Nonequilibrium states in attractive 1D Bose gases

Lorenzo Piroli
École Normale Supérieure

Benasque
May 3, 2022

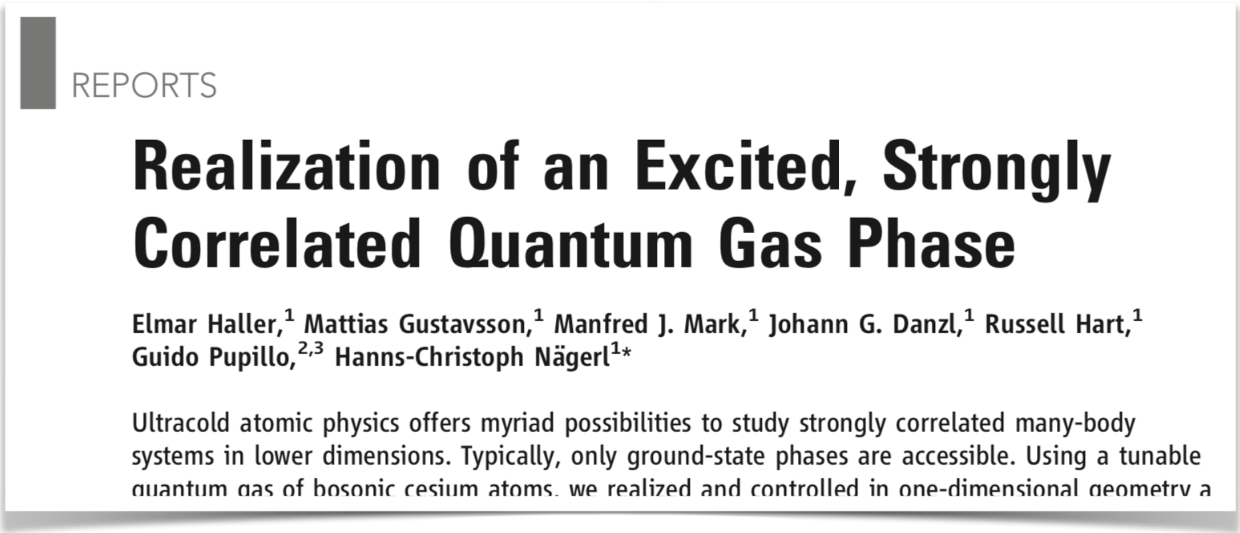
- L. Piroli, P. Calabrese, and F. H. L. Essler,
  Quantum Quenches to the Attractive One-Dimensional Bose Gas: Exact Results,
  SciPost Phys. **1**, 001 (2016)

- L. Piroli, P. Calabrese, and F. H. L. Essler,
  Multiparticle Bound-State Formation Following a Quantum Quench to the One-Dimensional Bose Gas with Attractive Interactions,
  Phys. Rev. Lett. **116**, 070408 (2016)

See also

P. Calabrese, F. H. L. Essler, and G. Mussardo,
Introduction to ‘Quantum Integrability in Out of Equilibrium Systems,
J. Stat. Mech. 064001 (2016)

A. Bastianello, B. Bertini, B. Doyon, and R. Vasseur,
Introduction to the Special Issue on Emergent Hydrodynamics in Integrable Many-Body Systems,
J. Stat. Mech. 2022, 014001 (2022)

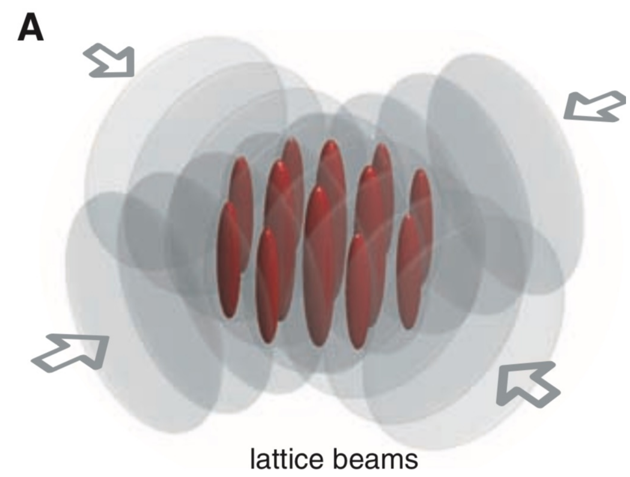


REPORTS

Realization of an Excited, Strongly Correlated Quantum Gas Phase

Elmar Haller,[1] Mattias Gustavsson,[1] Manfred J. Mark,[1] Johann G. Danzl,[1] Russell Hart,[1] Guido Pupillo,[2,3] Hanns-Christoph Nägerl[1]*

Ultracold atomic physics offers myriad possibilities to study strongly correlated many-body systems in lower dimensions. Typically, only ground-state phases are accessible. Using a tunable quantum gas of bosonic cesium atoms, we realized and controlled in one-dimensional geometry a



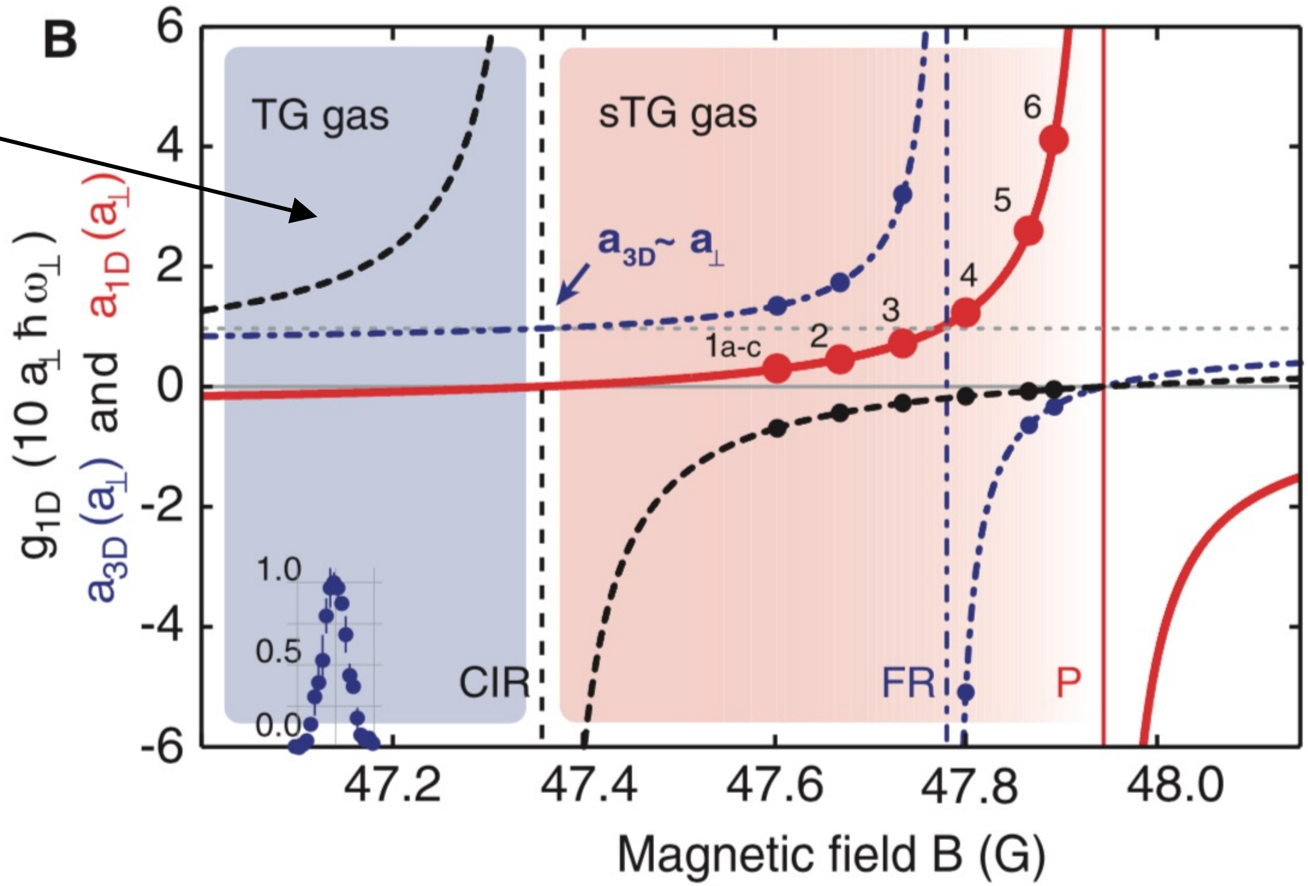
- Ultra-cold gas of of bosonic cesium atoms in 1D
- Model: point particles interacting via $U_{1\mathrm{D}}(z) = g_{1D}\delta(z)$
- Interaction is varied by tuning the scattering length

$$g_{1\mathrm{D}} = -\frac{2\hbar^2}{m a_{1\mathrm{D}}} = \frac{2\hbar^2 a_{3\mathrm{D}}}{m a_\perp^2} \frac{1}{1 - C a_{3\mathrm{D}}/a_\perp}$$

Olshanii, PRL **81**, 938 (1998)

Haller, Gustavsson, Mark, Danzl, Hart, Pupillo, Nägerl, Science 325 1224 (2009)

$g_{1\mathrm{D}}$



- $a_{3\mathrm{D}}$ varied across $1 - Ca_{3D}/a_\perp$

$$\Rightarrow g_{1\mathrm{D}} = +\infty \rightarrow g_{1\mathrm{D}} = -\infty$$

- super-Tonks Girardeau $\Longrightarrow$ non-thermal stationary state

Astrakharchik et al. PRL **95** (2005)

Quantum quench

- Initial state $|\psi\rangle$ ground-state of

$$H = \int_0^L \mathrm{d}x \left[\partial_x \Psi_x^\dagger \partial_x \Psi_x + c\Psi_x^\dagger \Psi_x^\dagger \Psi_x \Psi_x + V(x)\Psi_x^\dagger \Psi_x\right]$$

$$[\Psi_x^\dagger, \Psi_y] = \delta(x-y)$$

- Sudden variation $c \to c'$

- Problem: characterize post-quench equilibrium state

Quantum quench

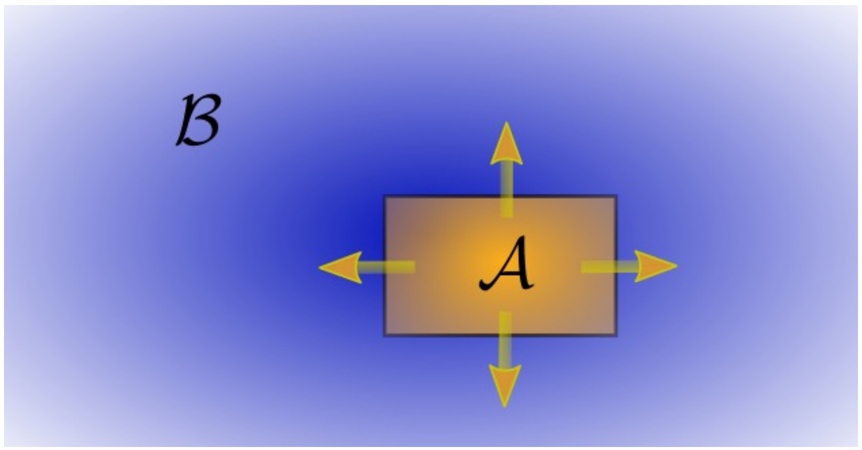
- Initial state $|\psi\rangle$ ground-state of

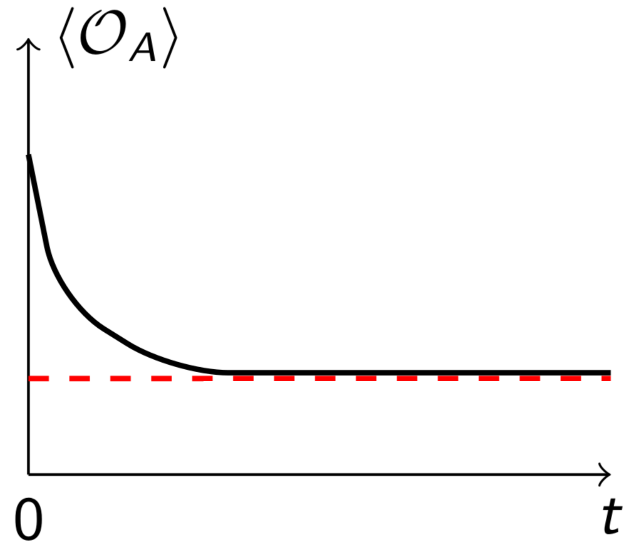
$$H = \int_0^L \mathrm{d}x \left[\partial_x \Psi_x^\dagger \partial_x \Psi_x + c\Psi_x^\dagger \Psi_x^\dagger \Psi_x \Psi_x + V(x)\Psi_x^\dagger \Psi_x\right]$$

- Sudden variation $c \to c'$

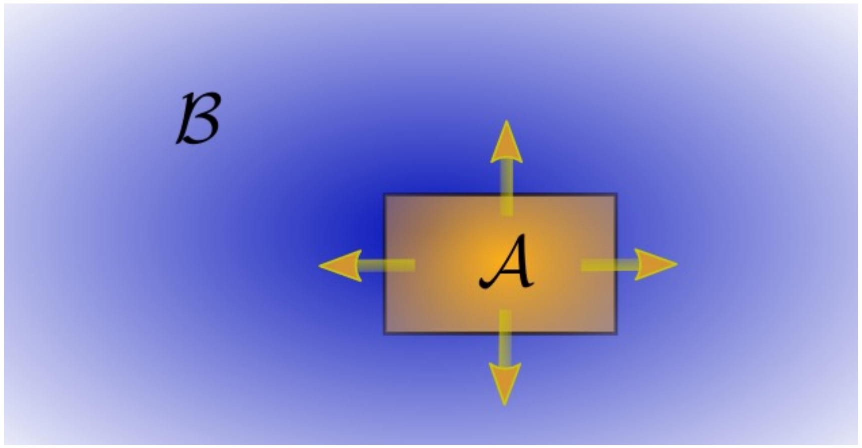
- Problem: characterize post-quench equilibrium state
- Idealized setting:
  - no trapping potential $V(x) \equiv 0$
  - isolated system
  - thermodynamic limit $L \to \infty$,
    $D = N/L$, $\gamma = c/D$ const.

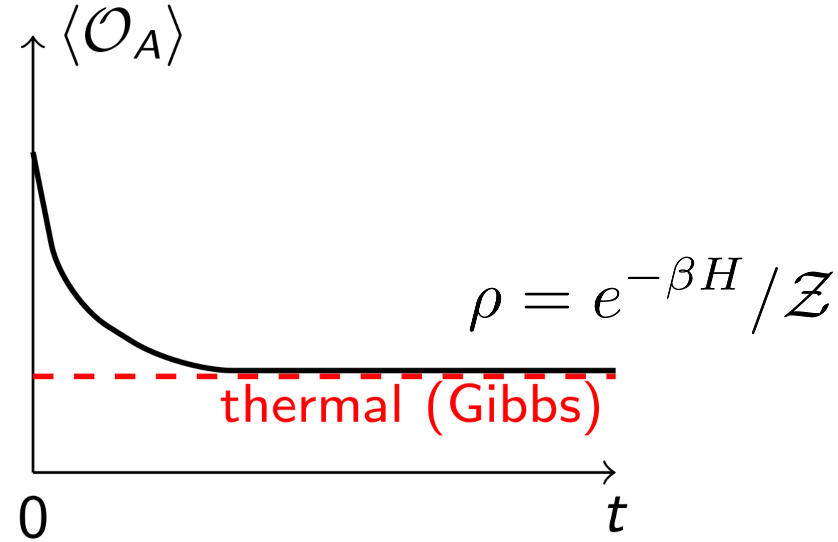
- System globally in a pure state, but relaxation possible locally





- System globally in a pure state, but relaxation possible locally





- Typical behavior: thermalization [Rigol et al. Nature 452 (2008)]

- However, exceptions exist in 1D!
- Ex: integrable systems

- Example:

$$H = \int_0^L \mathrm{d}x \left[\partial_x \Psi_x^\dagger \partial_x \Psi_x + c \Psi_x^\dagger \Psi_x^\dagger \Psi_x \Psi_x\right]$$

- Extensive number of integral of motions $[H, I_j] = 0$

- Int. of motions protect from thermalization (analogy with classical mechanics)

- Local relaxation to "Generalized Gibbs Ensembles"

$$\rho = \frac{e^{-\sum_j \beta_j I_j}}{\mathcal{Z}}$$

Cassidy, Clark, Rigol, PRL **106** (2011)
Langen et al. Science **348** (2015)

- GGE novel “non-equilibrium phases of matter”

- Non-thermal features (e.g. super-Tonks-Girardeau gas)

- How to characterize them?
  - Non-perturbative regime
  - Away from mean-field limit

- Past ten years:
  - systematic understanding of GGE physics based on Bethe Ansatz
  - new “generalized hydrodynamic” theory of integrable models

---

Calabrese, Essler, Mussardo, J. Stat. Mech. 064001 (2016)
Bastianello, Bertini, Doyon, Vasseur, J. Stat. Mech. 014001 (2022)

$$H = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{j>k} \delta\left(x_j - x_k\right)$$

$$\boxed{c > 0}$$

- Bethe Ansatz: exact wave-function

$$\Psi\left(x_1, \ldots, x_N | k_1, \ldots, k_N\right) = \sum_{\mathcal{P}} A_{\mathcal{P}}(x, k) e^{\mathrm{i} \sum_j k_{\mathcal{P}_j} x_j}$$

- Stable quasi-particles, quasi-momenta $\{k_j\}_j$
- Thermodynamic states defined by quasi-momenta distribution

$$\rho_T(k) \propto \frac{1}{1 + e^{\varepsilon(k)/K_B T}}$$

"dressed energy"

- Characterize the GGE = compute quasi-momenta distribution

$$\langle \Psi(t) | \mathcal{O} | \Psi(t) \rangle = \sum_{n,m} \langle E_n | \mathcal{O} | E_m \rangle \langle E_m \mid \Psi_0 \rangle \langle \Psi_0 \mid E_n \rangle \, e^{i(E_n - E_m)t}$$

- Characterize the GGE = compute quasi-momenta distribution

$$\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle=\sum_{n,m}\langle E_n|\mathcal{O}|E_m\rangle\,\langle E_m\mid\Psi_0\rangle\,\langle\Psi_0\mid E_n\rangle\,e^{i(E_n-E_m)t}$$

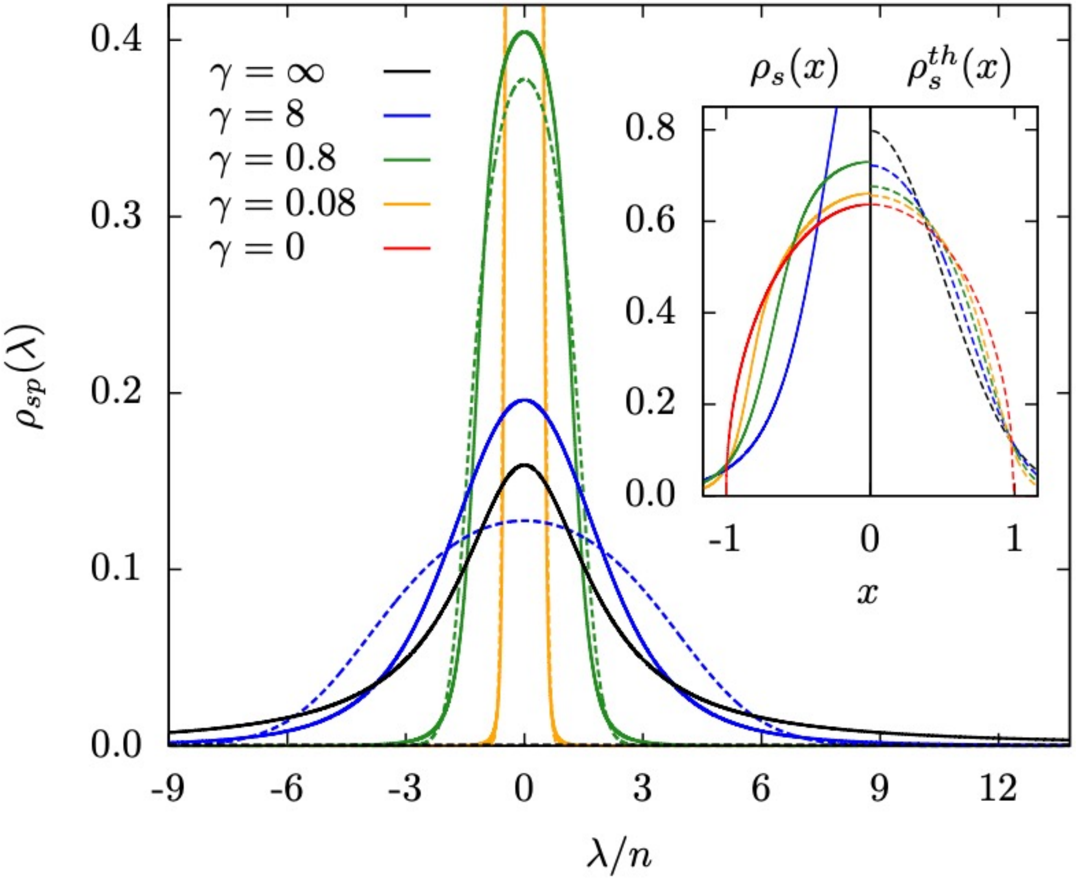
- *Quench action approach*: cast sum into functional integral
  [Caux, Essler, PRL 110, 257203 (2013)]

$\Longrightarrow$ exact GGE quasi-momenta distribution functions

- First example:
  interaction quenches in 1D repulsive and attractive Bose gas
  - De Nardis, Wouters, Brockmann, Caux, PRA 89 (2014)
  - Piroli, Calabrese, Essler, PRL 116 (2016)

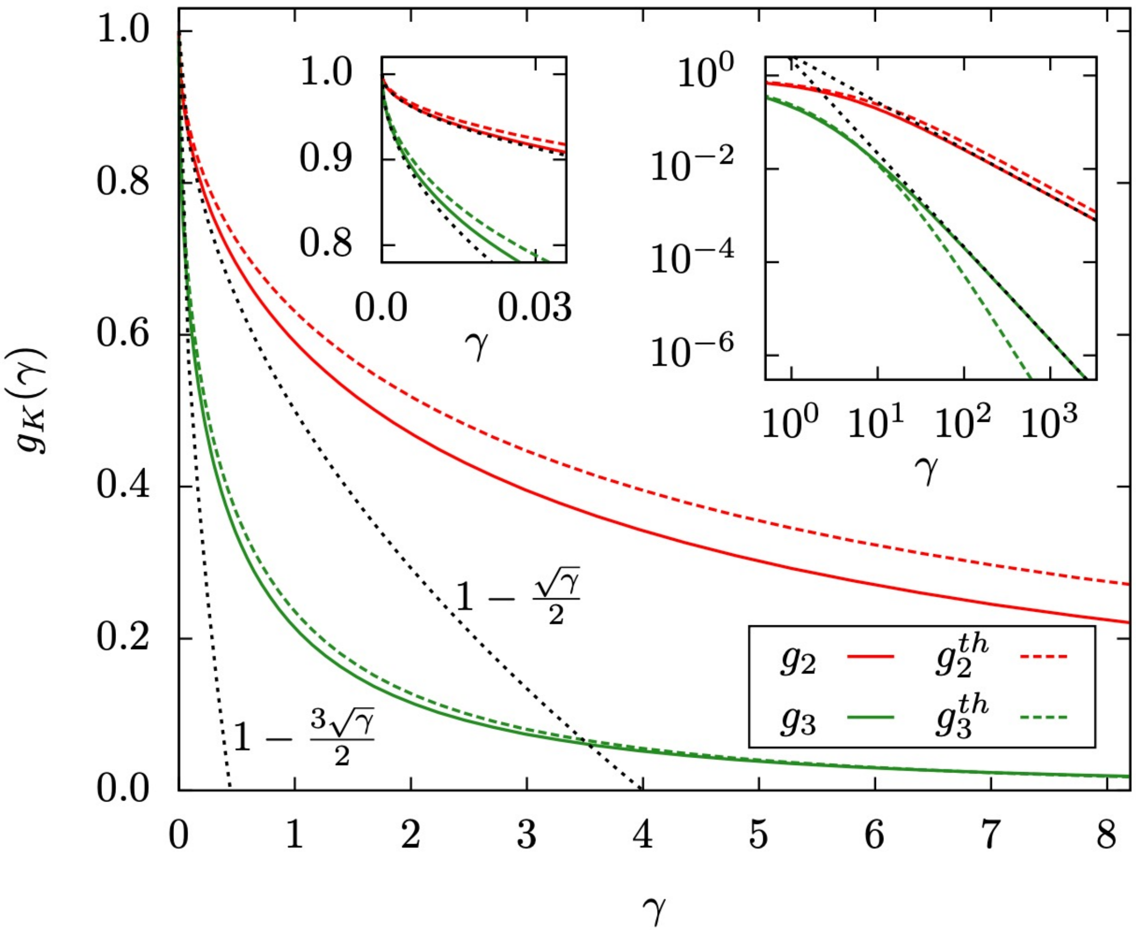
See also alternative approach by Natan Andrei and collaborators

- Interaction quench $c = 0 \rightarrow c > 0$
- Initial wave function $\psi(x_1, \ldots, x_N) = 1/\sqrt{L^N}$



De Nardis, Wouters, Brockmann, Caux, PRA 89 (2014)

$$g_K = \frac{\left\langle \left[\Psi^\dagger(0)\right]^K \Psi^K(0) \right\rangle}{D^K}$$



- GGE states particularly interesting for $c < 0$

- At equilibrium, GP regime $c = -\kappa/N^2$

- For c fixed, thermodynamics ill-defined (clustering)

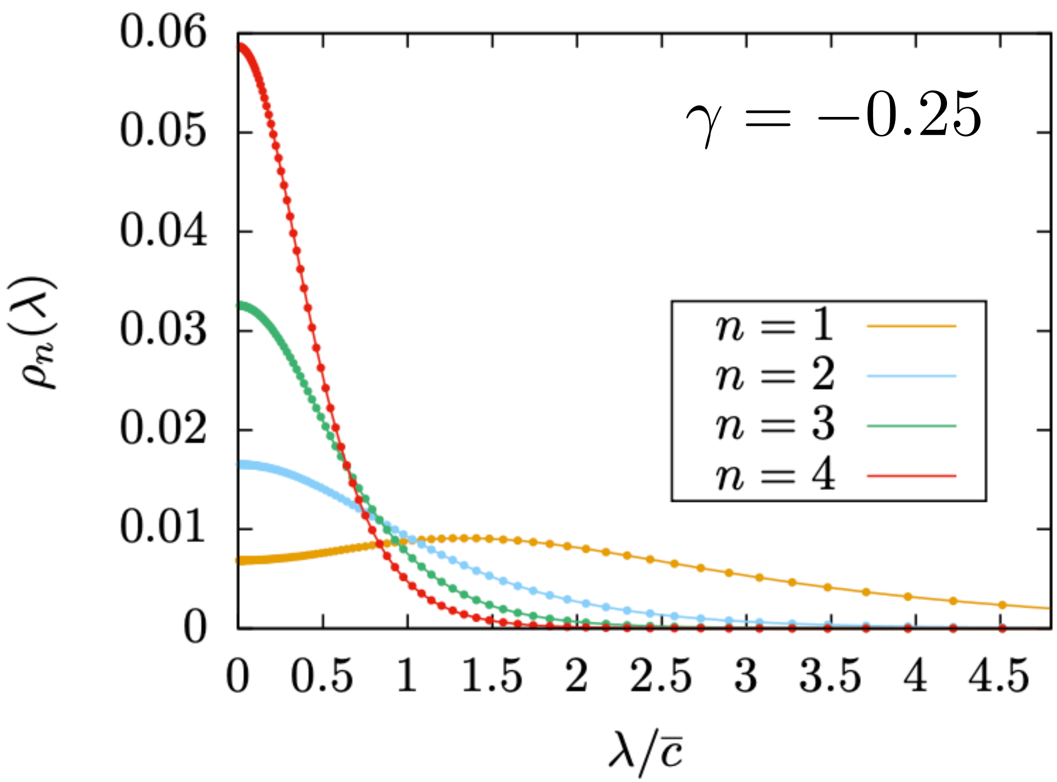
$$E_{\mathrm{GS}} \propto -|c|N^3$$

- Point of principle: Interaction quench $c = 0 \rightarrow c < 0$

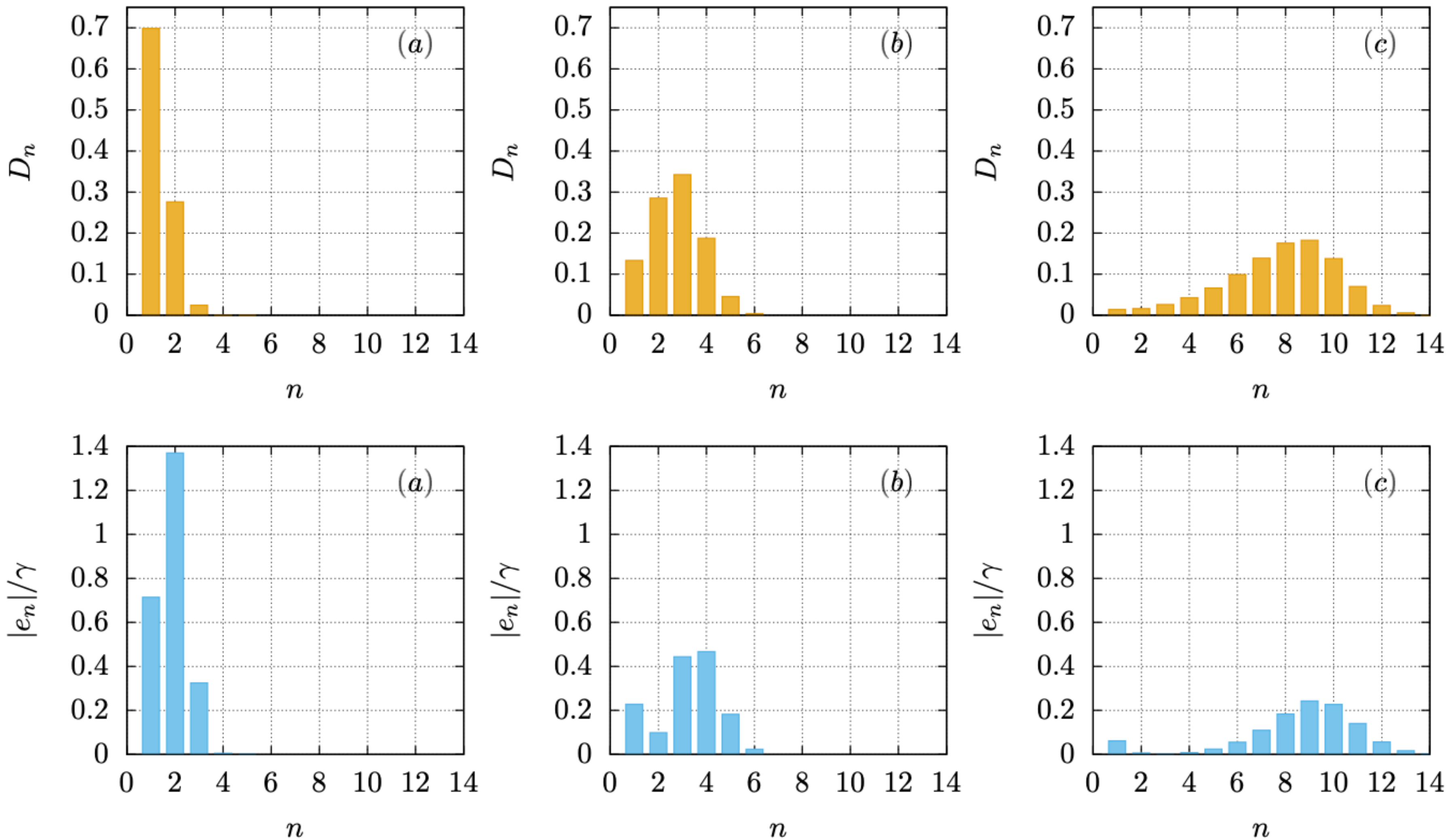
- GGE states stable. New phenomelogy beyond super-Tonks-Girardeau gas

Piroli, Calabrese, Essler, PRL 116 (2016)

- GGE characterized by non-trivial n-particle bound states $\rho_n(\lambda)$



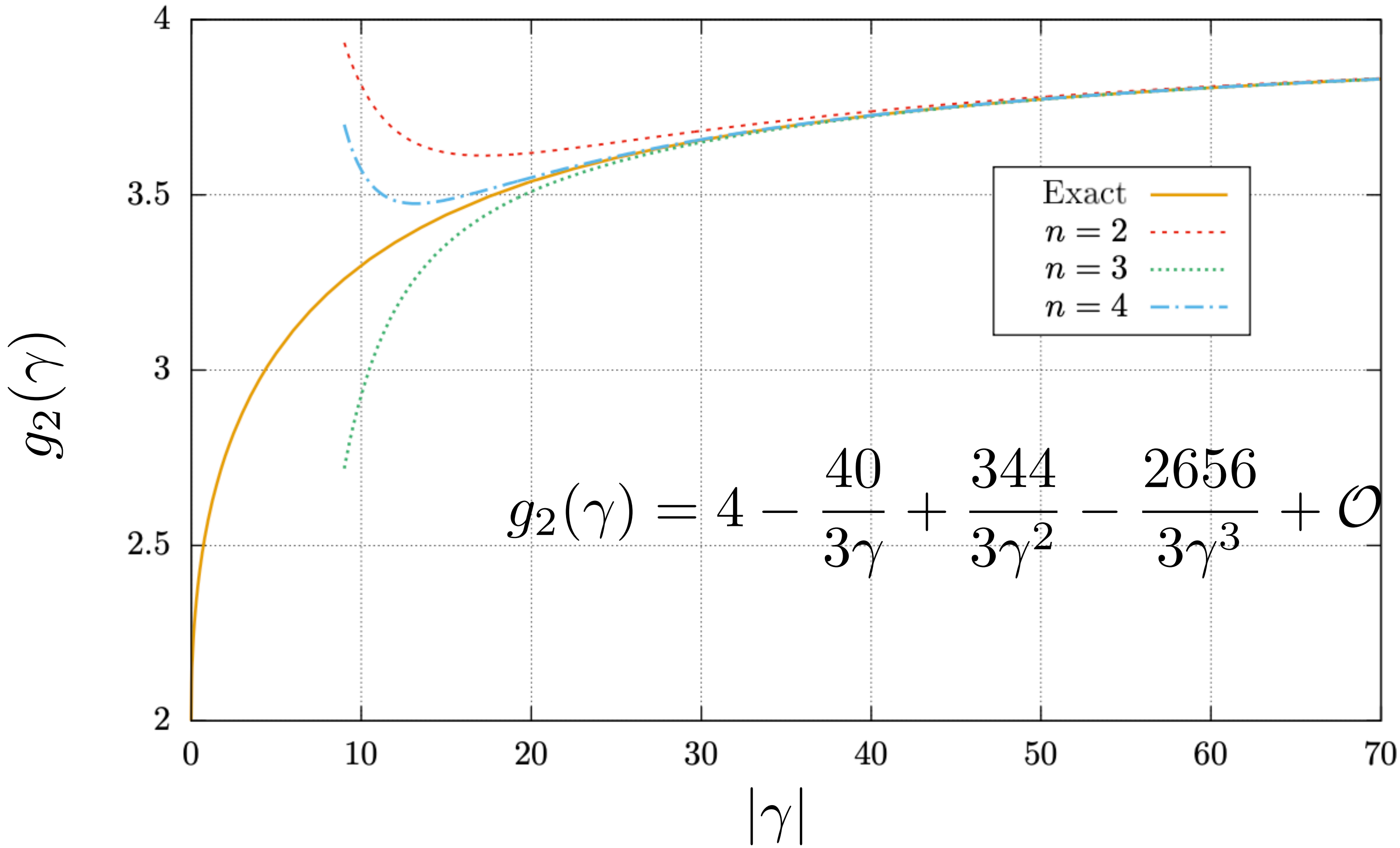
- Qualitatively different from super-Tonks-Girardeau (no bound-states)



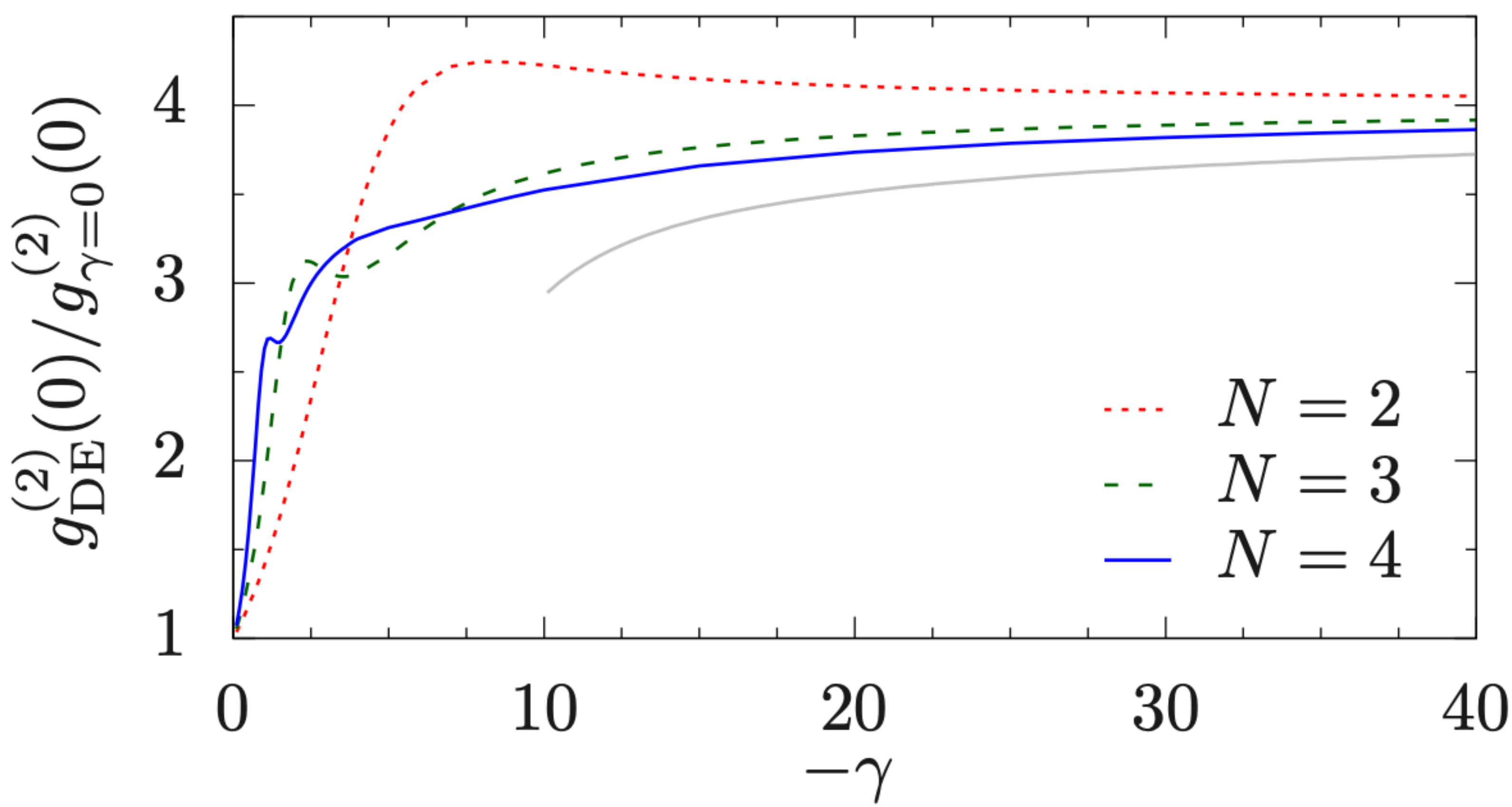
$\gamma = -20$ $\gamma = -2$ $\gamma = -0.2$

Pair correlation function

$$g_2 = \frac{\left\langle \left[\Psi^\dagger(0)\right]^2 \Psi^2(0) \right\rangle}{D^2}$$

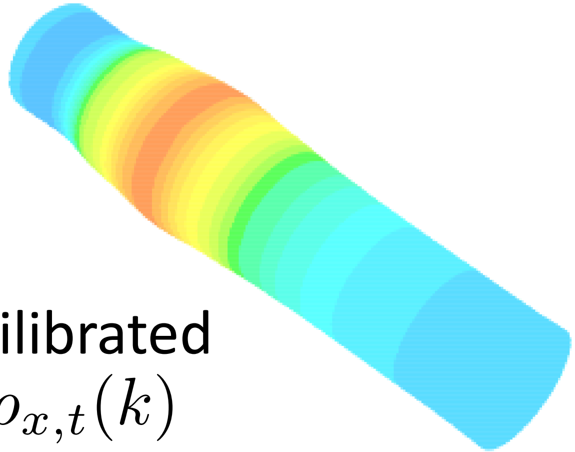


$$g_2(\gamma) = 4 - \frac{40}{3\gamma} + \frac{344}{3\gamma^2} - \frac{2656}{3\gamma^3} + \mathcal{O}\left(\gamma^{-4}\right)$$



Zill, Wright, Kheruntsyan, Gasenzer, Davis, SciPost Phys. 4 (2018)

Generalized hydrodynamics (2016)



- "LDA-like" approach:
  - system consists of "fluid cells", locally equilibrated
  - each cell state described by "GGE state" $\rho_{x,t}(k)$

- allows for spatial inhomogeneities at "hydrodynamic" scales:
  - trapping potential, finite systems
  - inhomogeneous densities and interactions

Castro-Alvaredo et al. Phys. Rev. X 6, 041065 (2016)
Bertini et al. Phys. Rev. Lett. 117, 207201 (2016)

Generalized hydrodynamics (2016)

- Fundamental continuity equation

$$\partial_t \rho_{x,t}(k) + \partial_x [v^{\text{eff}}_{x,t}(k) \rho_{x,t}(k)] = \left(\frac{\partial_x V}{m}\right) \partial_v \rho_{x,t}(k)$$

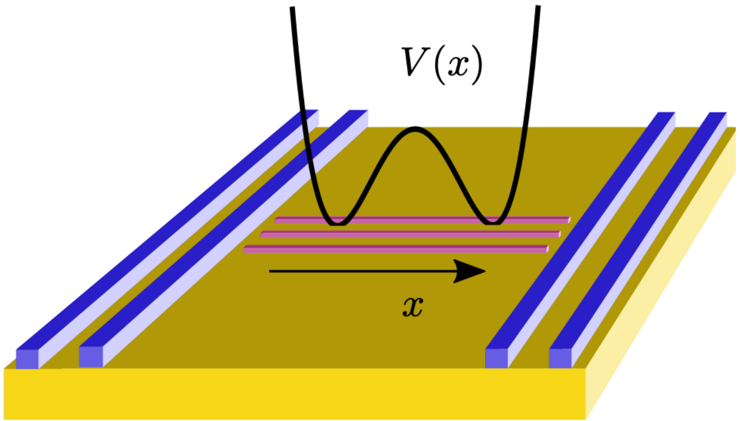
"Dressed velocity"

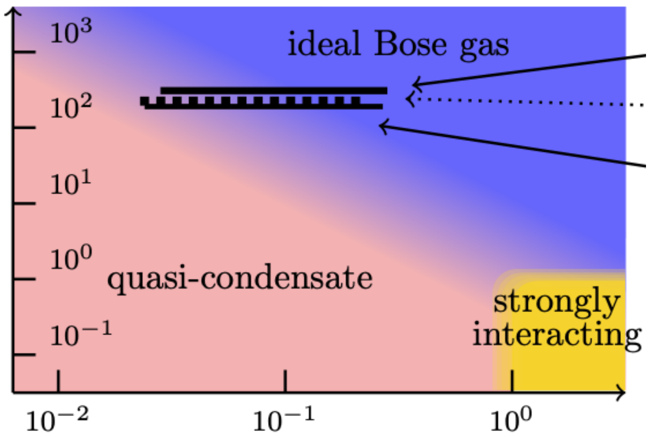
- Supersedes conventional hydrodynamic approaches

- Experimentally tested in recent cold-atom experiments:
  - Schemmer, Bouchoule, Doyon, Dubail, PRL **122** (2019)
  - Møller, Li, Mazets, Stimming, Zhou, Zhu, Chen, Schmiedmayer, PRL **126**, (2021)
  - Malvania, Zhang, Le, Dubail, Rigol, Weiss, Science **373** (2021)

GHD on an Atom Chip

$$\theta = 2\hbar^2 k_B T / \left(mc^2\right)$$





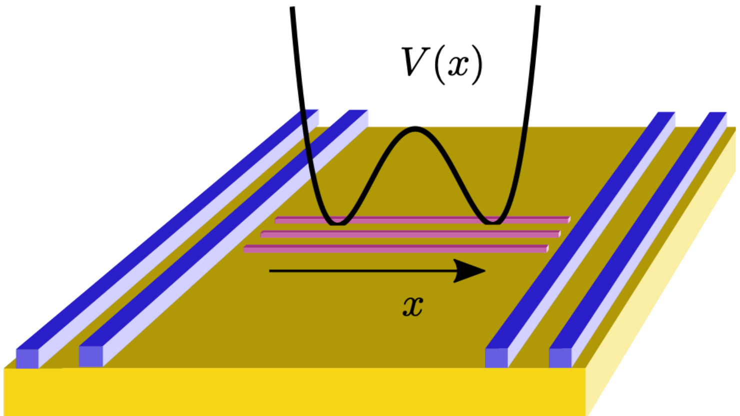
$$\gamma = mc / \left(\hbar^2 n\right)$$

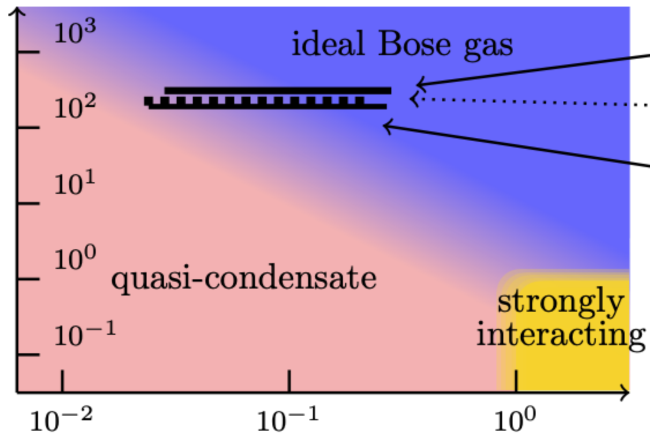
- ^{87}Rb atoms confined in magnetic traps produced by wires on chip
- Transverse confinement provided by wires with AC currents (red)
- Longitudinal $V(x)$ controlled by pairs of wires running DC currents (blue)

Schemmer, Bouchoule, Doyon, Dubail, Phys. Rev. Lett. **122** (2019),

GHD on an Atom Chip

$$\theta = 2\hbar^2 k_B T / \left(mc^2\right)$$



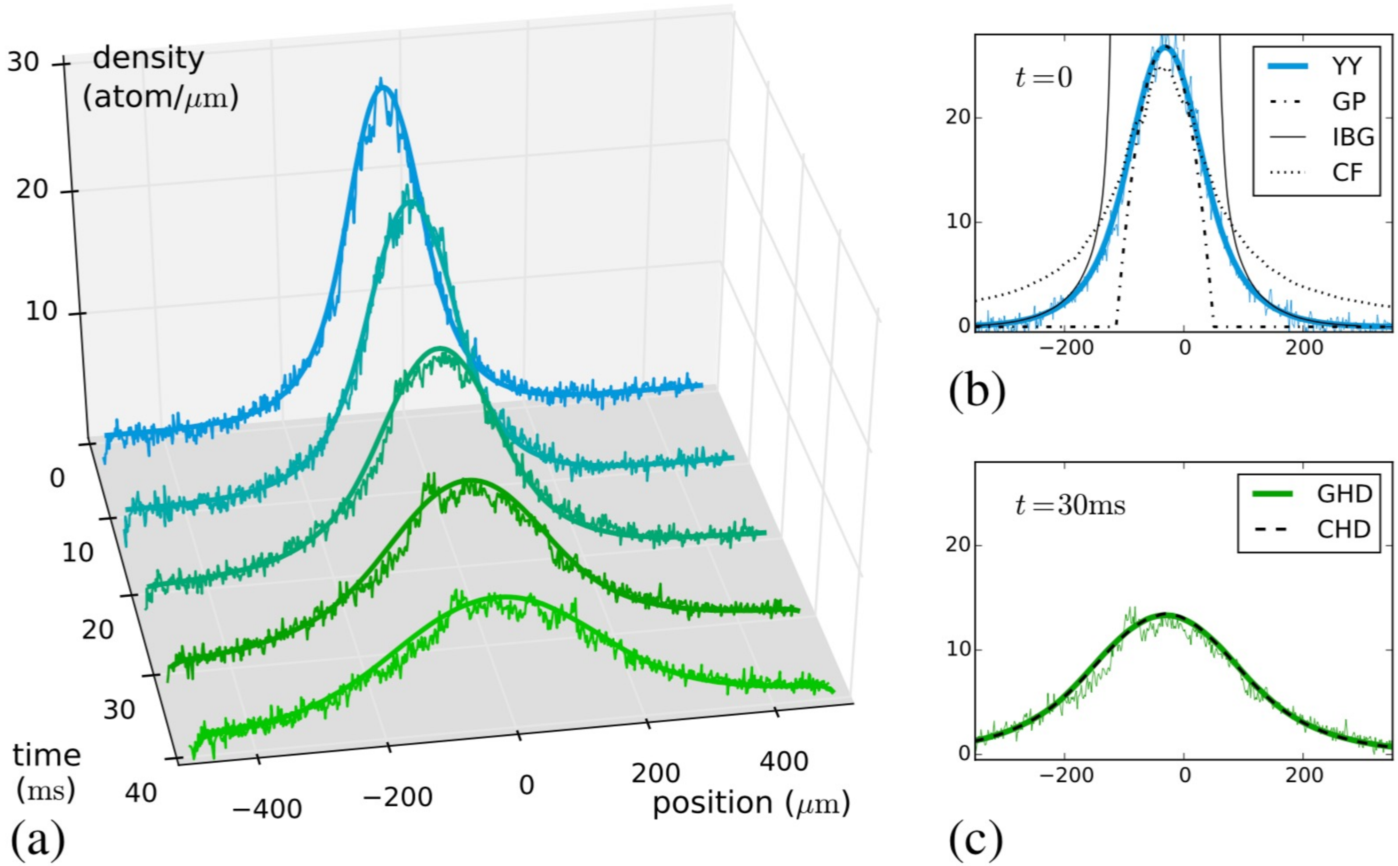


$$\gamma = mc / \left(\hbar^2 n\right)$$

- Model: $H = -\frac{\hbar}{2m}\sum_{i=1}^{N}\partial_{x_i}^2 + c\sum_{i<j}\delta(x_i - x_j) + \sum_{i=1}^{N} V(x_i)$ $\boxed{c > 0}$
- N~5000 ^{87}Rb atoms initially trapped $(T \sim 0.4\mu\mathrm{K})$
- Trap potential quenched at $t = 0$

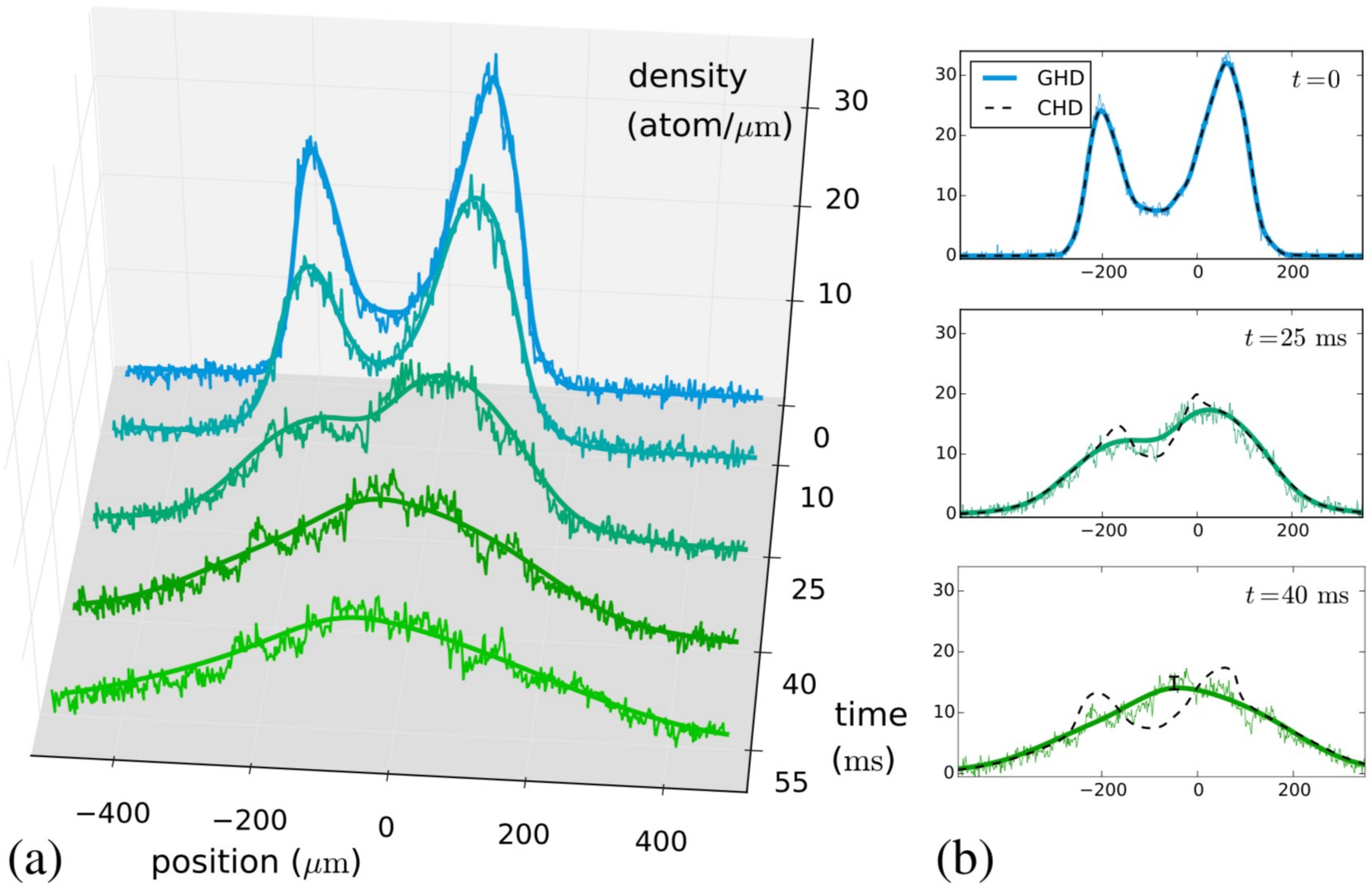
Schemmer, Bouchoule, Doyon, Dubail, Phys. Rev. Lett. **122** (2019),

Expansion from harmonic trap



Schemmer, Bouchoule, Doyon, Dubail, Phys. Rev. Lett. **122** (2019),

Expansion from double-well potential



- Quasi-momenta distribution also directly probed in subsequent experiments [Malvani et al. Weiss, Science **373** (2021)]

Schemmer, Bouchoule, Doyon, Dubail, Phys. Rev. Lett. **122** (2019),

Conclusions

- Integrable models display exceptional non-thermal, non-equilibrium states
- Prototypical examples in 1D repulsive and attractive Bose gases
- Recent theoretical breakthrough in exact Bethe ansatz description
- Further developments:
  - multi-component quantum gases
  - quasi-1D systems
  - inhomogenous interactions
  - transport settings
  - [...]

Calabrese, Essler, Mussardo, J. Stat. Mech. 064001 (2016)
Bastianello, Bertini, Doyon, Vasseur, J. Stat. Mech. 014001 (2022)

Thank you for your attention!