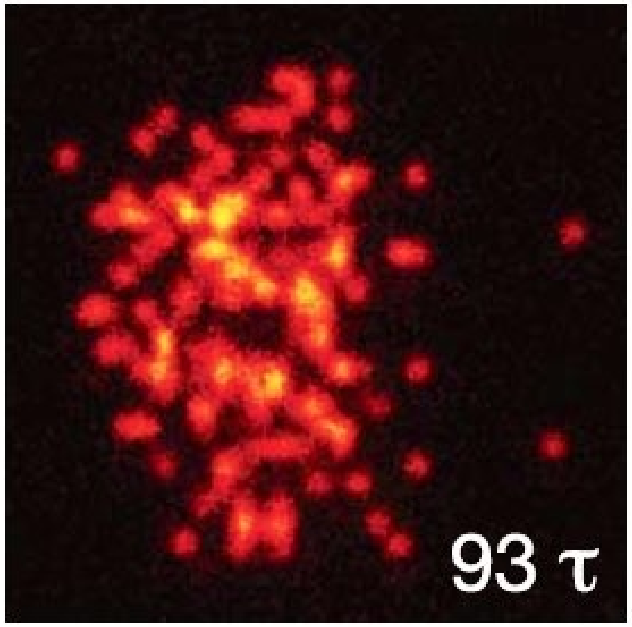
Probing mixed-state, symmetry-resolved, entanglement with randomized measurements

B. Vermersch (LPMMC Grenoble, & IQOQI Innsbruck)

Motivation: quantum technologies

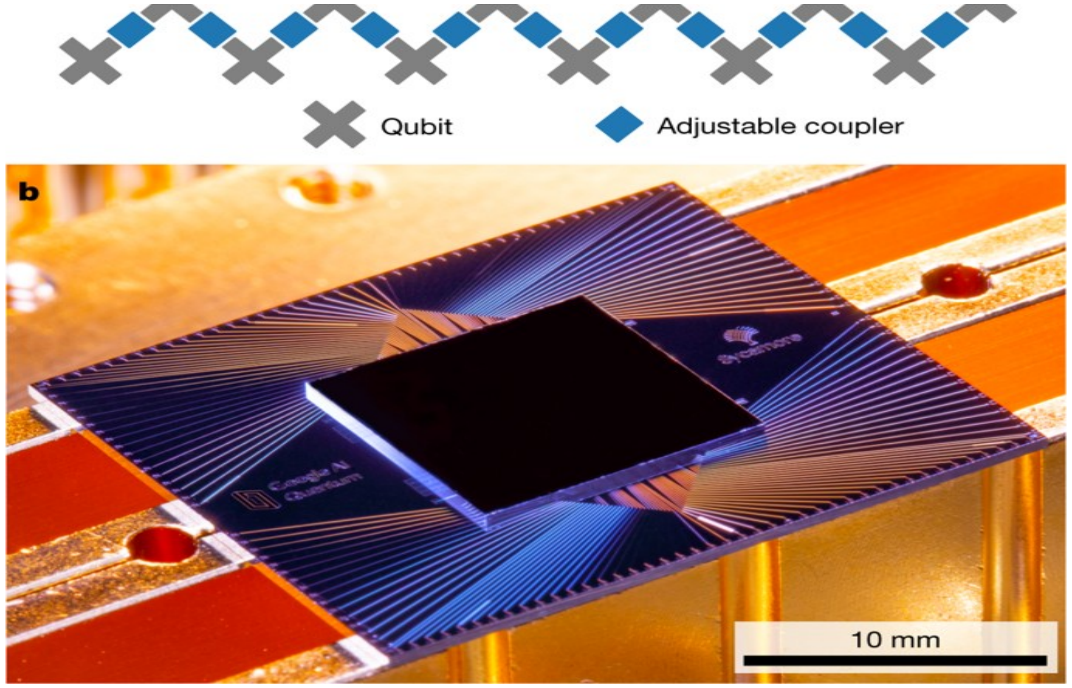
Quantum simulators



Fermi-Hubbard simulation (MPQ)

Understand quantum matter (superconductivity, topology, High energy physics,..)

Quantum computers

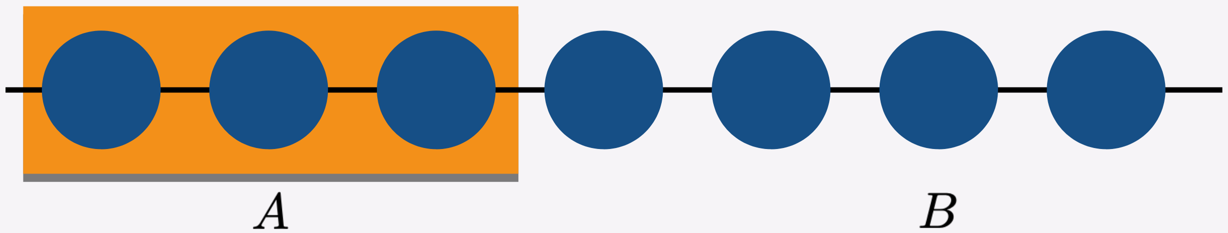


Google Sycamore chip

Quantum algorithms
Optimization problems (Annealing)

Key challenge: probe quantum properties of these many-body systems

Entanglement



Two subsystems A and B are **bipartite entangled** iff $|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle \quad \rho \neq \sum_j p_j \rho_j^{(A)} \otimes \rho_j^{(B)}$

Reduced density matrix $\rho_A = \mathrm{Tr}_B(\rho)$

Entanglement condition (Horodecki 1996) $\mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho_B^2\right] < \mathrm{Tr}\left[\rho^2\right]$

Quantifying entanglement for pure states → Entanglement entropies

$$S_A = -\mathrm{Tr}_A\left[\rho_A \log \rho_A\right]$$ von-Neumann

$$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}_A\left[\rho_A^n\right] \quad \leq S_A$$ Nth Rényi

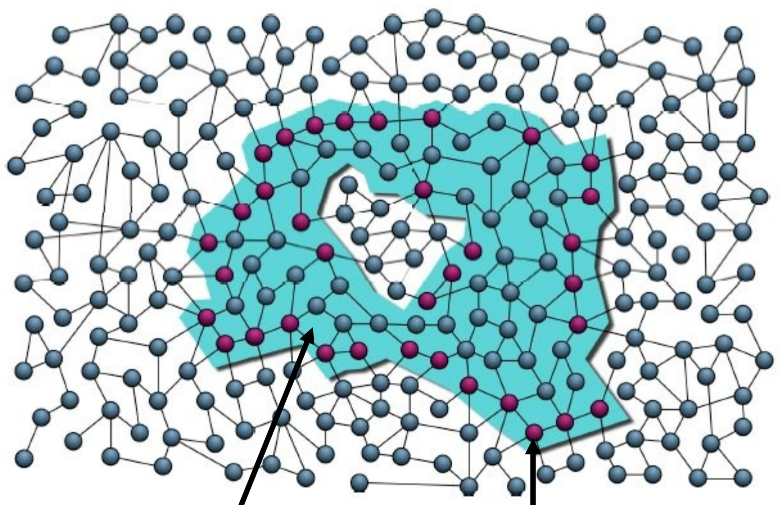
purity

$$S_A^{(2)} = -\log(\mathrm{Tr}_A(\rho_A^2))$$ **2nd Rényi**

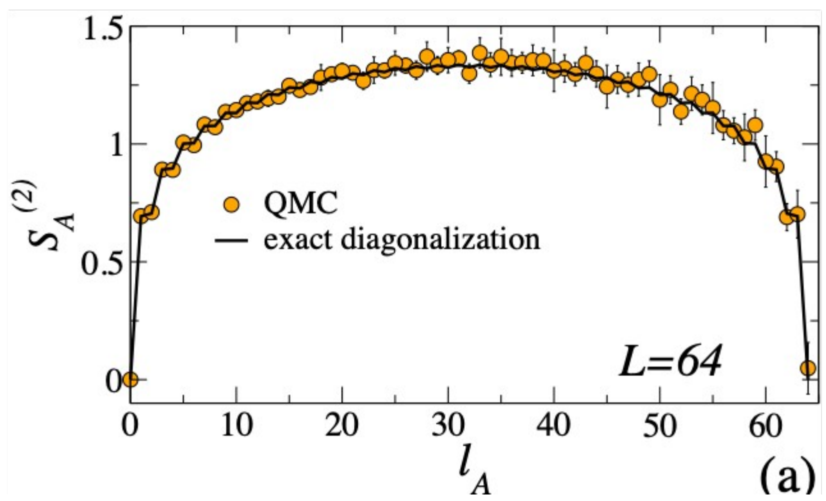
Measuring entanglement entropies: so what?

Measuring Entanglement entropies is fundamental for **Quantum Simulation**

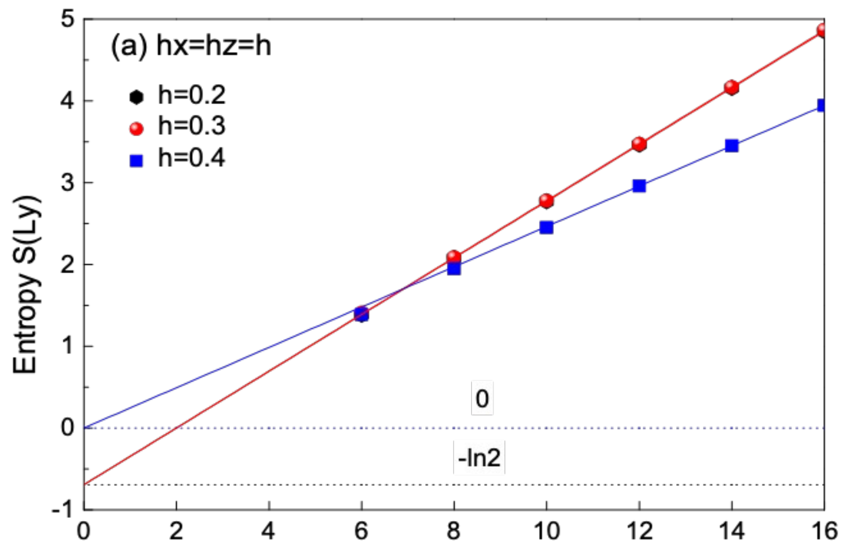
Many-body ground states | Quantum Phase transitions | Topological order



Amico et al., Rev.Mod.Phys, 80, 517 (2008)
Eisert et al., Rev. Mod. Phys. 82, 277 (2010)



P. Calabrese and J. Cardy, J. Stat. Mech (2004).
Humeniuk, Roscilde PRB (2012)



Kitaev, Preskill, PRL 2006
Levin, Wen, PRL 2006
Jian et al, NP 2012

Area law: $S_A^{(2)} \propto L_A^{D-1}$

$S_A^{(2)} \approx (c/4)\log(L_A)$

central charge

$S_A^{(n)} \approx \alpha_n L_A - \gamma$

Topological entanglement Entropy

Quantum Thermalization

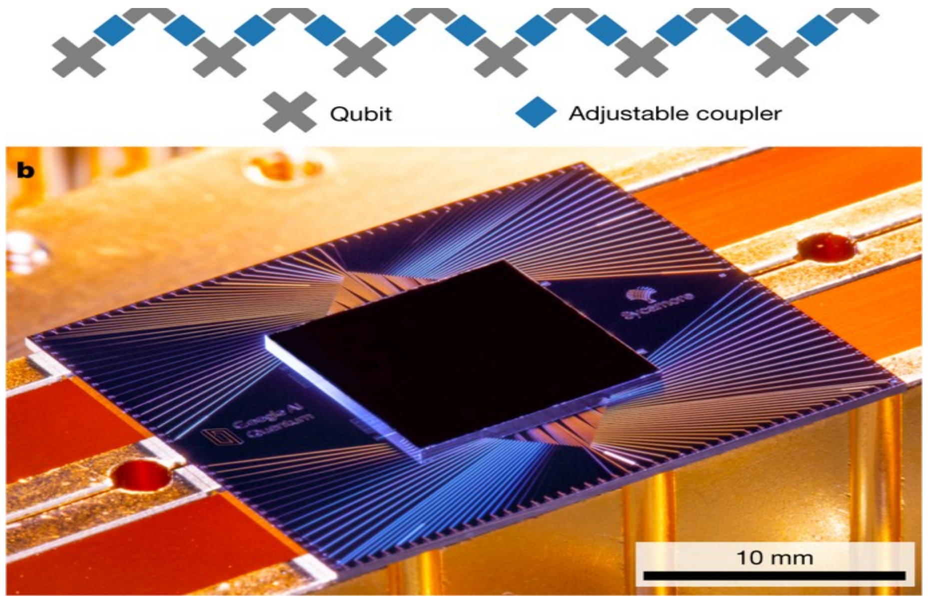
P. Calabrese and J. Cardy, PRL 2006
Badarson et al, PRL 2012

Measuring entanglement entropies: so what?

Measuring the entanglement "power" of quantum computers

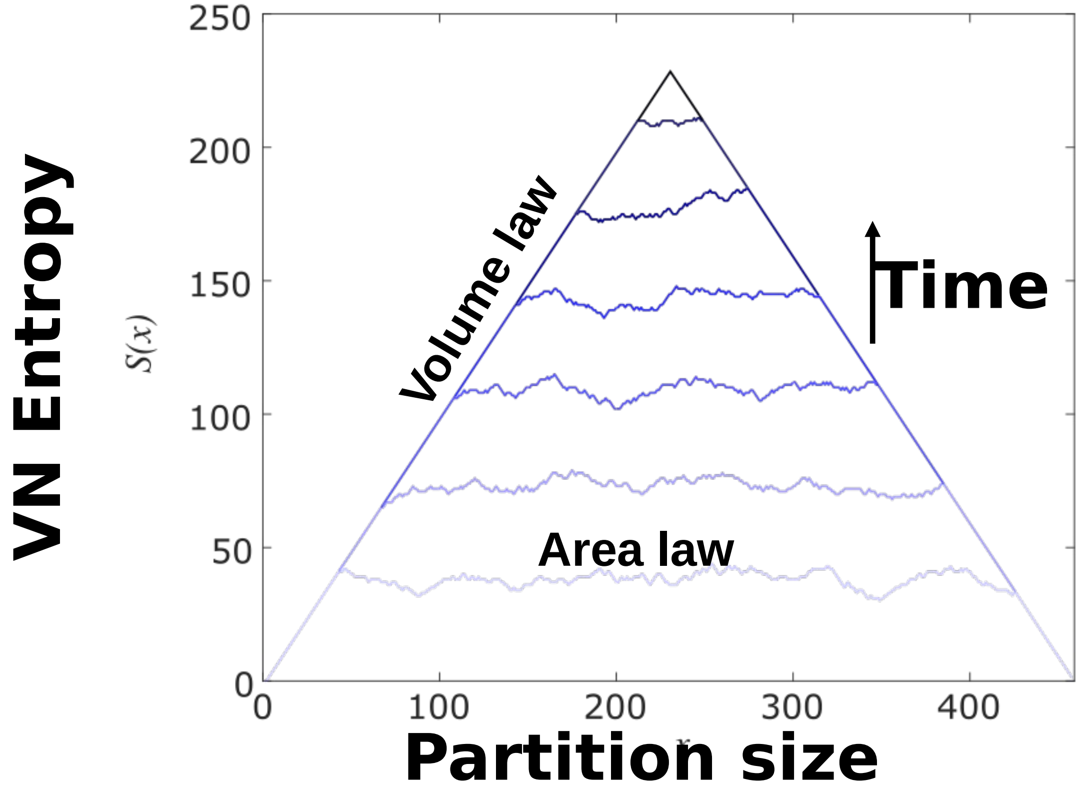
"Checks"

Purity checks
Entanglement checks



Google Sycamore chip

Universal behaviors

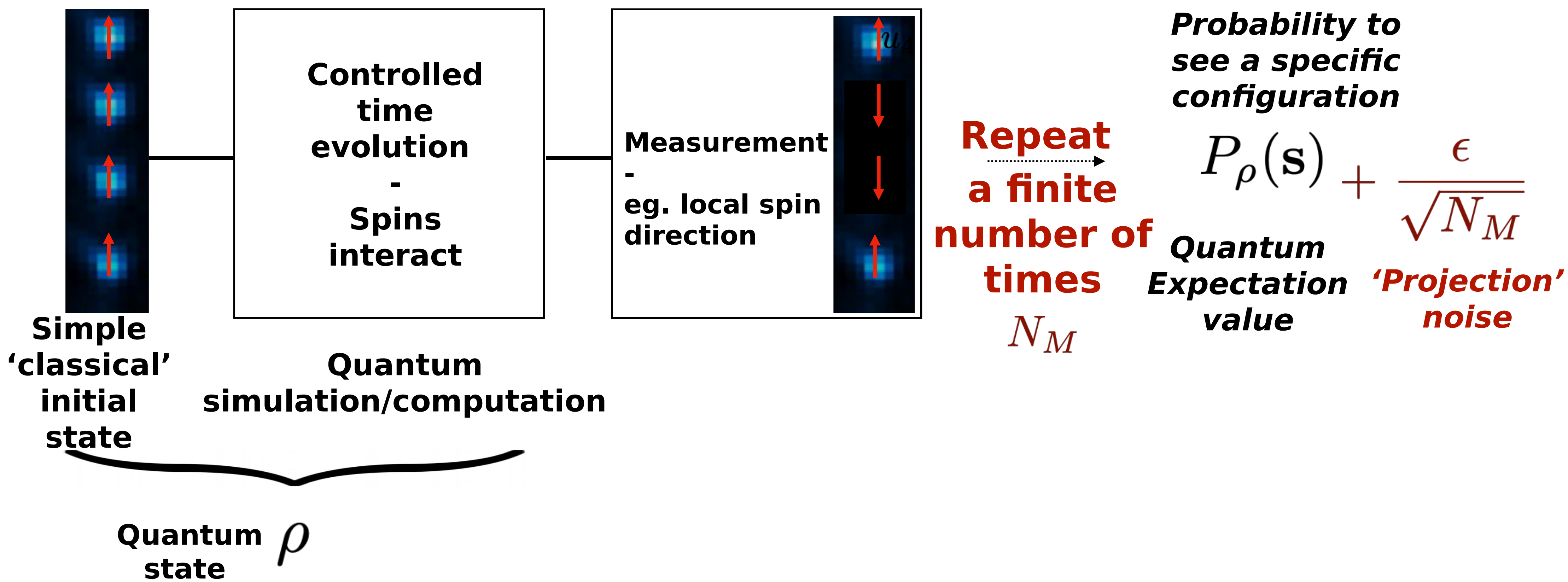


Nahum et al, Phys. Rev. X 7, 031016 (2017)

How to measure entanglement in such many-body quantum systems?

A new tool: randomized measurements

A standard measurement protocol



Limited to `observables', correlation functions, etc

Not applicable to Entanglement-related quantities, nonlinear functions w.r.t the density matrix

Example: $\mathrm{tr}(\rho^2)$

A new tool: randomized measurement protocols

Randomized measurement

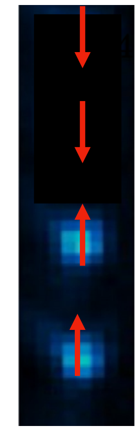
Simple ‘classical’ initial state

Controlled time evolution - Spins interact

Quantum Simulation

Controlled randomness U **random unitary**

Measurement - eg. local spin direction



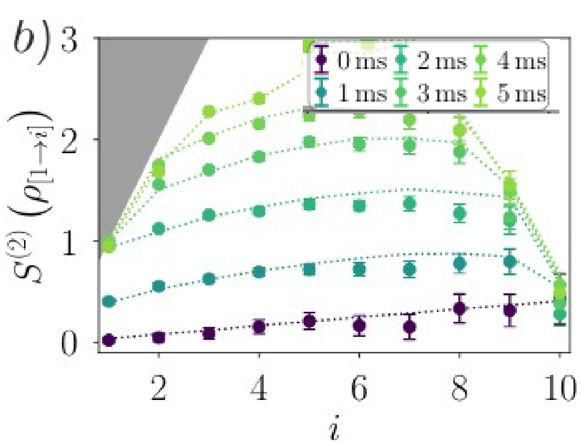
Repeat →

Measurement → random number $P_{\rho,U}(\mathbf{s})$

Ensemble average over random unitaries

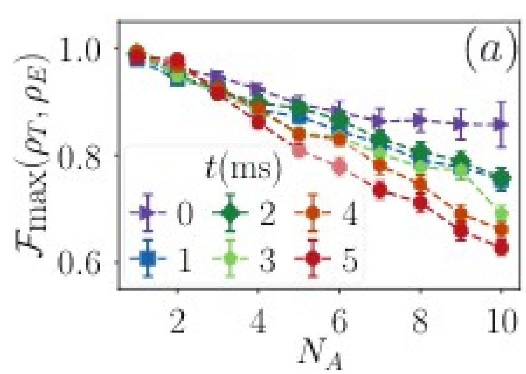
Correlations of probabilities $\overline{P_{\rho,U}(\mathbf{s}_1)P_{\rho,U}(\mathbf{s}_2)}$

Entanglement entropies



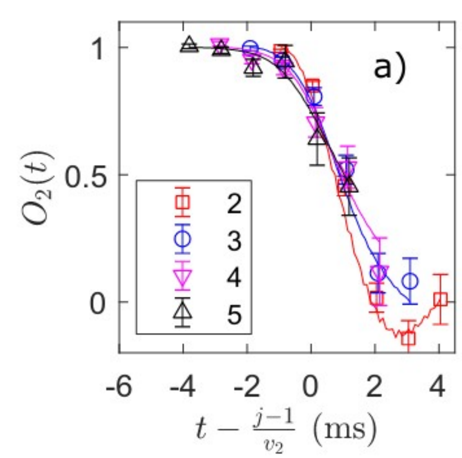
Van Enk, Beenakker PRL 2012
A.Elben, BV, et al.
PRL 2018, PRA 2018, PRA 2019
Brydges,..,BV,.., Science 2019
Huang et al Nature Physics 2020
Elben et al, arXiv:2101.07814

Fidelities



Elben, BV et al PRL 2020

Scrambling



BV et al., PRX 2019
Joshi, BV, et al PRL 2020

Topology

Elben,..,BV, Sci. Adv. 6 (2020)
ZP Cian, BV, et al, PRL 2021

Mixed-state entanglement

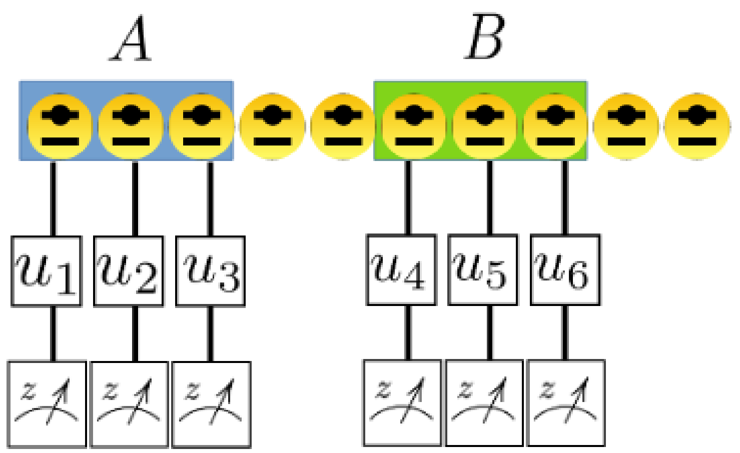
Zhou et al, PRL 2020
Elben, .., BV PRL 2020

See also works by Knips, Ketterer

Part I: Mixed-State Entanglement from Local Randomized Measurements

Phys. Rev. Lett. 125, 200501 (2020)

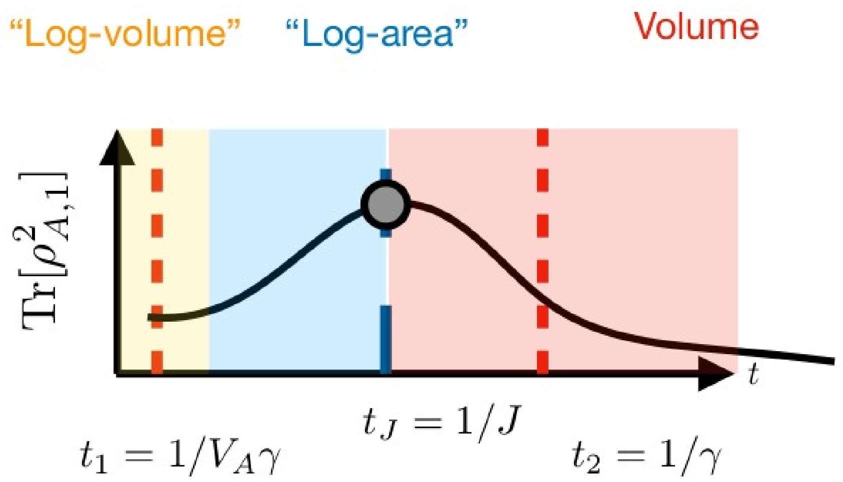
A. Elben (Innsbruck) R. Kueng (Caltech→ Linz), R. Huang (Caltech), R. van Bijnen (Innsbruck) C. Kokail (Innsbruck) , M. Dalmonte (Trieste), P. Calabrese (Trieste), B. Kraus, (Innsbruck) John Preskill (Caltech), Peter Zoller (Innsbruck), and BV



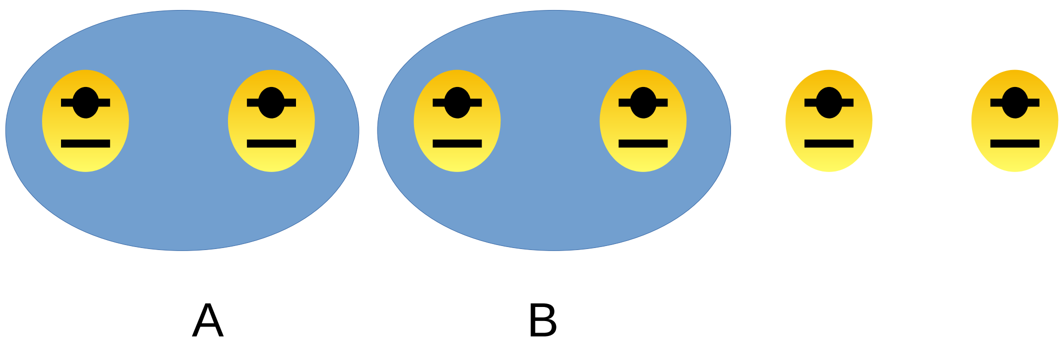
Part II: Symmetry-resolved dynamical purification in synthetic quantum matter

ArXiv:2101.07814 + arXiv:21xx.xxxx

V. Vitale, A. Elben, R. Kueng, A. Neven, J. Carrasco, B. Kraus, P. Zoller, P. Calabrese, B. Vermersch, M. Dalmonte



Mixed-state entanglement



Mixed-State Entanglement

$$\rho \neq \sum_j p_j \rho_j^{(A)} \otimes \rho_j^{(B)}$$

What kind of entanglement detection?

Purity test: $\mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho_B^2\right] < \mathrm{Tr}\left[\rho^2\right]$

Not very powerful for highly mixed states
(Brydges 2019)

Entanglement witness: $\mathrm{Tr}(O\rho_{AB}) < 0$

The relevant operator is state-dependent
(ex: CHSH inequalities..)

Not a quantifier of mixed-state entanglement

PPT condition $\rho_{AB}^{T_A}$ is not positive semi-definite

Powerful (ex: sufficient for two qubits)
Basis-independent
Entanglement monotone: negativity
Relevant in quantum field theories

Positive-Partial-Transpose (PPT) Condition for mixed state entanglement

If the state is separable $\rho_{AB} = \sum_k c_k \rho_A^{(k)} \otimes \rho_B^{(k)}$

Partial transposition

Then $\rho_{AB}^{T_A} = \sum_k c_k \left[(\rho_A^{(k)})^T \otimes \rho_B^{(k)} \right]$, Is positive semi-definite, i.e only has positive eigenvalues

Conversely, if $\rho_{AB}^{T_A}$ is not positive semi-definite → The state is entangled

Example: Bell state $\rho_{AB} = |Bell\rangle\langle Bell|$

=[[0. 0. 0. 0.]
 [0. 0.5 0.5 0.]
 [0. 0.5 0.5 0.]
 [0. 0. 0. 0.]]

00   01   10   11

$\rho_{AB}^{T_A}$ =[[0. 0. 0. 0.5]
 [0. 0.5 0. 0.]
 [0. 0. 0.5 0.]
 [0.5 0. 0. 0.]]

Spec = (0.5,0.5,0.5,-0.5)

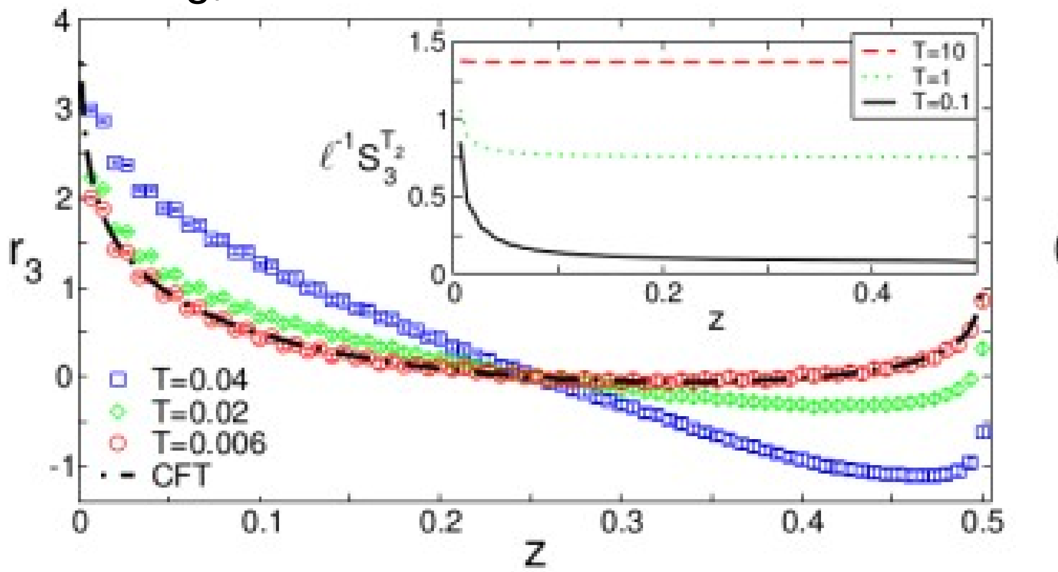
How to detect entanglement via the PPT condition in multi qubit systems??

Mixed-state entanglement

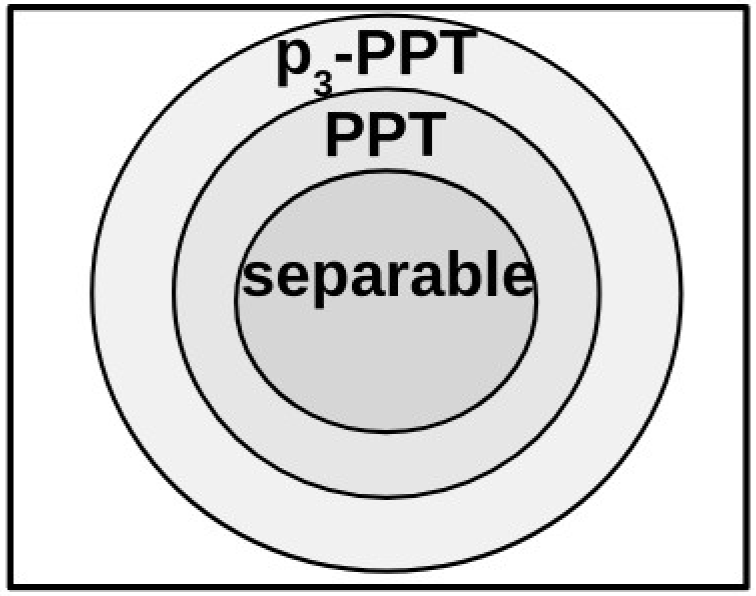
PT moments $$p_n = \mathrm{Tr}[(\rho_{AB}^{T_A})^n] \quad \text{for } n = 1, 2, 3, \ldots.$$

→ **Quantify mixed-entanglement in quantum-field theories:**
Works by P. Calabrese, etc

Chung, PRB 2014



→ **A measurable powerful entanglement condition**
Elben et al, PRL 2020



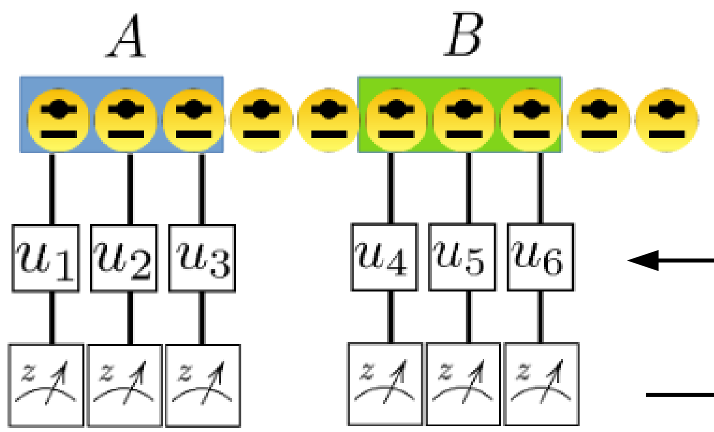
p_3 PPT condition

$p_3 < p_2^2$ implies PPT violation (implies entanglement)

Hint for the proof: large negative eigenvalues make p_3 small, and p_2 large

Measuring PT moments via local randomized measurements

Protocol



Random single qubit rotation (Haar distributed)

Measure bitstrings

See also, Zhou PRL 2020 with global unitaries

Randomized measurements are tomographically complete

Elben, et al PRA 2019, Huang et al, Nature Physics 2020 (see also Ohligher NJP 2013 for Hubbard models)

Measured bit strings

Ensemble average

$$\hat{\rho}_{AB}^{(r)} = \bigotimes_{i \in AB} \left[3 (u_i^{(r)})^\dagger \left| k_i^{(r)} \right\rangle \left\langle k_i^{(r)} \right| u_i^{(r)} - \mathbb{I}_2 \right] \qquad \mathbb{E}[\hat{\rho}_{AB}^{(r)}] = \rho_{AB}$$

Polynomials of the density matrix can be estimated via U-statistics

See also Huang et al, Nature Physics 2020 for the purity

$$p_3 = \mathbb{E} \left[\mathrm{Tr} \left((\rho_{AB}^{(r_1)})^{T_A} (\rho_{AB}^{(r_2)})^{T_A} (\rho_{AB}^{(r_3)})^{T_A} \right) \right]$$

→ Multi-linear post-processing of the data (no tomography)

→ Measurement budget ~$2^{N[AB]}$

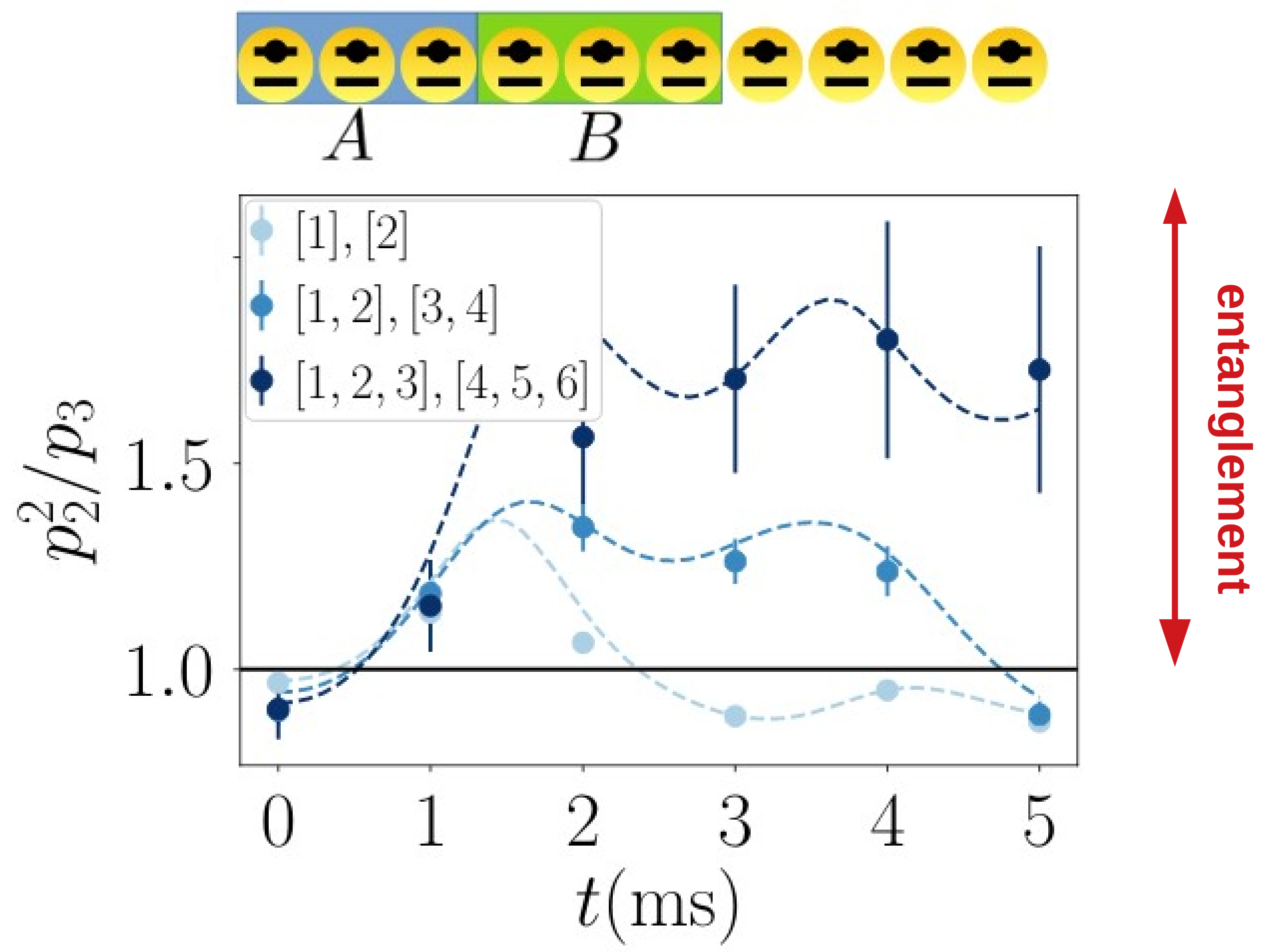
First experimental measurements of PT moments

Elben et al, Phys. Rev. Lett. 125, 200501 (2020)

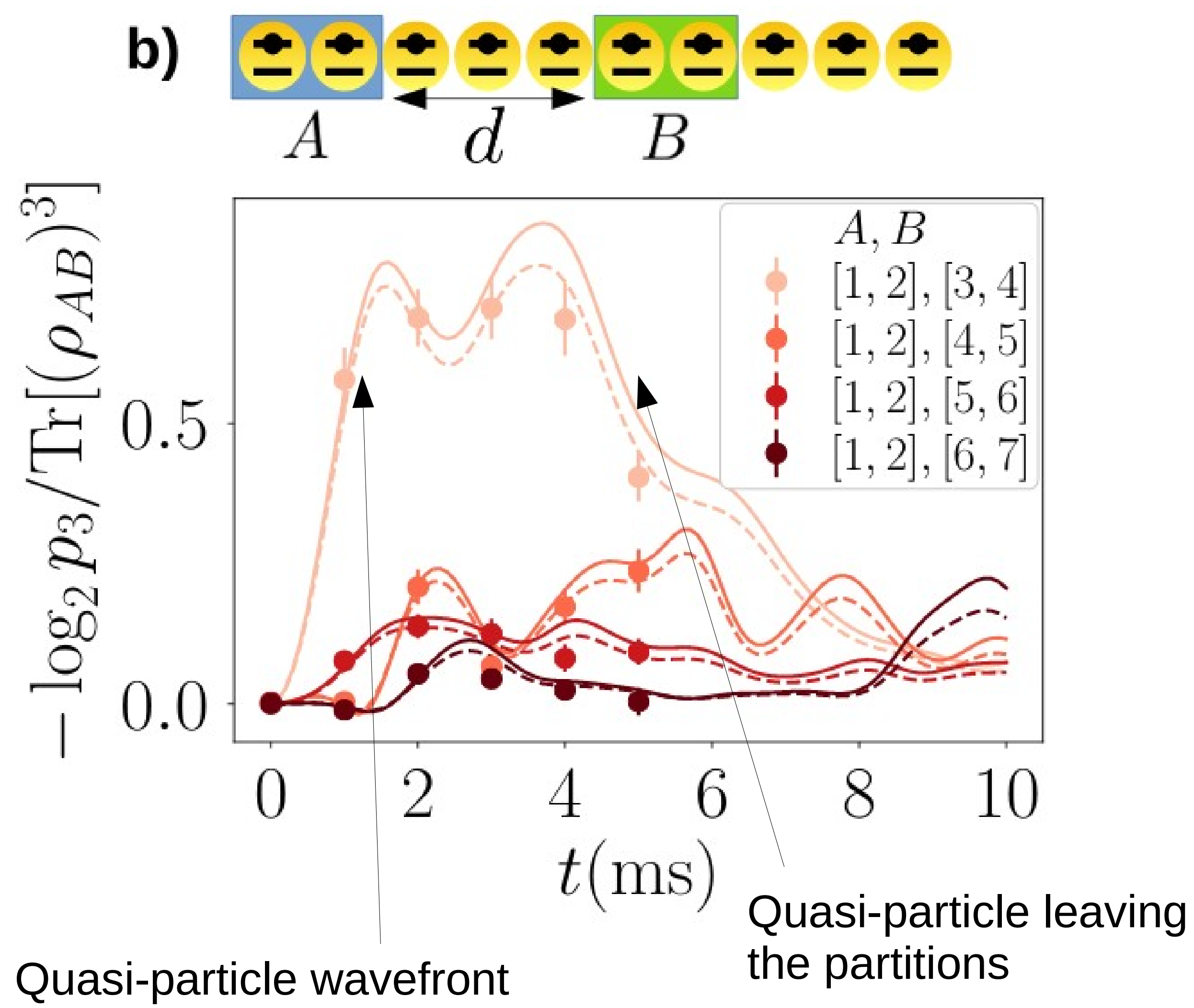
State: Quench of a Neel state with long range XY model

Data: Brydges , Science 2019 (reanalyzed)

Entanglement detection



Entanglement spreading
Quantum-field theory predictions: P. Calabrese et al



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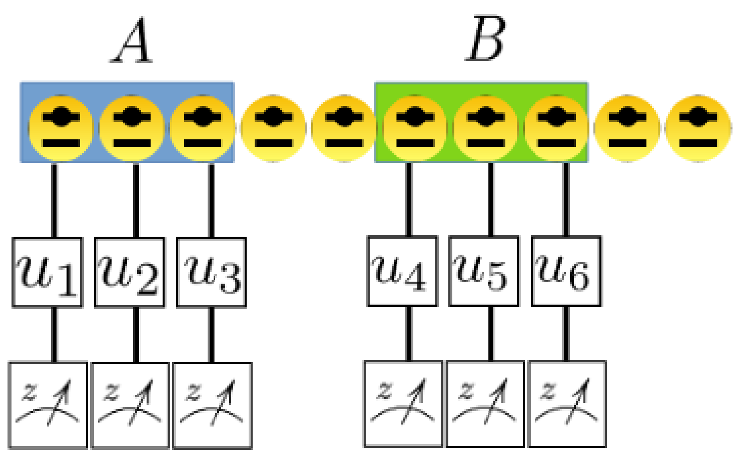

Part II: Symmetry-resolved dynamical purification in synthetic quantum matter

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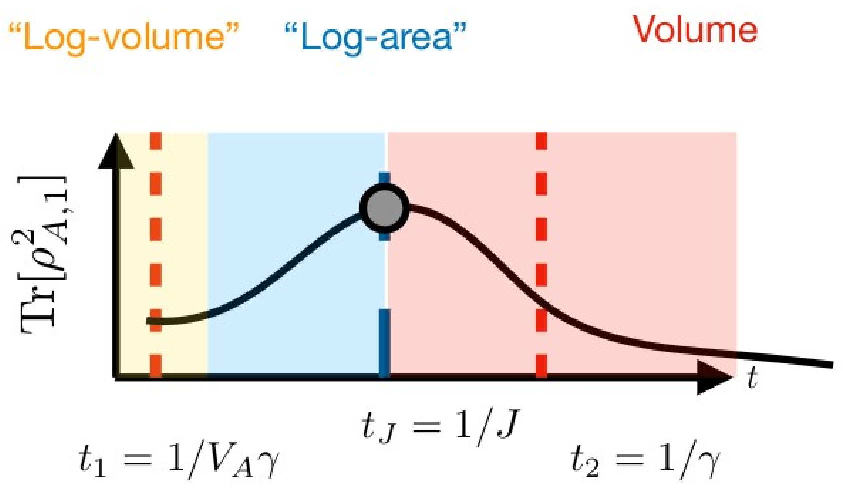
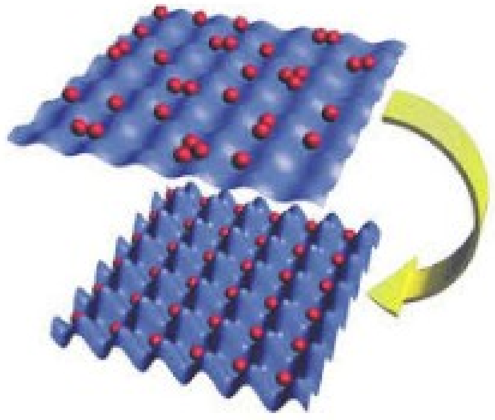
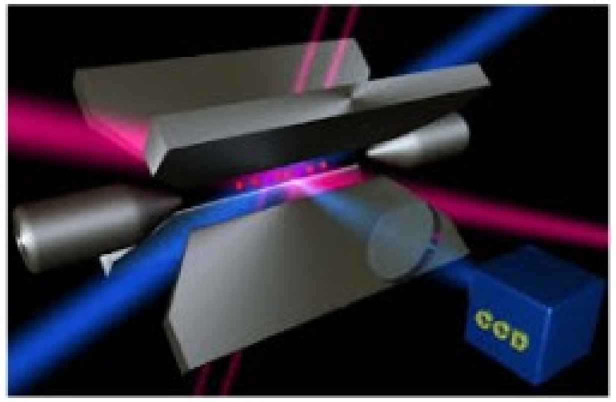


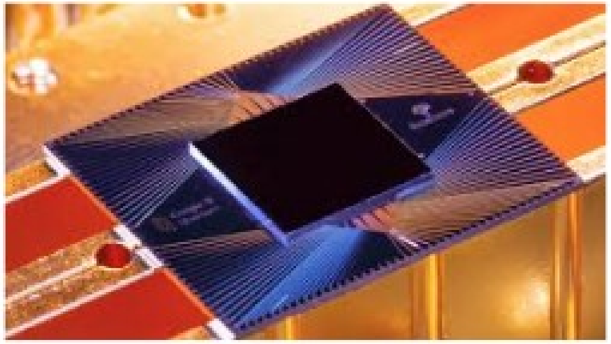
Part II: Symmetry-resolved dynamical purification in synthetic quantum matter

Motivation:

quantum simulators and noisy-intermediate scale quantum devices







The total number of excitations (atoms, spin up states) is conserved [**U(1) symmetry**]

Local interactions

Independent sources of dissipation [Spontaneous emission, particle loss, etc]

Dynamics from a product state (ex Neel State 01010101)

Can we observe universal short-time entanglement signatures of the competition between unitary versus decoherence dynamics?

Our model:

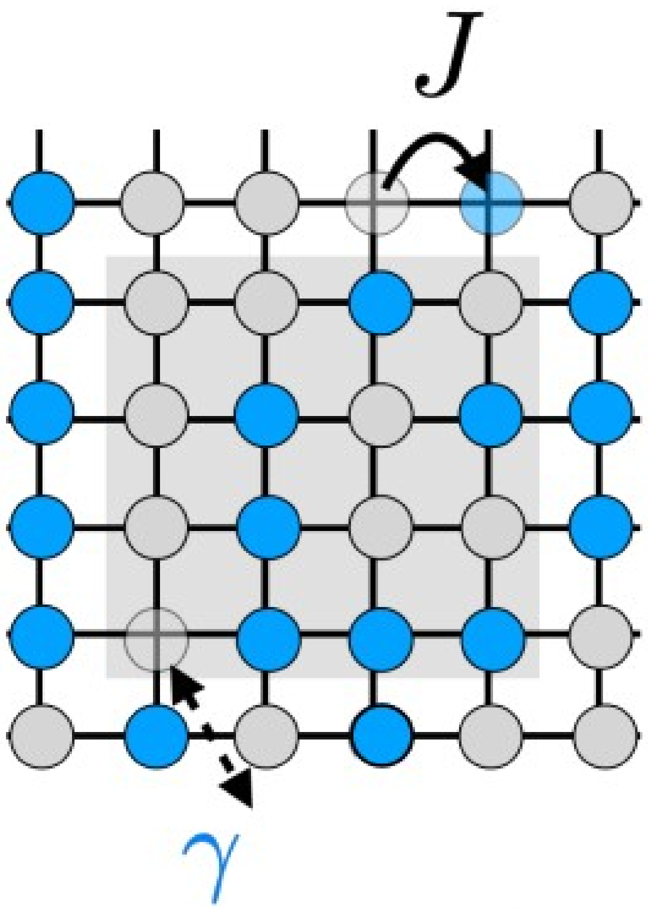
$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + \sum_k [L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}]$$

Hard-core bosons dynamics

$$H = J \sum_{<i,j>} (b_i^\dagger b_j + \text{h.c.})$$

Single particle loss

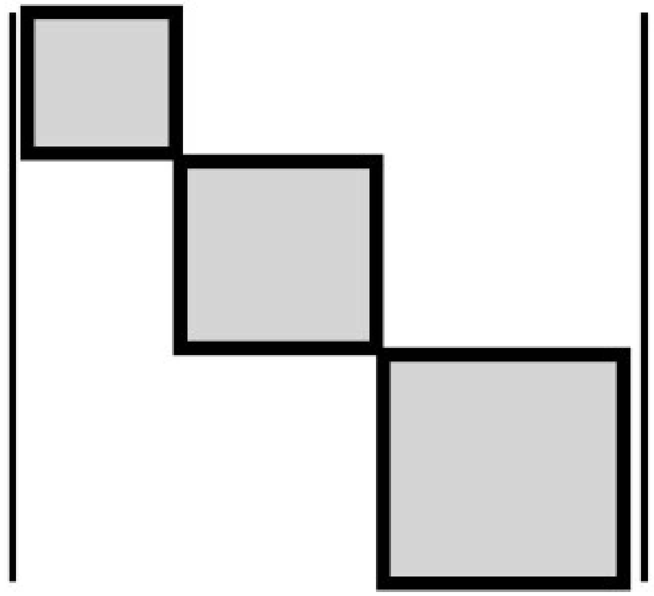
$$L_j = \sqrt{\gamma} b_j$$



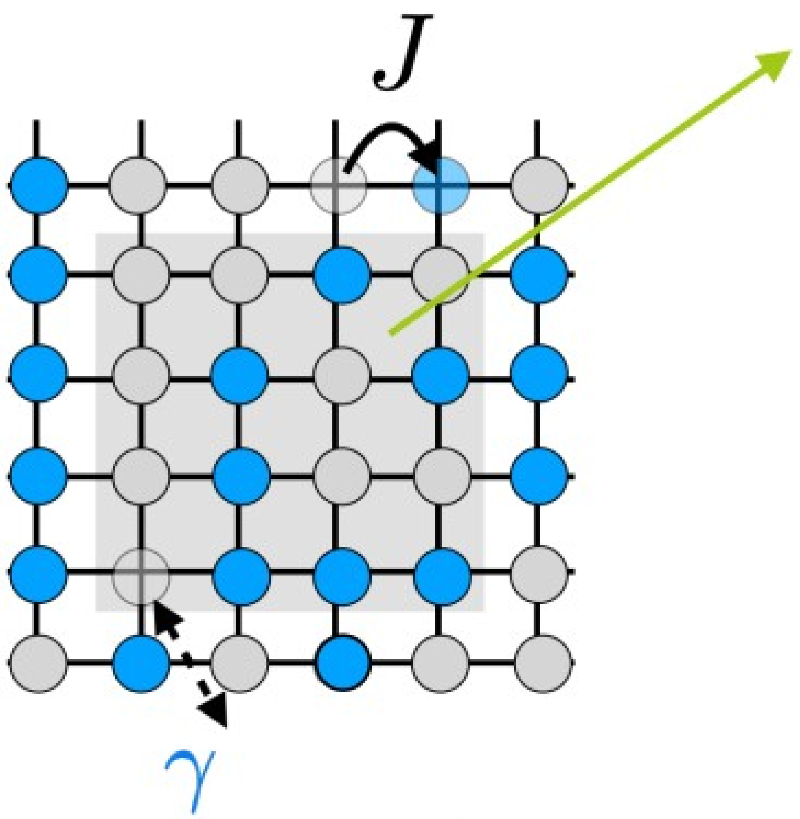
Part II: Symmetry-resolved dynamical purification in synthetic quantum matter

Consequence of U(1) Symmetry: Reduced density matrices are block-diagonal

$$\rho_A = \oplus_q \, p(q) \, \rho_A(q) =$$



q=number of bosons-
-initial value



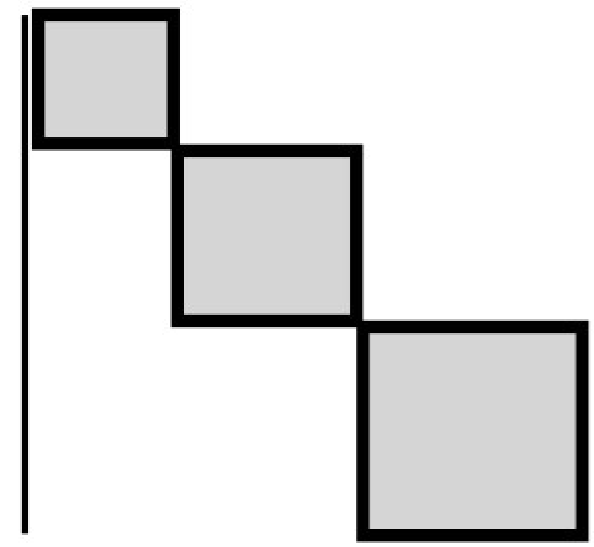
$$\rho_A(q) \equiv \frac{\Pi_q \rho_A \Pi_q}{\mathrm{Tr} \rho_A \Pi_q}, \quad \mathrm{Tr} \rho_A(q) = 1$$

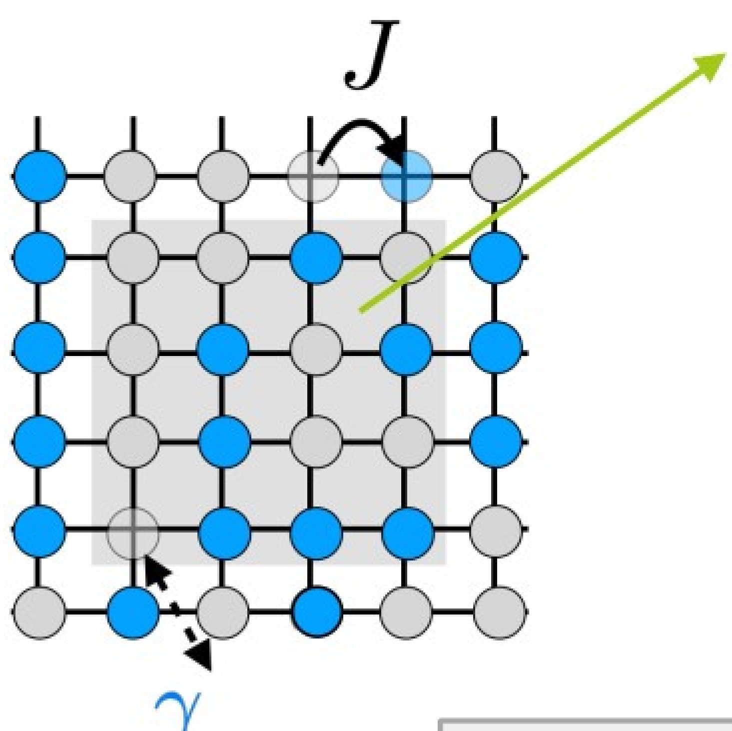
Description in terms of *m* *symmetry-resolved* reduced
(and normalized) density matrices

See early works in gauge theories + more recent by Calabrese, Goldstein, Laflorencie, Sela.

Consequence of U(1) Symmetry: Reduced density matrices are block-diagonal

$$\rho_A = \oplus_q \, p(q) \, \rho_A(q) =$$





q=number of bosons-
-initial value

$$\rho_A(q) \equiv \frac{\Pi_q \rho_A \Pi_q}{\mathrm{Tr}\rho_A \Pi_q}, \quad \mathrm{Tr}\rho_A(q) = 1$$

How to quantify SR dynamics?

SR-Renyi entropies $$S_A^{(n)}(q) \equiv \frac{1}{1-n} \log \mathrm{Tr}\rho_A(q)^n$$

SR-purity $$\mathcal{P}_A(q) \equiv \mathrm{Tr}\rho_A(q)^2$$

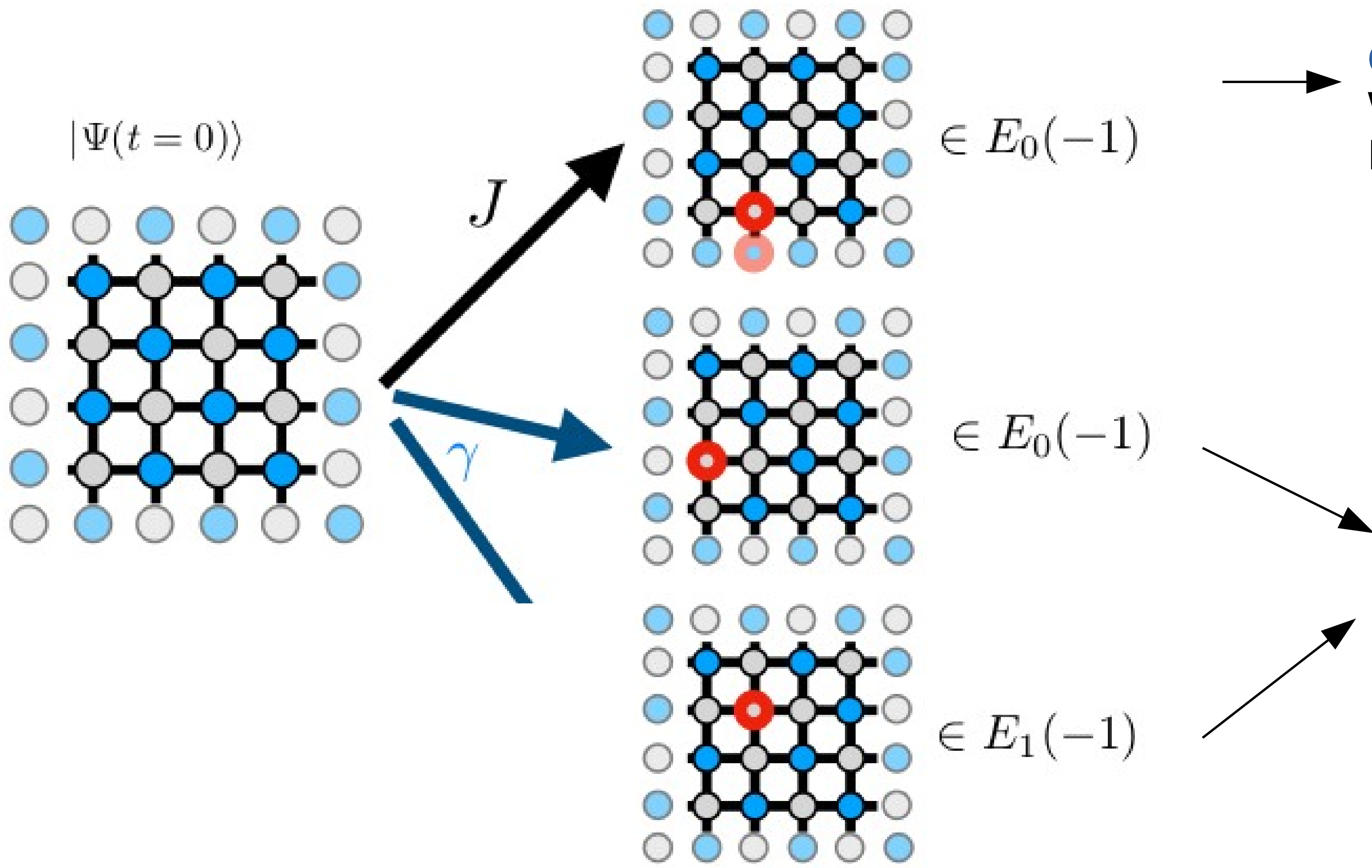
(~ inverse of the number of dominant eigenstates in each block)

And SR-PPT entanglement conditions...

Symmetry-resolved dynamical purification at short times

Tool: Second-Order Perturbation Theory at short times (w.r.t Lindblad rates)

In the q=-1 sector (I lost one particle)



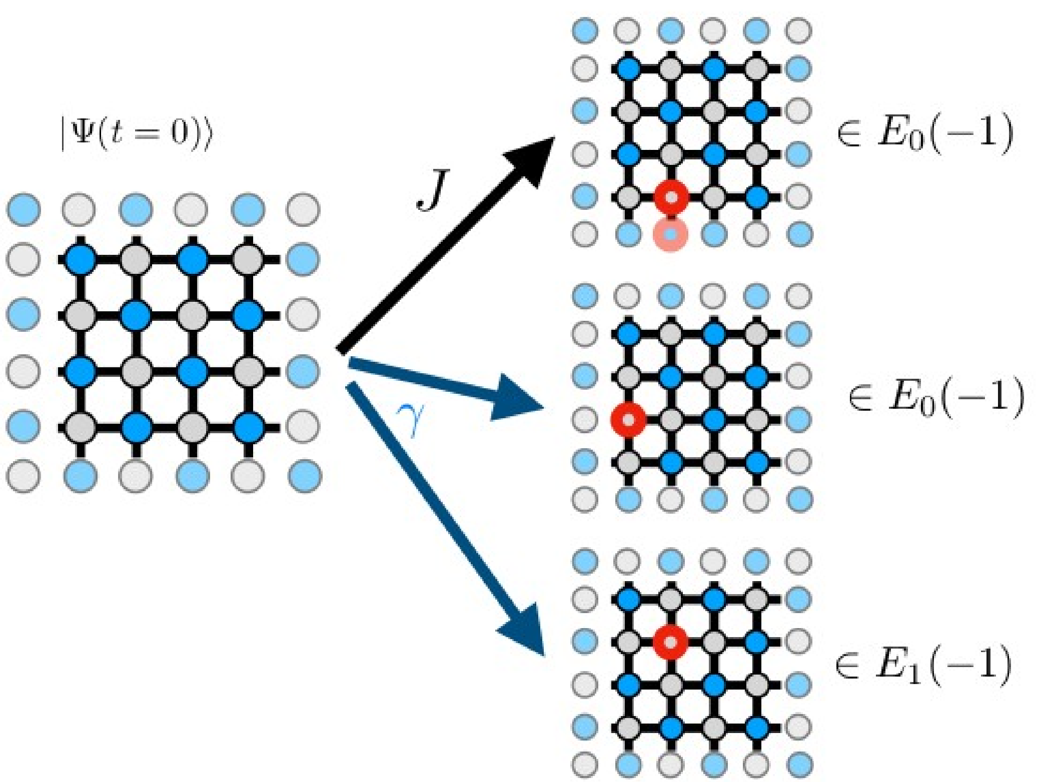
Coherent hopping
Weight of each term: J^2t^2
Number of terms: Area of partition

Loss
Weight of each term: γt
Number of terms: Volume partition

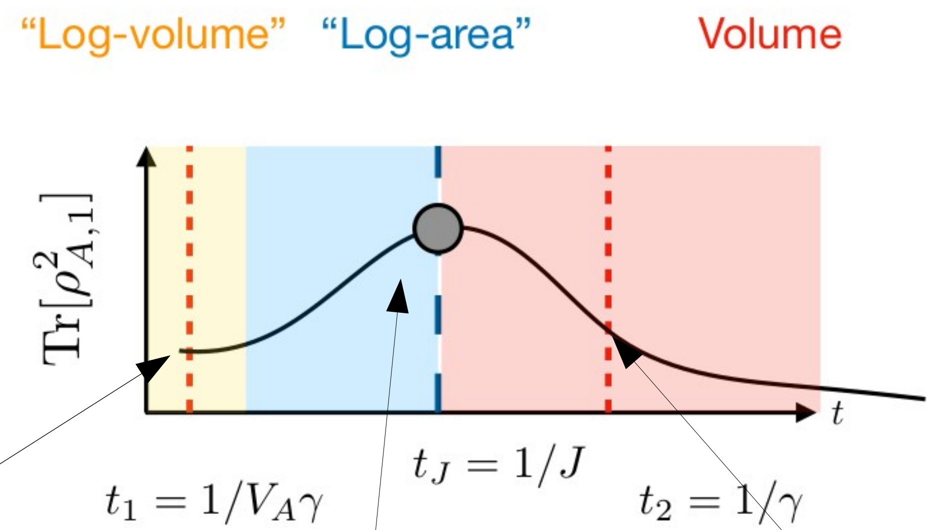
Loss terms always win at very short times!

Coherent terms will progressively beat the loss terms

Universal symmetry-resolved dynamical purification at short times



$$\mathcal{P}_A(-1) \propto \begin{cases} 1/\mathcal{V}_A & t_1 \gg t \geq 0 \text{ (short time)} \\ 1/(\partial \mathcal{V}_A) & t_J > t > t_1 \text{ (int. time)}, \\ 1/2^{\mathcal{V}_A} & t \gg t_j \text{ (long time)}. \end{cases}$$



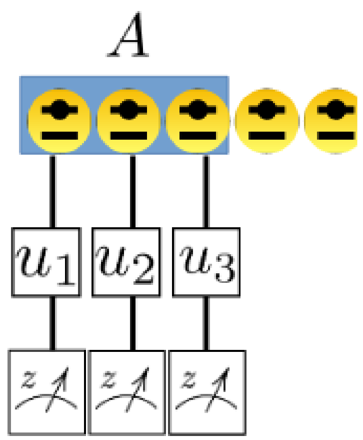
Dissipative processes dominant

Coherent 'boundary' terms dominant

Thermalization

Dynamical purification reveals the locality of interactions `on top' of a dissipative environment

How to measure SR entropies in an experiment via randomized measurements?



Random single qubit rotation (Haar distributed)

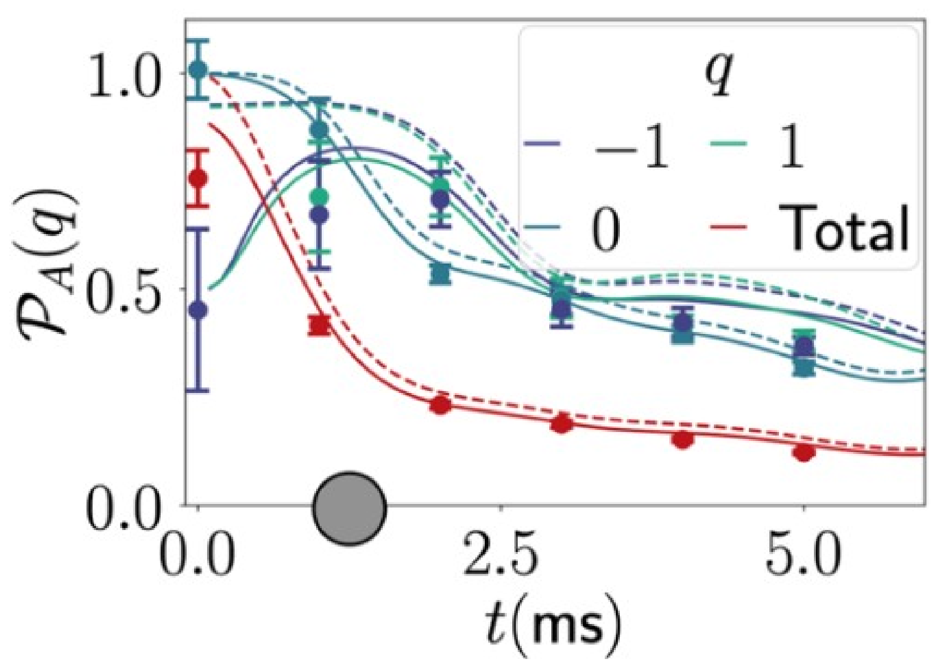
Measure bitstrings

Symmetry projection

$$\hat{\rho}_A^{(r)} = \bigotimes_{i\in A}\left[3(u_i^{(r)})^\dagger \left|k_i^{(r)}\right\rangle \left\langle k_i^{(r)}\right| u_i^{(r)} - \mathbb{I}_2\right] \longrightarrow \mathcal{P}_A(q)^{(r,r')} = \tfrac{1}{2}\mathrm{tr}[(\hat{\rho}_A^{(r)}\Pi_q)(\hat{\rho}_A^{(r')}\Pi_q)] + \tfrac{1}{2}\mathrm{tr}[(\hat{\rho}_A^{(r')}\Pi_q)(\hat{\rho}_A^{(r)}\Pi_q)]$$

Experimental observation of dynamical purification

State: Quench of a 10 qubit Neel state via a long range XY model
Data: Brydges , Science 2019 (reanalyzed)

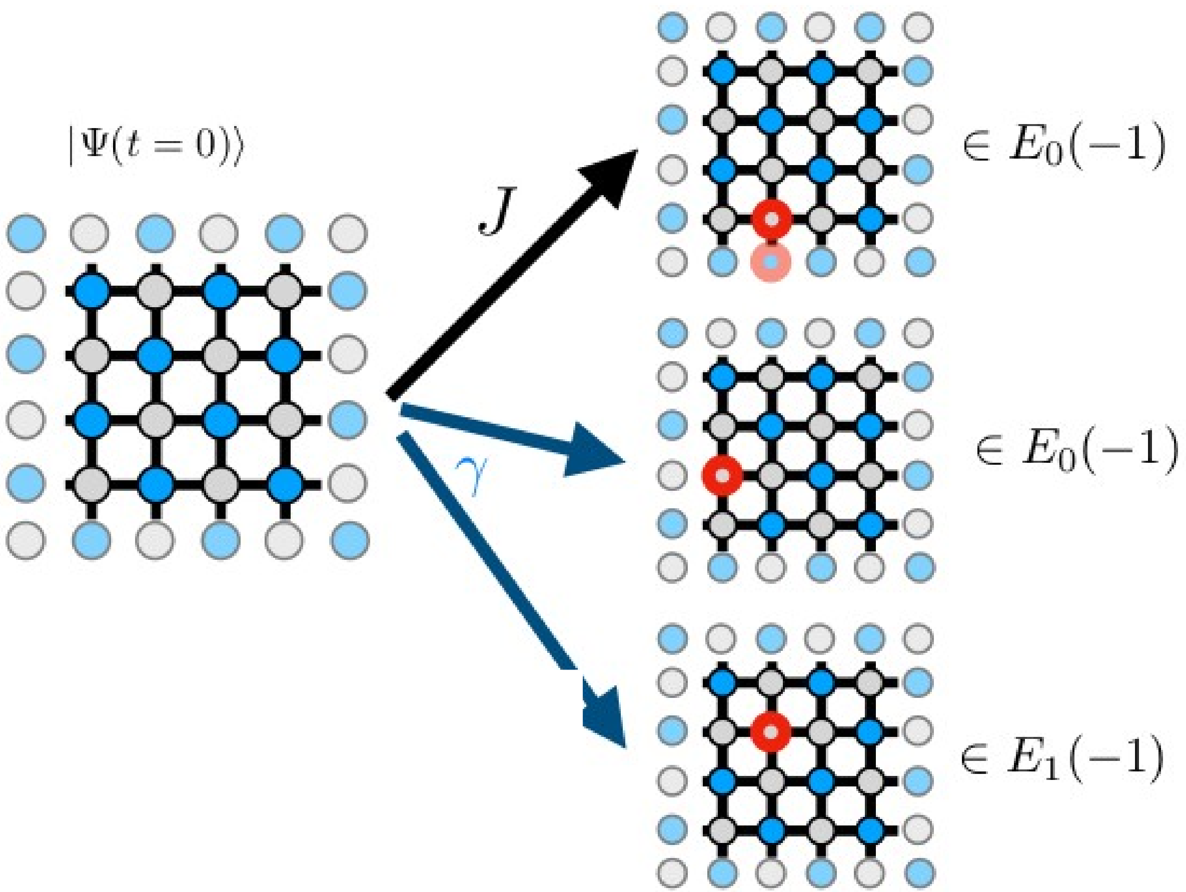


SR purity increases! Then decreases

The total purity decays

Part II: Symmetry-resolved dynamical purification in synthetic quantum matter

1) What about entanglement???



→ In one sector, the partition A **purifies**

→ **However, A is entangled with B**

→ **Proof: SR PPT condition**

2) Symmetry-resolved Partial transpose

Cornfeld, Goldstein and Sela, PRA 2018

$$\rho^{T_A}(\tilde{q}) \equiv \frac{\Pi_{\tilde{q}}\rho^{T_A}\Pi_{\tilde{q}}}{\mathrm{Tr}\rho^{T_A}\Pi_{\tilde{q}}}$$

3) Symmetry-resolved p3 PPT condition

$$p_3(\tilde{q}) < p_2(\tilde{q})^2$$

→ SR negative eigenvalue

→ entanglement

Perturbation theory: **SR p3 ppt detects entanglement at arbitrary short times**

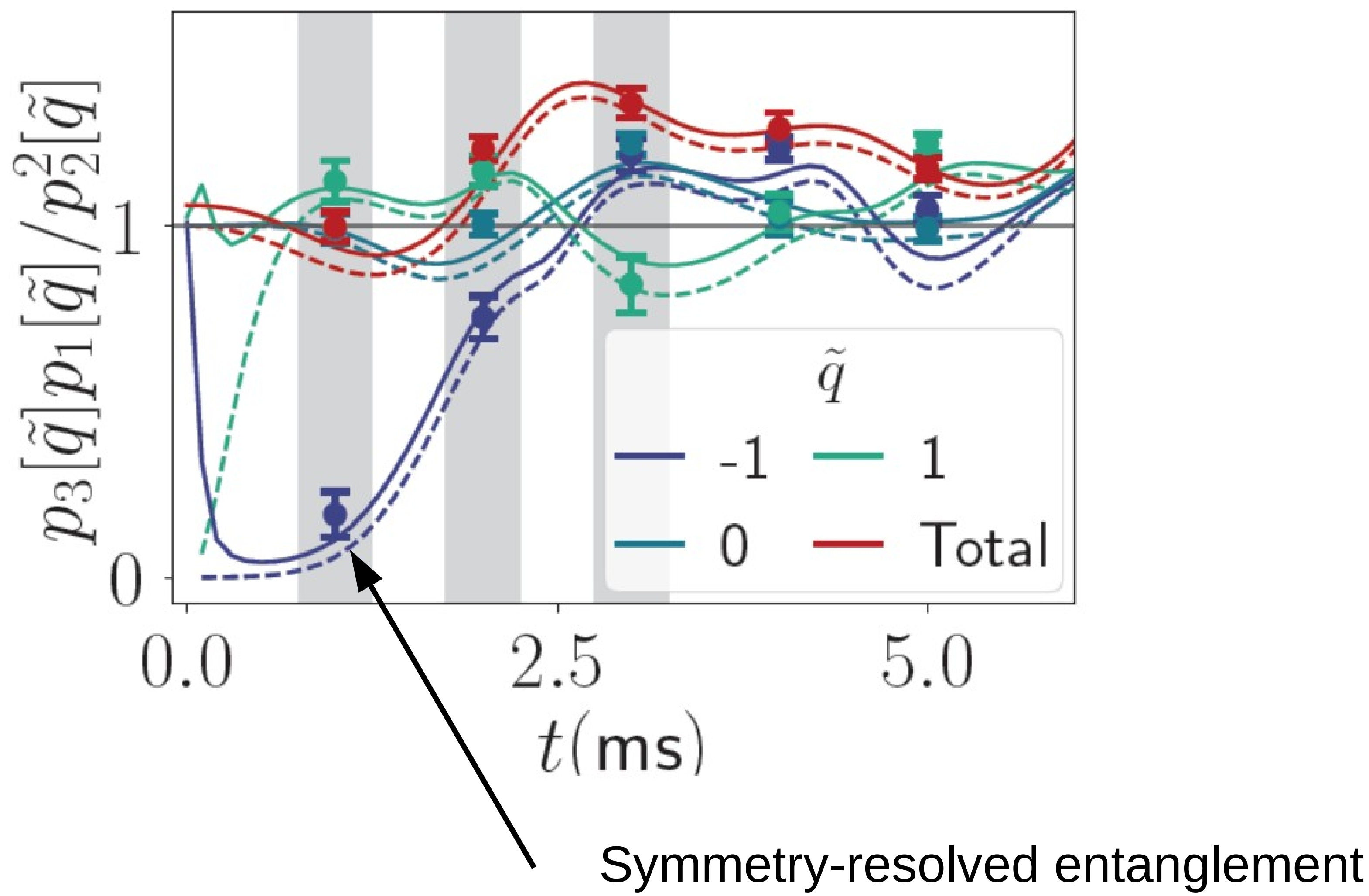
Neven, Carrasco et al, in preparation

Observation of symmetry-resolved entanglement

Neven, Carrasco et al, in preparation

State: Quench of a 10 qubit Neel state via a long range XY model

Data: Brydges , Science 2019 (reanalyzed)



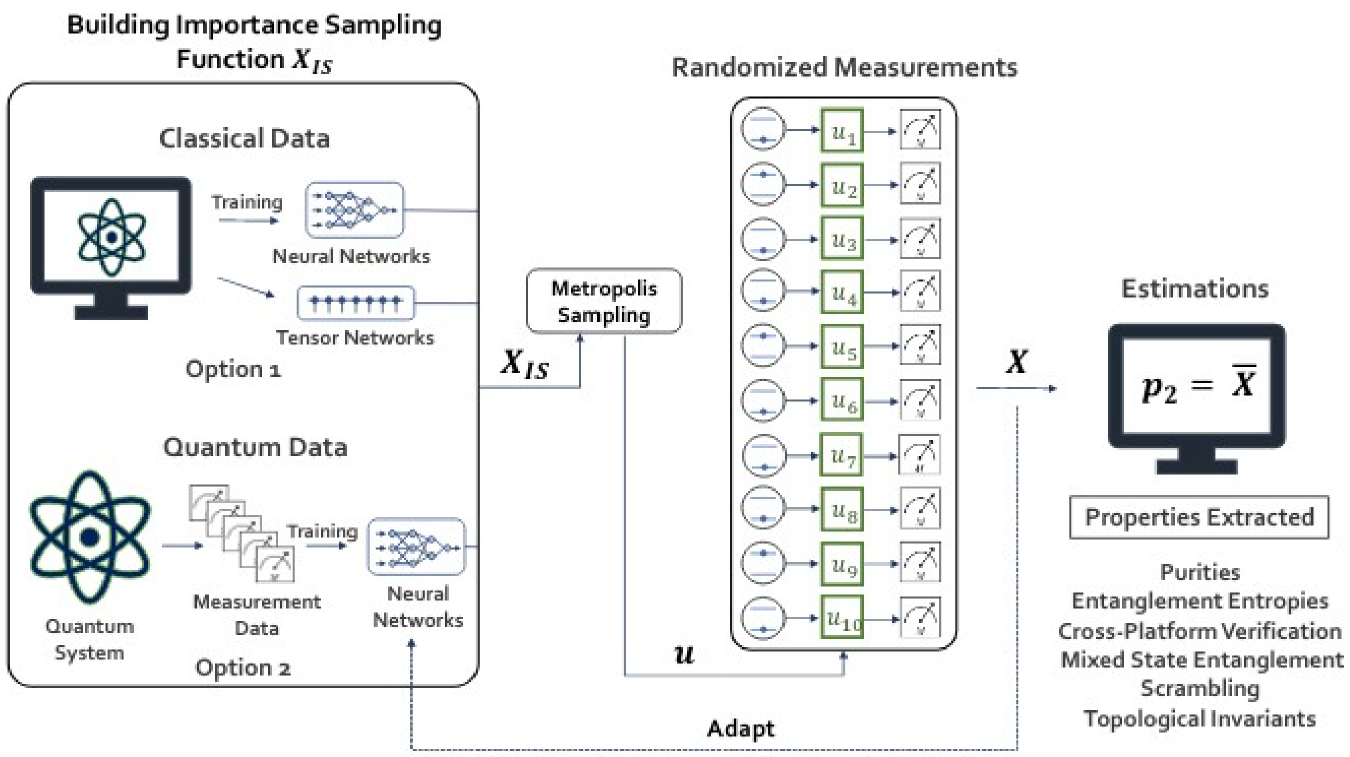
Conclusion

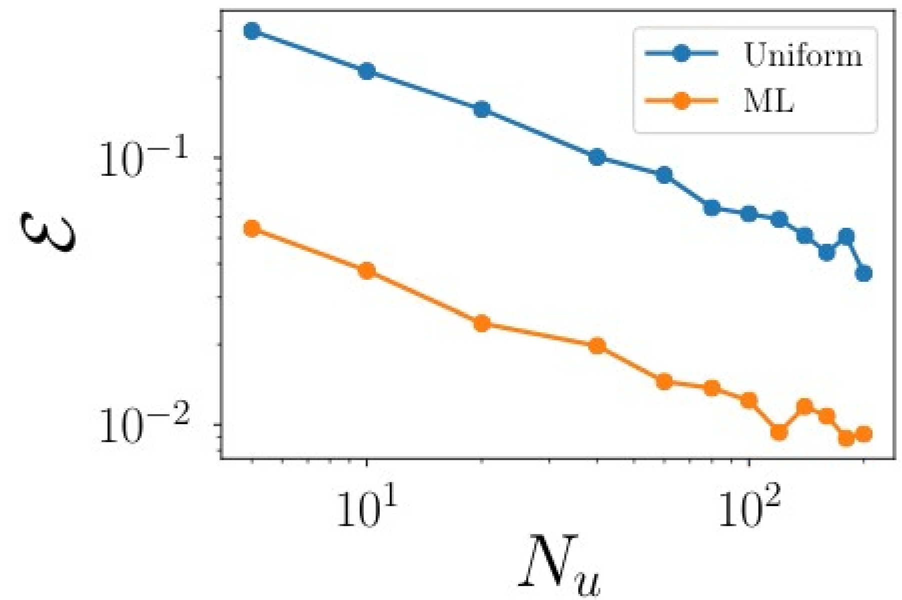
Randomized measurements: a versatile toolbox to probe many-body physics in quantum experiments

Current efforts

Optimized protocols

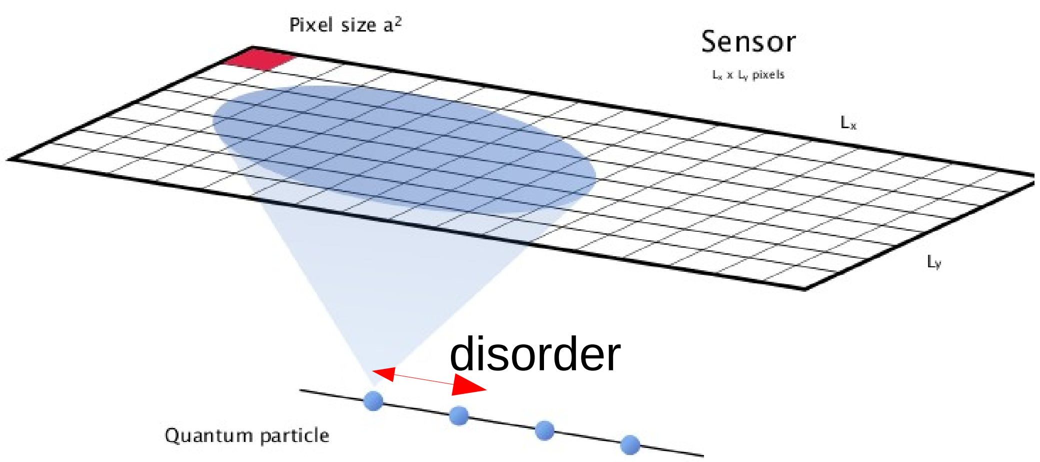
A.Rath, A. Elben, R. van Bijnen, P. Zoller





Random Time-of-flight Microscopy

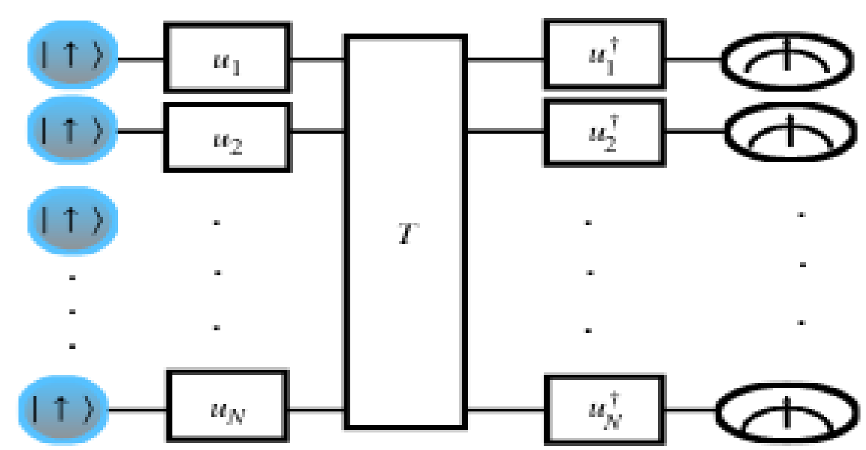
P. Naldesi, A. Elben, P. Zoller, A. Minguzzi



Random Hopping+Time of Flight

Measuring Spectral Form Factors

L. Joshi, A. Elben, P. Zoller



Thank you!

Funding available for PhD

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