Fractional Chiral Hinge Insulator

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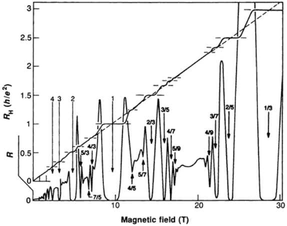
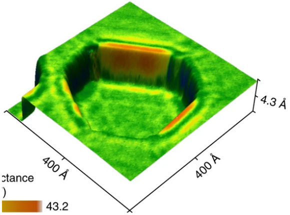
Reference: arXiv:2010.09728

Toward 3D topological order

2D TO: FQHE



Higher order topological insulators

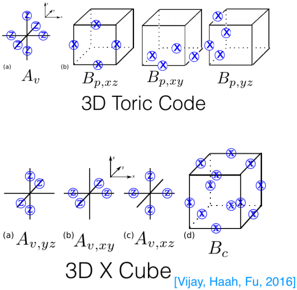


An intermediate step (3d but chiral hinge modes)

3D TO

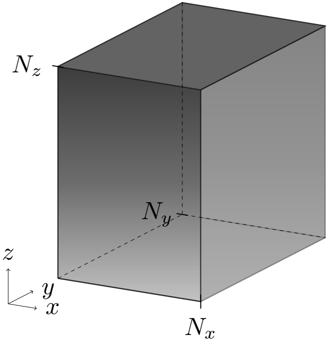
Toric code, fractons, ...

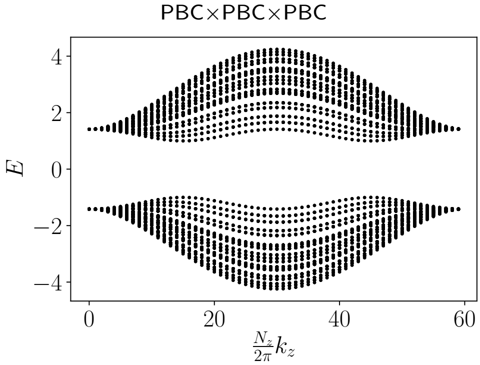
Any microscopic elctronic model?

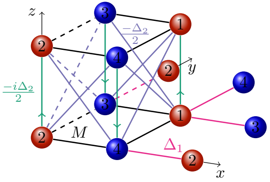


Building a model state for a FCHI

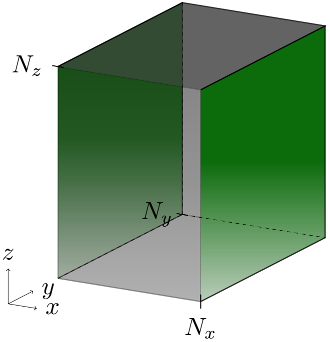
Chiral Hinge Insulator: 2nd order TI in 3D [Schindler et al. 2018]

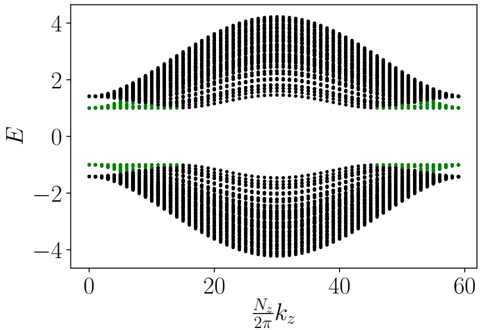


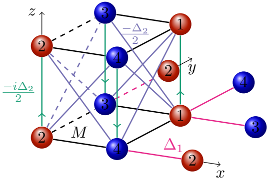




Chiral Hinge Insulator: 2nd order TI in 3D [Schindler et al. 2018]

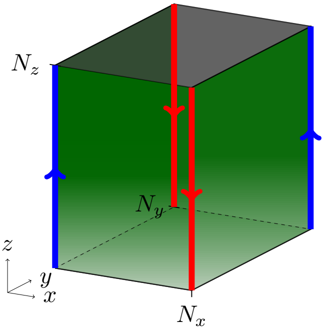


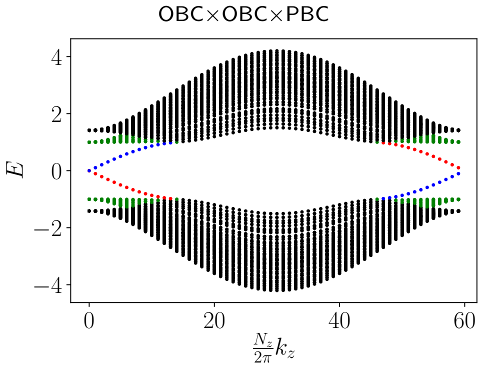


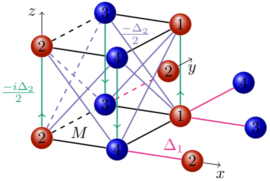


- Gapped vertical surfaces

Chiral Hinge Insulator: 2^{nd} order TI in 3D [Schindler et al. 2018]

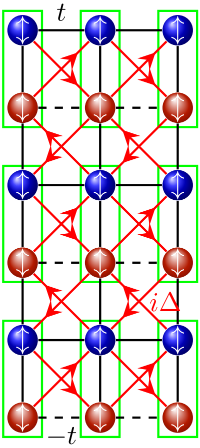






- Gapped vertical surfaces.
- Gapless chiral & antichiral hinge modes.
- Hinge modes = IQHE edge modes (BUT surfaces $\neq$ IQHE bulk).
- Protected by the $C_4 T$ symmetry (nbr hinges mod 2).

Fractional Chern insulator from Gutzwiller projection



- Two copies of CI with spin $s = \uparrow, \downarrow$
- On-site repulsion $U \sum_x n_{x,\uparrow} n_{x,\downarrow}$
- FCI phase if $U \gg t, \Delta$ (Laughlin 1/2)

Model wave function from limit $U \to \infty$:

- Double-occupancy forbidden, 1 particle per site: Spin s only remaining degree of freedom
- Ground state obtained by Gutzwiller projection

$$P_G = \prod_x (1 - n_{x,\uparrow} n_{x,\downarrow})$$

Zhang, T. Grover, and A. Vishwanath PRB (2011)

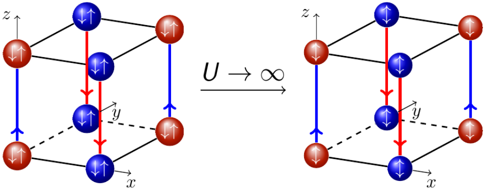
- Topological sectors by using PBC/APBC for the CIs

Interacting model wave function: Gutzwiller projection of two spin copies!

Fractional chiral hinge insulator (FCHI)

- $|\psi_s\rangle$ ground state of CHI at half-filling with spin $s = \uparrow, \downarrow$
- **FCHI model wave function:** Gutzwiller projection

$$|\Psi_{\mathrm{FCHI}}\rangle = P_G \left[|\psi_\uparrow\rangle \otimes |\psi_\downarrow\rangle \right]$$



Properties accessible in Monte Carlo simulations to characterize our model state:

- Second Renyi entanglement entropy: $S^{(2)} = -\ln \left(\mathrm{Tr}_{\mathcal{A}} \rho_{\mathcal{A}}^2 \right)$
- Spin fluctuations: $\mathrm{Var}(M_{\mathcal{A}}) = \mathrm{Tr}_{\mathcal{A}}(M_{\mathcal{A}}^2 \rho_{\mathcal{A}}) - \left(\mathrm{Tr}_{\mathcal{A}}(M_{\mathcal{A}} \rho_{\mathcal{A}}) \right)^2$
- Overlaps between different "topological sectors"

Monte Carlo simulations for $S^{(2)}$

Partition into $\mathcal{A}$ & complement $\mathcal{B}$, want $S^{(2)}_{\mathcal{A}}$:

- Spin configuration $|v\rangle = |v_{\mathcal{A}}, v_{\mathcal{B}}\rangle$
- Swap operator

$$\mathrm{SWAP}\left(|v_{\mathcal{A}}, v_{\mathcal{B}}\rangle \otimes \left|v'_{\mathcal{A}}, v'_{\mathcal{B}}\right\rangle\right) = \left|v'_{\mathcal{A}}, v_{\mathcal{B}}\right\rangle \otimes \left|v_{\mathcal{A}}, v'_{\mathcal{B}}\right\rangle$$

- Entanglement entropy

$$\frac{\langle\Psi \otimes \Psi|\,\mathrm{SWAP}\,|\Psi \otimes \Psi\rangle}{\langle\Psi \otimes \Psi|\Psi \otimes \Psi\rangle} = e^{-S^{(2)}_{\mathcal{A}}}$$

- Double-layer simulation
- Measured value decays **exponentially** with $|\partial\mathcal{A}|$
- **More than 2 million CPU hours**

Probing the FCHI

Luttinger liquids: 1D critical quantum system

Entanglement for subregion of size L scales as $\ln L$.

Chiral conformal field theory for finite size N

(Second Renyi) Entropy:

$$S^{(2)}_{\text{crit}}(L; N) = \frac{c}{8} \ln \left[\frac{N}{\pi} \sin \left(\frac{\pi L}{N} \right) \right]$$

Expect $c = 1$ for Luttinger liquid!

Fluctuations of U(1) current:

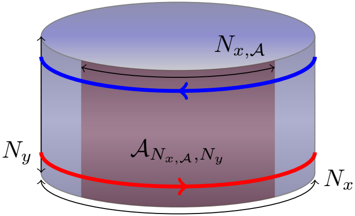
$$\mathrm{Var}(M_{\mathcal{A}}) = \frac{K}{2\pi^2} \ln \left[\frac{N}{\pi} \sin \left(\frac{\pi L}{N} \right) \right]$$

$K \neq 1$ indicates fractionalization!

Study scaling of $S^{(2)}$ and $\mathrm{Var}(M_{\mathcal{A}})$ with L

Disentangling the edge & hinge from the bulk contribution

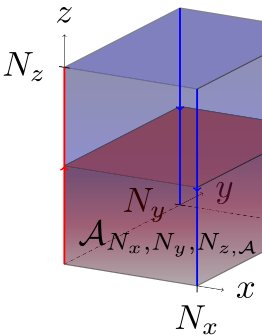
Edge states in 2D:



$$S^{(2)}(N_{x,\mathcal{A}}) \;=\; \alpha_{2D} + 2 \times S^{(2)}_{\text{crit}}(N_{x,\mathcal{A}}; N_x)$$

α_{2D} bulk area law (constant)

Hinge states in 3D:

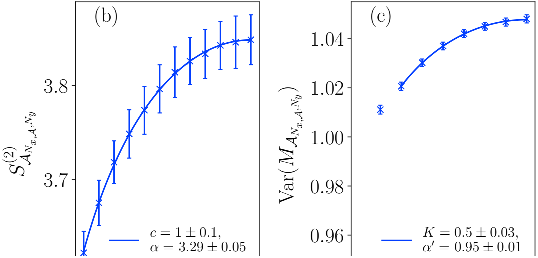


$$S^{(2)}(N_{z,\mathcal{A}}) \;=\; \alpha_{3D} + 4 \times S^{(2)}_{\text{crit}}(N_{z,\mathcal{A}}; N_z)$$

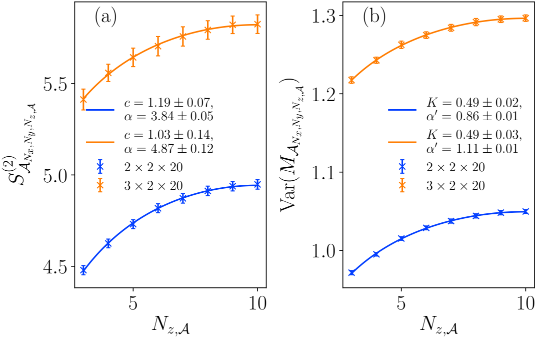
α_{3D} bulk area law (constant)

Monte-Carlo results

FCI:



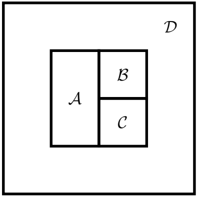
FCHI:



Both have $c = 1$ and $K = 1/2$: Fractionalized hinge states

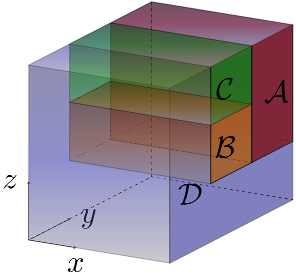
Topological entanglement entropy (TEE) γ

- Area law for entanglement entropy: $S_\mathcal{A} = \alpha|\partial\mathcal{A}| - \gamma$
- Kitaev-Preskill cut.
- $-\gamma = S_{\mathcal{ABC}} - S_{\mathcal{AB}} - S_{\mathcal{BC}} - S_{\mathcal{AC}} + S_\mathcal{A} + S_\mathcal{B} + S_\mathcal{C}$



Periodic boundary conditions in x

- $\gamma_{PBC} = -0.009 \pm 0.102 \approx 0$
- Same value if taken along z,
- $\gamma_{PBC} \approx 0$ for two system sizes $\rightarrow$ not a layered construction.



Open boundary conditions in x

- $\gamma_{OBC} = 0.32 \pm 0.16 \approx \ln(\sqrt{2}) = 0.35$

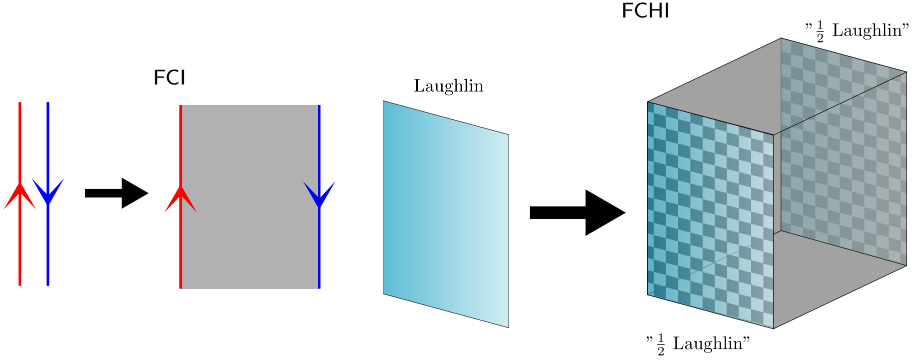
$$\gamma^{2D}_{\mathrm{FCHI}} = (\gamma_{OBC} - \gamma_{PBC})/2 \approx \ln(\sqrt{2})/2$$

Unconventional surface phase

$$\gamma^{2D}_{\mathrm{FCHI}} = \ln(\sqrt{2})/2$$

But: Strictly 2D topological phase has $\gamma = 0$ **or** $\gamma \geq \ln(\sqrt{2})$

Surfaces host unconventional topological phase that cannot be described by 2D TQFT

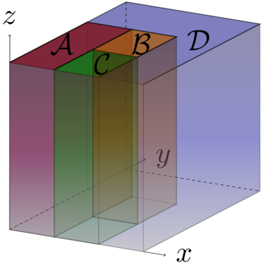


Signature from the bulk

What about the TEE in the bulk?

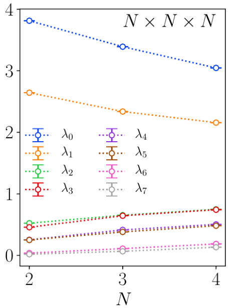
- $\gamma_{PBC} = -0.009 \pm 0.102 \approx 0$
- Same value if taken along z or x
- $\gamma_{PBC} \approx 0$ for two system sizes $\rightarrow$ not a layered construction.
- ... but the cut is macroscopic in one direction

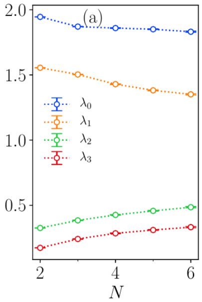
Zhang, Grover, Vishwanath PRB (2011).



Topological degeneracy?

- Can play with PBC/APBC for the CHI to generate 8 states (full PBC) or 4 states (OBC in x)
- Should be isotropic $\rightarrow$ size is a killer.



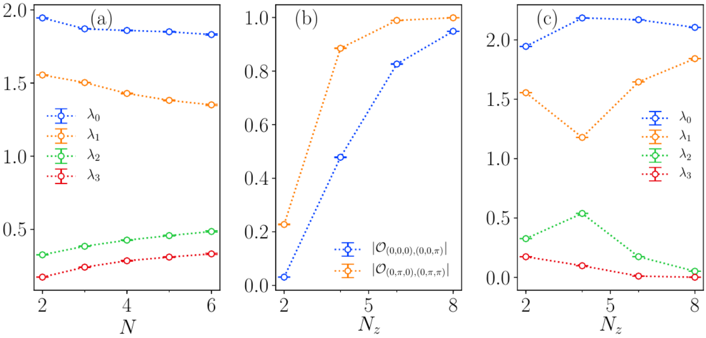


Conclusion

- Fractional CHI model wave function obtained from Gutzwiller projection.
- Fractionalized chiral hinge modes similar to edge modes of a fractional Chern insulator.
- Gapped vertical surfaces hosting a topological phase that cannot be realized in 2D.

- **Outlook:**
  - 3D is hard. Beyond Monte-Carlo?
  - What is the nature of the phase?
  - Fate of the Dirac cones in the interacting model?

Anisotropic limit OBC×PBC×PBC



- OBC×PBC×PBC: 4 states → 2 linearly independent states.
- PBC×PBC×PBC: 8 states → 4 linearly independent states.