Detecting fractional Chern insulators in few-boson systems

Cécile Repellin, LPMMC

Entanglement in strongly correlated systems

Benasque online workshop

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laboratoire de physique et de modélisation des milieux condensés

Acknowledgements

Nathan Goldman
ULB

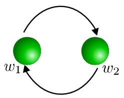
Julian Léonard
Harvard

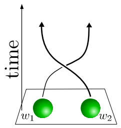
CR, J. Léonard, N. Goldman, Phys. Rev. A 102 (6), 063316 (2020)

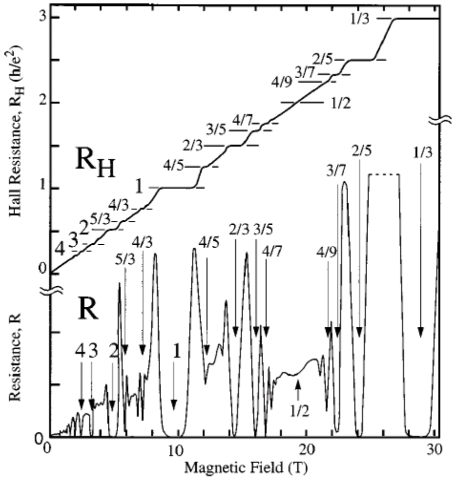
Fractional Chern Insulators

- **Fractional Quantum Hall effect**

  - Large magnetic field + strong interactions in 2D
  - Emergent collective excitations : anyons







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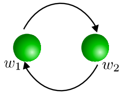
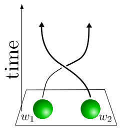
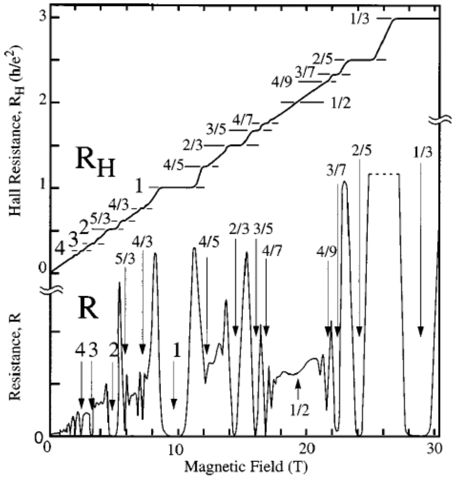




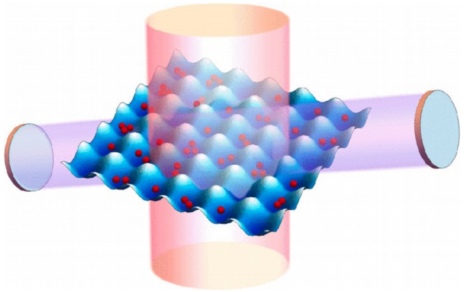


- **Fractional Chern insulators**
  - FQHE on a lattice with no magnetic field
  - More than FQHE

Engineering Fractional Chern Insulators

- Good understanding of emergence conditions of FCIs
- **Design platforms where we can control the microscopic parameters**

such as cold atom in optical lattices



Detection methods in engineered quantum matter ?

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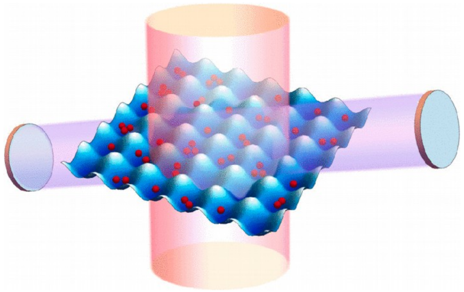

Detection methods in engineered quantum matter ?

$$\sigma_H = \frac{1}{2}\frac{e^2}{h}$$ → How to measure the Hall conductivity of fractional Chern insulators ?

Fractional Chern Insulators : why few-atom systems ?

The challenge of preparing a state with topological order

No direct cooling down to the topological ground state

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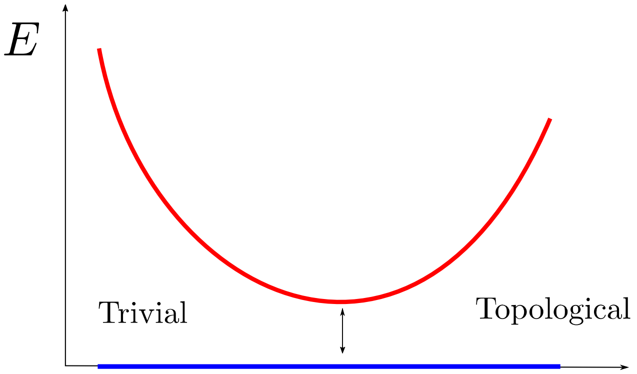

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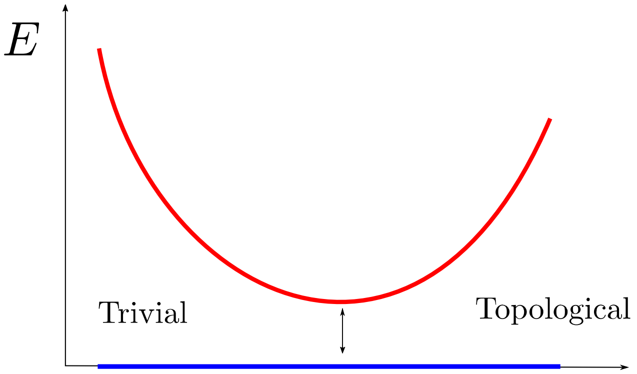
Strategy : adiabatic preparation of few-boson systems

N = 2, 3, 4…

[Tai et al (Greiner lab) Nature ‘17]



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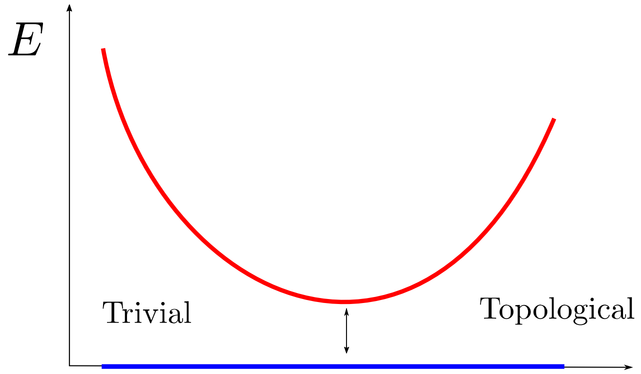

N = 2, 3, 4…

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σ_H not quantized for small systems...

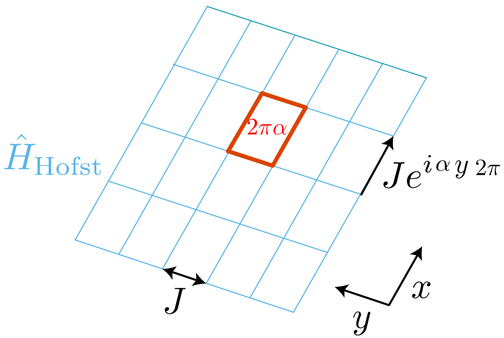
σ_H robust over a range of parameters ?

Outline

- Emergence of Fractional Chern Insulators in systems with edges
- Detecting FCIs from a local density measurement : application of Streda's formula
- Detecting FCIs from their Hall drift

Emergence of Fractional Chern Insulators in systems with edges

Hall effect on a lattice : the Hofstadter model



α : Magnetic flux density

$$\mathrm{H}{=}\text{-J}\left(\sum_{m,n} \hat{a}^{\dagger}_{m,n+1}\hat{a}_{m,n} + e^{i2\pi\alpha n}\hat{a}^{\dagger}_{m+1,n}\hat{a}_{m,n} + \mathrm{h.c.}\right)$$

Commensurate flux $\Phi = 2\pi\alpha = 2\pi\dfrac{p}{q}$

Magnetic unit cell : q lattice sites → flux $2\pi p$

Realised in cold atoms [Aidelsburger et al PRL ‘13, Miyake et al PRL ‘13]

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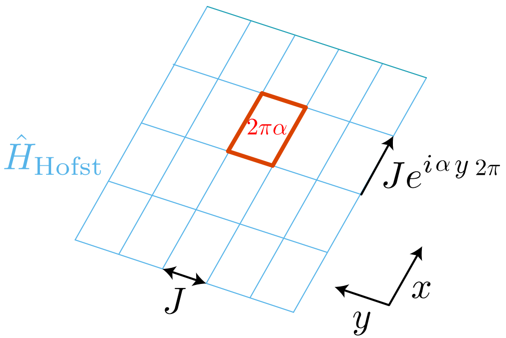

α : Magnetic flux density

$$\text{H=-J}\left(\sum_{m,n} \hat{a}^{\dagger}_{m,n+1}\hat{a}_{m,n} + e^{i2\pi\alpha n}\hat{a}^{\dagger}_{m+1,n}\hat{a}_{m,n} + \text{h.c.}\right)$$

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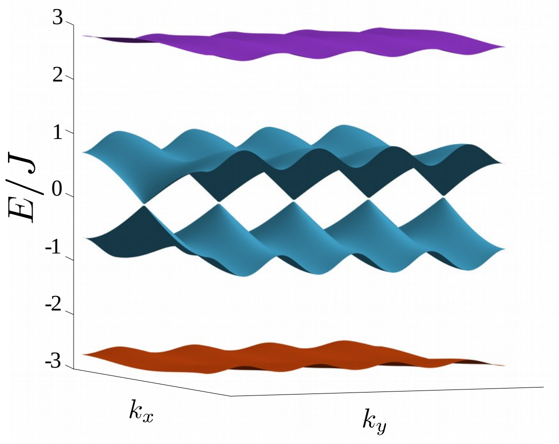
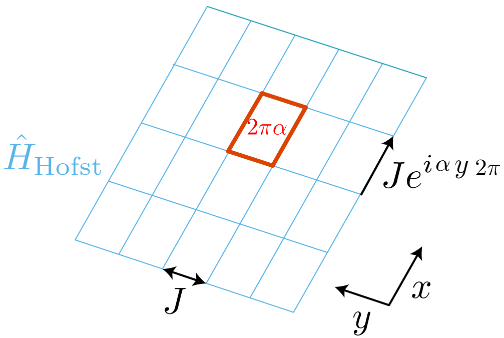
Example : $\alpha = 1/4$ $\Phi = \frac{\pi}{2}$

4 lattice sites per magnetic unit cell
4 bands



Hall effect on a lattice : the Hofstadter model



α : Magnetic flux density

$$H = -J\left(\sum_{m,n} \hat{a}^\dagger_{m,n+1}\hat{a}_{m,n} + e^{i2\pi\alpha n}\hat{a}^\dagger_{m+1,n}\hat{a}_{m,n} + \text{h.c.}\right)$$

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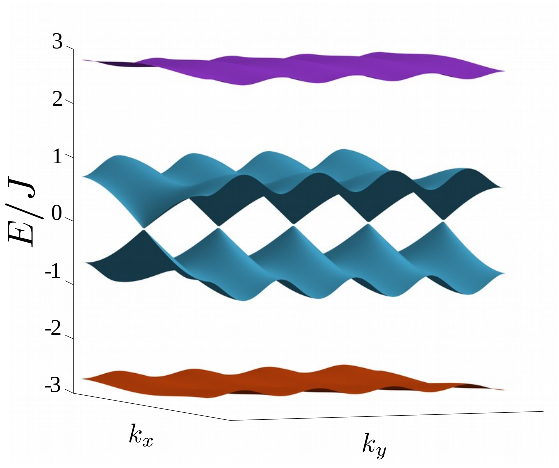

Generically

1 filled band → $\nu = 1 \rightarrow \sigma_H = e^2/h$
Nbr states in lowest band $\propto \alpha$

Fractional Chern insulators : from torus to geometry with edges

$$\hat{H}_0 = - J \left(\sum_{m,n} \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} + e^{i2\pi\alpha n} \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} + \text{h.c.} \right) + (U/2) \sum_{m} \hat{a}^\dagger_{m,n} \hat{a}_{m,n} (\hat{a}^\dagger_{m,n} \hat{a}_{m,n} - 1)$$

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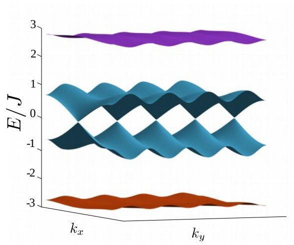
Torus geometry

Single-particle spectrum



$$\nu = 1/2$$

Filling fraction of lowest band

$$\nu = \frac{N}{\alpha N_s}$$

[Sorensen, Demler, Lukin PRL ‘05]

Emergence of FCI

Fractional Chern insulators : from torus to geometry with edges

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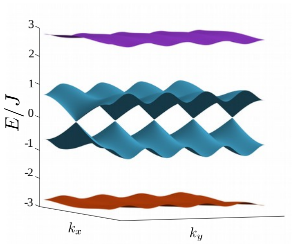
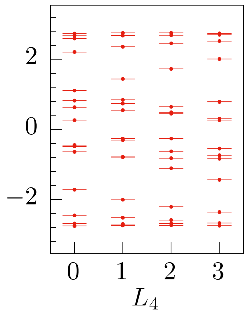
Torus geometry

Geometry with edges

Confinement : box potential

Single-particle spectrum





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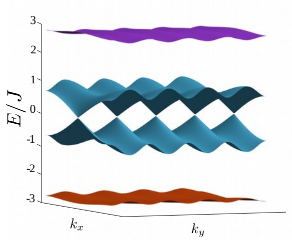

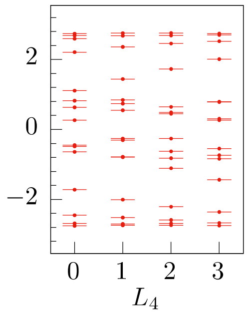

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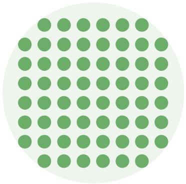
Emergence of FCI

[Sorensen, Demler, Lukin PRL ‘05]

→Preliminary study : FCI in the geometry with edges

Emergence of a FCI in a geometry with edges



4 hardcore bosons
60 lattice sites

[CR, J. Leonard, N. Goldman, PRA 2020]

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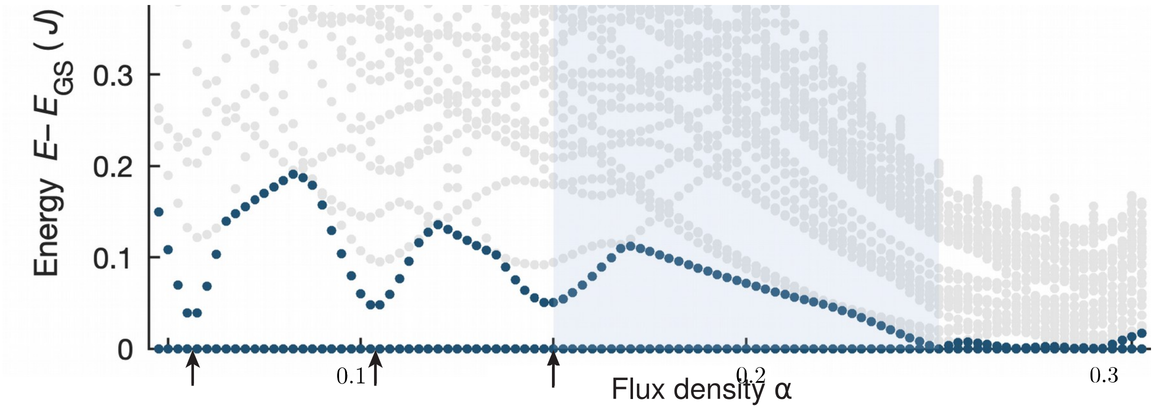

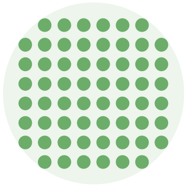

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Energy spectrum

No chiral edge modes (hard wall)

[Macaluso, Carusotto PRA '17]

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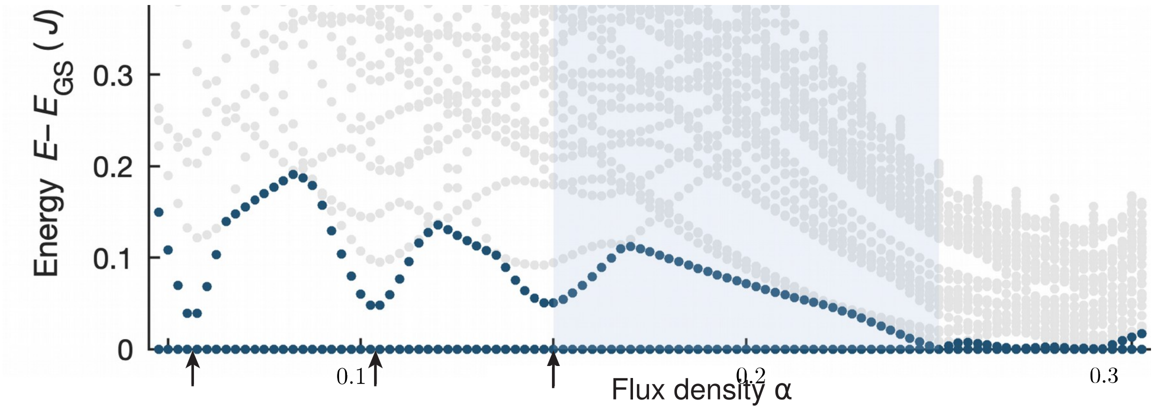

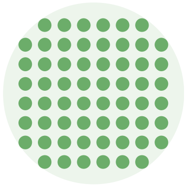

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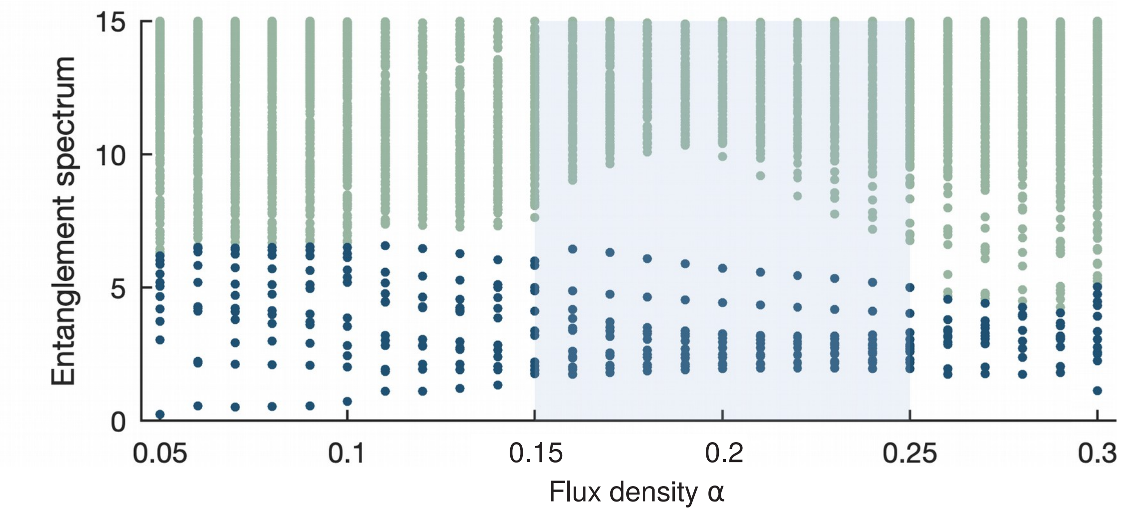


Particle Entanglement spectrum

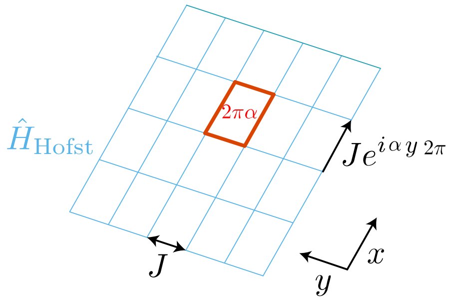
[Sterdyniak et al PRL ‘11]
Spectrum of $\rho_A = Tr_B|\Psi_{\rm GS}\rangle\langle\Psi_{\rm GS}|$
(trace over a subset of particles)

Reveals the degeneracy of quasiholes

[CR, J. Leonard, N. Goldman, PRA 2020]

Detecting FCIs from a local density measurement : application of Streda's formula

Bulk density of a fractional Chern insulator



Streda's formula for gapped GS: $$\sigma_{\perp} = \frac{\partial \rho_{\mathrm{bulk}}}{\partial \alpha}$$

What is the bulk density ρ_{bulk} for a system with few bosons?

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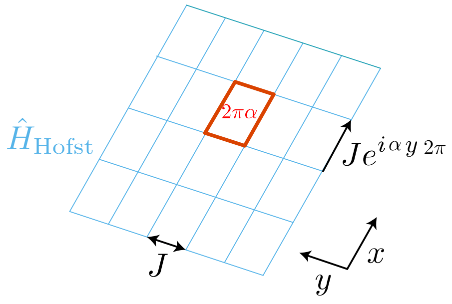
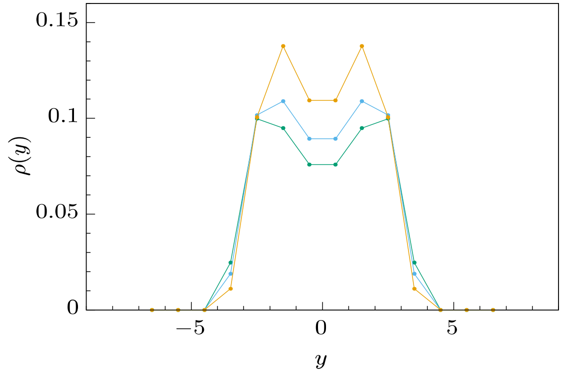
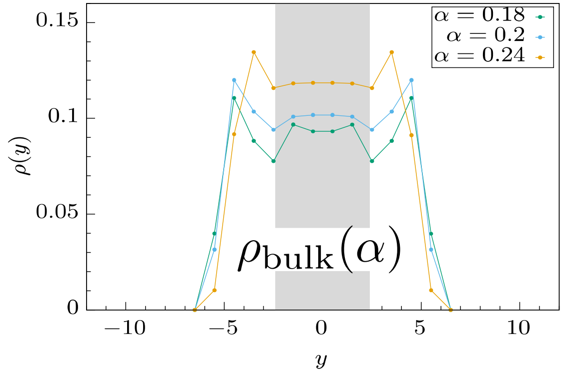


What is the bulk density $\rho_{\rm bulk}$ for a system with few bosons?

N=4, 60 sites



N=10 bosons, 120 sites



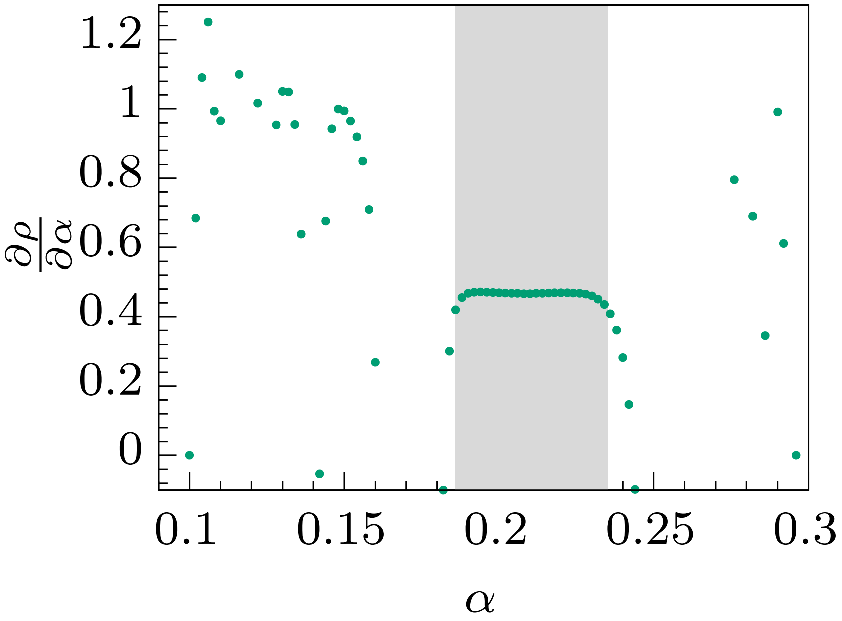
$$\langle \rho \rangle = \frac{N}{N_s}$$

Verifying Streda's formula numerically

N=10 hardcore bosons, 120 sites

Ψ_{GS} obtained using DMRG

Quantized $\sigma_\perp = \frac{\partial\rho}{\partial\alpha} = \frac{1}{2}$

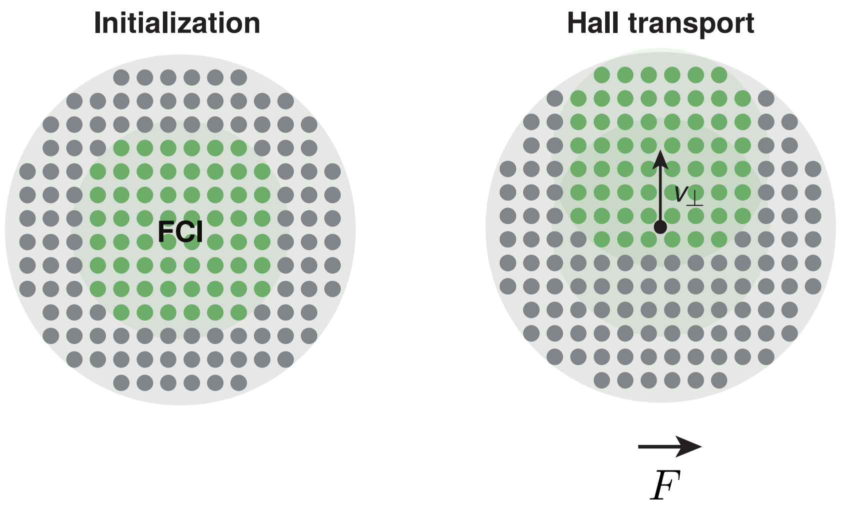


N < 10 : bulk not well defined enough

[CR, J. Leonard, N. Goldman, PRA 2020]

Detecting fractional Chern insulators using the Hall drift

Hall drift protocol



Prepared FCI released into larger trap

Linear response :

$$\sigma_\perp = \frac{\rho_{\text{bulk}}}{F} v_\perp$$

[CR, J. Leonard, N. Goldman, PRA 2020]

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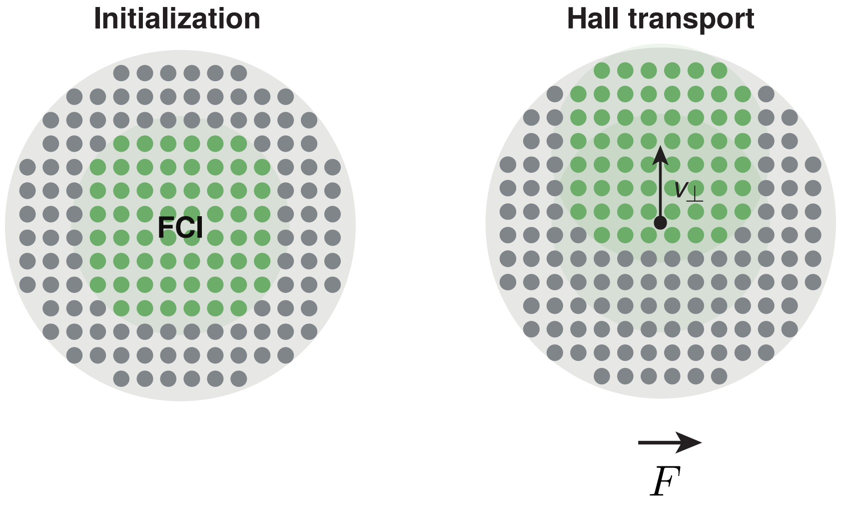

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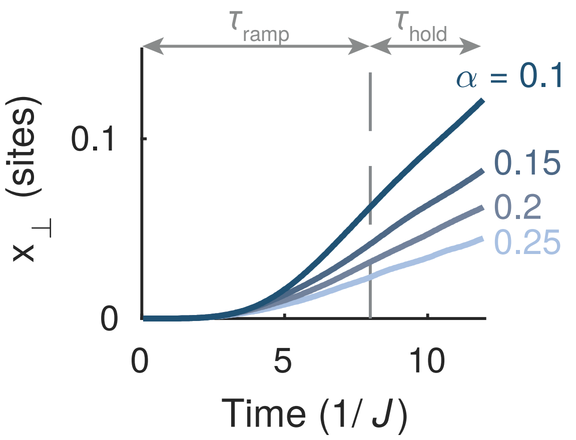


Linear response :

$$\sigma_\perp = \frac{\rho_{\text{bulk}}}{F} v_\perp$$

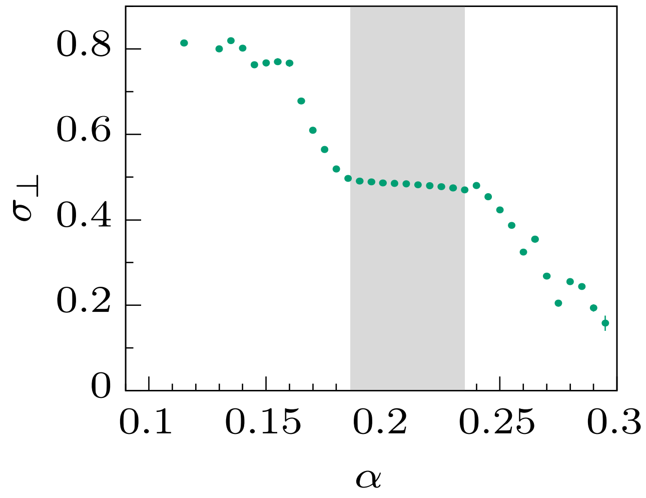
→ monitor transverse center-of-mass velocity

[CR, J. Leonard, N. Goldman, PRA 2020]

Numerical time-evolution : Hall drift

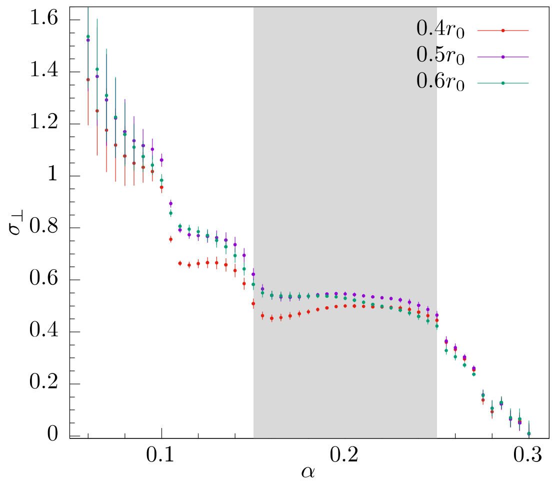
[CR, J. Leonard, N. Goldman, PRA 2020]

N=10 (DMRG)



time evolution method :
[Zaletel, Mong, Karrasch, Moore, Pollmann PRB '15]

N=4 (ED)



Quasi-quantized Hall conductivity for N ~ 10

Robust plateau

Hall plateau coincides with boundaries of FCI phase

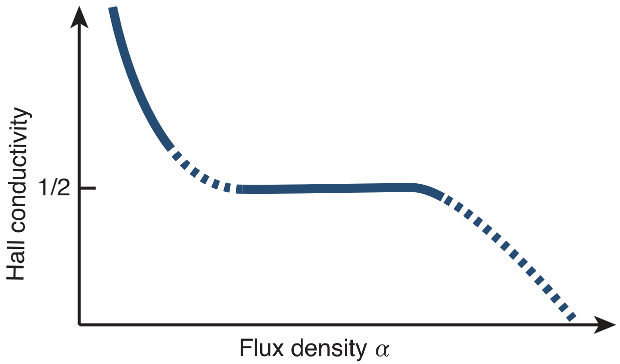
Conclusion

- FCIs can be detected from a bulk density measurement
  → Streda’s formula (from ~ 10 bosons)

- The Hall drift provides a clear signature of FCIs for even fewer particles

Hall conductivity : what do we expect ?

Thermodynamic limit :
Incompressible phase → quantized $\sigma_\perp$ → plateaus



Small system :

$\sigma_\perp$ not perfectly quantized

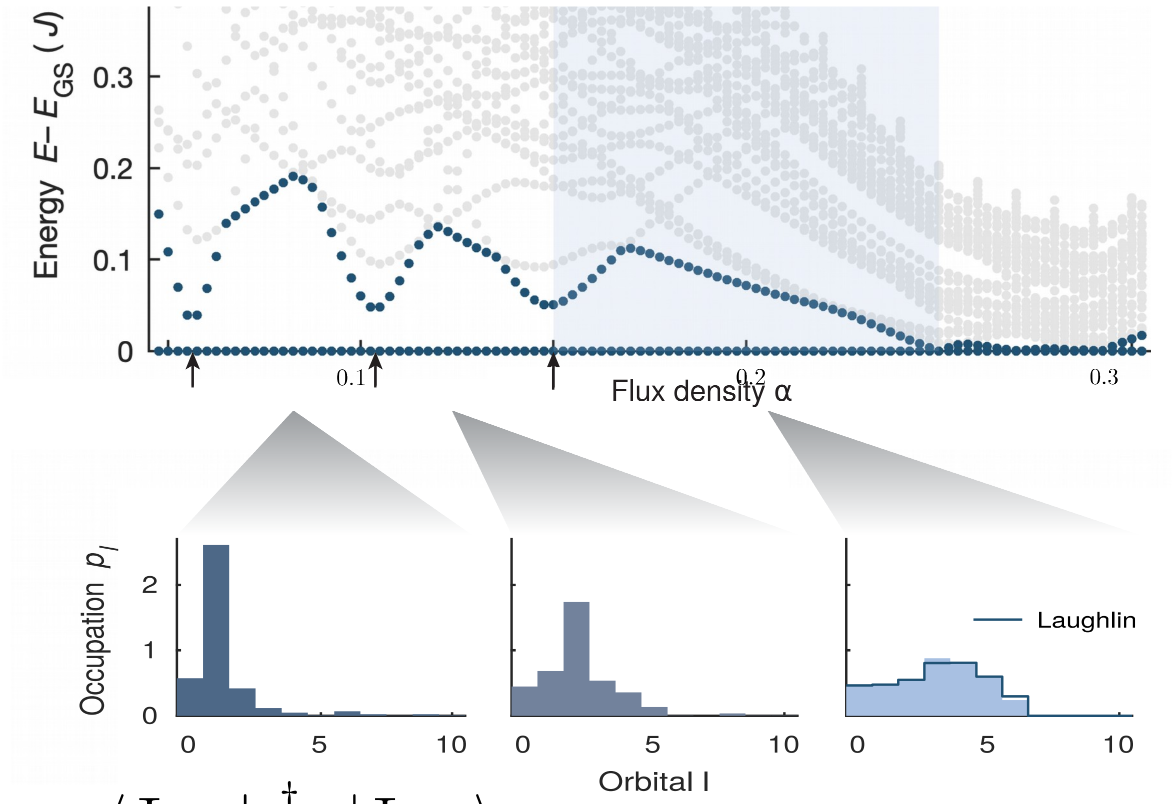
Near-constant far from the phase transitions ?

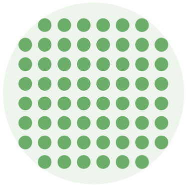
Variation of ν with α

Fixed number of particles N and number of sites
But the number of states in the lowest band increases with α

$\nu \searrow \qquad \alpha \nearrow$

Emergence of a FCI in a geometry with edges





4 hardcore bosons
60 lattice sites

Energy spectrum :
finite-size phase transitions

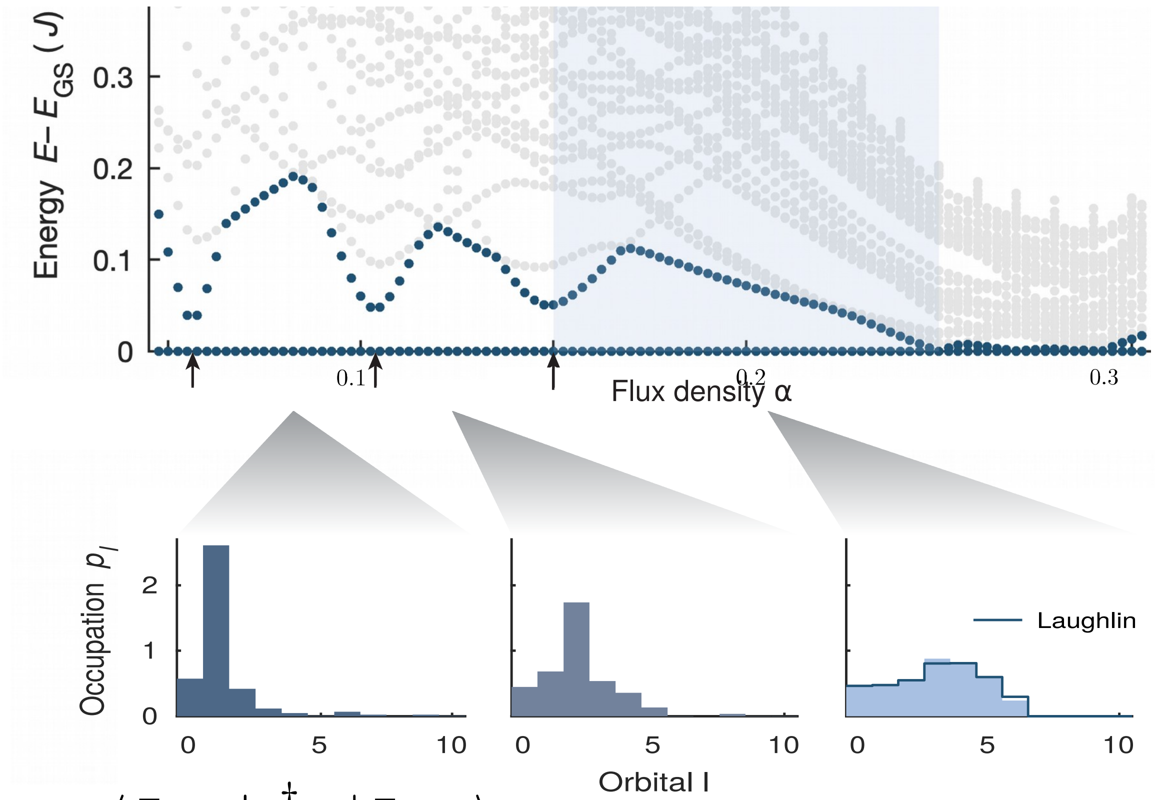
Orbital occupation :
Similarity with continuum Laughlin

$$p_l = \langle \Psi_{\mathrm{GS}} | c_l^\dagger c_l | \Psi_{\mathrm{GS}} \rangle$$

$\simeq 7$ filled orbitals

First hint of a FCI phase !

Emergence of a FCI in a geometry with edges



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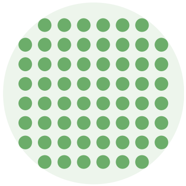

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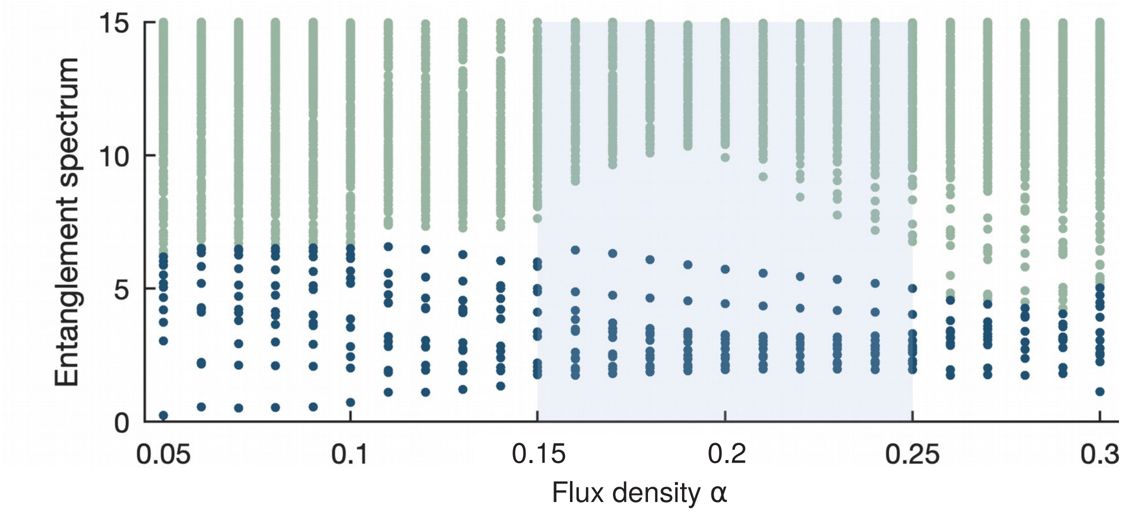
Emergence of a fractional quantum Hall state on the lattice

Topological fingerprint of the FCI phase in open geometry ?

- Chiral edge modes ? → not with our hard wall confinement *[Macaluso, Carusotto PRA '17]*
- Bulk quasihole excitations → revealed by the particle entanglement spectrum *[Sterdyniak et al PRL '11]*

Spectrum of $\rho_A = Tr_B |\Psi_{\mathrm{GS}}\rangle\langle\Psi_{\mathrm{GS}}|$ Trace over the coordinates of a subset of N_B particles



Number of states below the entanglement gap = Degeneracy of quasiholes in the system with N_A particles

[CR, J. Leonard, N. Goldman, PRA 2020]