Classical Dimers on Quasicrystals

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Entanglement in Strongly Correlated Systems
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Loyd, Biswas, Simon, SP, Flicker arXiv:2102.xxxx

Flicker, Simon, SP, *Phys. Rev. X* **10**, 011005 (2020)

erc StG: "Topological Matter and Crystalline Symmetries"

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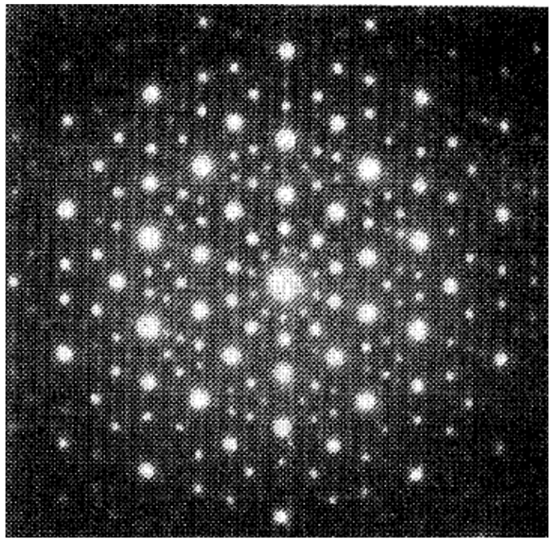
Quasicrystals

- Crystals without periodicity, but with regular Bragg peaks that show "forbidden" symmetries (e.g. 5- or 8-fold)

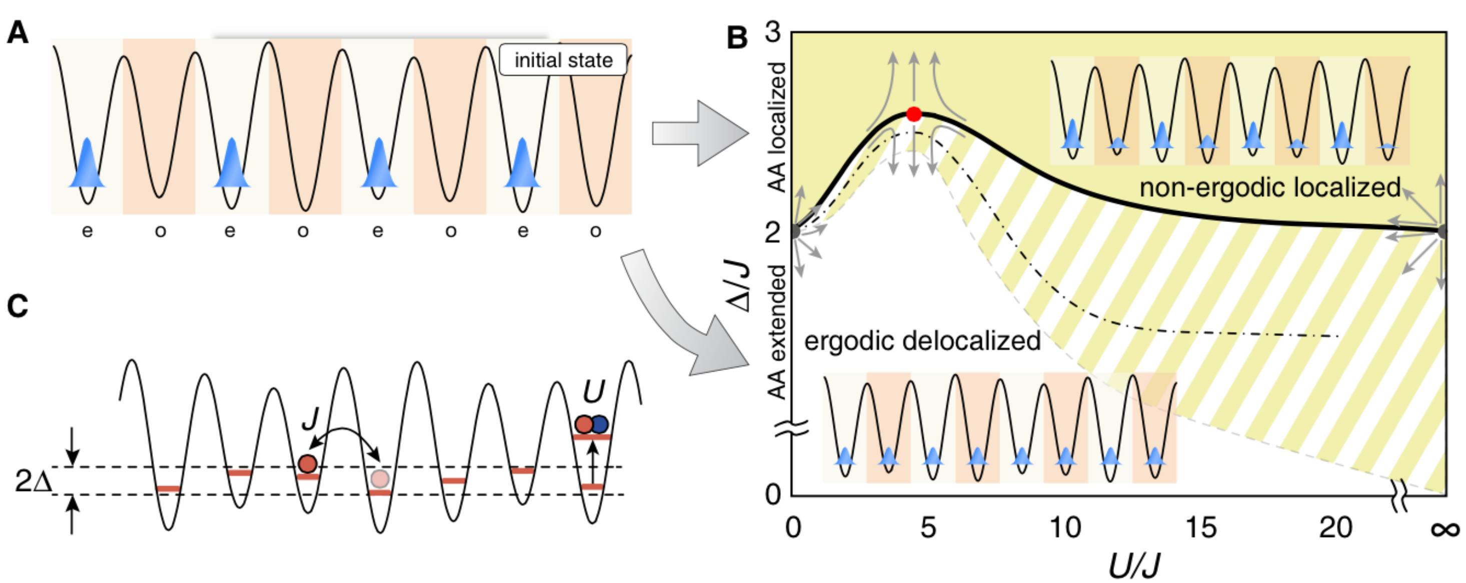


- Discovered via diffraction on Al-Mn alloys [Schechtman *et al* '84]
  - Old maths problem: aperiodic tiings

    [Penrose, Conway, Amman 1970s/ Wang 1960s/Kepler 1619 (!)]

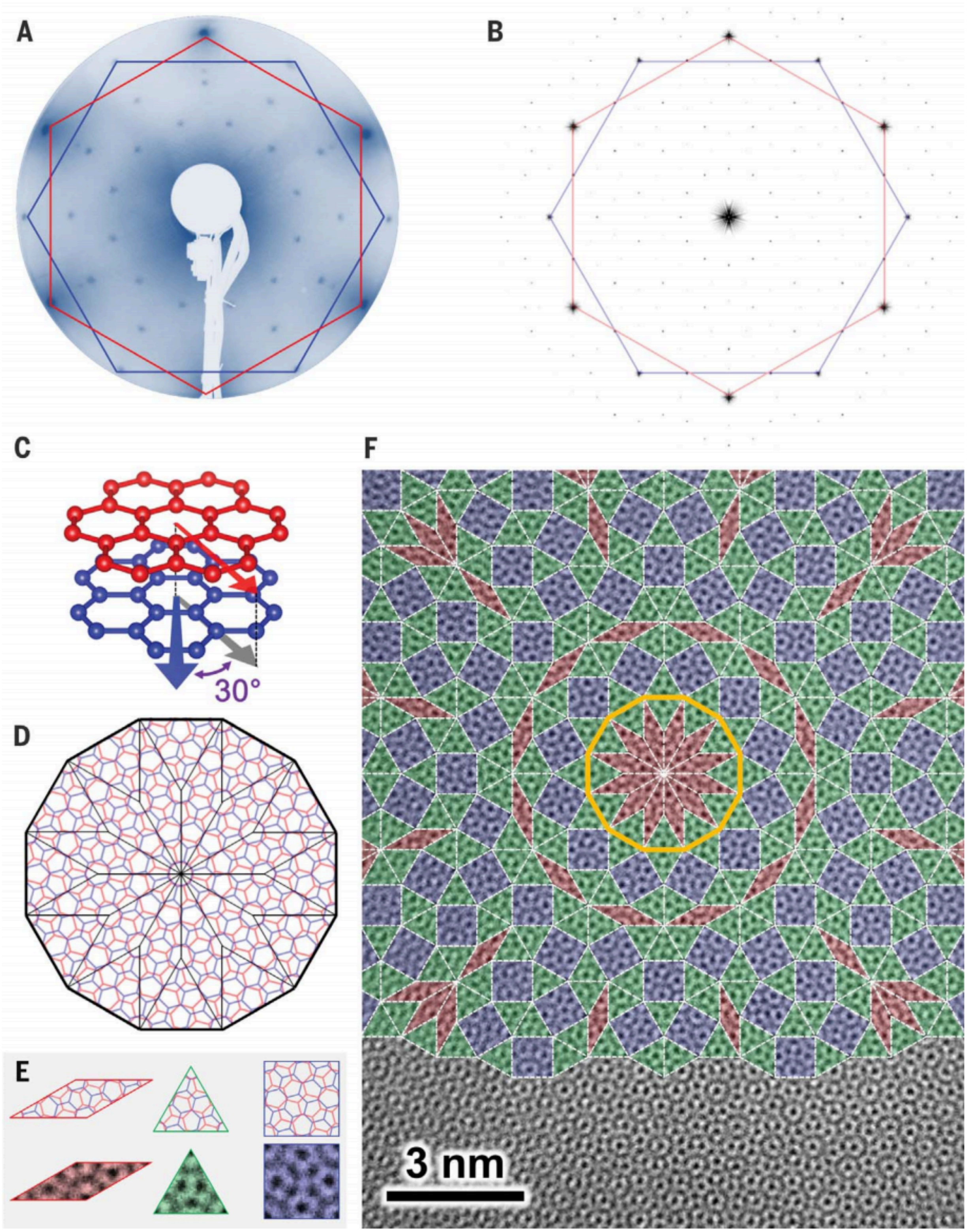
- Correlation effects: heavy-fermions, magnetism, superconductivity…

- Relevant to quantum dynamics: e.g. many-body localization in 1D optical lattices



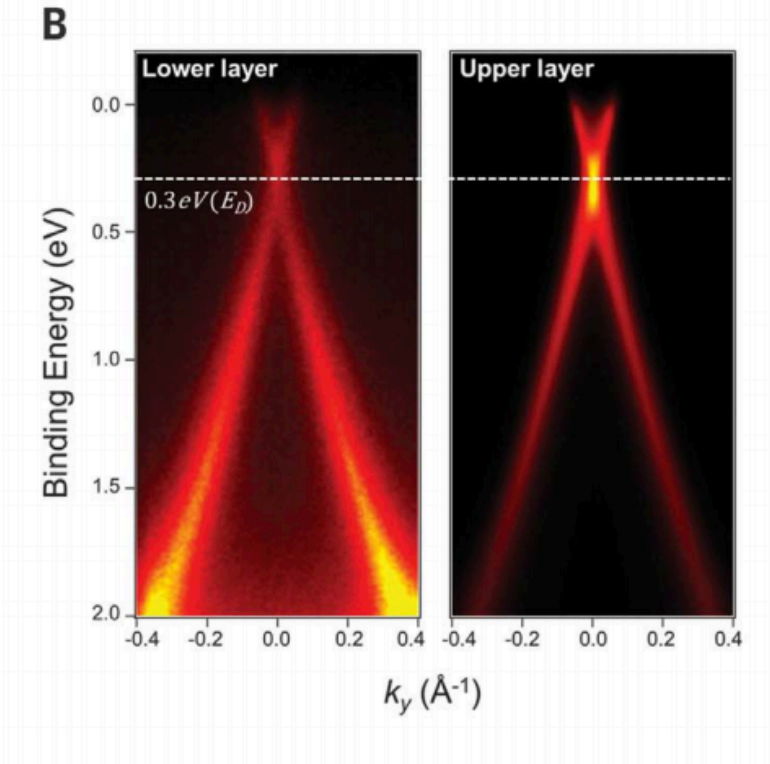
[Schreiber *et al* '15]

Moiré quasicrystals?



[Ahn *et al* '18]

- Two graphene sheets twisted by 30° wrt each other
- Aperiodic: 12-fold "dodecagonal" symmetry
- Dirac electrons, localization…



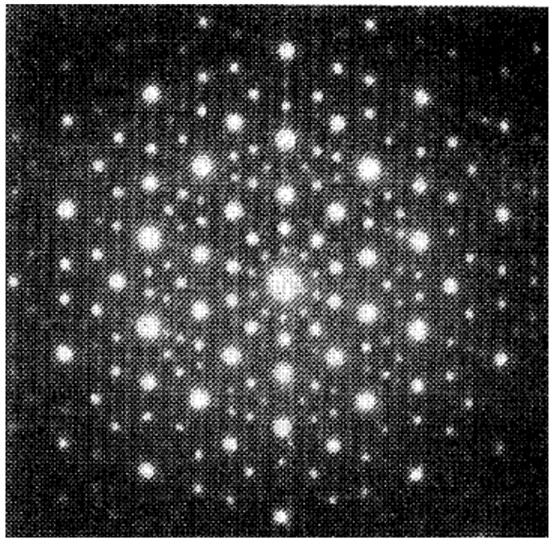
“Semi-Clean Dirt” or “Sort-of-Crystal”?

- Crystals without periodicity (no translational symmetry) but with regular Bragg peaks

  Sharp Bragg peaks

  ⇔ coherent interference of scattered waves

  ⇔ long-range correlations



[Schechtman *et al* ’84]

- Revisit usual translational invariance assumptions

- Perhaps a better term is “deterministic detuning” — no real randomness

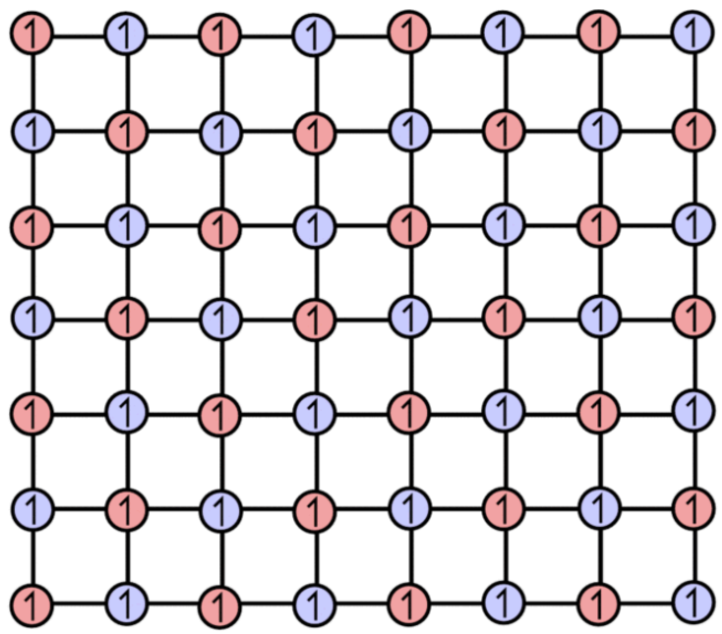
- Suppression of “rare region” effects

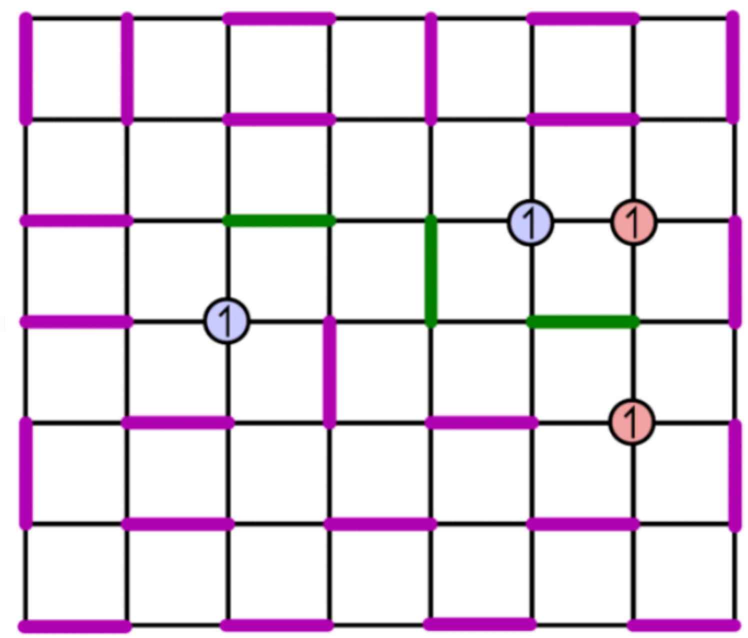
  - modifies “Harris criterion” for stability of critical points, and Chayes-Chayes-Fisher-Spencer bounds on correlation lengths

[Luck 1993]

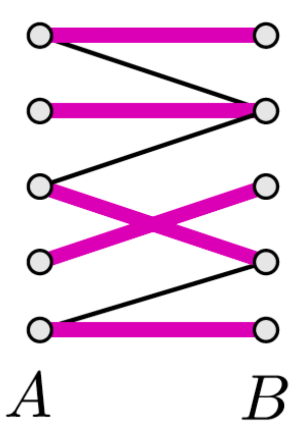
Dimer Models, Graph Matchings & Bipartiteness

- Classic problem in graph combinatorics: "matchings" or dimer coverings
  [Fisher-Kastelyn '61…]
- Approximate representation of singlets in quantum antiferromagnets
- Hard-core constraint ~ "Gauss law", monomers ~ gauge charges

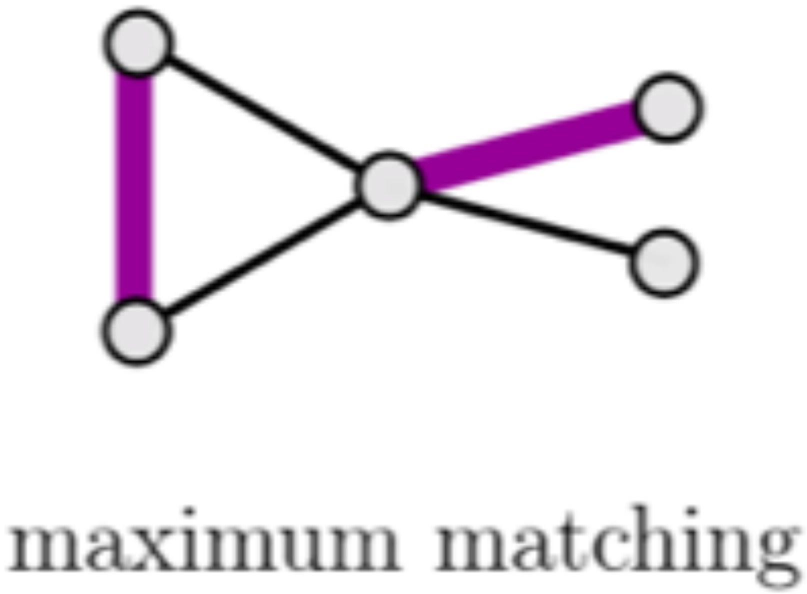




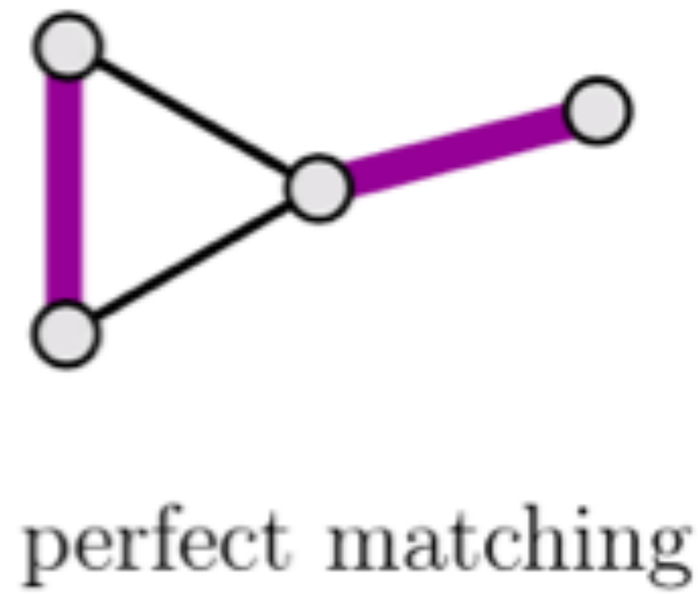
- Bipartite dimer models
  - two sublattices, bonds only connect different sublattices
  - Monomers remain on 1 sublattice while hopping ⇒ +/- charge
  - understand via mapping to 'height model' w/ local action



Maximum and Perfect



(max. dimers)



(every site has dimer)

Quantum Dimer Models

- Quantum fluctuations within dimer subspace ["resonances"] [Rokhsar & Kivelson '88]

$$\hat{H} = -t\left(|\square\rangle\langle\square| + |\square\rangle\langle\square|\right) + V\left(|\square\rangle\langle\square| + |\square\rangle\langle\square|\right)$$

constraint ~ quantum gauge theory
(toy model of quantum spin liquid)

At RK point t=V: exact g.s. is sum over dimer coverings $|\Psi_{\mathrm{RK}}\rangle = \sum_{\mathcal{C}} |\mathcal{C}\rangle$

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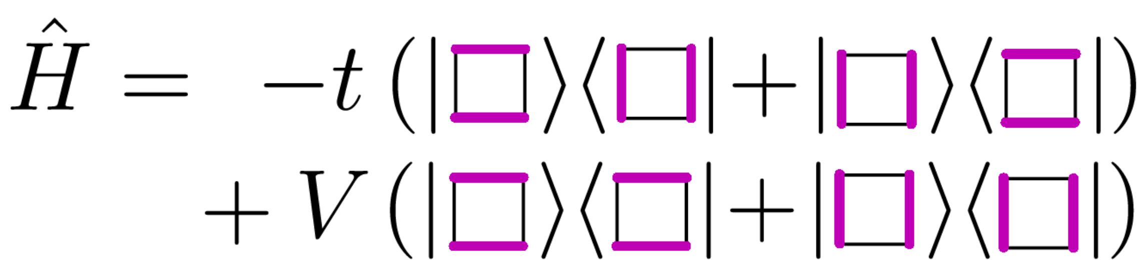

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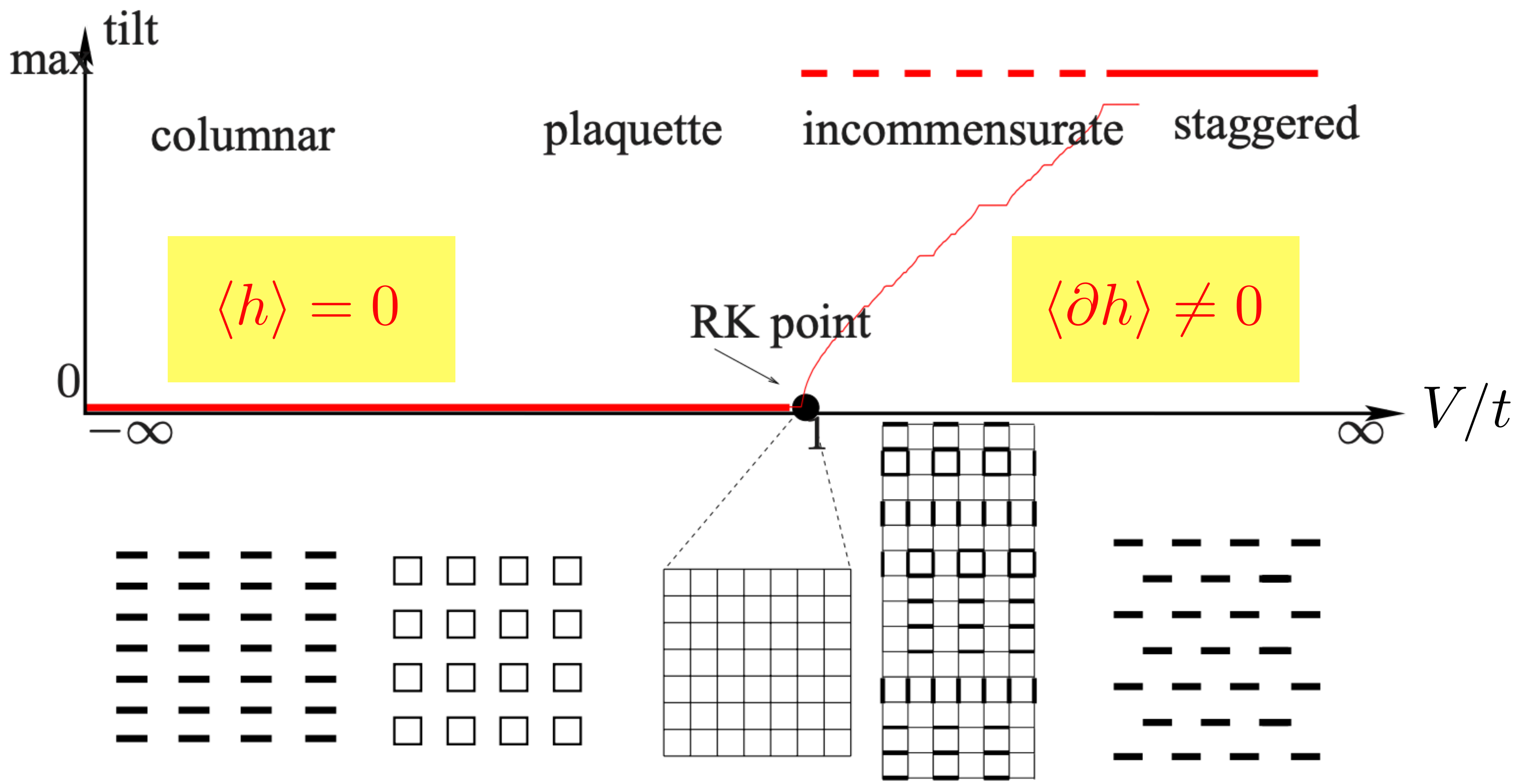
- Is “deconfined” dimer/RVB liquid a stable phase?
  - non-bipartite lattices, *d*=2,3: discrete gauge structure (Z_2 QSL)
  - bipartite lattice, *d*=3: continuous (e.g. Maxwell) gauge structure (U(1) QSL)
  - bipartite lattice *d*=2: ‘Polyakov’ confinement (instantons in height field)
    - RK point: z=2 quantum Lifshitz multicriticality between “staggered” and “columnar” valence-bond crystals

[Moessner, Sondhi, … ‘00s]

“Cantor Deconfinement” and Incommensuration

- Away from RK point: “tilt” in height field + instanton effects lead to dimer crystals
- “tilted” phases incommensurate with lattice ⇒ “Devil’s staircase” of critical pts



[Fradkin, Huse, Moessner, Oganesyan, Sondhi ’03; Vishwanath, Balents, Senthil ’03]

- Can quasicrystals “build in” incommensuration microscopically?

Today:

What is the interplay of quasiperiodicity + local constraints?

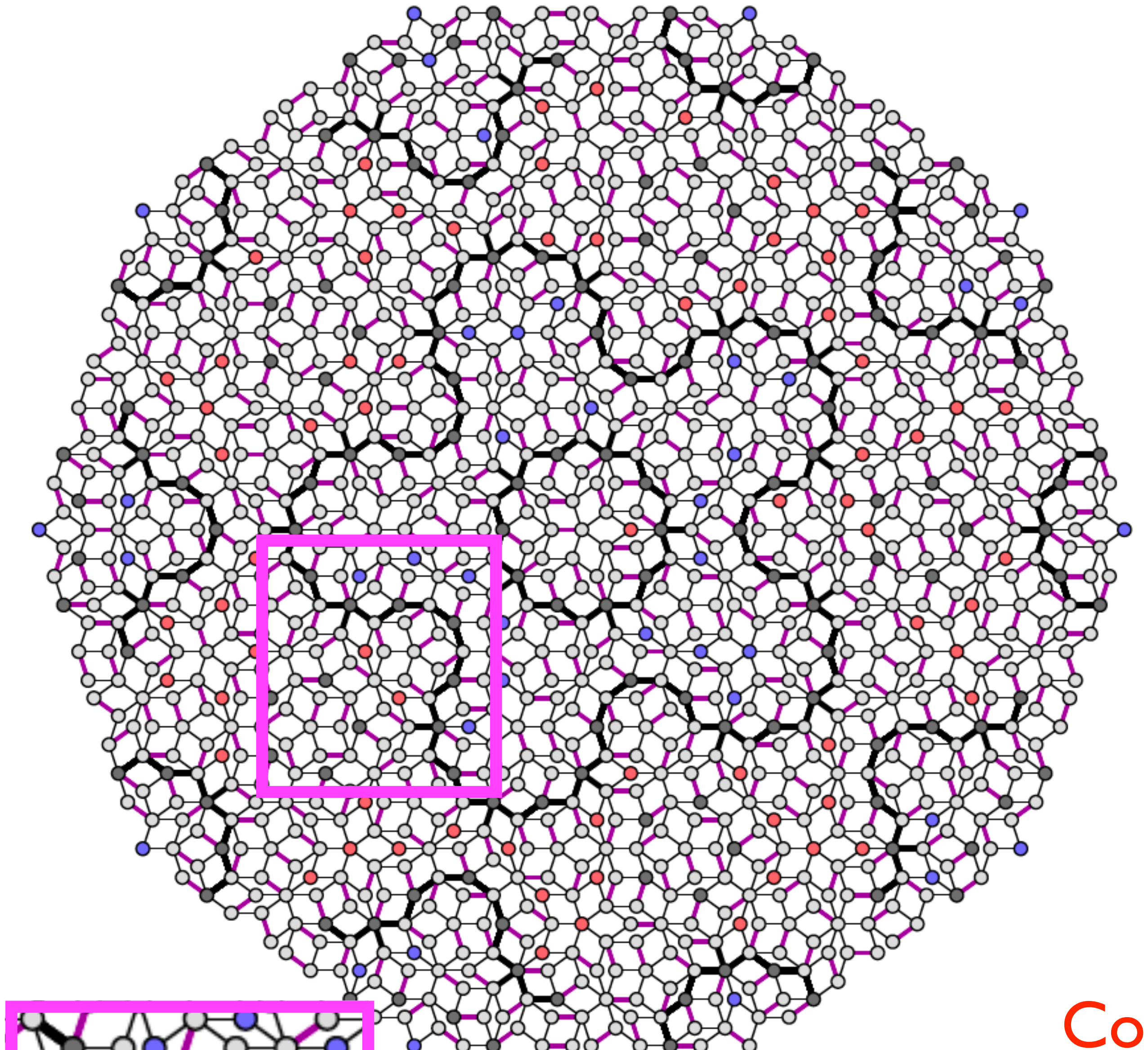
Caveat: $\hbar=0$ (already rich)

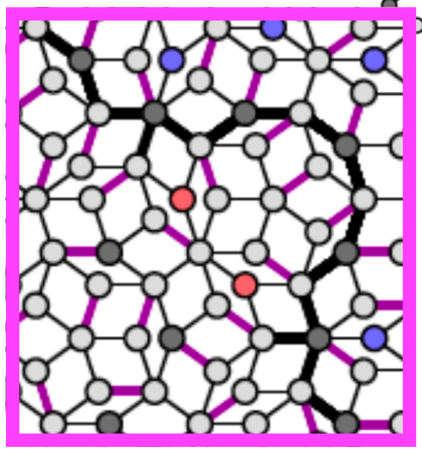
- naive coarse-graining forgets quasiperidicity
- possibility of new dimer phases
- implications for dynamics/MBL/fractons?

Other applications: chemical absorption, zero modes,…

Previous work: Penrose Tilings & Monomer Membranes

No perfect matchings; study maximum matchings





finite monomer # density ~ 0.098

vanishing monomer charge density

nested regions w/ opposite charge excess

"membranes" - no dimers in max-matching

fractal dimension $d_F = \frac{1}{\log_2 \varphi} \approx 1.44$

Connected correlations can't cross membranes

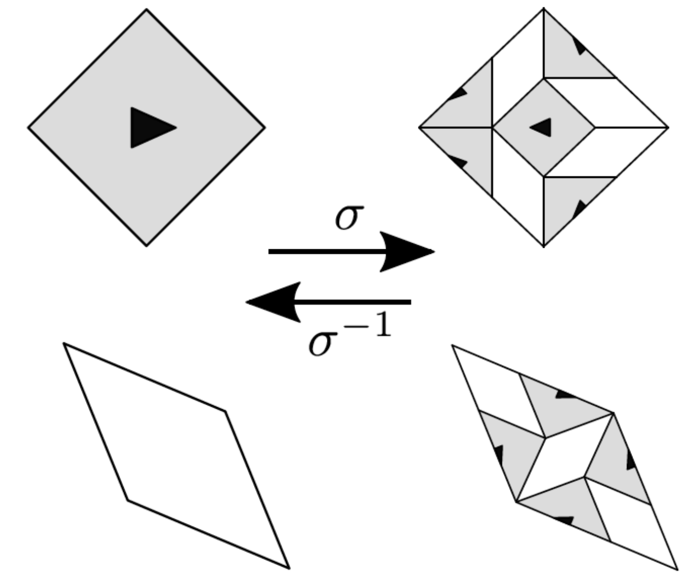
Any other possibilities?

[Flicker, Simon, SP, PRX 2020]

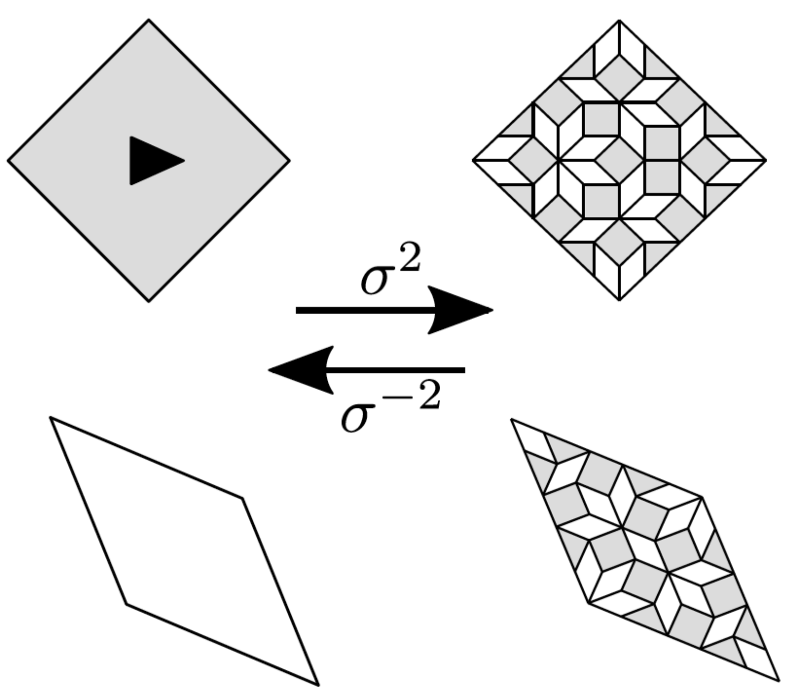
Amman-Beenker Tiling

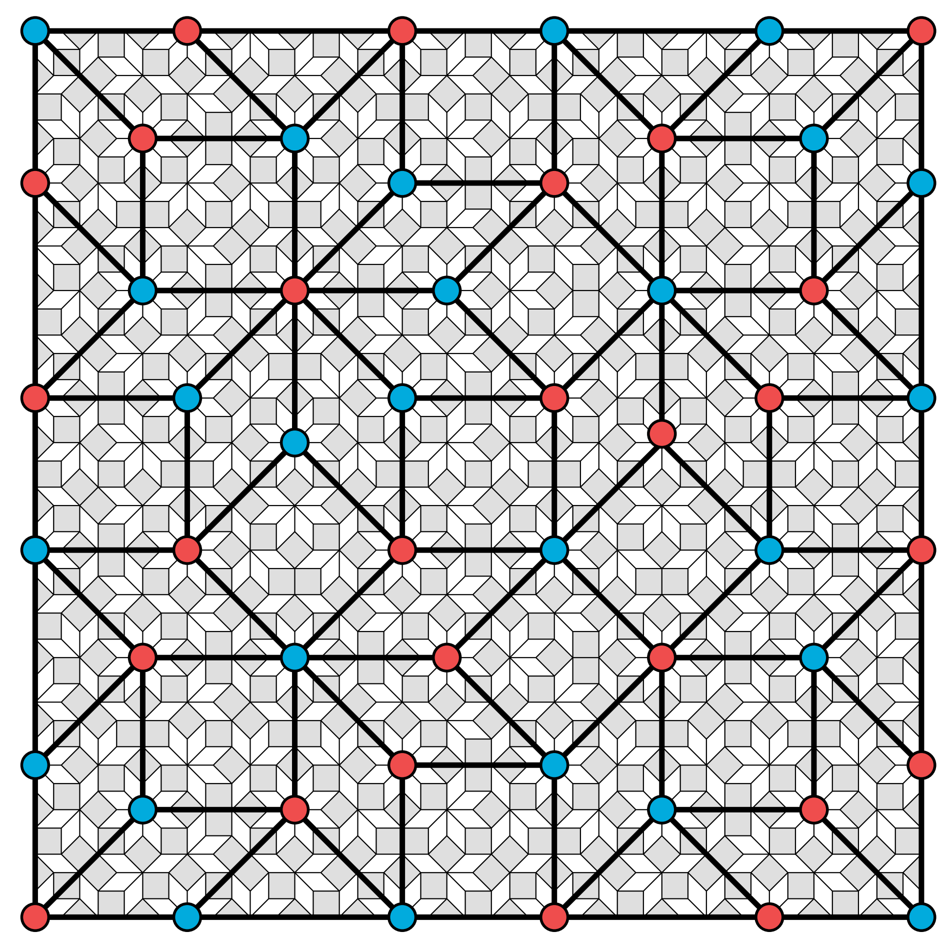
- Eight-fold symmetric tiling of square + rhombus tiles
- Inflation/deflation: discrete scaling by silver ratio

$$\delta_S = 1 + \sqrt{2} = 2.414\ldots$$



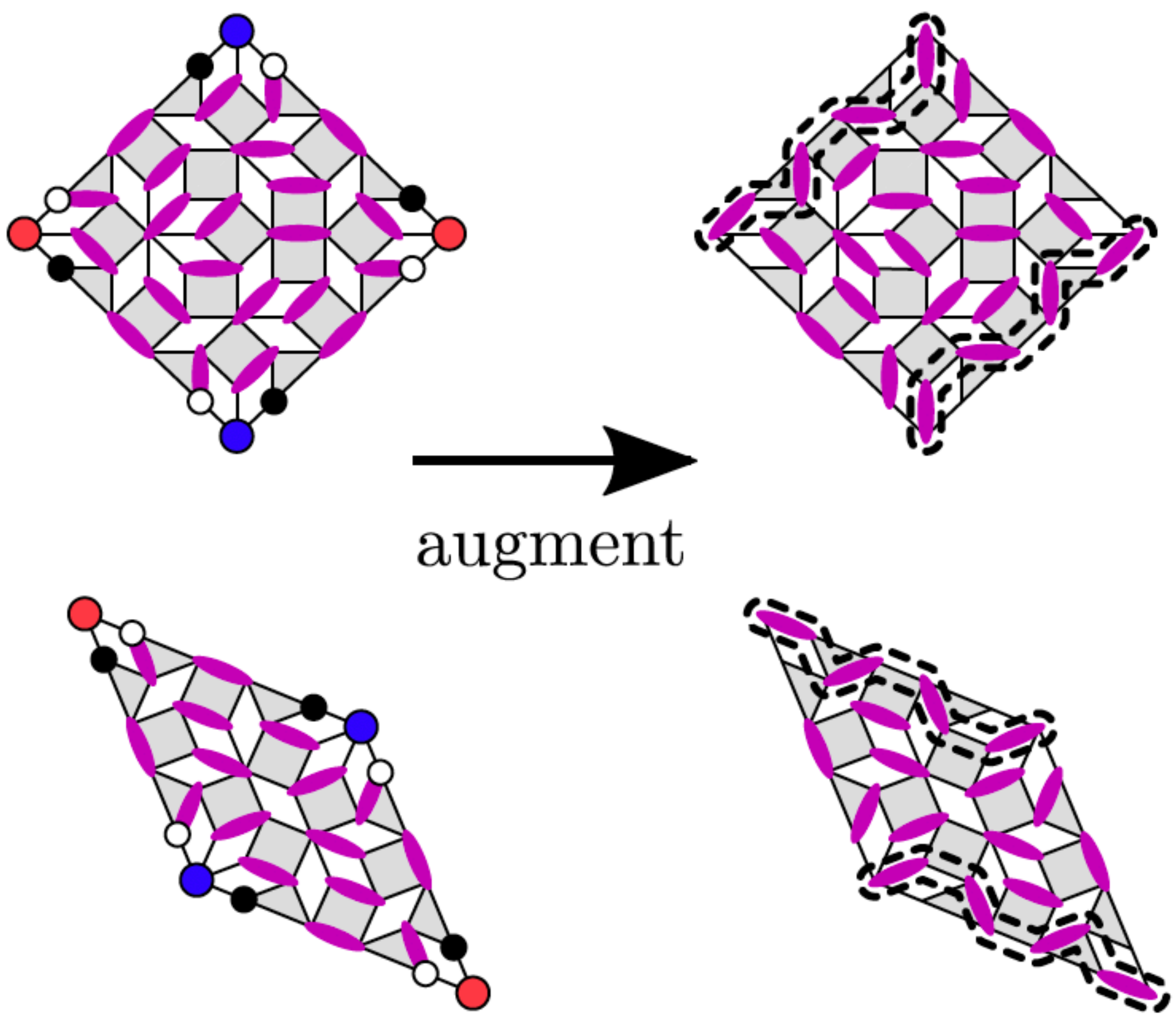
- Vertices with coordination 3, 4,…, 8
- 8-vertices lie on another AB tiling, bigger by δ_S^2
    - preserved by 2 flations





Perfect Matchings

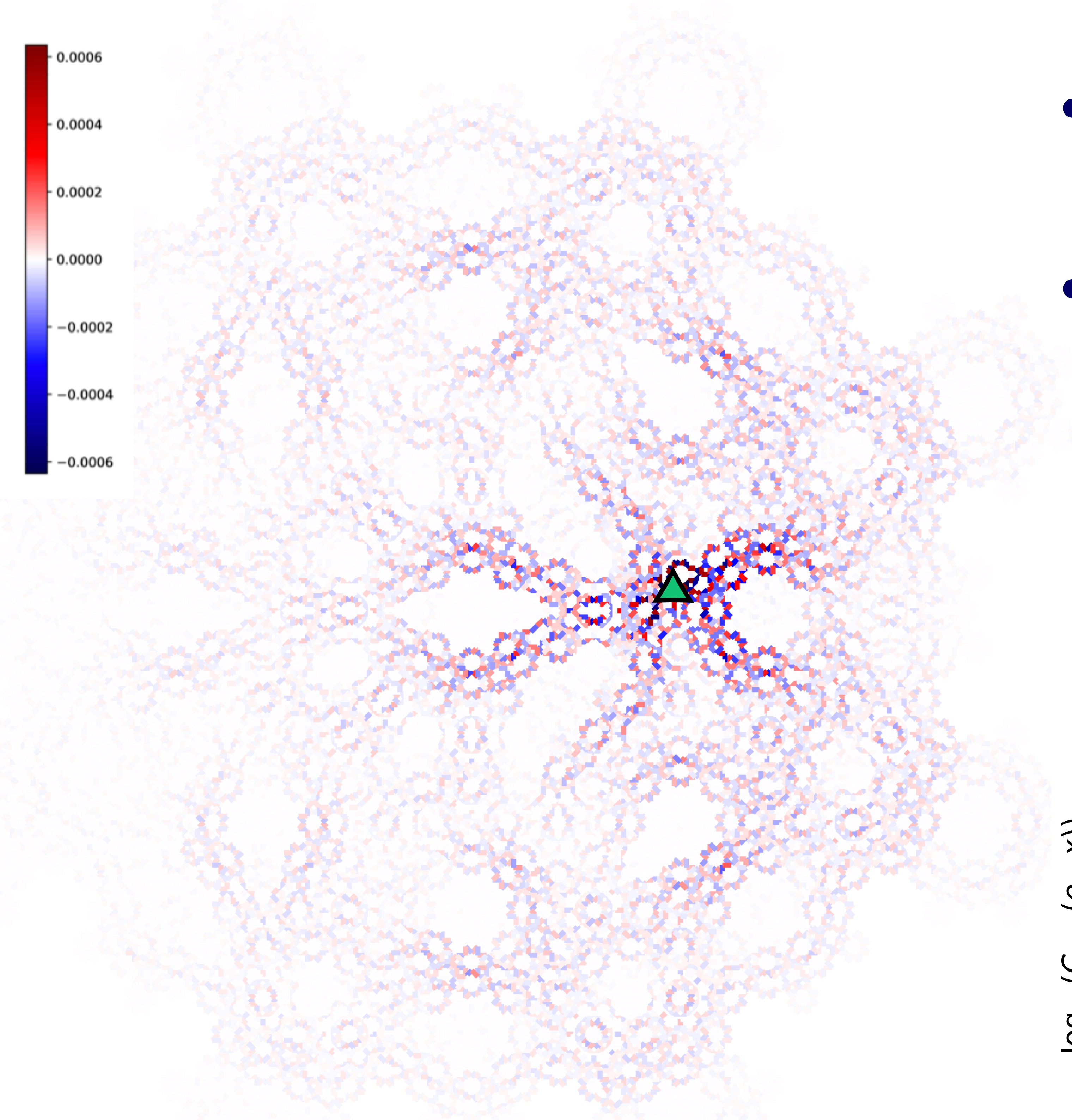
- Use DSI to show monomer density vanishes as $N \to \infty$
- Idea: inflate tiles decorated w/ dimers: matches all but 8-vertices
- Use tiling properties to match 8-vertices



Bound on monomer density after $2n$ inflations:

$$\rho^{(2n)} \leq \frac{1}{\delta_S^{8n}} \lesssim (0.03)^{2n}$$

Dimer Correlations in Perfect Matchings

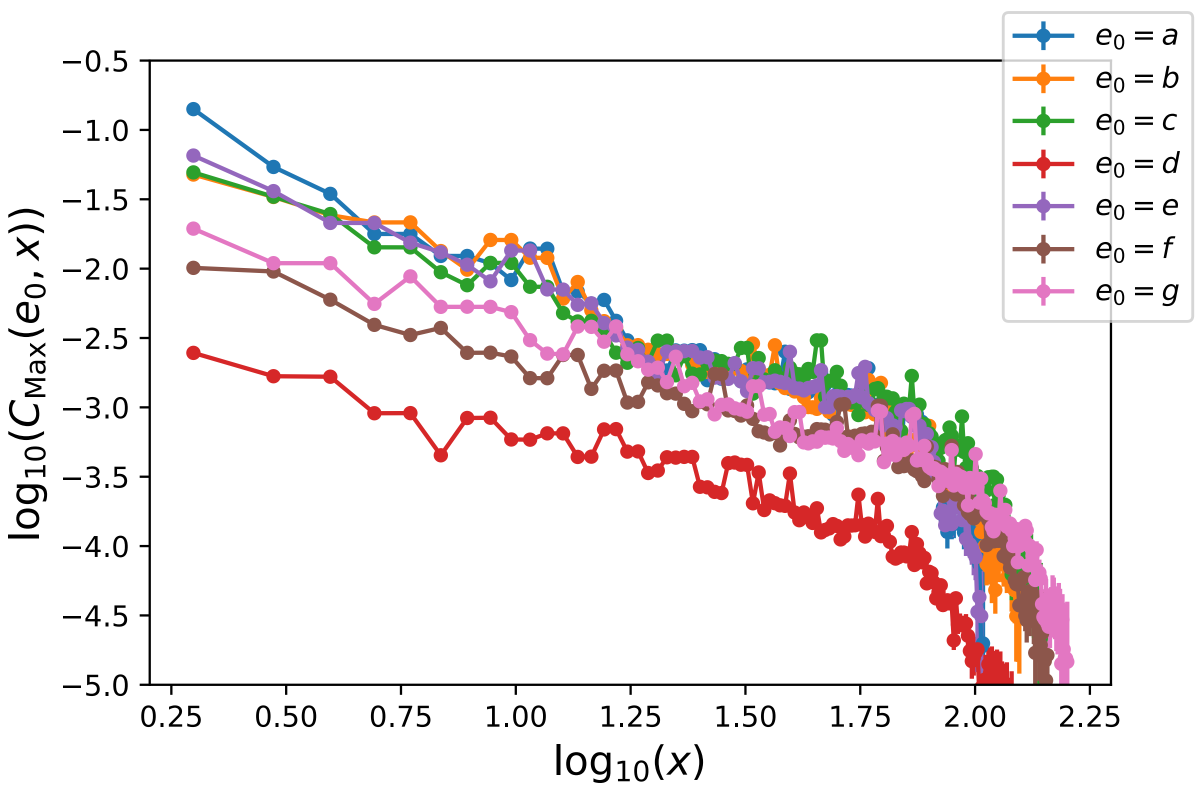


"Directed-loop" MC sampling

[Sandvik-Moessner PRB '05]

- Finite density of links w/ longer-ranged correlations
- Consistent w/ asymptotic power law (out to x~100)

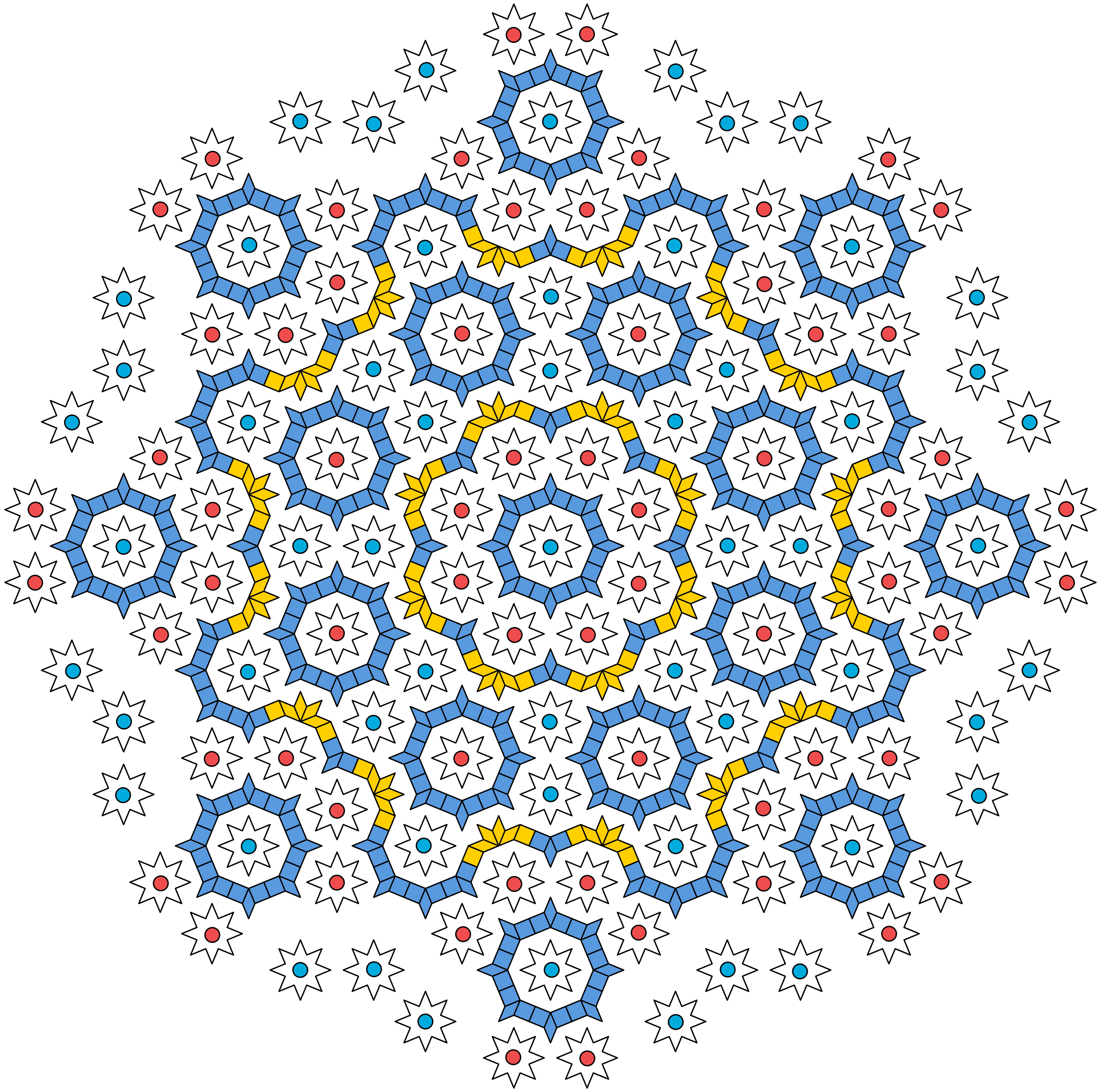
Can we say more?



Auxiliary Problem: AB* tiling

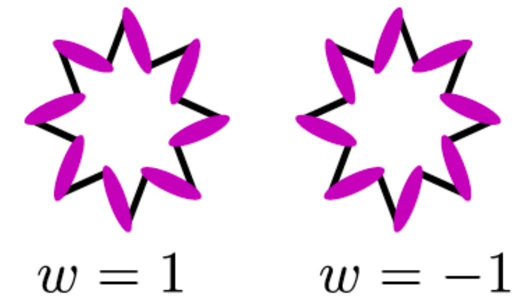
- Simplify problem: remove 8-vertices
- Exact membranes (cf Penrose) =
- No dimers on membrane links in perfect matching
- Dimer cover configs "disconnect"
- Partition function factorizes over 0D "stars" and 1D quasiperiodic "ladders"

$$Z_{\text{tiling}} = Z_*^{N_*} \prod_n (Z_{\text{lad}_n})^{N_{\text{lad}_n}}$$

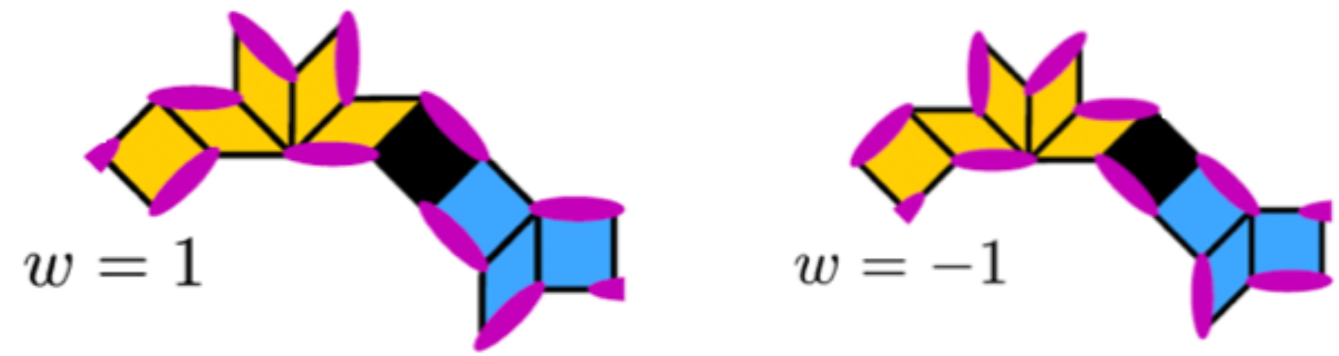


Exact Results on AB*

- Stars: two "staggered" configurations $Z_* = 2$

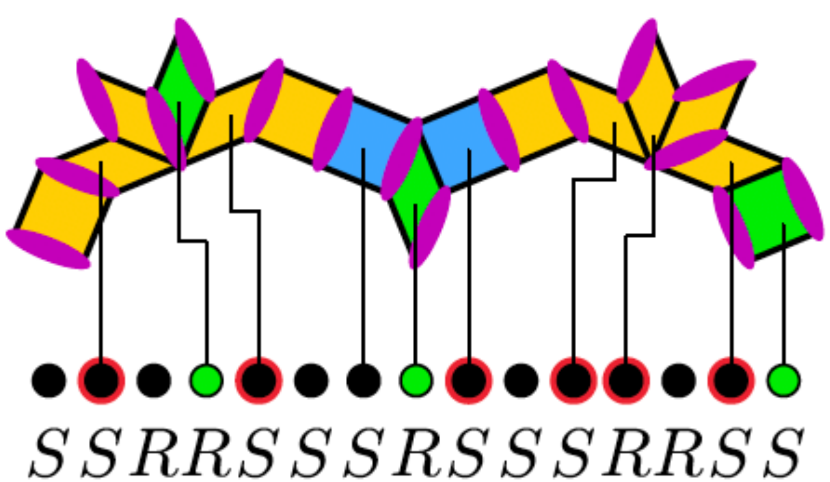


- Quasiperiodic ladders
  - two staggered configurations



  - columnar configs via transfer matrix (iteratively multiplying quasiperiodic strings of 2x2 matrices or using "trace map")

[trace map: Kohmoto, Kadanoff, Tang '83]



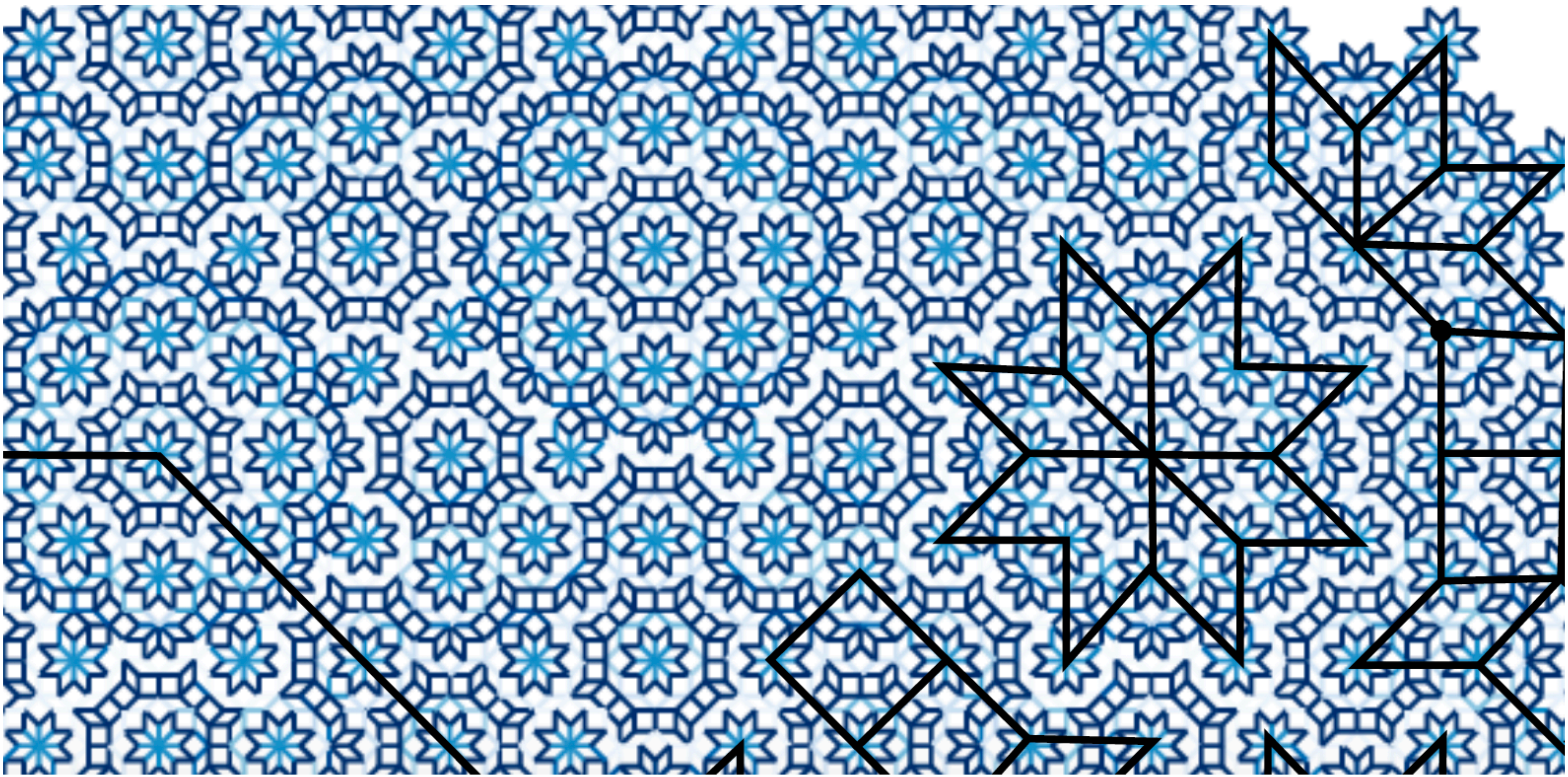
$$Z_{\mathrm{lad},n} = 2 + \mathrm{Tr}[\underbrace{RSSR\ldots}_{\text{q.p. string}}]^8$$

  - combine w/ density of stars/ladders: entropy per dimer ~ 0.436

cf. ~1.71 for square lattice [Kastelyn '61]

Back to AB: "Pseudomembranes" and Discrete Scaling

- Reinstate 8-vertices: remaining problem is to match 8-vertices to each other!
- Membranes no longer exact: no more than 1 dimer on "pseudomembrane" links
- DSI: 8-vertex dimer problem is "effective dimer" problem via pseudomembranes



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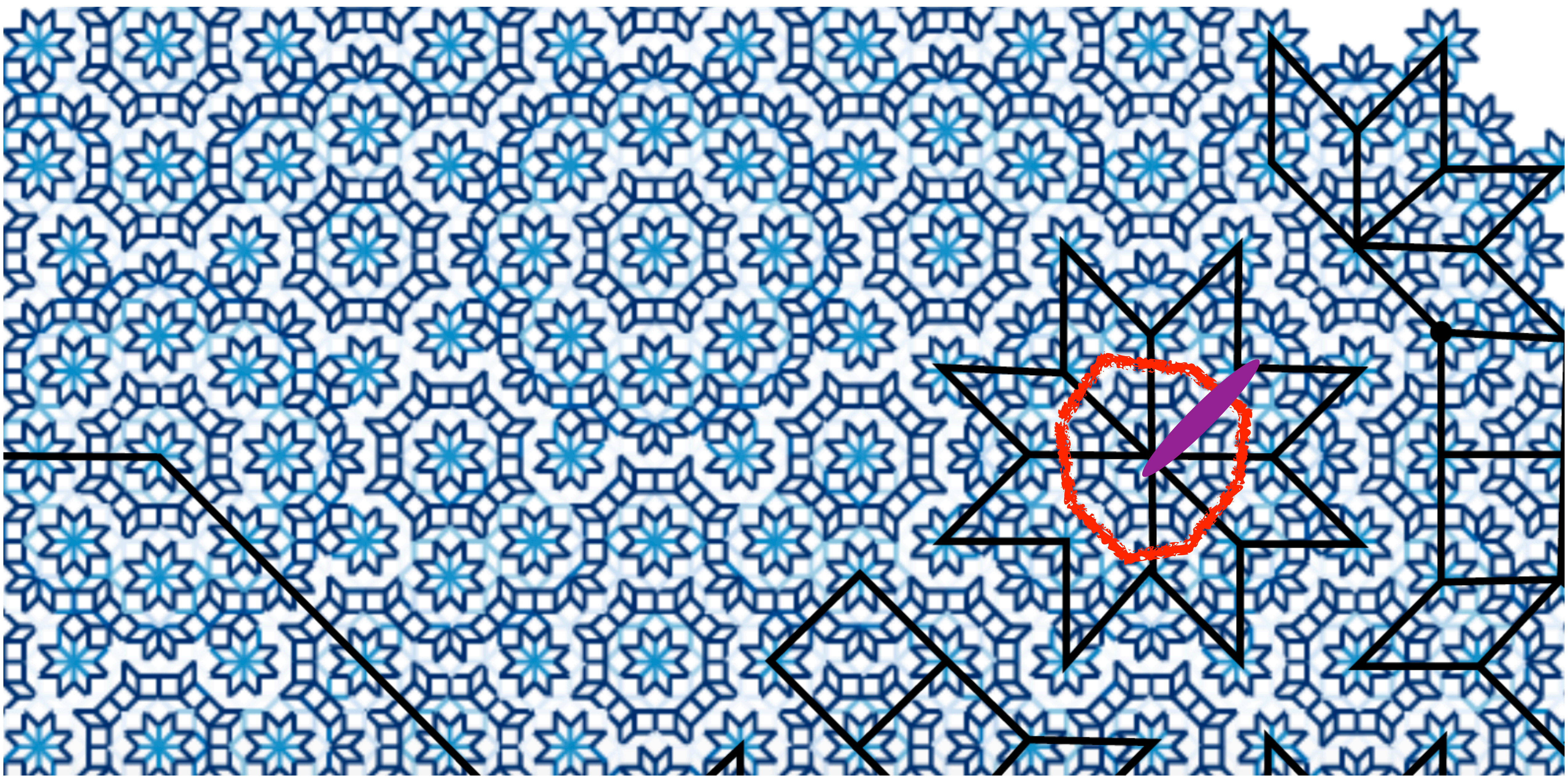

Membranes from graph theory

- Membranes linked to "coarse"/"fine" Dulmage-Mendelsohn graph decomposition

[Bhola-Biswas-Islam-Damle arxiv:2007.04974]

adjacency matrix

$$\mathcal{A} = \begin{pmatrix} 0 & \mathcal{G} \\ \mathcal{G}^{\mathcal{T}} & 0 \end{pmatrix}$$

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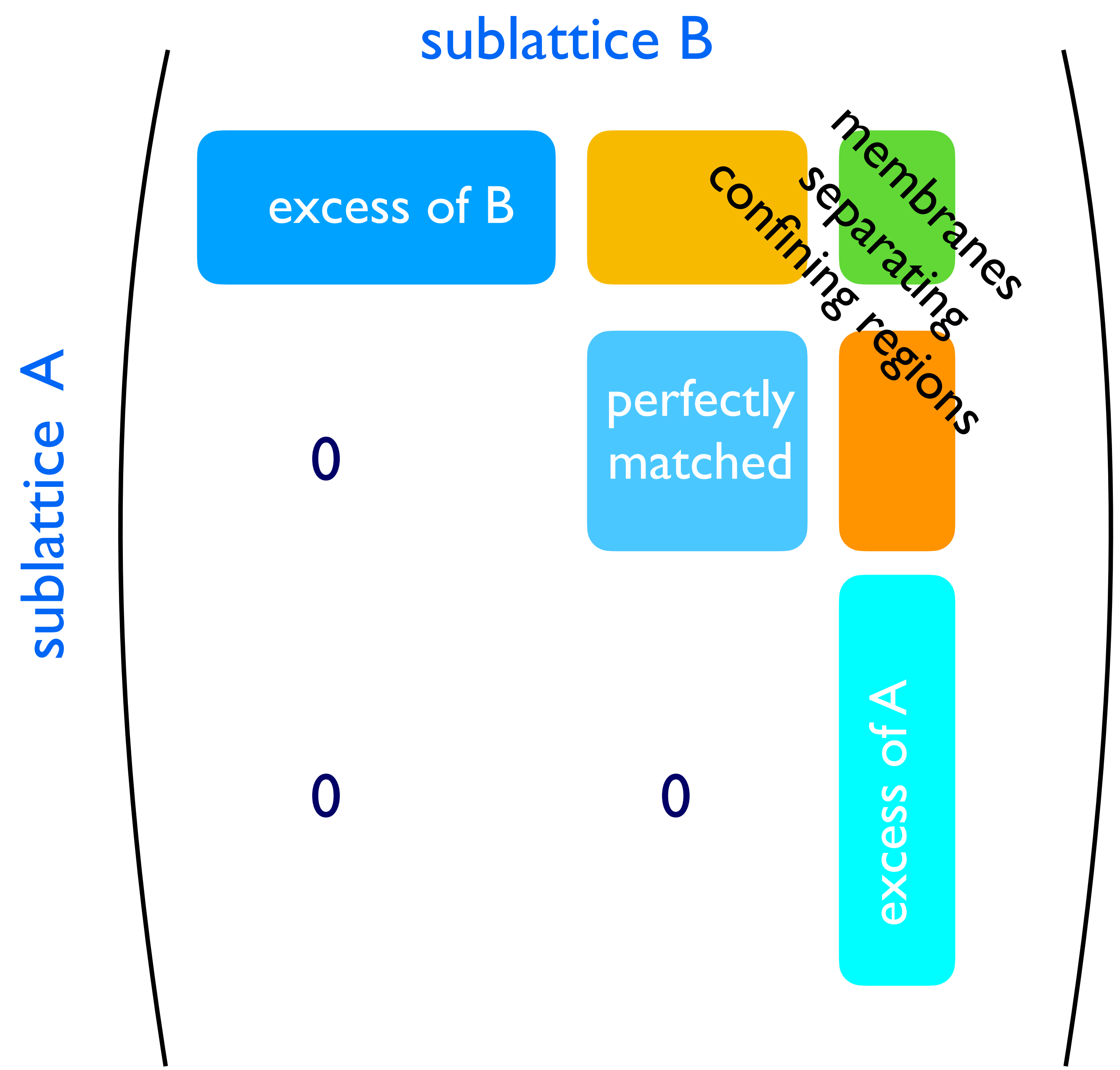
coarse

$\mathcal{G} =$



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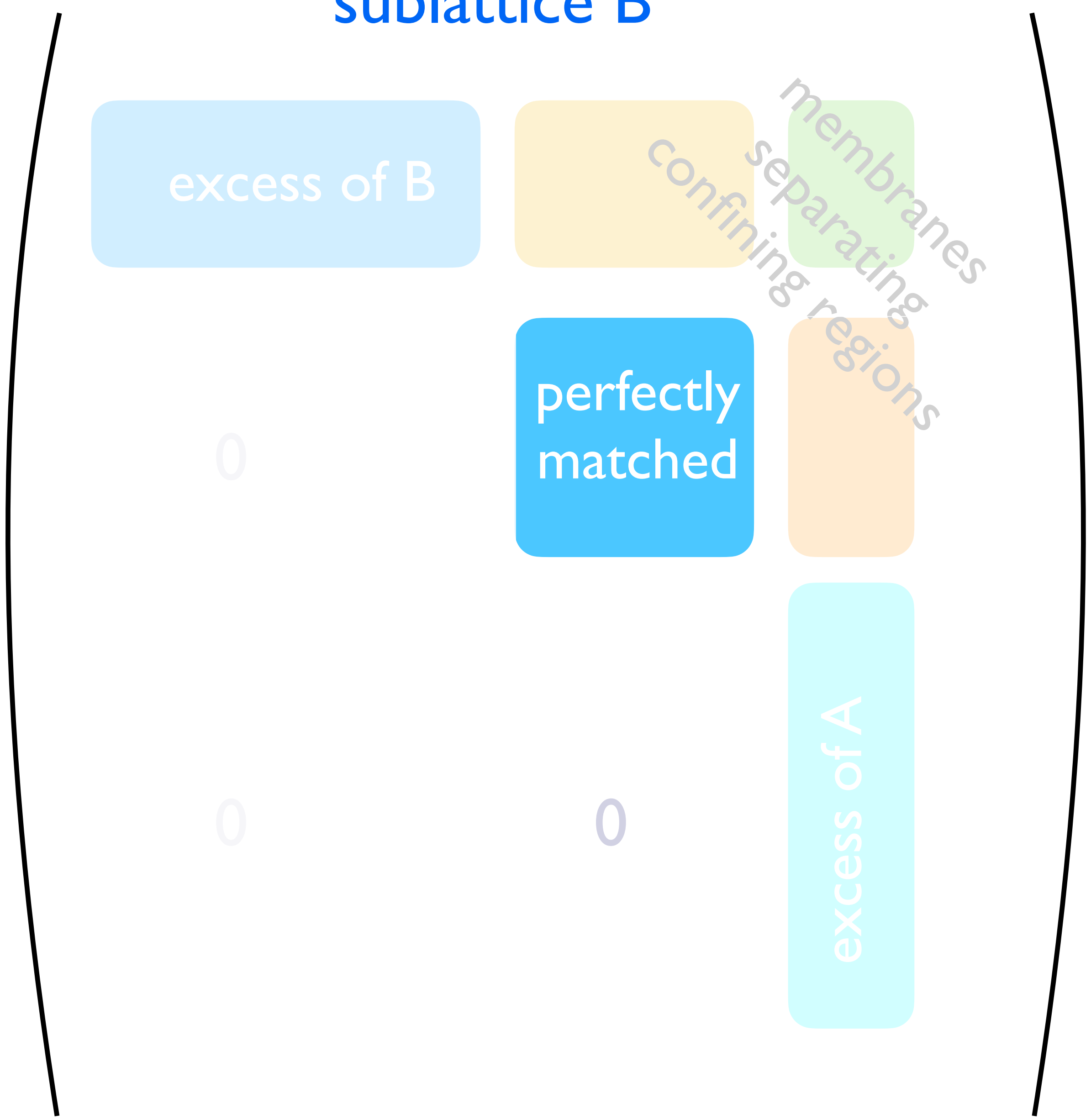
adjacency matrix $\mathcal{A} = \begin{pmatrix} 0 & \mathcal{G} \\ \mathcal{G}^T & 0 \end{pmatrix}$

coarse

sublattice B

$\mathcal{G} =$ sublattice A



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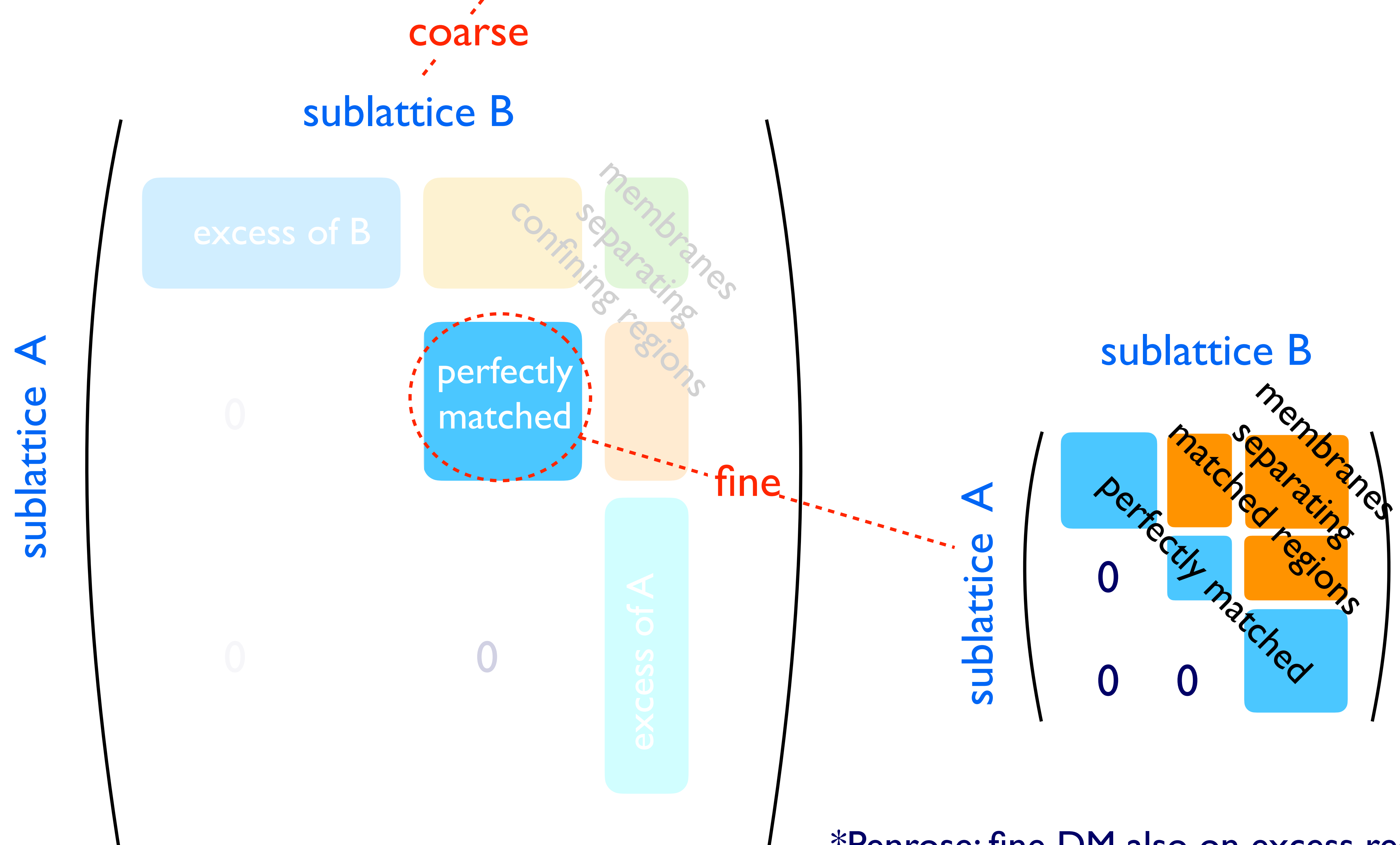
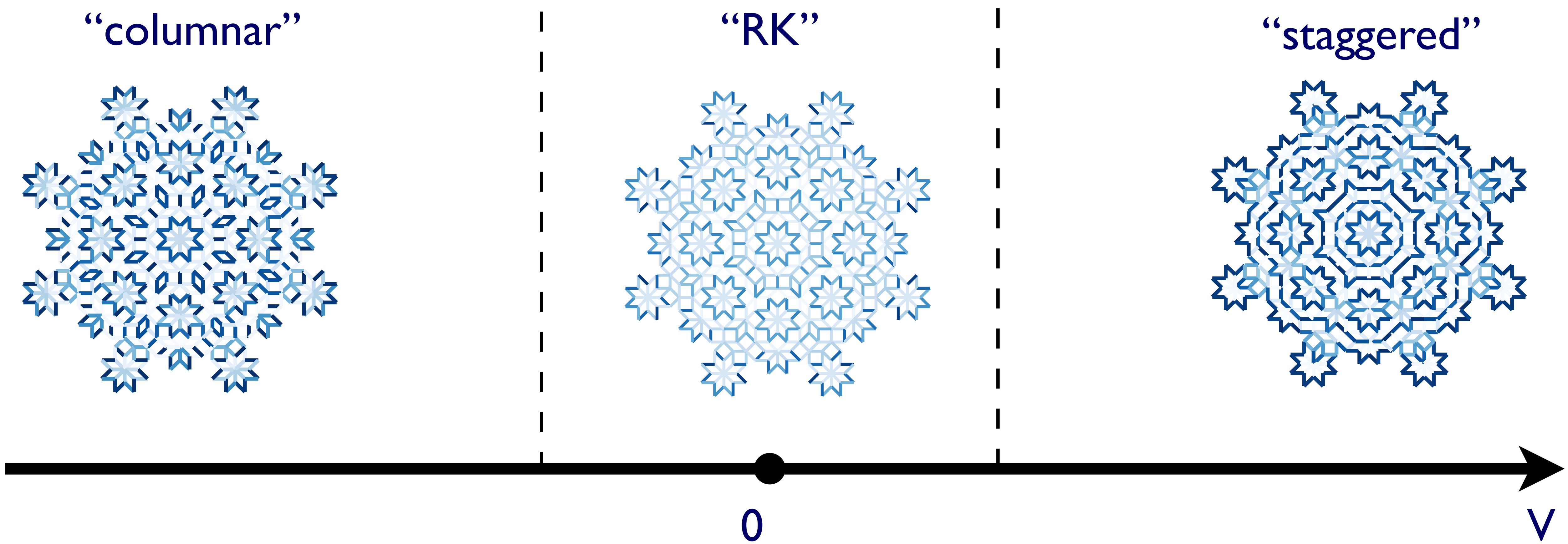


*Penrose: fine DM also on excess regions

Phase diagram w/ Aligning Interactions

- RK "potential" on flippable plaquettes $\hat{H} = V\left(|\square\rangle\langle\square| + |\square\rangle\langle\square|\right)$



- Crossovers as T is tuned
  - Can also compute V≠0 free energy exactly on ladders (3x3 transfer matrices)
  - Ladders have 1st order transition to staggered, but do not drive AB* transition

Summary

- Dimers on quasicrystals show rich physics
  - Fractally-confined monomers in maximum matchings (Penrose)

    [Flicker, Simon, SP, PRX 2020]
  - Slowly decaying dimer correlations in perfect matchings (Amman-Beenker)
    - Understand by proximity to AB* w/ "exact factorization" property
    - Dimer model apparently invariant under DSI: origin of power laws?
    - Aligning interactions ~ analog of "columnar" and "staggered" VBS

    [Loyd, Biswas, Simon, SP, Flicker arXiv to appear]
- Future: <u>quantum</u> dimers
  - QP ladders give some insight — DMRG ongoing
  - Can power-law correlations survive quantum fluctuations in $d=2$?
- Two intriguing connections: (1) hopping zero modes (2) fractons

Connection #1: Index Theorems and Zero Modes

- Bipartite Random Hopping ~ chiral symmetry class (AIII of Altland-Zirnbauer)

$$H = \begin{pmatrix} \mathbf{0}_{N_A \times N_A} & -\mathbf{t}_{AB} \\ -\mathbf{t}^*_{AB} & \mathbf{0}_{N_B \times N_B} \end{pmatrix}$$

$\mathbf{t}_{AB}$ ~ $N_A \times N_B$ matrix

- ε=0 is "special" b/c sublattice transformation takes $\mathbf{t}_{AB} \mapsto -\mathbf{t}_{AB}$

- central question: what is DoS near ε=0 ? $\rho(\varepsilon) = N_z \delta(\varepsilon) + \rho_{\rm smooth}(\varepsilon)$

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- central question: what is DoS near ε=0 ? $\rho(\varepsilon) = N_z \delta(\varepsilon) + \rho_{\text{smooth}}(\varepsilon)$
- N_z ~ number of exact zero modes
  - nontrivial bound (even if N_A=N_B) $N_z \geq N_A + N_B - 2N_{\text{dimer}} = N_{\text{monomer}}$

real-space index theorem [Longuet-Higgins 1950]

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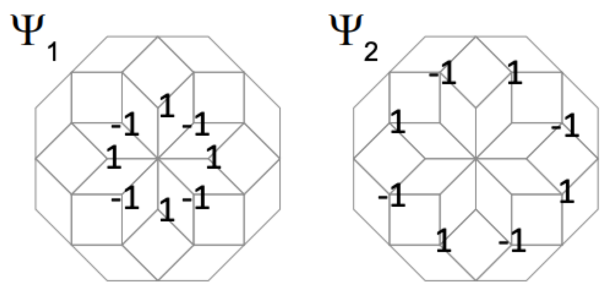
    real-space index theorem [Longuet-Higgins 1950]
- Index theorem distinguishes 2 types of zero modes on quasicrystals
  - confined monomers →"strong zero modes" (survive random *t*)

    [Flicker et al '20; Koga and Tsunetsugu '15, Day-Roberts et al '20]
  - local motifs → "fragile zero modes" (lost for random *t*)

    [Koga '21]



Connection #2: Fractons?

Fractons	QC Dimers
conventional 3d lattice	quasicrystal
unusual gauge structure (“subsystem symmetry”)	standard gauge structure (“dimer Gauss law”)
gauge charges at endpoints of fractals	gauge charges at endpoints of loops
mobility constraints on charges b/c “dipole conservation”	mobility constraints on charges b/c quasiperiodicity + dimer rules
groups of quasiparticles can move freely	pairs of monomers can cross membranes

Thanks for listening!