field Ansatz: $|\alpha\rangle = \mathcal{N}_\alpha \prod_p \left(\frac{\mathbb{1}+\alpha B_p}{2}\right)|1\rangle$ with $B_p = 2P_p - \mathbb{1}$

M. Levin, X.-G. Wen, Phys. Rev. B **71**, 045110 (2005)
C. Gils, S. Trebst, A. Kitaev, A. W. W. Ludwig, M. Troyer, Z. Wang, Nat. Phys. **5**, 834 (2009)
S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

String-net models with tension

Input data

- Set of labels (charges, superselection sectors, particles...)
- Fusion rules: $a \times b = \sum_c N_c^{ab} c$ (here $N_c^{ab} = 0, 1$)
- F-symbols
- Unitary Modular Tensor Category

String-net models with tension

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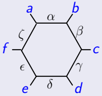
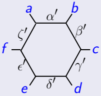
Constraints and projector

- Hilbert space: set of configurations respecting fusion rules at each vertex
- Trivalent graph mandatory (honeycomb lattice, two-leg ladder,...)
- Local projector onto a given plaquette:

P_p 

$$= \sum_s \frac{d_s}{D^2} \sum{}' F^{a\zeta\alpha}_{s\alpha'\zeta'} F^{b\alpha\beta}_{s\beta'\alpha'} F^{c\beta\gamma}_{s\gamma'\beta'} F^{d\gamma\delta}_{s\delta'\gamma'} F^{e\delta\epsilon}_{s\epsilon'\delta'} F^{f\epsilon\zeta}_{s\zeta'\epsilon'}$$



- d_s: quantum dimension of the string s $\quad D$: total quantum dimension

String-net models with tension

Mean-field description of the ground state

- Simple Ansatz: $|\alpha\rangle = \mathcal{N}_\alpha \prod_p \left(\frac{\mathbb{1}+\alpha B_p}{2}\right)|1\rangle$, with $B_p = 2P_p - \mathbb{1}$
- $\alpha = 0$: exact ground state for $J = 0$
- $\alpha = 1$: exact ground state for $h = 0$
- α is an order parameter
- Trivial perimeter law in the topological phase ($\alpha = 1$)
- Non-trivial area law in the non-topological phase ($0 \leqslant \alpha < 1$)
- Mean-field transition point: $h/J = \frac{D^2-1}{3D^2}$ $\rightarrow$ Only depends on D!

S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

String-net models with tension

Results for Abelian anyon theories

Mapping onto the transverse-field N-states Potts model (triangular lattice)

	$\mathbb{Z}_2$	$\mathbb{Z}_3$	...	$\mathbb{Z}_{N\to\infty}$
h/J (mean-field)	0.1667	0.2222	...	1/3
h/J (series)	0.2097	0.2466	...	1/3

H.-X. He, C. J. Hamer, J. Oitmaa, J. Phys. A **23**, 1775 (1990)
C. J. Hamer, J. Oitmaa, Z. Weihong, J. Phys. A **25**, 1821 (1992)
F. J. Burnell, S. H. Simon, J. K. Slingerland, Phys. Rev. B **84**, 125434 (2011)

Results for non-Abelian anyon theories

No exact mapping known!

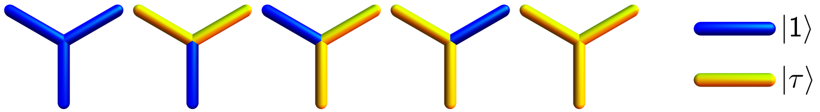
	Fibonacci	Ising
h/J (mean-field)	0.2412	0.25
h/J (series)	0.2618	0.267

M. D. Schulz, S. Dusuel, K. P. Schmidt, J. Vidal, Phys. Rev. Lett. **110**, 147203 (2013)
M. D. Schulz, S. Dusuel, G. Misguich, K. P. Schmidt, J. Vidal, Phys. Rev. B **89**, 201103 (2014)

String-net models with tension

The Fibonacci theory

- Two strings: $\{1, \tau\}$
- Fusion rules: $1 \times 1 = 1$, $1 \times \tau = \tau$, $\tau \times \tau = 1 + \tau$



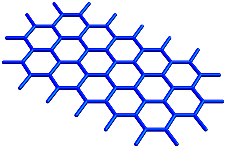
Hilbert space dimension for any graph with N_v trivalent vertices

- Dim $\mathcal{H} = (1+\varphi^2)^{\frac{N_v}{2}} + (1+\varphi^{-2})^{\frac{N_v}{2}}$, $\quad \varphi = \frac{1+\sqrt{5}}{2}$ (golden ratio)

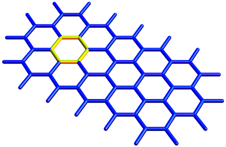
String-net models with tension

Pictorial representation of the mean-field Ansatz $|\alpha\rangle$

1

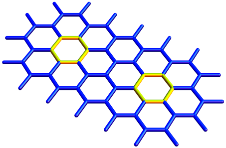


$+ \alpha \, d_\tau$

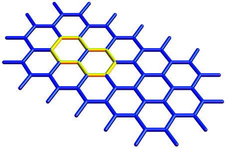


$+$

$\alpha^2 \, d_\tau^2$

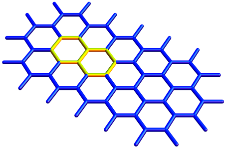


$+ \alpha^2 \, d_\tau$



$+$

$\alpha^2 \, d_\tau^{3/2}$

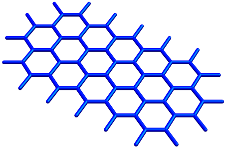
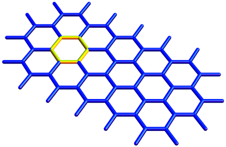


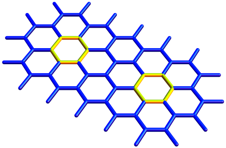
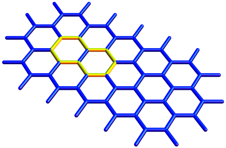
$+ \quad \cdots$

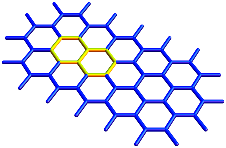
$d_\tau = \varphi$ is the quantum dimension of the label τ

String-net models with tension

Pictorial representation of the mean-field Ansatz $|\alpha, \beta\rangle$

1  $+ \, \alpha \, d_\tau \, \mathrm{e}^{-6\beta}$  $+$

$\alpha^2 \, d_\tau^2 \, \mathrm{e}^{-12\beta}$  $+ \, \alpha^2 \, d_\tau \, \mathrm{e}^{-10\beta}$  $+$

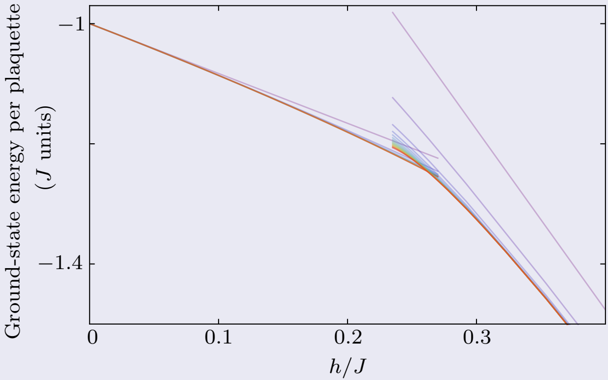
$\alpha^2 \, d_\tau^{3/2} \, \mathrm{e}^{-11\beta}$  $+ \quad \cdots$

$d_\tau = \varphi$ is the quantum dimension of the label τ

String-net models with tension

Ground-state energy (Fibonacci string-net model + tension)

Series $h \ll J$ (up to order 11) + Series $h \gg J$ (up to order 19)

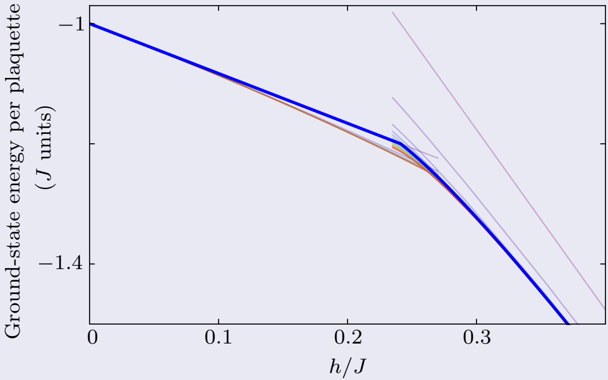


M. D. Schulz, S. Dusuel, K. P. Schmidt, J. Vidal, Phys. Rev. Lett. **110**, 147203 (2013)

String-net models with tension

Ground-state energy (Fibonacci string-net model + tension)

Series $h \ll J$ (up to order 11) + Series $h \gg J$ (up to order 19) + $|\alpha\rangle$



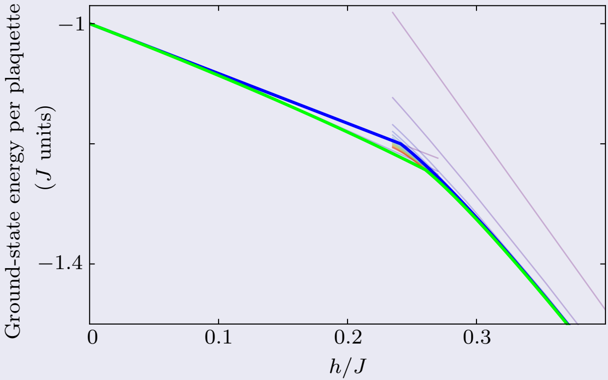
M. D. Schulz, S. Dusuel, K. P. Schmidt, J. Vidal, Phys. Rev. Lett. **110**, 147203 (2013)

S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

String-net models with tension

Ground-state energy (Fibonacci string-net model + tension)

Series $h \ll J$ (up to order 11) + Series $h \gg J$ (up to order 19) + $|\alpha\rangle$ + $|\alpha, \beta\rangle$



M. D. Schulz, S. Dusuel, K. P. Schmidt, J. Vidal, Phys. Rev. Lett. **110**, 147203 (2013)

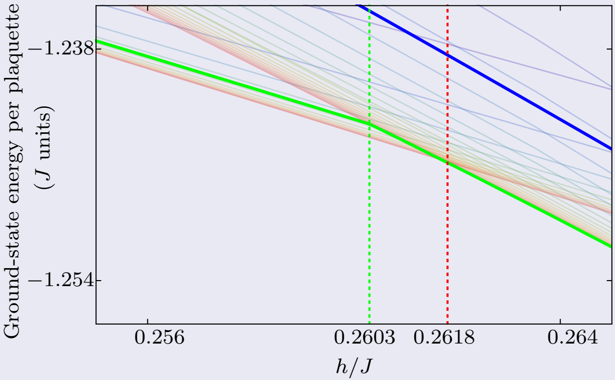
S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

A. Schotte, J. Carrasco, B. Vanhecke, L. Vanderstraeten, J. Haegeman, F. Verstraete, J. Vidal, Phys. Rev. B **100**, 245125 (2019)

String-net models with tension

Ground-state energy (Fibonacci string-net model + tension)

Series $h \ll J$ (up to order 11) + Series $h \gg J$ (up to order 19) + $|\alpha\rangle$ + $|\alpha, \beta\rangle$



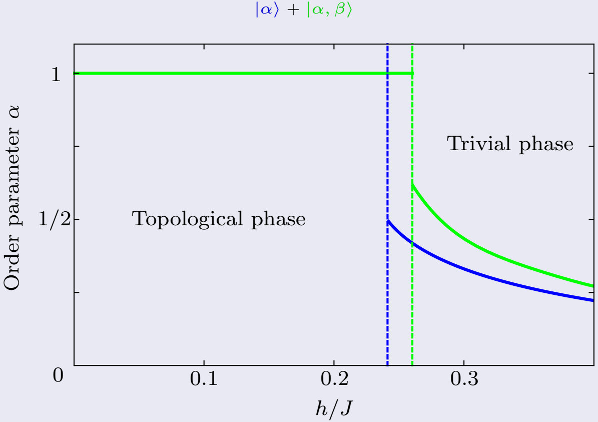
M. D. Schulz, S. Dusuel, K. P. Schmidt, J. Vidal, Phys. Rev. Lett. **110**, 147203 (2013)

S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

A. Schotte, J. Carrasco, B. Vanhecke, L. Vanderstraeten, J. Haegeman, F. Verstraete, J. Vidal, Phys. Rev. B **100**, 245125 (2019)

String-net models with tension

Order parameter (Fibonacci string-net model + tension)



α is discontinuous $\rightarrow$ First-order transition !

S. Dusuel, J. Vidal, Phys. Rev. B **92**, 125150 (2015)

A. Schotte, J. Carrasco, B. Vanhecke, L. Vanderstraeten, J. Haegeman, F. Verstraete, J. Vidal, Phys. Rev. B **100**, 245125 (2019)

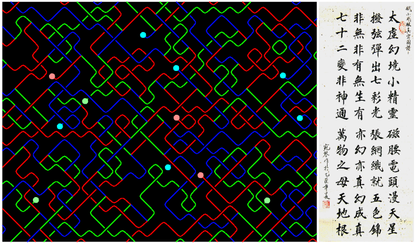
Outlook

Take-home messages

- Mean-field Ansatz $\rightarrow$ qualitatively good description
- Improved Ansatz (perturbative PEPS) $\rightarrow$ quantitatively good description
- Order parameter for topological phases (Chiral phases ?)
- Perimeter law and area law for Wilson loops
- To be tested in 3D models (Walker-Wang, X-cube, Haah's code,...)

D. Reiss, K. P. Schmidt, SciPost Phys. **6**, 078 (2019)

M. Mühlhauser, M. R. Walther, D. A. Reiss, K. P. Schmidt, arXiv:1911.13117



Artist view of string nets

X.-G.Wen, Science **363**, 6429 (2019)