

# Atomtronic implementation of SQUIDs and Josephson junctions

Ludwig Mathey

# Group

Caroline  
Nowozcyn



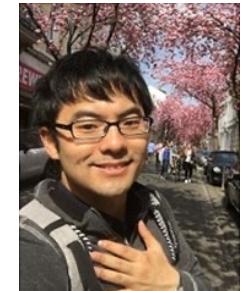
Jim Skulte



Lukas Broers



Lukas Freystatzky



Dapeng Li



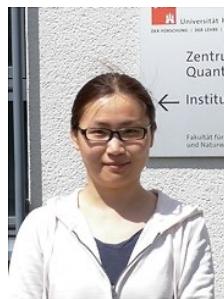
Guido Homann



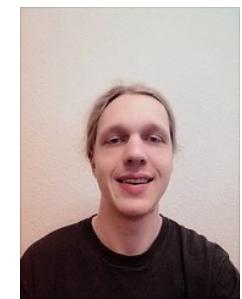
Lorenzo Oghittu



Nicolas Heimann



Beilei Zhu



Hannes Kiehn

## Alumni



Junichi Okamoto



Vijay Singh



Jayson Cosme



Kazuma Nagao



Marlon Nuske

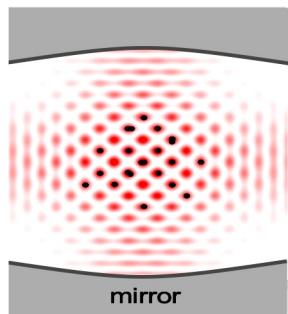
# Recent and current collaborators

Andreas Hemmerich  
Jayson Cosme  
Hans Keßler  
Christof Weitenberg  
Klaus Sengstock  
Christopher Foot  
James McIver  
Gregor Jotzu  
Andrea Cavalleri  
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Vijay Singh  
Junichi Okamoto

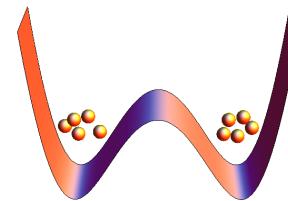
Henning Moritz  
Wen-Min Huang  
Guillaume Salomon  
Juliette Simonet  
Christoph Becker  
Marios Martin  
Wanzheng Hu  
Jean Dalibard  
Markus Heyl  
Jan Budich  
Darrick Chang  
Eite Tiesinga

# Research interests

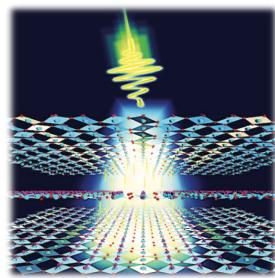
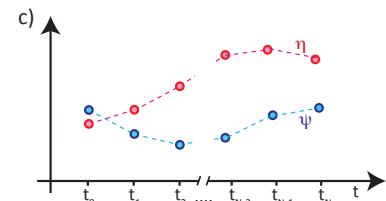
Atom-photon systems



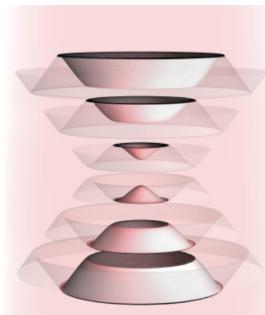
Cold atom dynamics



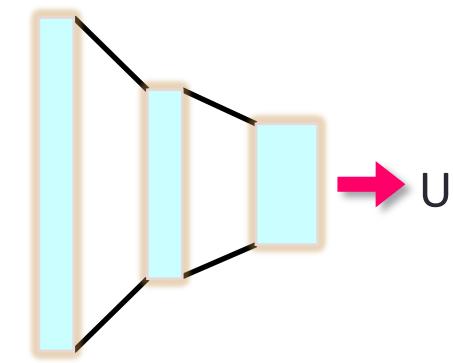
Path integrals



Superconductors



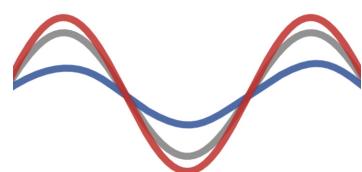
Graphene dynamics



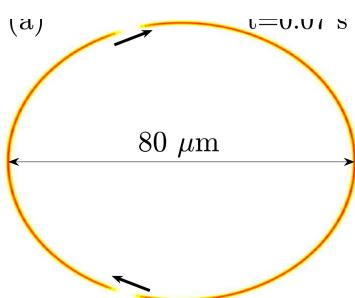
Quantum machine  
learning

# SQUIDs and Josephson junctions

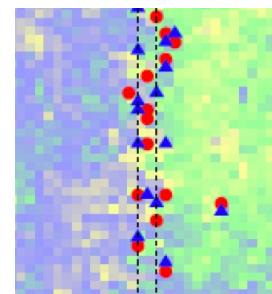
Dynamical control



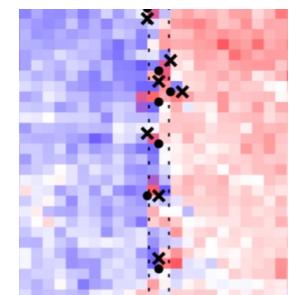
quasi-1d SQUID



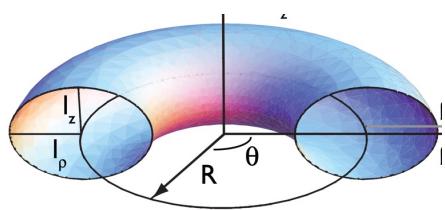
JJ dynamics, th



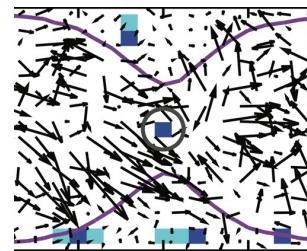
JJ dynamics, exp



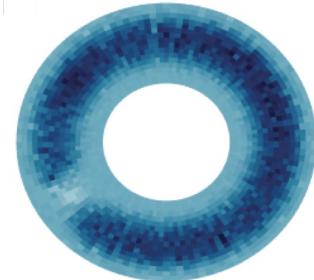
Phase fluctuations



Phase dynamics

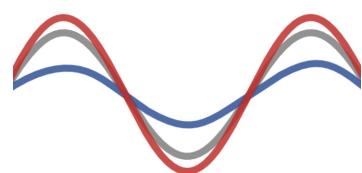


3D SQUID

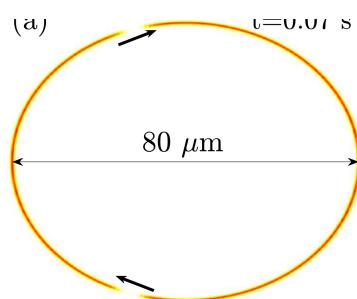


# SQUIDs and Josephson junctions

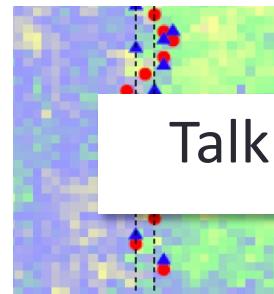
Dynamical control



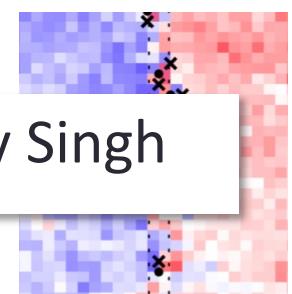
quasi-1d SQUID



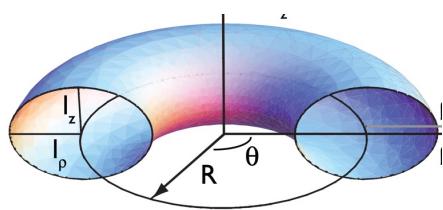
JJ dynamics, th



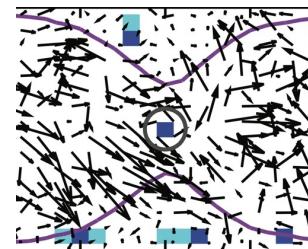
JJ dynamics, exp



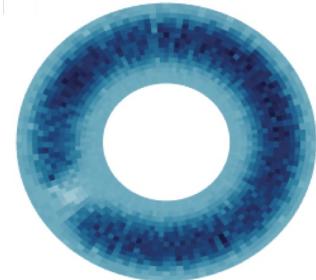
Phase fluctuations



Phase dynamics



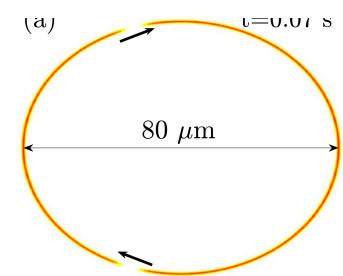
3D SQUID



# Overview

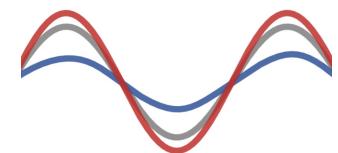
Implementation of an atomtronic SQUID in a strongly Confined toroidal condensate

*Hannes Kiehn, Vijay Singh, LM*

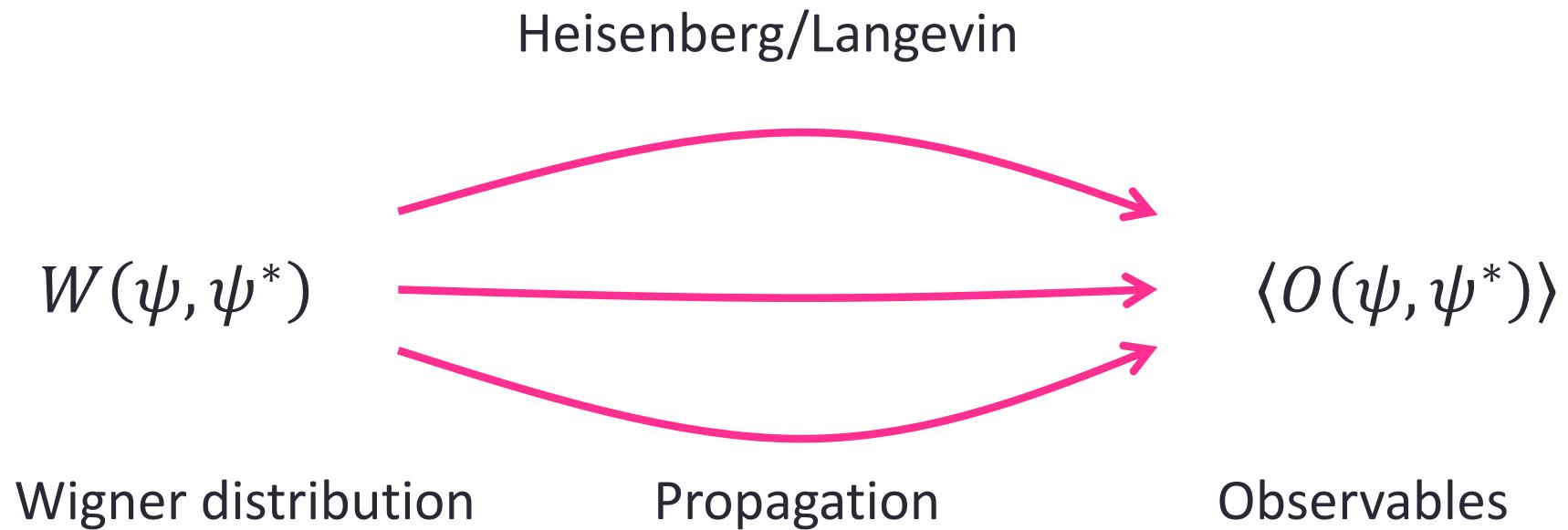


Dynamical control of an atomic Josephson junction

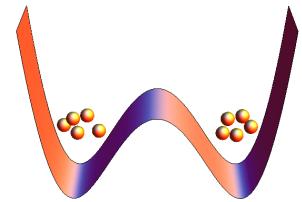
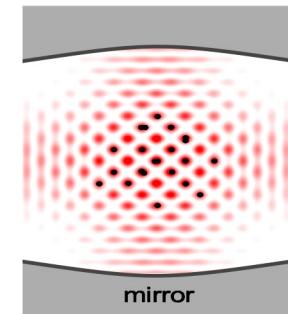
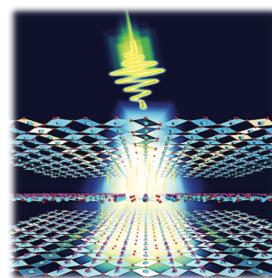
*Beilei Zhu, Vijay Singh, Junichi Okamoto, LM*



# C-field methods / Truncated Wigner



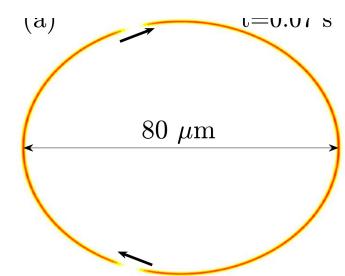
Physical  
systems:



# Overview

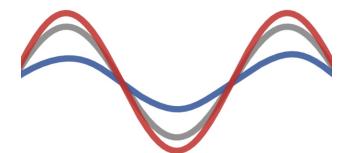
Implementation of an atomtronic SQUID in a strongly Confined toroidal condensate

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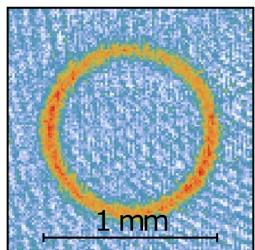


Dynamical control of an atomic Josephson junction

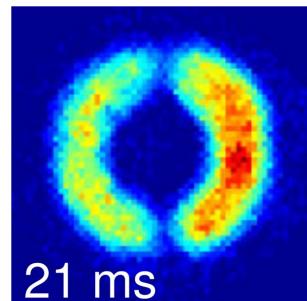
*Beilei Zhu, Vijay Singh, Junichi Okamoto, LM*



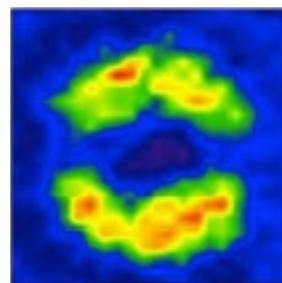
# Toroidal condensates



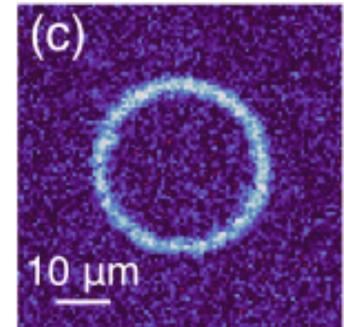
von Klitzing



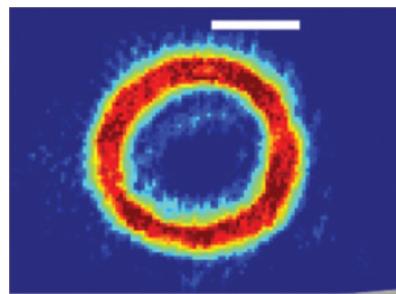
Campbell



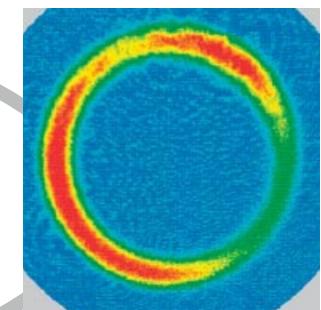
Boshier



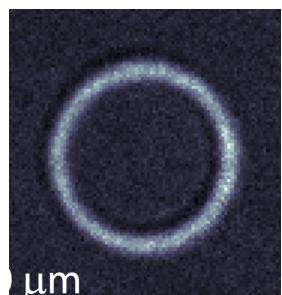
Roati



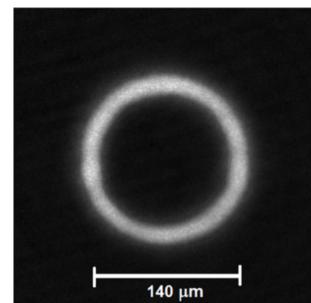
Hadzibabic



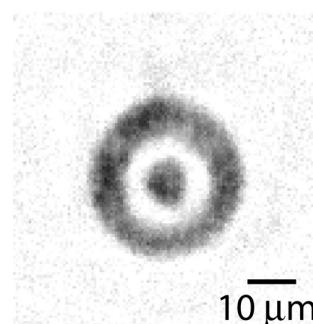
Stamper-Kurn



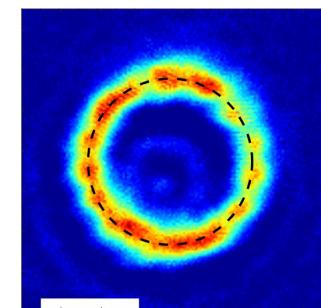
Wright



Rubinsztein-Dunlop

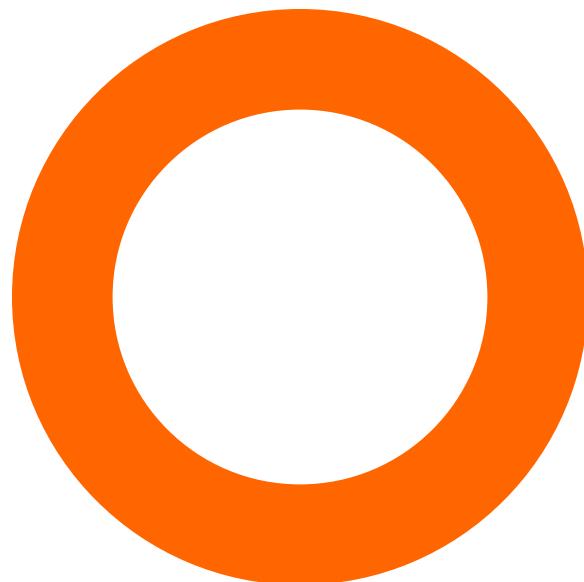


Dalibard



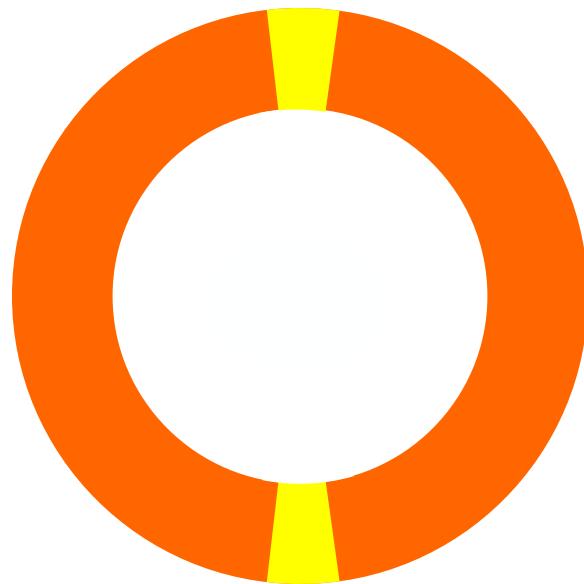
Perrin

# SQUIDs



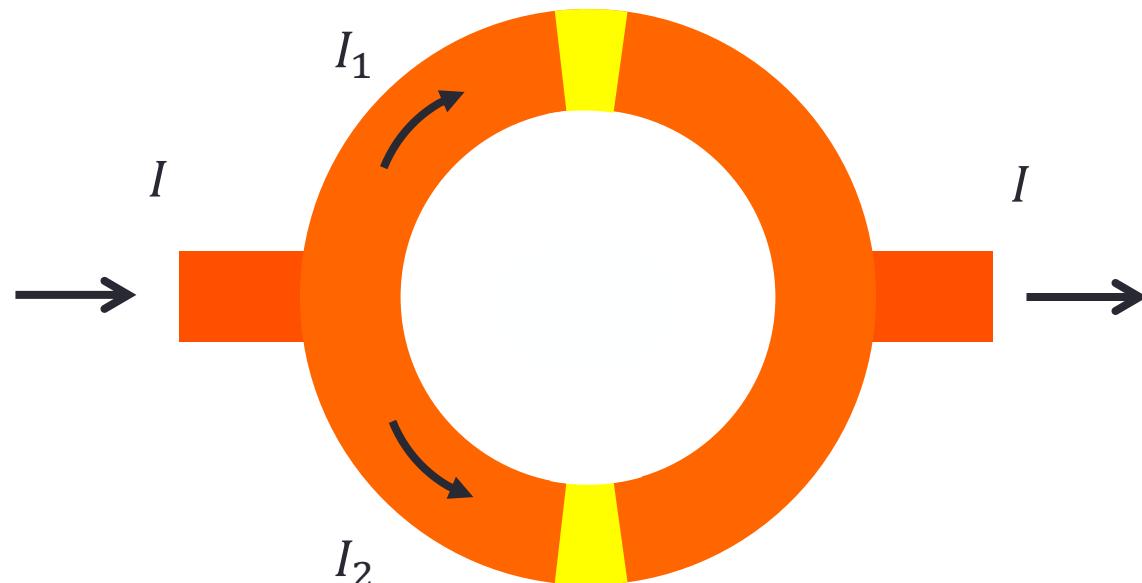
Superconducting ring

# SQUIDs



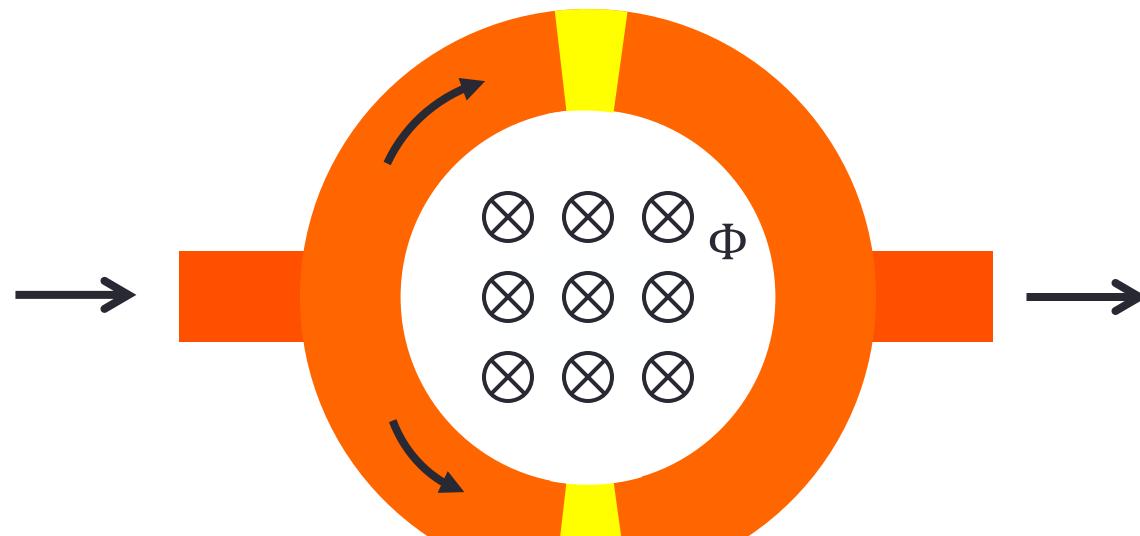
Josephson junctions

# SQUIDs



Leads

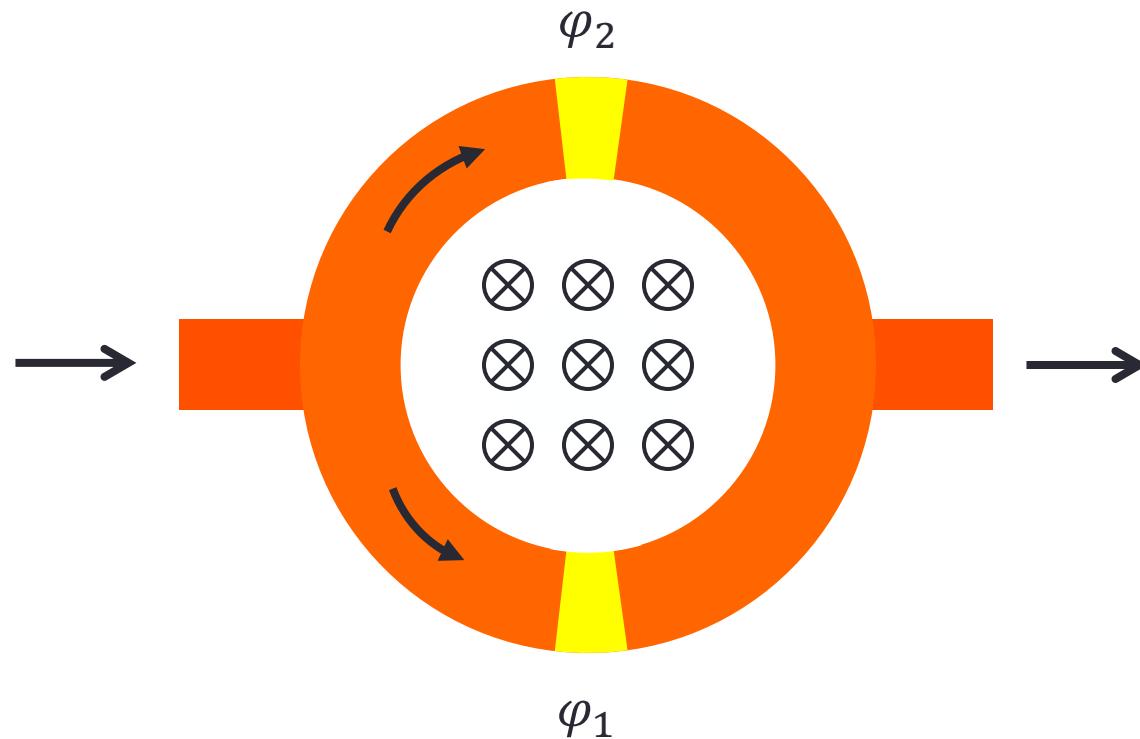
# SQUIDs



Magnetic flux

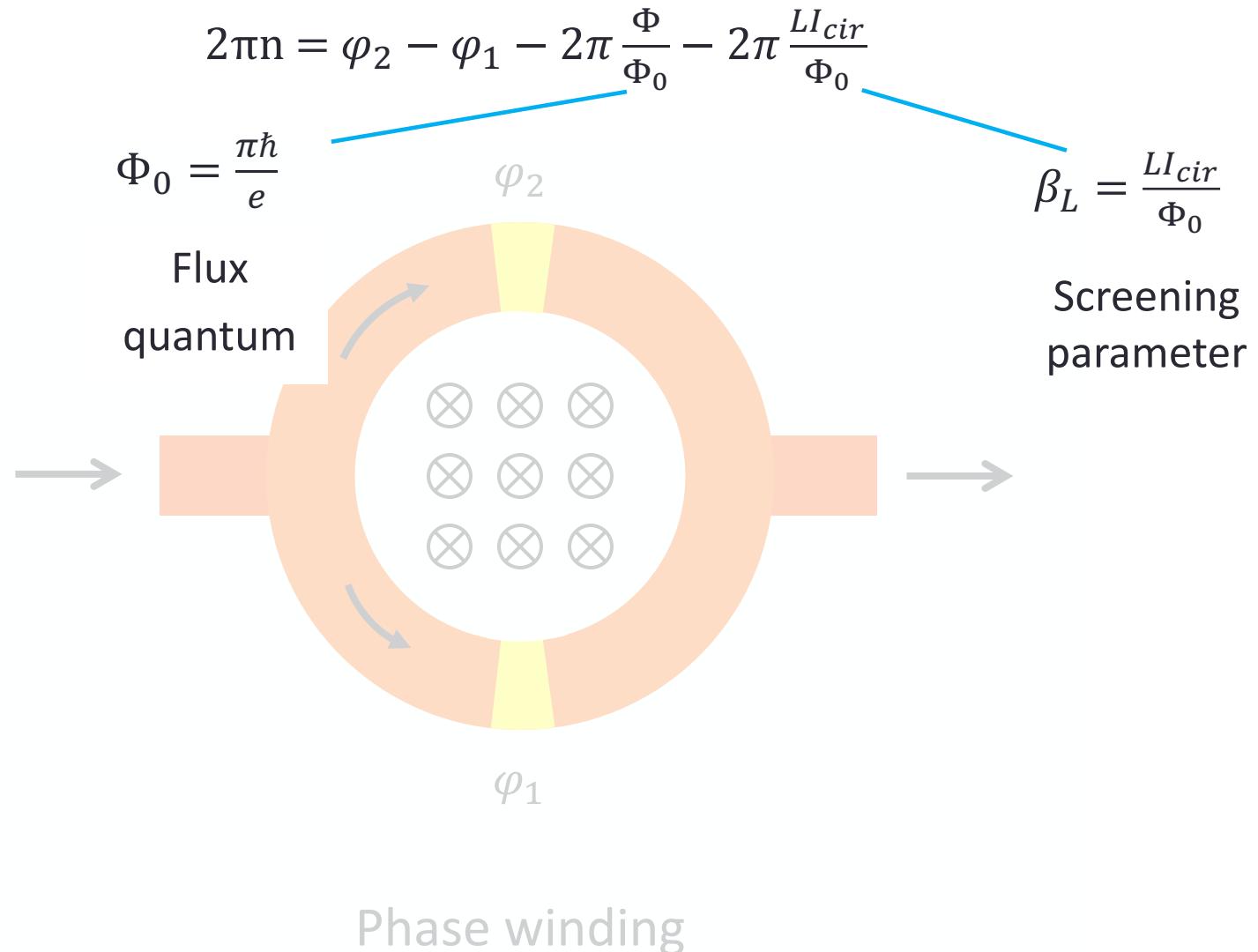
# SQUIDs

$$2\pi n = \varphi_2 - \varphi_1 - 2\pi \frac{\Phi}{\Phi_0} - 2\pi \frac{LI_{cir}}{\Phi_0}$$

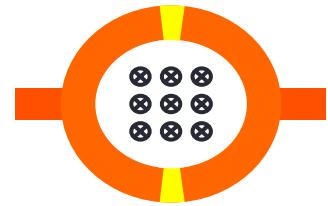


Phase winding

# SQUIDs



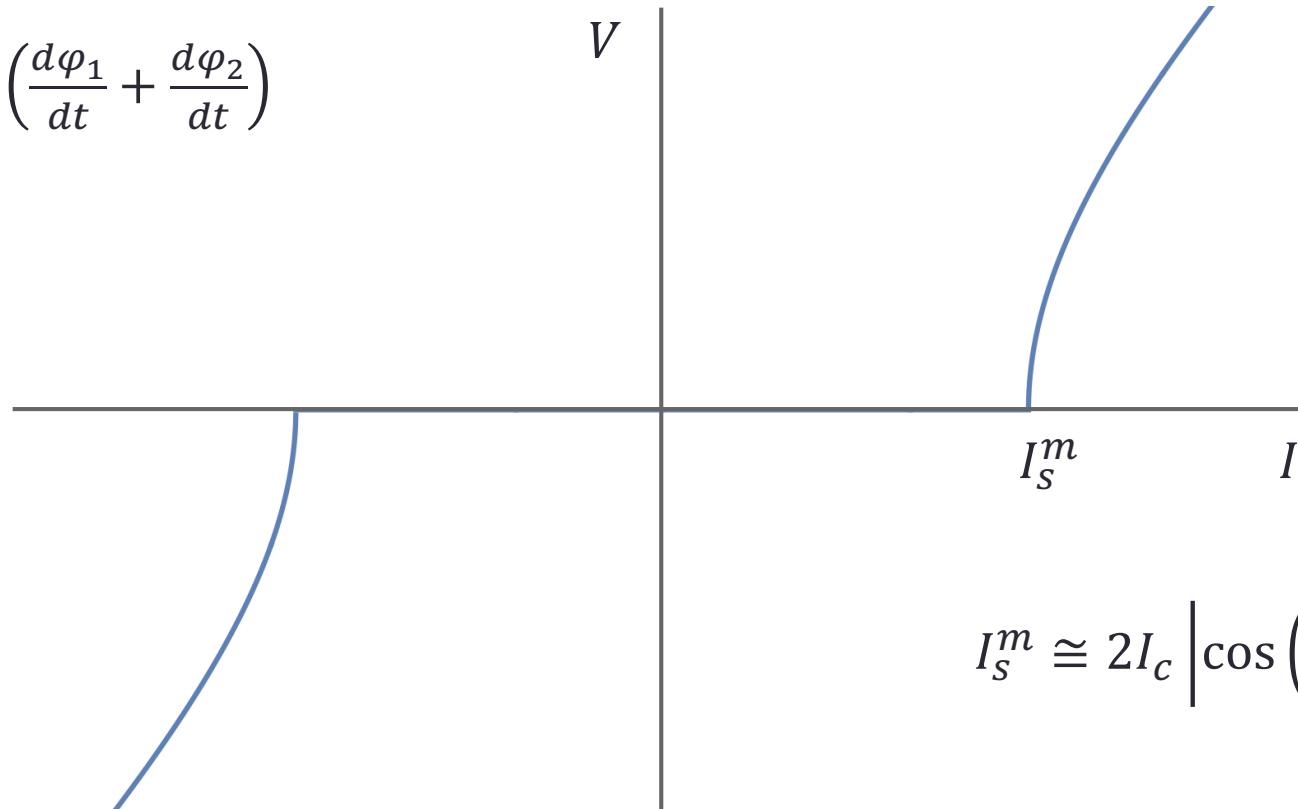
# SQUIDs



$$\frac{I}{2} = \frac{\hbar C}{2e} \frac{d^2\varphi_1}{dt^2} + \frac{\hbar}{2eR_N} \frac{d\varphi_1}{dt} + I_c \sin \varphi_1 - I_{cir}$$

$$\frac{I}{2} = \frac{\hbar C}{2e} \frac{d^2\varphi_2}{dt^2} + \frac{\hbar}{2eR_N} \frac{d\varphi_2}{dt} + I_c \sin \varphi_2 + I_{cir}$$

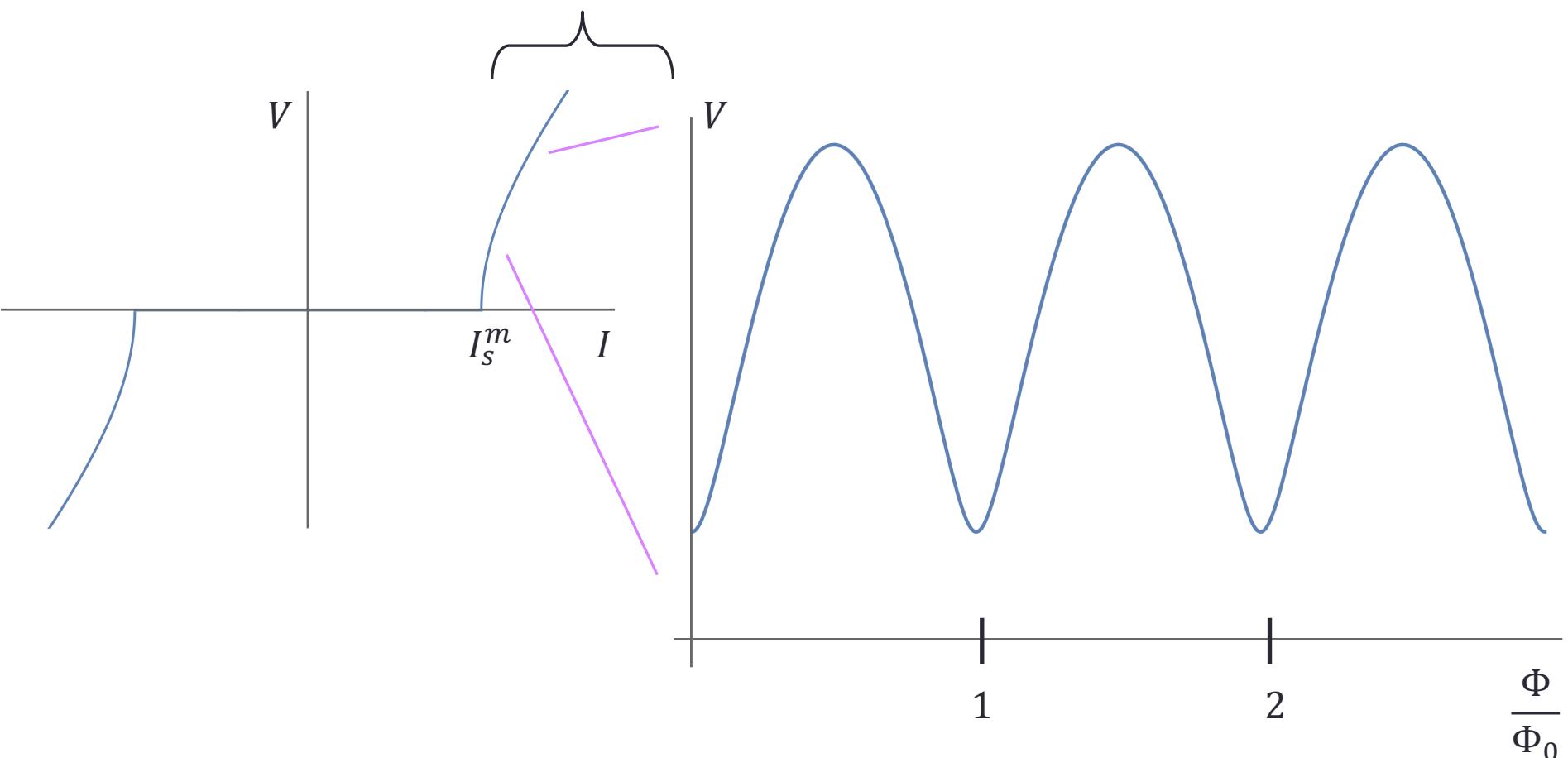
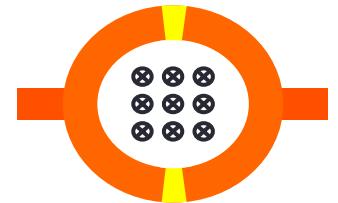
$$V = \frac{\Phi_0}{4\pi} \left( \frac{d\varphi_1}{dt} + \frac{d\varphi_2}{dt} \right)$$



$$I_s^m \cong 2I_c \left| \cos \left( \pi \frac{\Phi}{\Phi_0} \right) \right|$$

# SQUIDs

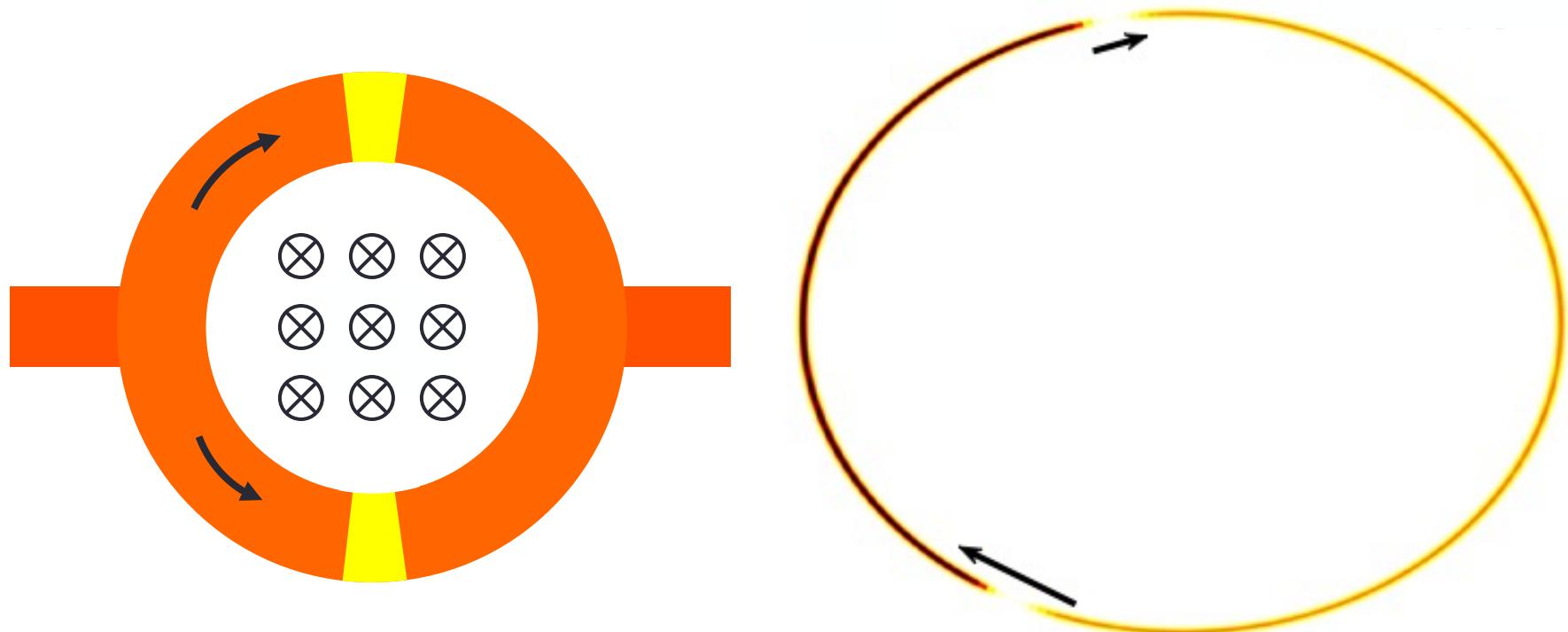
Resistive  
regime



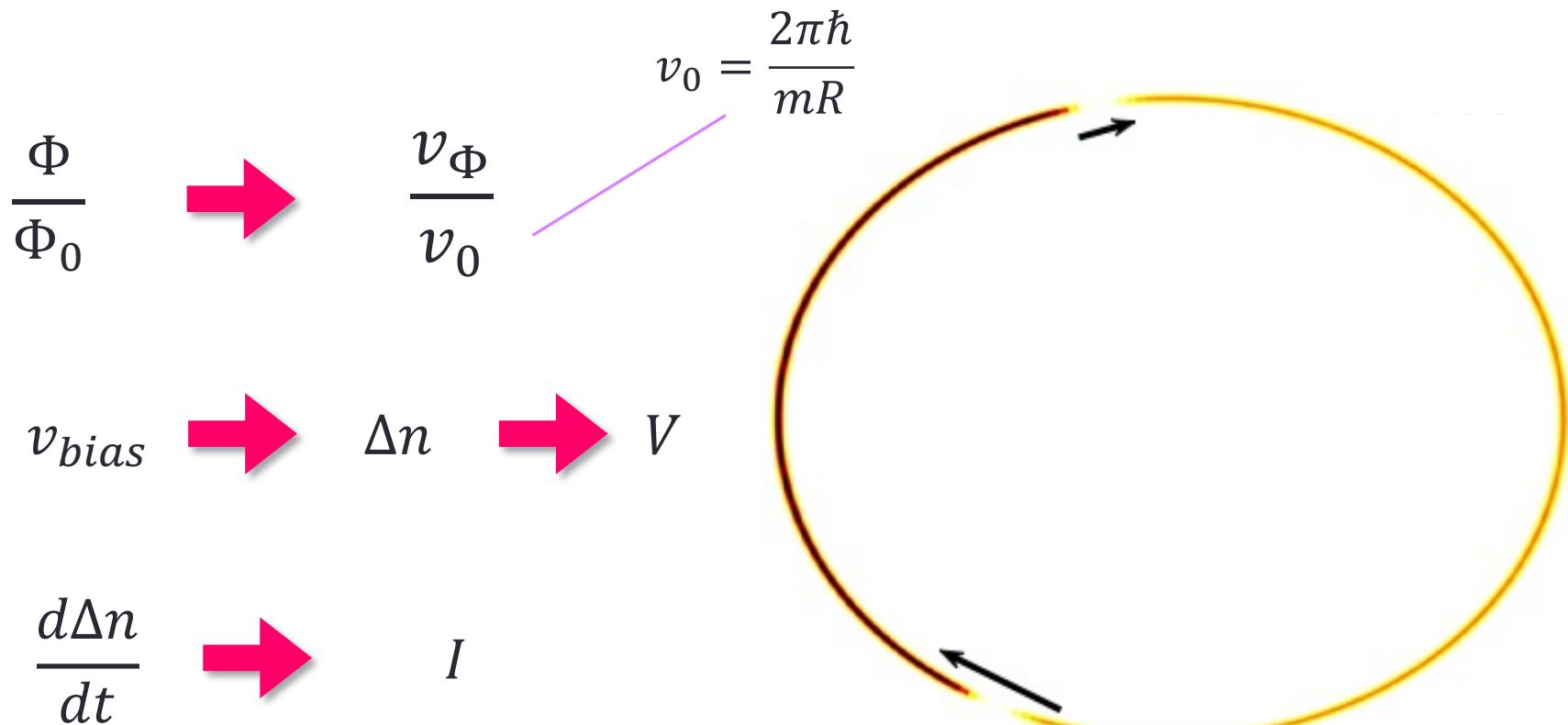
Overdamped  
limit

$$\langle V(t) \rangle = I_c R_N \sqrt{\frac{I}{2I_c} - \left( \cos \left( \pi \frac{\Phi}{\Phi_0} \right) \right)^2}$$

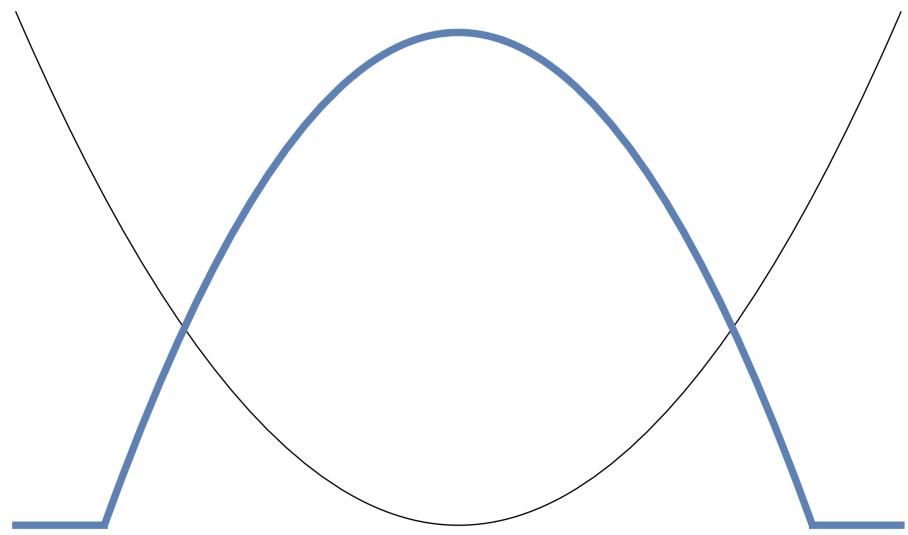
# Atomtronic realization



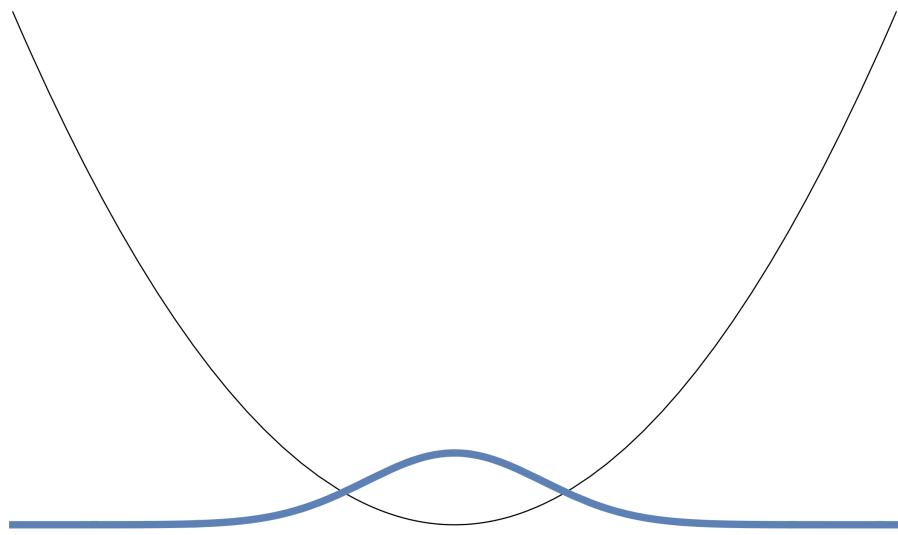
# Atomtronic realization



# Dimensionality

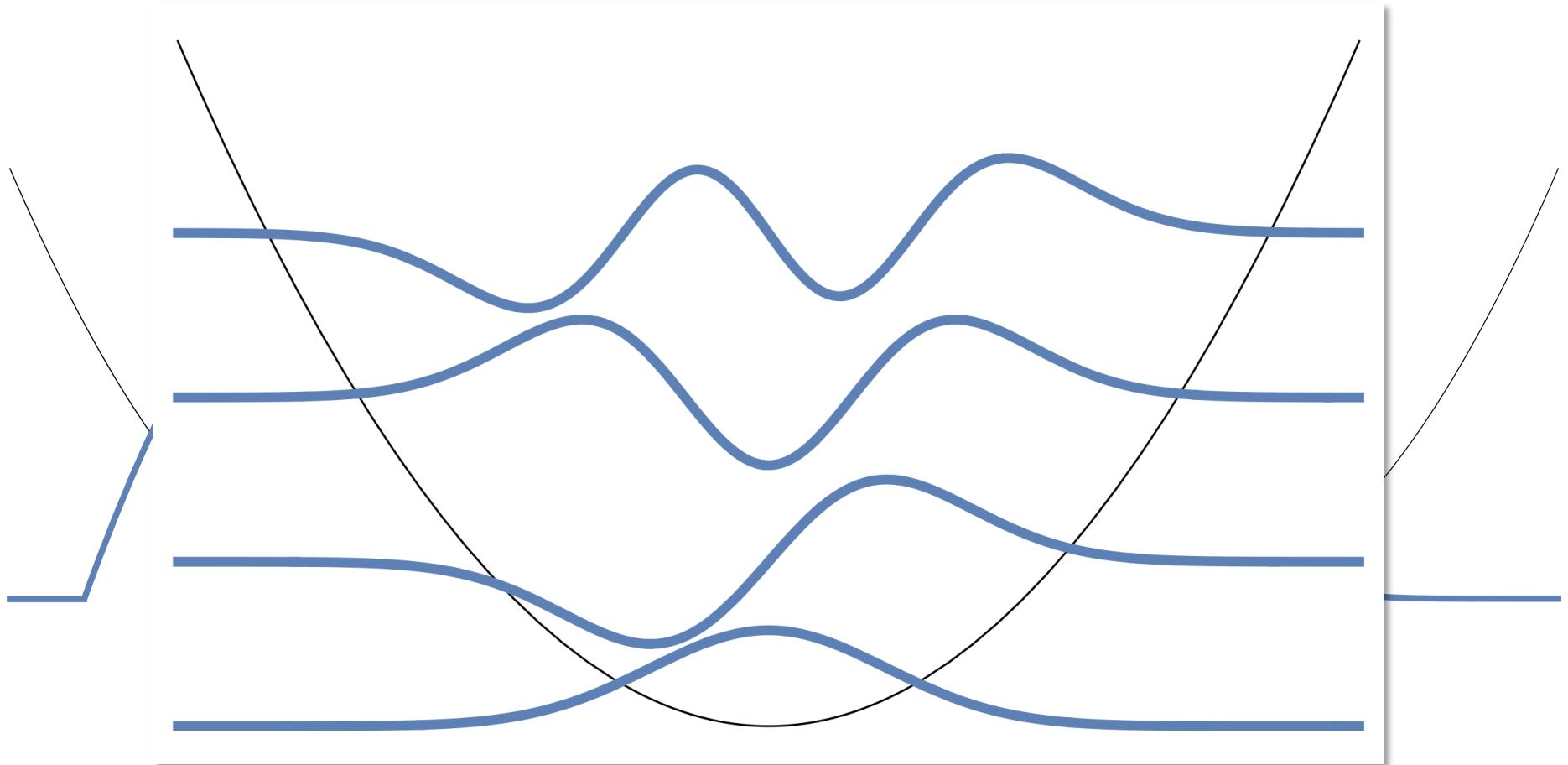


3D



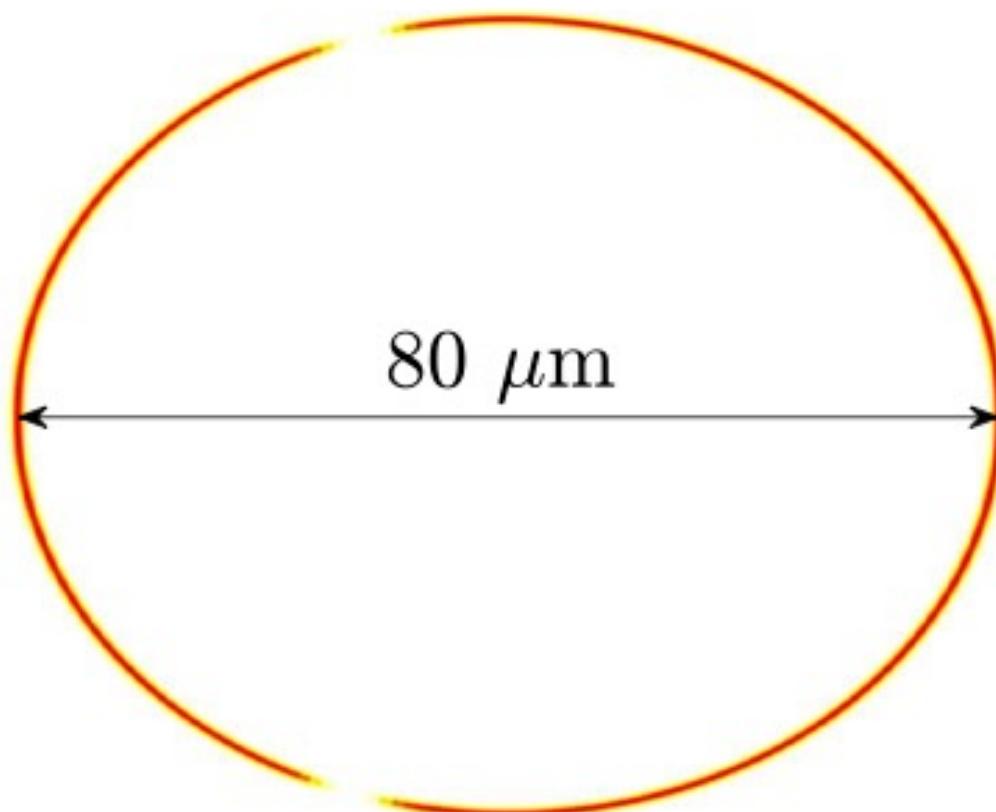
1D

# Dimensionality

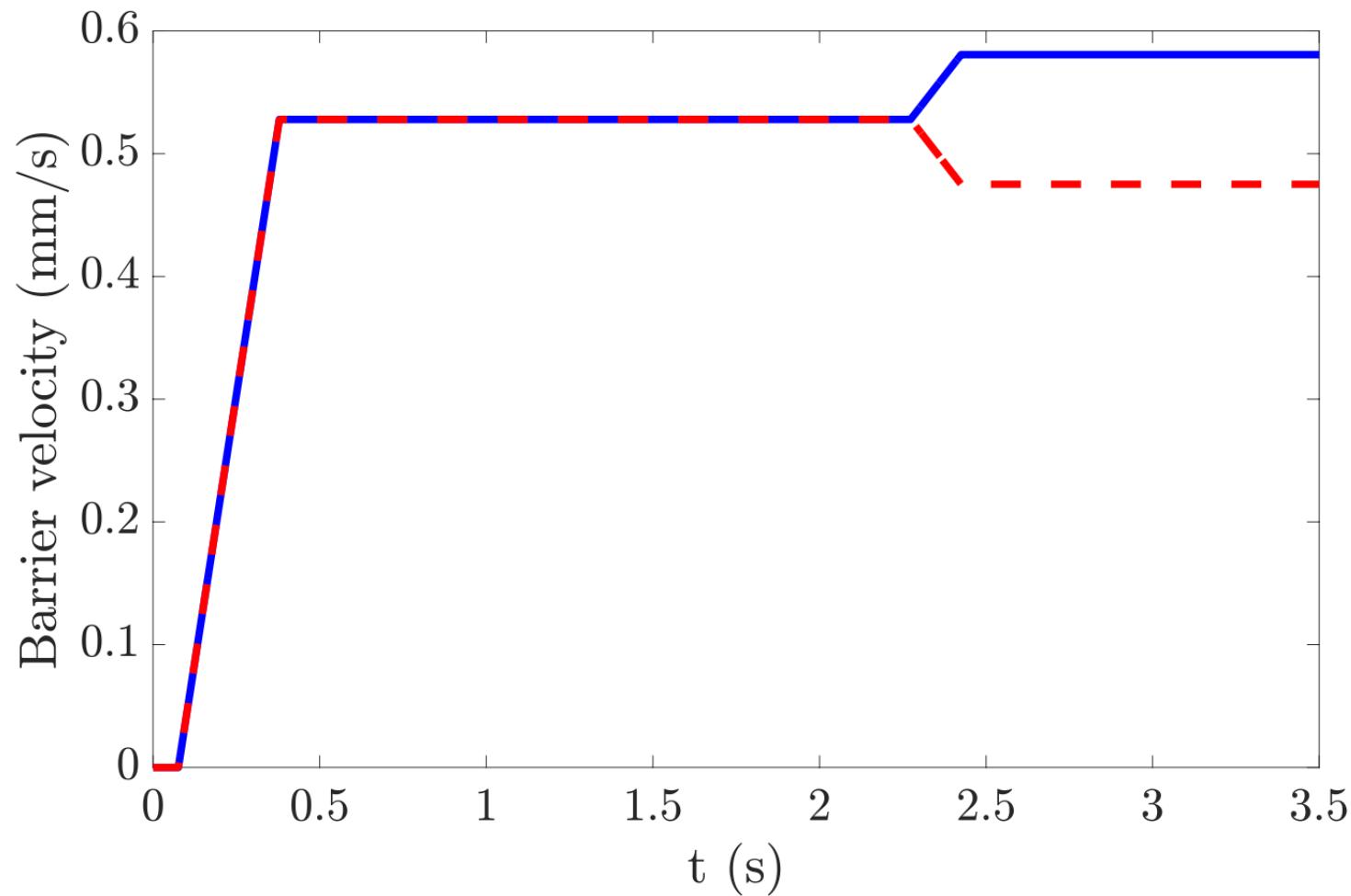


Quasi-1D

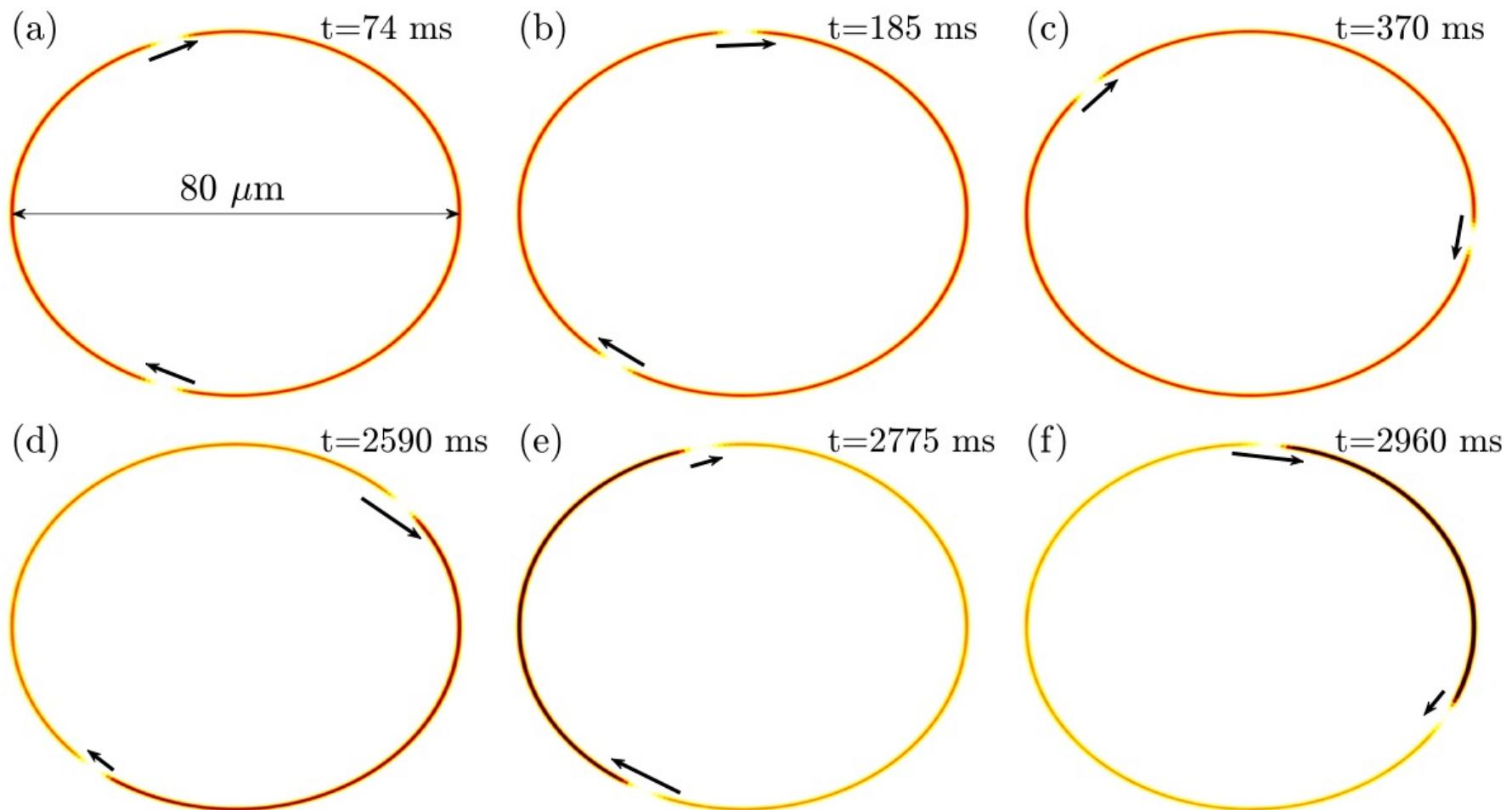
# Quasi-1D condensates



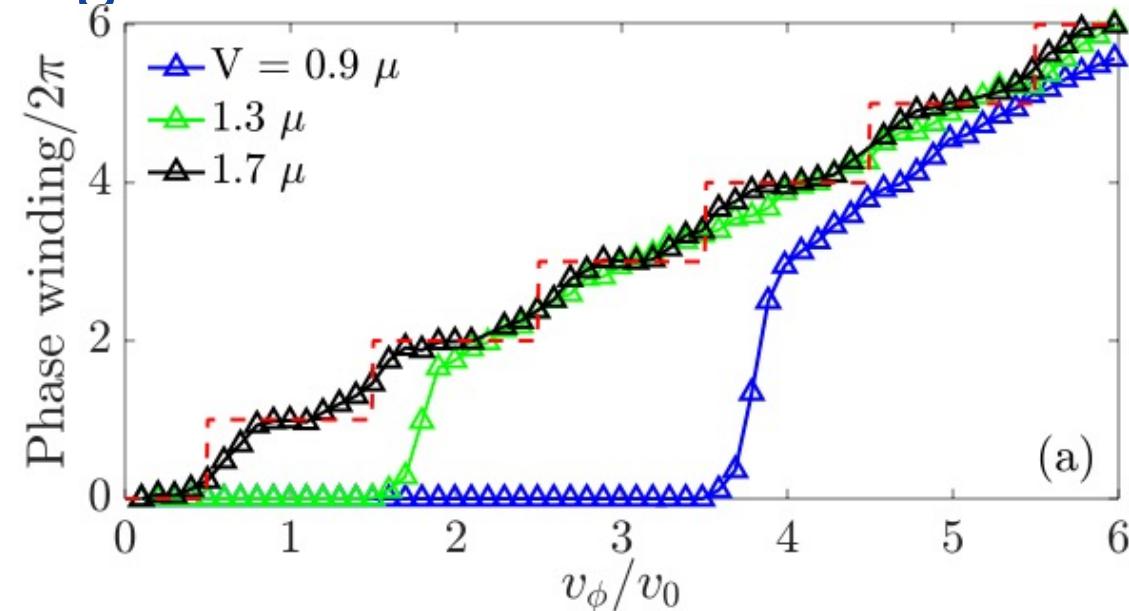
# Protocol



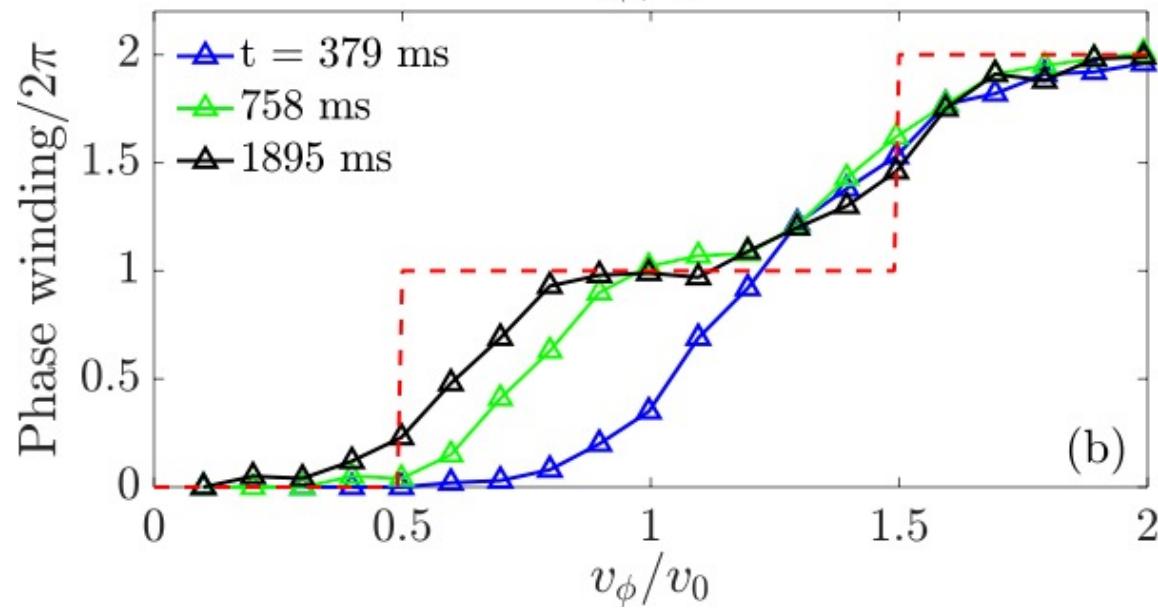
# SQUID dynamics



# Phase winding

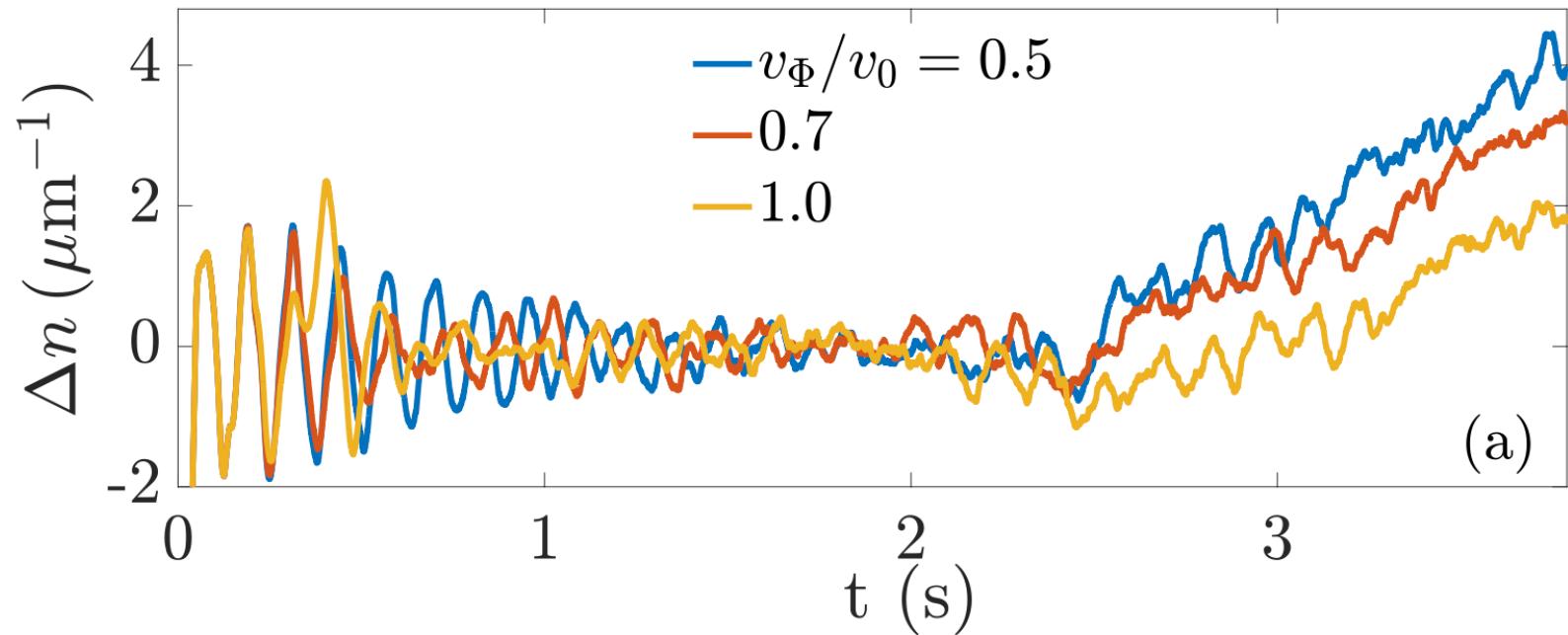


(a)

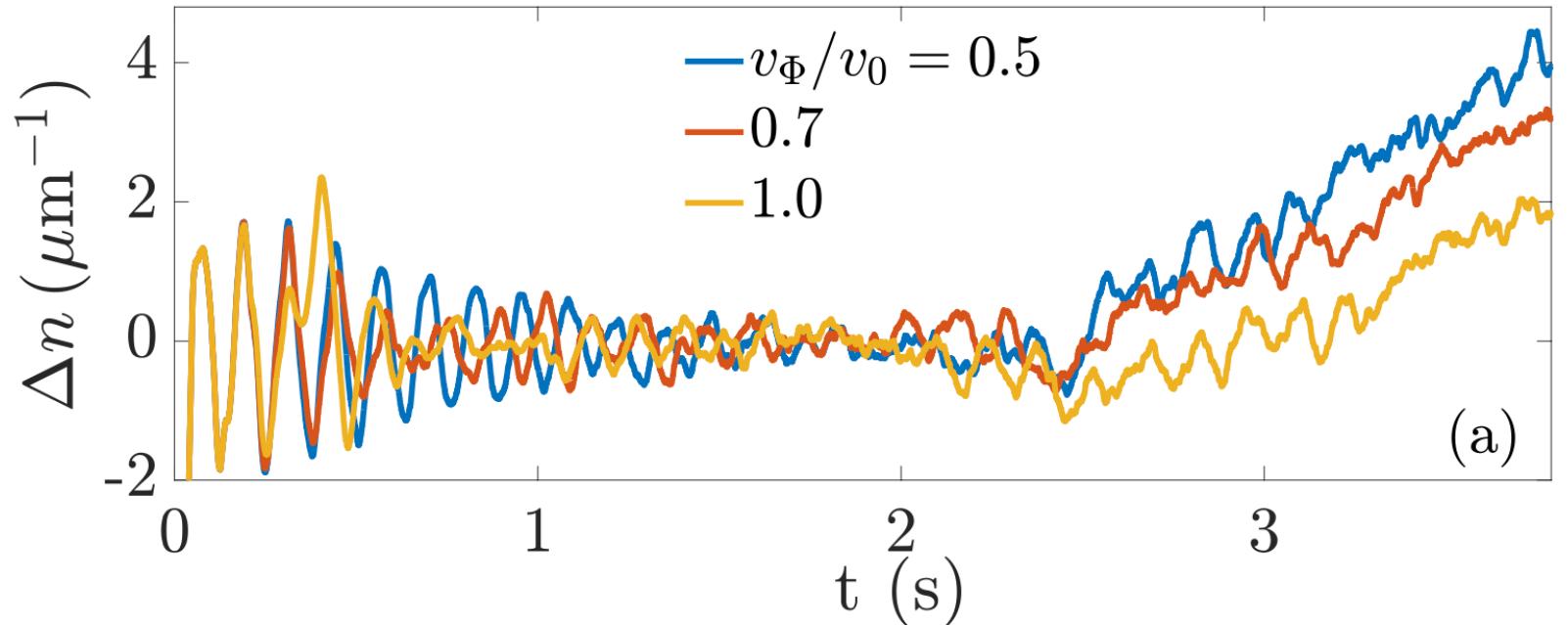


(b)

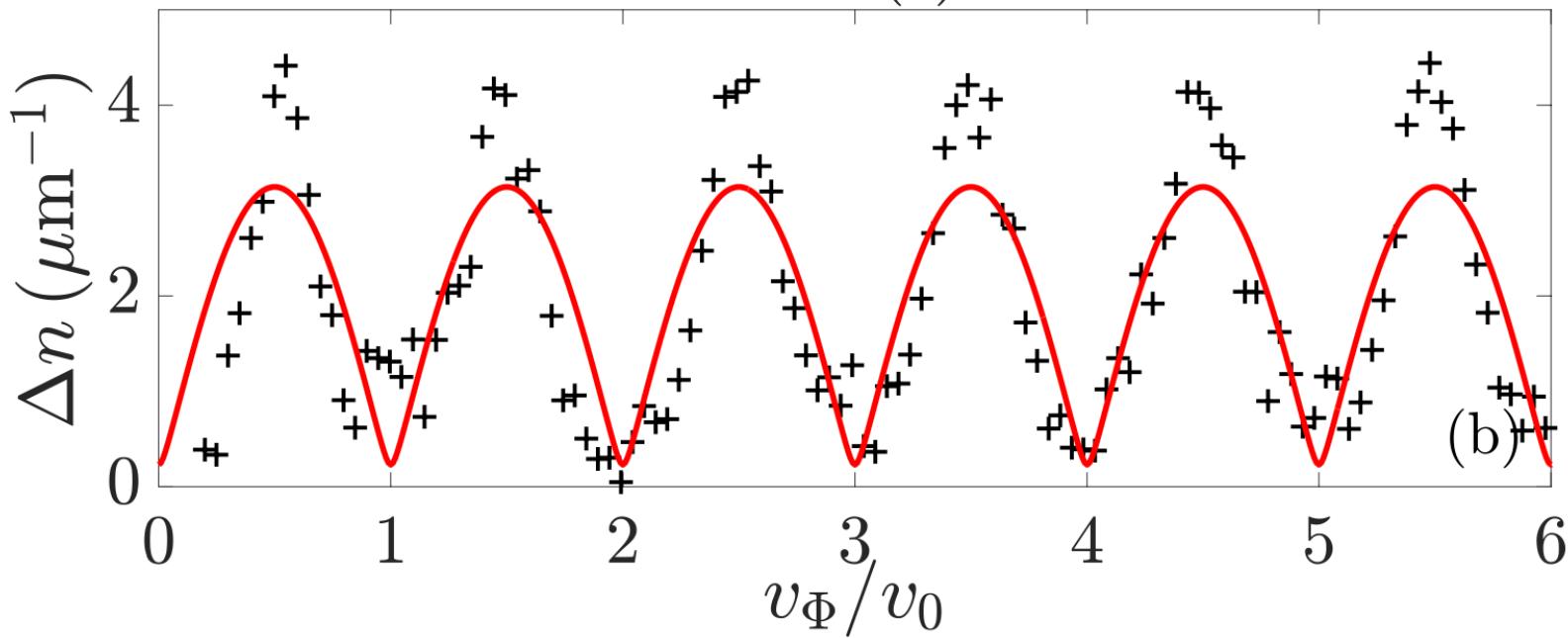
# Density imbalance



# Voltage-flux relation



(a)

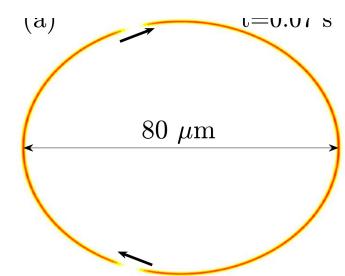


(b)

# Overview

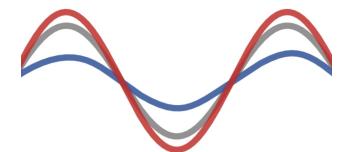
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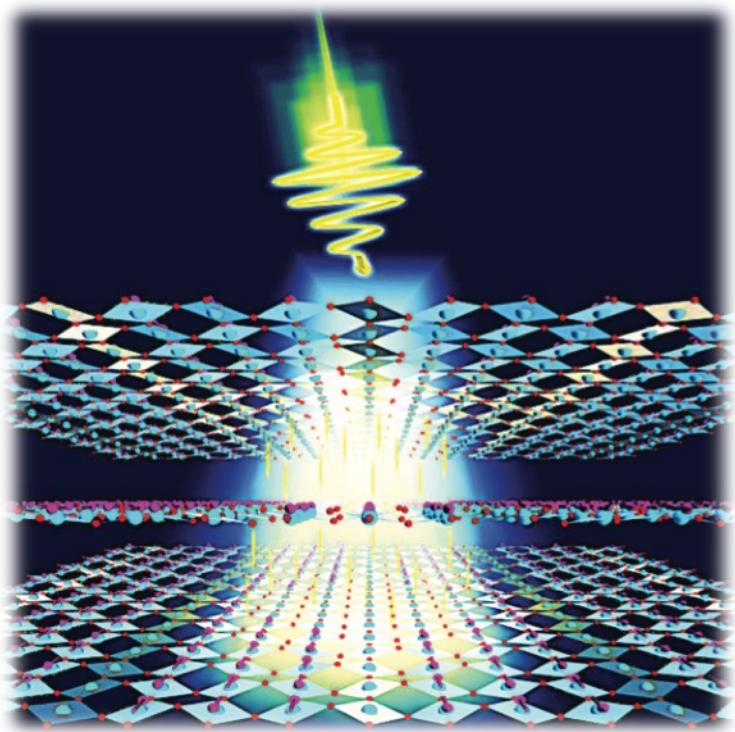


Dynamical control of an atomic Josephson junction

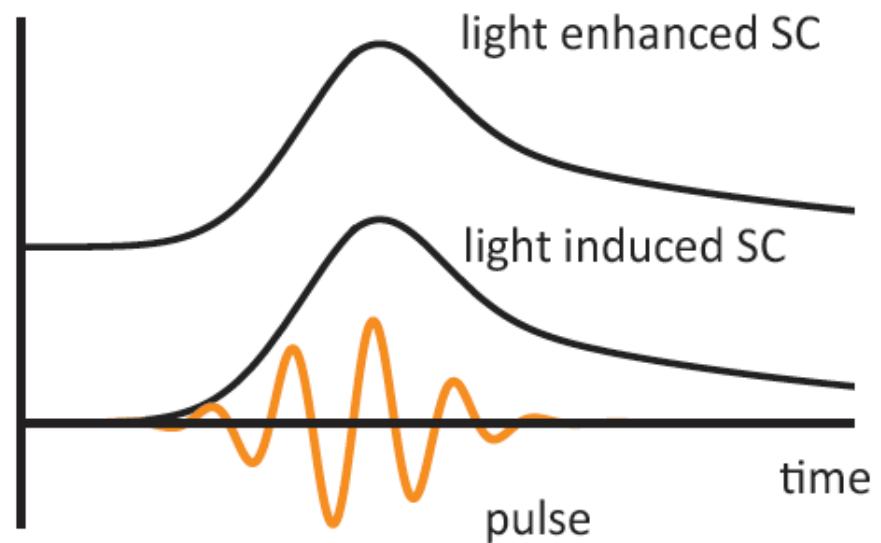
*Beilei Zhu, Vijay Singh, Junichi Okamoto, LM*



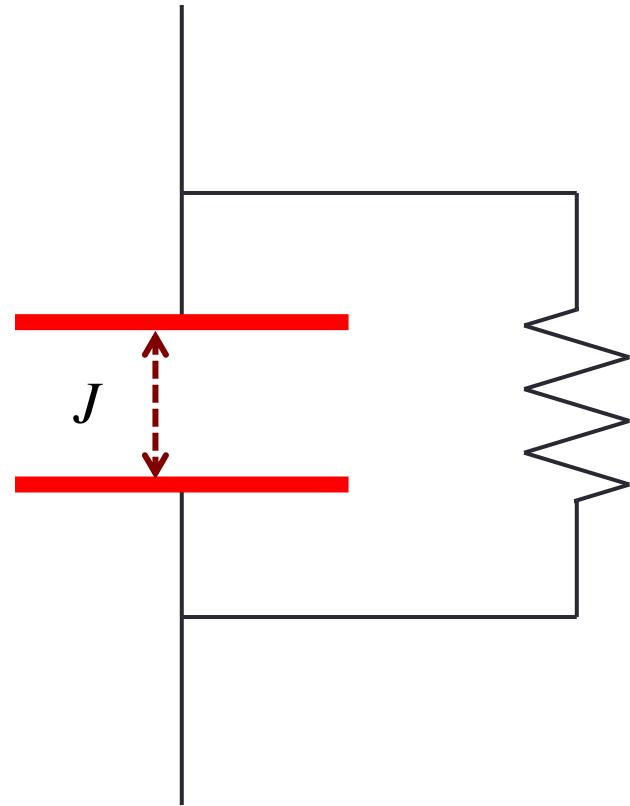
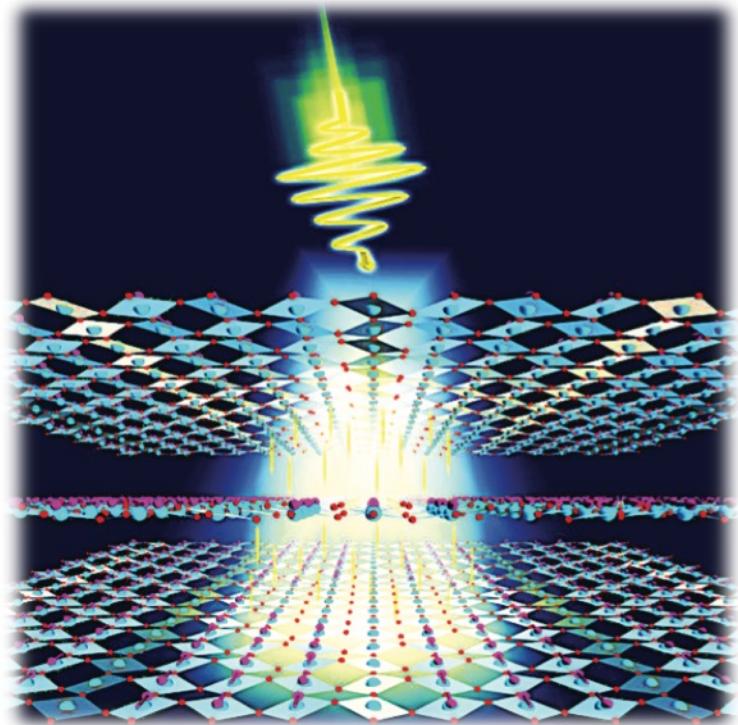
# Light-induced dynamics



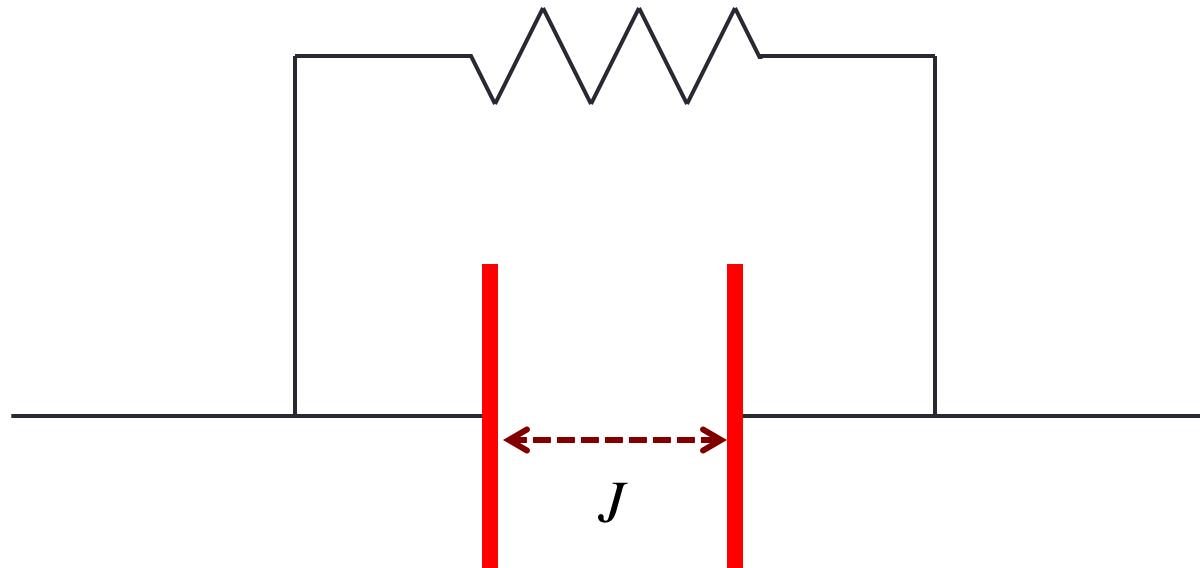
Pump-probe experiments



# Parametric Control



# Resistively and capacitively shunted Josephson junction

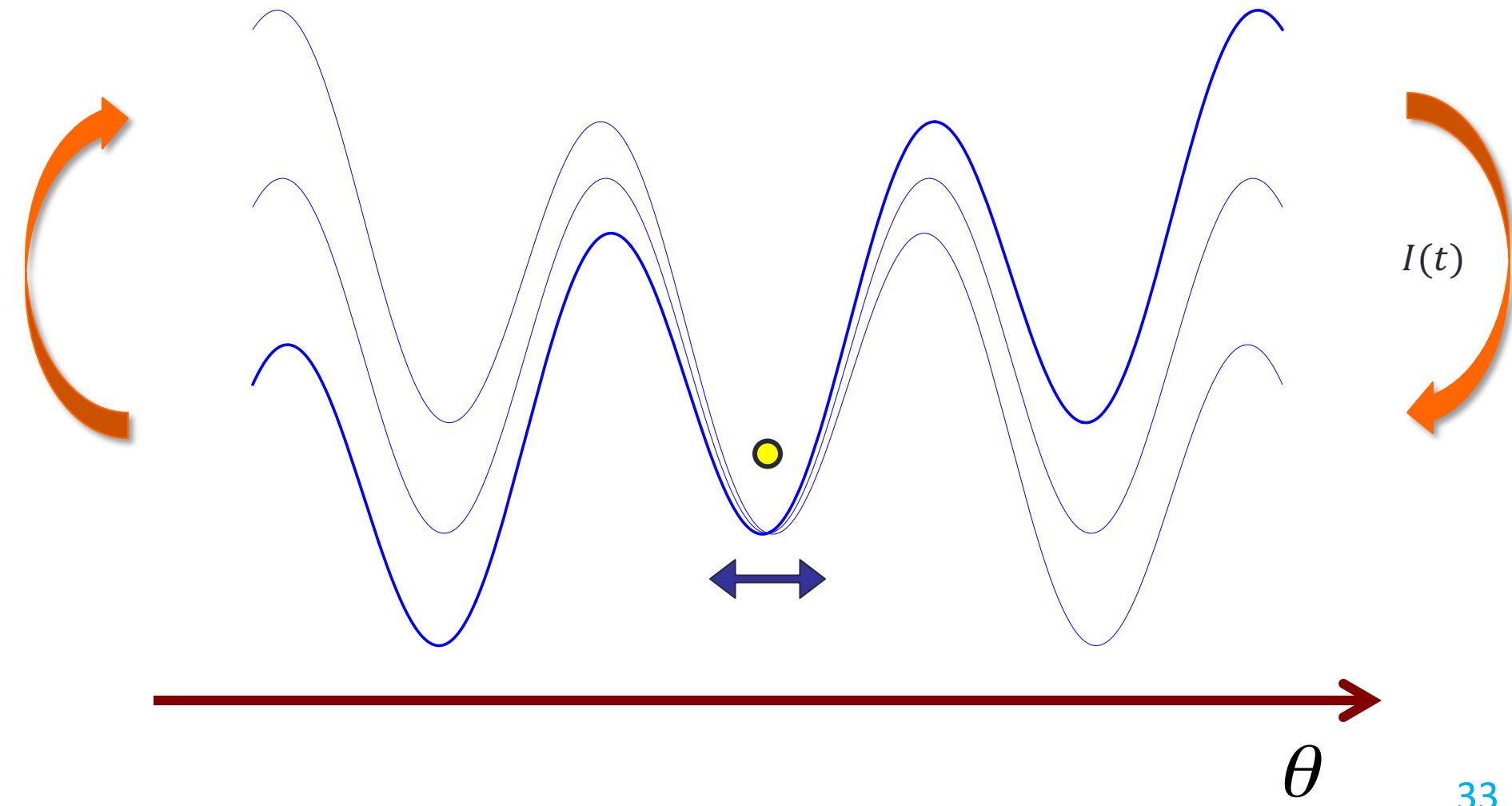


$$I(t) = Jn_0 \sin(\theta) + \frac{\hbar}{qR} \dot{\theta} + \frac{\hbar C}{q} \ddot{\theta}$$

$$\dot{\theta} = \dot{\theta}_2 - \dot{\theta}_1 = \frac{qV(t)}{\hbar} \propto E(t)$$

# Washboard potential

$$I(t) = Jn_0 \sin(\theta) + \frac{\hbar}{qR} \dot{\theta} + \frac{\hbar C}{q} \ddot{\theta}$$

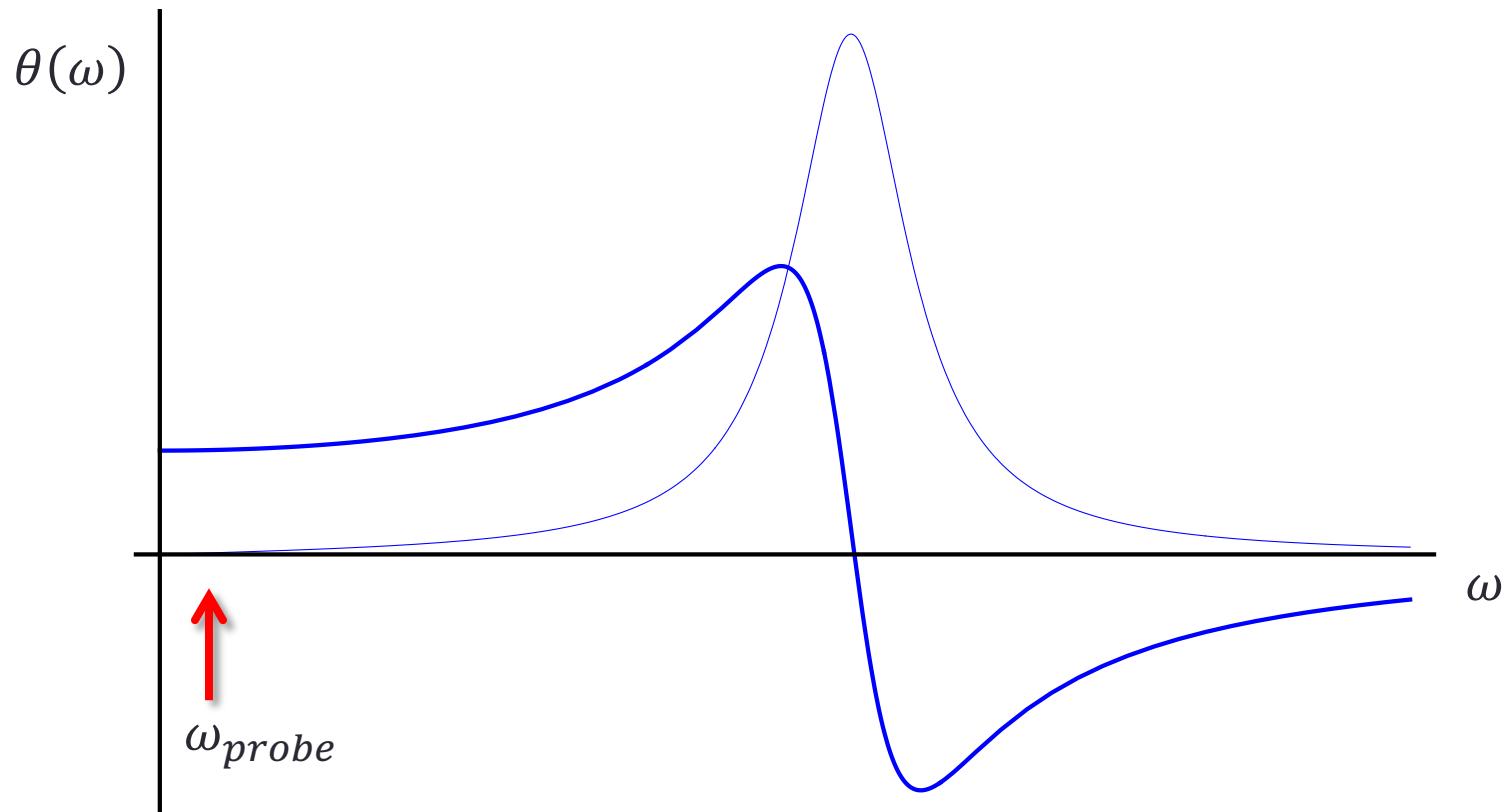


# Phase-current relation

$$I(t) \sim Jn_0\theta + \frac{\hbar}{qR}\dot{\theta} + \frac{\hbar C}{q}\ddot{\theta}$$



$$\theta(\omega) \propto \frac{I(\omega)}{\omega_0^2 - i\gamma\omega - \omega^2}$$

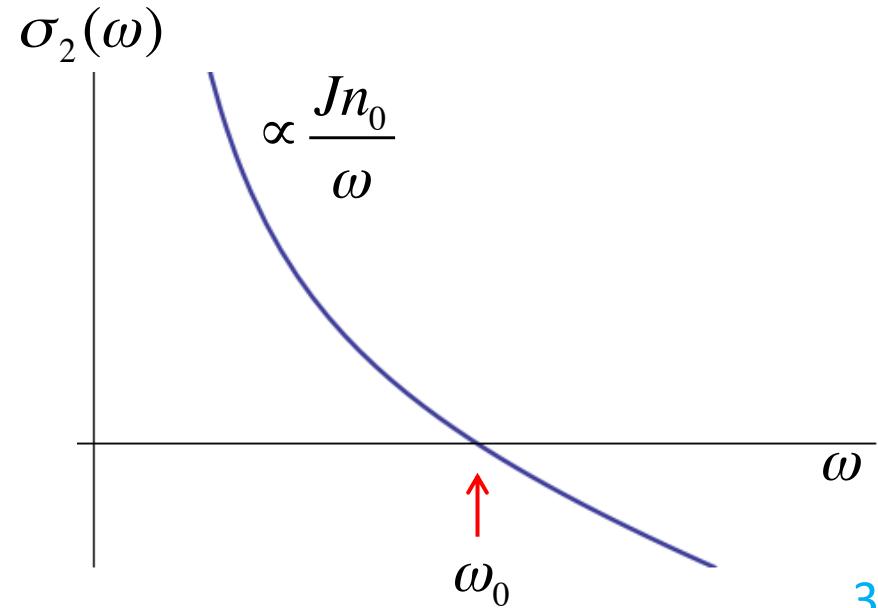
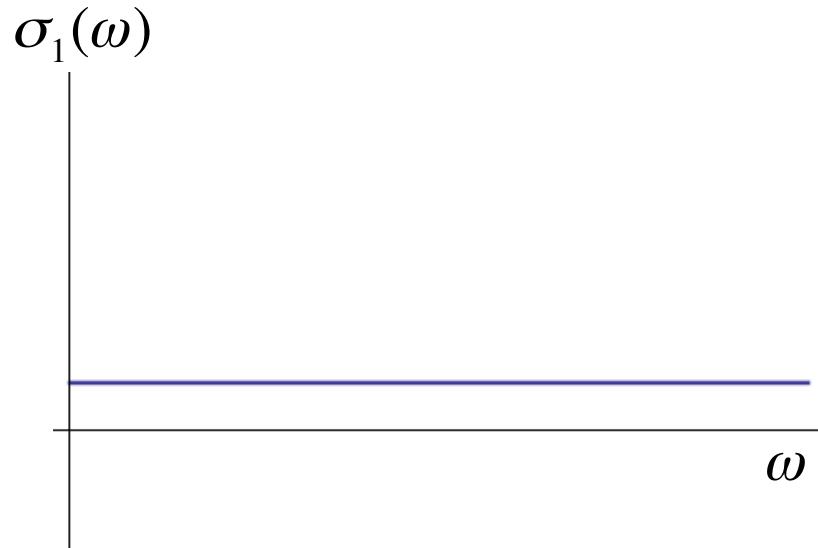


# Conductivity

$$E(\omega) \propto -i\omega\theta(\omega) \propto \frac{i\omega I(\omega)}{\omega_0^2 - i\gamma\omega - \omega^2}$$

$$\underbrace{\frac{\omega_0^2 - i\gamma\omega - \omega^2}{-i\omega}}_{E(\omega)} \propto I(\omega)$$

$$\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)$$



# Comparison to Drude model

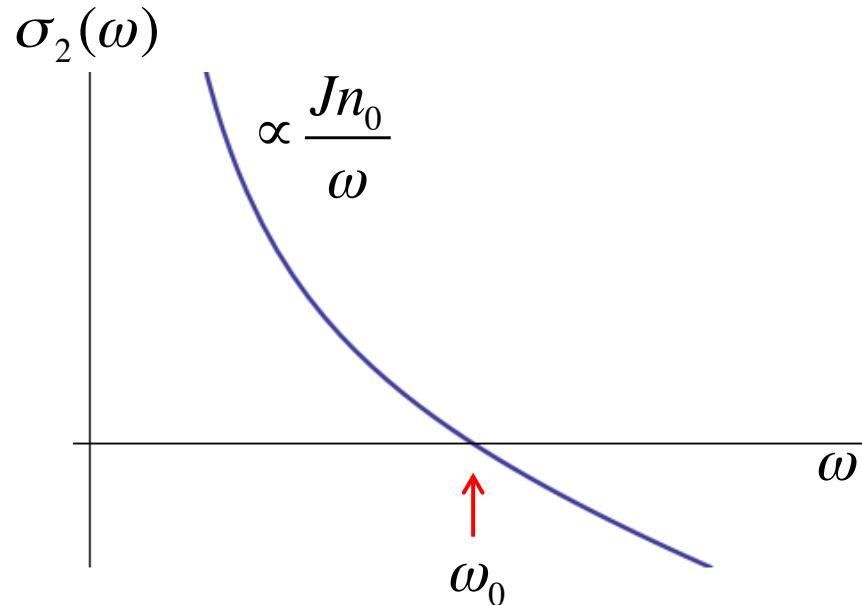
perfect  
conductor

$$\sigma(\omega) = \frac{ne^2}{m} \frac{1}{1/\tau - i\omega}$$

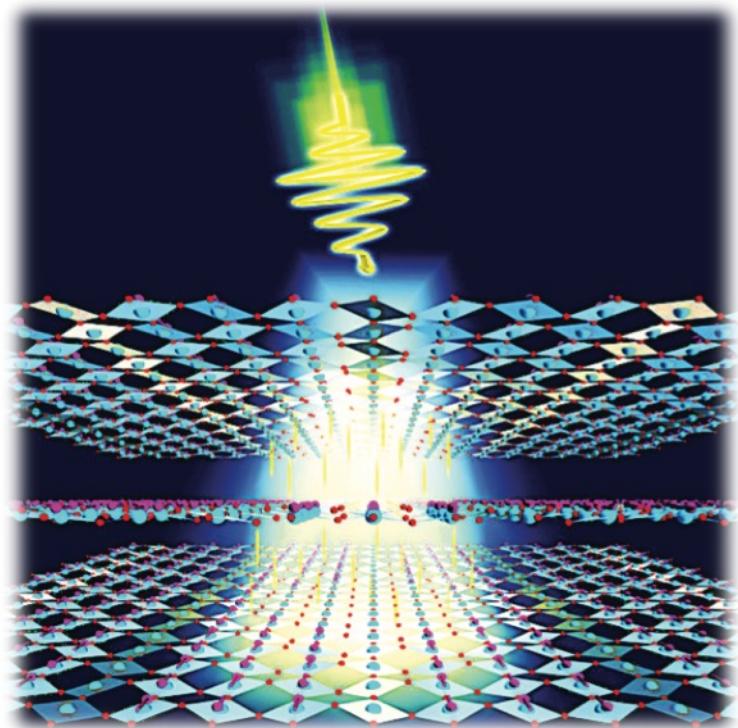


$$\tau \rightarrow \infty$$

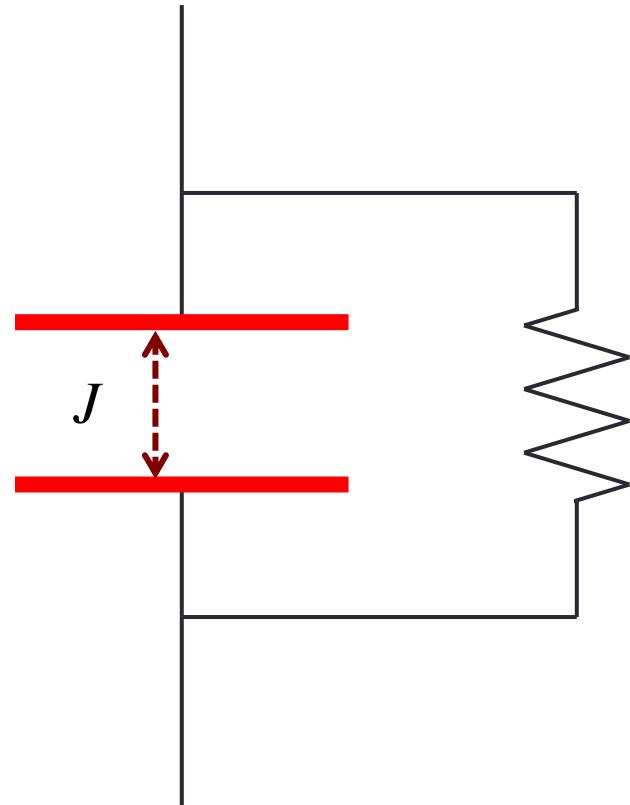
$$\sigma(\omega) = i \frac{ne^2}{m} \frac{1}{\omega} = i\sigma_2(\omega)$$



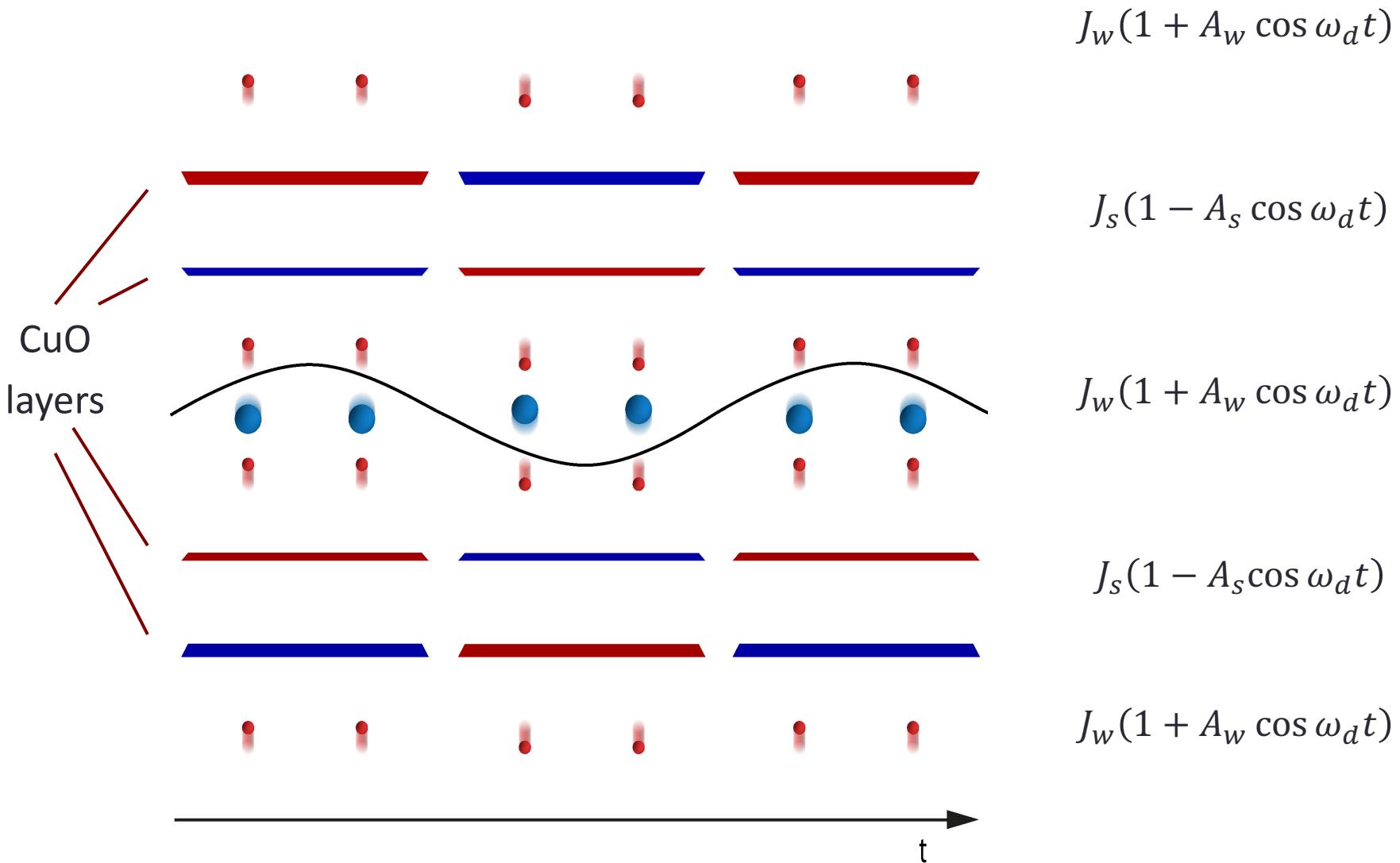
# Parametrically driven junction



$$J(1 + A \cos \omega_d t)$$

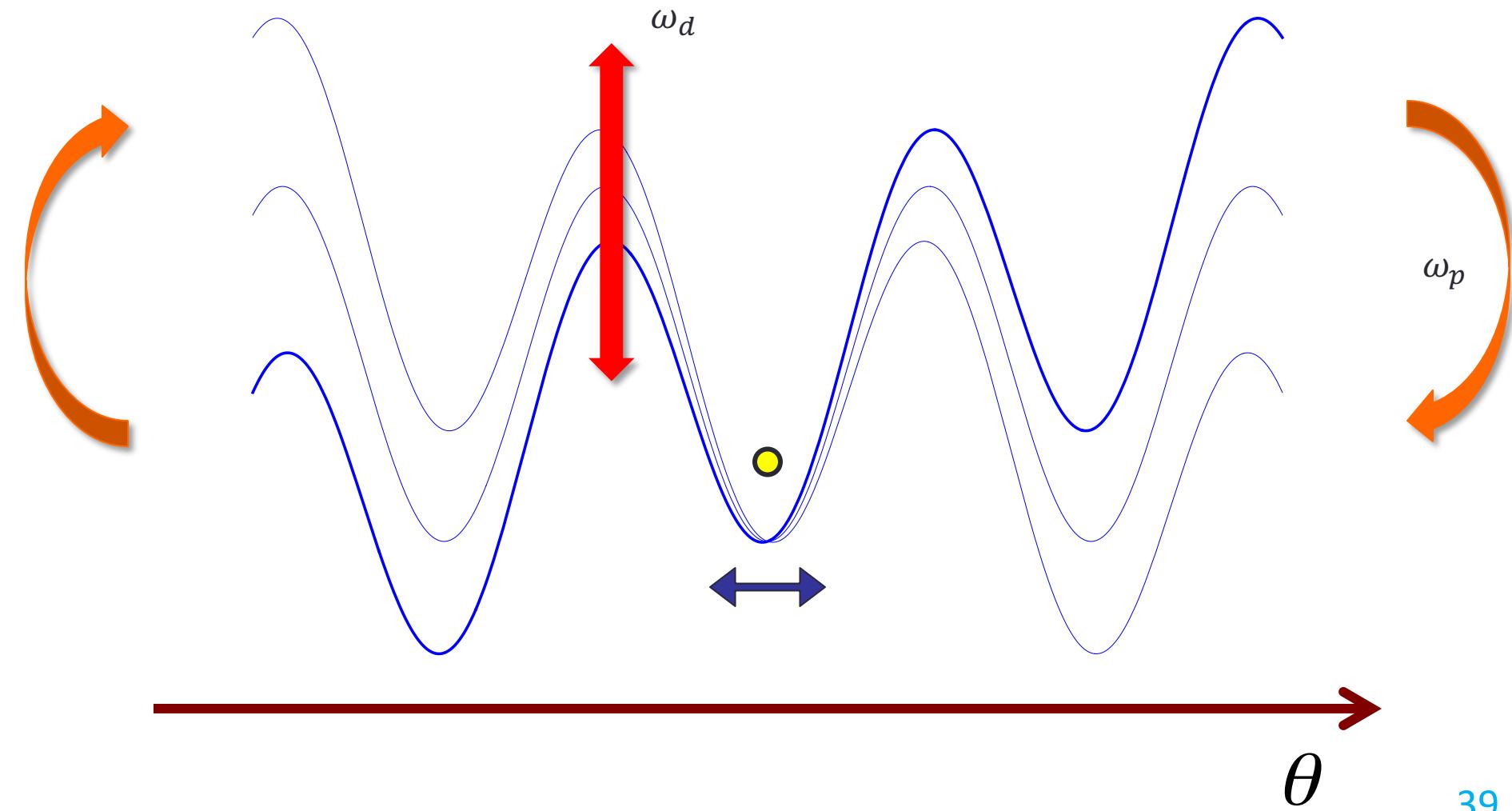


# Phonon-induced modulation



# Parametrically modulated washboard potential

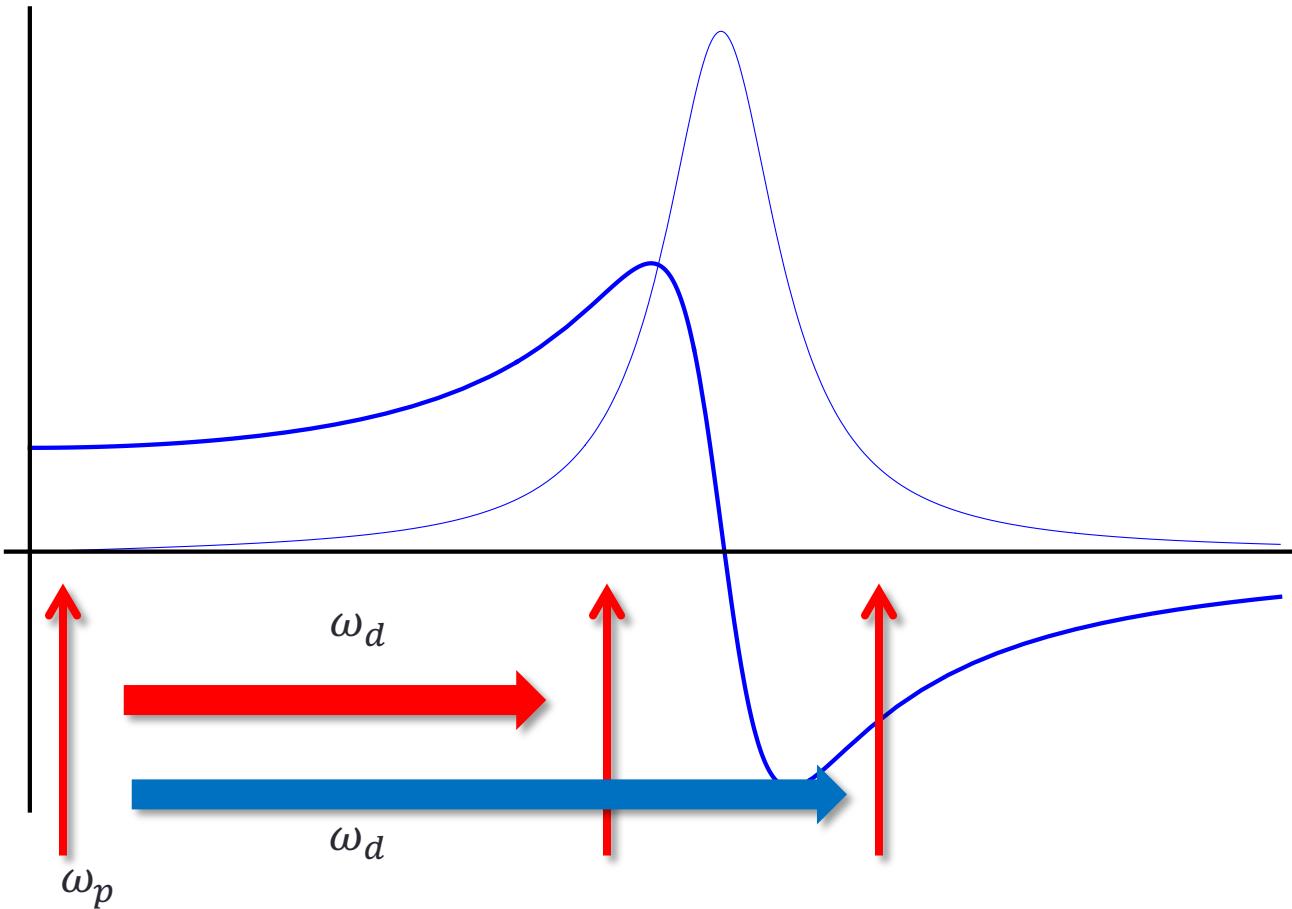
$$\omega_0^2(1 + A \cos \omega_d t)\theta + \gamma\dot{\theta} + \ddot{\theta} \propto I(t)$$



# First order correction



$$\theta_0 + A\theta_1 + A^2\theta_2$$

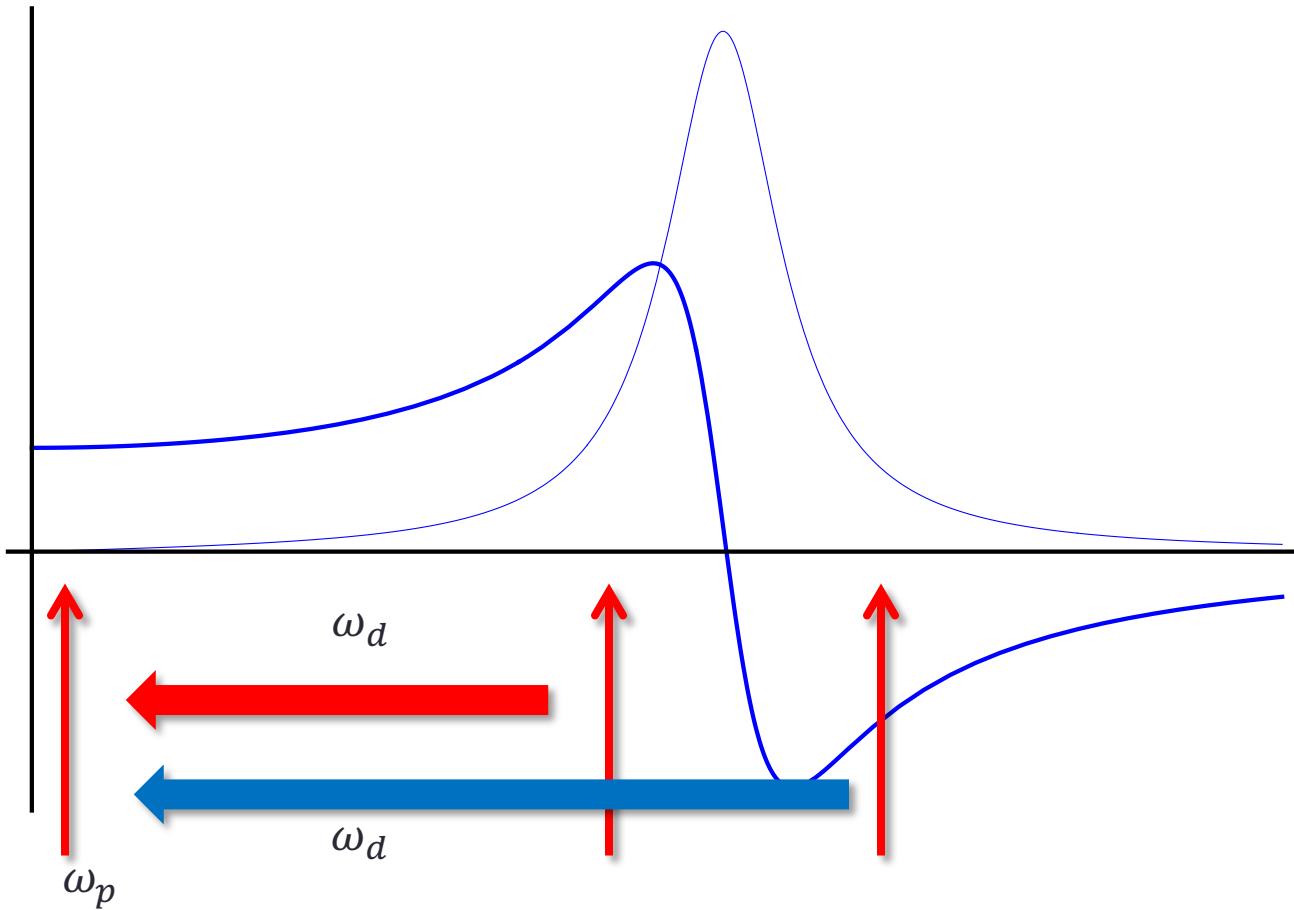


$$\theta_1(t) \propto -e^{-i\omega t} \cos \omega_d t \frac{\omega_0^2 - \omega_d^2}{(\omega_0^2 - \omega_d^2)^2 - \gamma^2 \omega_d^2}$$

## Second order correction



$$\theta_0 + A\theta_1 + A^2\theta_2$$

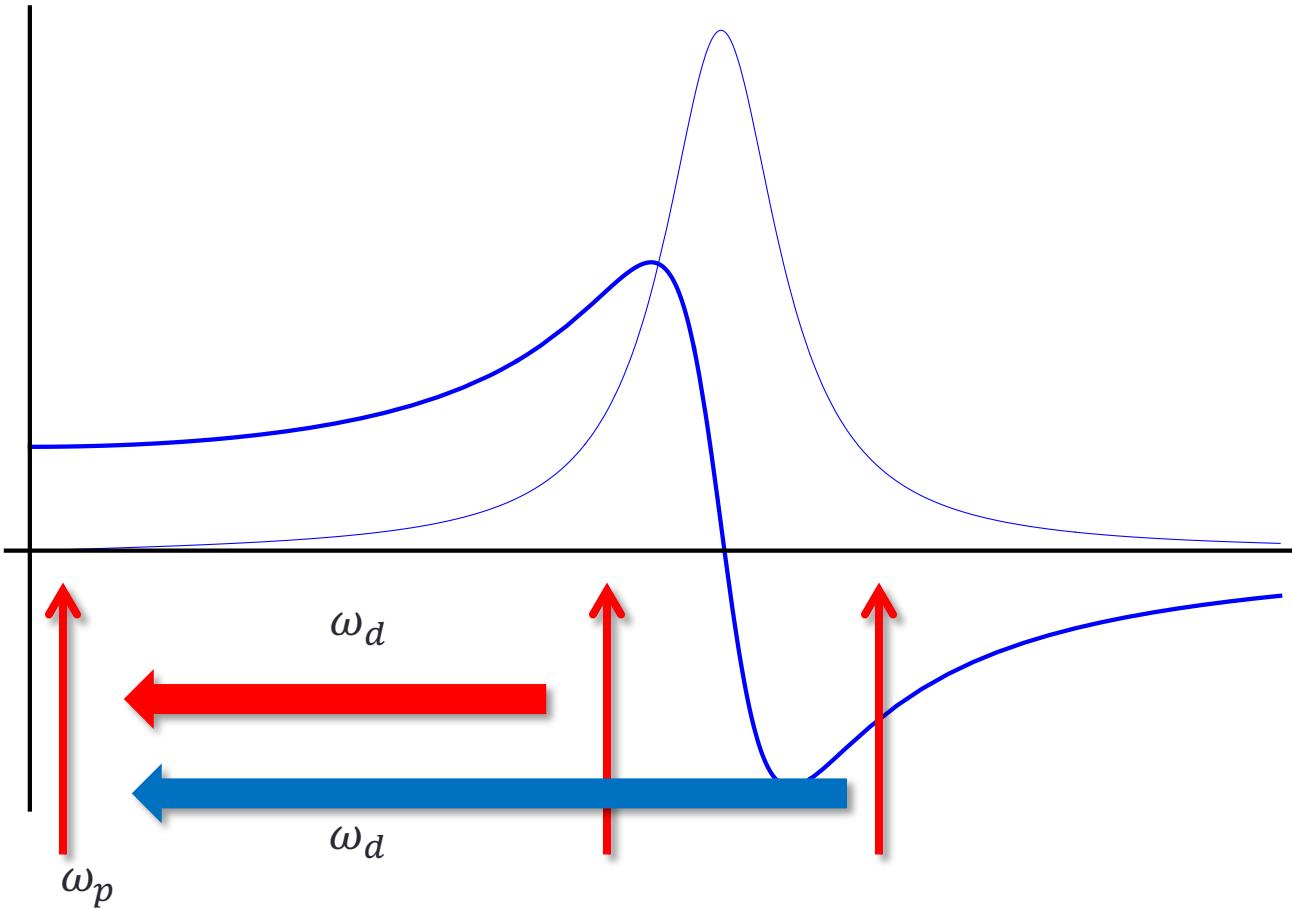


$$\theta_2(t) \propto e^{-i\omega t} (\cos \omega_d t)^2 \frac{\omega_0^2 - \omega_d^2}{(\omega_0^2 - \omega_d^2)^2 + \gamma^2 \omega_d^2}$$

## Second order correction



$$\theta_0 + A\theta_1 + A^2\theta_2$$

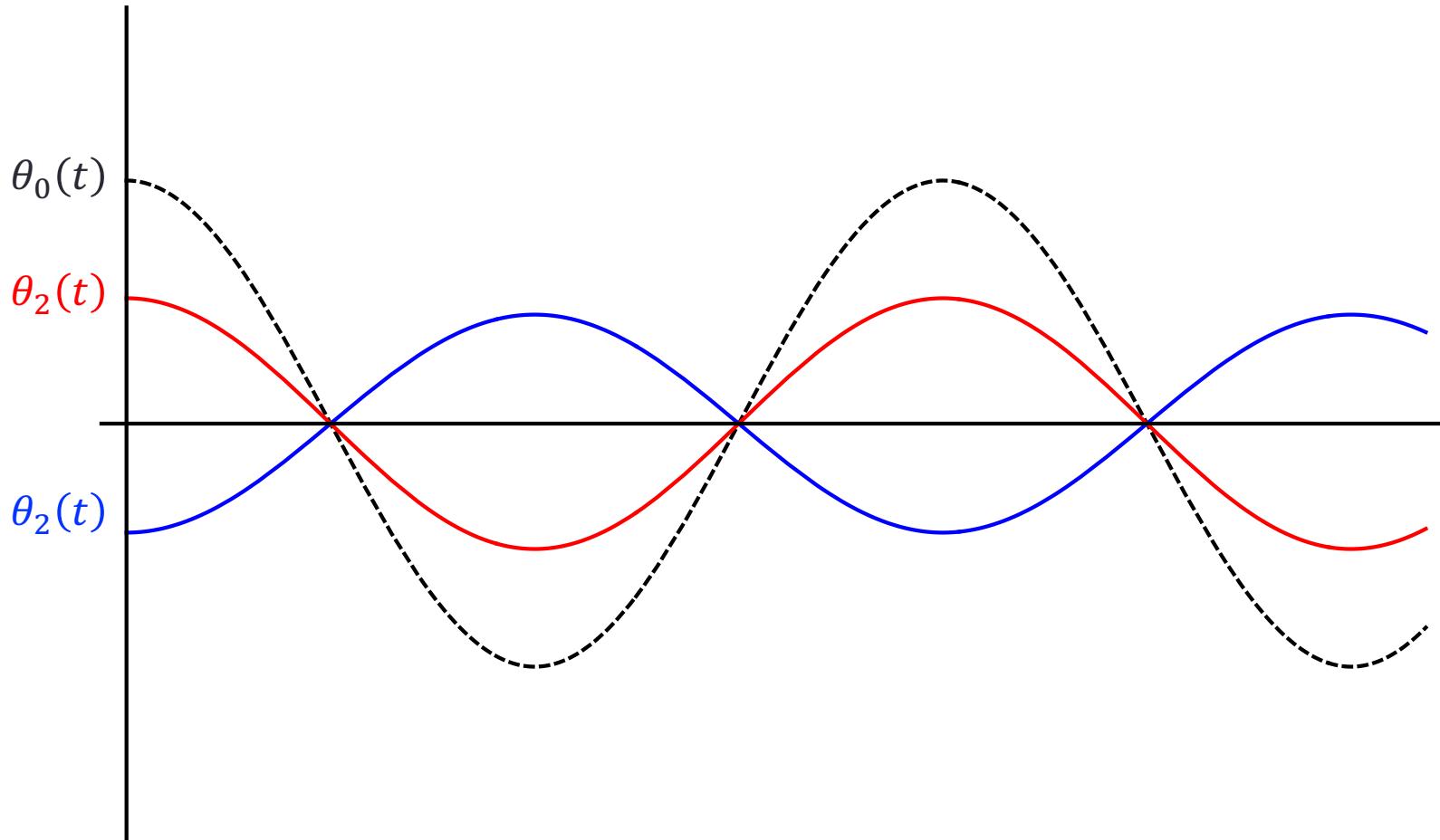


$$\theta_2(t) \propto e^{-i\omega t} \frac{1}{2} \frac{\omega_0^2 - \omega_d^2}{(\omega_0^2 - \omega_d^2)^2 + \gamma^2 \omega_d^2}$$

## Second order correction



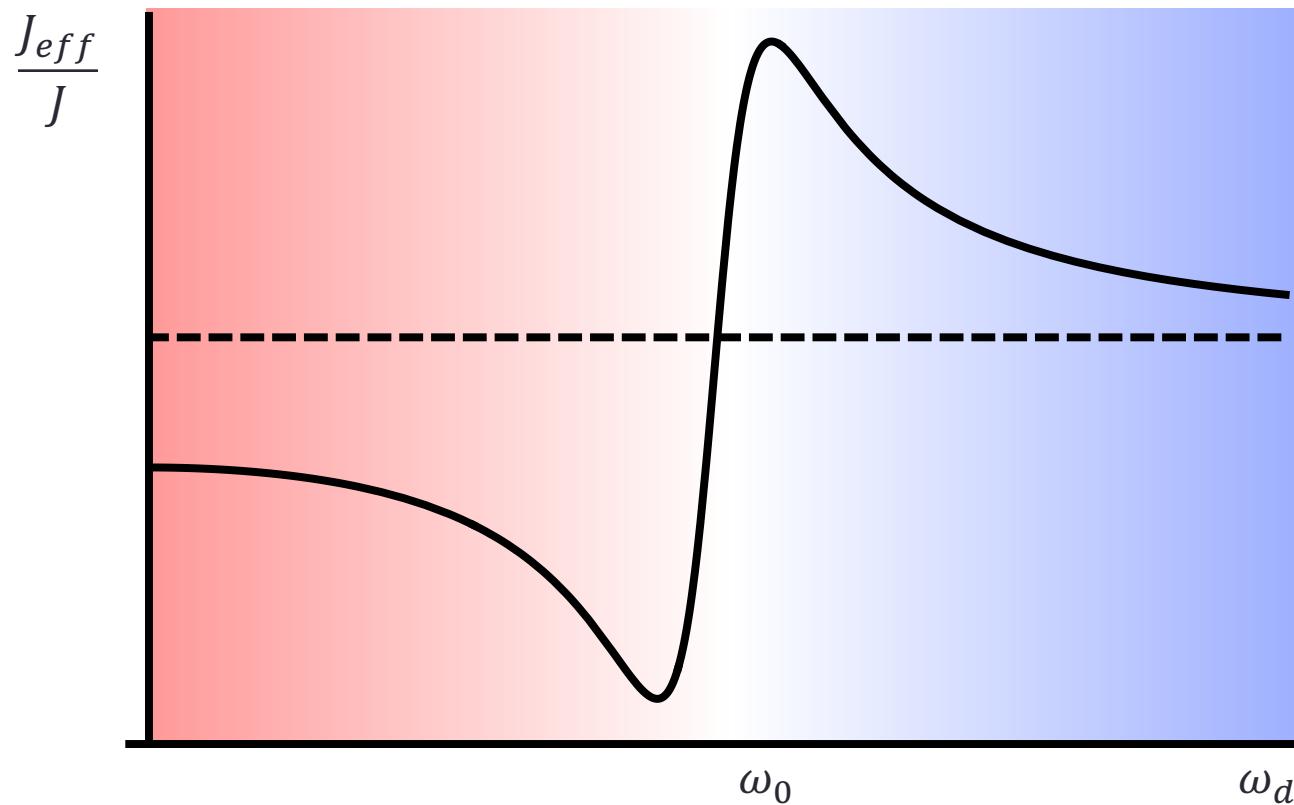
$$\theta_0 + A\theta_1 + A^2\theta_2$$



Red detuning: destabilization

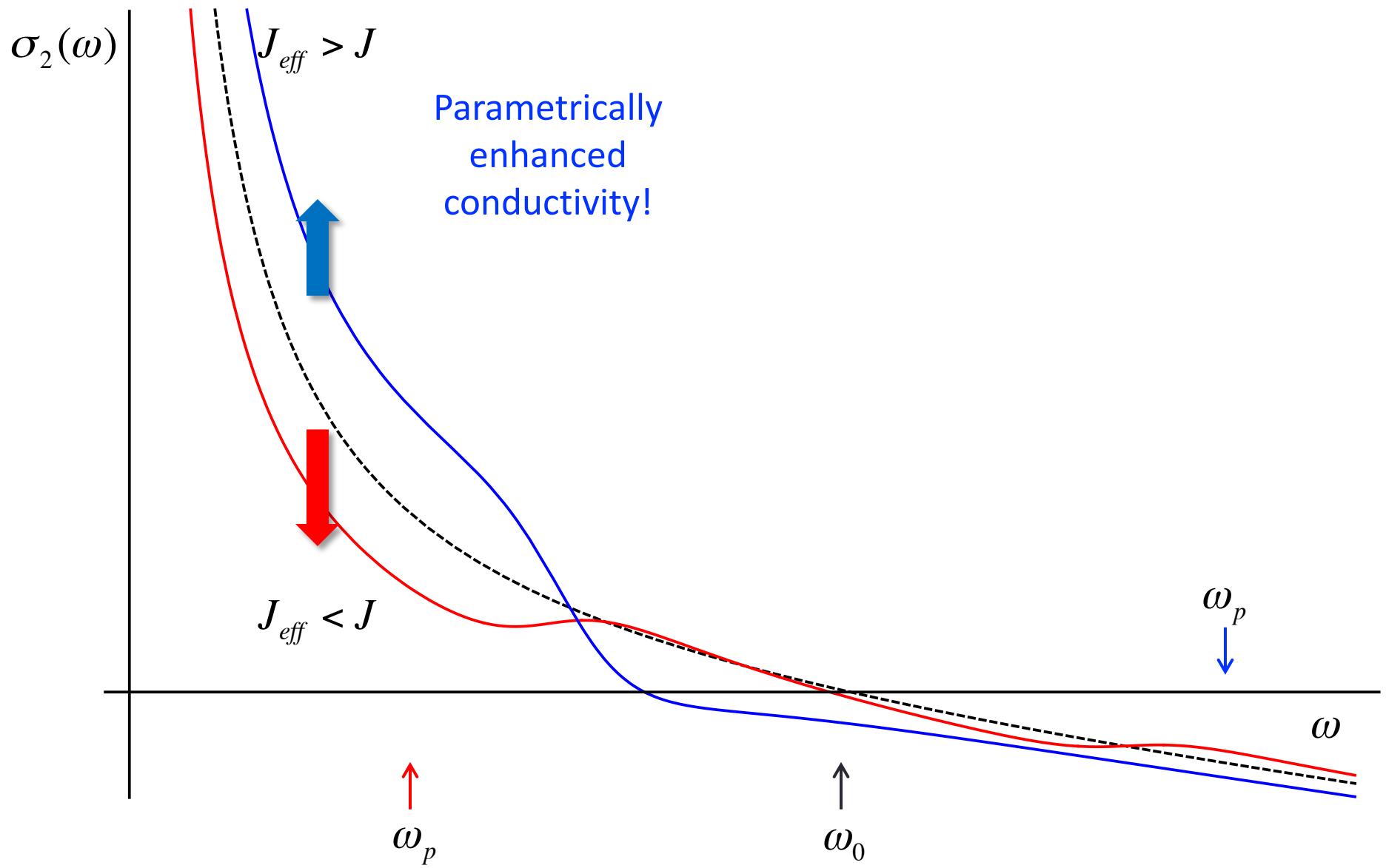
Blue detuning: stabilization

# Effective Josephson energy

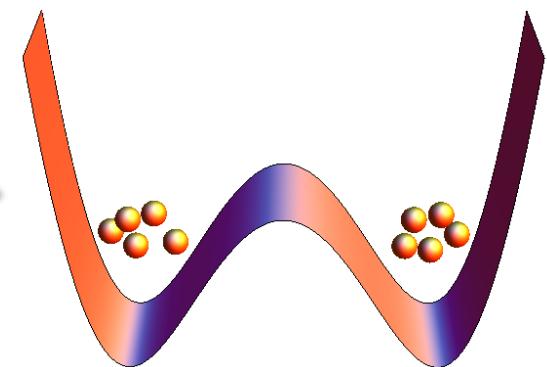
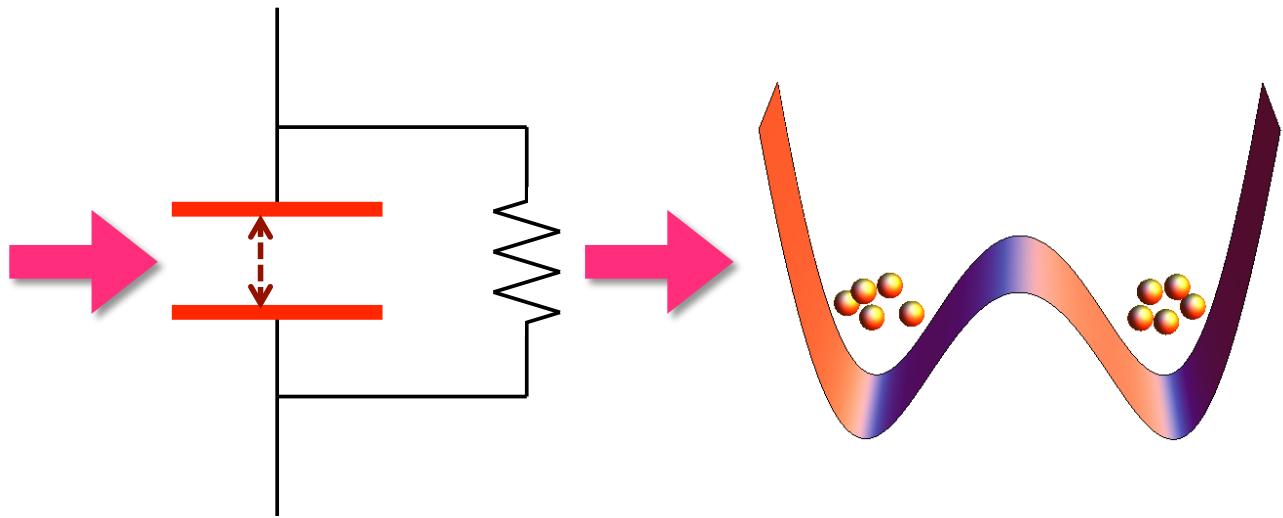
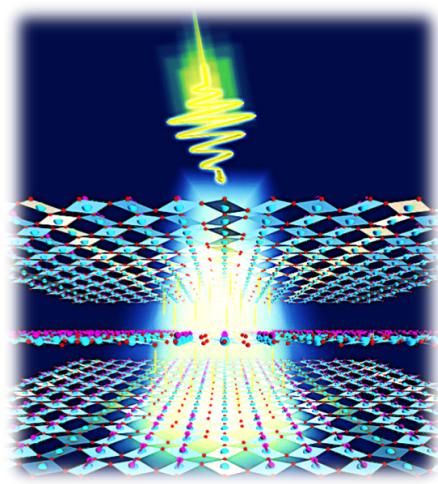


$$\frac{J_{eff}}{J} \propto 1 - \frac{A^2}{2} \frac{\omega_0^2(\omega_0^2 - \omega_d^2)}{(\omega_0^2 - \omega_d^2)^2 - \gamma^2 \omega_d^2}$$

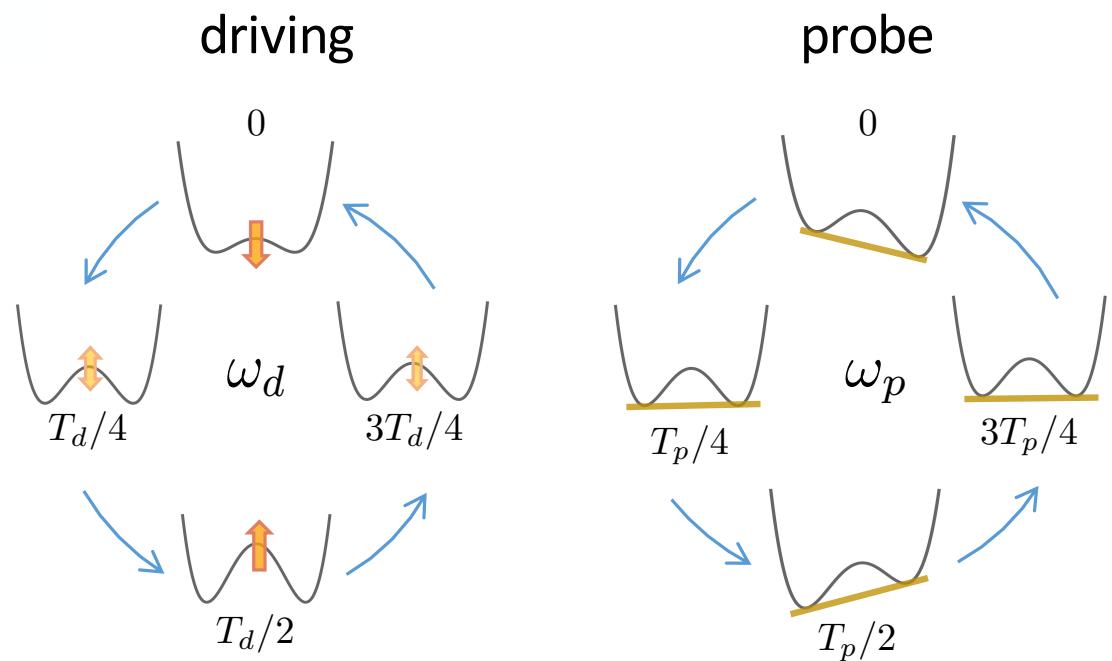
# Conductivity of the driven state



# Atomic Josephson junction

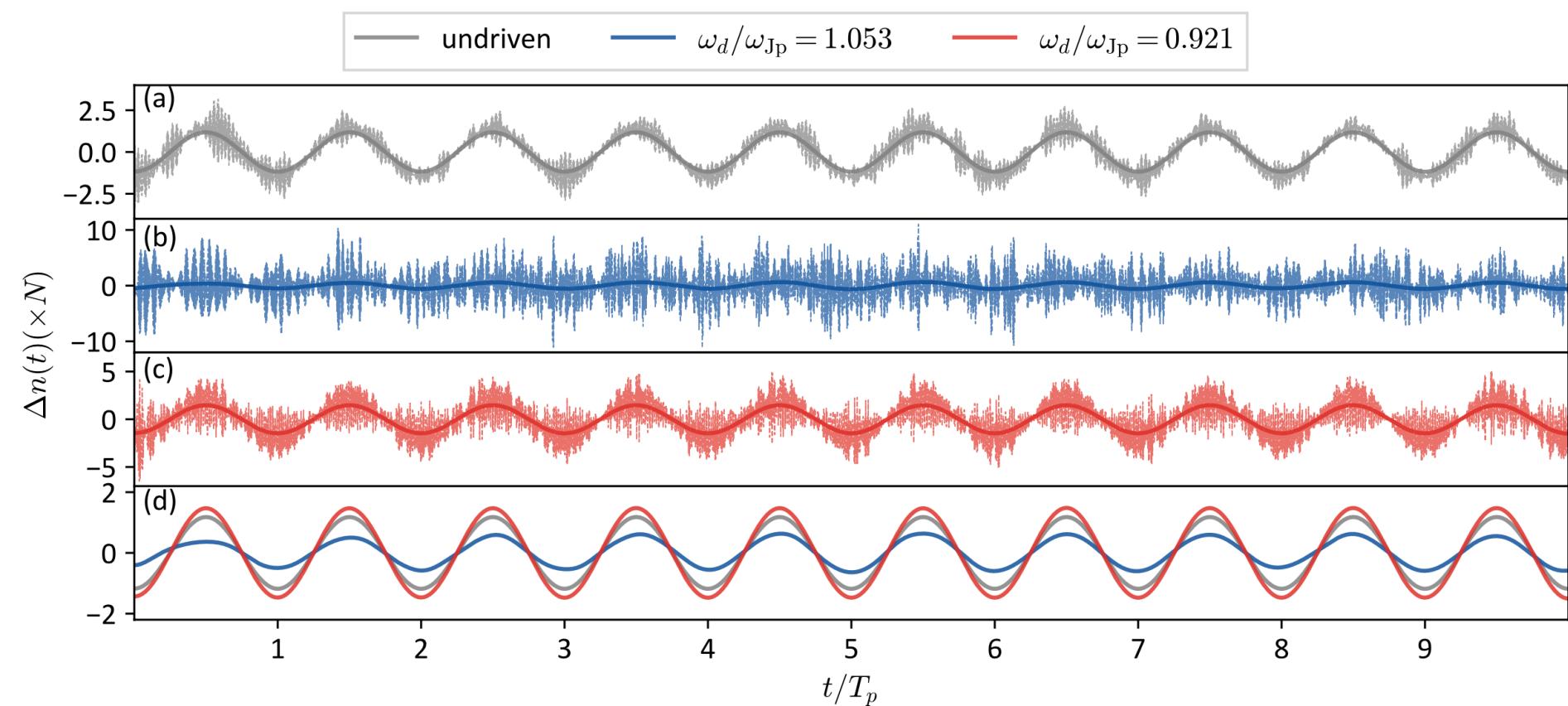


# Atomic Josephson junction



Condensates in a  
double-well

# Density response



## Neutral particles

$$-\dot{V}(t) \sim \omega_J^2 \theta + \gamma \dot{\theta} + \ddot{\theta}$$



$$\sigma^n(\omega) = \frac{j(\omega)}{V(\omega)}$$



$$\sigma^n(\omega) \sim \frac{i\omega}{\omega^2 - \omega_J^2 + i\gamma\omega}$$

## Charged particles

$$I(t) \sim J n_0 \theta + \frac{\hbar}{qR} \dot{\theta} + \frac{\hbar C}{q} \ddot{\theta}$$



$$\sigma^c(\omega) = \frac{j(\omega)}{E(\omega)}$$

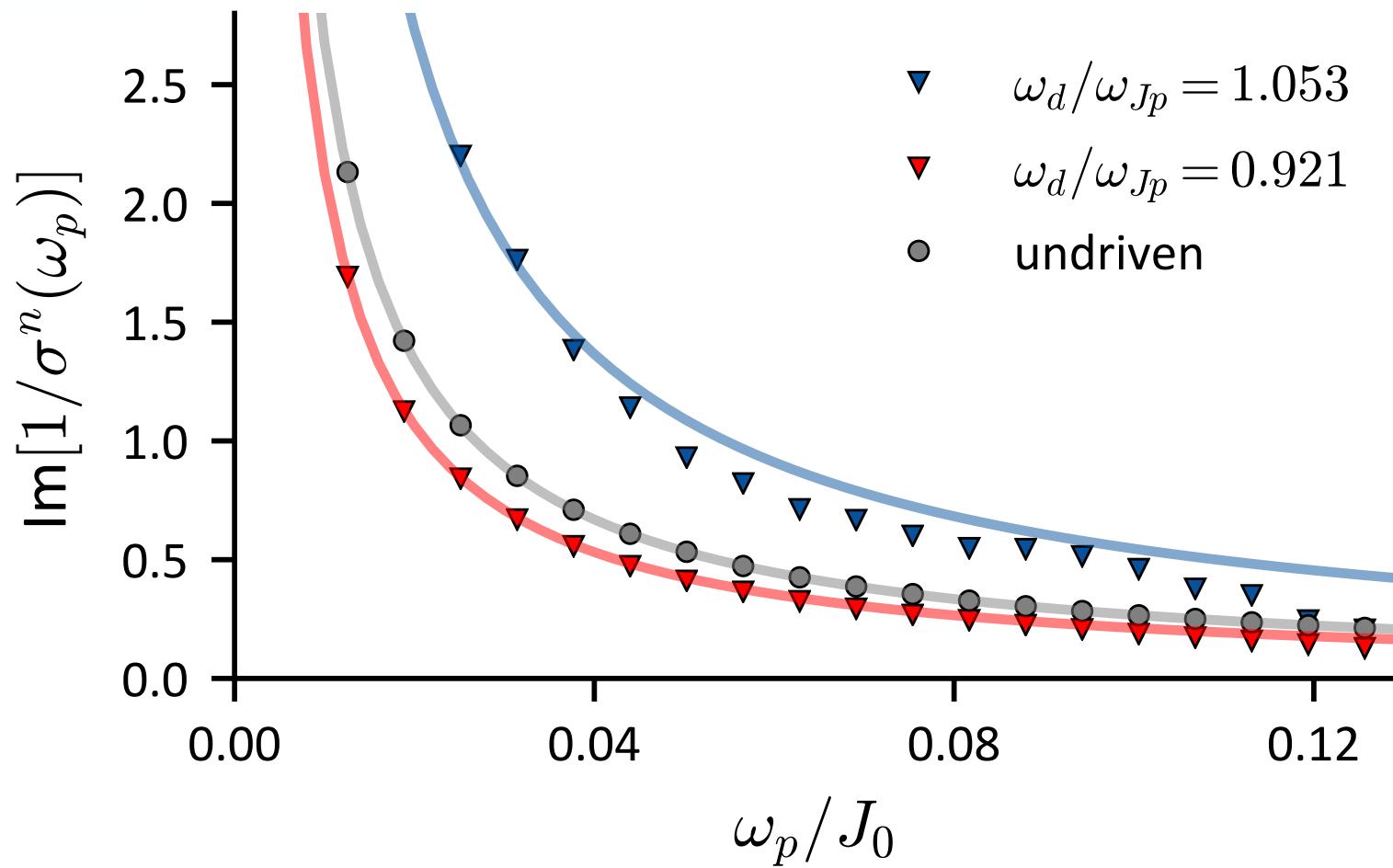


$$\sigma^c(\omega) \sim \frac{\omega^2 - \omega_J^2 + i\gamma\omega}{i\omega}$$



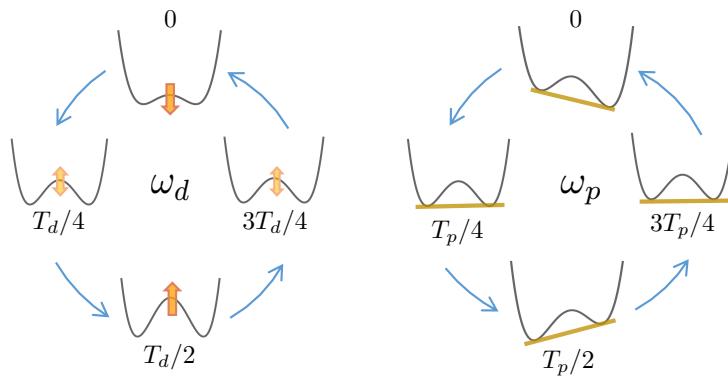
$$\frac{1}{\sigma^n(\omega)} \sim \sigma^c(\omega)$$

# Conductivity

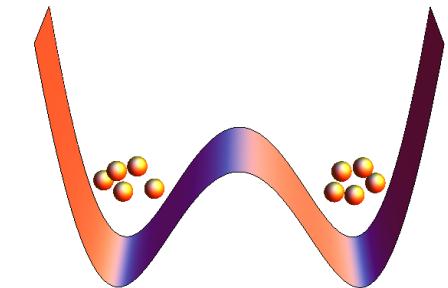


# Conclusions on atomic Josephson junctions

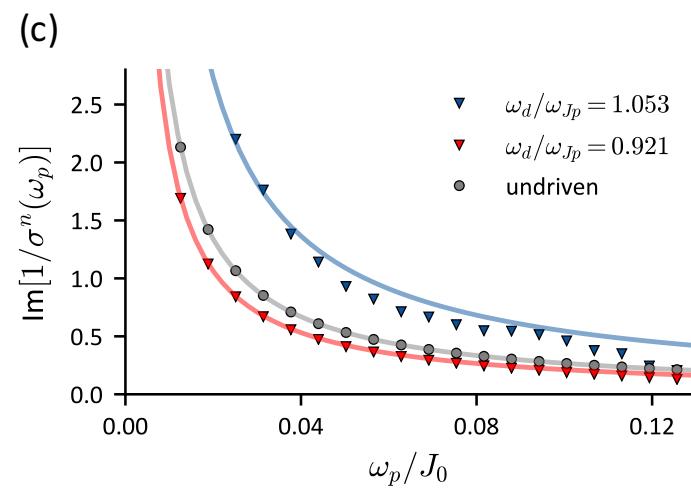
Atomic junction of coupled condensates



Parametric control  
of conductivity



Implementation of pump  
and probe mechanism

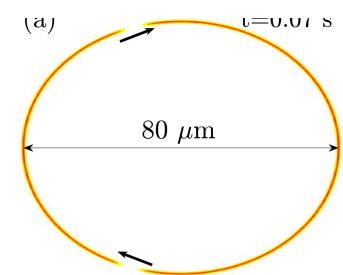


# Overview

Implementation of an atomtronic SQUID in a strongly Confined toroidal condensate

*Hannes Kiehn, Vijay Singh, LM*

arXiv:2204.03000



Dynamical control of an atomic Josephson junction

*Beilei Zhu, Vijay Singh, Junichi Okamoto, LM*

PRR 3, 013111, '21

