Phase diagram for two SU(4)-fermions on the square lattice with NN interaction

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arXiv:2002.05572 from today!

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Special thanks:
P. Nataf

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Model for two SU(4) fermions

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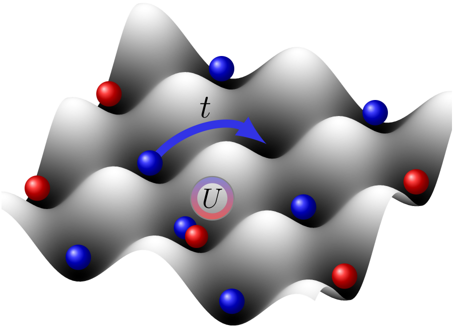
Phase diagram

SU(N) physics

- Generalization of SU(2) spin physics
- SU(4): 4 colors for each particle
- 2 fermions per site: irrep **6**

$$\left.\begin{array}{l} |AB\rangle - |BA\rangle \\ |AC\rangle - |CA\rangle \\ |AD\rangle - |DA\rangle \\ |BC\rangle - |CB\rangle \\ |BD\rangle - |DB\rangle \\ |CD\rangle - |DC\rangle \end{array}\right\} \mathbf{6}$$



Cazalilla and Rey, RPP **77** (2014)
Hofrichter et al., PRX **6** (2016)

Previously in Benasque

PHYSICAL REVIEW B **99**, 241112(R) (2019)

Rapid Communications

SU(4) topological resonating valence bond spin liquid on the square lattice

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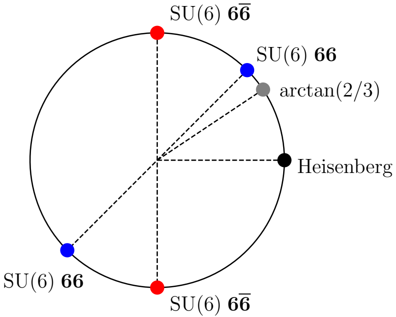
- constructed generalized RVB QSL for this system
- probed $\mathbb{Z}_2$ topological order
- our goal: find a Hamiltonian that stabilizes it

Nearest neighbor Hamiltonian

fusion rule: $\mathbf{6} \otimes \mathbf{6} = \mathbf{1} \oplus \mathbf{15} \oplus \mathbf{20}$

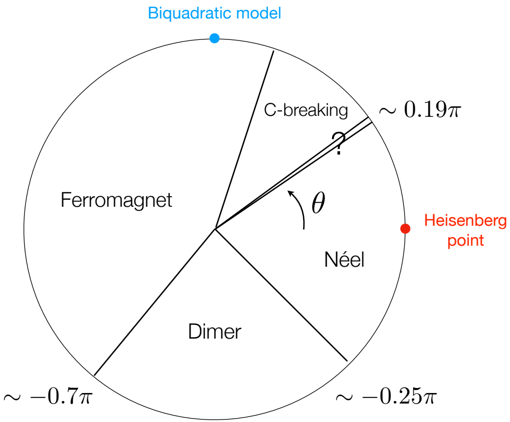
$$\mathcal{H}(\theta) = \cos\theta \mathbf{S}\cdot\mathbf{S} + \frac{\sin\theta}{4}(\mathbf{S}\cdot\mathbf{S})^2$$

$$\begin{array}{rc}
\theta = 0: & \mathcal{H} = \mathbf{S}\cdot\mathbf{S} \\
\theta = \arctan(2/3): & \mathbf{1} \leftrightarrow \mathbf{15} \\
\theta = \pi/4: & \mathbf{6}\otimes\mathbf{6} = \mathbf{15}\oplus\mathbf{21}, \mathrm{AFM} \\
\theta = \pi/2: & \mathbf{6}\otimes\overline{\mathbf{6}} = \mathbf{1}\oplus\mathbf{35}, \mathrm{FM} \\
\theta = -3\pi/4: & \mathbf{6}\otimes\mathbf{6} = \mathbf{15}\oplus\mathbf{21}, \mathrm{FM} \\
\theta = -\pi/2: & \mathbf{6}\otimes\overline{\mathbf{6}} = \mathbf{1}\oplus\mathbf{35}, \mathrm{AFM}
\end{array}$$



Affleck et al., NPB **366** (1991)

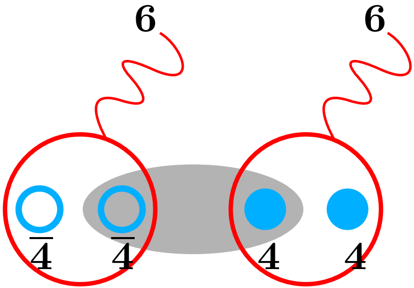
Previous work



Harada et al., PRL **90** (2003)
Paramekanti & Marston, JoP **19** (2007)
Kim et al., PRB **96** (2017)
Kim et al., PRB **100** (2019)

C-breaking phase?

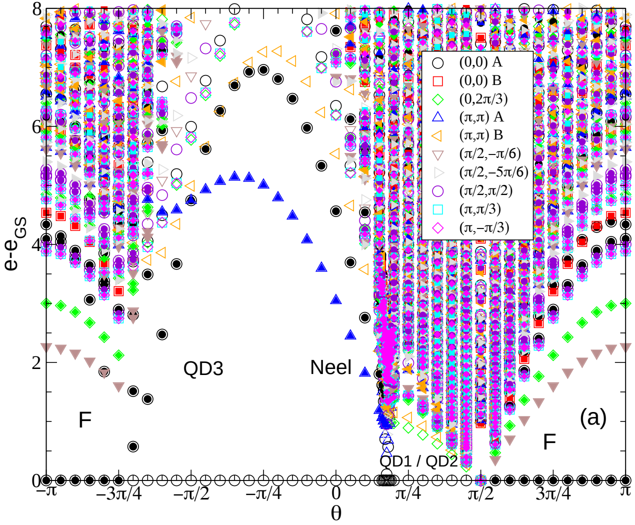
- SU(2): C is π-rotation
- SU(4): non-trivial C, $\mathbf{4} \leftrightarrow \overline{\mathbf{4}}$
- $\mathbf{6}$ is self-conjugate



exact ground state 1D for
$\theta = \arctan(2/3)$

Apply C on B-sites: translation invariant MPS for translation breaking wavefunction!

ED results

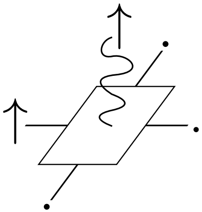


$N = 12$ sites, PBC

FM transition at SU(6) points $\theta = \pi/2, -3\pi/4$

SU(N) symmetric tensors

- Choose physical space S and virtual space V
- impose S **and** V to be SU(N) representations
- S has to appear in $V^{\otimes 4}$
- also impose C_{4v} symmetry
- describe QSL phases



Ex: RVB tensor

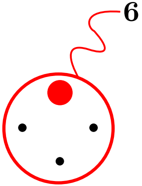
$$S = 1/2$$
$$V = 1/2 \oplus 0$$

$$A_{\uparrow}[\uparrow, 0, 0, 0] = 1$$
$$A_{\downarrow}[\downarrow, 0, 0, 0] = 1$$

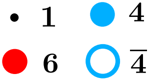
Mambrini et al., PRB **94** (2016)

PEPS Ansätze

$$V = \mathbf{6} \oplus \mathbf{1}$$

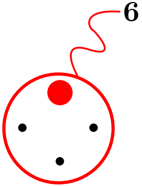


T_0

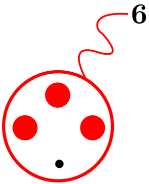


PEPS Ansätze

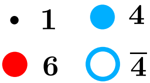
$$V = \mathbf{6} \oplus \mathbf{1}$$



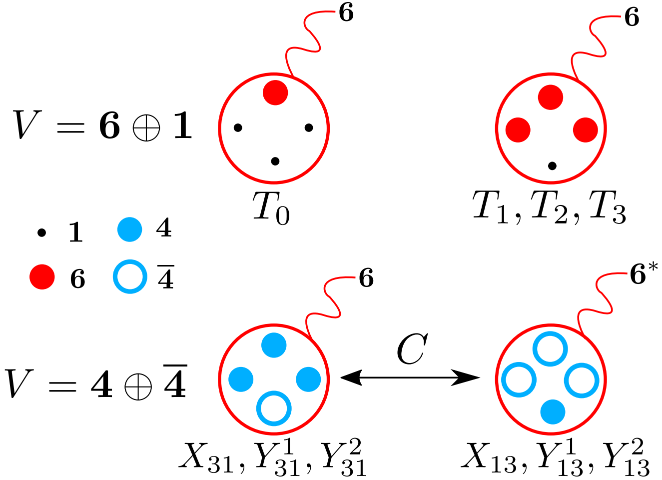
T_0



T_1, T_2, T_3



PEPS Ansätze



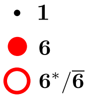
$$A = T_0 + a_1 T_1 + a_2 T_2 + i a_3 T_3$$

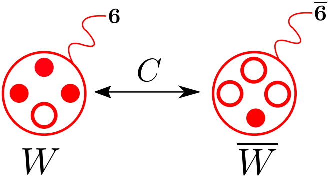
$$A = X_{31} + \alpha X_{13} + i\beta^j_{31} Y^j_{31} + i\beta^j_{13} Y^j_{13}$$

SU(4) & SU(6) symmetric PEPS

Principle: identify SU(4) and SU(6) irreps
$\rightarrow T_0$ is SU(6) $\mathbf{6}-\overline{\mathbf{6}}$-symmetric

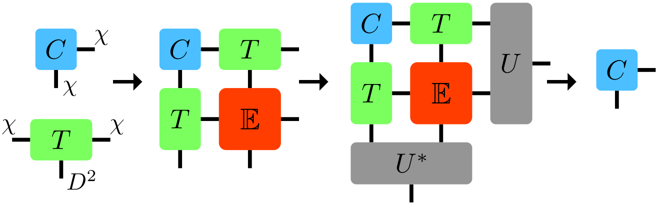
$$V = \mathbf{6} \oplus \overline{\mathbf{6}} \oplus \mathbf{1}$$





- SU(4): transaltion invariant, self-conjugate $\mathbf{6} = \mathbf{6}^*$
- SU(6): staggered $\mathbf{6} - \overline{\mathbf{6}}$
- restrain to W_1: virtual leg permutation (S_4) invariant
- SU(6) **66** requires D=43

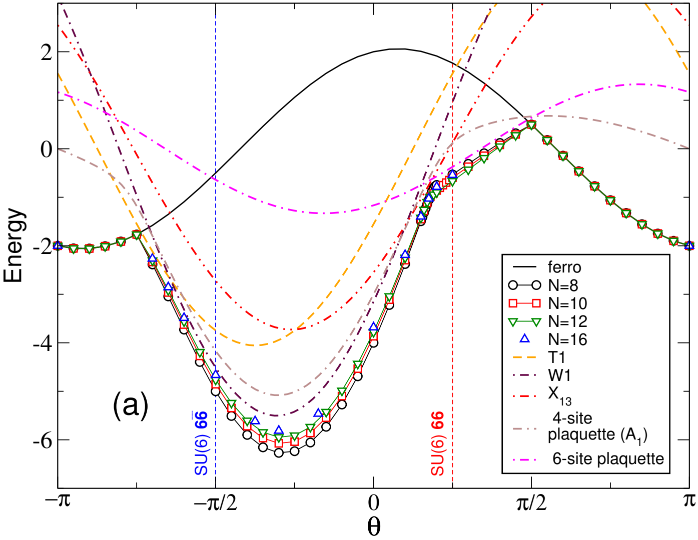
TN algorithm



- use CTMRG to converge environment
- implement $\mathbb{Z}_2^N$ symmetry to speed-up
- optimize coefficients with finite difference gradient
- extrapolate corner dimension $\chi \to \infty$
- compare with DMRG and ED

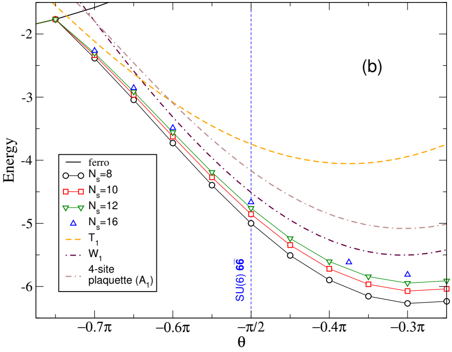
Nishino and Okunishi, JPSP **65** (1996)
Orús and Vidal, PRB **80** (2009)

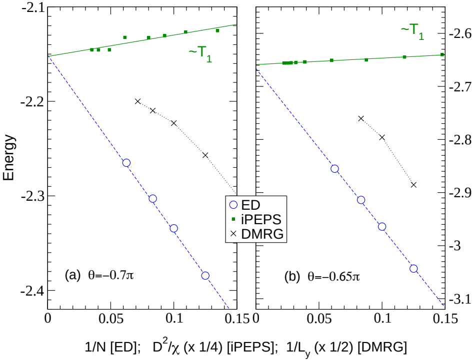
Energies



Energy
θ
−π
−π/2
0
π/2
π
2
0
-2
-4
-6
(a)
SU(6) $\overline{66}$
SU(6) 66
ferro
N=8
N=10
N=12
N=16
T1
W1
X_{13}
4-site plaquette (A_1)
6-site plaquette

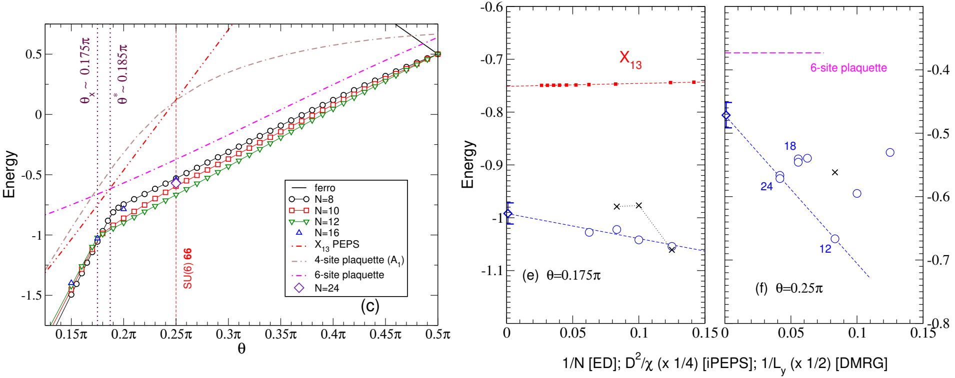
Energies $\theta \sim -0.7\pi$



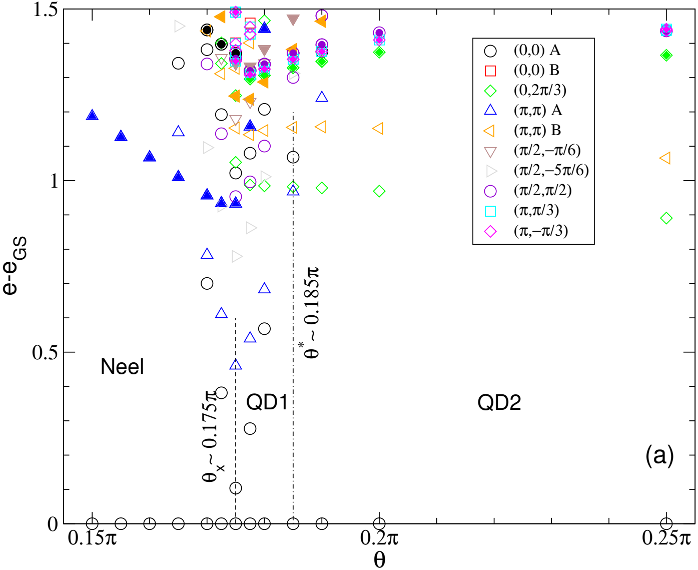


Excellent agreement ED, PEPS and DMRG
Ansatz is better than dimerized state

Energies $\theta \sim 0.18\pi$

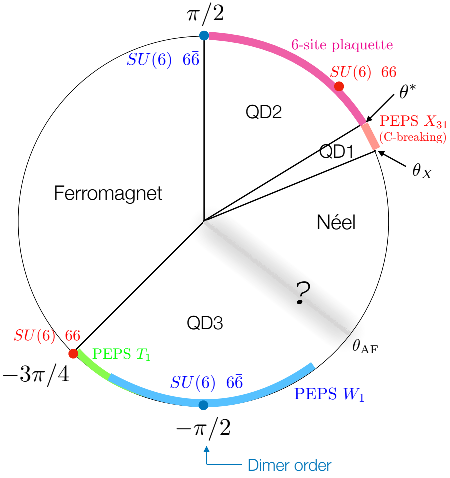


ED $\theta \sim 0.18\pi$



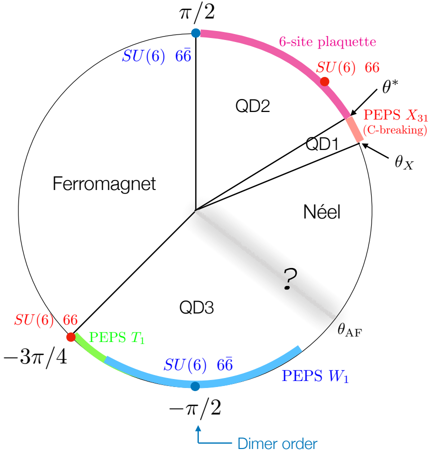
Phase diagram

- $\theta \sim 0$: AFM
- $\theta \sim 0.18\pi$: C-breaking (?)
- $\sim 0.18\pi < \theta < \pi/2$: 6-sites plaquette
- $\pi/2 < \theta < 5\pi/4$: FM
- $-5\pi/4 < \theta < \theta_{AF}$: quantum disordered, dimer/QSL

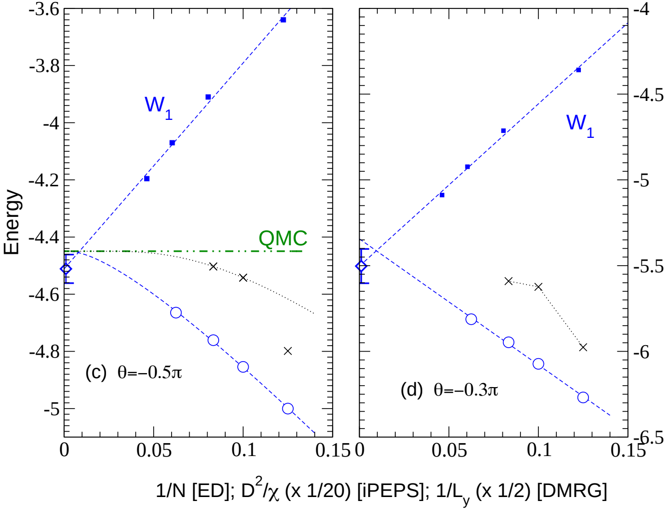


Conclusion

- use ED, PEPS and DMRG to investigate two SU(4) fermions NN Hamiltonian
- symmetric PEPS give excellent variational energies with very few parameters
- expect QSL phase for $\theta \sim -0.7\pi$ ($\neq$ SU(2))



W_1 scaling



Energy
-3.6
-3.8
-4
-4.2
-4.4
-4.6
-4.8
-5
W1
QMC
(c) θ=−0.5π
-4
-4.5
-5
-5.5
-6
-6.5
W1
(d) θ=−0.3π
0
0.05
0.1
0.15
1/N [ED]; D²/χ (x 1/20) [iPEPS]; 1/Ly (x 1/2) [DMRG]