TECHNISCHE UNIVERSITÄT DRESDEN

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SFB 1143

Tensor network representations of parton wave functions

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Y.-H. Wu, L. Wang & HHT, arXiv:1910.11011

Entanglement in Strongly Correlated Systems
Benasque, February 21st, 2020

Background

- Parton wave functions have been extensively used as variational ansatz in strongly correlated systems (e.g. high-T_c, quantum Hall, spin liquids).

Example: fermionic/bosonic representation for spin-1/2

$$\vec{S}_j = \frac{1}{2} \sum_{\alpha\beta=\uparrow,\downarrow} c^\dagger_{j\alpha} \vec{\sigma}_{\alpha\beta} c_{j\beta}$$

constraint: $\sum_{\alpha=\uparrow,\downarrow} c^\dagger_{j\alpha} c_{j\alpha} = 1$

$$|\psi\rangle = P_{\mathrm{G}} \prod_{|\mathbf{k}|<k_{\mathrm{F}}} \prod_{\alpha=\uparrow,\downarrow} c^\dagger_{\mathbf{k}\alpha} |0\rangle$$

$$|\psi\rangle = P_{\mathrm{G}} \exp\left[\sum_{j<l} g_{jl} (c^\dagger_{j\uparrow} c^\dagger_{l\downarrow} - c^\dagger_{j\downarrow} c^\dagger_{l\uparrow}) \right] |0\rangle$$

P_{G}: Gutzwiller projector (imposing the local constraint)

Background

- General construction: fermionic/bosonic Gaussian states subject to local projections

- Parton wave functions are determinant/Pfaffian/permanent wave functions (numerical technique: Variational Monte Carlo).

Advantage:
- Physically motivated (with very few parameters)
- (Possible) connection to low-energy effective theory

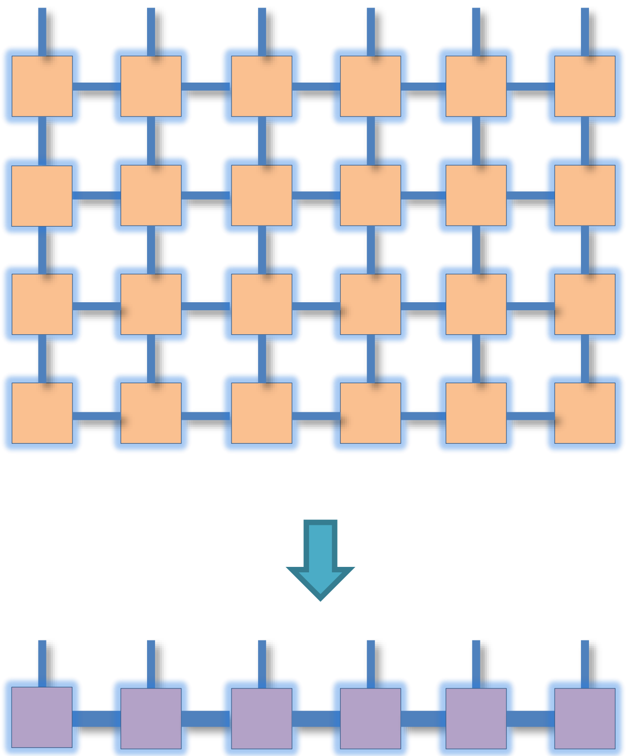
Drawback:
- Computationally expensive (sometimes impossible, e.g. most bosonic RVB states => permanent)
- Characterization tools rare (e.g. entanglement spectrum/entropy not available)

Representing parton wave functions as tensor networks

This talk:

- Derive exact tensor network representations

- Compress tensor networks into Matrix Product States



Motivation:

- Computations and characterizations become easy/possible

- Good ansatz for initializing DMRG

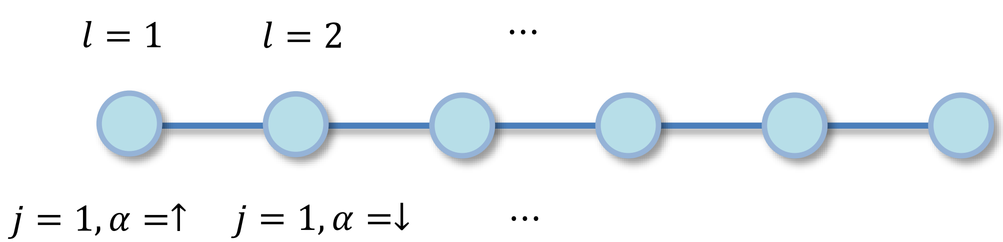
Gutzwiller projected Fermi sea

Example: $$|\psi\rangle = P_{\mathrm{G}} \prod_{m=1}^{N} d_m^\dagger |0\rangle$$

Occupied single-particle orbitals:

$$d_m^\dagger = \sum_{j=1}^{N} \sum_{\alpha=\uparrow,\downarrow} A_{m,j\alpha} c_{j\alpha}^\dagger = \sum_{l=1}^{2N} A_{ml} c_l^\dagger$$

$$l = (j, \alpha)$$



Single-particle orbital as Matrix Product Operator

$$d_m^\dagger = \sum_{l=1}^{2N} A_{ml} c_l^\dagger$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ A_{m1} c_1^\dagger & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ A_{m2} c_2^\dagger & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & 0 \\ A_{m,2N} c_{2N}^\dagger & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Single-particle orbital as Matrix Product Operator

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$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ A_{m1} \sigma_1^+ & \sigma_1^z \end{pmatrix} \begin{pmatrix} 1 & 0 \\ A_{m2} \sigma_2^+ & \sigma_2^z \end{pmatrix} \cdots \begin{pmatrix} 1 & 0 \\ A_{m,2N} \sigma_{2N}^+ & \sigma_{2N}^z \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

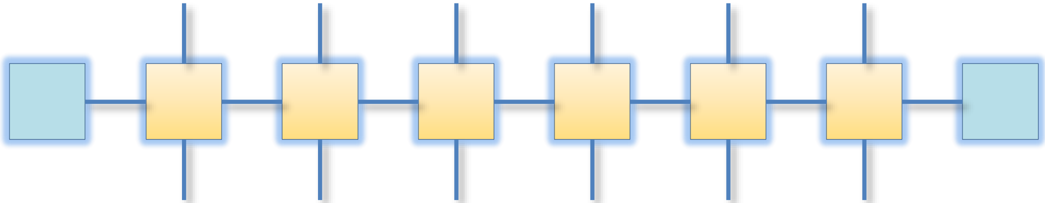
Jordan-Wigner mapping: $c_l^\dagger = \sigma_1^z \cdots \sigma_{l-1}^z \sigma_l^+$

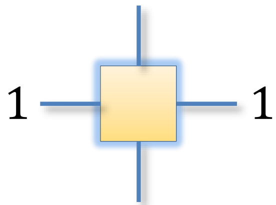
Single-particle orbital as Matrix Product Operator

$$d_m^\dagger = \sum_{l=1}^{2N} A_{ml} c_l^\dagger$$

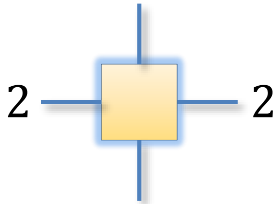
$$= (0 \quad 1) \begin{pmatrix} 1 & 0 \\ A_{m1}\sigma_1^+ & \sigma_1^z \end{pmatrix} \begin{pmatrix} 1 & 0 \\ A_{m2}\sigma_2^+ & \sigma_2^z \end{pmatrix} \cdots \begin{pmatrix} 1 & 0 \\ A_{m,2N}\sigma_{2N}^+ & \sigma_{2N}^z \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

=

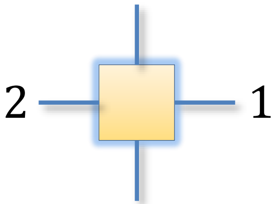




$= 1$



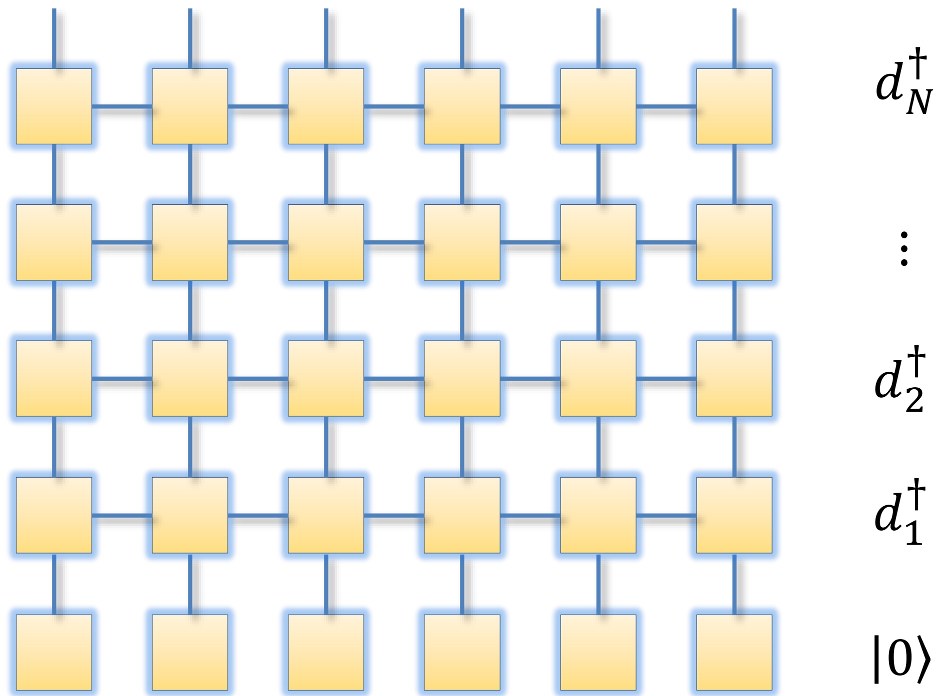
$= \sigma^z$



$= A_{ml}\sigma^+$

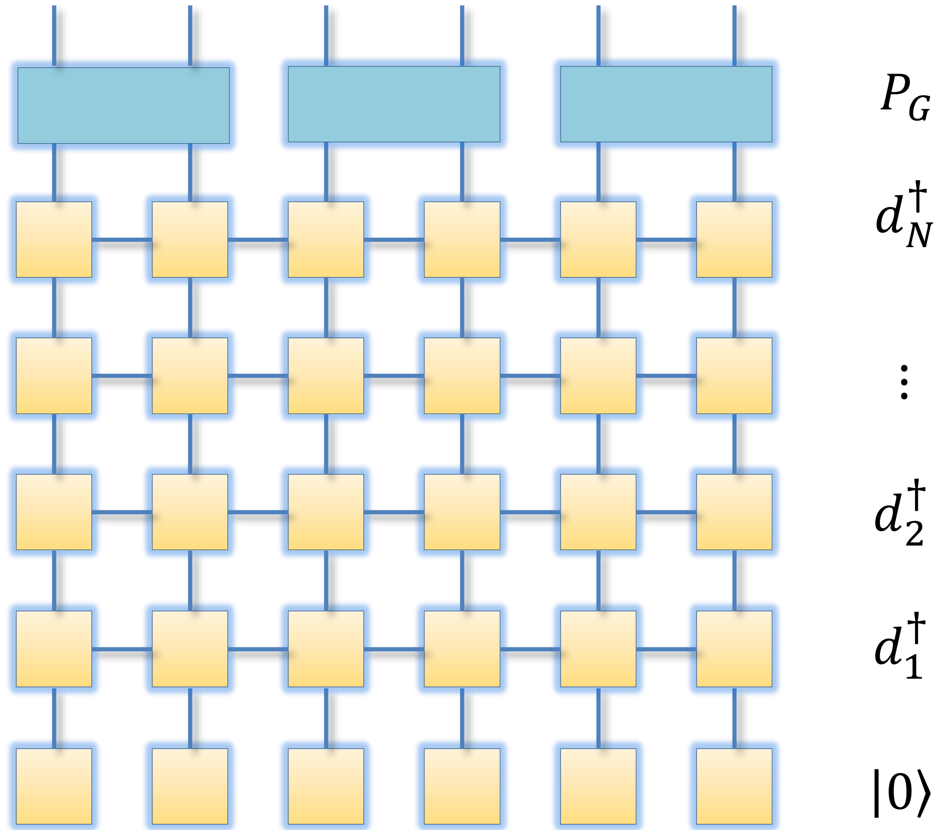
Tensor network representation of Fermi sea

$$\prod_{m=1}^{N} d_m^\dagger \, |0\rangle =$$



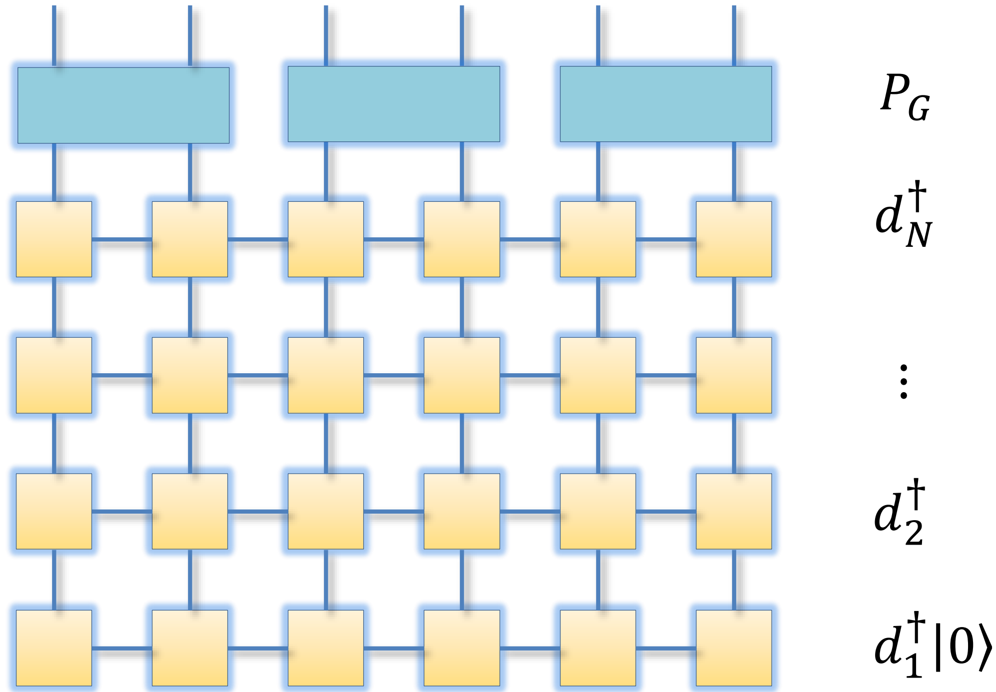
Tensor network representation of projected Fermi sea

$$P_{\mathrm{G}} \prod_{m=1}^{N} d_m^\dagger |0\rangle =$$



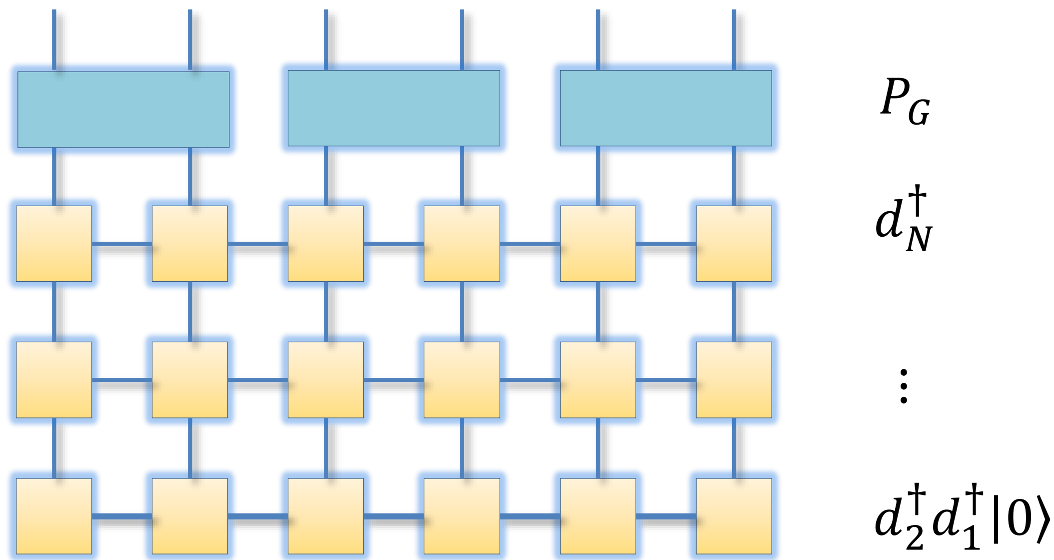
Compressing MPO-MPS tensor network into MPS

$$P_G \prod_{m=1}^{N} d_m^\dagger |0\rangle =$$



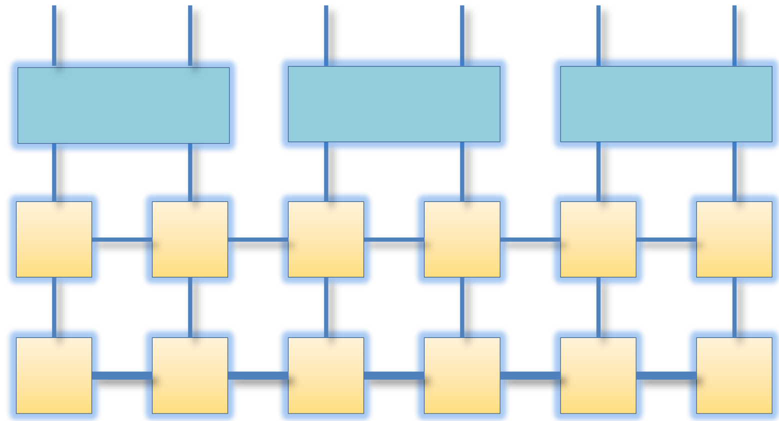
Compressing MPO-MPS tensor network into MPS

$$P_{\mathrm{G}} \prod_{m=1}^{N} d_m^{\dagger} \,|0\rangle =$$



Compressing MPO-MPS tensor network into MPS

$$P_G \prod_{m=1}^{N} d_m^\dagger |0\rangle =$$



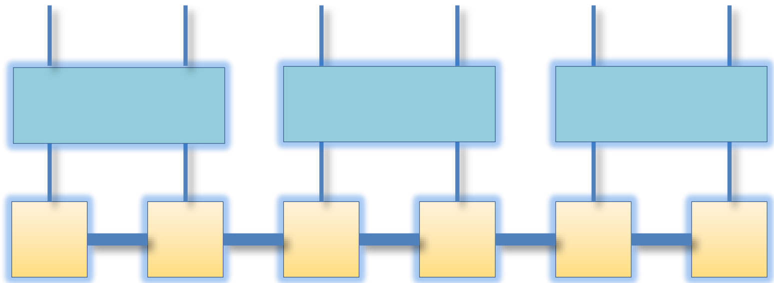
P_G

$\vdots$

$d_3^\dagger d_2^\dagger d_1^\dagger |0\rangle$

Compressing MPO-MPS tensor network into MPS

$$P_{\mathrm{G}} \prod_{m=1}^{N} d_m^{\dagger} |0\rangle =$$

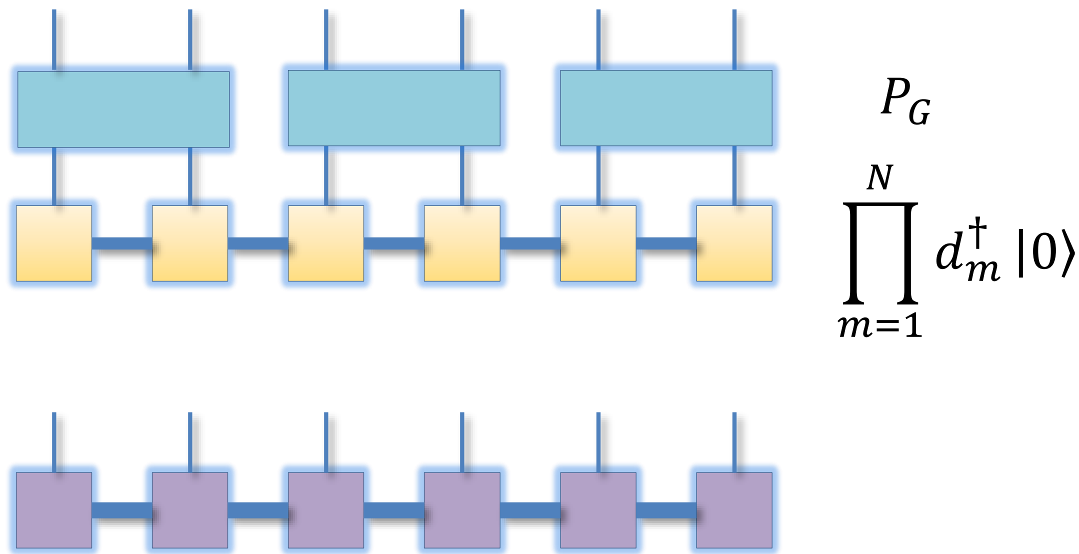


P_G

$$\prod_{m=1}^{N} d_m^{\dagger} |0\rangle$$

Compressing MPO-MPS tensor network into MPS

$$P_G \prod_{m=1}^{N} d_m^\dagger |0\rangle =$$



$$=$$

- Truncation needed in intermediate steps

High fidelity compression requires low-entanglement “intermediate” states!

Maximally localized Wannier orbitals

$$\prod_{m=1}^{N} d_m^\dagger |0\rangle = \prod_{r=1}^{N} f_r^\dagger |0\rangle$$

Wannier orbitals $\quad f_r^\dagger = \sum_{m=1}^{N} B_{rm} d_m^\dagger = \sum_{l=1}^{2N} (BA)_{rl} c_l^\dagger$

- Determination of maximally localized Wannier orbitals:

Position operator: $\quad X = \sum_{l=1}^{2N} l c_l^\dagger c_l$

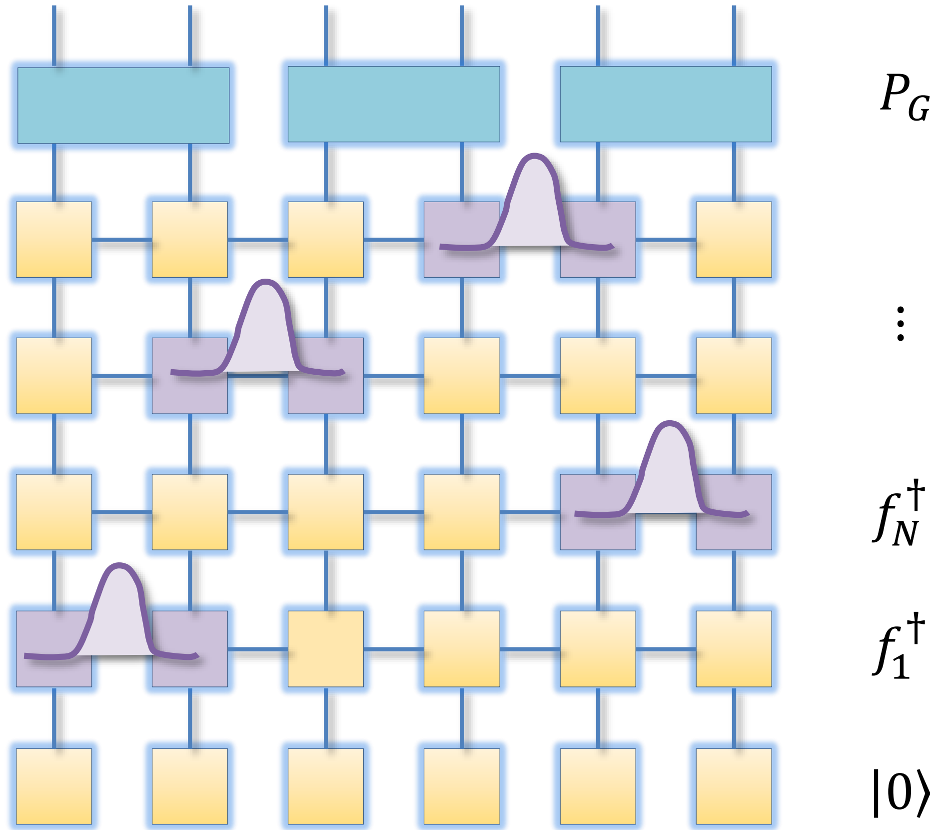
Diagonalization of the “projected” position operator (within the subspace of occupied single-particle states) → $f_r^\dagger$

$$X_{mn} = \langle 0 | d_m X d_n^\dagger | 0 \rangle$$

S. Kivelson, PRB (1982); X.-L. Qi, PRL (2011).

Compression with Wannier orbitals: left-meet-right

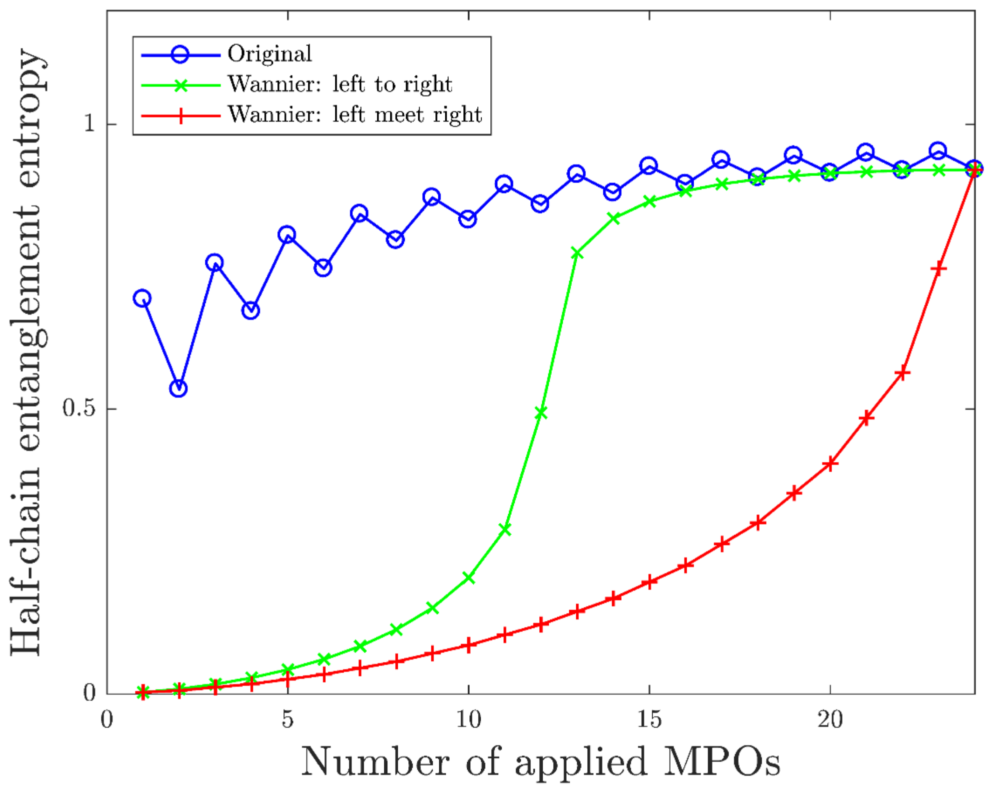
$$P_{\mathrm{G}} \prod_{r=1}^{N} f_r^{\dagger} |0\rangle =$$



- Accelerated MPO-MPS evolution and substantially suppressed truncation errors (due to gradually built-up entanglement)!

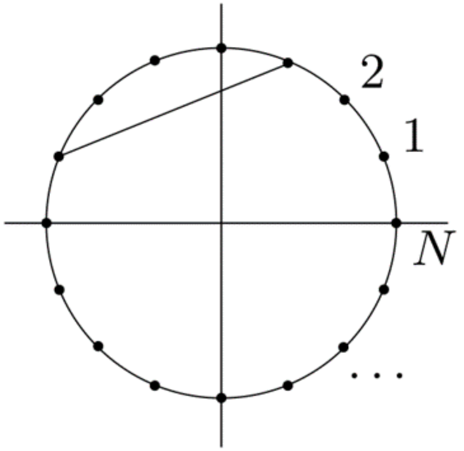
Example: 1D half-filled Fermi sea with OBC

$$d_m^\dagger = \sqrt{\frac{2}{N+1}} \sum_{j=1}^{N} \sin\left(\frac{\pi m j}{N+1}\right) c_j^\dagger \qquad 1 \le m \le N/2$$



Benchmark: 1D Haldane-Shastry model

$$|\psi\rangle = P_{\mathrm{G}} \prod_{|k|<\pi/2} \prod_{\alpha=\uparrow,\downarrow} c_{k\alpha}^{\dagger} |0\rangle$$



- Ground state of the Haldane-Shastry model

$$H_{\mathrm{HS}} = \sum_{i<j} \frac{\vec{S}_i \cdot \vec{S}_j}{\left(\frac{N}{\pi}\right)^2 \sin^2 \frac{\pi}{N}(i-j)}$$

with energy $E_{\mathrm{GS}} = -\frac{\pi^2}{24}\left(N + \frac{5}{N}\right)$

F.D.M. Haldane, PRL (1988);
B.S. Shastry, PRL (1988).

- Benchmark: $N = 100$, $E_{\mathrm{GS}} =$ -41.1439133...

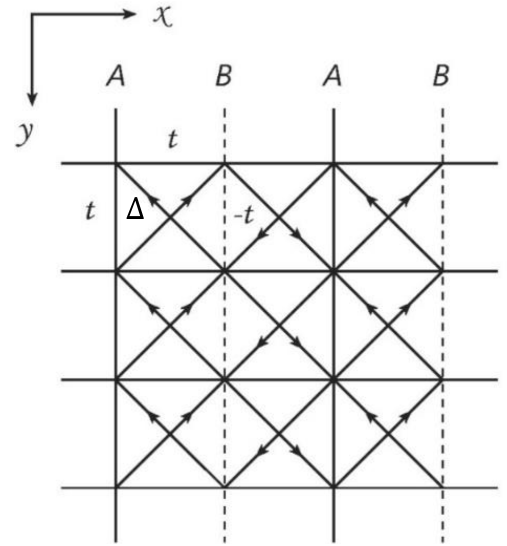
MPS: $D = 1000$, $E =$ -41.1435412...
$D = 3000$, $E =$ -41.1439061...
$D = 5000$, $E =$ -41.1439125...

$$\left|\frac{E - E_{\mathrm{GS}}}{E_{\mathrm{GS}}}\right| \sim 10^{-7}$$

Benchmark: 2D Laughlin chiral spin liquid

- Parton “mean-field” Hamiltonian:

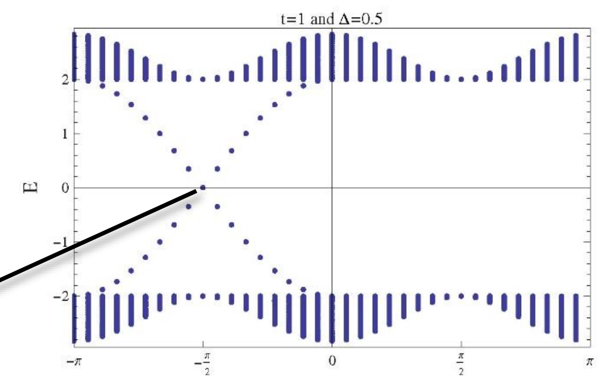
$$H_{\mathrm{MF}} = \sum_{\langle ij \rangle, \alpha} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{\langle\langle ij \rangle\rangle, \alpha} \Delta_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha}$$



- Parton wave functions of two topological sectors (identity & semion) on a cylinder:

$$|\psi_I\rangle = P_{\mathrm{G}} \gamma_{L\uparrow}^{\dagger} \gamma_{L\downarrow}^{\dagger} |\mathrm{FS}\rangle$$

$$|\psi_s\rangle = P_{\mathrm{G}} \gamma_{L\uparrow}^{\dagger} \gamma_{R\downarrow}^{\dagger} |\mathrm{FS}\rangle$$

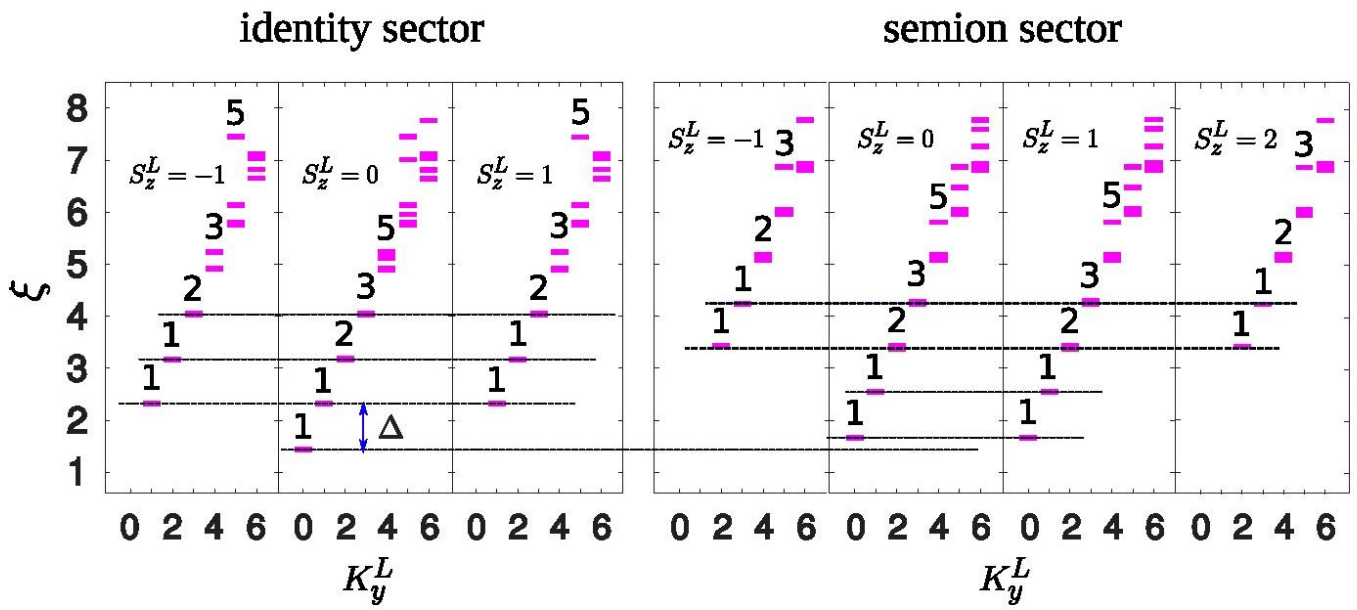


$\gamma_{L\alpha}^{\dagger}$ and $\gamma_{R\alpha}^{\dagger}$

HHT, Y. Zhang & X.-L. Qi, PRB (2013)

Benchmark: 2D Laughlin chiral spin liquid

Benchmark: $N_x = 16,\ N_y = 10$ MPS: $D = 9000$



- Entanglement spectrum agrees with the $\mathrm{SU(2)}_1$ CFT
- Topological spin of semion: $h_s \approx 0.2617$ (expected: $h_s = 1/4$)

Summary and outlook

- We have obtained exact tensor network representations of parton wave functions.
- For the projected Fermi sea, maximally localized Wannier orbitals allow a high-fidelity compression into MPS.
  (See H.-K. Jin, HHT & Y. Zhou, arXiv:2001.04611 for projected BCS states)
- Outlook: Projected bosonic paired states (bosonic RVB), continuum limit, excitations…

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Thank you for your attention!