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Weak ergodicity breaking and quantum many-body scars

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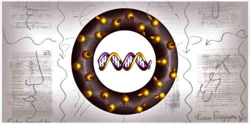

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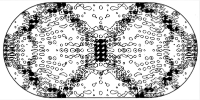
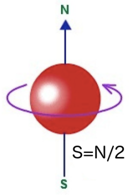
PRB 98, 155134
PRL 122, 220603

Outline

1. Introduction
2. Phenomenology of weak ergodicity breaking
3. Quantum scars and quasi-modes
4. Stabilising a hidden large spin
5. Wider context and conclusions





Quantum dynamics and thermalisation

Quantum ergodicity: from a non-equilibrium initial condition $|\psi\rangle$ local observable expectation values converge on thermal values as if

$$U(t) : |\psi\rangle \mapsto \rho \sim \mathrm{e}^{-\beta H}.$$

As a consequence, eigenstates should look *typical* for their energy (eigenstate thermalisation).

Exceptions:-

- Non-interacting,
- integrable,
- and many-body localised systems.

$\Bigg\}$ Generically fails to equilibrate. Ergodicity is completely broken.

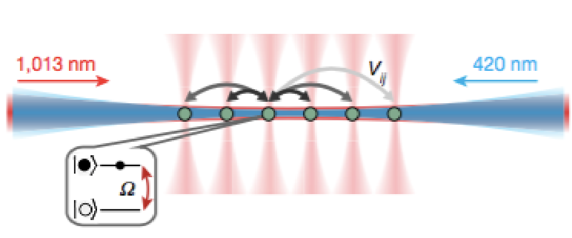
What would be weak ergodicity breaking?

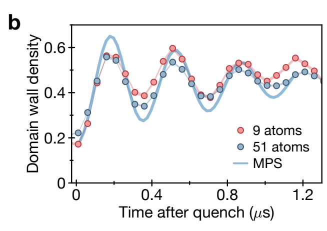
• Sensitivity to initial conditions, • some atypical eigenstates.

Experimental surprise!

Bernien *et al.* Nature 551(7682), 579 (2017)

A surprising finding in a Rydberg atom chain experiment!





Each atom has two states — a ground state $\circ$ and an excited Rydberg state $\bullet$.

If the interaction energy is made very large, then we get the Rydberg blockade where adjacent excited atoms $\bullet\bullet$ are forbidden.

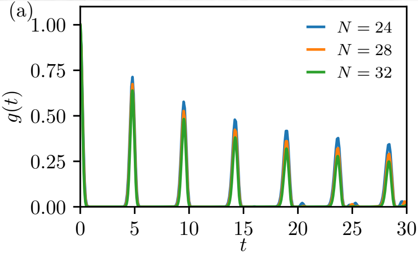
Starting from a Rydberg crystal state, $|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\cdots\rangle$, robust oscillations were observed, but other initial conditions thermalise much more rapidly.

Effective PXP model

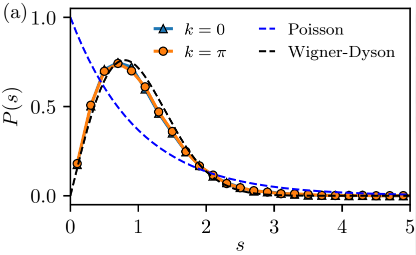
In the blockade regime, the effective Hamiltonian is

$$H = \sum_{j=1}^{N} P_{j-1} X_j P_{j+1}. \qquad \circ\circ\circ \leftrightarrow \circ\bullet\circ \text{(c.f. Toffoli gate)}$$

Oscillations in local observables is reflected in periodic quantum revival in the many-body wavefunction. This is shown with the fidelity $g(t)$.



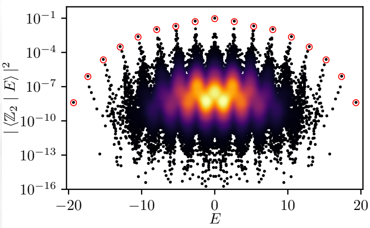
The level statistics is Wigner-Dyson. This rules out the previously known examples of non-ergodic quantum systems.

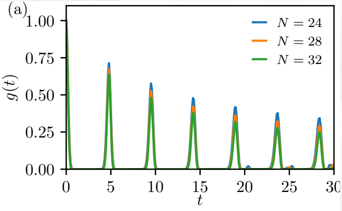


What's going on?

Turner *et al.* Nat. Phys. 14, 745 (2018)
Turner *et al.* PRB 98, 155134 (2018)

From quantum revival to special eigenstates



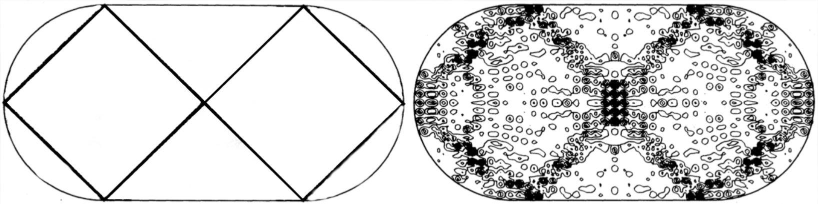


- A special subspace spanned by a small number of special eigenstates which account for most of the Néel state.
- These have approximately equally spaced eigenvalues, and converging with system size.
- Explains the oscillatory dynamics.

So, what's behind these atypical eigenstates?
Where else have we seen something like this?

Quantum scars for quantum billiards

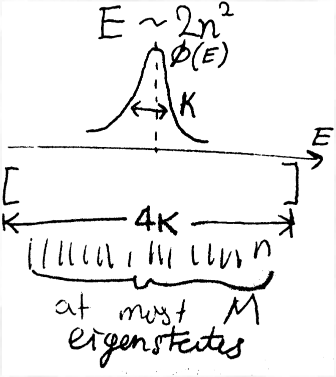
Heller PRL 53(16), 1515 (1984)



- Unstable periodic orbits of the chaotic classical billiards (left) imprint upon a wavefunction (right) after quantisation.
- This is surprising! One might expect unstable classical period orbits to be lost in the transition to quantum mechanics as the particle becomes “blurred”.

How can we understand what happens in this system?

Quasi-modes and scarred eigenstates



Approximate eigenfunctions of the form $\phi = \chi(x) \sin(ny)$ for suitable χ [O'Connor and Heller 1988]. These are like "bouncing ball" trajectories. But, how to relate this to the exact eigenstates?

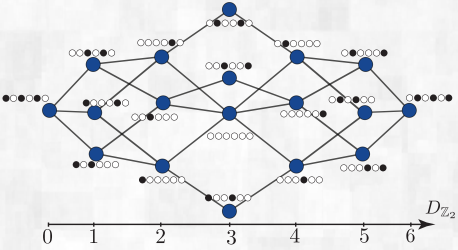
If the quasi-mode has an energy variance K^2 and there's at most M eigenstates in a $4K$ interval around the quasi-mode, then there exists eigenstates with anomalously large overlap [Zelditch 2004].This is the corresponding scarred eigenstate.

Constructing the quasi-modes was simple enough, but showing that the density of states is non-pathological was *much* more challenging [Hassell 2010].

Could we follow this recipe for our many-body system?

Quasi-modes for the PXP model

Split the Hamiltonian into two parts, $H = H_+ + H_-$, where H_+ is a raising operator for the Hamming distance $D_{\mathbb{Z}_2}$ from the initial state. This forms a ladder algebra somewhat like a large spin.

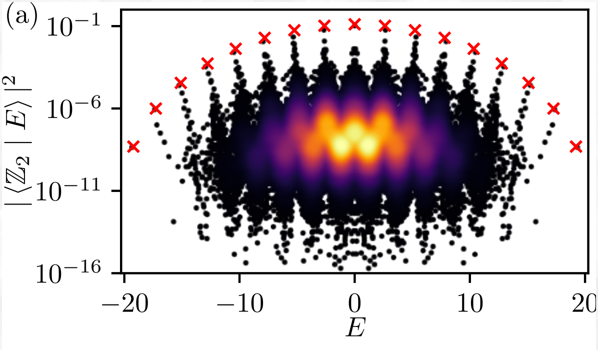


Using H_+ we can generate a low-dimensional Krylov subspace

$$\mathcal{K} = \mathrm{span}\left\{ (H_+)^k \left|\mathbb{Z}_2\right\rangle \right\} \qquad\qquad H \mapsto \mathcal{K}^\dagger H \mathcal{K} \qquad (1)$$

and produce approximate eigenstates. This entire construction is solvable in $\mathrm{poly}(N)$ time.

Quasi-modes for the PXP model

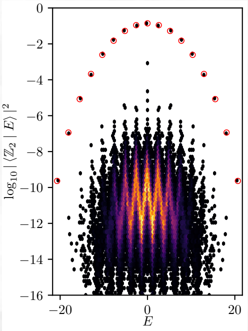


The crosses are the quasi-modes. From these we can approximate expectations values for eigenstates and for time evolution from a Néel state.

Could we deform the Hamiltonian to straighten-out the spin?

Deformed Hamiltonian: enhanced revivals

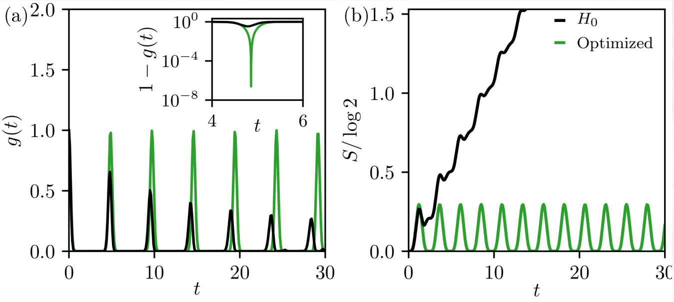
Choi *et al.* PRL 122, 220603 (2019)



We can improve the quantum revivals by adding a perturbation (c.f. Khemani *et al.* 2018),

$$H = H_0 + \sum_d h_d P_{j-1} X_j P_{j+1} (Z_{j+d} + Z_{j-d}),$$

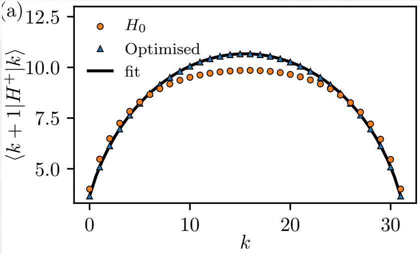
where $h_d \approx c\phi^{-2d}$. Most of the effect is obtained by adding just the $d = 2$ correction.



Deformed Hamiltonian: effective large spin

Choi *et al.* PRL 122, 220603 (2019)

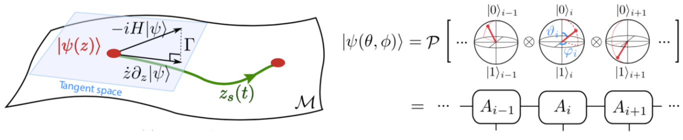
We can look at matrix elements of H_+ and $H^z = [H_+, H_-]$ and compare against predictions for an SU(2) representation.



- The quasi-modes have become essentially exact eigenstates – the subspace energy variance is around 10^{-7} for $N = 32$ sites.
- Outside of the space of scarred states, the dynamics remains complicated. Level statistics remains WD.

Wider context: PXP model

- TDVP dynamics for PXP model. Periodic orbits in the Hamiltonian flow on the variational manifold suggested to be analogous to the periodic orbits of classical billiard systems. [Ho *et al.* PRL 2019] and [Michailidis *et al.* PRX 2020].



- A couple of exact highly-excited eigenstates have been constructed [Lin *et al.* PRL 2019]. This proves PXP is not fully ergodic, but is insufficient to explain the experiment.

Wider context: other models

- Constructing models with atypical eigenstates through a method of embedding. [Shiraishi *et al.* PRL 2017] Many examples by now [1,2,3,...].
- Exact atypical eigenstates have been discovered in the AKLT model. [Moudgalya *et al.* PRB 2018]
- Also interesting are models of 'fractons' in 1D (effects of charge-and-dipole conservation). There is a 'fracturing' of the Hilbert space beyond of local symmetry classification. [Sala *et al.* arXiv:1904.04266] and [Khemani *et al.* arXiv:1904.04815]

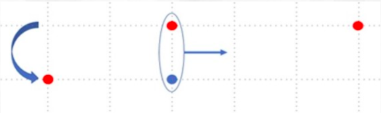


Diagram from M. Pretko's thesis.

Summary and outlook

A strange many-body system which displays a new kind of ergodicity breaking. An analogy:–

Quantum many-body scar	$\leftrightarrow$	Single particle quantum scar
Quantum revival	$\leftrightarrow$	Oscillatory wavepackets
Atypical eigenstates	$\leftrightarrow$	Scarred wavefunctions
TDVP dynamics regular trajectories	$\leftrightarrow$	Classical periodic orbits
$\mathcal{K}$-subspace quasimodes	$\leftrightarrow$	'bouncing-ball' quasimodes

- How generic? Necessary or sufficient conditions.
- Can we find a precise quantum–classical correspondence? Hint: Yes! [Desaules *et al.* Benasque poster 2020]