APPLYING TNS TO LATTICE GAUGE THEORIES

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Benasque SCS February 2021

early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004
Tagliacozzo PRB2011; Sugihara JHEP2005
Meurice PRB2013

3+1 dimensions

Magnifico et al. 2011.10658

2+1 dimensions

Falser et al. arXiv:1911.09693
Robaina et al. PRL126, 050401 (2021)
Emonts et al. PRD102, 074501 (2020)

Schwinger model U(1) in 1D

precise equilibrium simulations, feasibility of QSim

MCB et al JHEP11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review: Dalmonte, Montangero, Cont. Phys. 2016
MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838

Non-Abelian in 1D

string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
Silvi et al., Quantum 2017
S. Kühn et al. PRX 2017

SU(3)QLM

Silvi et al, PRD 2019

finite density

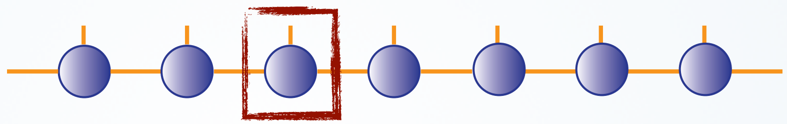
S. Kuehn et al, PRL118 (2017) 071601

MCB, K. Cichy 1910.00257
QTFLAG Collab.1911.00003

SPECTRUM

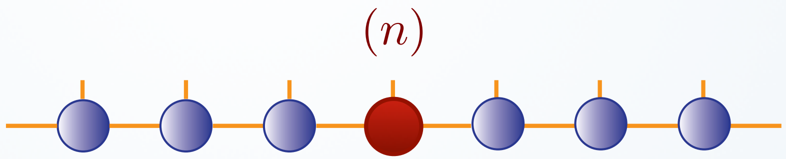
COMPUTING THE LOW ENERGY LEVELS

Efficient algorithms to find ground state and excitations

$|E_0\rangle \simeq$ 

variational, imaginary time…

work directly in the TD limit

$\sum_n e^{ikn}$ 

excitation ansatz

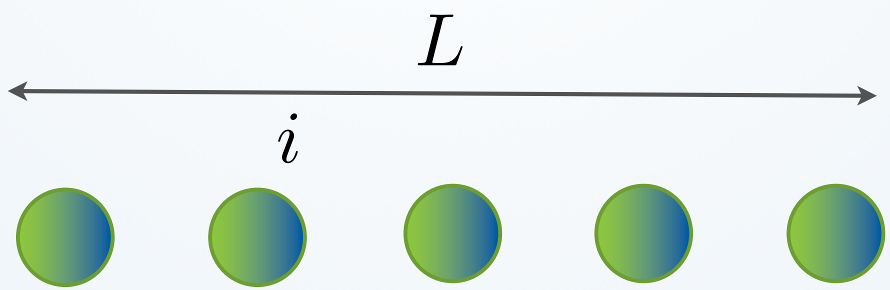
Different strategies possible for LGT

truncate the gauge dof, integrate out

explicit symmetries in tensors

How to do the continuum calculation?

discrete system → finite lattice spacing

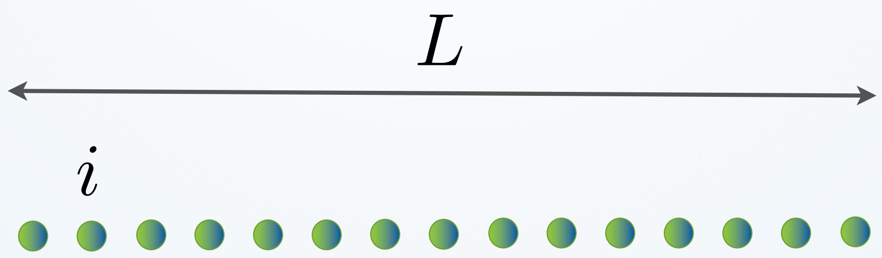


How to do the continuum calculation?

discrete system → finite lattice spacing

reduce lattice spacing → need larger size (N)

alternative: infinite size



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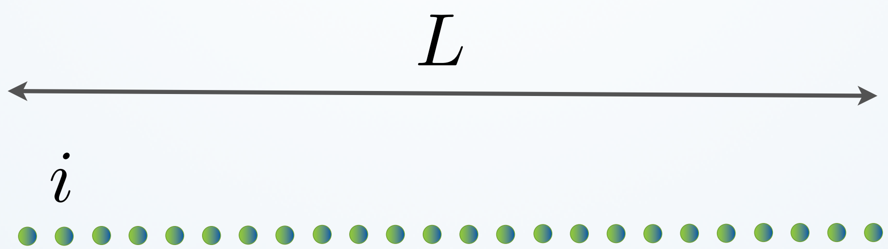


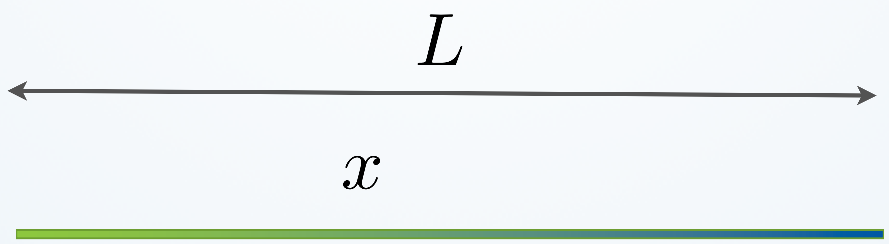
How to do the continuum calculation?

discrete system → finite lattice spacing

reduce lattice spacing → need larger size (N)

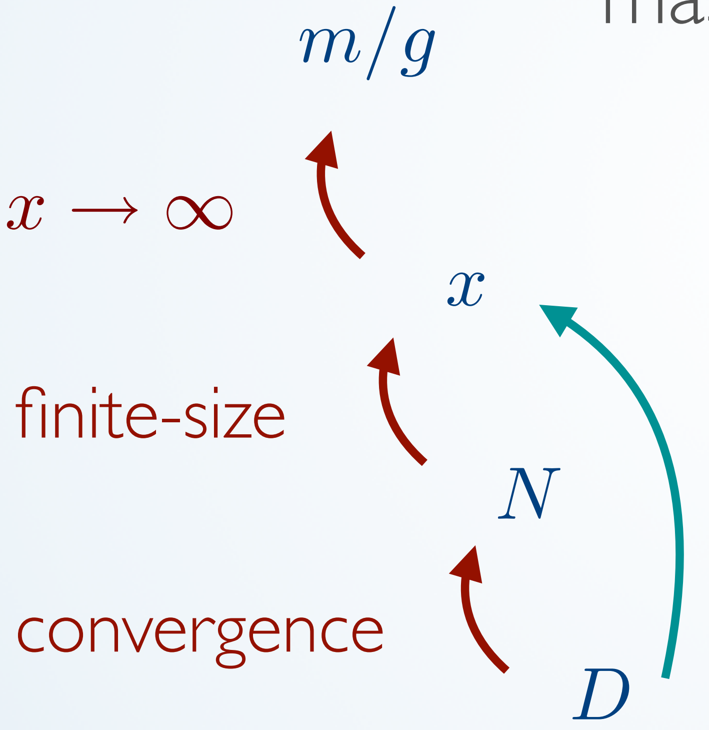
alternative: infinite size

extrapolate to vanishing spacing → possible divergences



COMPUTING CONTINUUM QUANTITIES

Scan parameters



mass gaps and GS energy density in the continuum $x \to \infty$

$x \lesssim 600$

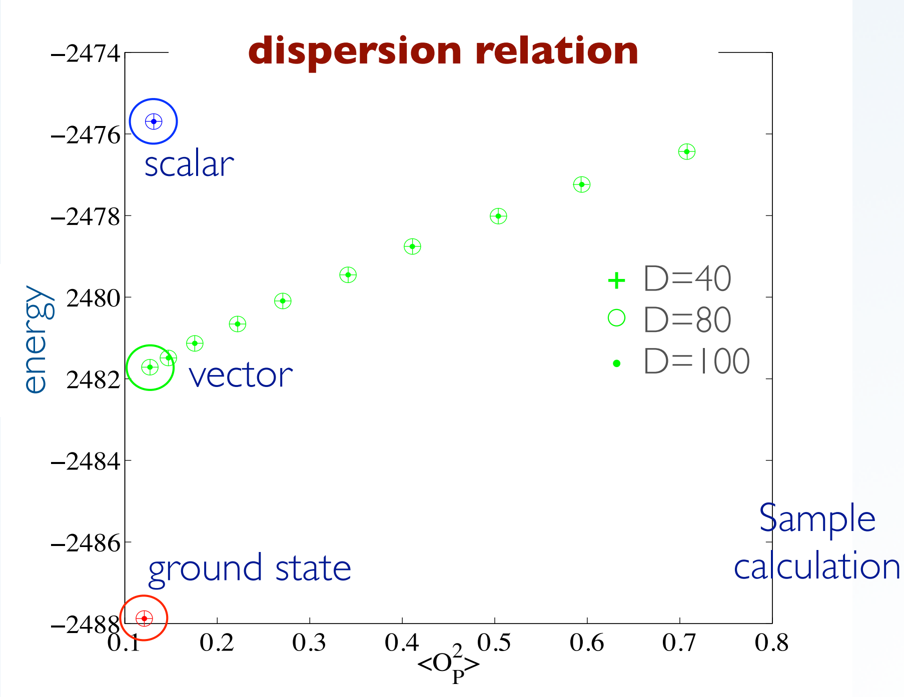
$N \propto \sqrt{x}$ (up to ~850)

alternative: infinite size

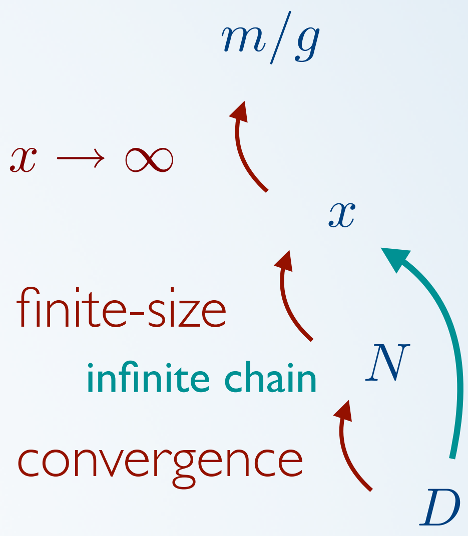
$D \approx O(100)$

MCB, Cichy, Cirac, Jansen JHEP11(2013)158

Buyens et al. PRL113 (2014) 091601



after the continuum limit



better precision than any earlier numerics

m/g	DMRG	MPS with OBC [1]	gauge inv. uMPS [2]	SCE	MPS with OBC [1]	gauge inv. uMPS [2]
0	0.5641859	0.56414(26)	0.56418(2)	1,128379	1.1283(10)	-
125	0.53950(7)	0.53946(20)	0.539491(8)	1.22(2)	1.2155(28)	1.222(4)
0.25	0.51918(5)	0.51915(14)	0.51917(2)	1.24(3)	1.2239(22)	1.2282(4)
0.5	0.48747(2)	0.48748(6)	0.487473(7)	1.20(3)	1.1998(17)	1.2004(1)

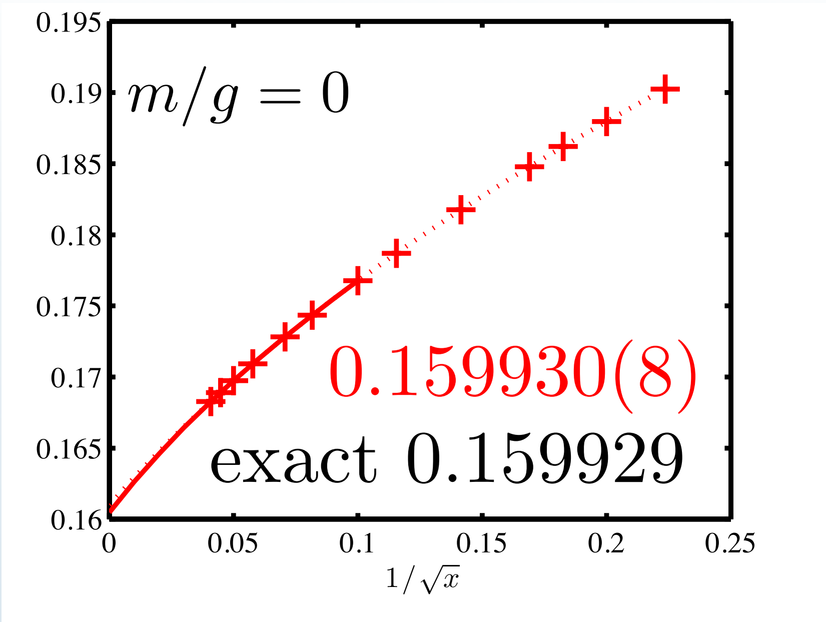
[1] MCB, Cichy, Cirac, Jansen JHEP11(2013)158
[2] Buyens et al. PRL113 (2014) 091601

MPS give us access to observables:
expectation values

MPS STATES → OBSERVABLES

chiral condensate in the GS: order parameter for chiral symmetry breaking (m/g=0) $\frac{\Sigma}{g} = \frac{\langle \bar{\Psi}\Psi \rangle}{g}$

in the spin language $\frac{\sqrt{x}}{L}\sum_n (-1)^n \frac{1+\sigma_n^3}{2}$



MPS STATES → OBSERVABLES

chiral condensate in the GS: order parameter for chiral symmetry breaking ($m/g=0$)

$$\frac{\Sigma}{g} = \frac{\langle \bar{\Psi}\Psi \rangle}{g}$$

in the spin language $$\frac{\sqrt{x}}{L} \sum_n (-1)^n \frac{1+\sigma_n^3}{2}$$

no exact value known for $m/g \neq 0$

only estimations

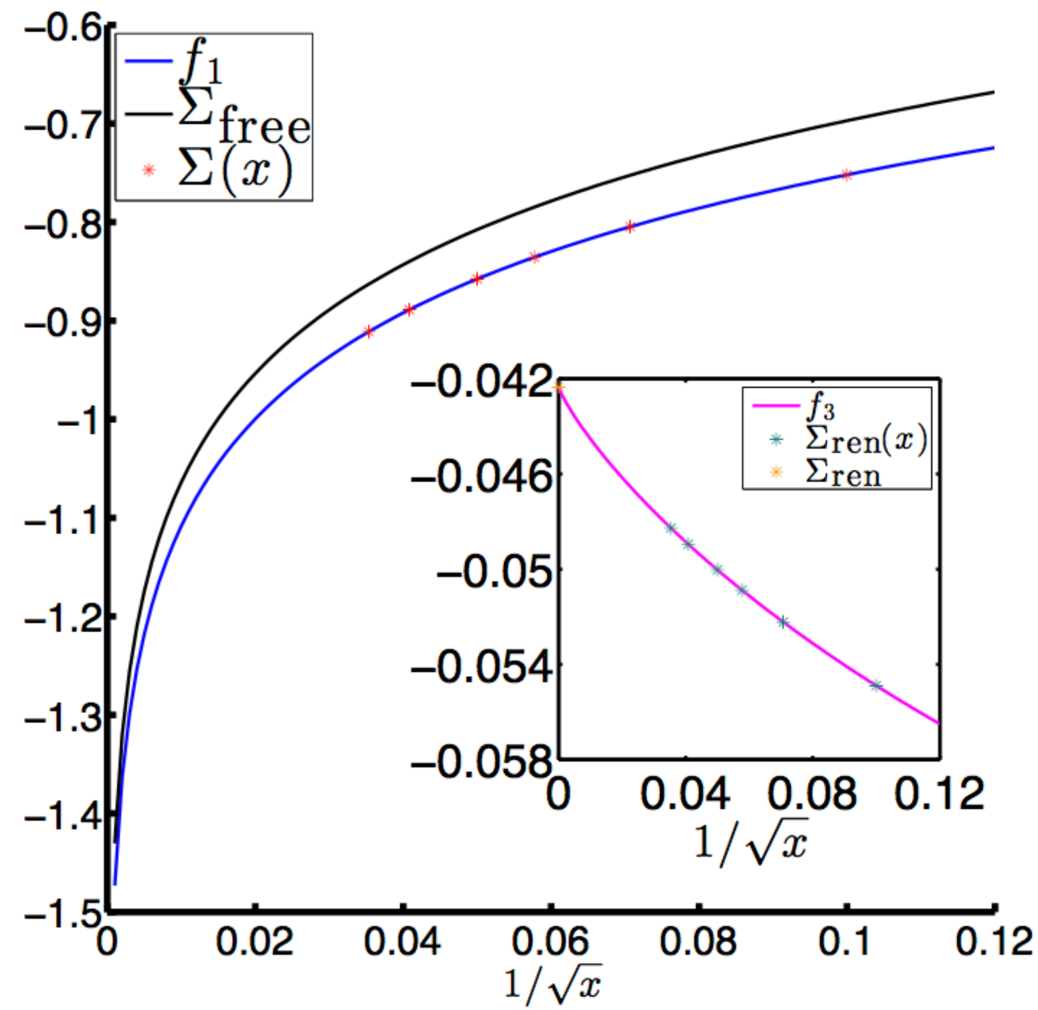
de Forcrand et al. 97
Hosotani 97

logarithmic divergence → same as in non-interacting case

MPS STATES → OBSERVABLES

chiral conde ... er $\dfrac{\Sigma}{g} = \dfrac{\langle \bar{\Psi}\Psi \rangle}{g}$

for chiral sy



from Buyens et al. arXiv:1411.0020

in the spin l

no exact va

Forcrand et al. 97

sotani 97

logarithmic ... racting case

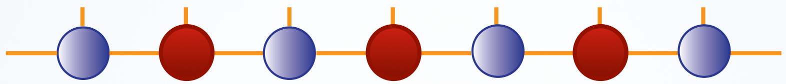
nteracting

condensate

uMPS: how important is the truncation of gauge dof?

uniform MPS (uMPS)

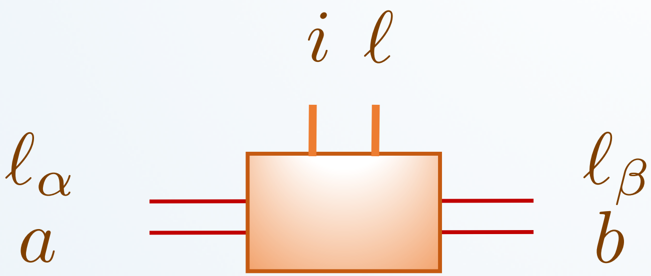
cannot integrate out gauge $|\ldots s_e\,\ell\;s_o\,\ell\;s_e\,\ell\;s_o\ldots\rangle$



truncate

but physical states have to satisfy Gauss' law

$\Rightarrow$ symmetries

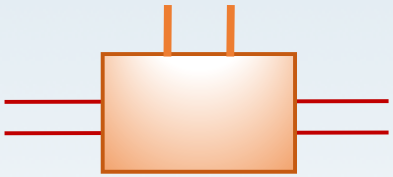


$A^{i\,\ell}_{\alpha\beta}$

$$\alpha = (\ell_\alpha\, a)$$
$$\beta = (\ell_\beta\, b)$$

$$\ell = \ell_\beta \qquad \ell_\beta = \ell_\alpha + \frac{(-1)^i + (-1)^n}{2}$$

symmetric MPS has block structure



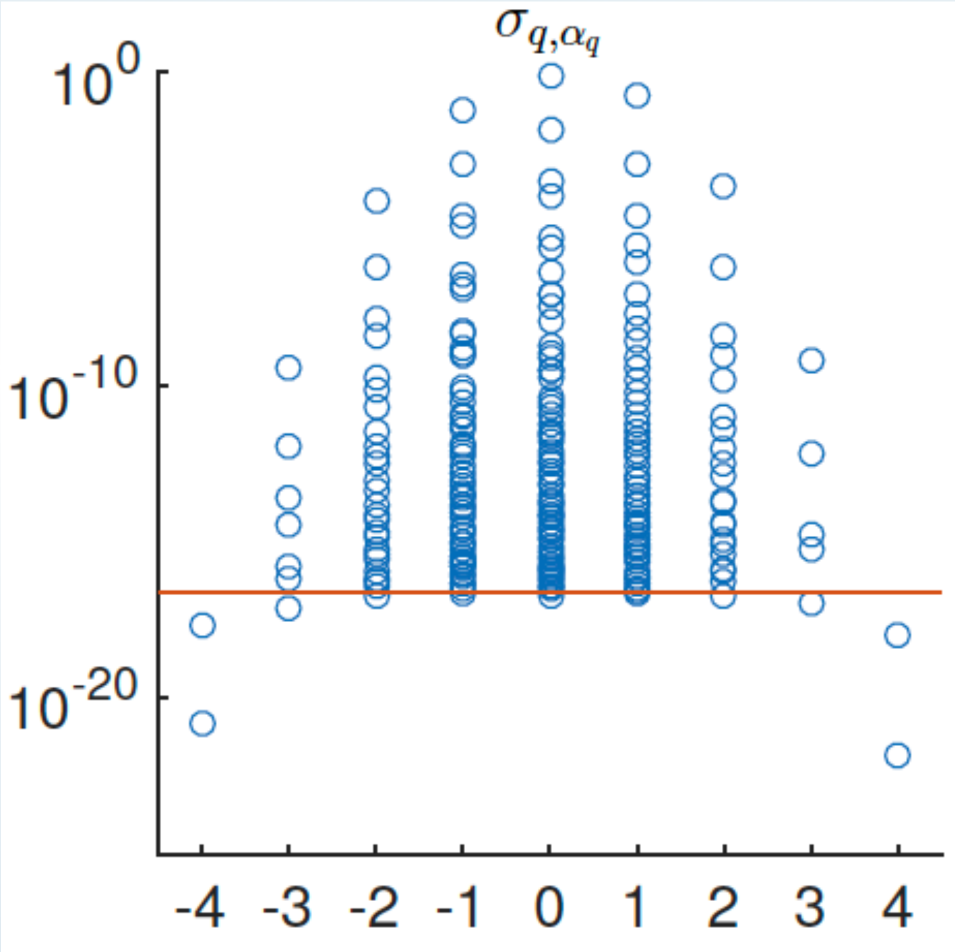
$$\ell = \ell_\beta \qquad\qquad \ell_\beta = \ell_\alpha + \frac{(-1)^i + (-1)^n}{2}$$

$$A_1^{1,p} = \begin{array}{c} q\backslash r \\ \vdots \\ p \\ p+1 \\ p+2 \\ \vdots \end{array} \begin{array}{c|ccc|ccc|ccc|c} \dots & & p-1 & & & p & & & p+1 & & \dots \\ \hline \ddots & & \vdots & & & \vdots & & & \vdots & & \iddots \\ \hline & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & \\ \dots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \dots \\ & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & \\ \hline & 0 & \dots & 0 & & & & 0 & \dots & 0 & \\ \dots & \vdots & \ddots & \vdots & & a_1^{q,1} & & \vdots & \ddots & \vdots & \dots \\ & 0 & \dots & 0 & & & & 0 & \dots & 0 & \\ \hline & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & \\ \dots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \dots \\ & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & \\ \hline \iddots & & \vdots & & & \vdots & & & \vdots & & \ddots \end{array}$$

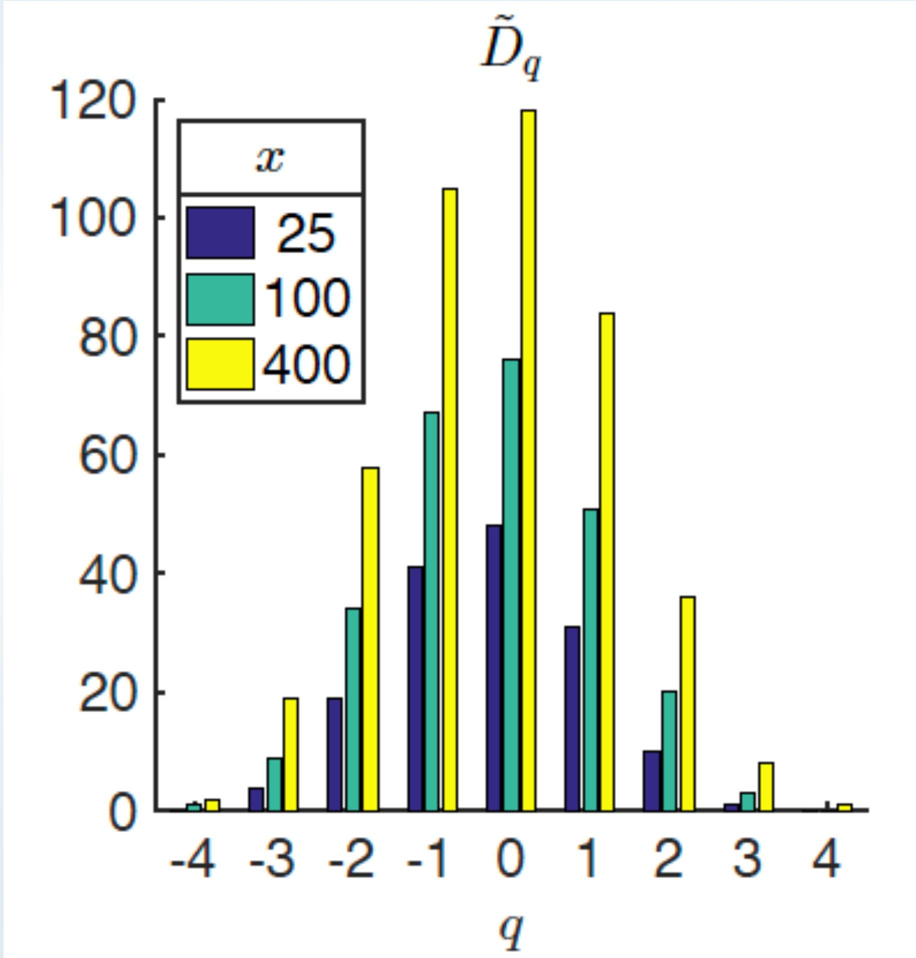
a maximum bond dimension per block

from B. Buyens

decay of Schmidt values



required D towards cont



Buyens et al PRD 95 (2017) 094509

D increases as $x^{1/2}$

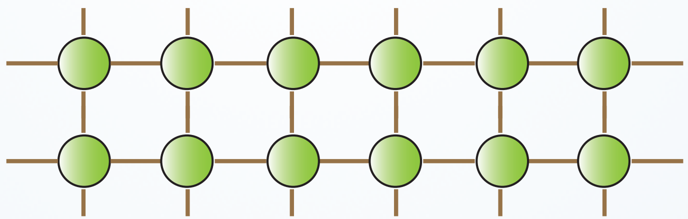
THERMAL EQUILIBRIUM

THERMAL PROPERTIES WITH MPO

$$\rho_{th}(\beta) = e^{-\frac{\beta}{2}H}\mathbf{1}e^{-\frac{\beta}{2}H}$$

$$\langle O\rangle_{th} = \mathrm{tr}(Oe^{-\frac{\beta}{2}H}\mathbf{1}e^{-\frac{\beta}{2}H})$$

combining real time evolution can compute thermal response functions



$\rho_{th}(\beta)$

Barthel, NJP 2013
Karrasch et al NJP 2013

alternative method: METTS

White, PRL 2009
Binder, Barthel, PRB 2015

THERMAL PROPERTIES WITH MPO

Scan parameters; perform extrapolations for each β

m/g chiral condensate as a function of temperature, in the continuum $x \to \infty$

$x \to \infty$

x $x \in [9,\ 1024]$

finite-size

N $N \propto \sqrt{x}$ (up to ~800)

extrapolation

δ sufficiently small for resolution

convergence

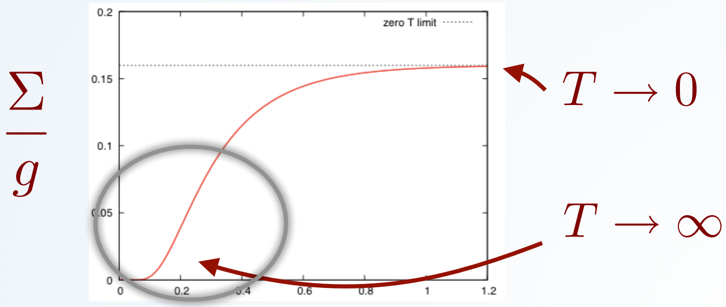
D $D \in [80,\ 160]$

PRD 92, 034519 (2015); PRD93, 094512 (2016)

Buyens PRD 94, 085018 (2016)

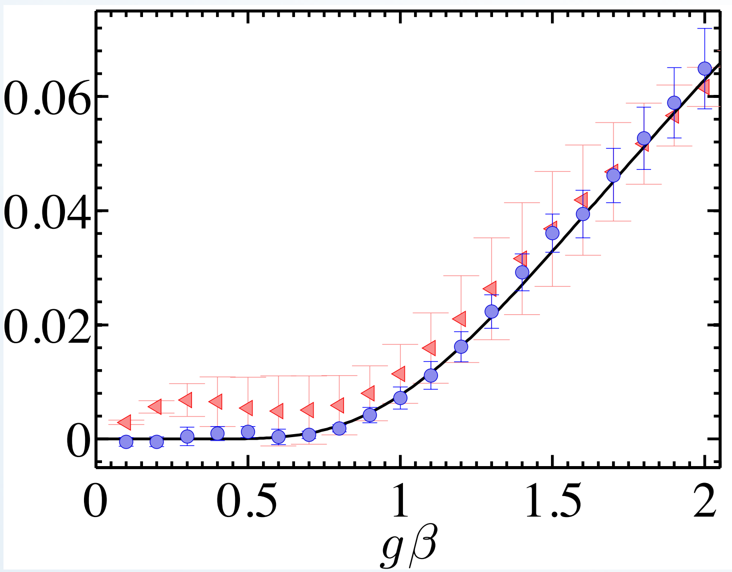
THERMAL PROPERTIES SCHWINGER

chiral condensate at finite T: analytical for m/g=0

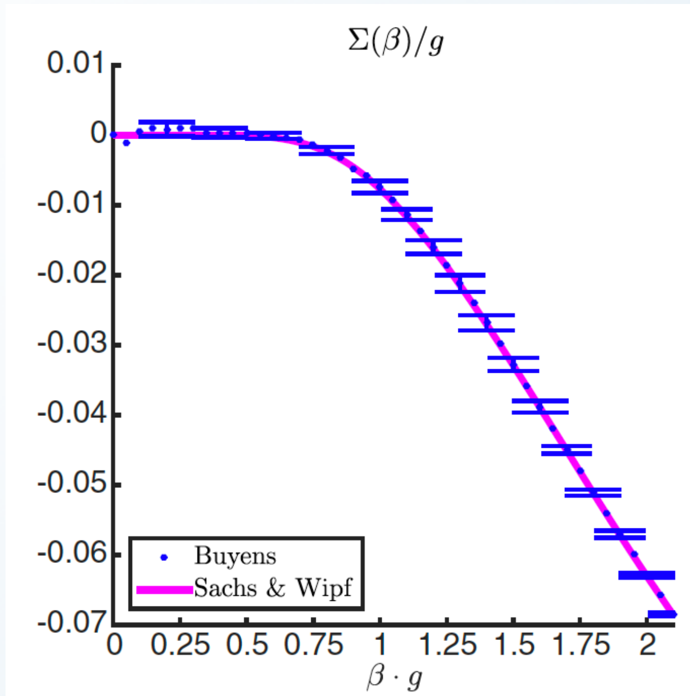


smooth
restoration of
chiral symmetry

Sachs, Wipf 92



PRD 92, 034519 (2015); PRD 93, 094512 (2016)



Buyens PRD 94, 085018 (2016)

REAL TIME

BASIC EVOLUTION ALGORITHMS

Reliable for moderate times, or in some setups

Useful for quantum simulation

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)

S. Kühn et al., JHEP 07 (2015) 130

Buyens et al., PRL 2014; PRX 2016

Rico et al., PRL 2014; NJP 2014; PRX 2016

No full continuum extrapolation yet, but results near the continuum limit

QUENCH SCENARIO

Interacting vacuum (TI)

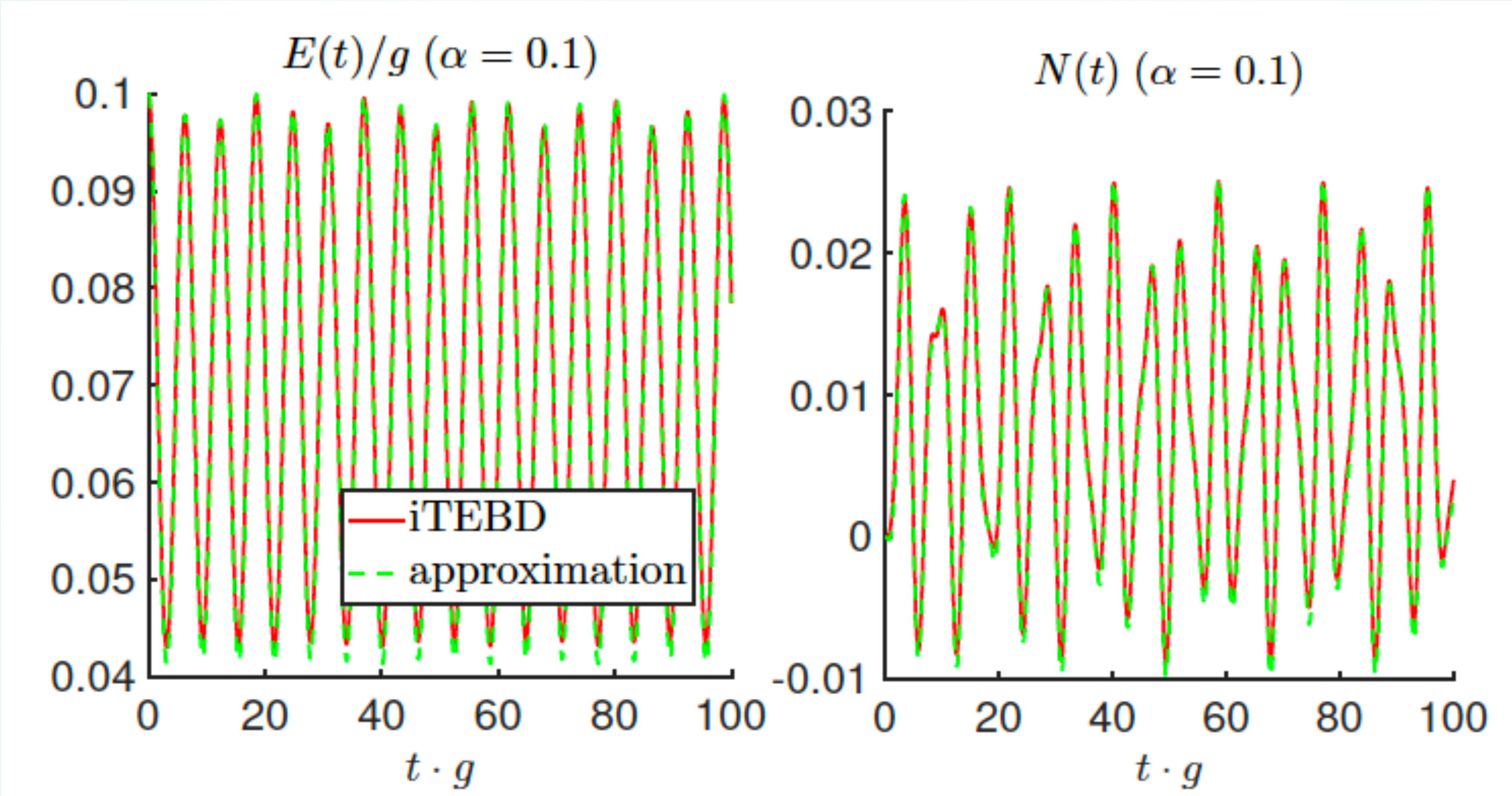
switch on background electric field α

$\Rightarrow$ pair production

iTEBD evolution

B. Buyens et al, PRD96, 114501 (2017)

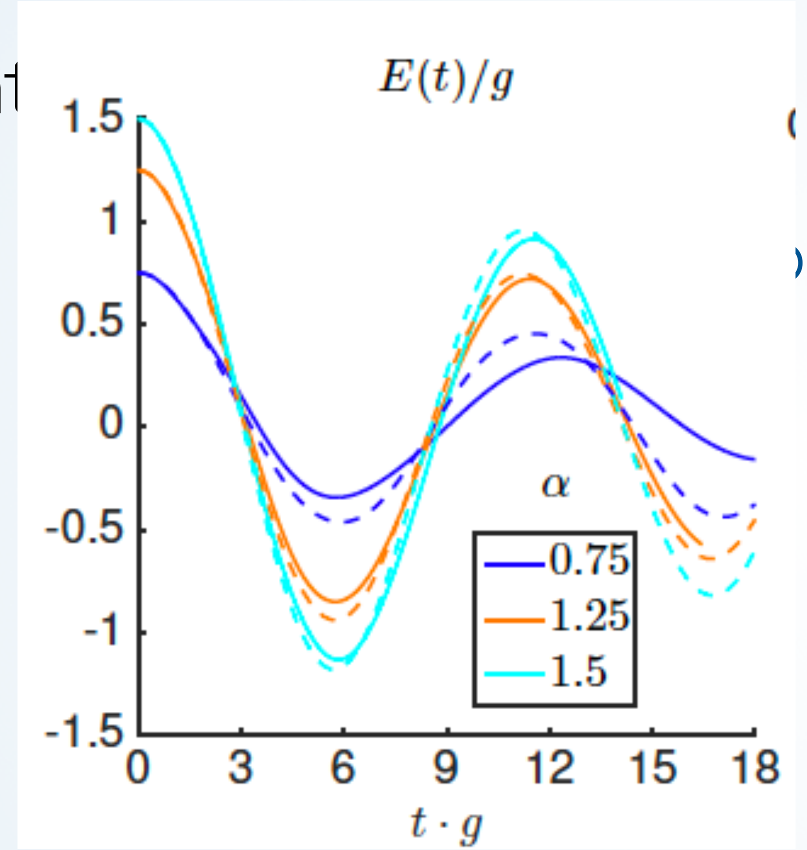
QUENCH SCENARIO

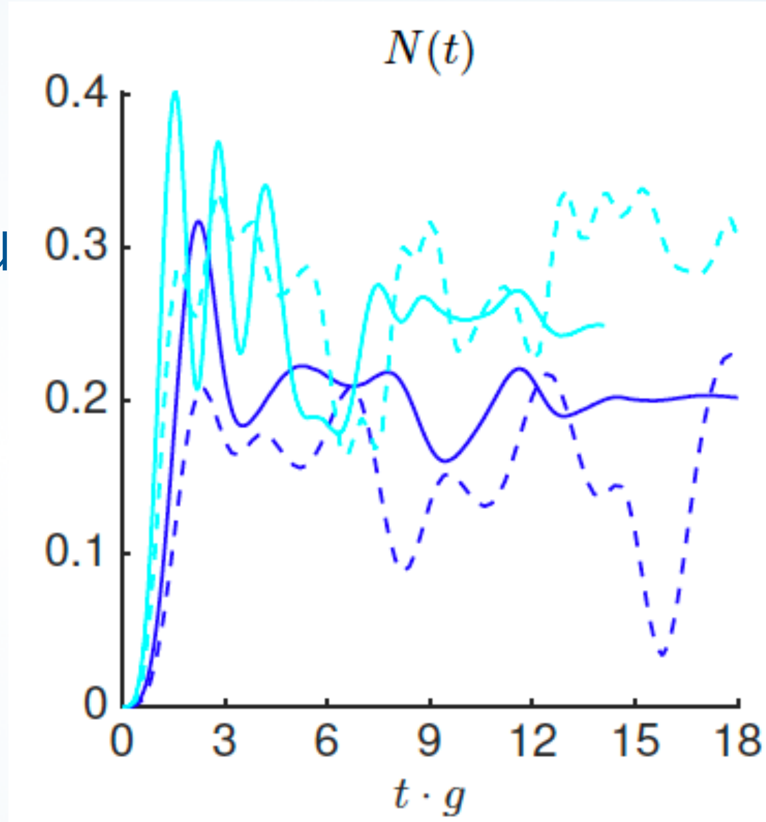


weak field

B. Buyens et al, PRD96, 114501 (2017)

QUENCH SCENARIO





strong field

B. Buyens et al, PRD96, 114501 (2017)

BEYOND SCHWINGER

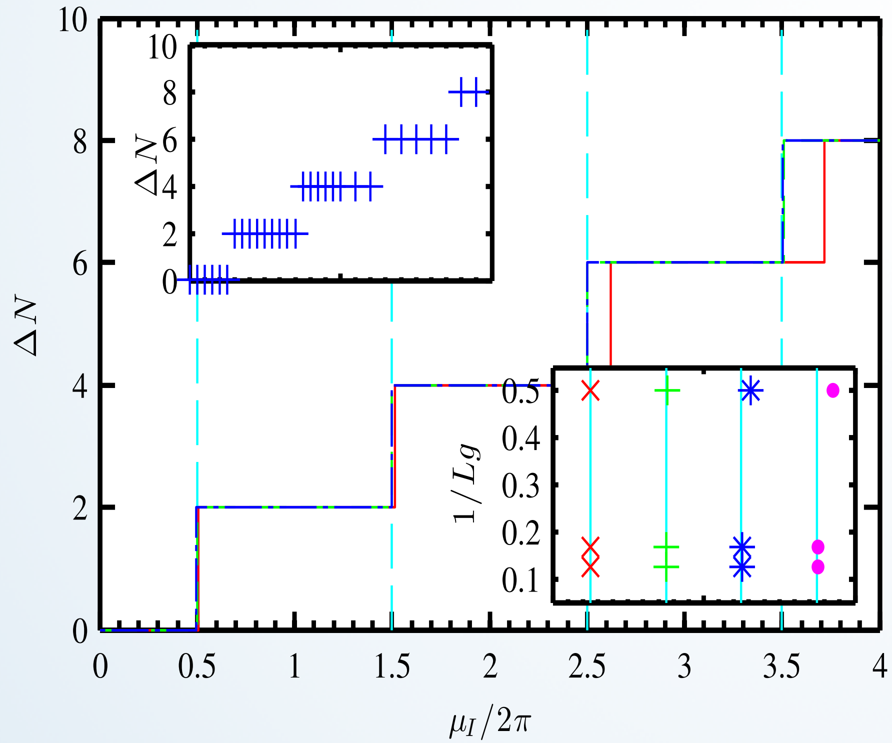
CHEMICAL POTENTIAL:
OVERCOMING THE SIGN PROBLEM

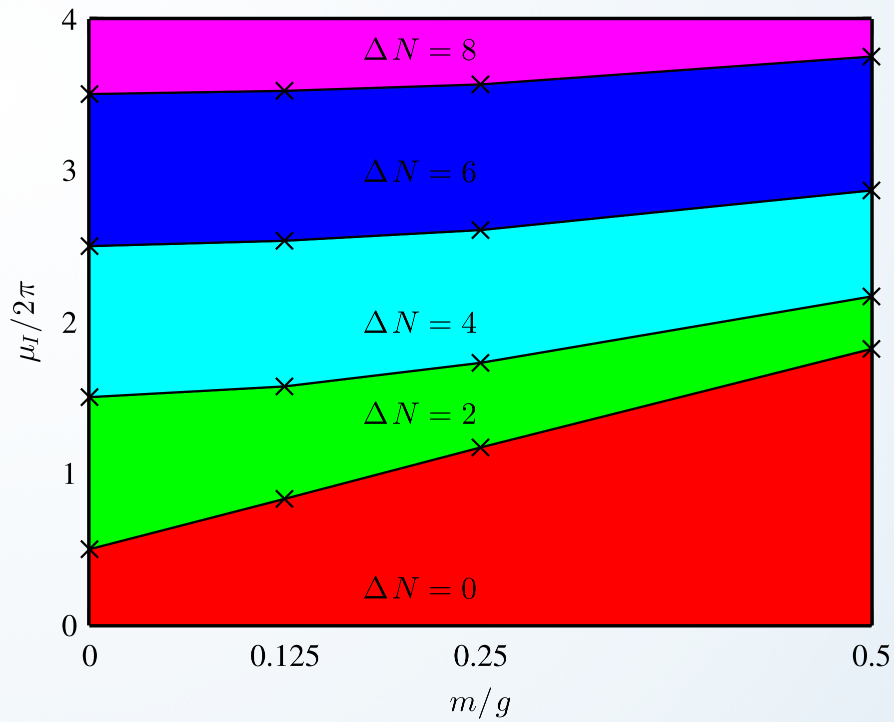
FINITE DENSITY WITH MPS

Several fermion flavors, different chemical potentials

ground state density changes (first order PT)

Montecarlo has sign problem





S. Kuehn et al, PRL118 (2017) 071601

BEYOND SCHWINGER

NON-ABELIAN 1D MODELS

SU(2) MODEL

SU(2) matrices

$$H = \frac{1}{2a} \sum_{n,b,c} \left(\phi_n^{b\,\dagger} U_n^{bc} \phi_{n+1}^c + \text{h.c.} \right) + m \sum_{n,b} (-1)^n \phi_n^{b\,\dagger} \phi_n^b + \frac{a g^2}{2} \sum_n \mathbf{J}_n^2$$

Kogut-Susskind '75

plus non-Abelian Gauss' Law $G_n^\nu |\Psi_{\text{phys}}\rangle = 0$

$$G_n^\nu = L_n^\nu - R_{n-1}^\nu - Q_n^\nu,$$

$$Q_n^\nu = \sum_{bc} \frac{1}{2} \phi_n^{b\,\dagger} \sigma_{bc}^\nu \phi_n^c$$

a proper tensor product basis for MPS…

$$|\ldots n^1 n^2 \; j \ell \ell' \; n^1 n^2 \ldots \rangle$$

- truncate gauge dof
- integrate out gauge

also P. Sala et al, PRB 98, 034505 (2018)

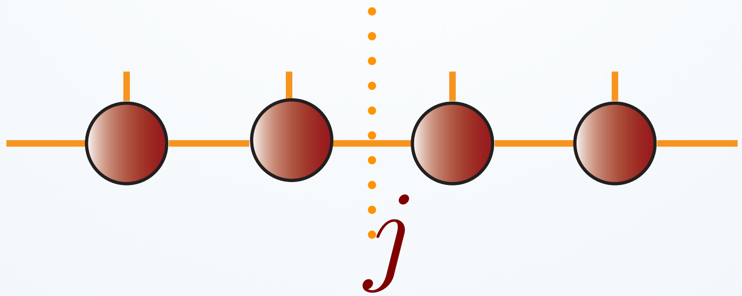
ENTROPY

SU(2) MODEL

Entropy can be efficiently computed from MPS state

gauge constraints not purely local ⇒ not all entropy physical

$$S(\rho) = -\underbrace{\sum_j p_j \log_2(p_j)}_{S_{\text{class}}} + \underbrace{\sum_j p_j \log_2(2j+1)}_{S_{\text{repr}}} + \underbrace{\sum_j p_j S(\bar{\rho}_j)}_{S_{\text{dist}}}$$



Gosh et al JHEP 2015
Soni, Trivedi JHEP 2016
van Acoleyen et al PRL 2016

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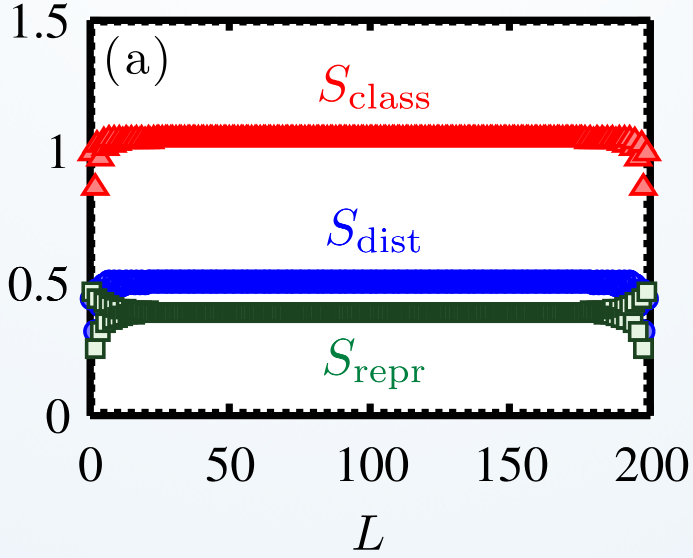
S_{class} S_{repr} S_{dist}



$m/g = 0.8$
$j_{\max} = 2$

Gosh et al JHEP 2015
Soni, Trivedi JHEP 2016
van Acoleyen et al PRL 2016

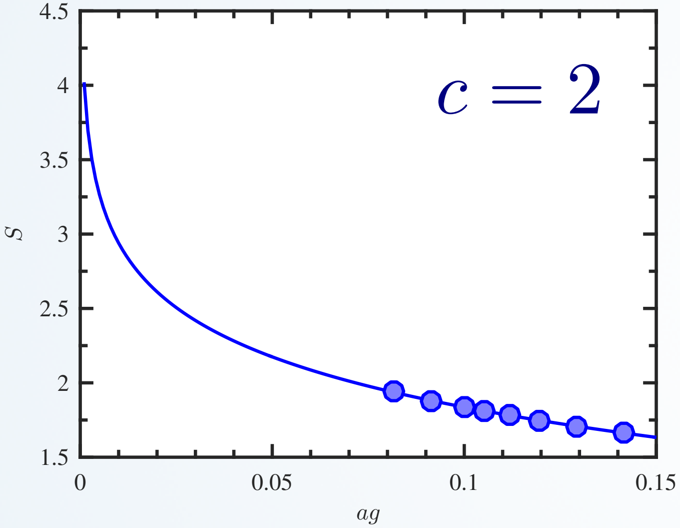
PRX7, 041046 (2017)

Entropy can be efficiently computed from MPS state

gauge constraints not purely local ⇒ not all entropy physical

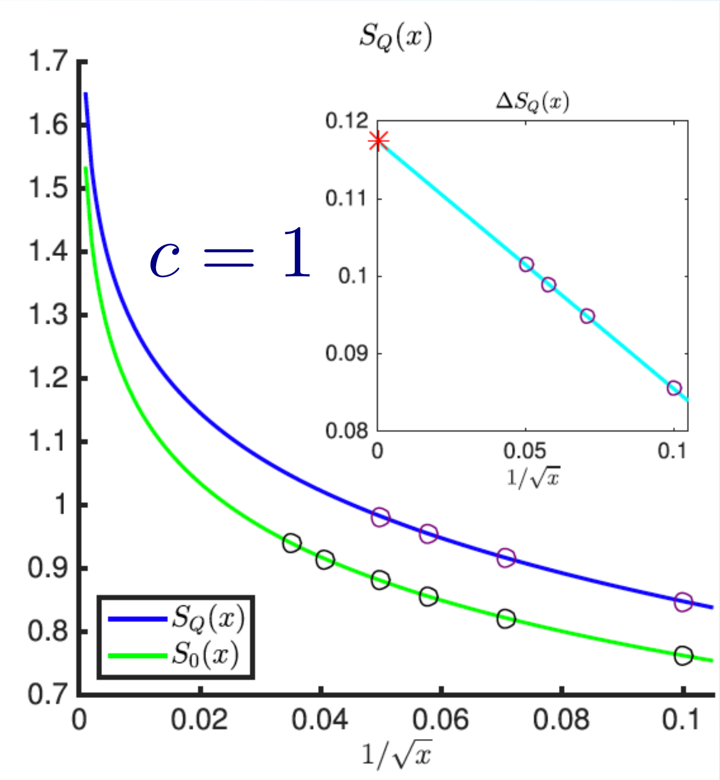
Casini et al 2014; Gosh et al JHEP 2015
Soni, Trivedi JHEP 2016; van Acoleyen et al PRL 2016

SU(2)



PRX7, 041046 (2017)

earlier for Schwinger



Buyens PRX6, 041040 (2016)

divergence in the continuum limit

$$S \propto \frac{c}{6} \log_2 \frac{\xi}{a}$$

Calabrese, Cardy JStatMech 2004

BEYOND ID

PEPS FOR 2+1 D LGT

some results with other TNS

Tagliacozzo, Vidal PRB 2011
Felser et al. PRX 2020
Magnifico et al. 2011.10658

explicitly gauge invariant PEPS

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

restricted ansatz calculations

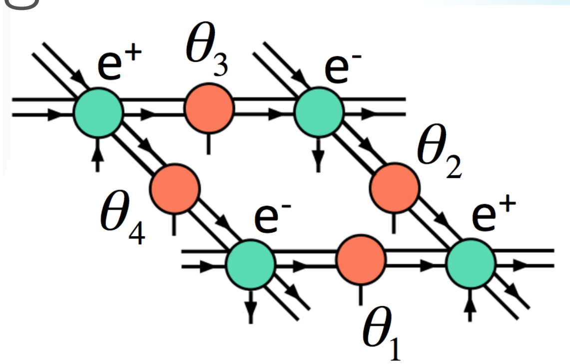
also Gaussian PEPS

Zohar, Cirac PRD 2018
Emonts et. al., PRD 102, 074501 (2020)

standard PEPS toolbox contains all ingredients for full variational computation

computational cost, required D

plaquette terms



Zapp, Orús PRD 2017

PEPS FOR LGT

computational cost, required D

great recent progress

Corboz PRB94, 035133 (2016)
Vanderstraeten et al. PRB 94, 155123 (2016)

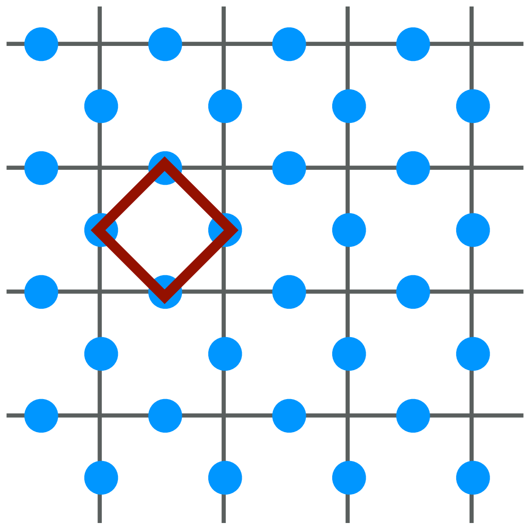
outperform other methods

Corboz PRB93, 045116 (2016)
PRL 113, 046402 (2014)

plaquette terms

optimizing four tensors at once ⇒ much larger cost

Dusuel et al. PRL 2011
Schulz et al. NJP 2012



A NEW STRATEGY

generalizes a strategy proposed for digital quantum simulation

Zohar et al. PRL 118, 070501 (2017); PRA 95, 023604 (2017); JPA 50, 085301(2017)

plaquette terms reduced to nearest neighbour at the cost of introducing ancillary dof

can also be used to impose Gauss' law

incorporate in iPEPS imaginary time evolution

use it to study the phase diagram of Z_3 pure LGT

D. Robaina, MCB, J. I. Cirac, PRL126, 050401 (2021)

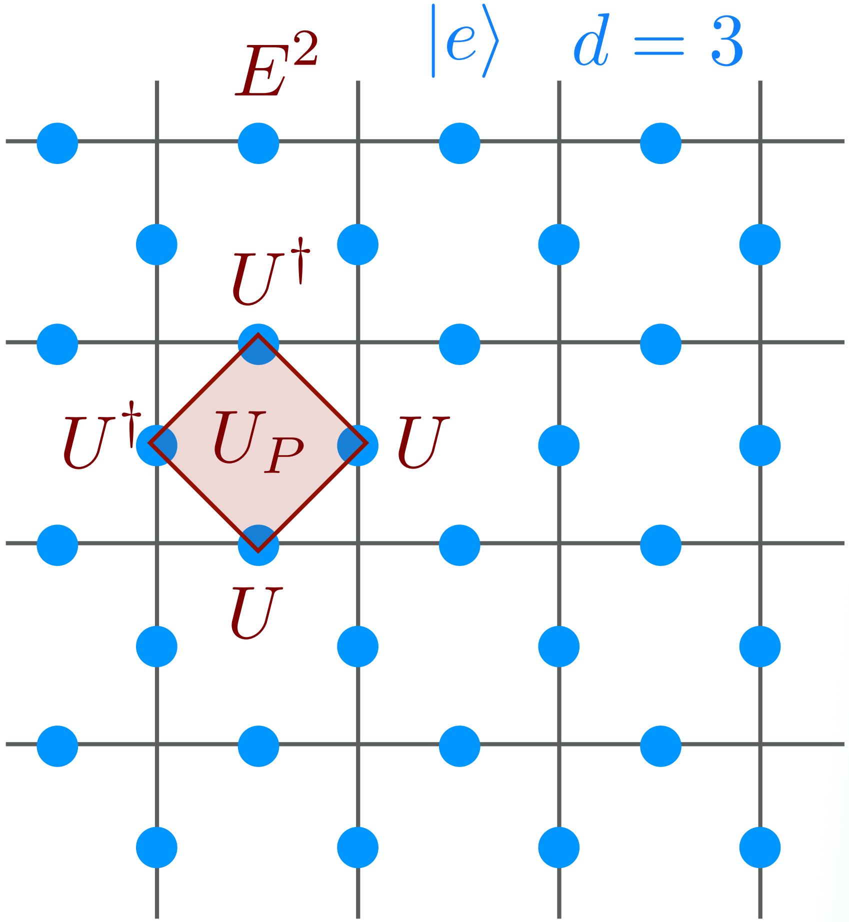
Z_3 LGT MODEL

Z_3 LGT

$$E|e\rangle = e|e\rangle$$

$$e \in \{-1, 0, 1\}$$

$$U|e\rangle = |e-1\rangle$$



$$H = H_{\mathrm{E}} + H_{\Box}$$

electric

$$H_{\mathrm{E}} = \frac{g^2}{2}\sum_{\ell} E_{\ell}^2$$

magnetic

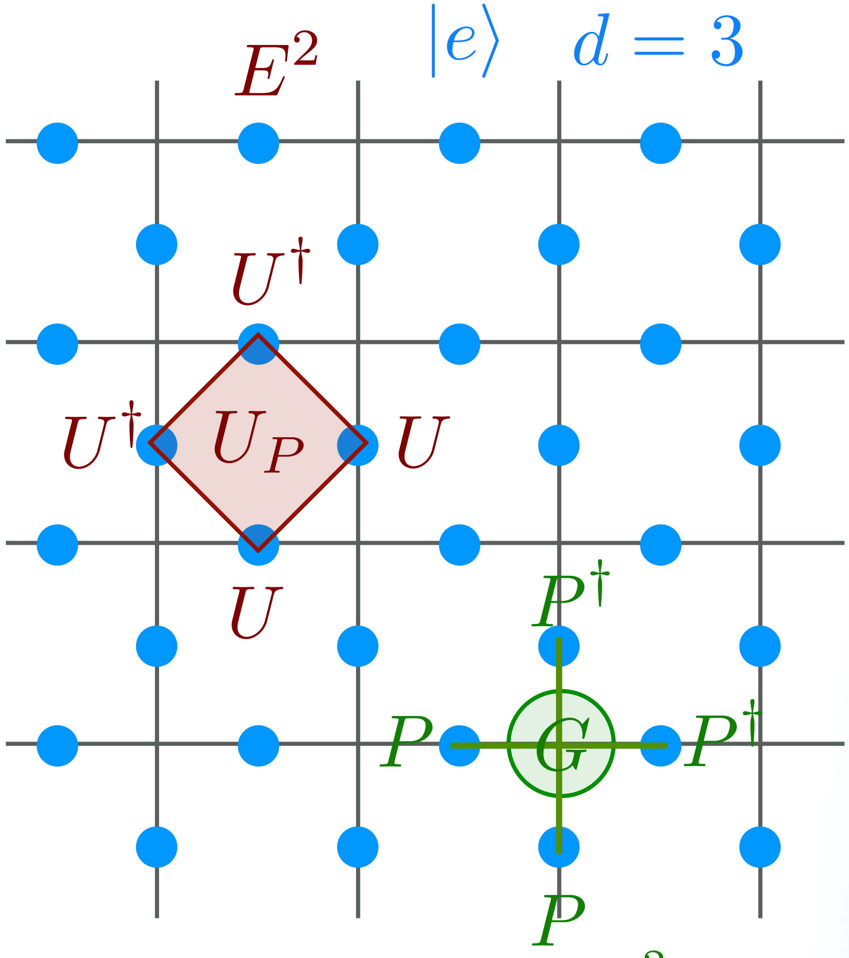
$$H_{\Box} = -\frac{1}{2g^2}\sum_{\Box} U_P + U_P^\dagger$$

$Z_N \to U(1)$ as $N \to \infty$

deconfined confined

g^2

Z_3 LGT



$$H = H_{\mathrm{E}} + H_{\square}$$

$$[H, G(x)] = 0$$

physical states: Gauss' law

$$G(\mathbf{x})|\psi\rangle = e^{\frac{2\pi i}{3} q(\mathbf{x})}|\psi\rangle$$

$q \in \{-1, 0, 1\}$

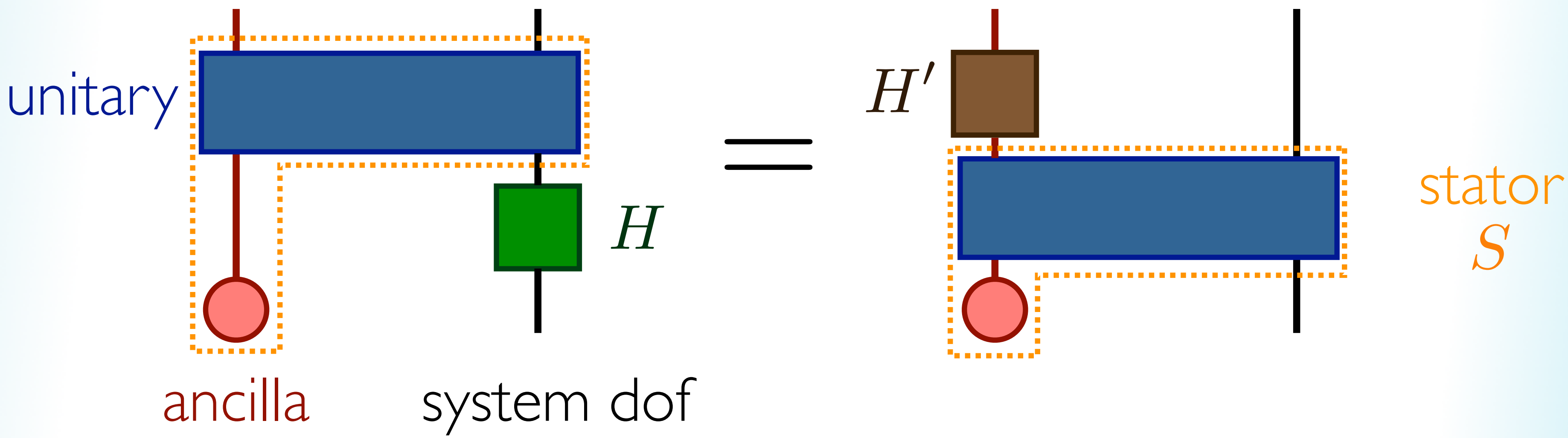
external charges

$$P = e^{i\frac{2\pi}{3}E}$$

$$e^{i\frac{2\pi}{3}(E_\ell + E_d - E_r - E_u)}$$

EFFECTIVE PLAQUETTE TERMS

strategy proposed for digital quantum simulation

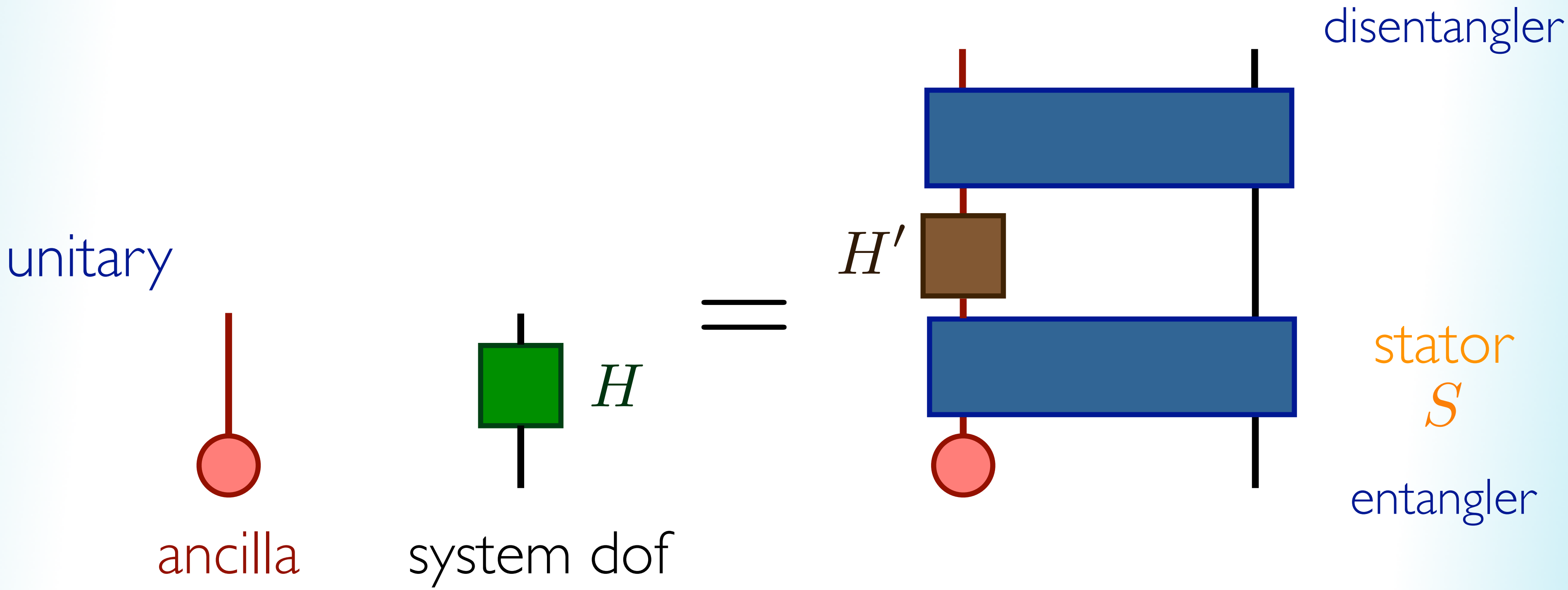


$$SH = H'S$$

Zohar et al. PRL 118, 070501 (2017)
Zohar et al. PRA 95, 023604 (2017)
Zohar, JPA 50, 085301(2017)

EFFECTIVE PLAQUETTE TERMS

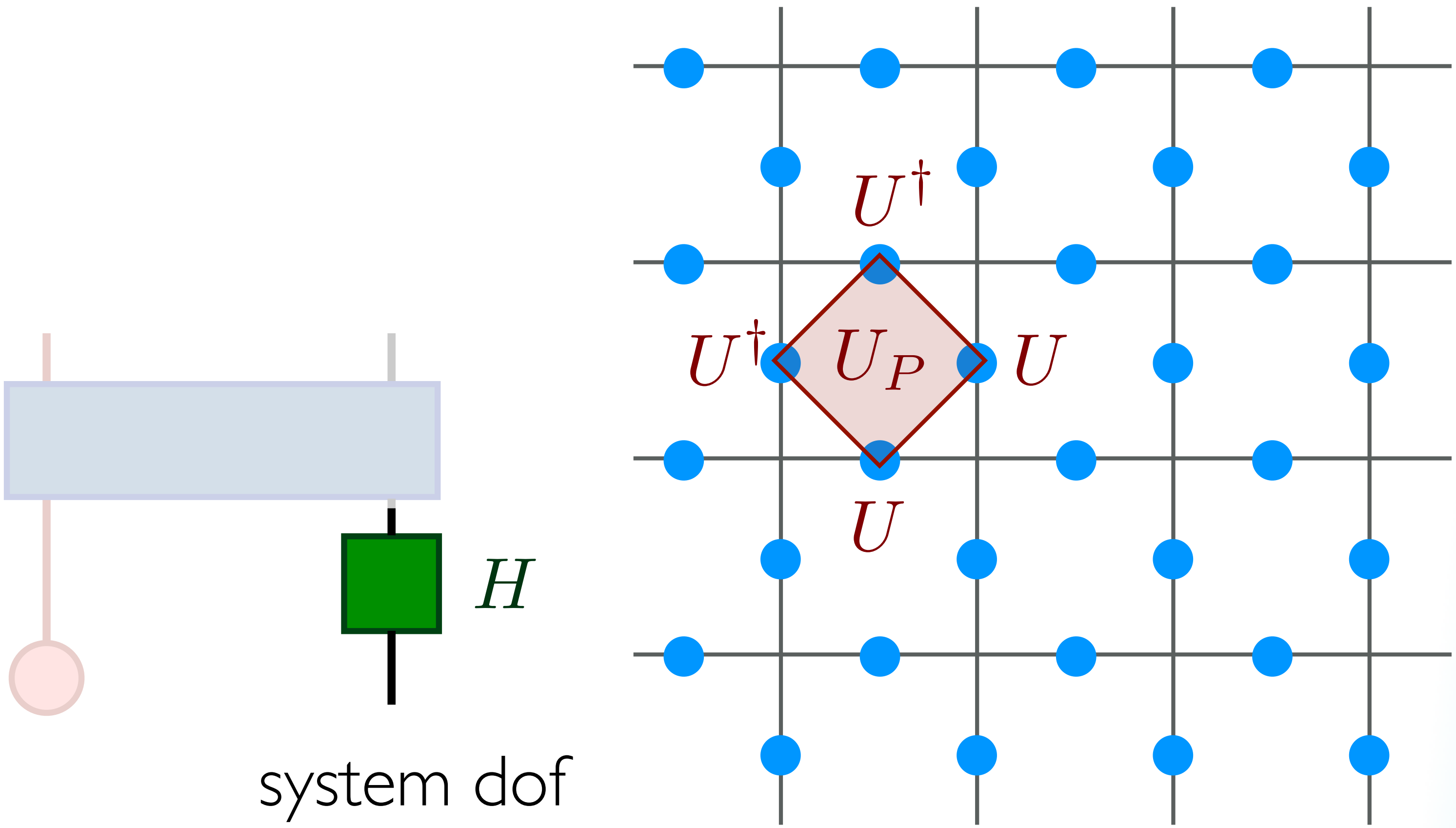
strategy proposed for digital quantum simulation



$$SH = H'S$$

Zohar et al. PRL 118, 070501 (2017)
Zohar et al. PRA 95, 023604 (2017)
Zohar, JPA 50, 085301(2017)

EFFECTIVE PLAQUETTE TERMS

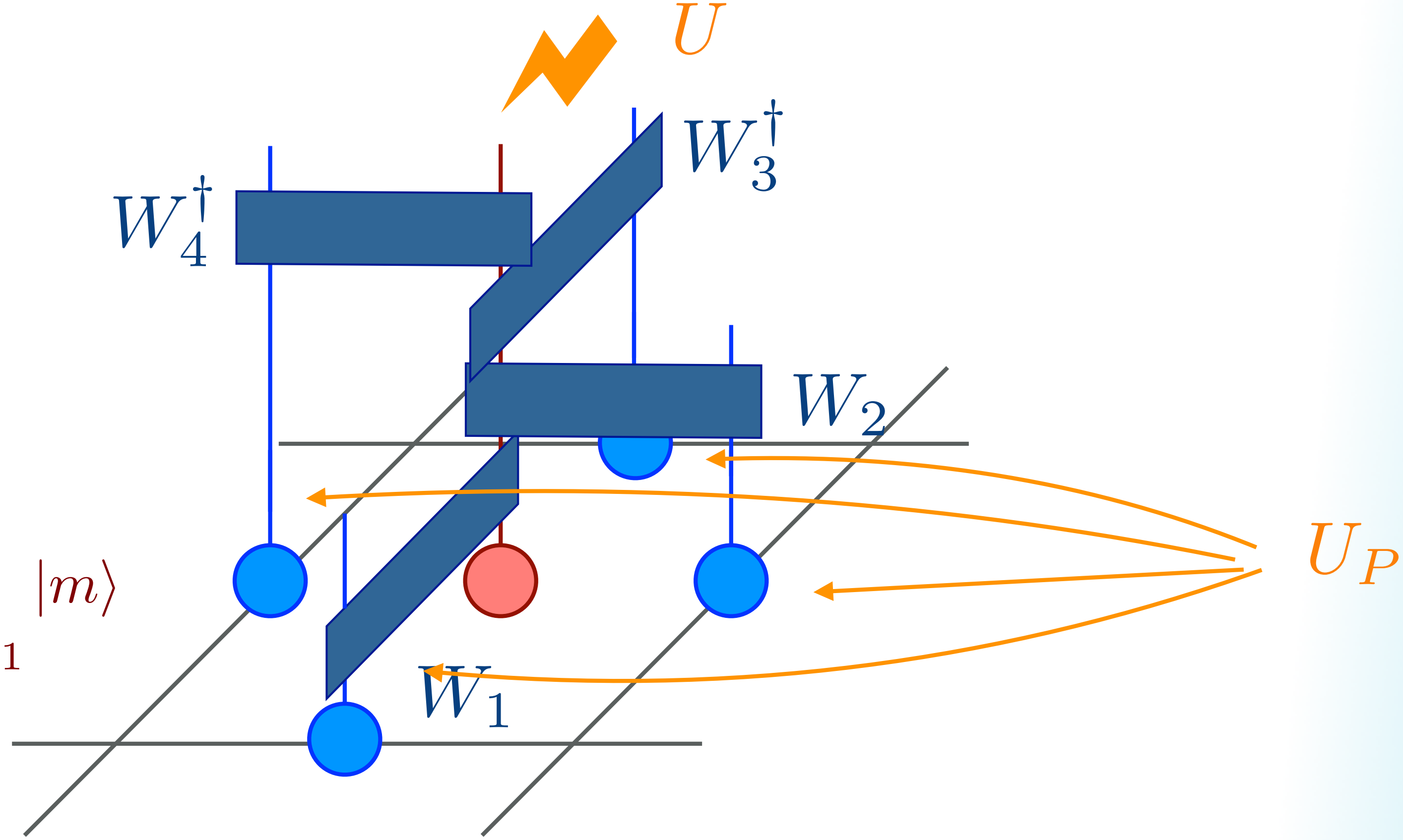


Zohar et al. PRL 118, 070501 (2017)
Zohar et al. PRA 95, 023604 (2017)
Zohar, JPA 50, 085301(2017)

EFFECTIVE PLAQUETTE TERMS

$$S = W_{\Box}|\mathrm{in}\rangle$$

$$|\mathrm{in}\rangle = \frac{1}{\sqrt{3}} \sum_{m=-1,0,1} |m\rangle$$



$$W = U \otimes P_1^{\mathrm{anc}} + \mathbb{I} \otimes P_0^{\mathrm{anc}} + U^\dagger \otimes P_{-1}^{\mathrm{anc}}$$

Zohar et al. PRL 118, 070501 (2017)
Zohar et al. PRA 95, 023604 (2017)
Zohar, JPA 50, 085301(2017)

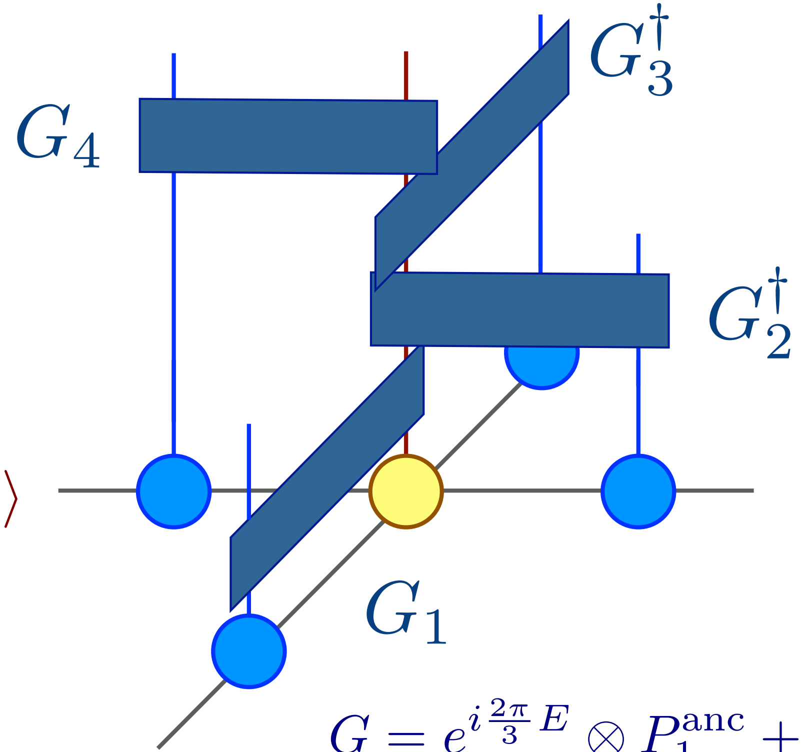
GAUSS' LAW

projector for external charge on a vertex

$$P_q = \frac{1}{3} \sum_{m=-1,0,1} e^{im\frac{2\pi}{3}(E_1 - E_2 - E_3 + E_4 - q(\mathbf{x}))}$$

$$S_G = G_+|\mathrm{in}\rangle$$

$$|\mathrm{in}\rangle = \frac{1}{\sqrt{3}} \sum_{m=-1,0,1} |m\rangle$$



$$G = e^{i\frac{2\pi}{3}E} \otimes P_1^{\mathrm{anc}} + \mathbb{I} \otimes P_0^{\mathrm{anc}} + e^{-i\frac{2\pi}{3}E} \otimes P_{-1}^{\mathrm{anc}}$$

$$S_G P_q = \tilde{P}_q S_G$$

$$\tilde{P}_q = \mathbb{I} + e^{-i\frac{2\pi q}{3}} U + e^{i\frac{2\pi q}{3}} U^\dagger$$

Robaina, MCB, Cirac, PRL126, 050401 (2021)

iPEPS SIMULATION

imaginary time evolution

$$\frac{e^{-\tau H}|\Phi\rangle}{\|e^{-\tau H}|\Phi\rangle\|} \to |E_0\rangle$$

local

using ancilla construction

$$e^{-\tau H} = \lim_{n\to\infty}\left(e^{-\frac{\delta_\tau}{2}H_E}e^{-\delta_\tau H_\square}e^{-\frac{\delta_\tau}{2}H_E}\right)^n$$

simple update used to evolve the tensors

environment contraction with CTM

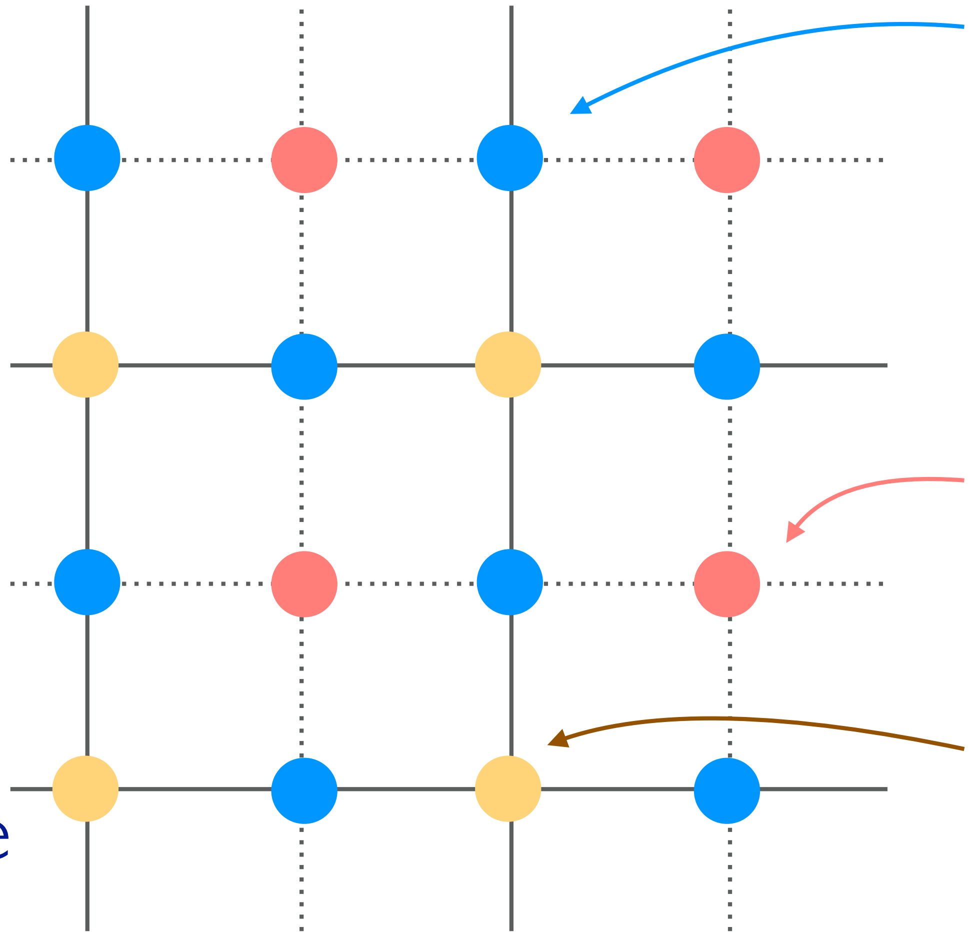
SyTen toolkit by C. Hubig https://syten.eu/

Robaina, MCB, Cirac, PRL126, 050401 (2021)

iPEPS SIMULATION

unit cell

4×4



links $d = 3$

plaquette ancillas

$d = 3$

vertex ancillas

$d = 3$

charge sector imposed at the beginning with vertex projector

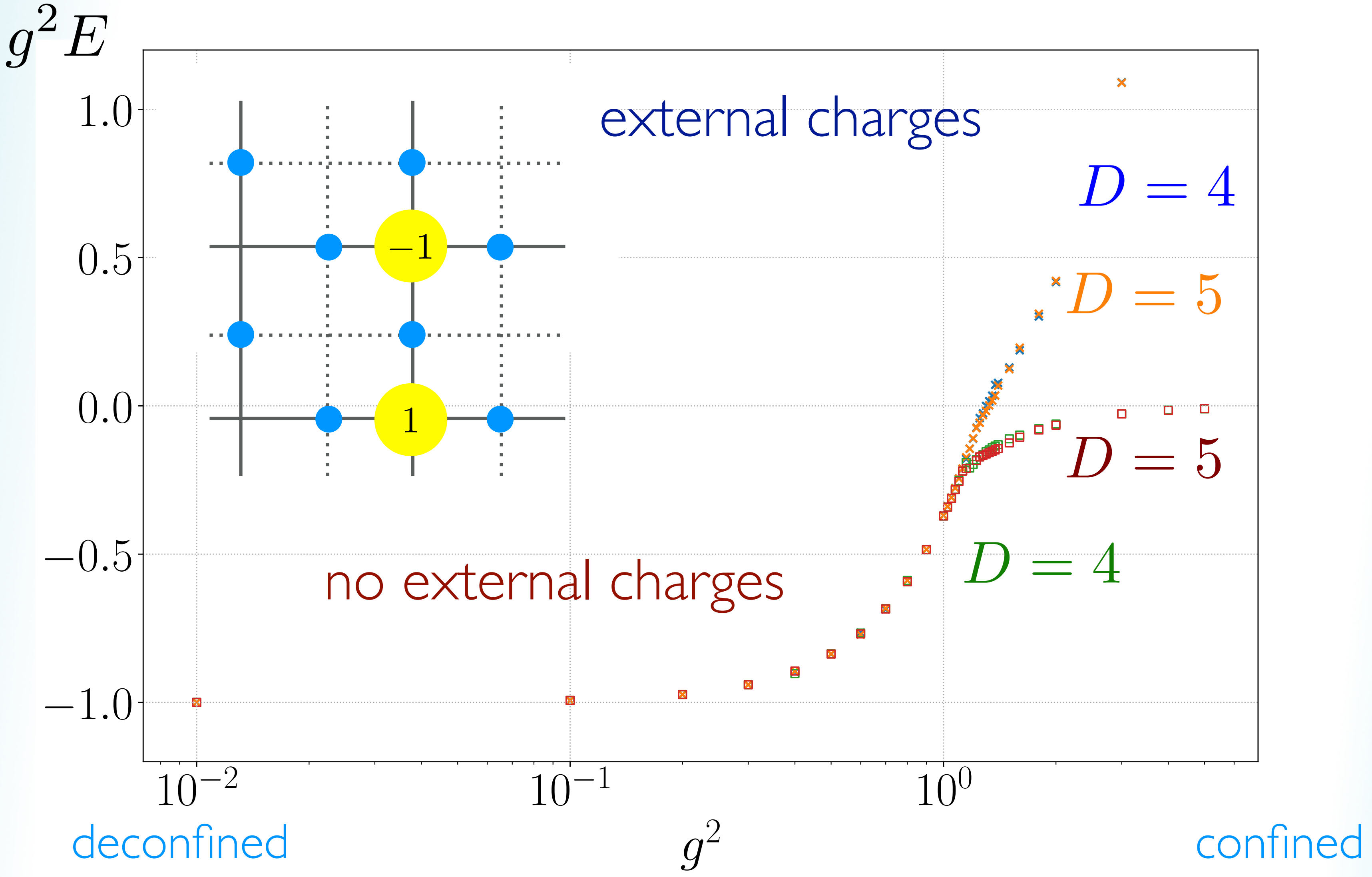
Robaina, MCB, Cirac, PRL 126, 050401 (2021)

RESULTS

PHASE DIAGRAM

first order phase transition

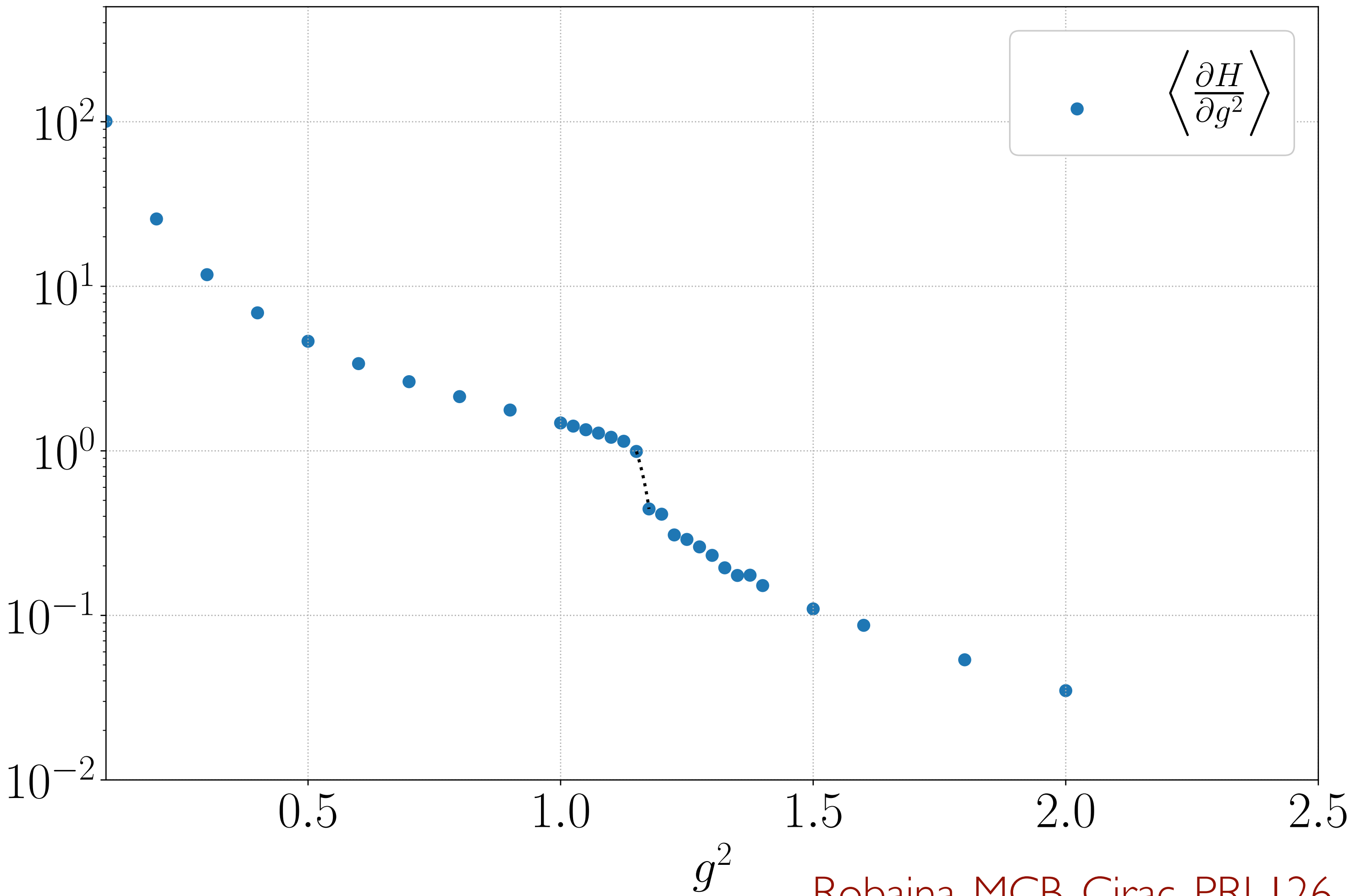
Blöte, Swendsen, PRL 1979
Bhanot, Creutz, PRD 1980



PHASE DIAGRAM

first order phase transition

discontinuity in derivative of energy



Robaina, MCB, Cirac, PRL126, 050401 (2021)

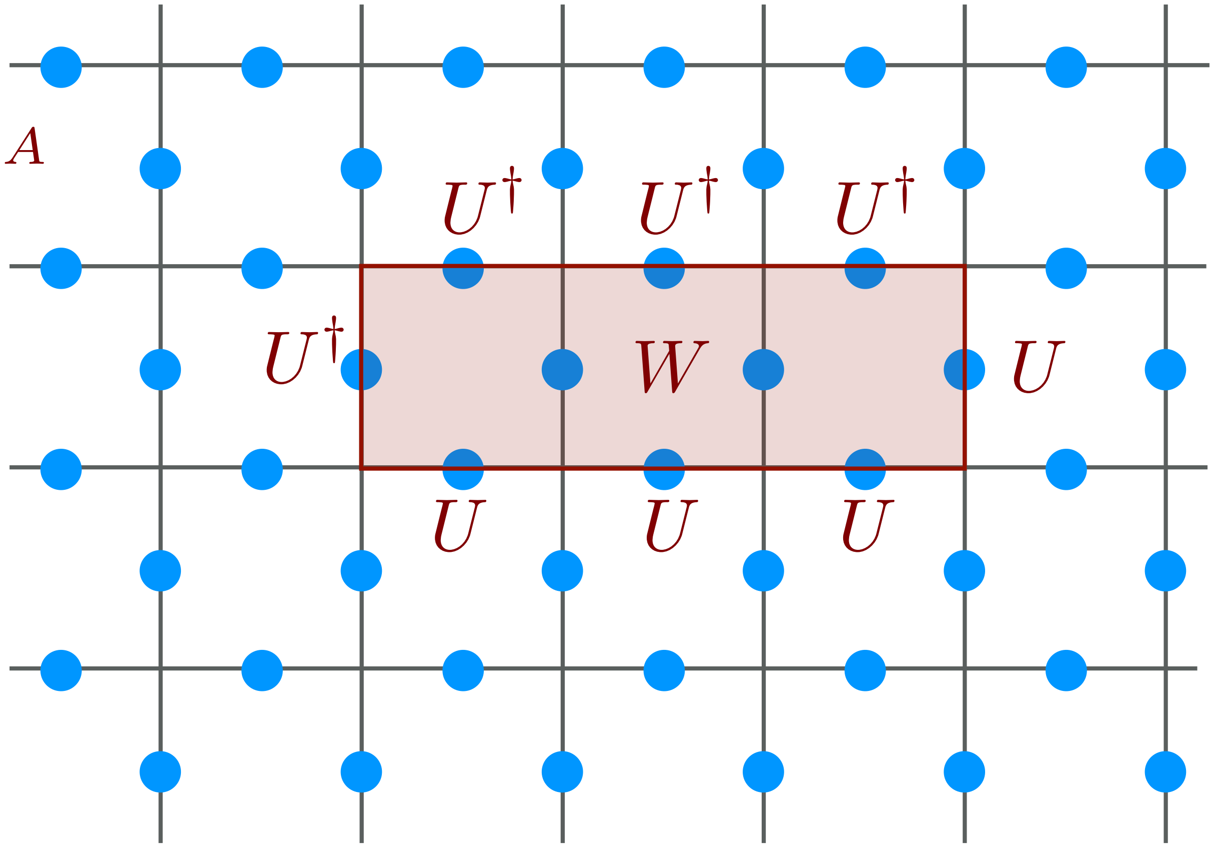
WILSON LOOP

to distinguish between deconfined and confined phases

decay with area

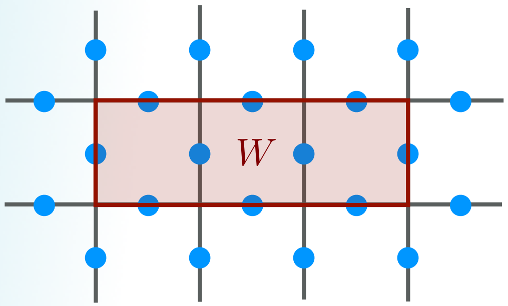
$$\langle W_A \rangle \approx e^{-\sigma A}$$

string tension

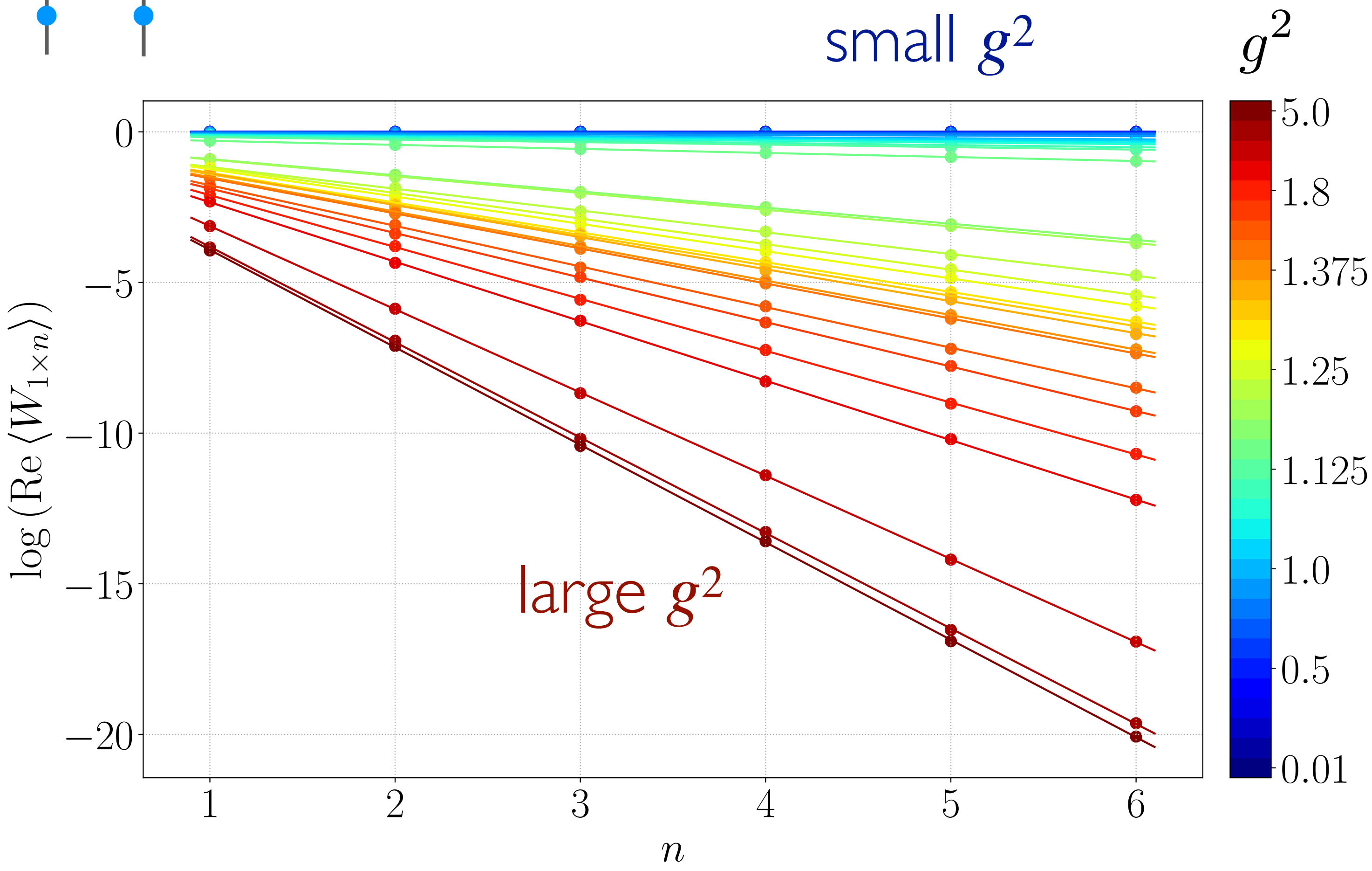


Robaina, MCB, Cirac, PRL126, 050401 (2021)

WILSON LOOP

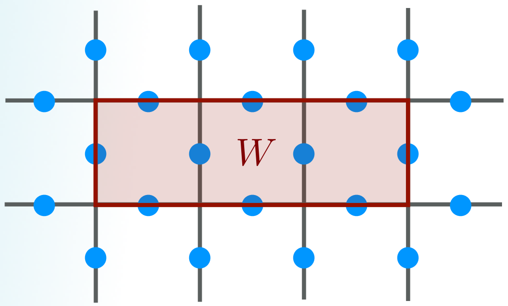


$W_{1\times n}$

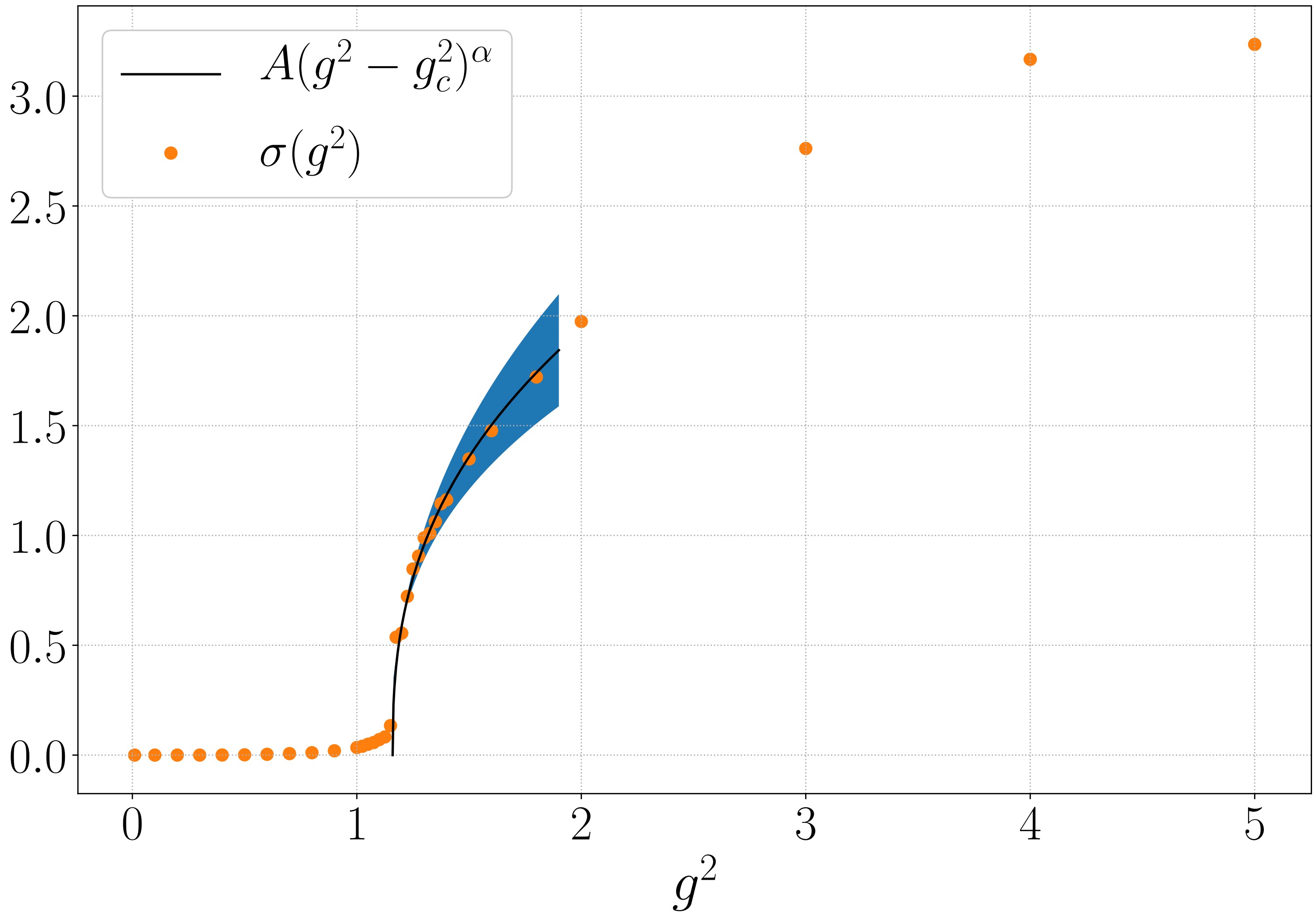


Robaina, MCB, Cirac, PRL126, 050401 (2021)

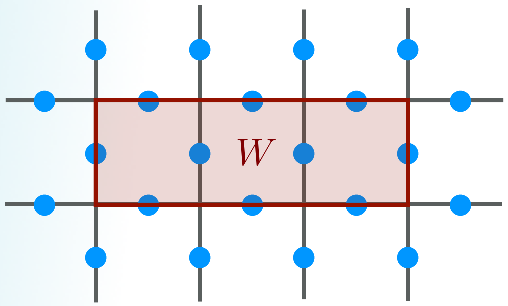
WILSON LOOP



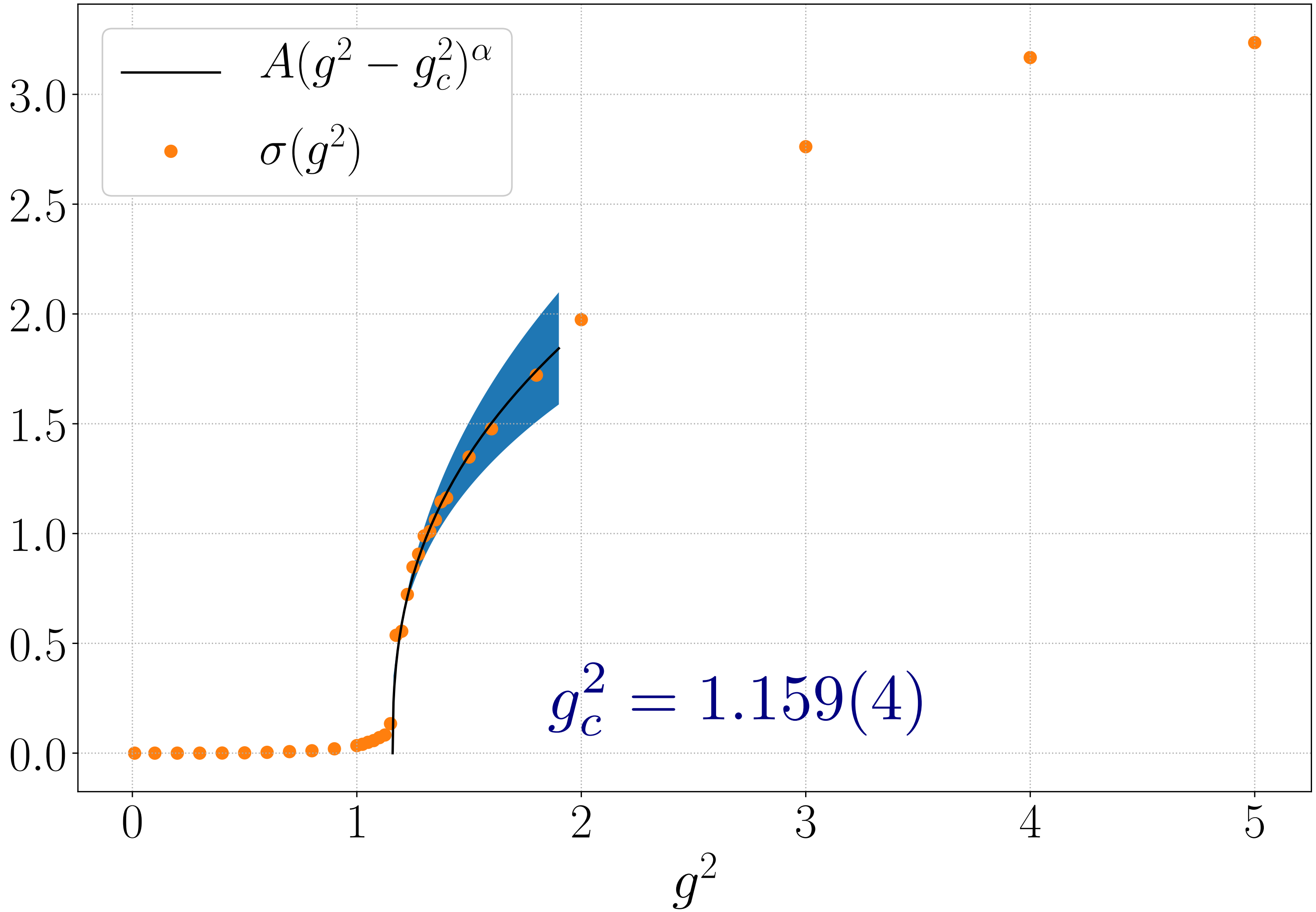
$W_{1\times n}$



WILSON LOOP

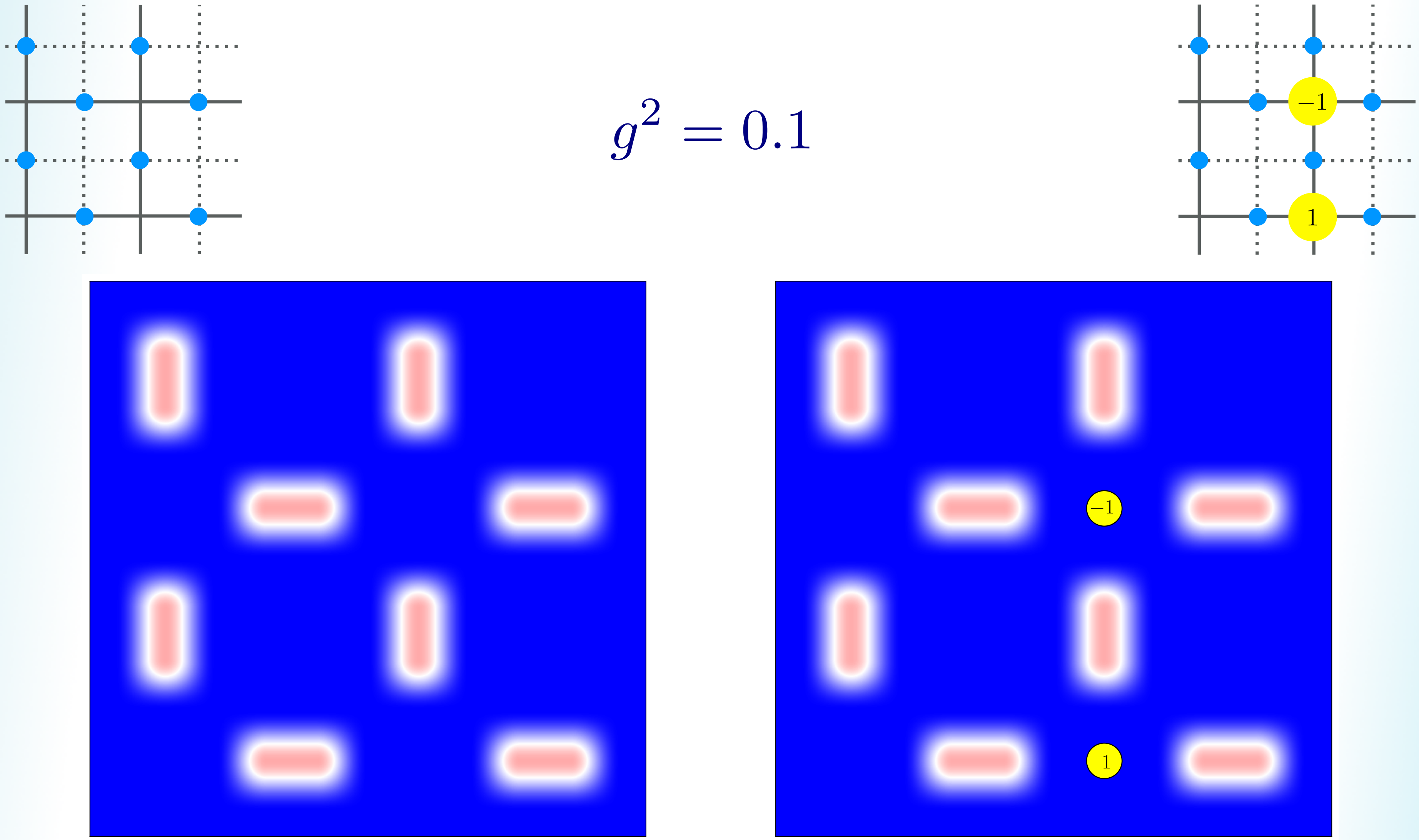


$W_{1\times n}$



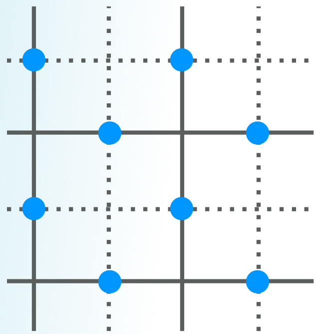
CONFINEMENT OF ELECTRIC FIELD

$$g^2 = 0.1$$

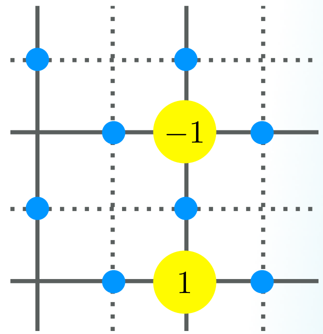


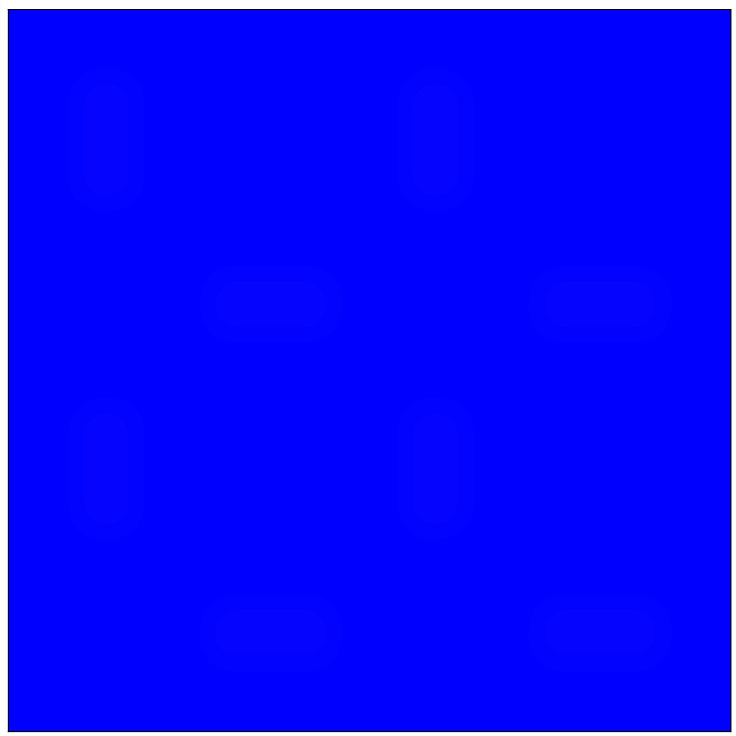
Robaina, MCB, Cirac, PRL126, 050401 (2021)

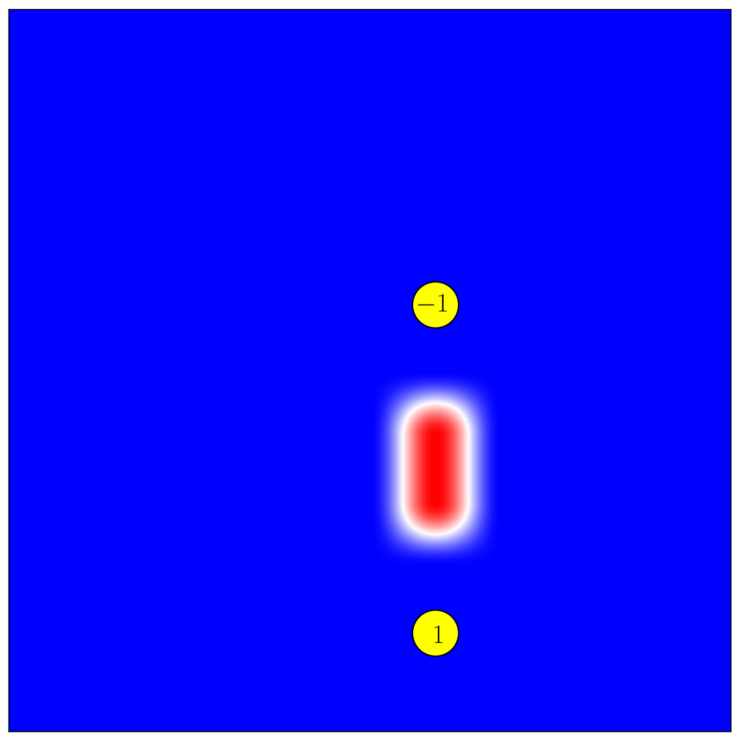
CONFINEMENT OF ELECTRIC FIELD



$$g^2 = 5$$







Robaina, MCB, Cirac, PRL126, 050401 (2021)

ANOTHER APPROACH

A FAMILY OF PEPS

development from formal approach

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

explicitly gauge invariant PEPS

numerical calculations limited

computational cost

restricted families of PEPS allow for more efficient contraction/optimization

using Monte Carlo techniques

Zohar, Cirac, PRD97, 034510 (2018)

accurate enough for certain regimes

Emonts, MCB, Cirac, Zohar, PRD102, 074501 (2020)

ESSENTIALS

gauging PEPS

whole state: matter + gauge dof

$$|\Psi\rangle = \sum_{\mathcal{G}} |\mathcal{G}\rangle_{\mathbf{gauge}} |\Psi(\mathcal{G})\rangle_{\mathbf{matter}}$$

not normalized

observable diagonal in gauge variables

more general also possible

$$\langle O\rangle = \sum_{\mathcal{G}} \langle\mathcal{G}|O|\mathcal{G}\rangle \frac{\langle\Psi(\mathcal{G})|\Psi(\mathcal{G})\rangle}{\sum_{\mathcal{G}'}\langle\Psi(\mathcal{G}')|\Psi(\mathcal{G}')\rangle}$$

probability

computable for specific $|\Psi(\mathcal{G})\rangle$

Zohar, Cirac, PRD97, 034510 (2018)

ESSENTIALS

gauged Gaussian PEPS

- fermionic matter → globally symmetric Gaussian fermionic PEPS Kraus et al., PRA 81, 052338 (2010)
- additional gauge dof → entangled with virtual dof via controlled group operation
- contraction of matter part is efficient (Gaussian formalism)

whole state not Gaussian

Zohar, Burrello, NJP18, 043008 (2016)
Zohar, Cirac, PRD97, 034510 (2018)

APPLIED TO Z_3

considered pure gauge theory

ansatz incorporates translational, rotational symmetry

finite PBC lattices

layered construction to control number of variational parameters

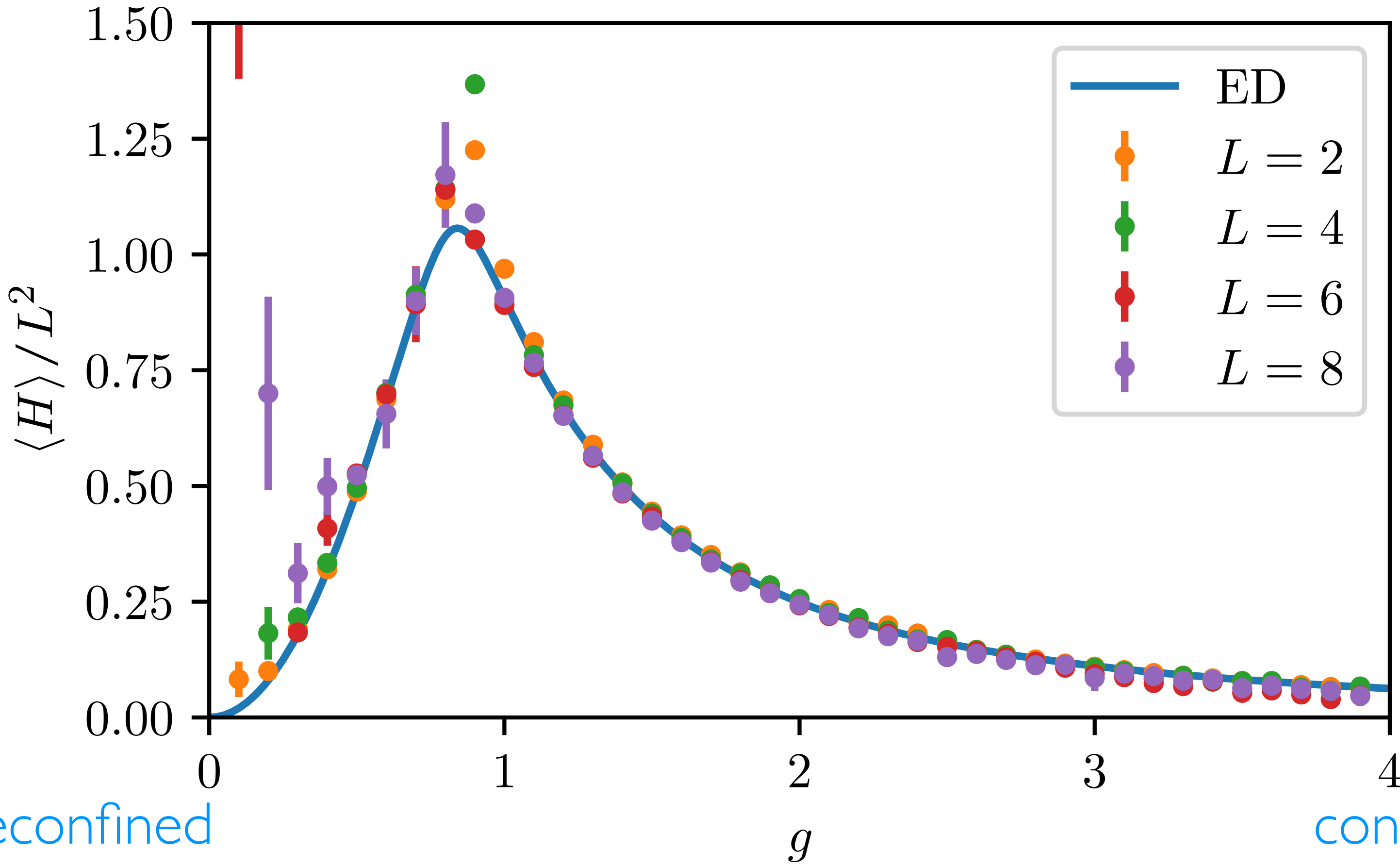
variational Monte Carlo to optimize parameters

Emonts, MCB, Cirac, Zohar, PRD102, 074501 (2020)

RESULTS

LxL lattice

small g^2
more
layers
needed



deconfined

confined

large g^2
very good
description

Related, but I didn't talk about…

TNS for other field theories

Thirring model, PRD100, 094504 (2019)
Thermal QFT dynamics, PRR2, 033301 (2020)

continuous TNS for QFT

Verstraete, Cirac PRL104, 190405 (2010)
Tilloy, Cirac PRX9, 021040 (2019)

proposals for quantum simulation of LGT with ultracold atoms

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012

Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015

http://heptnseminar.org

CONCLUSION

feasibility of TNS for LQFT demonstrated in 1+1D

- high numerical precision attainable (controlled errors)
- spectrum, thermal equilibrium, finite density, (some) dynamics
- Abelian and non-Abelian models

MCB, K. Cichy 1910.00257
QTFLAG Collab.1911.00003

already results for 2+1D LGT (and higher)

- first fully variational iPEPS study of 2+1D LGT

  Robaina, MCB, Cirac, 2007.11630
- finite PBC lattice →variational MC and gauged Gaussian PEPS

  Emonts, MCB, Cirac, Zohar, PRD102, 074501 (2020)
- first 3+1D simulations available

  Magnifico et al. 2011.10658

progress in higher D brings us closer to more complicated theories/scenarios

MPQ MCQST

http://heptnseminar.org