APPLYING TNS TO LATTICE GAUGE THEORIES

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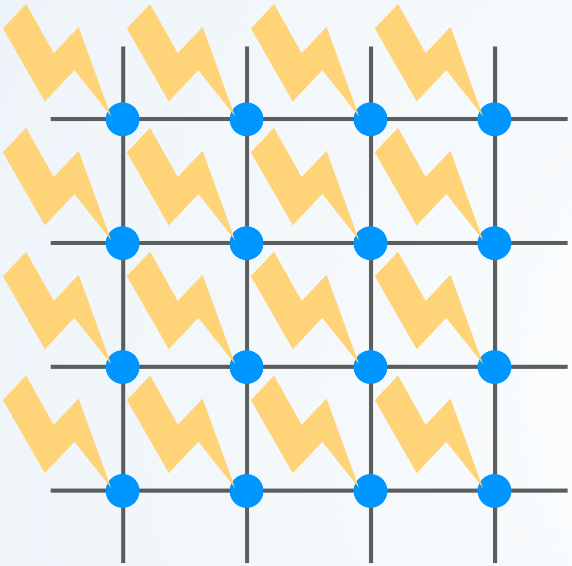
Benasque SCS February 2021

WHAT ARE LGT?

theory with a local
(gauge) symmetry

ISING MODEL



$$H = \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + g \sum_i \sigma_i^x$$

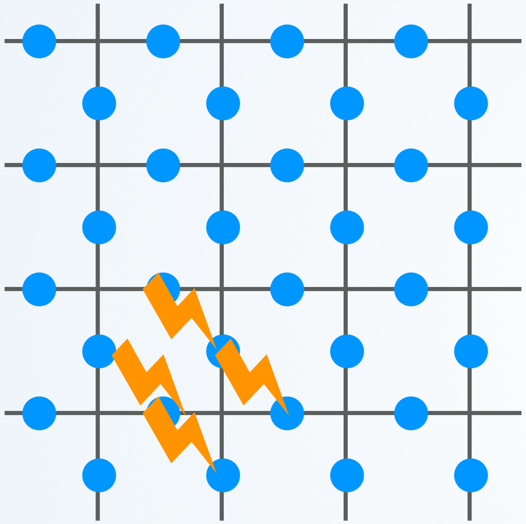
global $\mathbf{Z}_2$ symmetry

$$U = \otimes_i \sigma_i^x$$

$U \sigma_k^z U^\dagger = -\sigma_k^z$ flips all spins at once

ISING LATTICE GAUGE THEORY

Wenger 1971

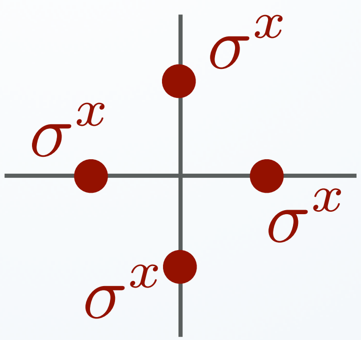


$$H = \sum_{\square} \sigma^z_{\ell_1}\sigma^z_{\ell_2}\sigma^z_{\ell_3}\sigma^z_{\ell_4} + g\sum_{\ell}\sigma^x_{\ell}$$

local Z_2 symmetry

$$G_i \sigma^z_\ell G_i = -\sigma^z_\ell$$

$G =$



$$[G_i, H] = 0 \qquad \forall i$$

$G_i|\Psi\rangle = |\Psi\rangle$ Gauss' law

phase transition: deconfined / confined

promoting a global symmetry to local

gauging the symmetry

fundamental role in HEP

QED

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$ Dirac fermion

global $\mathrm{U}(1)$ symmetry $\quad \psi(x) \to e^{i\theta}\psi(x)$

global phase

local $\mathrm{U}(1)$ $\quad \psi(x) \to e^{i\theta(x)}\psi(x)$

$$\bar{\psi}\partial^\mu\psi \to \bar{\psi}\left(\partial^\mu + i\partial^\mu\theta\right)\psi$$

solution $\quad \partial^\mu \to D^\mu = \partial^\mu + iA^\mu$

$$A^\mu \to A^\mu + \partial^\mu\theta$$ gauge field

gauge invariant dynamical term

$$\mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

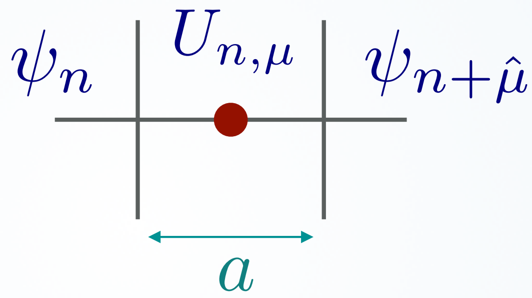
WILSON'S LGT

Wilson 1974

discretized action → loses gauge invariance

discrete derivatives

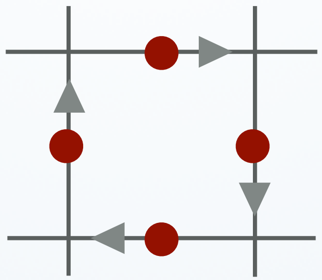
$$\frac{\psi_{n+\hat{\mu}} - \psi_{n-\hat{\mu}}}{2a}$$



$$\bar{\psi}_n \gamma_\mu U_{n,\mu} \psi_{n+\hat{\mu}}$$

$$U_{n,\mu} \to e^{ig\alpha_n} U_{n,\mu} e^{-ig\alpha_{n+\hat{\mu}}}$$

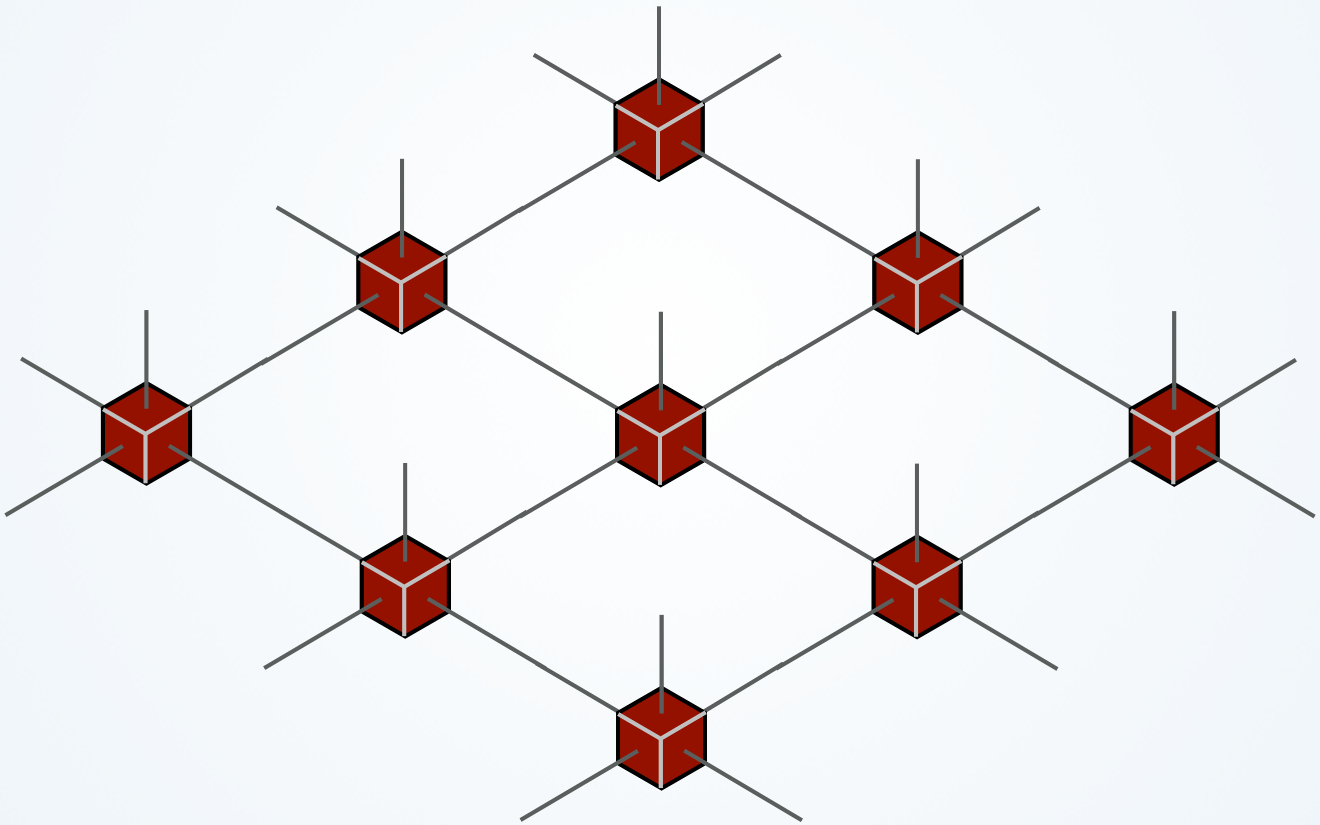
dynamics of gauge dof



$$\mathrm{Tr}\left(U_\square\right) + \mathrm{h.c.}$$

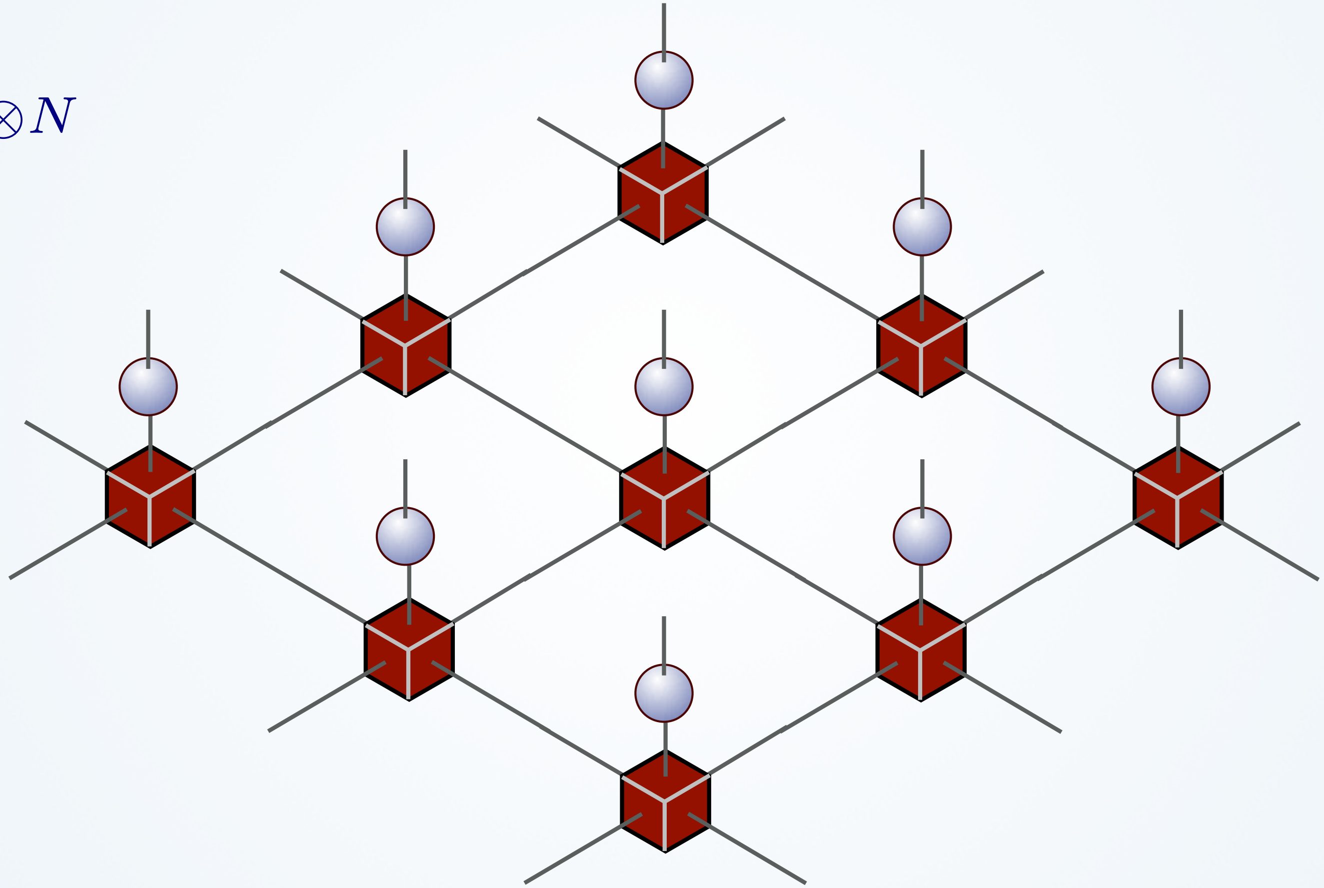
$a \to 0$ continuum QFT

GAUGING PEPS



GAUGING PEPS

$U_g^{\otimes N}$



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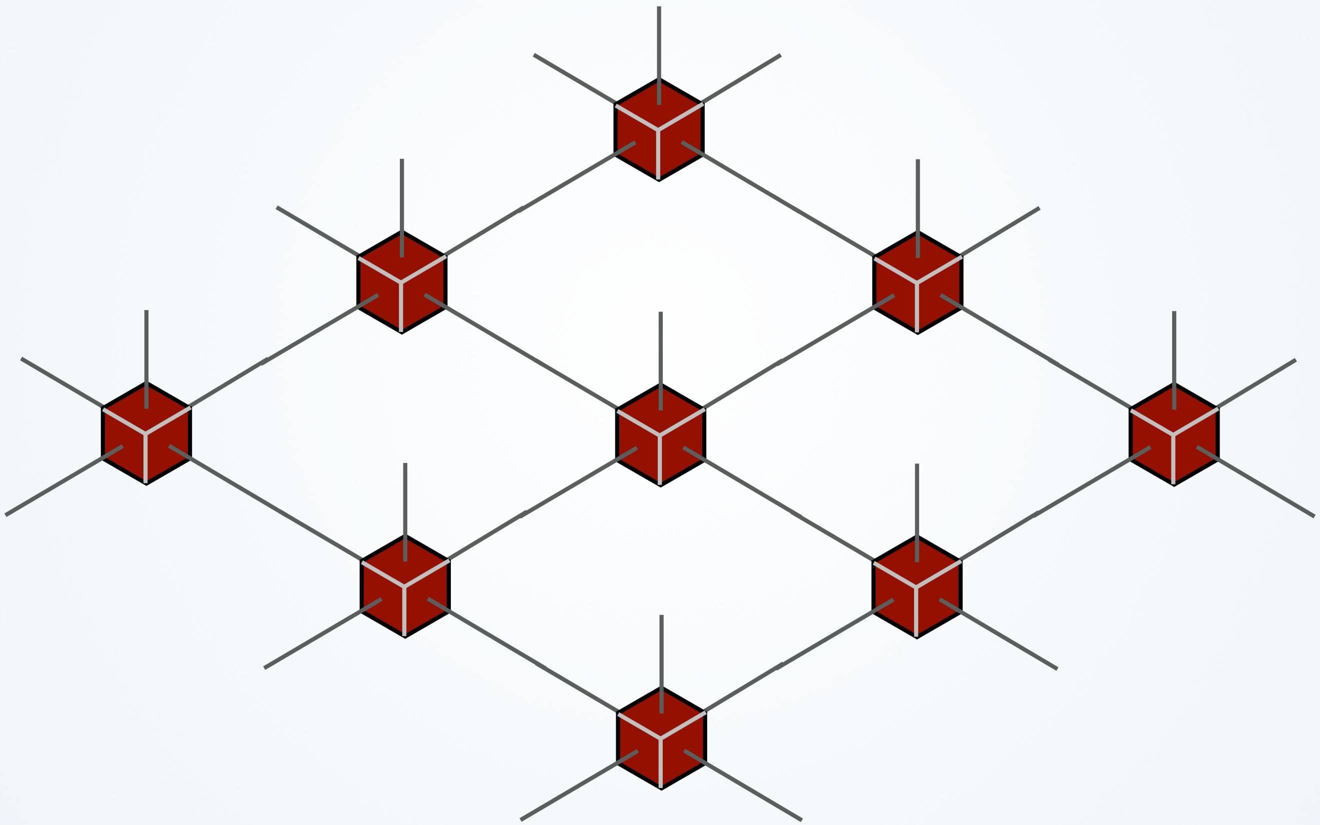

GAUGING PEPS

state (globally) invariant ⇔

Pérez-García et al., PRL 2008
Sanz et al., PRA 2009
Schuch et al., Ann. Phys. 2010
Singh et al., NJP 2007, PRA 2010

GAUGING PEPS

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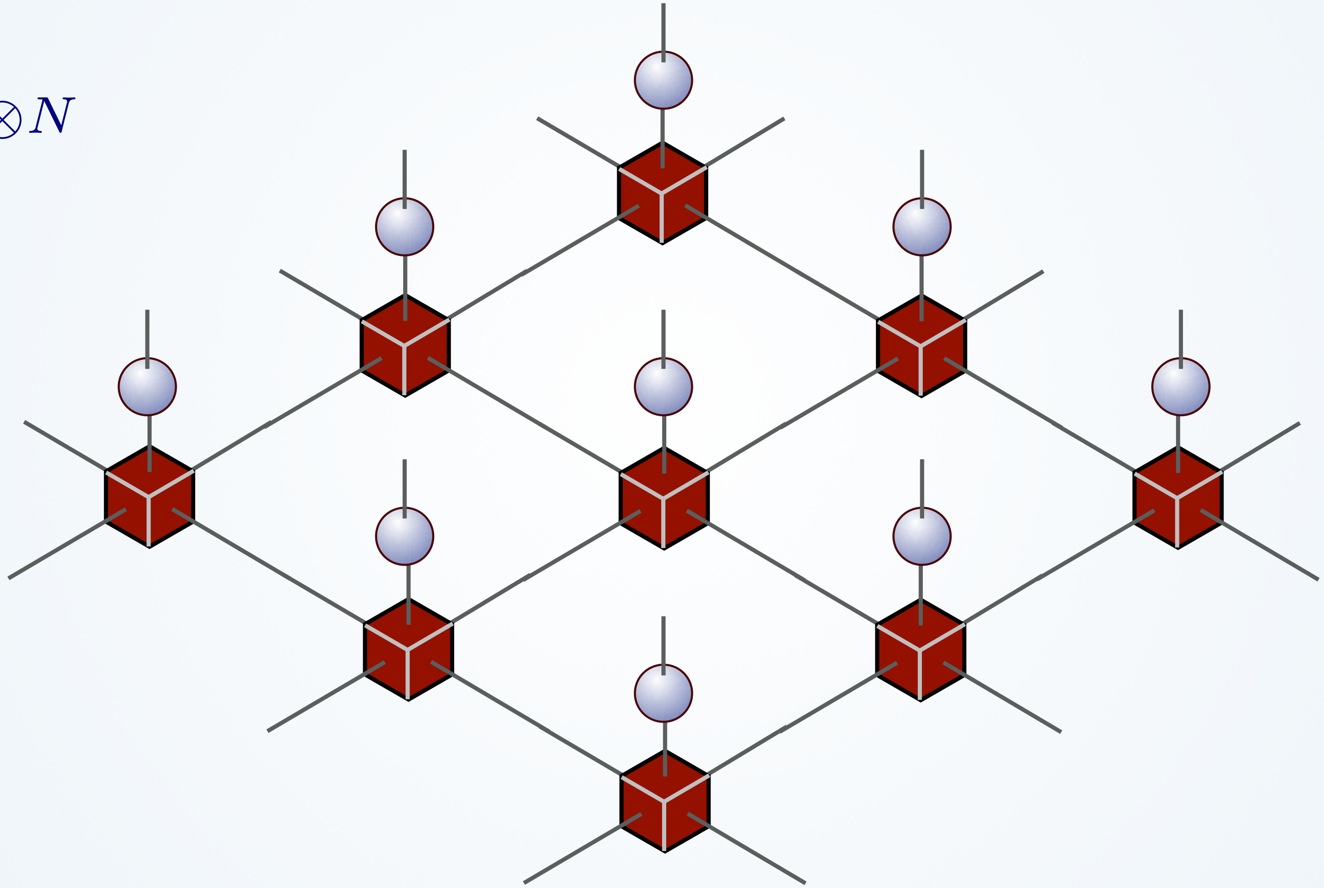

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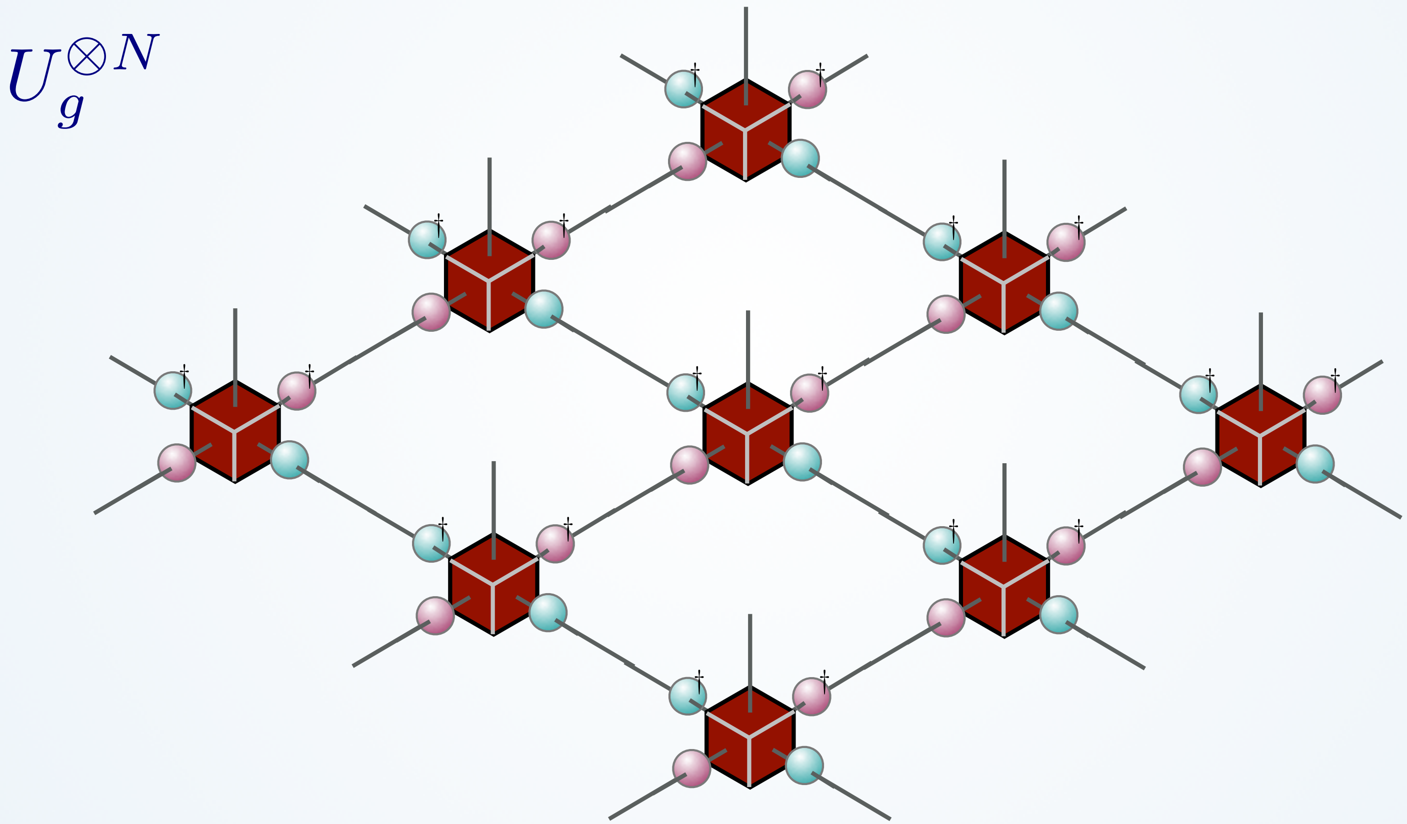

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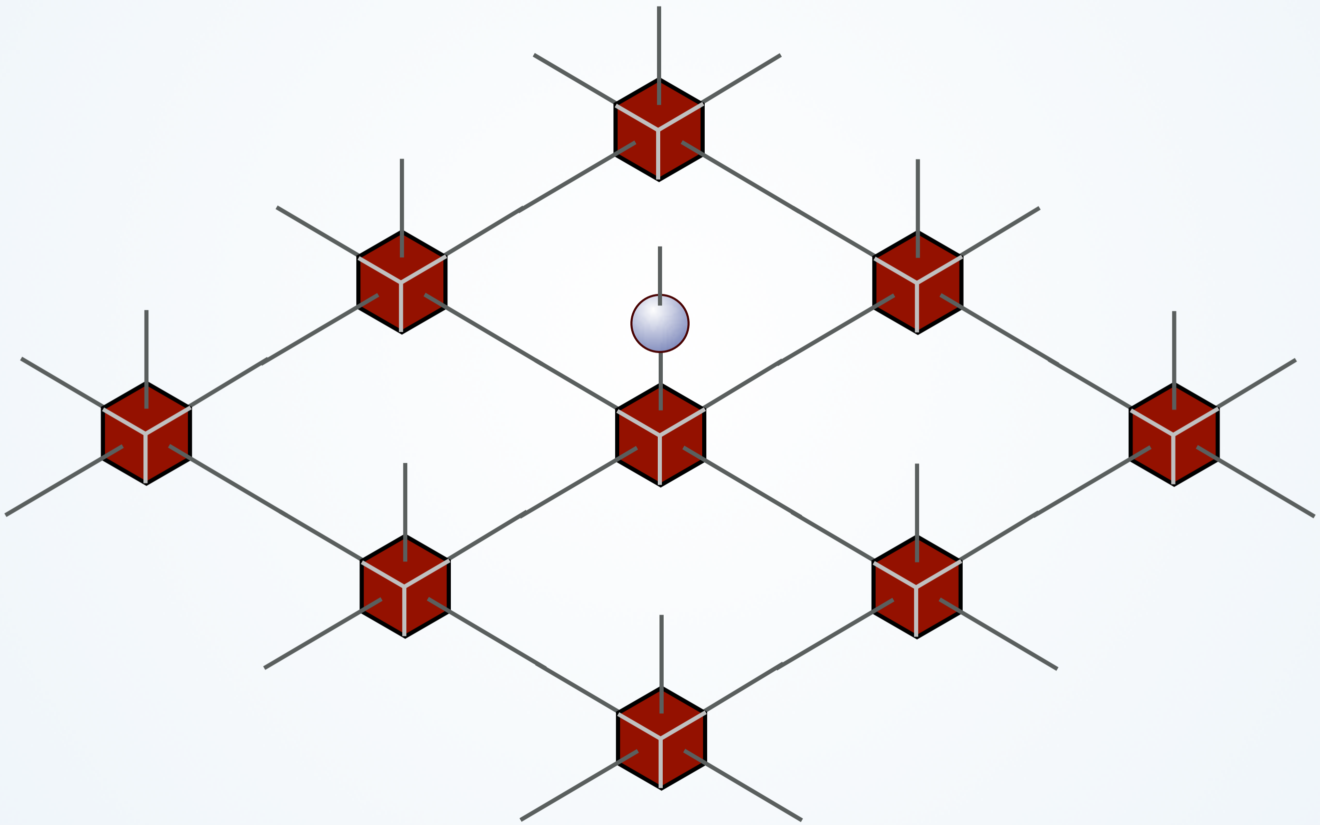

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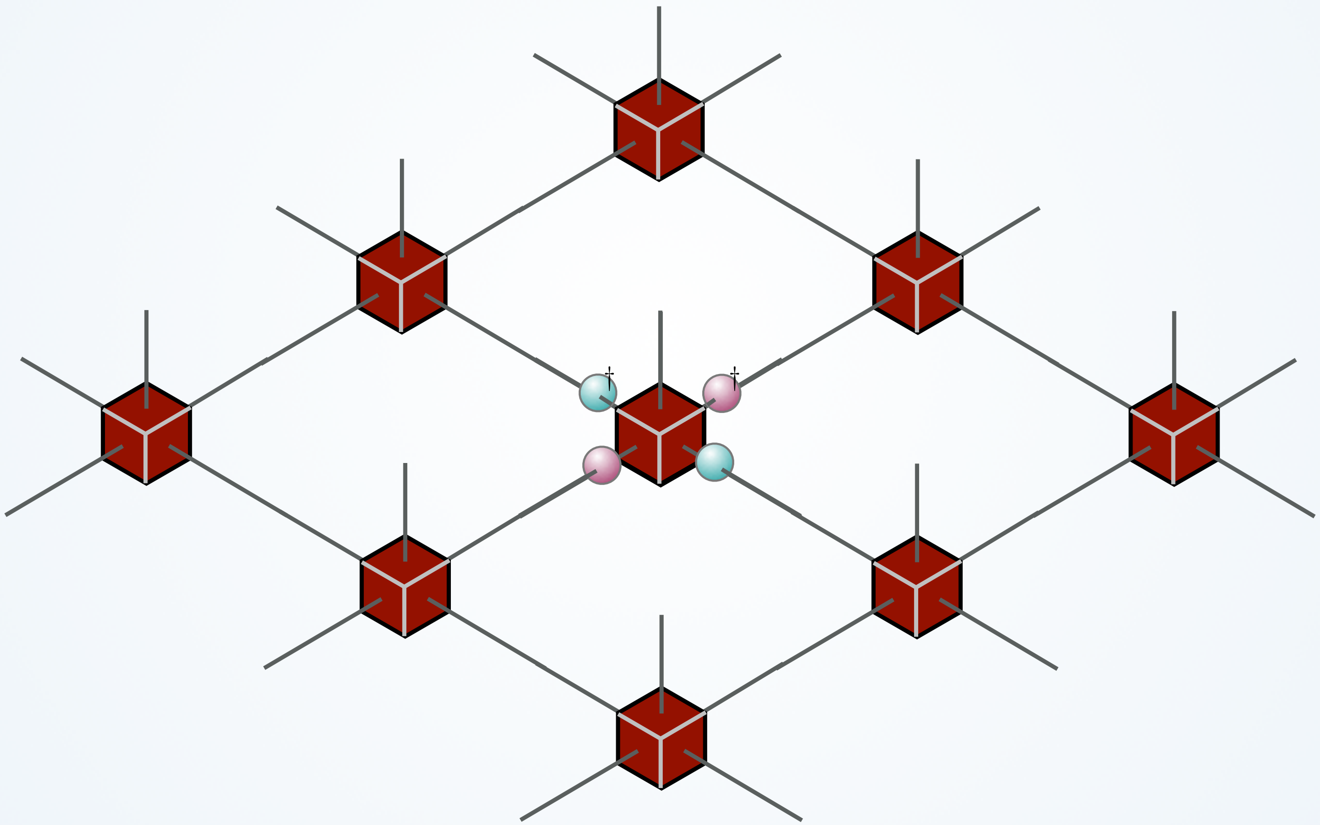

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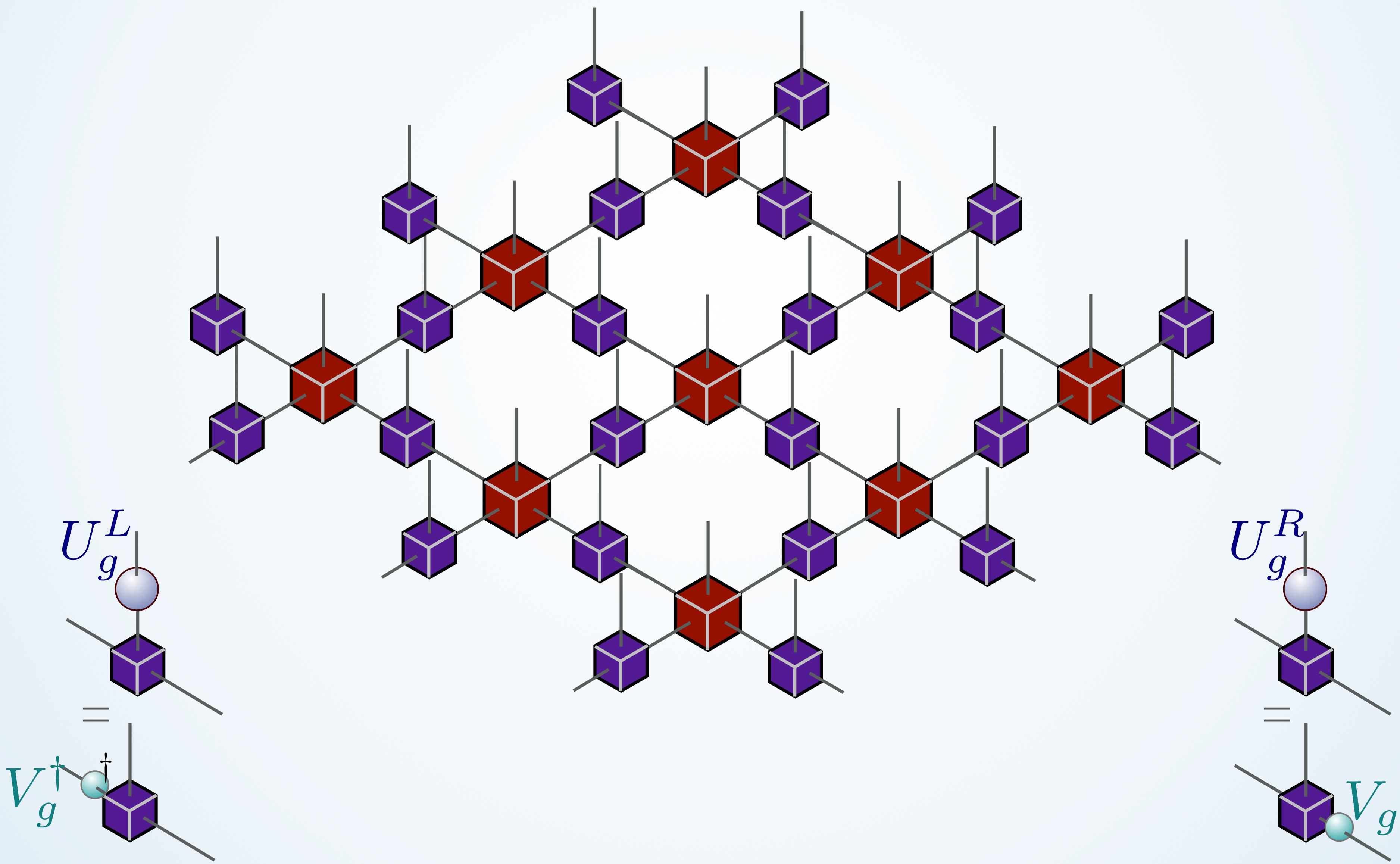

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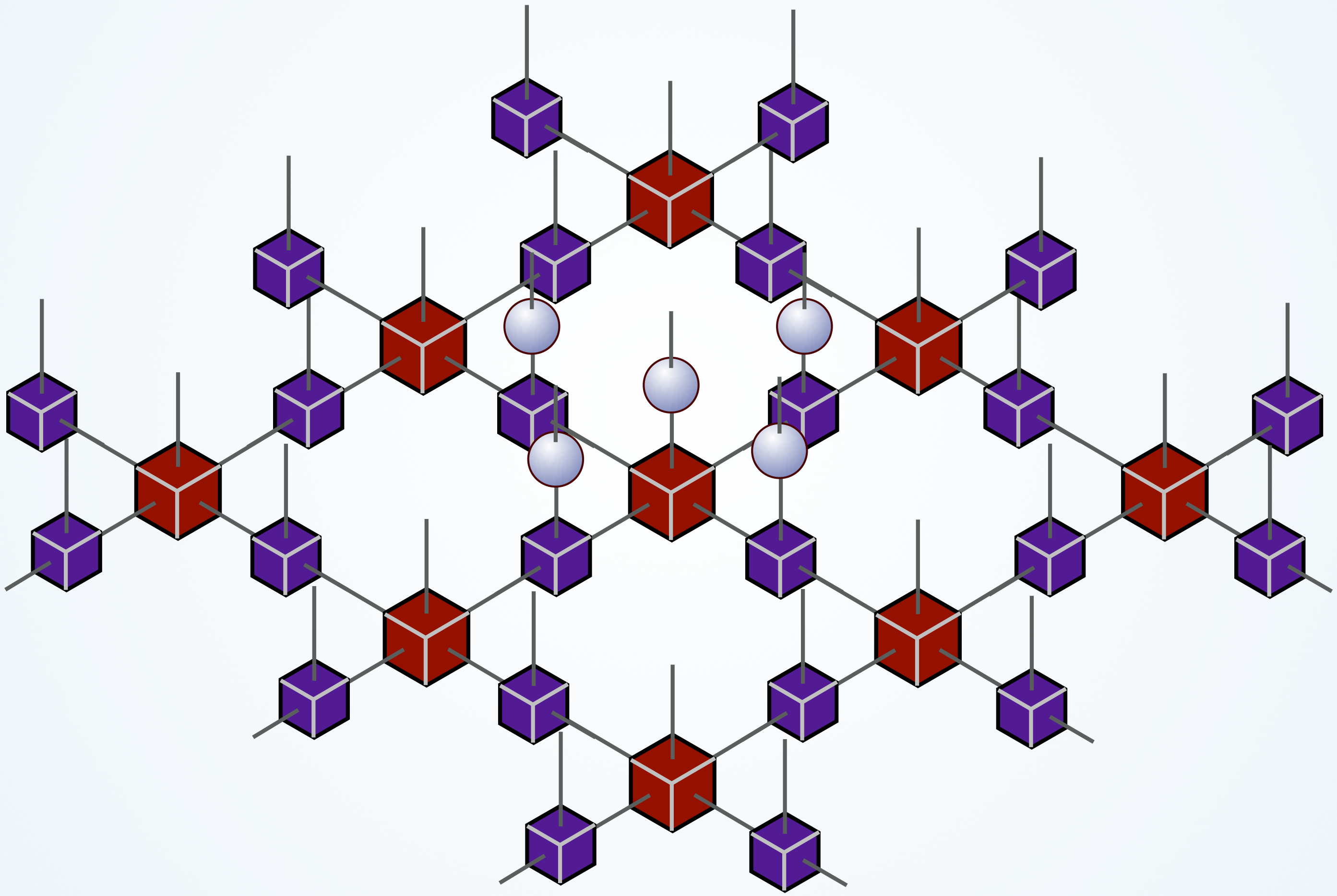
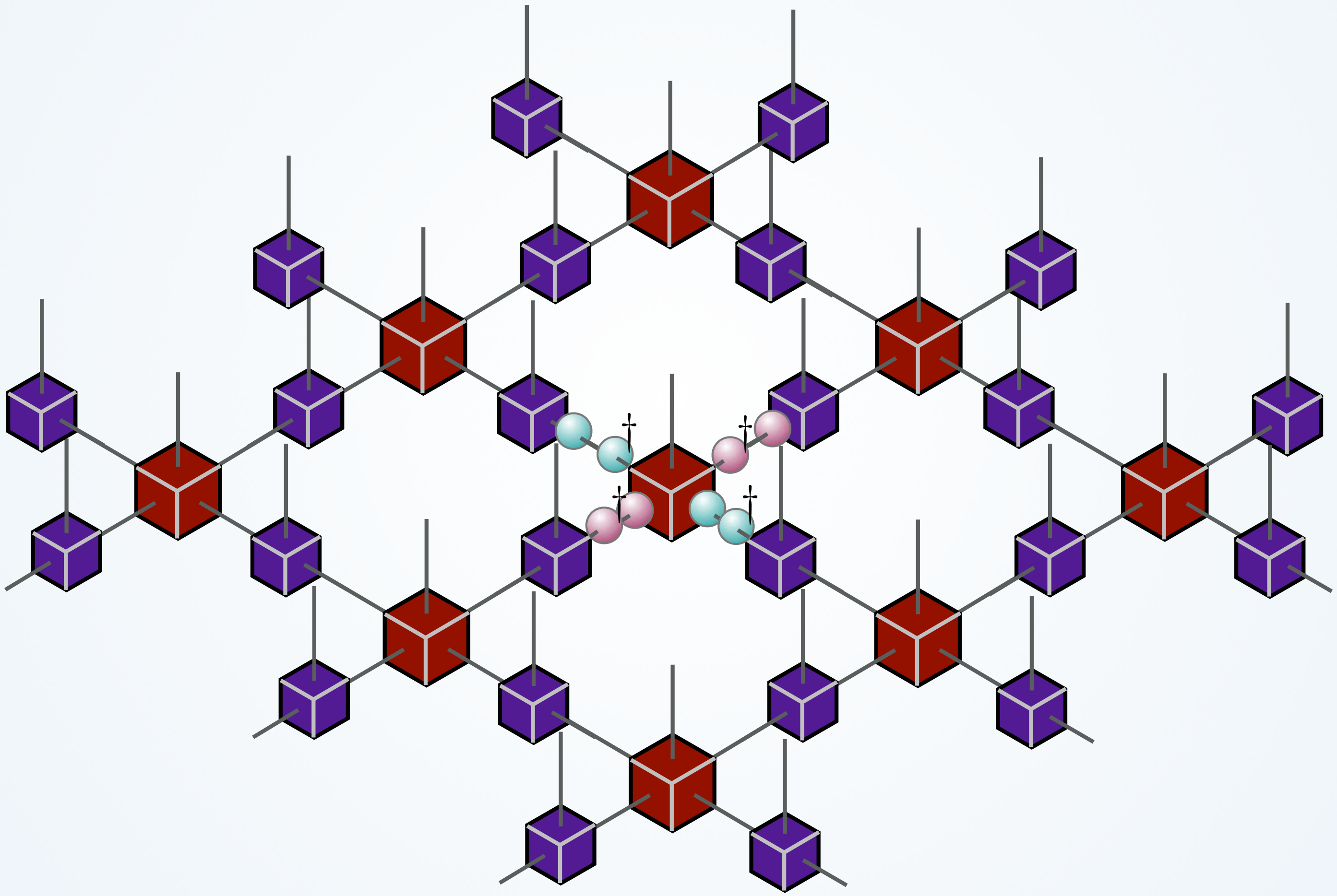


GAUGING PEPS

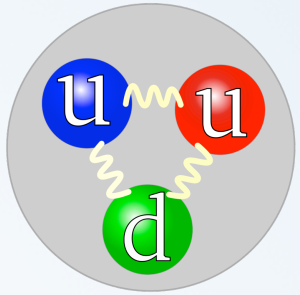


WHY LGT???

Motivation for LGT: QCD

Wilson, 1974

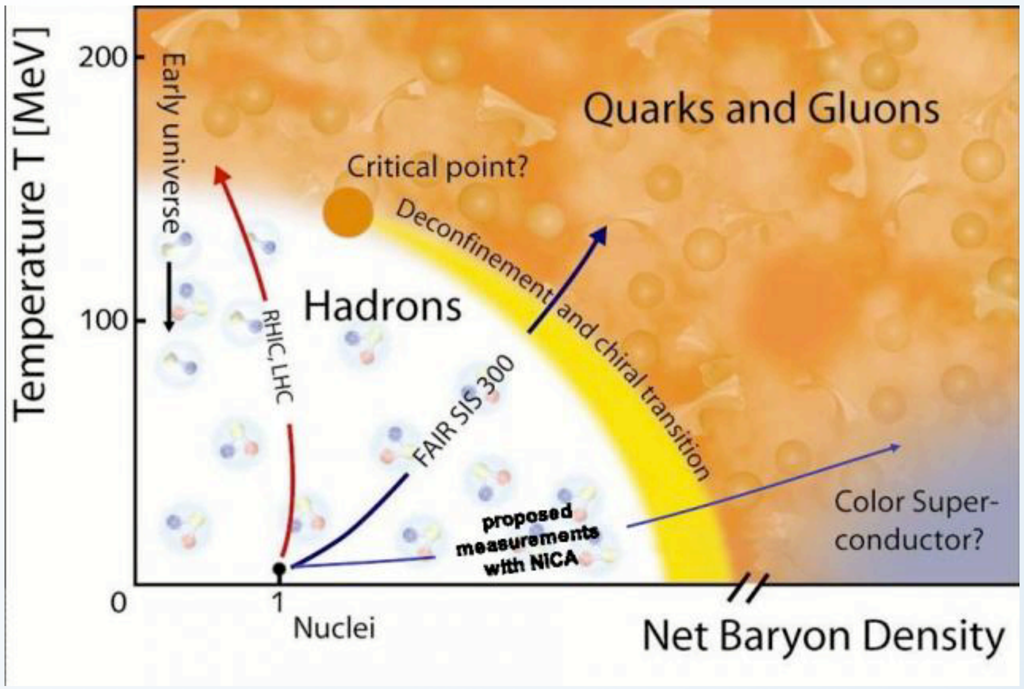
non perturbative at low energy



LQCD

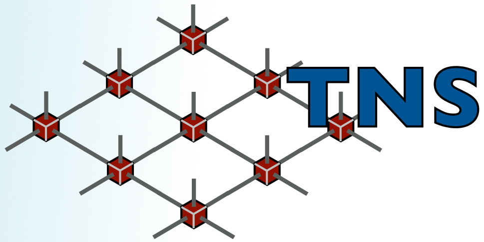
successful spectral calculations

limitations: time, finite density



WHY WITH TNS?

A NATURAL CONNECTION

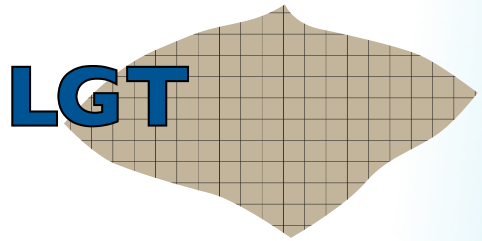


Non-perturbative for Hamiltonian systems

Extremely successful for 1D systems (MPS)

Consistent developments for higher dimensions

ground states
low-lying excitations
thermal states
time evolution



Non-perturbative way of solving QFT (QCD)

Mostly path-integral formalism & MC

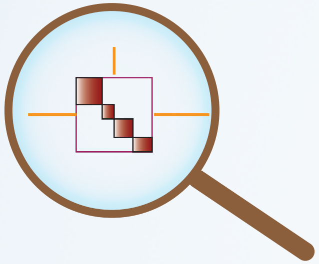
4D lattice

spectrum
finite T
big 3+1 dimensional
chemical potential
time evolution

HOW TNS FOR LGT???

USING TNS FOR QMB

a formal approach



classifying tensors

constructing states

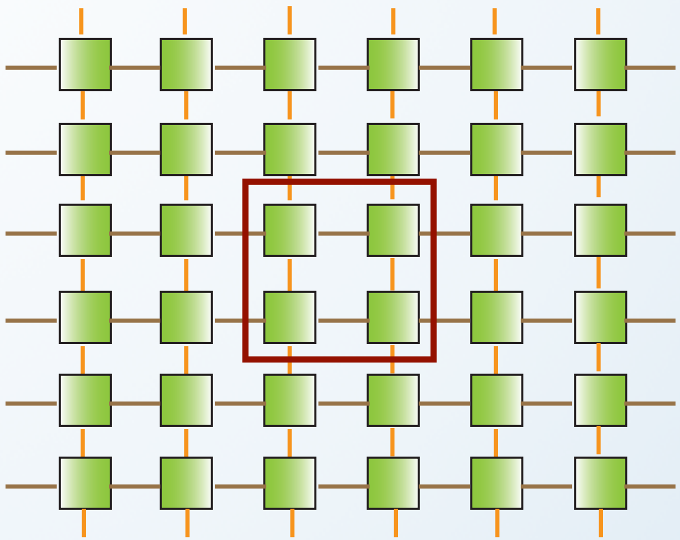
great descriptive power: phases,
topological chiral states, anyons...

Chen et al PRB 2011
Schuch et al PRB 2011
Wahl et al PRL 2013; Yang et al PRL 2015
Haegeman et al, Nat. Comm. 2015

no sign problem

numerical algorithms

tensor networks describe
partition functions (observables)

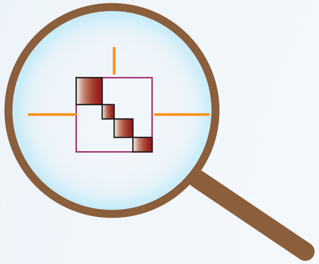


need to contract a TN
TRG approaches

Nishino, JPSJ 1995
Levin & Wen PRL 2008
Xie et al PRL2009; Zhao et al PRB 2010

USING TNS FOR QMB

a formal approach



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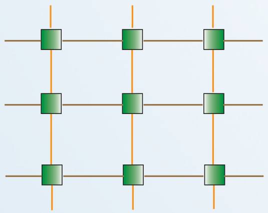

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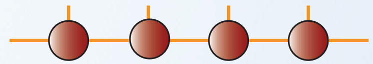


TNS as ansatz for the state

efficient algorithms for GS, low excited states, thermal, dynamics

White PRL 1992; Schollwöck RMP 2011
Vidal PRL 2003; Verstraete et al PRL 2004
Verstraete et al Adv Phys 2008; Orús Ann Phys 2014

USING TNS FOR LGT

formal approach

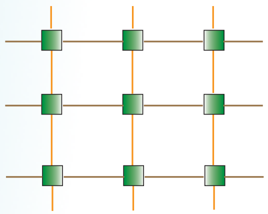
gauging the symmetry

explicitly invariant states

general prescriptions, U(1), SU(2)

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

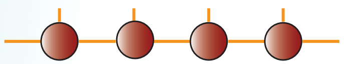
numerical simulations **no sign problem**



TN describe partition functions (observables)

TRG approaches to classical and quantum models

Liu et al PRD 2013; Shimizu, Kuramashi, PRD 2014; Kawauchi, Takeda 2015; review Meurice et al. 2010.06539



TN describe states


there is long way to go until LQCD
journey begins with 1+1D steps
Photo by Maria Teneva on Unsplash

early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004
Tagliacozzo PRB2011; Sugihara JHEP2005
Meurice PRB2013

Schwinger model
U(1) in 1D

precise equilibrium simulations, feasibility of QSim

MCB et al JHEP11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016
MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838

MCB, K. Cichy 1910.00257
QTFLAG Collab.1911.00003

3+1 dimensions

Magnifico et al. 2011.10658

2+1 dimensions

Falser et al. arXiv:1911.09693
Robaina et al. PRL126, 050401 (2021)
Emonts et al. PRD102, 074501 (2020)

Non-Abelian in 1D

string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
Silvi et al., Quantum 2017
S. Kühn et al. PRX 2017

SU(3)QLM

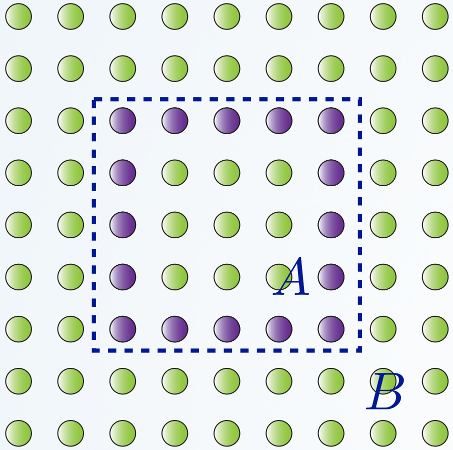
Silvi et al, PRD 2019

finite density

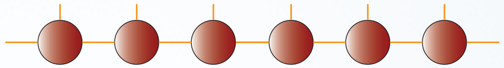
S. Kuehn et al, PRL118 (2017) 071601

TNS = entanglement based ansatz

area law

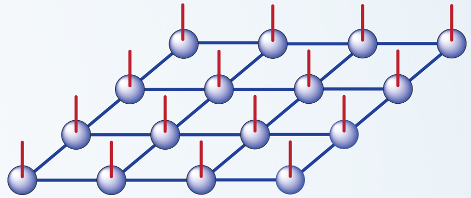


MPS



Schollwöck Ann.Phys.2011

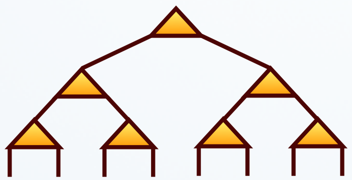
PEPS



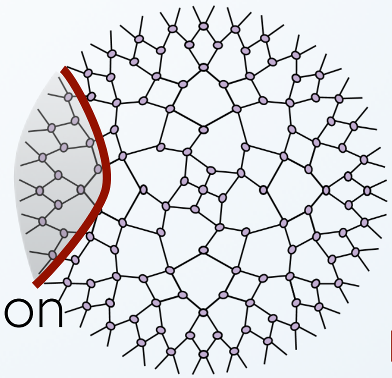
Verstraete et al. Adv. Phys. 2008

other TNS

TTN

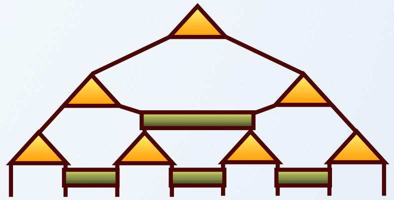


Shi et al PRA 2006



suggested connection
to AdS/CFT

Vidal PRL 2007 MERA



Swingle PRD 2012
Molina JHEP 2013
Nozaki et al JHEP 2012
Bao et al PRD 2015

in principle, they can all be used for LGT simulations

GENERAL STRATEGY

Hamiltonian formulation

acting on a Hilbert space

⟶ choose proper basis

Finite dimensional degrees of freedom

fermions

⟶ ✓ no sign problem

gauge bosons require attention

⟶ truncating, integrating out (also QLinks)

★ Common ingredients for quantum simulation

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012
Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015

early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004
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Schwinger model U(1) in 1D

precise equilibrium simulations, feasibility of QSim

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Emonts et al. PRD102, 074501 (2020)

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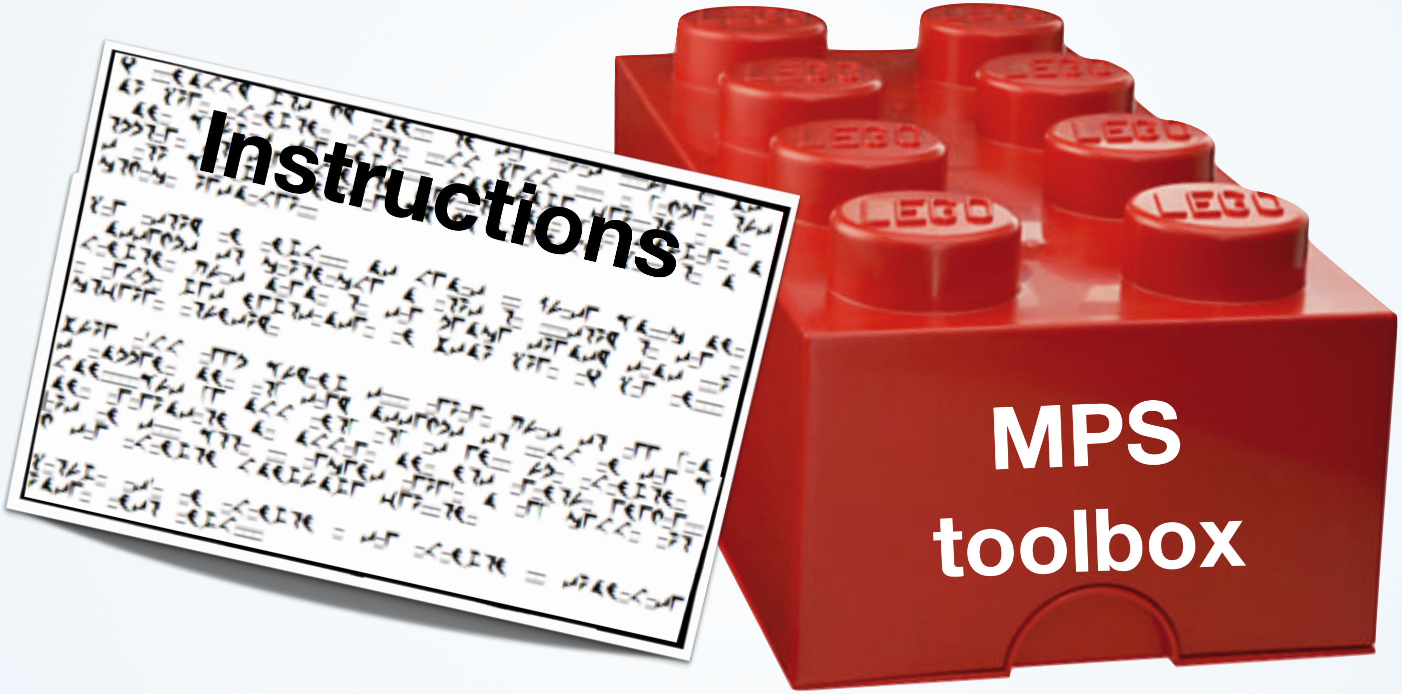


start with the simplest case: 1D (1+1) LGT

MPS PROPERTIES

- MPS = Matrix Product States

MPS

good approximation of ground states

Verstraete, Cirac, PRB 2006

Hastings, J. Stat. Phys 2007

gapped finite range Hamiltonian ⇒
area law (ground state)

Cramer, Eisert, Plenio, RMP 2009

efficient calculation of expectation values

exponentially decaying correlations

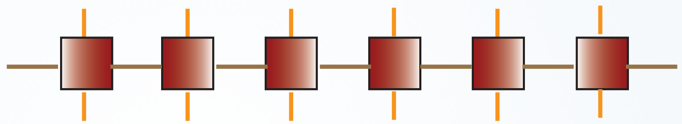
can be prepared efficiently

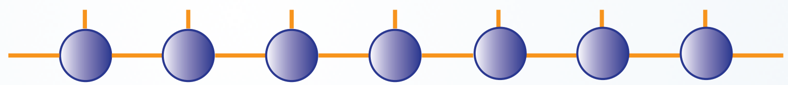
WHAT CAN WE DO WITH THEM?

BASIC ALGORITHMS

variational minimization of energy

local
Hamiltonian

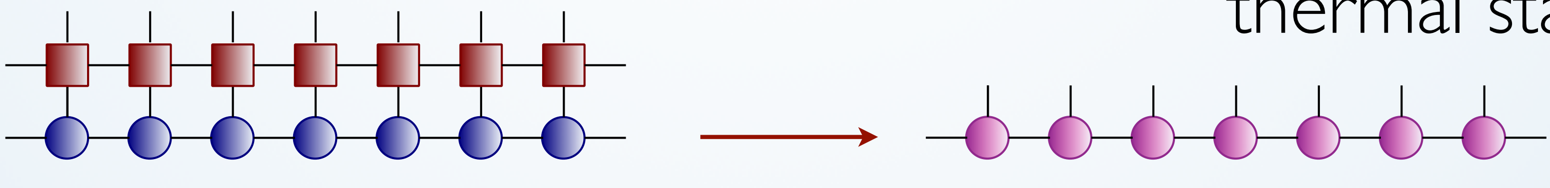
$H =$ 

$|E_0\rangle \simeq$ 

ground state
excitations

apply local operators → simulate time evolution

imaginary time → ground state
thermal state

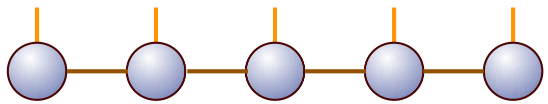


alternatively: TDVP

REGARDING DYNAMICS

basic time evolution algorithms

initial MPS



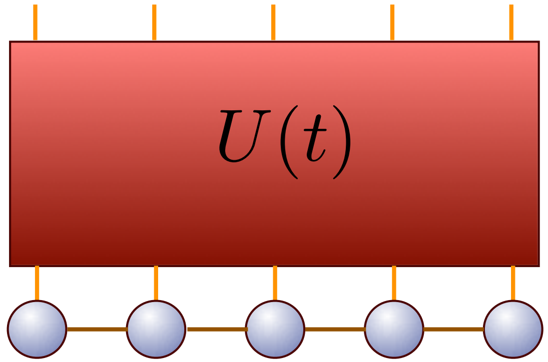
TEBD, t-DMRG

Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

basic time evolution algorithms

initial MPS



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Vidal, PRL 2003, 2004

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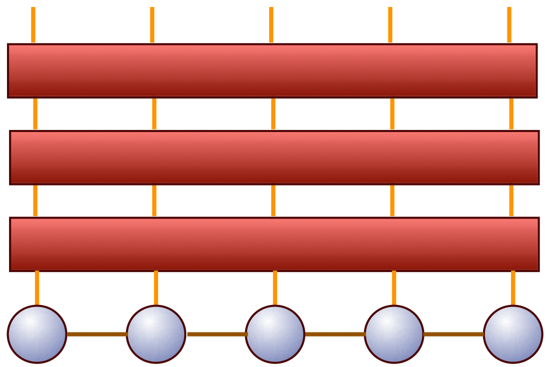
basic time evolution algorithms

initial MPS

discrete time

$$U(t) \to [U(\delta)]^M$$



TEBD, t-DMRG

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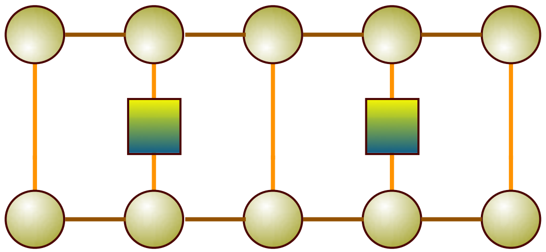
basic time evolution algorithms

time evolved state approximated by MPS



TEBD, t-DMRG

Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

alternative: TDVP Haegeman et al, PRL 2011

initial MPS

discrete time

$$U(t) \rightarrow [U(\delta)]^M$$

Suzuki-Trotter expansion

$$U(\delta) \approx e^{-iH_e\delta}e^{-iH_o\delta}$$

truncate bond dimension

iterate

compute observables

ALSO FOR MIXED STATES

MIXED STATES

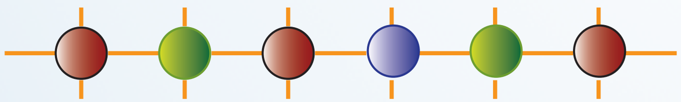
- MPO = Matrix Product Operator

Similar problems can be attacked

equilibrium → thermal states

imaginary time evolution

time-dependent → real time evolution



unitary $\rho(t) = U(t)\rho(0)U(t)^\dagger$

non-unitary $$\frac{d\rho(t)}{dt} = \mathcal{L}(\rho)$$

Verstraete et al., PRL 2004
Prosen, Znidaric PRL 2008
Cai, Barthel, PRL 2013,...


Instructions
MPS
toolbox

In the rest of this talk…

Why using TNS/MPS for LGT? ✓

testbench:
Schwinger model

- spectral calculations
- finite temperature
- real-time
- chemical potential

higher dimensional problems

SCHWINGER MODEL AS LABORATORY

SCHWINGER MODEL

Schwinger '62

Simplest gauge theory with matter

QED in 1+1 dimensions

electrons & photons

Shows some of the features of *full* QCD

confinement → bound states (massive bosons)

fermion condensate

A testbench for lattice techniques

SCHWINGER MODEL

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

in 1+1 D single adimensional parameter m/g

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$$

U(1) gauge invariance

$$\Psi(x) \to e^{-ig\phi(x)}\Psi(x)$$

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \phi(x)$$

equations of motion

$$\partial_\alpha \frac{\partial \mathcal{L}}{\partial \Phi_{,\alpha}} - \frac{\partial \mathcal{L}}{\partial \Phi} = 0 \quad \text{for} \quad \Phi = A_\mu,\ \Psi$$

$$(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\,\Psi = 0$$

$$\partial_\mu F^{\mu\nu} = g\bar{\Psi}\gamma^\nu \Psi$$

SCHWINGER MODEL

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Hamiltonian formulation

$$A^0 = 0$$

$$E = -\dot{A}^1$$

$$H = \int dx \left[-i\bar{\Psi}\gamma^1\partial_1\Psi + g\bar{\Psi}\gamma^1 A_1\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2\right]$$

fermion kinetic term — fermion-photon coupling — fermion mass — electrostatic energy

plus a constraint: $\partial_1 E = g\bar{\Psi}\gamma^0\Psi$ Gauss' law

quantization

$$\{\Psi_i(x), \Psi_j^\dagger(y)\} = \delta_{ij}\delta(x-y)$$
$$\{\Psi_i(x), \Psi_j(y)\} = 0$$
$$[A_1(x), E(y)] = i\delta(x-y)$$

discretize

SCHWINGER MODEL

on the lattice

discrete Hamiltonian (staggered) formulation

$$x$$

$$\begin{pmatrix} \Psi^{(1)}(x) \\ \Psi^{(2)}(x) \end{pmatrix}$$

SCHWINGER MODEL

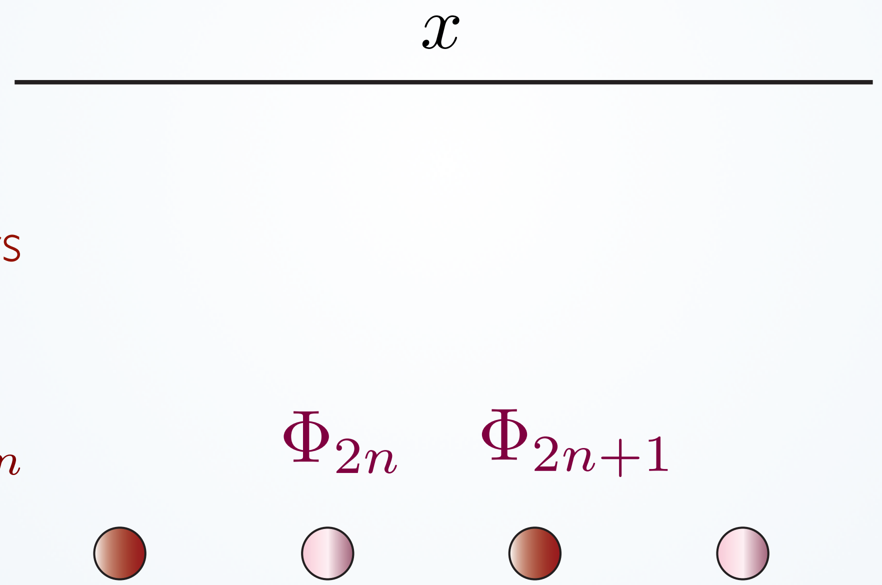
on the lattice

discrete Hamiltonian (staggered) formulation

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

$$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$$



SCHWINGER MODEL

on the lattice

discrete Hamiltonian (staggered) formulation

x

$$U(x, x+\epsilon) = e^{ig\epsilon A_1(x)}$$

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

$$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$$

Φ_{2n} Φ_{2n+1}

SCHWINGER MODEL

on the lattice

discrete Hamiltonian (staggered) formulation

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

$$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$$

Φ_{2n} Φ_{2n+1}

$$U_{n,n+1} = e^{i\theta_n}$$

$$\frac{1}{ga}\theta_n \to -A^1(x)$$

$$gL_n \to E(x)$$

$$[\theta_n, L_m] = ig\delta_{nm}$$

SCHWINGER MODEL

on the lattice

discretized

$$H = -\frac{i}{2a}\sum_n \left(\phi_n^\dagger e^{i\theta_n}\phi_{n+1} - \text{h.c.}\right) + m\sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

plus constraint: Gauss' Law

spinless fermions

$$L_n - L_{n-1} = \phi_n^\dagger \phi_n - \frac{1}{2}\left[1 - (-1)^n\right]$$

1D spins $\Longleftrightarrow$ fermions: Jordan-Wigner $\quad \phi_n = \prod_{k<n} (i\sigma_k^z)\sigma_n^-$

notice: not strictly necessary for TNS

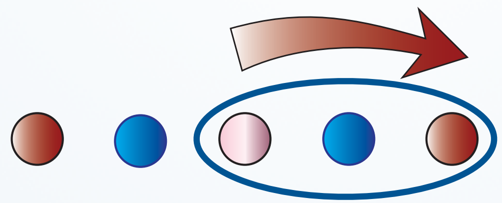
SCHWINGER MODEL

on the lattice

Jordan-Wigner → spin model

$$H = \frac{1}{2a}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right)$$

$$+\frac{m}{2}\sum_n \left(1+(-1)^n\sigma_n^3\right) + \frac{ag^2}{2}\sum_n L_n^2$$



hopping

$$U_{n,n+1} = e^{i\theta_n}$$

$$\frac{1}{ga}\theta_n \to -A^1(x)$$

$$gL_n \to E(x)$$

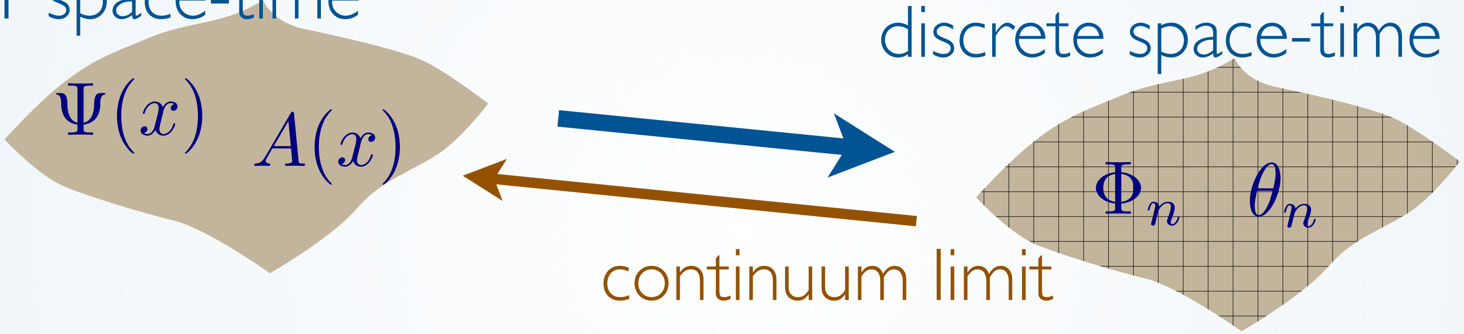
SCHWINGER MODEL

continuum QED

on the lattice

1+1 space-time

discrete space-time



fermions

photons

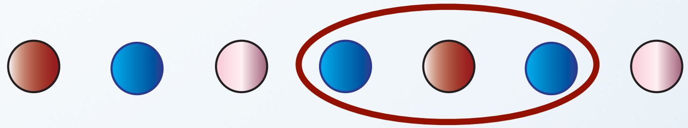
lattice spacing $ag \rightarrow 0$

spin Hamiltonian

+ Gauss law

fermion mass m/g

adimensional



Hamiltonian

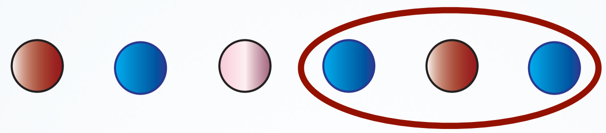
+ Gauss law

for a TNS we need a basis

SCHWINGER MODEL

basis $\quad |\ldots s_e\ \ell\ \ s_o\ \ell\ \ s_e\ \ell\ \ s_o \ldots\rangle$

SCHWINGER MODEL



basis $\quad |\ldots s_e \;\ell\; s_o \;\ell\; s_e \;\ell\; s_o \ldots\rangle$

but Gauss' law fixes *photon* content

$$L_n - L_{n-1} = \frac{1}{2}[\sigma_n^3 + (-1)^n]$$

SCHWINGER MODEL

basis $|\ldots s_e \quad s_o \quad s_e \quad s_o \ldots\rangle$

but Gauss' law fixes *photon* content

$$L_n = \ell_0 + \frac{1}{2}\sum_{k\leq n}\sigma_n^3 + \ldots$$

$\Rightarrow$ eliminate gauge dof

introducing long range interactions

SCHWINGER MODEL

MPS representation for OPEN BOUNDARIES

$$|\ell_0 \ldots s_e \; s_o \; s_e \; s_o \ldots\rangle$$

non-local terms

$$L_n = \ell_0 + \frac{1}{2} \sum_{k \leq n} \sigma_n^3$$

Long range interactions

$$\sum_n \sum_{k<n} (N-n) \sigma_k^3 \sigma_n^3$$

can be written as a MPO of D=5

both possibilities

SCHWINGER MODEL

on the lattice

basis $|\ldots s_e\, \ell\, s_o\, \ell\, s_e\, \ell\, s_o \ldots\rangle$ all terms are local

infinite dimensional: truncation

Gauss' law needs to be imposed

works by Buyens et al., PRL 2014; arXiv:1509.00246
Rico et al., PRL 2014; NJP 2014

or integrating out the gauge dof

basis $|\ell_0 \ldots s_e\, s_o\, s_e\, s_o \ldots\rangle$ non-local terms

SCHWINGER MODEL

on the lattice

basis $|\ldots s_e\ \ell\ s_o\ \ell\ s_e\ \ell\ s_o \ldots\rangle$ all terms are local

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or integrating out the gauge dof

basis $|\ell_0 \ldots s_e\ s_o\ s_e\ s_o \ldots\rangle$ non-local terms

exact physical subspace

✗ does not generalize to bigger dimensions