$SU(3)_1$ chiral spin liquid on the square lattice
—A view from symmetric PEPS

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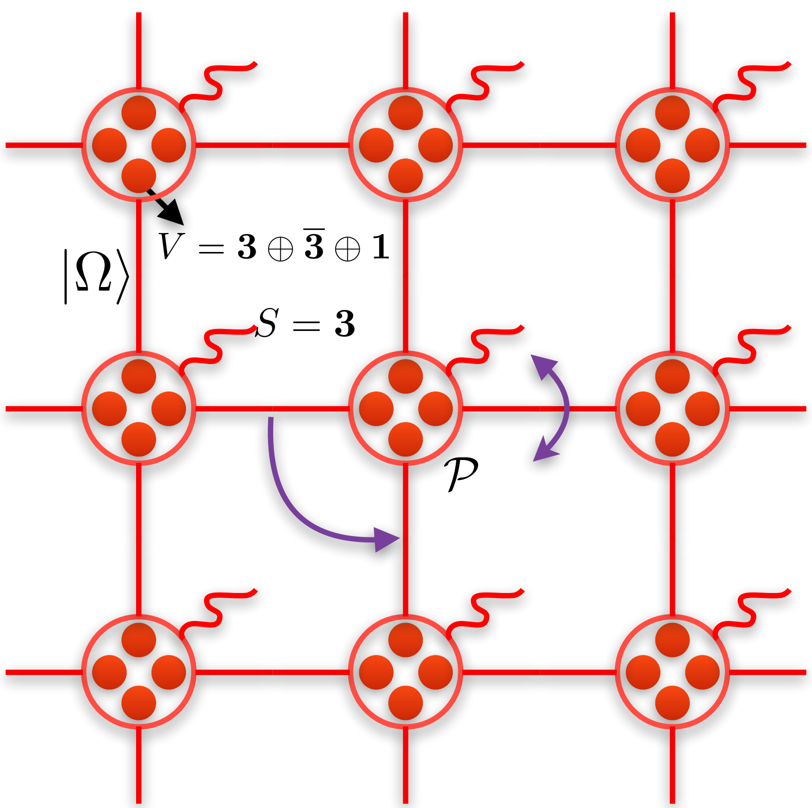
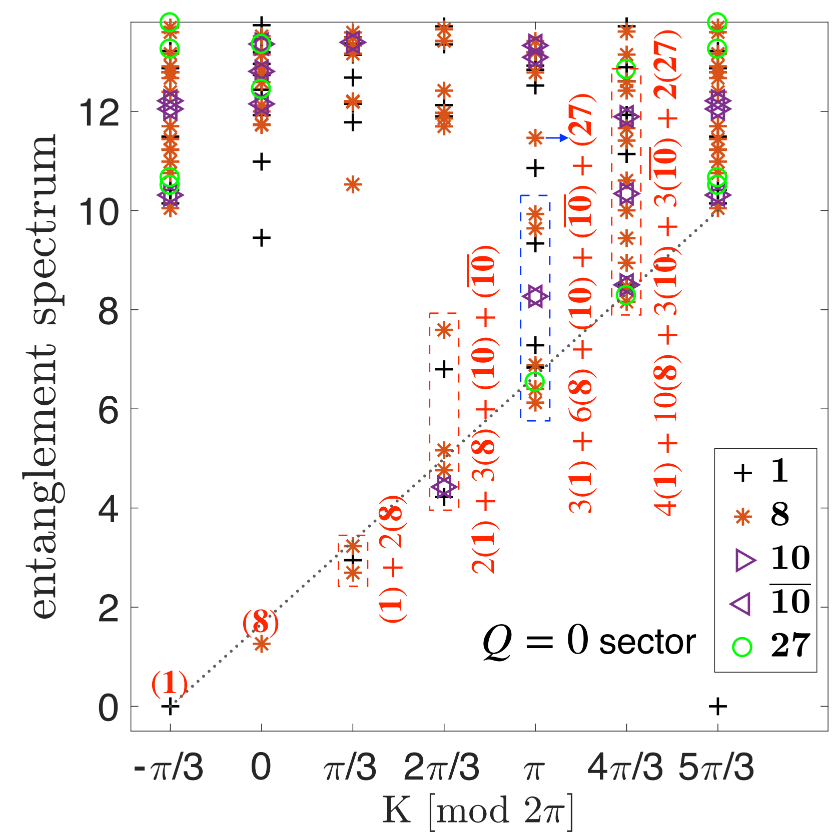




JYC, S. Capponi, A. Wietek, M. Mambrini, N. Schuch, and D. Poilblanc, arXiv: 1912.13393

Motivation

- Chiral spin liquid
  analog of fractional quantum Hall states in Mott insulators

- SU(N) system
  relevant for cold atoms, where CSL could naturally appear with large N

- PEPS description of chiral topological phase
  critical RVB —> critical chiral RVB: $\mathrm{SU}(2)_1, \mathrm{SU}(2)_2$ CSL
  gapped trimer RVB —> ?

Outline

- Model and exact diagonalization
- Symmetric PEPS ansatz
- PEPS optimization
- Property: entanglement spectrum, bulk correlations
- Degeneracy structure of topological chiral PEPS

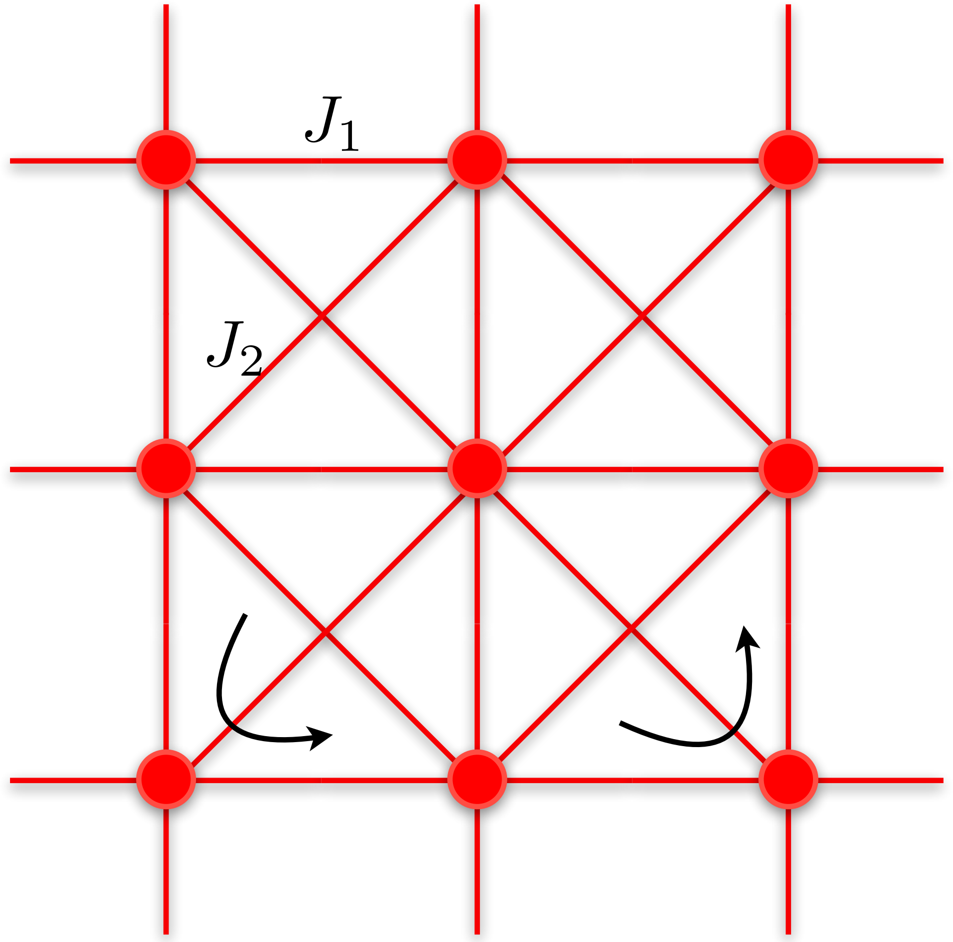
V. Kalmeyer and R. B. Laughlin (1987); M. Hermele, V. Gurarie, and A. M. Rey (2009); T. Wahl, et al (2013)
D. Poilblanc, J. I. Cirac, and N. Schuch (2015); JYC, et al (2018); X.-Y. Dong, JYC, and H.-H. Tu (2018)

Model

On every site, we put a $\mathrm{SU}(3)$ fundamental representation.

Inspired by work on $\mathrm{SU}(2)_1$ CSL on various lattices, and $\mathrm{SU}(N)_1$ CSL on triangular lattice, we propose to study the following Hamiltonian:

$$H = J_1 \sum_{\langle i,j \rangle} P_{ij} + J_2 \sum_{\langle\langle k,l \rangle\rangle} P_{kl} + J_{\mathrm{R}} \sum_{\triangle_{ijk}} (P_{ijk} + P_{ijk}^{-1}) + iJ_{\mathrm{I}} \sum_{\triangle_{ijk}} (P_{ijk} - P_{ijk}^{-1})$$



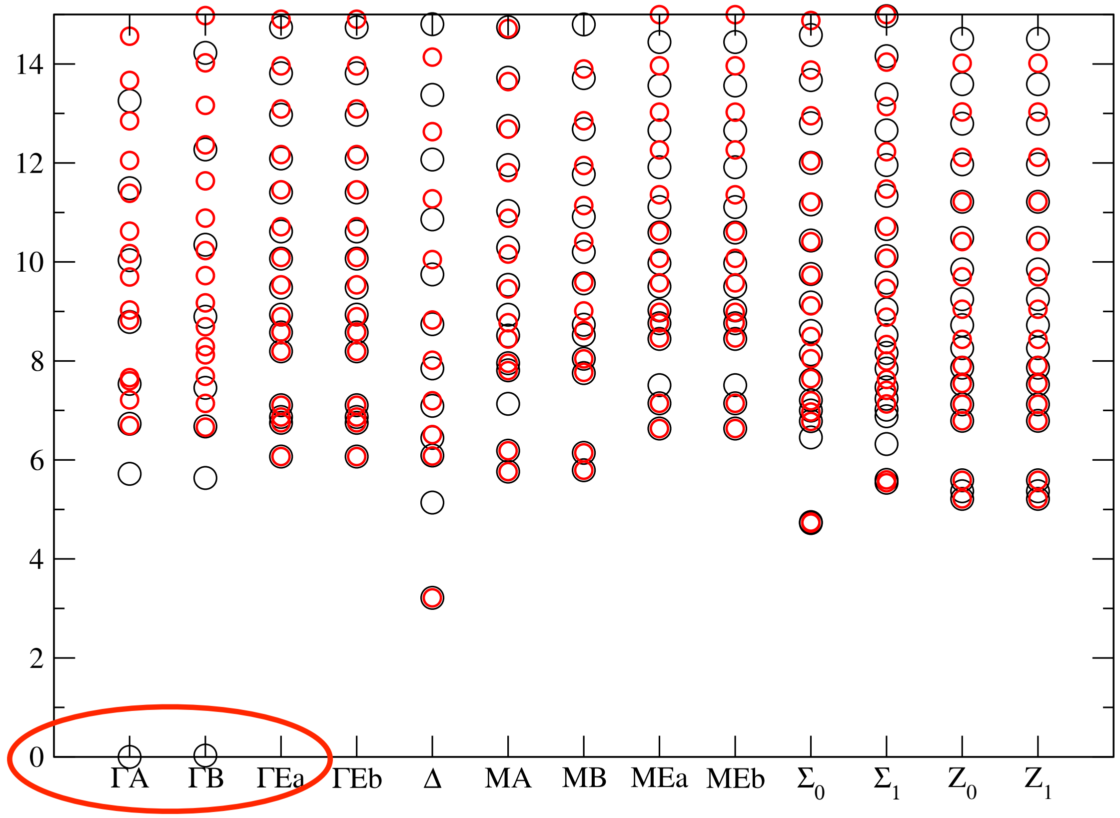
Reparametrization:

$$J_1 = 2J_2 = \frac{4}{3}\cos\theta\sin\phi$$
$$J_{\mathrm{R}} = \cos\theta\cos\phi$$
$$J_{\mathrm{I}} = \sin\theta$$

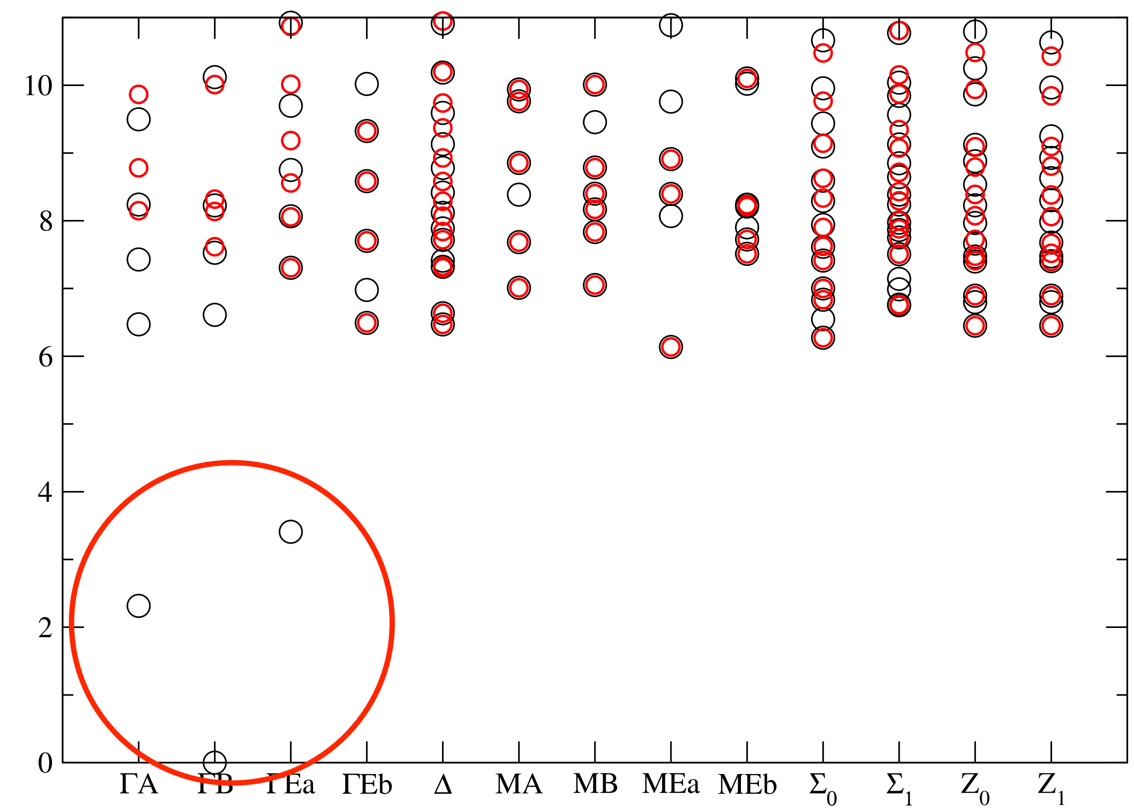
A. Nielsen, G. Sierra, and J. I. Cirac (2013); P. Nataf, M. Lajko, A. Wietek, K. Penc, F. Mila, and A. Lauchli (2016)

Exact diagonalization

On a 18-site torus, we expect three quasi degenerate singlets below spinful excited states (topological degeneracy).



$$\theta = 0, \phi = \pi/4$$

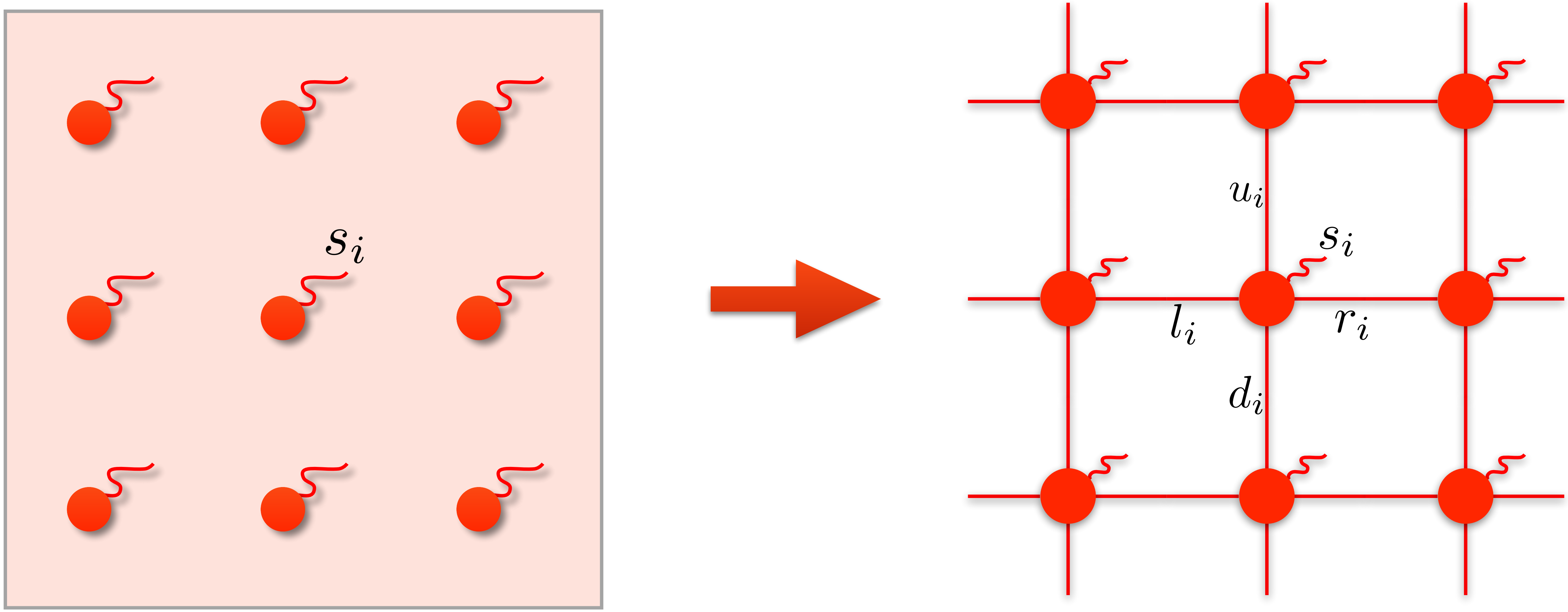


$$\theta = \phi = \pi/4$$

Further evidence can be found by studying the model on various system size and different boundary condition. (related to Halperin 221 state)

Symmetric PEPS ansatz

Encode symmetries into local tensors.



$$|\psi\rangle = \sum_{\cdots,s_i,\cdots} c_{\cdots,s_i,\cdots} |\cdots, s_i, \cdots\rangle$$

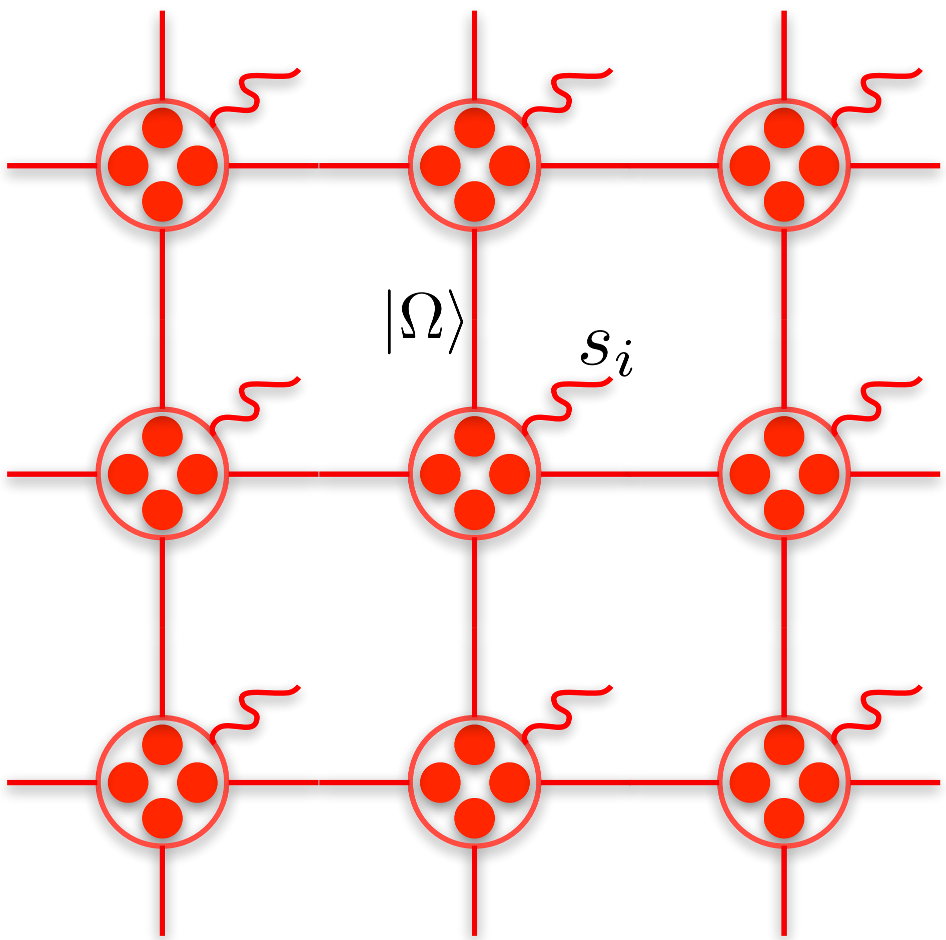
$$|\psi\rangle = \sum_{\cdots,s_i,\cdots} \mathrm{tr}(\cdots A^{s_i}_{l_i,r_i,u_i,d_i} \cdots) |\cdots, s_i, \cdots\rangle$$

rank dD^4 tensor, constrained by symmetry

F. Verstraete, J. I. Cirac (2004)

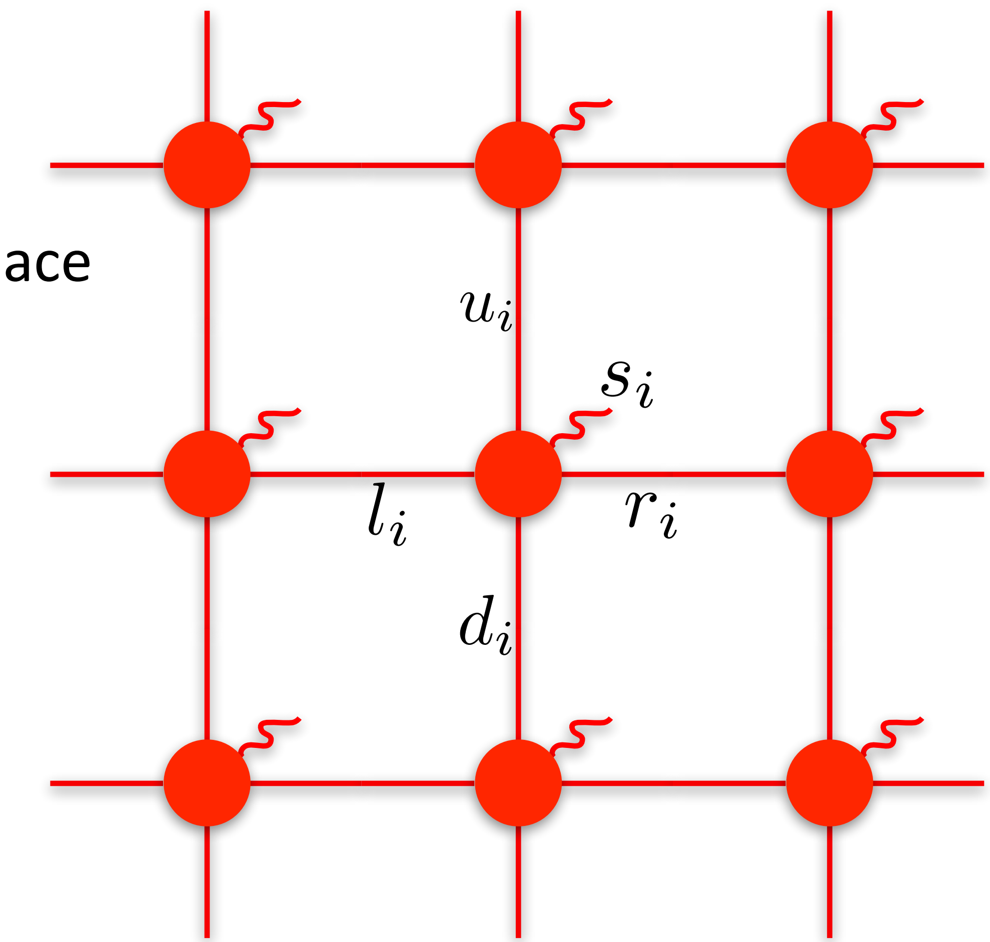
Symmetric PEPS ansatz: SU(3) symmetry

Encode symmetries into local tensors.



Choose virtual space

$\mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1}$



$$|\psi\rangle = \bigotimes_{i=1} \hat{P}^{(i)} \bigotimes_{\langle i_1,i_2\rangle} |\Omega\rangle_{i_1,i_2}$$

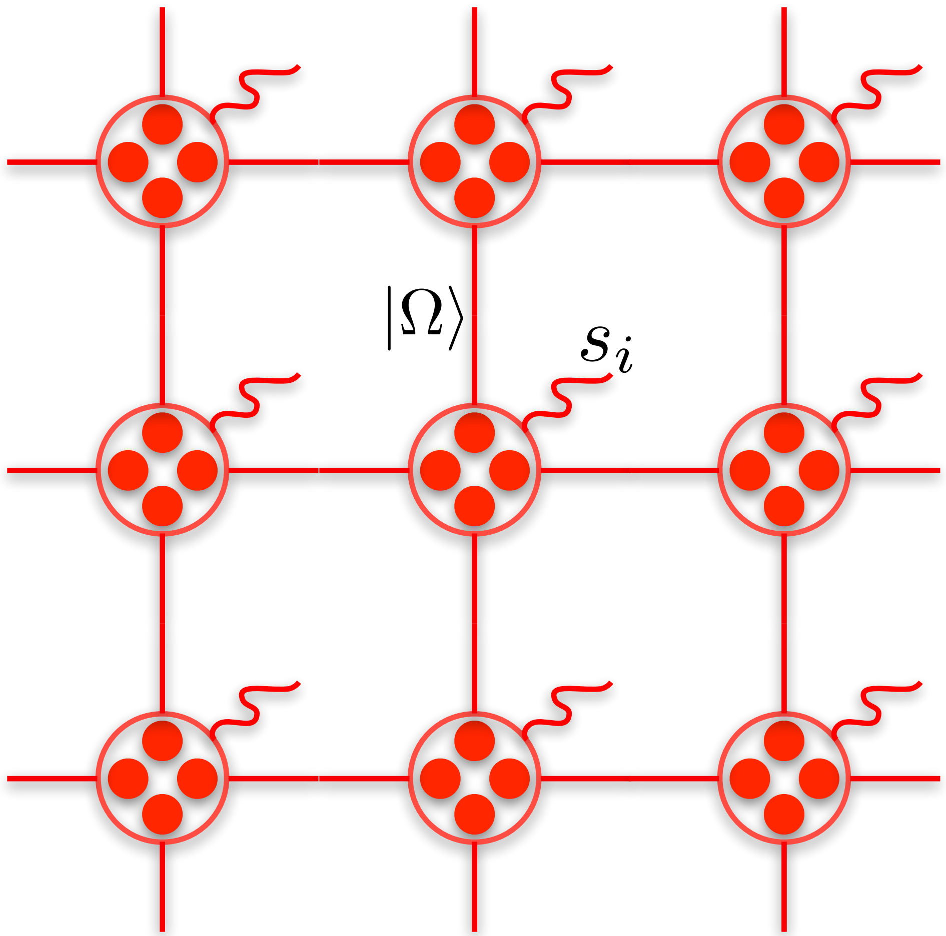
projector — virtual singlet

$$|\psi\rangle = \sum_{\cdots,s_i,\cdots} \mathrm{tr}(\cdots A^{s_i}_{l_i,r_i,u_i,d_i} \cdots)|\cdots,s_i,\cdots\rangle$$

rank dD^4 tensor, constrained by symmetry

D. Perez-Garcia, M. Sanz, C. E. Gonzalez-Guillen, M. M. Wolf, and J. I. Cirac (2010)

Symmetric PEPS ansatz: lattice symmetry

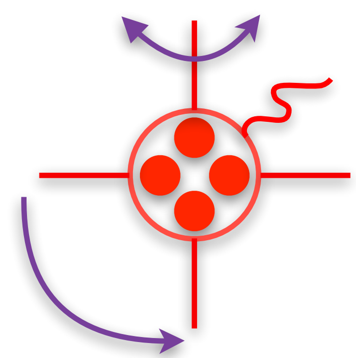


$$|\psi\rangle = \bigotimes_{i=1} \hat{P}^{(i)} \bigotimes_{\langle i_1, i_2 \rangle} |\Omega\rangle_{i_1, i_2}$$

projector

virtual singlet

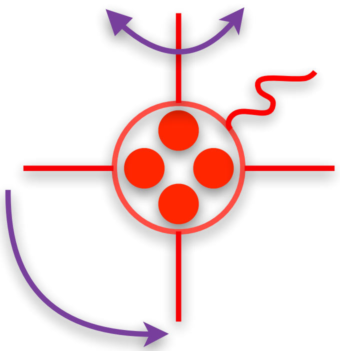
Choose $|\Omega\rangle$ to be reflection symmetric, maximally entangled



$\hat{P}$ should be rotation invariant, become complex conjugate under reflection/time reversal

S. Jiang, Y. Ran (2015)

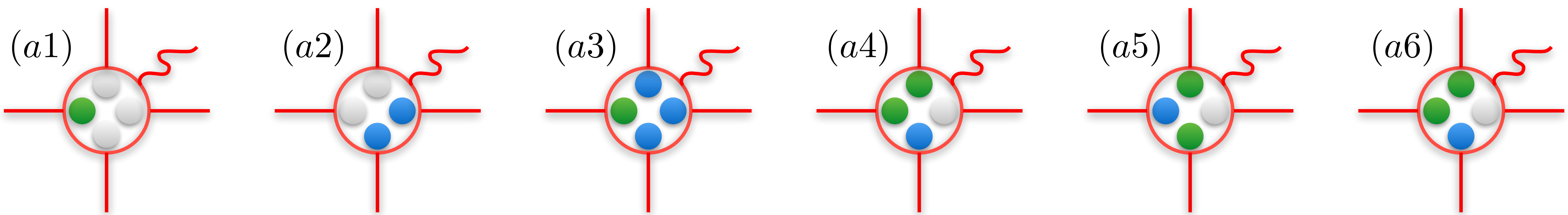
A classification of projector



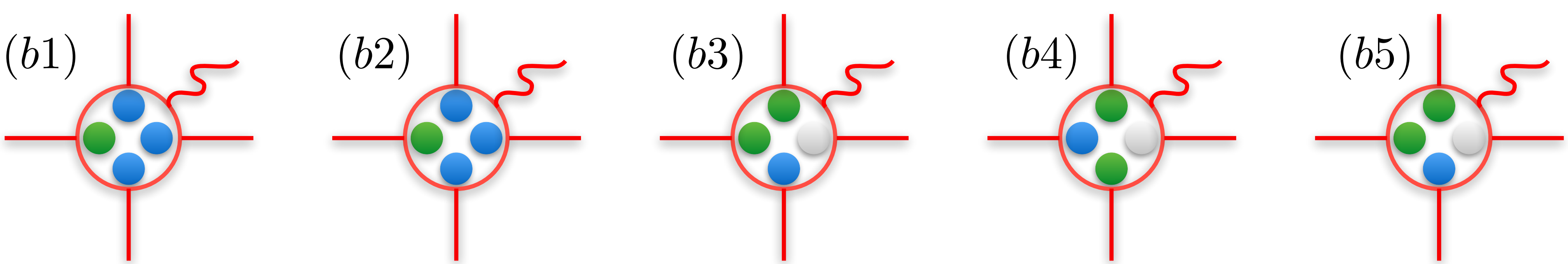
classify $\hat{P}$ according to $C_{4\mathrm{v}}$ group IRREP: A_1, A_2, B_1, B_2, E

$\mathbf{3}$ (green) $\bar{\mathbf{3}}$ (blue) $\mathbf{1}$ (white)

B_1 class



B_2 class

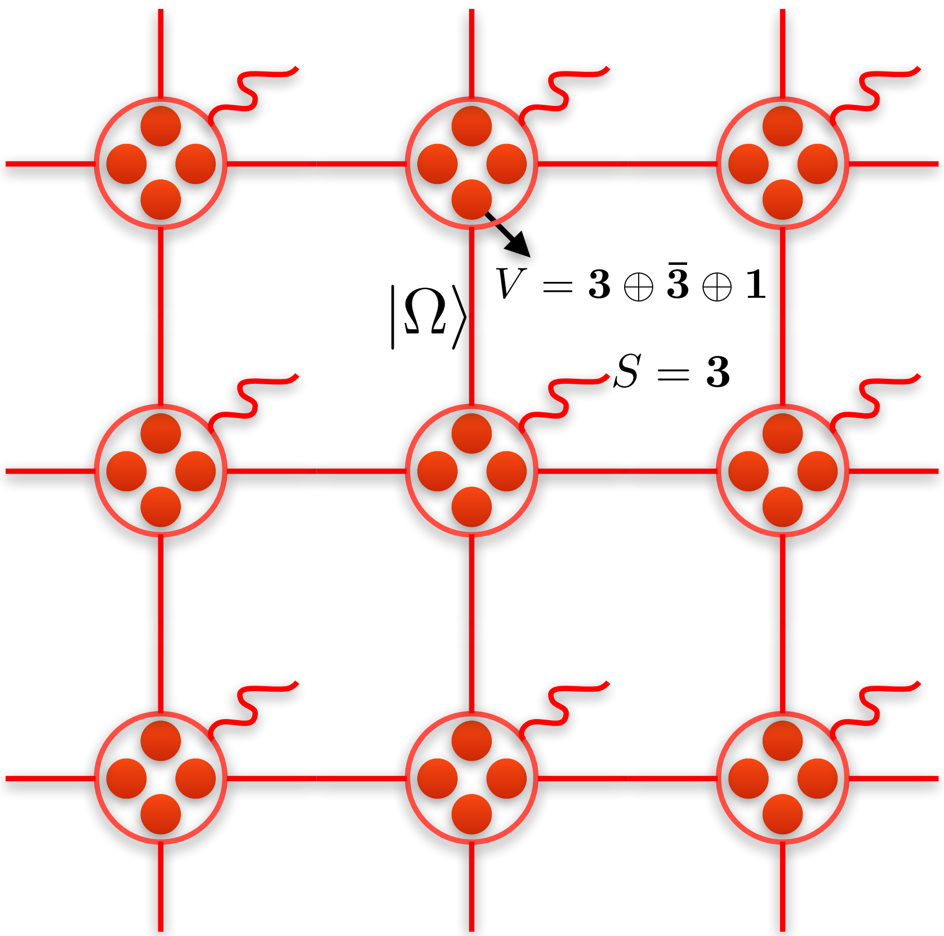


A significant reduction of parameters: 3×7^4—> 11

M. Mambrini, R. Orus, and D. Poilblanc (2016)

Symmetric PEPS ansatz

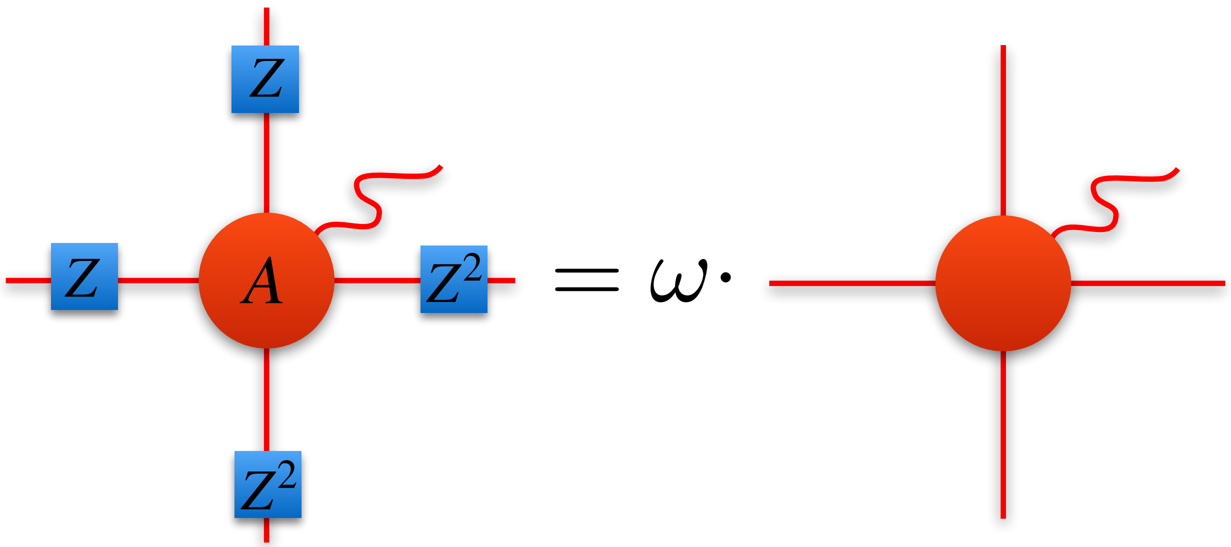
$$\hat{P} = B_1 + iB_2 = \sum_{a=1}^{N_1} \lambda_1^a B_1^a + i \sum_{b=1}^{N_2} \lambda_2^b B_2^b$$



Center of SU(3) acts as gauge symmetry.

$\{I, Z, Z^2\}$,

$Z = \mathrm{diag}(\omega, \omega, \omega, \omega^2, \omega^2, \omega^2, 1), \omega = \exp(i2\pi/3)$



N. Schuch, J. I. Cirac, and D. Perez-Garcia (2010)

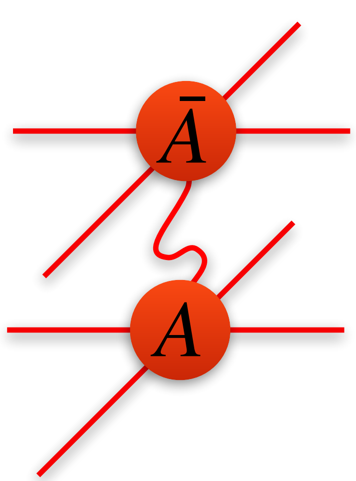
Optimization

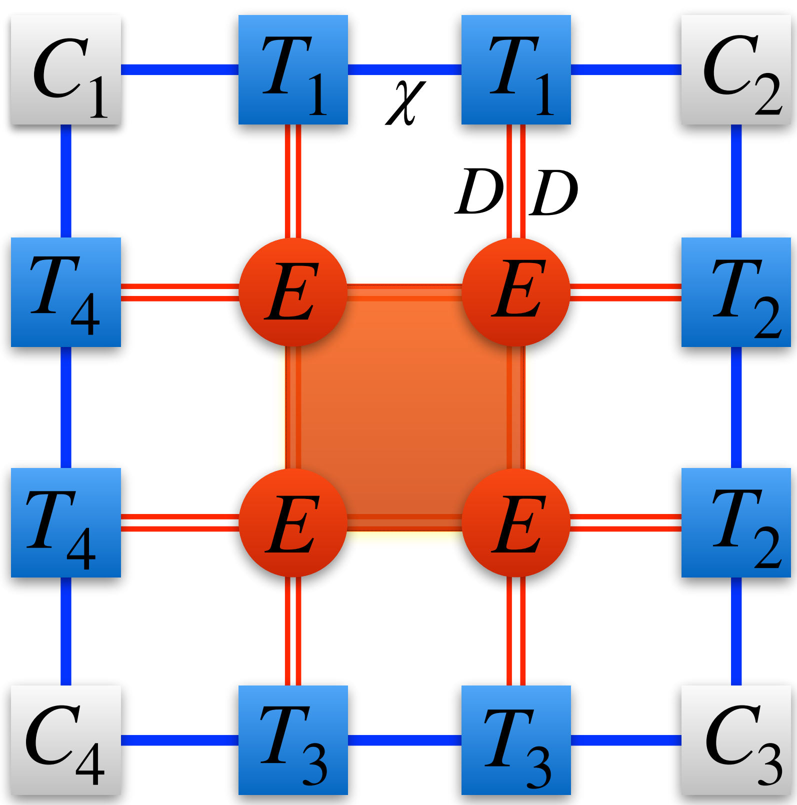
Use corner transfer matrix renormalization group to compute effective environment tensors.

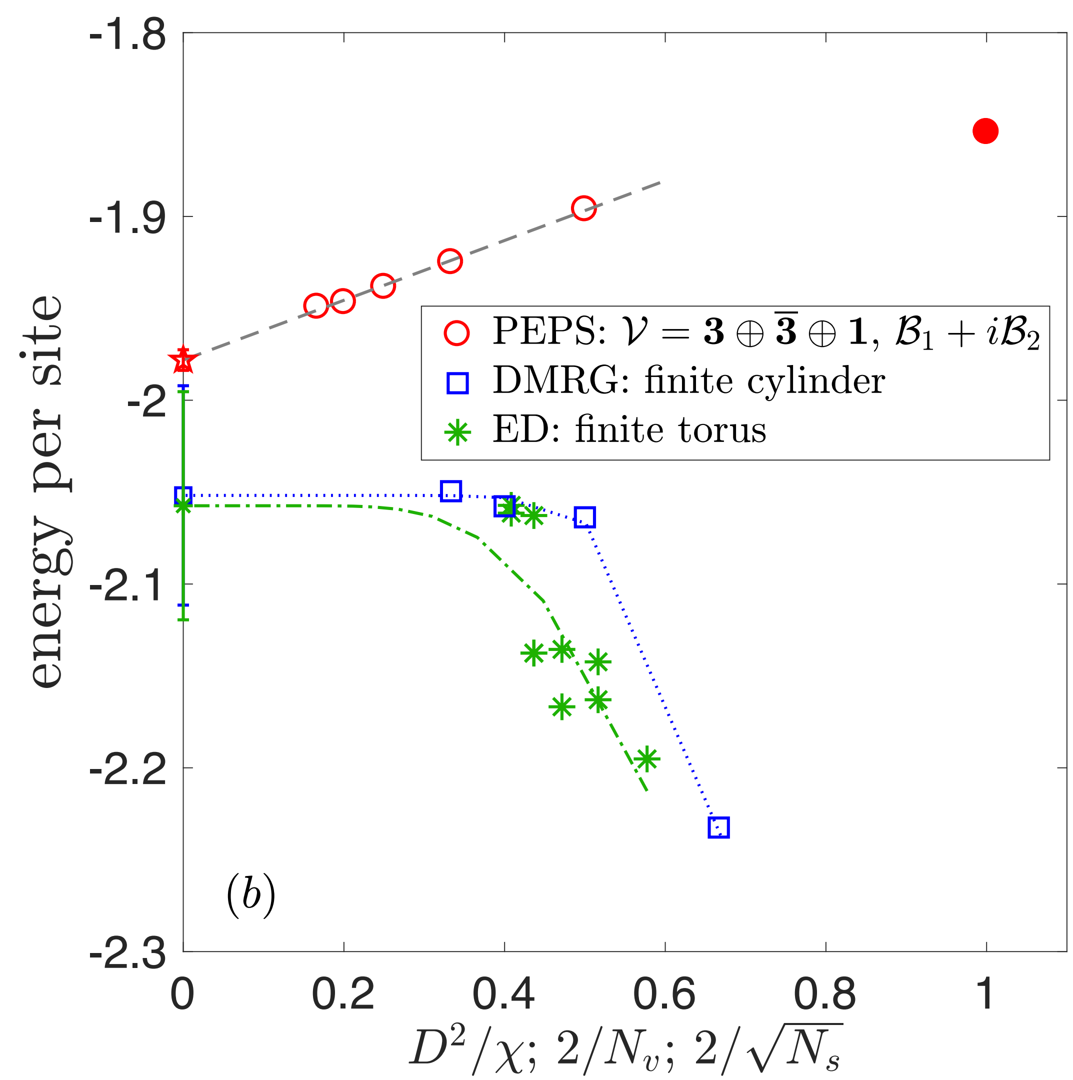
U(1) symmetry is used to speed up.

Simple finite difference method is used to compute energy gradient.

Conjugate gradient method is used to find optimal $\{\lambda_1^a, \lambda_2^b\}$.



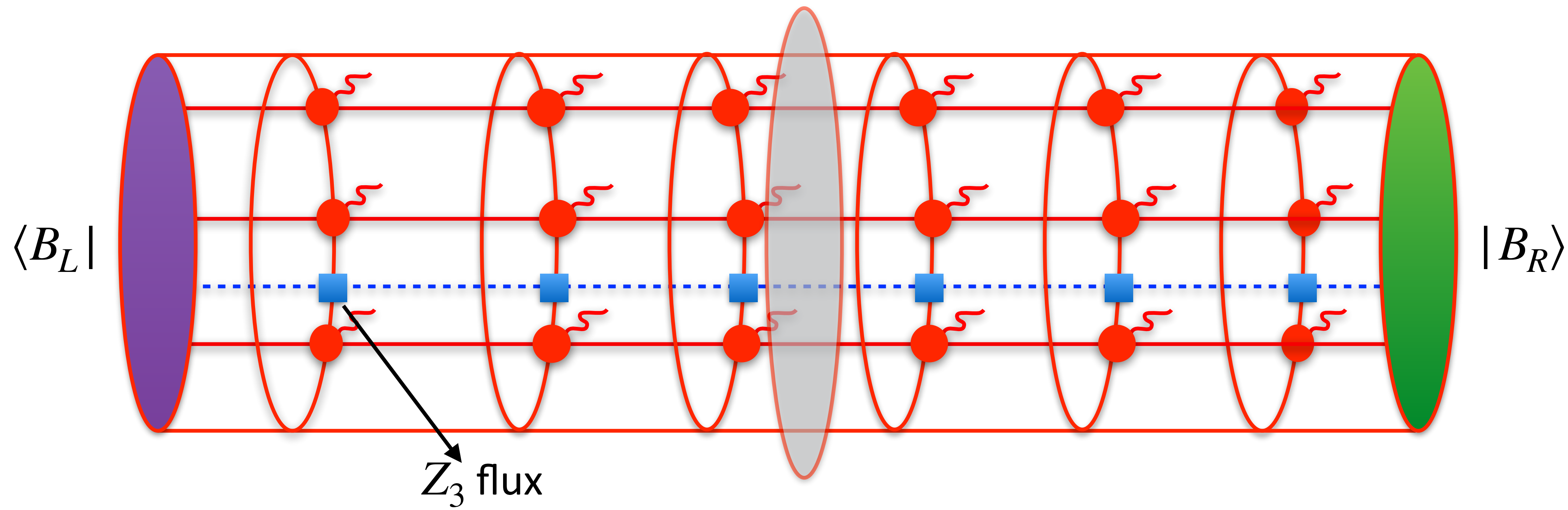




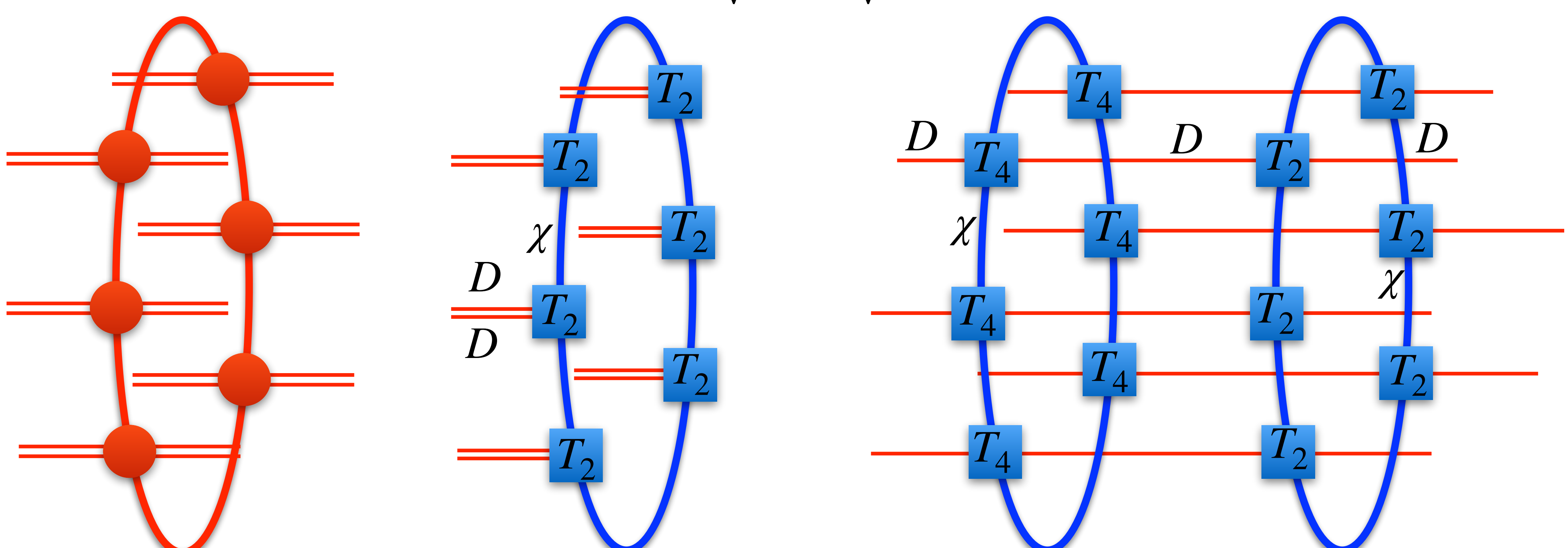
T. Nishino, K. Okunishi (1996); R. Orus, G. Vidal (2009); P. Corboz, et al (2014)

Entanglement spectrum: setup

Z_3 gauge symmetry indicates 9 states on torus or infinite long cylinder.



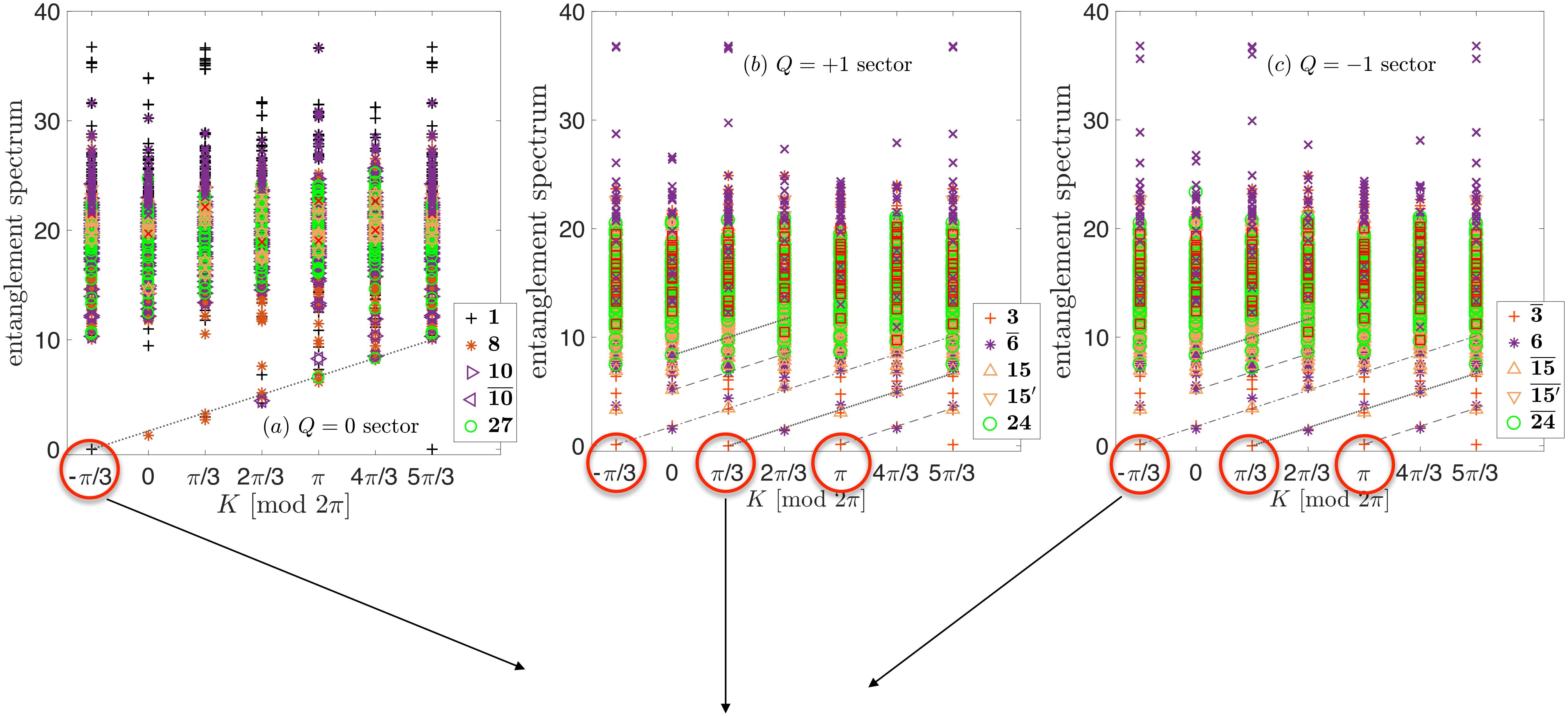
$$\rho_L = U\sqrt{\sigma_L}\sigma_R^T\sqrt{\sigma_L}U^\dagger$$



Li and Haldane (2008); J. I. Cirac, D. Poilblanc, N. Schuch, and F. Verstraete (2011)

Entanglement spectrum

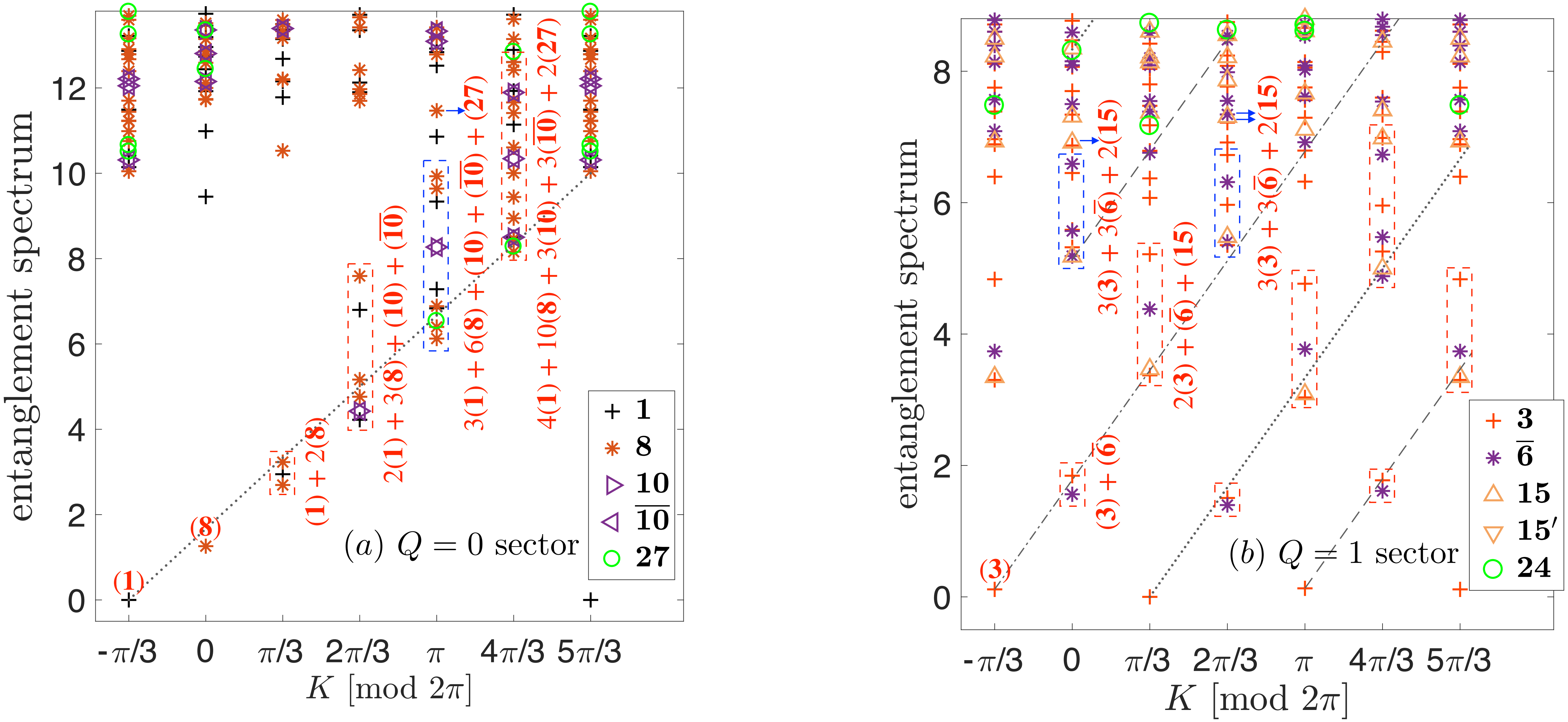
$\chi = 343$, full spectrum



Incommensurate with $N_{\mathrm{v}} = 3{,}9$

Entanglement spectrum

$\chi = 343$, low-energy level contents



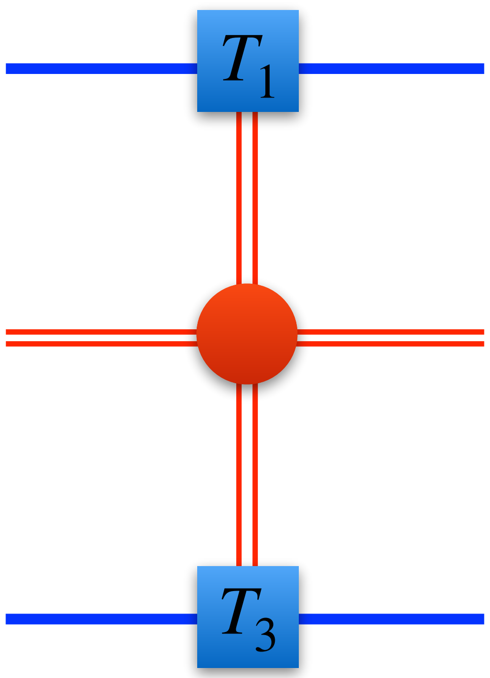
$SU(3)_1$ Wess-Zumino-Witten conformal field theory

Bulk correlation: transfer matrix spectrum

Transfer matrix spectrum: t_a $(a = 0,1,2,...)$

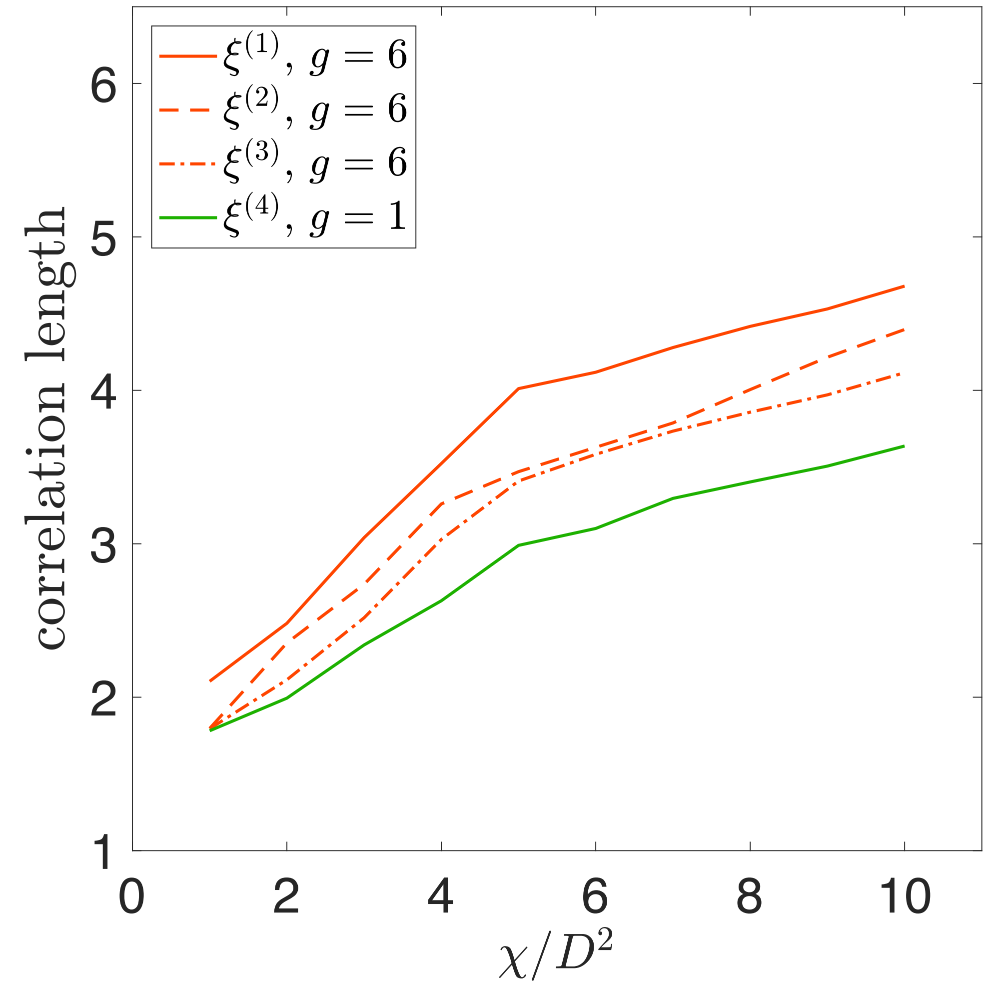
$$|t_0| > |t_1| > |t_2| > |t_3| > |t_4| > \ldots$$

singlet | 6-fold degenerate | singlet



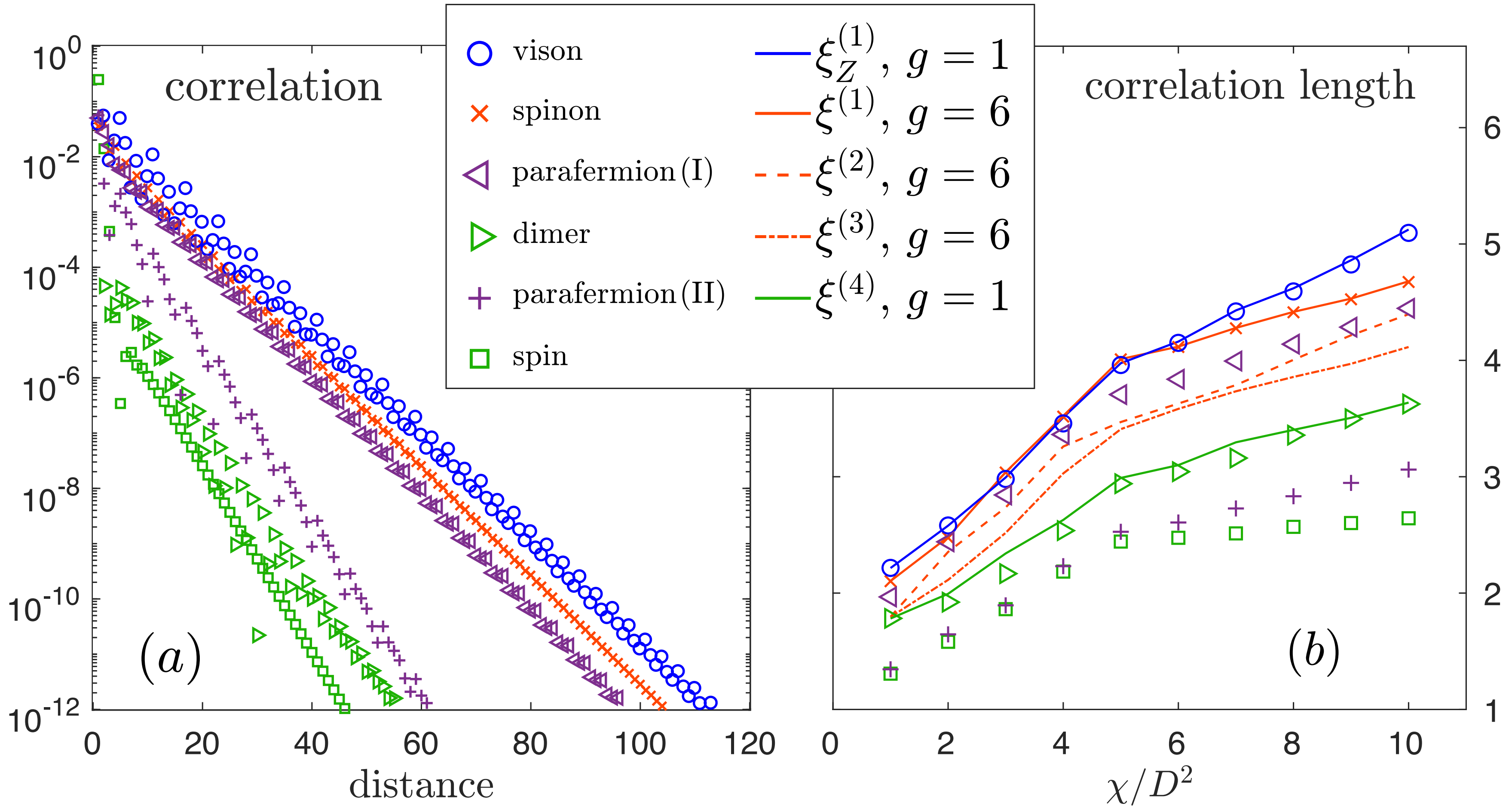
Correlation length:

$$\xi^{(a)} = -1/\ln(|t_a/t_0|),\ a = 1,2,3,...$$



Bulk correlation: different type of correlation

Various types of correlations can be computed with environment tensors:
spin, dimer (topologically trivial)
spinon, vison, and their bound state

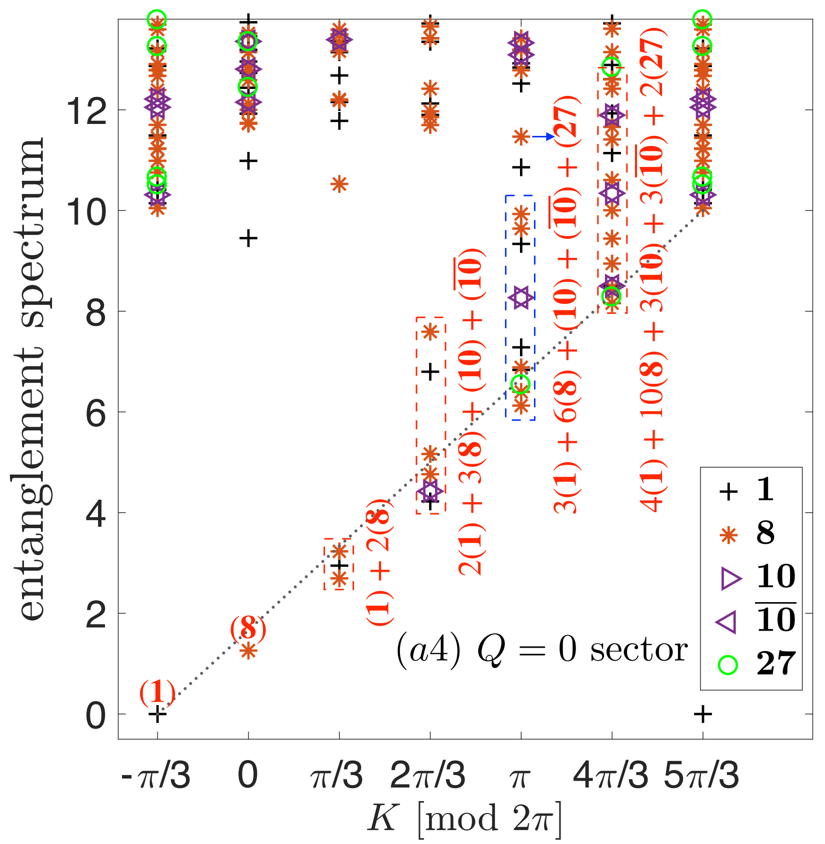


All correlation length, except ξ_{spin}, have no sign of saturation with increasing χ.

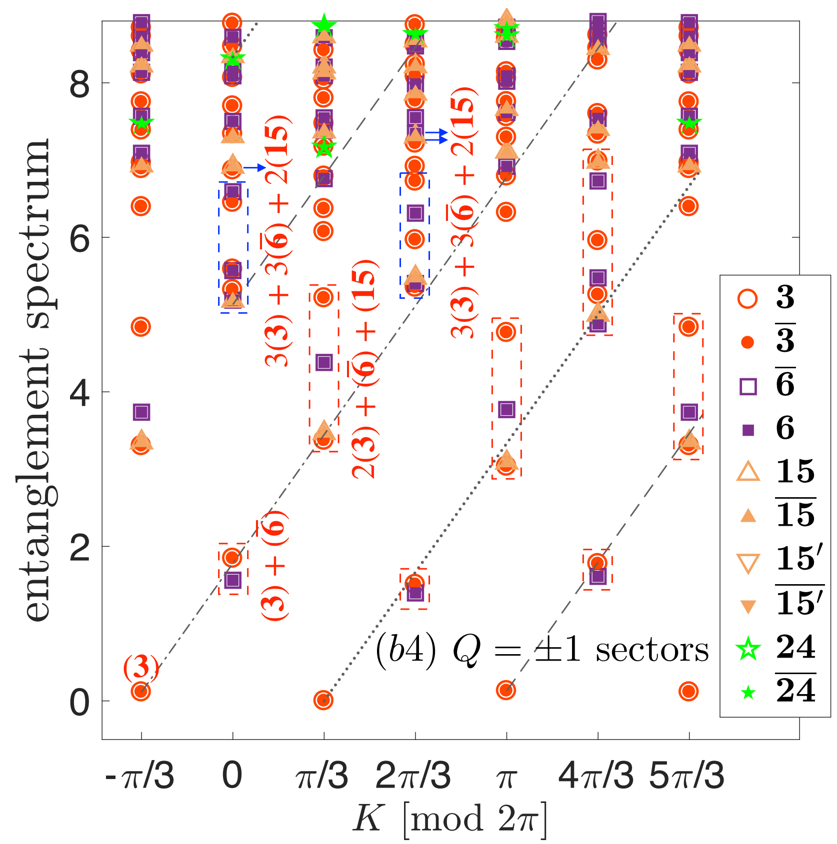
Degeneracy structure of topological chiral PEPS

Correspondence between transfer matrix spectrum and entanglement spectrum:

$t_0 : \mathbf{1}$



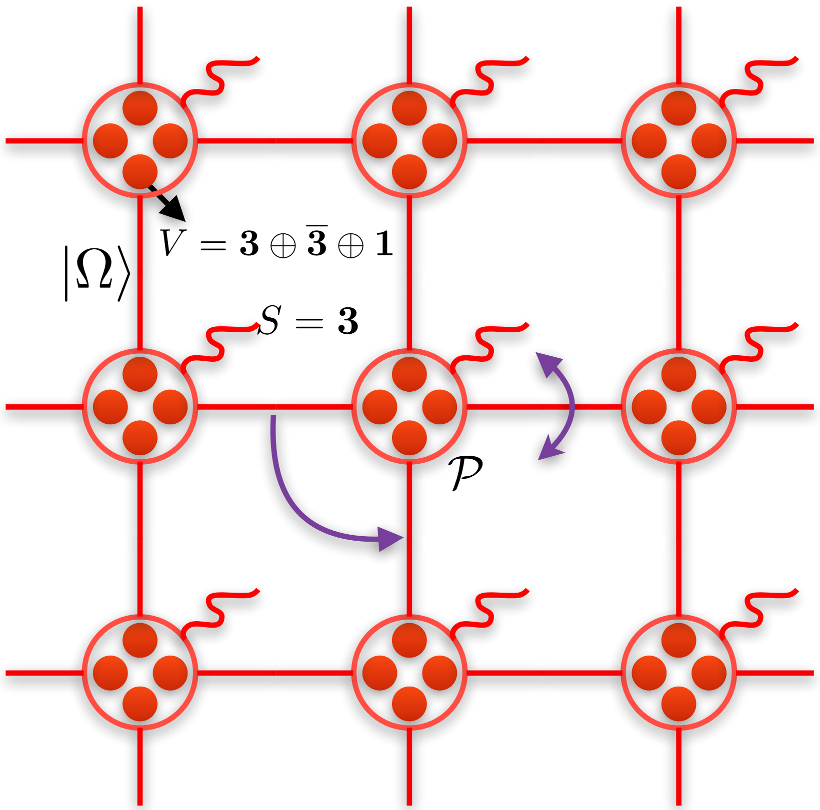
$t_1, t_2, t_3 : \mathbf{3} \oplus \overline{\mathbf{3}}$

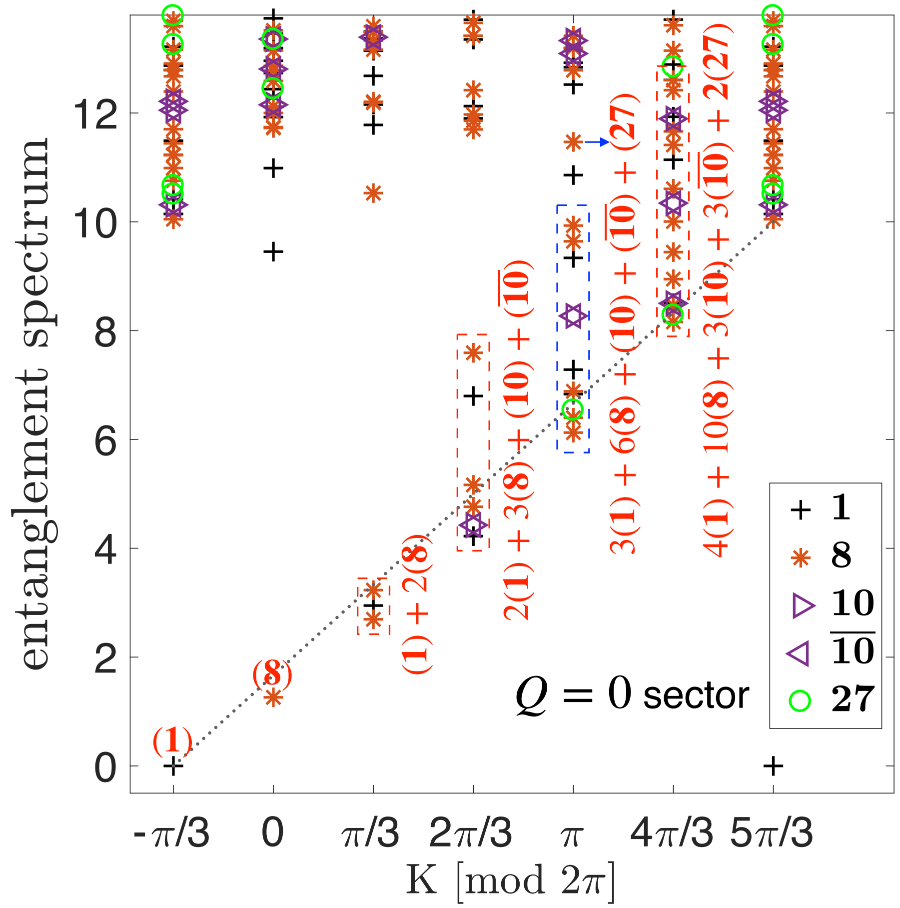


This degeneracy structure also holds in cases of chiral PEPS with $\mathrm{SU}(2)_1$ or $\mathrm{SU}(2)_2$ ES.

D. Poilblanc (2017); A. Hackenbroich, A. Sterdyniak, and N. Schuch (2018);
JYC, L. Vanderstraeten, S. Capponi, and D. Poilblanc (2018)

Conclusion





- A hamiltonian for $\mathrm{SU}(3)_1$ CSL on the square lattice is proposed.
- A representative PEPS wave function for $\mathrm{SU}(3)_1$ CSL is found, which appears to be critical.
- A correspondence between bulk correlation and entanglement spectrum is observed. It will be verified in further study of $\mathrm{SU}(N)_k$ CSL.

Open problem

- Statistics of anyon excitation in chiral phases is hard to extract from PEPS.

Collaborators

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