Classifying topological many-body localized phases

Thorsten B. Wahl

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Benasque, 20.02.2020

European Commission

Marie Skłodowska-Curie Actions

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Amos Chan, and Thorsten B. Wahl, arXiv:1808.05656 (→ J. Phys. Cond. Mat.)
Zheyu Li, Amos Chan, and Thorsten B. Wahl, arXiv:1908.03928.
Thorsten B. Wahl, and Benjamin Béri, arXiv:2001.03167.

Many-body localization in one dimension

Sufficiently strong disorder in 1D ⇒ ergodicity breaking:

Many-body localization (MBL)

D. Basko, I. Aleiner, and B. Altshuler, Ann. Phys. **321**, 1126 (2006).

I. Gornyi, A. Mirlin, and D. Polyakov, Phys. Rev. Lett. **95**, 206603 (2005).

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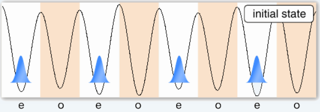

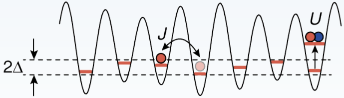

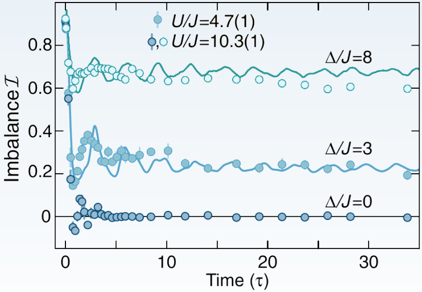

taken from: M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Science **349**, 842 (2015)

Many-body localization (MBL)

Disordered Heisenberg antiferromagnet: MBL for $W > W_c \approx 3.5J$

$$H = \sum_{i=1}^{N} (J\,\mathbf{S}_i \cdot \mathbf{S}_{i+1} + h_i S_i^z), \qquad h_i \in [-W, W]$$

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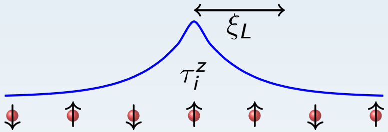


Local integrals of motion (LIOM):

$$H = U H_{\mathrm{diag}} U^\dagger$$
$$\tau_i^z = U \sigma_i^z U^\dagger$$
$$[H, \tau_i^z] = [\tau_i^z, \tau_j^z] = 0$$

M. Serbyn, Z. Papić, and D. A. Abanin, Phys. Rev. Lett. **110**, 260601 (2013)

D. A. Huse, and V. Oganesyan, Phys. Rev. B **90**, 174202 (2014)

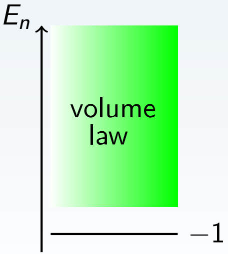
Topological Many-body Localized Phases

Cluster model:

Clean system

$$H = \sum_{j=1}^{N} \left(\sigma_{j-1}^{x} \sigma_{j}^{z} \sigma_{j+1}^{x} + h \sigma_{j}^{z} \right)$$

topological index: $ww^* = \pm \mathbb{1}$



Y. Bahri, R. Vosk, E. Altman and A. Vishwanath, Nat. Commun. **6**, 7341 (2015)
K. S. C. Decker, D. M. Kennes, J. Eisert, and C. Karrasch, Phys. Rev. B **101**, 014208 (2020)

Topological Many-body Localized Phases

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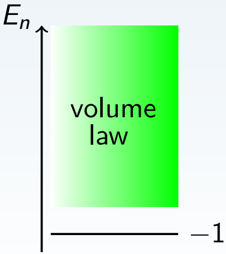

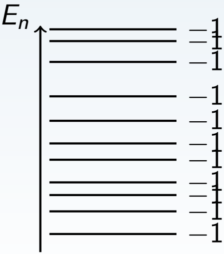

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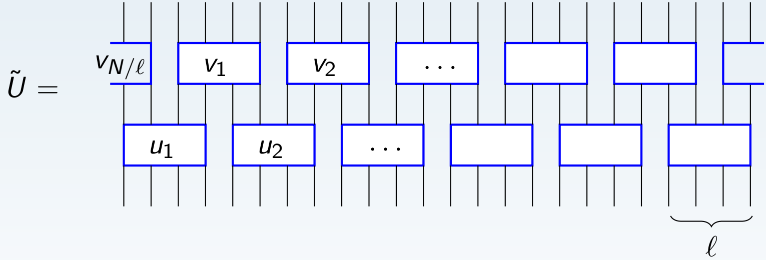
Representation by Quantum Circuits

Goal

$\tilde{U} H \tilde{U}^\dagger \approx$ diagonal matrix



$$\text{error} \sim e^{-\ell/\xi_L}$$

F. Pollmann, V. Khemani, J. I. Cirac, and S. L. Sondhi, Phys. Rev. B **94**, 041116 (2016),

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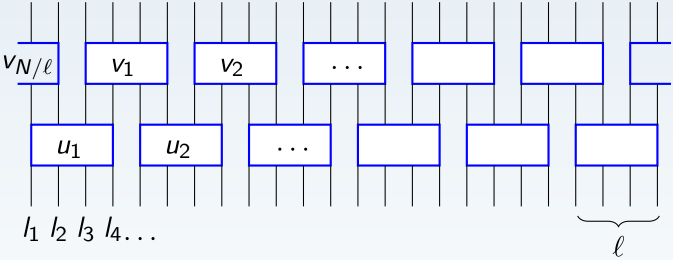
$|\tilde{\psi}_{I_1 \dots I_N}\rangle =$



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Ground states	All eigenstates
Translationally invariant, gapped	Disordered, many-body localized

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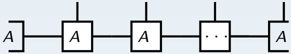
$\vert\tilde{\psi}\rangle =$ 	
Bond dimension D	

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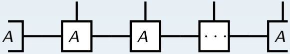
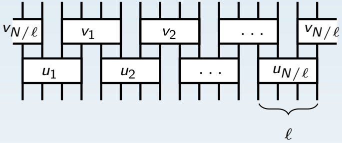
$\|\tilde{\psi}\rangle =$ 	$\tilde{U} =$ 
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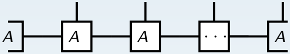
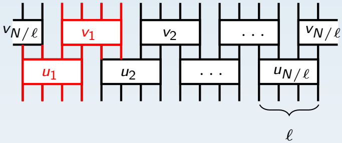
$\vert\tilde{\psi}\rangle =$ 	$\tilde{U} =$ 
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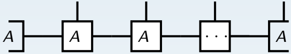
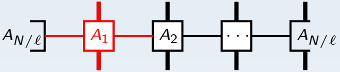
$|\tilde{\psi}\rangle =$ 	$\tilde{U} =$ 
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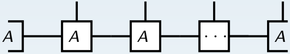
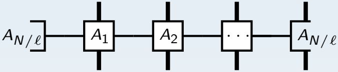
$\lvert\tilde{\psi}\rangle =$ 	$\tilde{U} =$ 
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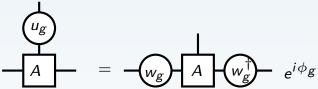
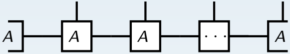
Symmetry $g \in G$: $H = u_g^{\otimes N} H\, u_g^{\dagger \otimes N}$	
	

MPS

Ground states

Translationally invariant, gapped

$\lvert\tilde{\psi}\rangle =$



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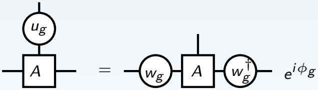
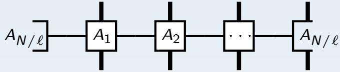


Quantum Circuit

All eigenstates

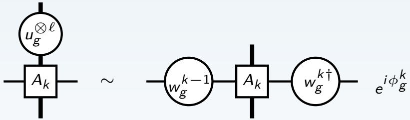
Disordered, many-body localized

$\tilde{U} =$



Length of unitary gates ℓ ($D = 2^{\ell/2}$)

Abelian symmetry group G : $H = u_g^{\otimes N} H\, u_g^{\dagger \otimes N}$



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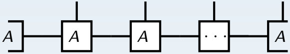
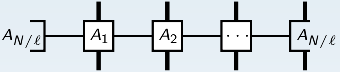
$\vert\tilde{\psi}\rangle =$ 	$\tilde{U} =$ 
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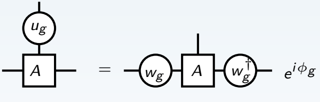	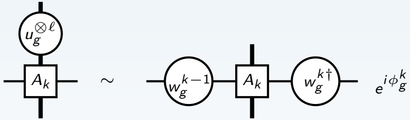
$w_g w_h = w_{gh} e^{i\beta(g,h)}$	
$w'_g = w_g e^{i\chi_g} \Rightarrow \beta'(g,h) = \beta(g,h) - \chi_{gh} + \chi_g + \chi_h$	
$\vert\psi\rangle$: second cohomology group $H^2(G, U(1))$	

N. Schuch, D. Pérez-García, and J. I. Cirac, Phys. Rev. B **84**, 165139 (2011)

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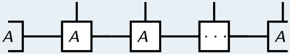
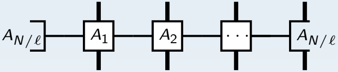
$\vert\tilde{\psi}\rangle =$ 	$\tilde{U} =$ 
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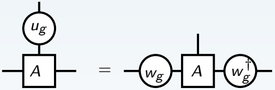
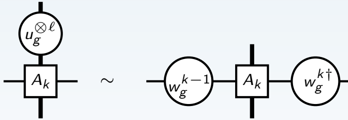
 $e^{i\phi_g}$	 $e^{i\phi_g^k}$
$w_g w_h = w_{gh} e^{i\beta(g,h)}$	$w_g^k w_h^k = w_{gh}^k e^{i\beta(g,h)}$
$w_g' = w_g e^{i\chi_g} \;\Rightarrow\; \beta'(g,h) = \beta(g,h) - \chi_{gh} + \chi_g + \chi_h$	$w_g^{k\prime} = w_g^k e^{i\chi_g} \;\Rightarrow\; \beta'(g,h) = \beta(g,h) - \chi_{gh} + \chi_g + \chi_h$
$\vert\psi\rangle$: second cohomology group $H^2(G, U(1))$	$\tilde{U} \ni \vert\tilde{\psi}_{l_1 \dots l_N}\rangle$: second cohomology group $H^2(G, U(1))$

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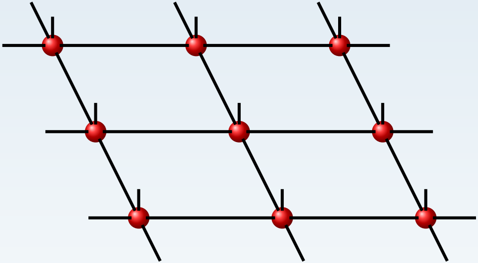
In two dimensions

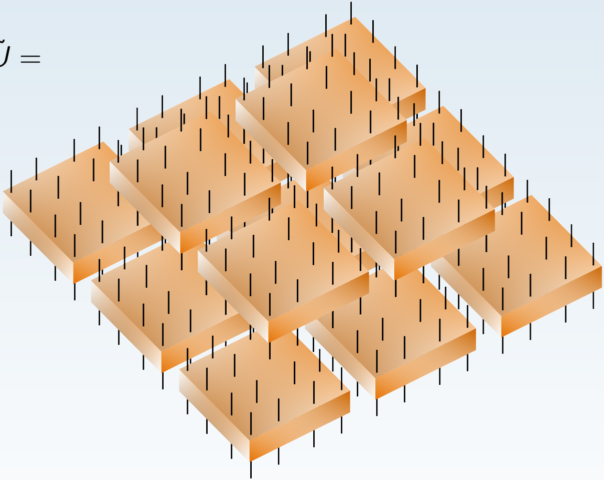
$$|\tilde{\psi}\rangle =$$

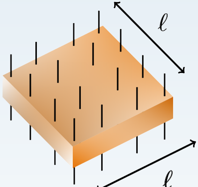


becomes ...

In two dimensions

$\tilde{U} =$





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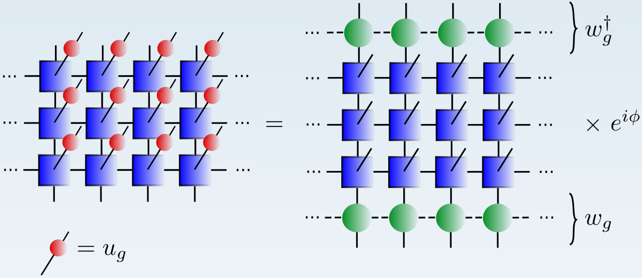
Classification of 2D symmetry-protected MBL phases



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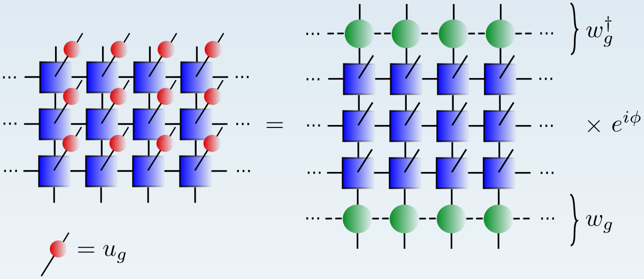

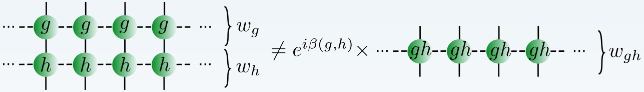

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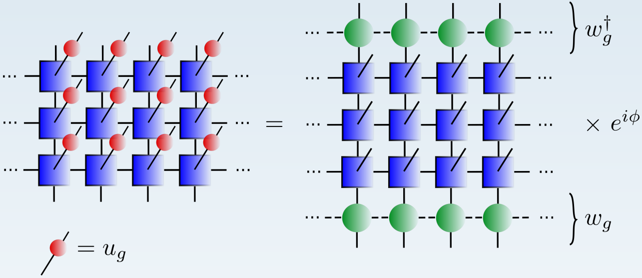
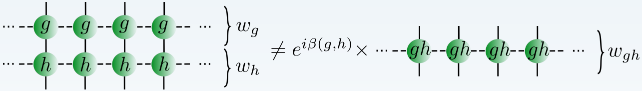
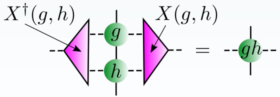




w_g injective $\Rightarrow$



classify complex phases of $X(g,h) \Rightarrow H^3(G, U(1))$

D. J. Williamson, N. Bultinck, M. Mariën, M. B. Şahinoğlu, J. Haegeman, and F. Verstraete, Phys. Rev. B **94**, 205150 (2016)

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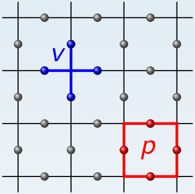
Topologically ordered ground states

Example: **Toric code**

$$H = -\sum_v A_v - \sum_p B_p$$

$$A_v = \prod_{i \in v} \sigma_i^x, \quad B_p = \prod_{i \in p} \sigma_i^z$$

$$[H, A_v] = [H, B_p] = [A_v, B_p] = 0$$



- four ground states on the torus: $|\psi_j\rangle, \; j = 1, 2, 3, 4$
- anyonic excitations
- cannot be connected to product state via local unitary U_{loc}: $|\psi_{\mathrm{prod}}\rangle \neq U_{\mathrm{loc}} |\psi_j\rangle$

Topologically ordered many-body localization

Example: **Random coupling toric code**

$$H = -\sum_v J_v A_v - \sum_p K_p B_p$$

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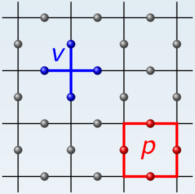

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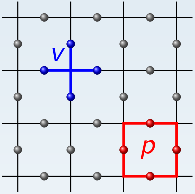


~~Local~~ integrals of motion:

$$H = U H_{\mathrm{diag}} U^\dagger$$

$$\tau_i^z = U \sigma_i^z U^\dagger$$

$$[H, \tau_i^z] = [\tau_i^z, \tau_j^z] = 0$$

Alternative choice:

$$S_i = A_v, B_p$$

$$\Rightarrow [H, S_i] = [S_i, S_j] = 0$$

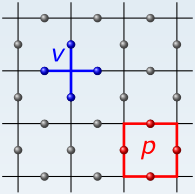
Topologically ordered many-body localization

Example: **Random coupling toric code + perturbation**

$$H = -\sum_v J_v A_v - \sum_p K_p B_p + h \sum_i \sigma_i^z$$

$$A_v = \prod_{i \in v} \sigma_i^x, \;\; B_p = \prod_{i \in p} \sigma_i^z$$

$$[H, A_v] = [H, B_p] = [A_v, B_p] = 0$$



~~Local~~ integrals of motion:

$$H = U H_{\mathrm{diag}} U^\dagger$$

$$\tau_i^z = U \sigma_i^z U^\dagger$$

$$[H, \tau_i^z] = [\tau_i^z, \tau_j^z] = 0$$

Alternative choice:

$$S_i = A_v, B_p$$

$$\Rightarrow [H, S_i] = [S_i, S_j] = 0$$

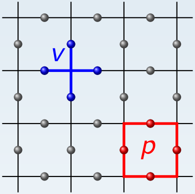
Topologically ordered many-body localization

Example: **Random coupling toric code + perturbation**

$$H = -\sum_v J_v A_v - \sum_p K_p B_p + h \sum_i \sigma_i^z$$

$$A_v = \prod_{i \in v} \sigma_i^x, \quad B_p = \prod_{i \in p} \sigma_i^z$$

$$[H, A_v] = [H, B_p] = [A_v, B_p] = 0$$



Topological local integrals of motion:

stabilizers S_i (Abelian, non-chiral)

J. C. M. de la Fuente, N. Tarantino, and J. Eisert, arXiv:2001.11516

$$T_i = U_{\mathrm{loc}} S_i U_{\mathrm{loc}}^\dagger$$

$$[H, T_i] = [T_i, T_j] = 0$$

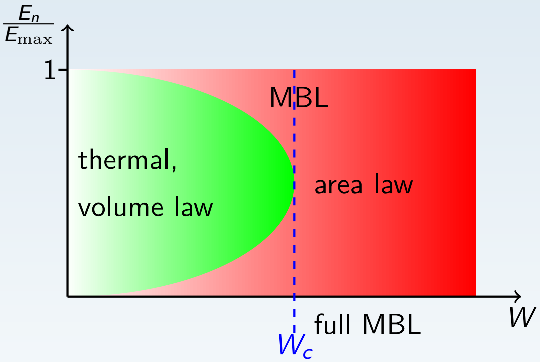
1. all eigenstates are in same topological phase
2. stable unless perturbations are strong enough to destroy MBL

Summary

- **SPT:** classification by second cohomology group in 1D ($\mathbb{Z}_2$-extension for fermions), classification by third cohomology group in 2D
- **top. order:** extended definition of local integrals of motion
- all eigenstates are in the same topological phase
- protected by MBL (and symmetry)

Thorsten B. Wahl, Phys. Rev. B **98**, 054204 (2018).
Amos Chan, and Thorsten B. Wahl, arXiv:1808.05656 (→ J. Phys. Cond. Mat.)
Zheyu Li, Amos Chan, and Thorsten B. Wahl, arXiv:1908.03928.
Thorsten B. Wahl, and Benjamin Béri, arXiv:2001.03167.

Mobility edge



D. J. Luitz, N. Laflorencie, and F. Alet, Phys. Rev. B **91**, 081103 (2015)

However:

W. De Roeck, F. Huveneers, M. Müller, and M. Schiulaz, Phys. Rev. B **93**, 014203 (2016)

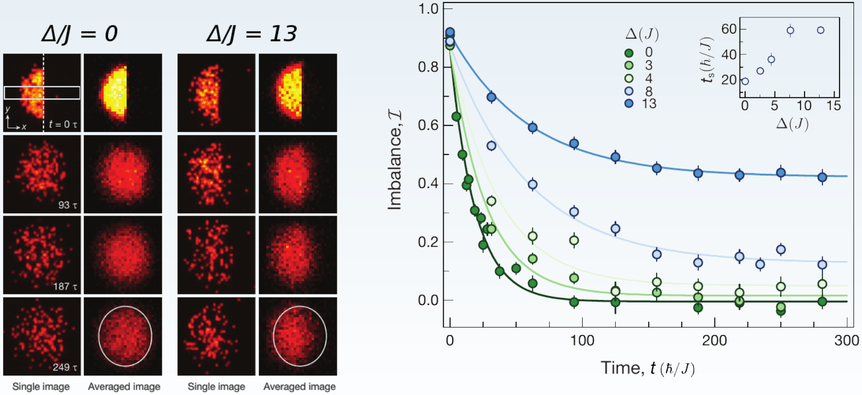
Many-body localization in higher dimensions?

Theralizing behavior in higher dimensions

W. De Roeck, J. Z. Imbrie, Phil. Trans. R. Soc. A **375**, 20160422 (2017).

See however: I.-D. Potirniche, S. Banerjee, and E. Altman, arxiv:1805.01475

But:



taken from: J.-y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, Science **352**, 1547 (2016).